IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received 15 December 2023, accepted 29 December 2023, date of publication 5 January 2024, date of current version 11 January 2024.

Digital Object Identifier 10.1109/ACCESS.2024.3350189

RESEARCH ARTICLE

Cooperative Adaptive Tracking Control Based on Quantized States for Multiple Surface Vessels

QINXUE XU, LIKANG FENG, AND LIQIANG YAO
School of Mathematics and Informational Science, Yantai University, Yantai, Shandong 264005, China
Corresponding author: Liqiang Yao (liqiangyao1978@ytu.edu.cn)

This work was supported by the Shandong Provincial Natural Science Foundation of China under Grant ZR2020MA033.

ABSTRACT The cooperative adaptive tracking control based on full states quantization is investigated in this paper for multiple uncertain surface vessel systems. Firstly, an uncertain surface vessel system model based on disturbance accelerations is presented. Next, using the quantized system states, a series of cooperative tracking controllers and adaptive laws are constructed such that the closed-loop system of every follower is practically stable and the tracking error between every follower and the leader can be adjusted arbitrarily small by parameter adjustment technique. Finally, a simulation example is given to verify the effectiveness of the proposed control strategy.

INDEX TERMS Practical stability, states quantization, cooperative control, adaptive control.

I. INTRODUCTION

Surface vessels was broadly used in ocean survey, ocean exploration, and military missions, then they have attracted significant interests and attentions from researchers, and relevant results have been reported ([1], [2], [3]). Note that the complexity of task, single surface vessel may not effectively accomplish the desired control objective, researchers begin to consider multiple surface vessels to realize complex control task in many cases. For multiple surface vessels, signals need to be transmitted among multiple vessels by means of digital platforms, but the bandwidth of data transmission is limited, then the discontinuous control techniques such as quantization control have been applied to reduce the communication load. The so-called quantization is to use a digital processor (i.e., quantizer) to manage the signal so that the signal remains at the same level on some intervals, that is, the continuous signal is transformed into a piecewise constant signal. This process can be seen as a discontinuous map from a continuous space to a finite set. An important fact is that the selection of quantization scheme can relieve the pressure of network bandwidth, it also can affect the performance of the system. Compare with the uniform quantizer ([4], [5]), the logarithmic quantizer ([6], [7]) can improve the accuracy of the quantized signal and make up for the lack of uniform quantizer. It is worth noting that chattering always occur when the output of quantizer is transferred from one interval to the next, which may add the burden of signal transmission and lead to the instability of system. To solve this problem, hysteresis quantizer was introduced in [8], [9], and [10] to avoid the chattering signal by the introduction of additional quantization levels and the dwell time before a new transition.

The quantization control main contains input quantization and state quantization. For input quantization, [11] constructed a new quantizer based on hysteresis quantizer and uniform quantizer to study the adaptive tracking control of uncertain nonlinear systems. Reference [12] proposed a switching quantization mechanism to complete the sliding mode tracking problem of surface vessel. For surface vessel with prescribed performance, the trajectory tracking control was studied in [13] by radial basis neural networks. More results can be found in [14], [15], [16], and [17]. When system state cannot be sampled or only the quantized system state can be obtained, [18] solved the control performance degradation problem caused by the inherent state quantization error. [19] investigated the adaptive tracking control of mobile robots with quantized states by unknown slippage effects. The quantization feedback tracking control of mobile robots with state quantization and input quantization were studied in [20]. A novel quantized extended state observer is firstly

The associate editor coordinating the review of this manuscript and approving it for publication was Nasim Ullah.

proposed in [21] to study the tracking control of autonomous underwater vehicles. For the trajectory tracking problem of surface vessel, a quantized state-feedback robust controller is established in [22]. Due to the number of agents, the spatial distribution of actuators, limited sensing capability of sensors, and short wireless communication ranges, it is difficult that the implementation of centralized control. Thus, the distributed cooperative control, depending only on the local information of every agents, is a promising tool to stabilization multi-agent systems. Designing appropriate distributed cooperative controllers is generally a challenging task, especially for multi-agent systems with complex dynamics, because of the interconnected effect of the agent dynamics and the interaction graph among agents ([23]).

For the cooperative control of multiple surface vessels with quantized states, [24] studied the distributed formation tracking control based on logarithmic quantizer by some performance functions. The uniform quantizer was used to solve the H_∞ formation control in [25]. However, they didn't solve the chattering phenomenon. How should we design a cooperative state feedback tracking controller for every follower together with the communication topology is a challenge when the system states are quantized. Reference [26] studied the adaptive control problem for nonlinear uncertain system with quantized states, but this result cannot be directly applied to the cooperative tracking control problem for multiple surface vessels. Therefore, how to avoid indifferentiable caused by quantized system states and further analyze the performance of the closed-loop system by Lyapunov function is a difficulty. Based on the above analysis, the cooperative adaptive tracking control problem under full states quantization for multiple uncertain surface vessel systems is discussed in this paper. The main contributions of this paper are as follows:

1) The disturbance accelerations in Body-fixed frame, which caused by wind, waves and ocean currents, is decomposed in the Earth-fixed frame, further the uncertain surface vessel system model is proposed.
2) For multiple surface vessel systems with one leader, by vector backstepping method, a number of cooperative controllers and adaptive laws are constructed such that every follower's closed-loop system is practically stable and every follower's tracking error can be made enough small by adjusting mutually independent controller parameters.

The remainder structures of this paper are structured as follows. Section II provides the model of surface vessel and the related graph theory. For multiple surface vessel systems with one leader, the cooperative adaptive tracking control problem based on quantized system states is researched in Section III. Section IV analyzes the closed-loop system performance of every follower. Section V illustrates the feasibility of proposed control schemes by a simulation example. Section VI concludes this paper.

Notions: The real n-dimensional space is represented by $\mathcal{R}^n$, $\mathcal{R}^{n\times m}$ stands for the real $n\times m$ matrix space and $\mathcal{R}_+$ represents the set of all nonnegative real numbers. $|x|$ denotes the Euclidean norm of vector x and $|X|_F$ is the Frobenius norm of matrix X. $\lambda_{\max}(X)$ and $\lambda_{\min}(X)$ stand for the maximum and the minimum eigenvalues of matrix X, respectively. The family of all functions with continuous k-th partial derivative is marked as C^k. $C^{1,1}(\mathcal{R}^n\times[t_0,\infty);\mathcal{R}_+)$ represents the set of all functions $w(x,t)\in\mathcal{R}_+$ on $\mathcal{R}^n\times[t_0,\infty)$ which are C^1 in x and C^1 in t. For $\vartheta_1,\vartheta_2\in\mathcal{R}$, define $\vartheta_1\vee\vartheta_2=\max\{\vartheta_1,\vartheta_2\}$.

II. PRELIMINARIES

Consider the multiple surface vessel systems including $\mathcal{N}$ followers and one leader, where i-th follower's system model can be described by

$$\begin{cases}\dot{\eta}_i=J_i(\psi_i)v_i,\\ M_i\dot{v}_i+C_i(v_i)v_i+D_i(v_i)v_i=\tau_i+M_iJ_i^T(\psi_i)\xi_i,\end{cases}\tag{1}$$

where $\eta_i=(x_i,y_i,\psi_i)^T$ denotes the i-th follower's displacement, $v_i=(v_{i1},v_{i2},\omega_{i3})^T$ represent its velocity, and $\tau_i=(\tau_{i1},\tau_{i2},\tau_{i3})^T$ is its control input. M_i stands for its symmetric positive definite inertia matrix, $C_i(v_i)$ represents its Coriolis-centripetal force matrix, $J_i(\psi_i)$ is its rotation matrix and satisfies $J_i^T(\psi_i)J_i(\psi_i)=J_i(\psi_i)J_i^T(\psi_i)=I$, and $D_i(v_i)$ satisfies $D_i(v_i)=\theta_i\Gamma_i(v_i)$ with $\Gamma_i(v_i)\in\mathcal{R}^{3\times3}$ being its damping matrix and θ_i being a unknown constant, i.e.,

$$M_i=\begin{pmatrix}m_{i11}&0&0\\0&m_{i22}&m_{i23}\\0&m_{i32}&m_{i33}\end{pmatrix},$$

$$C_i(v_i)=\begin{pmatrix}0&0&-c_{i1}(v_i)\\0&0&c_{i2}(v_i)\\c_{i1}(v_i)&-c_{i2}(v_i)&0\end{pmatrix},$$

$$J_i(\psi_i)=\begin{pmatrix}\cos\psi_i&-\sin\psi_i&0\\\sin\psi_i&\cos\psi_i&0\\0&0&1\end{pmatrix},$$

$$\Gamma_i(v_i)=\begin{pmatrix}d_{i11}(v_i)&0&0\\0&d_{i22}(v_i)&d_{i23}(v_i)\\0&d_{i32}(v_i)&d_{i33}(v_i)\end{pmatrix}.$$

Furthermore, $\xi_i=(\xi_{i1},\xi_{i2},\xi_{i3})^T$ is the disturbance acceleration of i-th follower and satisfies $\max_{1\le i\le\mathcal{N}}|\xi_i|\le K_1$ with $K_1>0$, where ξ_{i1}, ξ_{i2} and ξ_{i3} represent disturbance accelerations along X_{i1} axis, Y_{i1} axis and Z_{i1} axis, respectively. Figure 1 shows the decomposition diagram of disturbance accelerations.

The communication relationship among $\mathcal{N}$ followers and one leader can be expressed by a topology graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$. $\mathcal{V}=\{0,1,2,\cdots,\mathcal{N}\}$ is the set of nodes, where 0 and $i(i=1,\cdots,\mathcal{N})$ denote the leader and the i-th follower, respectively. $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ represents the set of edges. $\mathcal{A}=(a_{ij})_{\mathcal{N}\times\mathcal{N}}$ is the adjacency matrix of $\mathcal{N}$ followers, $a_{ij}>0$ if and only if node j can send information to node i, directly, otherwise $a_{ij}=0$, and we define that $a_{ii}=0$. $\mathcal{B}=diag(b_1,b_2,\cdots,b_{\mathcal{N}})$ denotes the leader-weighted matrix associated with $\mathcal{G}$ and if the i-th follower can directly receive information from the leader,

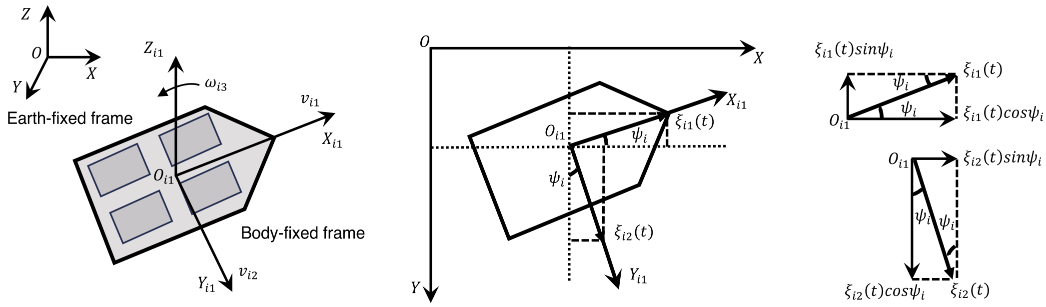

FIGURE 1. The decompositions of disturbance accelerations.

then $b_i > 0$, otherwise $b_i = 0$, where $i = 1, \cdots, \mathcal{N}$. $\mathcal{L} = \mathcal{D} - \mathcal{A}$ is the Laplacian matrix associated with $\mathscr{G}$, where $\mathcal{D} = diag(\sum_{j=1}^{\mathcal{N}} a_{1j}, \sum_{j=1}^{\mathcal{N}} a_{2j}, \cdots, \sum_{j=1}^{\mathcal{N}} a_{\mathcal{N}j})$ is an in-degree matrix. In order to ensure that every follower can receive from the leader's information, it is supposed that there exists a path from the leader (the node 0) to every follower (the node $i(i = 1, \cdots, \mathcal{N})$) in $\mathscr{G}$.

Remark 1: It is assumed that the motion of surface vessel on the ocean only has 3 degrees of freedom on the horizontal plane, then the studied surface vessel system in this paper only includes surging-direction, swaying-direction and yawing-direction by [27].

III. COOPERATIVE ADAPTIVE TRACKING CONTROL BASED ON QUANTIZED STATES

In this section, we consider the cooperative adaptive tracking control problem for multiple surface vessel systems, where only quantized system states can be obtained to design controller τ_i in (1). Let $\bar{v}_i = J_i(\psi_i)v_i$ and $\tau_i = u_i$, we can rewrite (1) as

$$\begin{cases} \dot{\eta}_i = \bar{v}_i, \\ M_i J_i^T \dot{\bar{v}}_i + M_i \dot{J}_i^T \bar{v}_i + \bar{C}_i \bar{v}_i + \bar{D}_i \bar{v}_i = u_i + M_i J_i^T \xi_i, \\ p_i = \eta_i, \quad i = 1, \cdots, \mathcal{N}, \end{cases} \tag{2}$$

where $\bar{C}_i = C_i(J_i^T \bar{v}_i)J_i^T$, $\bar{D}_i = D_i(J_i^T \bar{v}_i)J_i^T$ satisfies $\bar{D}_i = \theta_i \bar{\Gamma}_i(\bar{v}_i)J_i^T$ with $\bar{\Gamma}_i \in \mathcal{R}^{3\times3}$ being a smooth matrix function and $\dot{J}_i(\psi_i) = \dot{\psi}_i J_i(\psi_i)S = \omega_{i3} J_i(\psi_i)S$, in which $S = (s_{jk})_{3\times3} = -S^T$ with $s_{21} = -s_{12} = 1$ and $s_{jj} = s_{3k} = s_{k3} = 0 (j = 1, 2, 3, k = 1, 2)$. Note that only quantized system states $q(\eta_i)$ and $q(\bar{v}_i)$ are obtained, where $q(\cdot)$ is a state quantizer, that is to say, the cooperative state feedback tracking controller u_i must be generated by

$$u_i = u_i(q(\eta_i), q(\bar{v}_i)). \tag{3}$$

Remark 2: In this section, we construct the cooperative adaptive controller only through the quantized states $q(\eta_i)$, $q(\bar{v}_i)$, the quantizer can be a uniform quantizer, a logarithm quantizer or a hysteresis quantizer. In fact, the design method of cooperative adaptive tracking controller based on quantized states proposed in this paper is applicable to uniform quantizer, logarithm quantizer and hysteresis quantizer.

The goal of this section is to construct a series of cooperative adaptive state feedback tracking controllers $u_i(i = 1, \cdots, \mathcal{N})$ for multiple surface vessel systems such that the closed-loop system of every follower is practically stable, and the output $p_i(t)$ of i-th follower can practically track the leader's output $p_0(t)$, where only quantized system states can be obtained and $|p_0(t)| \vee |\dot{p}_0(t)| \le K$ with $K > 0$. Moreover, other signals of every follower's closed-loop system are ultimately bounded. Figure 2 gives the flow diagram of cooperative adaptive tracking control based on quantized states.

To this end, first of all, define the coordinate transformations

$$\begin{cases} e_{i1} = b_i(\eta_i - p_0) + \sum_{k=1}^{\mathcal{N}} a_{ik}(\eta_i - \eta_k), \\ e_{i2} = \bar{v}_i - \alpha_i, \quad i = 1, \cdots, \mathcal{N}, \end{cases} \tag{4}$$

where $\alpha_i \in \mathcal{R}^3 (i = 1, \cdots, \mathcal{N})$ are stabilizing functions to be designed.

Step 1: Let $V_{i1} = e_{i1}^T e_{i1}/2$, it follows from (2), (4) and Young's inequality that

$$\begin{aligned} \dot{V}_{i1} &= e_{i1}^T (b_i(\dot{\eta}_i - \dot{p}_0) + \sum_{k=1}^{\mathcal{N}} a_{ik}(\dot{\eta}_i - \dot{\eta}_k)) \\ &= e_{i1}^T (h_i \bar{v}_i - b_i \dot{p}_0 - \sum_{k=1}^{\mathcal{N}} a_{ik} \bar{v}_k) \\ &\le e_{i1}^T (h_i \alpha_i + h_i e_{i2} + \frac{b_i^2}{4d_K} e_{i1} - \sum_{k=1}^{\mathcal{N}} a_{ik} \alpha_k) \\ &\quad - e_{i1}^T \sum_{k=1}^{\mathcal{N}} a_{ik} e_{k2} + d_K K^2, \end{aligned} \tag{5}$$

where $d_K > 0$ is a designed constant and $h_i = b_i + \sum_{k=1}^{\mathcal{N}} a_{ik} > 0$. Choosing the following stabilizing vector functions

$$\begin{aligned} &(\alpha_1^T, \cdots, \alpha_{\mathcal{N}}^T)^T \\ &\quad = -(\mathcal{H} \otimes I_3)^{-1} ((\bar{C}_{11} e_{11})^T, \cdots, (\bar{C}_{\mathcal{N}1} e_{\mathcal{N}1})^T)^T, \end{aligned} \tag{6}$$

where $\bar{C}_{i1} = C_{i1} + (\mathcal{N}/4 + b_i^2/4d_K) I_3 (i = 1, \cdots, \mathcal{N})$ and C_{i1} is a diagonal matrix with positive diagonal elements to be

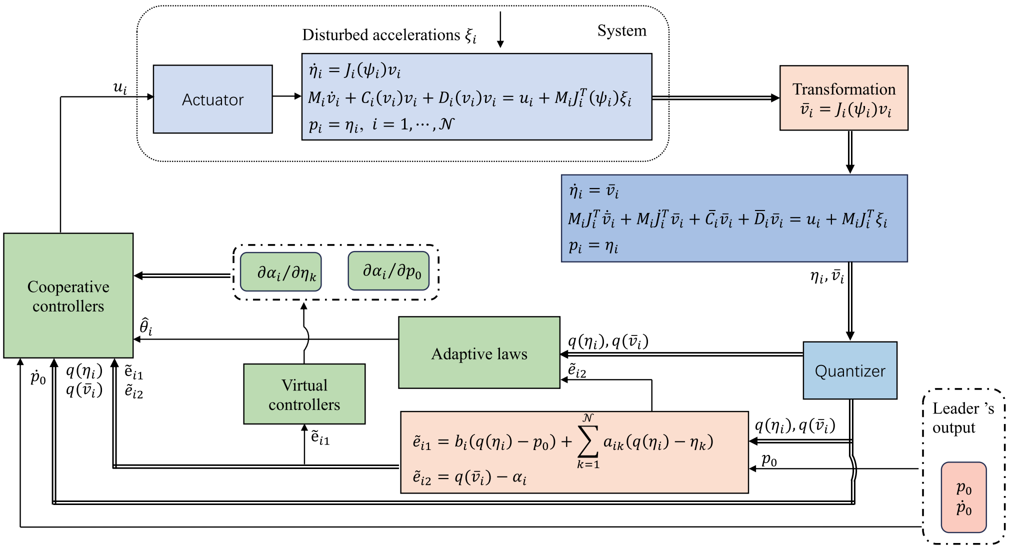

FIGURE 2. Block diagram of cooperative adaptive tracking control based on quantized states.

designed later. (6) can further lead that

$$\alpha_i(\eta_1, \cdots, \eta_{\mathcal{N}}, p_0) = -\frac{1}{h_i}(\bar{C}_{i1}e_{i1} - \sum_{k=1}^{\mathcal{N}} a_{ik}\alpha_k). \tag{7}$$

Substituting (7) into (5) yields that

$$\dot{V}_{i1} \leq -e_{i1}^T(C_{i1} + \frac{\mathcal{N}}{4}I_3)e_{i1} + h_ie_{i1}^Te_{i2} - e_{i1}^T\sum_{k=1}^{\mathcal{N}} a_{ik}e_{k2} + d_KK^2. \tag{8}$$

Step 2: Let $V_{i2} = V_{i1} + e_{i2}^Te_{i2}/2 + \tilde{\theta}_i^2/2\gamma_i$, where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ is the estimation error of θ_i and $\gamma_i > 0 (i = 1, \cdots, \mathcal{N})$ are gain constants. Then,

$$\begin{aligned}\dot{V}_{i2} &\leq e_{i2}^T(J_i(-\dot{J}_i^T\bar{v}_i - M_i^{-1}\bar{C}_i\bar{v}_i - M_i^{-1}\bar{D}_i\bar{v}_i + M_i^{-1}u_i \\ &\quad + J_i^T\xi_i) - \dot{\alpha}_i) - e_{i1}^T(C_{i1} + \frac{\mathcal{N}}{4}I_3)e_{i1} + h_ie_{i1}^Te_{i2} \\ &\quad - e_{i1}^T\sum_{k=1}^{\mathcal{N}} a_{ik}e_{k2} + d_KK^2 - \frac{1}{\gamma_i}\tilde{\theta}_i\dot{\hat{\theta}}_i \\ &= e_{i2}^T(J_i(-\dot{J}_i^T\bar{v}_i - M_i^{-1}\bar{C}_i\bar{v}_i - \hat{\theta}_iM_i^{-1}\bar{\Gamma}_iJ_i^T\bar{v}_i + M_i^{-1}u_i \\ &\quad + J_i^T\xi_i) - \dot{\alpha}_i + h_ie_{i1}) - e_{i1}^T(C_{i1} + \frac{\mathcal{N}}{4}I_3)e_{i1} + d_KK^2 \\ &\quad + \frac{c_{\theta_i}}{\gamma_i}\tilde{\theta}_i\hat{\theta}_i - \tilde{\theta}_i(\frac{1}{\gamma_i}\dot{\hat{\theta}} + \kappa_i) - e_{i1}^T\sum_{k=1}^{\mathcal{N}} a_{ik}e_{k2},\end{aligned} \tag{9}$$

where $\dot{\alpha}_i = \sum_{k=1}^{\mathcal{N}} \partial\alpha_i/\partial\eta_k\bar{v}_k + \partial\alpha_i/\partial p_0\dot{p}_0$ and $\kappa_i = e_{i2}^TM_i^{-1}\bar{\Gamma}_iJ_i^T\bar{v}_i + c_{\theta_i}\hat{\theta}_i/\gamma_i$ with $c_{\theta_i} > 0$. By Young's inequality, we can get that

$$\begin{cases} -e_{i1}^T\sum_{k=1}^{\mathcal{N}} a_{ik}e_{k2} \leq \frac{\mathcal{N}}{4}e_{i1}^Te_{i1} + \sum_{k=1}^{\mathcal{N}} a_{ik}^2e_{k2}^Te_{k2}, \\ e_{i2}^T\xi_i \leq \frac{1}{4d_1}e_{i2}^Te_{i2} + d_1K_1^2, \end{cases}$$

where $d_1 > 0$ is a designed constant. Then we can rewrite (9) as

$$\begin{aligned}\dot{V}_{i2} &\leq e_{i2}^T(J_i(-\dot{J}_i^T\bar{v}_i - M_i^{-1}\bar{C}_i\bar{v}_i - \hat{\theta}_iM_i^{-1}\bar{\Gamma}_iJ_i^T\bar{v}_i + M_i^{-1}u_i \\ &\quad + \frac{1}{4d_1}e_{i2}) - \dot{\alpha}_i + h_ie_{i1}) - e_{i1}^TC_{i1}e_{i1} - \tilde{\theta}_i(\frac{1}{\gamma_i}\dot{\hat{\theta}} + \kappa_i) \\ &\quad + \frac{c_{\theta_i}}{\gamma_i}\tilde{\theta}_i\hat{\theta}_i + \sum_{k=1}^{\mathcal{N}} a_{ik}^2e_{k2}^Te_{k2} + d_KK^2 + d_1K_1^2.\end{aligned} \tag{10}$$

Let

$$\begin{aligned}u_i &= -M_iJ_i^T\bar{C}_{i2}e_{i2} - h_iM_iJ_i^Te_{i1} - \frac{1}{4d_1}M_ie_{i2} \\ &\quad + M_iJ_i^T\dot{\alpha}_i + M_i\dot{J}_i^T\bar{v}_i + \bar{C}_i\bar{v}_i + \hat{\theta}_i\bar{\Gamma}_iJ_i^T\bar{v}_i \\ &= \sigma_i + H_i + \hat{\theta}_i\bar{\Gamma}_iJ_i^T\bar{v}_i, \quad i = 1, \cdots, \mathcal{N},\end{aligned} \tag{11}$$

and

$$\dot{\hat{\theta}}_i = -\gamma_i\kappa_i, \quad i = 1, \cdots, \mathcal{N}, \tag{12}$$

where $\sigma_i = -M_iJ_i^T\bar{C}_{i2}e_{i2} - h_iM_iJ_i^Te_{i1} - M_ie_{i2}/4d_1 + M_iJ_i^T\dot{\alpha}_i$ and $\bar{C}_{i2} = C_{i2} + \sum_{k=1}^{\mathcal{N}} a_{ki}^2I_3 (i = 1, \cdots, \mathcal{N})$ with C_{i2} being a diagonal matrix to be determined later, $H_i = M_i\dot{J}_i^T\bar{v}_i + \bar{C}_i\bar{v}_i = \omega_{i3}M_iS^TJ_i^T(\psi_i)\bar{v}_i + \bar{C}_i\bar{v}_i$. Then, (10) can be turned into

$$\begin{aligned}\dot{V}_{i2} &\leq -e_{i1}^TC_{i1}e_{i1} - e_{i2}^T\bar{C}_{i2}e_{i2} + d_KK^2 + d_1K_1^2 \\ &\quad + \frac{c_{\theta_i}}{\gamma_i}\tilde{\theta}_i\hat{\theta}_i + \sum_{k=1}^{\mathcal{N}} a_{ik}^2e_{k2}^Te_{k2} \\ &\leq -e_{i1}^TC_{i1}e_{i1} - e_{i2}^T\bar{C}_{i2}e_{i2} - \frac{c_{\theta_i}}{2\gamma_i}\tilde{\theta}_i^2 + \frac{c_{\theta_i}}{2\gamma_i}\theta_i^2 \\ &\quad + d_KK^2 + d_1K_1^2 + \sum_{k=1}^{\mathcal{N}} a_{ik}^2e_{k2}^Te_{k2}.\end{aligned} \tag{13}$$

Let Lyapunov function

$$V=\sum_{i=1}^{\mathcal{N}}V_{i2}=\frac{1}{2}\sum_{i=1}^{\mathcal{N}}e_{i1}^{T}e_{i1}+\frac{1}{2}\sum_{i=1}^{\mathcal{N}}e_{i2}^{T}e_{i2}+\sum_{i=1}^{\mathcal{N}}\frac{1}{2\gamma_i}\tilde{\theta}_i^2,$$

then,

$$\begin{aligned}r_1|\bar{e}|^2&\le\frac{1}{2}\sum_{i=1}^{\mathcal{N}}e_{i1}^{T}e_{i1}+\frac{1}{2}\sum_{i=1}^{\mathcal{N}}e_{i2}^{T}e_{i2}+\frac{1}{2\gamma_{\max}}\sum_{i=1}^{\mathcal{N}}\tilde{\theta}_i^2\\&\le V\\&\le\frac{1}{2}\sum_{i=1}^{\mathcal{N}}e_{i1}^{T}e_{i1}+\frac{1}{2}\sum_{i=1}^{\mathcal{N}}e_{i2}^{T}e_{i2}+\frac{1}{2\gamma_{\min}}\sum_{i=1}^{\mathcal{N}}\tilde{\theta}_i^2\\&\le r_2|\bar{e}|^2,\end{aligned}\tag{14}$$

where $r_1=\min\{1/2,1/2\gamma_{\max}\}$ with $\gamma_{\max}=\max_{1\le i\le\mathcal{N}}\{\gamma_i\}$, $r_2=\max\{1/2,1/2\gamma_{\min}\}$ with $\gamma_{\min}=\min_{1\le i\le\mathcal{N}}\{\gamma_i\}$, $\bar{e}=(e_1^T,e_2^T,\tilde{\theta}^T)^T$ with $e_j=(e_{1j}^T,\cdots,e_{\mathcal{N}j}^T)^T,\tilde{\theta}=(\tilde{\theta}_1,\cdots,\tilde{\theta}_{\mathcal{N}})^T$ and $j=1,2$. On the other hand, it follows from (13) that

$$\begin{aligned}\dot{V}&\le-\sum_{i=1}^{\mathcal{N}}e_{i1}^{T}C_{i1}e_{i1}-\sum_{i=1}^{\mathcal{N}}e_{i2}^{T}\bar{C}_{i2}e_{i2}-\sum_{i=1}^{\mathcal{N}}\frac{c_{\theta_i}}{2\gamma_i}\tilde{\theta}_i^2\\&\quad+\sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{N}}a_{ik}^2e_{k2}^Te_{k2}+d\\&\le-\sum_{i=1}^{\mathcal{N}}e_{i1}^{T}C_{i1}e_{i1}-\sum_{i=1}^{\mathcal{N}}e_{i2}^{T}(\bar{C}_{i2}-\sum_{k=1}^{\mathcal{N}}a_{ki}^2)e_{i2}\\&\quad-\sum_{i=1}^{\mathcal{N}}\frac{c_{\theta_i}}{2\gamma_i}\tilde{\theta}_i^2+d\\&\le-\sum_{i=1}^{\mathcal{N}}e_{i1}^{T}C_{i1}e_{i1}-\sum_{i=1}^{\mathcal{N}}e_{i2}^{T}C_{i2}e_{i2}-\sum_{i=1}^{\mathcal{N}}\frac{c_{\theta_i}}{2\gamma_i}\tilde{\theta}_i^2+d\\&\le-cV+d,\end{aligned}\tag{15}$$

where $d=\sum_{i=1}^{\mathcal{N}}c_{\theta_i}\theta_i^2/2\gamma_i+\mathcal{N}d_KK^2+\mathcal{N}d_1K_1^2$, $c=\min\{2c_1,2c_2,\min_{1\le i\le\mathcal{N}}\{c_{\theta_i}\}\}$ and $c_j=\min_{1\le i\le\mathcal{N}}\{c_{ij}^*\}$ with c_{ij}^* being the minimum value of diagonal elements in C_{ij}, $i=1,\cdots,\mathcal{N},j=1,2$.

From (14) and (15), we know that the i-th follower's closed-loop system (including (1), (2), (4), (6), (11) and (12)) is practically stable. Further, it follows from (15) that

$$e^{ct}(\dot{V}+cV)\le e^{ct}d,$$

which can further lead that

$$e^{ct}V(t)-e^{ct_0}V(t_0)\le\int_{t_0}^{t}e^{cs}d\,ds,$$

i.e.,

$$V\le e^{-c(t-t_0)}V(t_0)+\frac{d}{c}.\tag{16}$$

Combining with (14) and (16), we have

$$\begin{aligned}r_1|\bar{e}(t)|^2&\le V\\&\le e^{-c(t-t_0)}V(t_0)+\frac{d}{c}\\&\le r_2|\bar{e}(t_0)|^2e^{-c(t-t_0)}+\frac{d}{c},\end{aligned}\tag{17}$$

which yields that $\limsup_{t\to\infty}|\bar{e}(t)|\le(d/cr_1)^{1/2}$. This shows that all signals in every follower's closed-loop system are ultimately bounded.

In addition, let $e_1=(e_{11}^T,e_{21}^T,\cdots,e_{\mathcal{N}1}^T)^T$, then we have

$$\begin{aligned}e_1&=\Big(b_1(\eta_1^T-p_0^T)+\sum_{k=1}^{\mathcal{N}}a_{1k}(\eta_1^T-\eta_k^T),\cdots,\\&\quad b_{\mathcal{N}}(\eta_{\mathcal{N}}^T-p_0^T)+\sum_{k=1}^{\mathcal{N}}a_{\mathcal{N}k}(\eta_{\mathcal{N}}^T-\eta_k^T)\Big)^T\\&=\Big((b_1+\sum_{k=1}^{\mathcal{N}}a_{1k})(\eta_1^T-p_0^T)-\sum_{k=1}^{\mathcal{N}}a_{1k}(\eta_k^T-p_0^T),\cdots,\\&\quad(b_{\mathcal{N}}+\sum_{k=1}^{\mathcal{N}}a_{\mathcal{N}k})(\eta_{\mathcal{N}}^T-p_0^T)-\sum_{k=1}^{\mathcal{N}}a_{\mathcal{N}k}(\eta_k^T-p_0^T)\Big)^T\\&=(\mathcal{H}\otimes I_3)(\eta-p_0\otimes 1_{\mathcal{N}}),\end{aligned}\tag{18}$$

where $\eta=(\eta_1^T,\cdots,\eta_{\mathcal{N}}^T)^T\in\mathcal{R}^{3\mathcal{N}}$, $1_{\mathcal{N}}=(1,\cdots,1)^T\in\mathcal{R}^{\mathcal{N}}$ and $\mathcal{H}=\mathcal{B}+\mathcal{D}-\mathcal{A}$. From

$$|e_1|^2=|e_{11}|^2+\cdots+|e_{\mathcal{N}1}|^2\le 2V,$$

together with (17) and (18), we can obtain that

$$\begin{aligned}|\eta-p_0\otimes 1_{\mathcal{N}}|^2&\le|\mathcal{H}^{-1}\otimes I_3|_F^2|e_1|^2\\&\le 2|\mathcal{H}^{-1}\otimes I_3|_F^2V\\&\le 2|\mathcal{H}^{-1}\otimes I_3|_F^2(r_2|\bar{e}(t_0)|e^{c(t-t_0)}+\frac{d}{c}),\end{aligned}$$

which means that

$$\begin{aligned}\limsup_{t\to\infty}|p_i(t)-p_0(t)|&=\limsup_{t\to\infty}|\eta_i(t)-p_0(t)|\\&\le\limsup_{t\to\infty}|\eta-p_0\otimes 1_{\mathcal{N}}|\\&\le\sqrt{\frac{2|\mathcal{H}^{-1}\otimes I_3|_F^2d}{c}}.\end{aligned}\tag{19}$$

Accroding to the definitions of c and d, if we make the design constants d_K and d_1 as small as possible and the design parameters $\gamma_i(i=1,\cdots,\mathcal{N})$ large enough, the right hand of (19) can be adjusted arbitrarily small, where these parameters are independent of each other.

Based on the above analysis, we know that we can design the cooperative tracking controller (11) and adaptive law (12) for the i-th follower ($i=1,\cdots,\mathcal{N}$) such that every follower's closed-loop system is practically stable, all signals of every follower's closed-loop system are ultimately bounded and the tracking error of every follower can be regulated arbitrarily small by parameter adjustment scheme.

Next, we consider to construct the cooperative adaptive tracking controller based on quantized system states $q(\eta_i(t))$ and $q(\bar{v}_i(t))$. Choose

$$u_i(t)=\tilde{\sigma}_i+\tilde{H}_i+\hat{\theta}_i\tilde{\Gamma}_i\tilde{J}_i^T(q(\psi_i))q(\bar{v}_i)$$

$$
\begin{aligned}
&= -M_i\tilde{J}_i^T(q(\psi_i))\bar{C}_{i2}\tilde{e}_{i2} - h_iM_i\tilde{J}_i^T(q(\psi_i))\tilde{e}_{i1} \\
&\quad - \frac{1}{4d_1}M_i\tilde{e}_{i2} + M_i\tilde{J}_i^T(q(\psi_i))\dot{\tilde{\alpha}}_i + \tilde{C}_i(q(\bar{v}_i))q(\bar{v}_i) \\
&\quad + q(\omega_{i3})M_iS^T\tilde{J}_i^T(q(\psi_i))q(\bar{v}_i) \\
&\quad + \hat{\theta}_i\tilde{\Gamma}_i(q(\bar{v}_i))\tilde{J}_i^T(q(\psi_i))q(\bar{v}_i), \qquad (20)
\end{aligned}
$$

$$\dot{\hat{\theta}} = -\gamma_i\tilde{\kappa}_i, \qquad (21)$$

$$\tilde{e}_{i1} = b_i(q(\eta_i) - p_0) + \sum_{k=1}^{\mathcal{N}} a_{ik}(q(\eta_i) - q(\eta_k)), \qquad (22)$$

$$\tilde{e}_{i2} = q(\bar{v}_i) - \tilde{\alpha}_i, \qquad (23)$$

$$\tilde{\alpha}_i == -\frac{1}{h_i}(\bar{C}_{i1}\tilde{e}_{i1} - \sum_{k=1}^{\mathcal{N}} a_{ik}\tilde{\alpha}_k), \qquad (24)$$

$$\tilde{\kappa}_i = \tilde{e}_{i2}^T M_i^{-1}\tilde{\Gamma}_i(q(\bar{v}_i))\tilde{J}_i^T(q(\psi_i))q(\bar{v}_i) + c_{\theta_i}\hat{\theta}_i/\gamma_i, \qquad (25)$$

$$\dot{\tilde{\alpha}}_i = \sum_{k=1}^{\mathcal{N}} \frac{\partial\alpha_i}{\partial\eta_k}q(\bar{v}_k) + \frac{\partial\alpha_i}{\partial p_0}\dot{p}_0, \qquad (26)$$

where $\tilde{\sigma}_i = -M_i\tilde{J}_i^T(q(\psi_i))\bar{C}_{i2}\tilde{e}_{i2} - h_iM_i\tilde{J}_i^T(q(\psi_i))\tilde{e}_{i1} - M_i\tilde{e}_{i2}/4d_1 + M_i\tilde{J}_i^T(q(\psi_i))\dot{\tilde{\alpha}}_i$, $\tilde{\Gamma}_i = \tilde{\Gamma}_i(q(\bar{v}_i))$ and $\tilde{H}_i = q(\omega_{i3})M_iS^T\tilde{J}_i^T(q(\psi_i))q(\bar{v}_i) + \tilde{C}_i(q(\bar{v}_i))q(\bar{v}_i)$.

Note that we only use quantized syatem states $q(\eta_i)$ and $q(\bar{v}_i)$ to replace states η_i and $\bar{v}_i$, respectively, in (4), (6), (11) and (12), then (20)-(26) can be obviously obtained. In fact, by the cooperative tracking controller (20) and adaptive law (21), which are made up of quantized system states $q(\eta_i)$ and $q(\bar{v}_i)$, we can still guarantee the performance of evey follower.

IV. STABILITY ANALYSIS

To analyze the stability of every follower's closed-loop system and tracking performance of every follower, the following Lemma 1 is proposed as a preparation and the proof of Lemma 1 can be found in **Appendix**.

Lemma 1: For $i = 1, \cdots, \mathcal{N}$, we have

$$|\tilde{e}_{i1}(q(\eta_i)) - e_{i1}(\eta_i)| \leq \epsilon_{e_1}, \qquad (27)$$
$$|\tilde{\alpha}_i(q(\eta_i)) - \bar{\alpha}_i(\eta_i)| \leq \epsilon_\alpha, \qquad (28)$$
$$|\tilde{e}_{i2}(q(\eta_i), q(\bar{v}_i)) - e_{i2}(\eta_i, \bar{v}_i)| \leq \epsilon_{e_2}, \qquad (29)$$
$$|\tilde{\sigma}_i(q(\eta_i), q(\bar{v}_i)) - \sigma_i(\eta_i, \bar{v}_i)| \leq \epsilon_\sigma, \qquad (30)$$
$$|\tilde{H}_i(q(\bar{v}_i)) - H_i(\bar{v}_i)| \leq \epsilon_H, \qquad (31)$$
$$|\tilde{\Gamma}_i(q(\bar{v}_i))\tilde{J}_i^T(q(\psi_i))q(\bar{v}_i) - \bar{\Gamma}_i(\bar{v}_i)J_i^T(\psi_i)\bar{v}_i| \leq \epsilon_{\bar{\Gamma}}, \qquad (32)$$
$$|\tilde{\kappa}_i(q(\eta_i), q(\bar{v}_i)) - \kappa_i(\eta_i, \bar{v}_i)| \leq \epsilon_\kappa, \qquad (33)$$

where $\epsilon_{e_1}, \epsilon_\alpha, \epsilon_{e_2}, \epsilon_\sigma, \epsilon_H, \epsilon_{\bar{\Gamma}}$ and ϵ_κ are known constants and depend on $\mathscr{G}$, quantization bound δ and designed parameters.

Theorem 1: For multiple surface vessel systems with $\mathcal{N}$ followers and one leader, and the dynamical equation of every follower is described by (1), the cooperative adaptive tracking controller (20) and (21) based on quantized system states can be constructed and can guarantee that the closed-loop system ((1), (2) and (20)-(26)) of i-th follower is practically stable, all the signals in every follower's closed-loop system are ultimately bounded, and the tracking error of i-th follower satisfies

$$\limsup_{t\to\infty} |p_i(t) - p_0(t)| \leq \sqrt{\frac{2d_0|\mathcal{H}^{-1}\otimes I_3|_F^2}{\rho}}, \qquad (34)$$

where $\mathcal{H} = \mathcal{B} + \mathcal{D} - \mathcal{A}$. By parameter adjustment technique, the right side of (34) can be made arbitrarily small.

Proof: Let Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{\mathcal{N}} e_{i1}^T e_{i1} + \frac{1}{2}\sum_{i=1}^{\mathcal{N}} e_{i2}^T e_{i2} + \sum_{i=1}^{\mathcal{N}} \frac{1}{2\gamma_i}\tilde{\theta}_i^2,$$

together with (13), we have

$$
\begin{aligned}
\dot{V} \leq& \sum_{i=1}^{\mathcal{N}} e_{i2}^T(J_i(-\dot{J}_i^T\bar{v}_i - M_i^{-1}\bar{C}_i\bar{v}_i + M_i^{-1}u_i + \frac{e_{i2}}{4d_1} \\
&- M_i^{-1}\hat{\theta}_i\bar{\Gamma}_iJ_i^T(\psi_i)\bar{v}_i) - \dot{\alpha}_i) - \sum_{i=1}^{\mathcal{N}} e_{i1}^T C_{i1}e_{i1} \\
&+ \sum_{i=1}^{\mathcal{N}} h_ie_{i1}^T e_{i2} - \sum_{i=1}^{\mathcal{N}} \tilde{\theta}_i(\frac{\dot{\hat{\theta}}}{\gamma_i} + \kappa_i) + \sum_{i=1}^{\mathcal{N}} \frac{c_{\theta_i}}{\gamma_i}\tilde{\theta}_i\hat{\theta}_i \\
&+ \mathcal{N}d_1K_1^2 + \mathcal{N}d_KK^2 + \sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{N}} a_{ik}^2 e_{k2}^T e_{k2}. \qquad (35)
\end{aligned}
$$

Substituting (20) and (21) into (35) yields that

$$
\begin{aligned}
\dot{V} \leq& \sum_{i=1}^{\mathcal{N}} e_{i2}^T(J_i(-\dot{J}_i^T\bar{v}_i - M_i^{-1}\bar{C}_i\bar{v}_i - \hat{\theta}_iM_i^{-1}\bar{\Gamma}_iJ_i^T(\psi_i)\bar{v}_i \\
&+ M_i^{-1}(\tilde{\sigma}_i + \tilde{H}_i + \hat{\theta}_i\tilde{\Gamma}_i\tilde{J}_i^T(q(\psi_i))q(\bar{v}_i) - \sigma_i + \sigma_i) \\
&+ \frac{e_{i2}}{4d_1}) - \dot{\alpha}_i + h_ie_{i1}) - \sum_{i=1}^{\mathcal{N}} e_{i1}^T C_{i1}e_{i1} + \sum_{i=1}^{\mathcal{N}} \frac{c_{\theta_i}}{\gamma_i}\tilde{\theta}_i\hat{\theta}_i \\
&- \sum_{i=1}^{\mathcal{N}} \tilde{\theta}_i(\frac{\dot{\hat{\theta}}}{\gamma_i} + \kappa_i) + \mathcal{N}d_KK^2 + \mathcal{N}d_1K_1^2 \\
&+ \sum_{i=1}^{\mathcal{N}}\sum_{k=1}^{\mathcal{N}} a_{ik}^2 e_{k2}^T e_{k2} \\
=& \sum_{i=1}^{\mathcal{N}} e_{i2}^T(J_iM_i^{-1}(\hat{\theta}_i\tilde{\Gamma}_i\tilde{J}_i^T(q(\psi_i))q(\bar{v}_i) - \hat{\theta}_i\bar{\Gamma}_iJ_i^T(\psi_i)\bar{v}_i)) \\
&+ \sum_{i=1}^{\mathcal{N}} e_{i2}^T(J_iM_i^{-1}(\tilde{\sigma}_i - \sigma_i)) + \sum_{i=1}^{\mathcal{N}} \tilde{\theta}_i(\tilde{\kappa}_i - \kappa_i) \\
&+ \sum_{i=1}^{\mathcal{N}} e_{i2}^T(J_iM_i^{-1}(\tilde{H}_i - H_i)) + \sum_{i=1}^{\mathcal{N}} \frac{c_{\theta_i}}{\gamma_i}\tilde{\theta}_i\hat{\theta}_i + \mathcal{N}d_KK^2 \\
&- \sum_{i=1}^{\mathcal{N}} e_{i1}^T C_{i1}\bar{e}_{i1} - \sum_{i=1}^{\mathcal{N}} e_{i2}^T C_{i2}e_{i2} + \mathcal{N}d_1K_1^2.
\end{aligned}
$$

(36)

By (30)-(33) and Young's inequality, we can obtain that

$$\begin{cases} e_{i2}^T(J_iM_i^{-1}(\tilde{\sigma}_i-\sigma_i)) \le \frac{1}{4}\lambda_{\max}(\tilde{M}_i)e_{i2}^Te_{i2}+\epsilon_\sigma^2, \\ e_{i2}^T(J_iM_i^{-1}(\tilde{H}_i-H_i)) \le \frac{1}{4}\lambda_{\max}(\tilde{M}_i)e_{i2}^Te_{i2}+\epsilon_H^2, \\ \tilde{\theta}_i(\tilde{\kappa}_i-\kappa_i) \le \frac{1}{4}\tilde{\theta}_i^2+\epsilon_\kappa^2, \\ \frac{c_{\theta_i}}{\gamma_i}\tilde{\theta}_i\hat{\theta}_i = -\frac{c_{\theta_i}}{\gamma_i}\tilde{\theta}_i^2+\frac{c_{\theta_i}}{\gamma_i}\tilde{\theta}_i\theta_i \le -\frac{c_{\theta_i}}{2\gamma_i}\tilde{\theta}_i^2+\frac{c_{\theta_i}}{2\gamma_i}\theta_i^2, \end{cases} \tag{37}$$

and

$$\begin{aligned} & e_{i2}^T(J_iM_i^{-1}(\hat{\theta}_i\tilde{\Gamma}_i\tilde{J}_i^T(q(\psi_i))q(\bar{v}_i)-\hat{\theta}_i\bar{\Gamma}_iJ_i^T(\psi_i)\bar{v}_i)) \\ & \le \frac{1}{4}\lambda_{\max}(\tilde{M}_i)e_{i2}^Te_{i2}+\epsilon_\Gamma^2\hat{\theta}_i^2 \\ & \le \frac{1}{4}\lambda_{\max}(\tilde{M}_i)e_{i2}^Te_{i2}+\epsilon_\Gamma^2(\theta_i-\tilde{\theta}_i)^2 \\ & \le \frac{1}{4}\lambda_{\max}(\tilde{M}_i)e_{i2}^Te_{i2}+\epsilon_\Gamma^2(\theta_i^2+\tilde{\theta}_i^2-2\theta_i\tilde{\theta}_i) \\ & \le \frac{1}{4}\lambda_{\max}(\tilde{M}_i)e_{i2}^Te_{i2}+2\epsilon_\Gamma^2\theta_i^2+2\epsilon_\Gamma^2\tilde{\theta}_i^2, \end{aligned} \tag{38}$$

where $\tilde{M}_i = M_i^{-1}(M_i^{-1})^T$. Let $c_{\theta_i} = \gamma_i\varsigma_i(i = 1,\cdots,\mathcal{N})$ with $\varsigma_i > 0$ being a designed constant, substituting (37) and (38) into (36) yields that

$$\begin{aligned} \dot{V} \le & \sum_{i=1}^{\mathcal{N}}\frac{c_{\theta_i}}{2\gamma_i}\theta_i^2+2\epsilon_\Gamma^2\sum_{i=1}^{\mathcal{N}}\theta_i^2+\epsilon_\sigma^2+\mathcal{N}d_KK^2+\epsilon_H^2 \\ & +\mathcal{N}d_1K_1^2-\sum_{i=1}^{\mathcal{N}}e_{i1}^TC_{i1}e_{i1}-\sum_{i=1}^{\mathcal{N}}\frac{c_{\theta_i}}{2\gamma_i}\tilde{\theta}_i^2+\frac{1}{4}\sum_{i=1}^{\mathcal{N}}\tilde{\theta}_i^2 \\ & -\sum_{i=1}^{\mathcal{N}}e_{i2}^T(C_{i2}-\frac{3}{4}\lambda_{\max}(\tilde{M}_i)I_3)e_{i2}+2\epsilon_\Gamma^2\sum_{i=1}^{\mathcal{N}}\tilde{\theta}_i^2+\epsilon_\kappa^2 \\ \le & \sum_{i=1}^{\mathcal{N}}\frac{\varsigma_i}{2}\theta_i^2+\mathcal{N}d_1K_1^2+\mathcal{N}d_KK^2+2\epsilon_\Gamma^2\sum_{i=1}^{\mathcal{N}}\theta_i^2 \\ & -c_1\sum_{i=1}^{\mathcal{N}}e_{i1}^Te_{i1}-(c_2-\frac{3}{4}\lambda_{\max}^*)\sum_{i=1}^{\mathcal{N}}e_{i2}^Te_{i2} \\ & -\sum_{i=1}^{\mathcal{N}}(\varsigma_i-\frac{1}{2}-4\epsilon_\Gamma^2)\gamma_i\frac{1}{2\gamma_i}\tilde{\theta}_i^2+\epsilon_\sigma^2+\epsilon_H^2+\epsilon_\kappa^2 \\ \le & -\rho V+d_0, \end{aligned} \tag{39}$$

where $\lambda_{\max}^* = \max_{1\le i\le\mathcal{N}}\{\lambda_{\max}(\tilde{M}_i)\}$, $d_0 = \sum_{i=1}^{\mathcal{N}}\varsigma_i\theta_i^2/2 + 2\epsilon_\Gamma^2\sum_{i=1}^{\mathcal{N}}\theta_i^2+\mathcal{N}d_KK^2+\mathcal{N}d_1K_1^2+\epsilon_\sigma^2+\epsilon_H^2+\epsilon_\kappa^2$, $\rho = \min\{2c_1, 2(c_2-3\lambda_{\max}^*/4), \varsigma^*\}$ with $c_j = \min_{1\le i\le\mathcal{N}}\{c_{ij}^*\}$ and c_{ij}^* being the minimum value of diagonal elements in $C_{ij}, j = 1, 2$, and $\varsigma^* = \min_{1\le i\le\mathcal{N}}\{(\varsigma_i-1/2-4\epsilon_\Gamma^2)\gamma_i\}$.

This means that the closed-loop system of every follower, including (1), (2), (20)-(26), is practically stable by (16) and (39).

Furthermore, we can also obtain the following inequality

$$r_1|\bar{e}|^2 \le e^{-\rho(t-t_0)}V(t_0)+\frac{d_0}{\rho} \le r_2|\bar{e}(t_0)|e^{-\rho(t-t_0)}+\frac{d_0}{\rho},$$

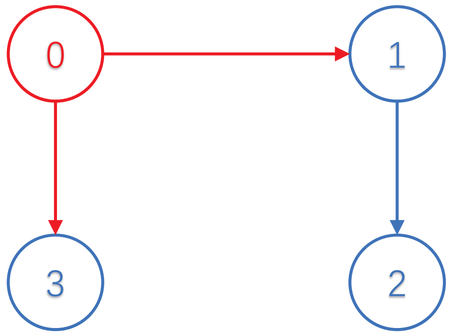

FIGURE 3. The communication topology graph $\mathscr{G}$.

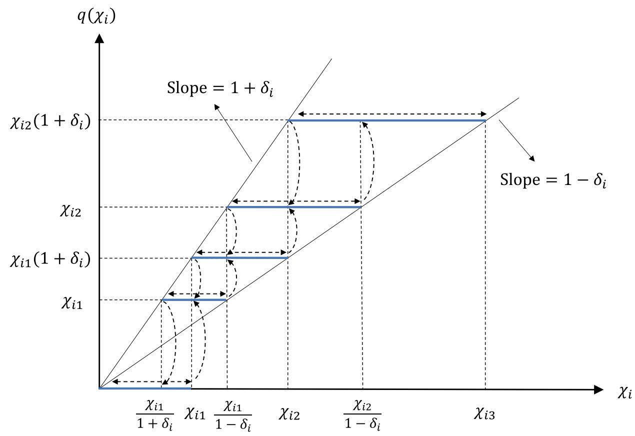

FIGURE 4. The hysteresis quantizer.

that is to say,

$$\limsup_{t\to\infty}|\bar{e}(t)| \le \sqrt{\frac{d_0}{r_1\rho}}, \tag{40}$$

which implies that all the signals in every follower's closed-loop system are ultimately bounded.

Note that

$$\begin{aligned} |e_1|^2 &= |e_{11}|^2+\cdots+|e_{\mathcal{N}1}|^2 \\ &\le 2V \\ &\le 2r_2|\bar{e}(t_0)|e^{-\rho(t-t_0)}+\frac{2d_0}{\rho}, \end{aligned} \tag{41}$$

combining with $e_1 = (\mathcal{H}\otimes I_3)(\eta-p_0\otimes 1_{\mathcal{N}})$, we can obtain that

$$\begin{aligned} & |\eta-p_0\otimes 1_{\mathcal{N}}|^2 \\ & \le |\mathcal{H}^{-1}\otimes I_3|_F^2|e_1|^2 \\ & \le 2|\mathcal{H}^{-1}\otimes I_3|_F^2V \\ & \le 2(r_2|\bar{e}(t_0)|e^{-\rho(t-t_0)}+\frac{d_0}{\rho})|\mathcal{H}^{-1}\otimes I_3|_F^2, \end{aligned} \tag{42}$$

which means that

$$\begin{aligned} \limsup_{t\to\infty}|p_i(t)-p_0(t)| &= \limsup_{t\to\infty}|\eta_i(t)-p_0(t)| \\ &\le \limsup_{t\to\infty}|\eta-p_0\otimes 1_{\mathcal{N}}| \\ &\le \sqrt{\frac{2d_0|\mathcal{H}^{-1}\otimes I_3|_F^2}{\rho}}. \end{aligned} \tag{43}$$

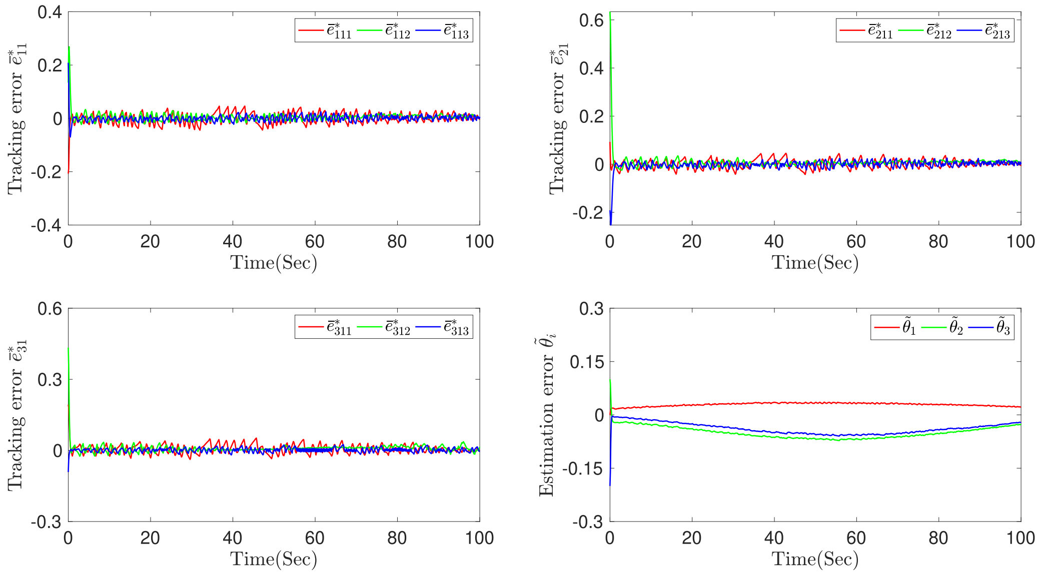

FIGURE 5. The tracking error and estimation error of every follower.

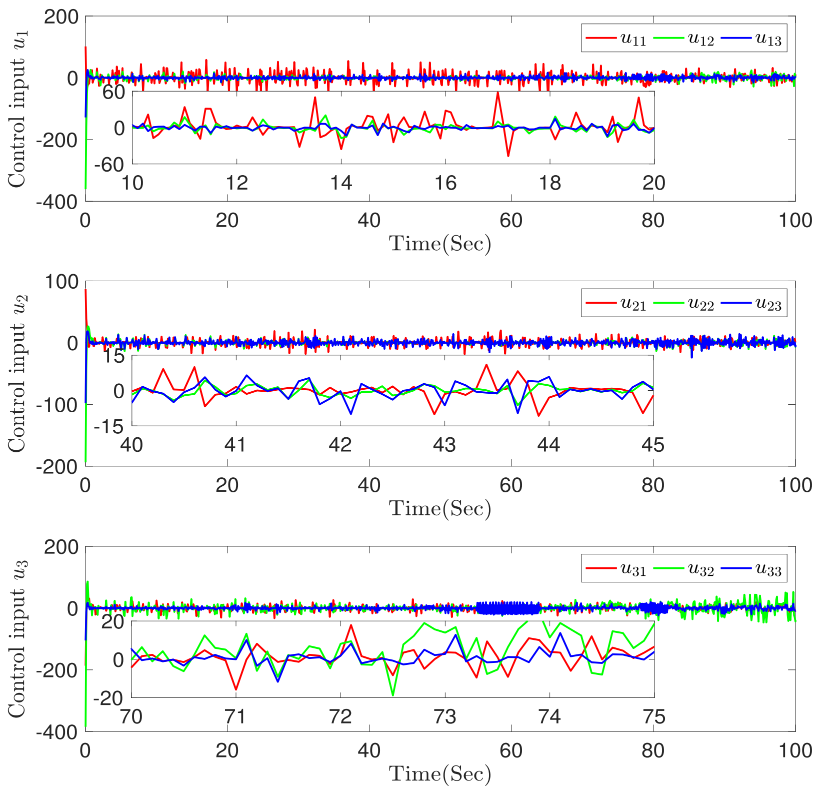

FIGURE 6. The control input of every follower.

From (43), we know that the tracking error of i-th follower can be regulated by selecting the appropriate parameters. In other words, for the definitions of ρ and d_0, if we make the diagonal elements $c^*_{ij}(1 \leq i \leq \mathcal{N}; 1 \leq j \leq 2)$ and $\varsigma_i(1 \leq i \leq \mathcal{N})$ as large as possible, the right side of (43) can be adjusted small enough. This means that the output of i-th follower can practically track the leader'output by parameter adjustment technique.

The above analysis shows that the cooperative tracking controller (20) and adaptive law (21) can make every follower's closed-loop system achieve the desired performance. ∎

V. SIMULATION RESULT

Consider the multiple surface vessel systems with 3 followers and one leader, and the dynamic behavior of

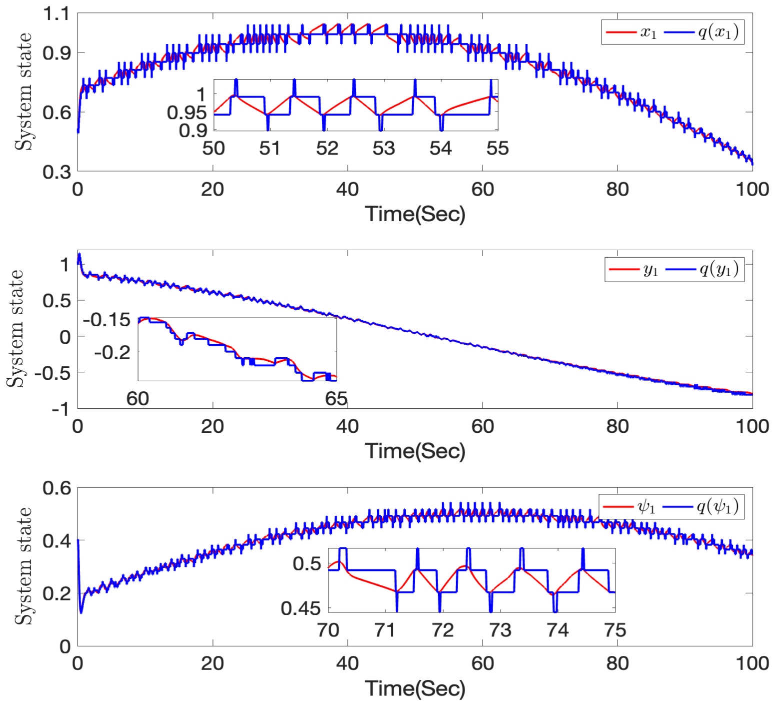

FIGURE 7. The system state η_1 and quantized state $q(\eta_1)$ of the first follower.

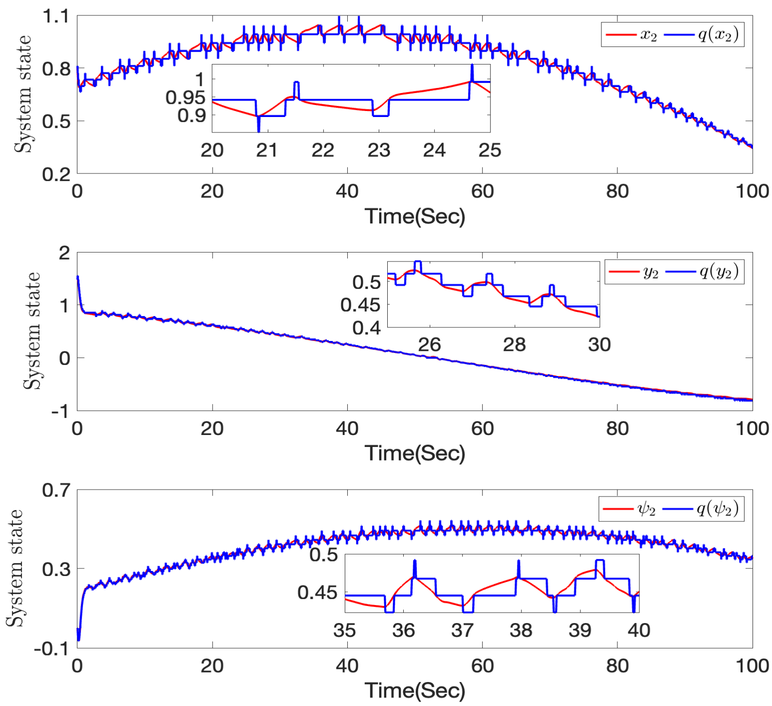

FIGURE 8. The system state η_2 and quantized state $q(\eta_2)$ of the second follower.

every follower is described by (1). The communication topology graph $\mathscr{G}$ of multiple surface vessel systems is shown in Figure 3. The elements of every follower's Coriolis-centripetal force matrix are given as $c_{11}(v_1(t) = 14.95v_{12}(t) + 1.49\omega_{13}(t)$, $c_{12}(v_1(t)) = 14.97v_{11}(t)$, $c_{21}(v_2(t)) = 9.98v_{22}(t)+3.95\omega_{23}(t)$, $c_{22}(v_2(t)) = 9.99v_{21}(t)$, $c_{31}(v_3(t)) = 15.05v_{22}(t) + 2.97\omega_{23}(t)$ and $c_{32}(v_3(t)) = 14.99v_{31}(t)$, and inertia matrix M_i and matrix $\Gamma_i(v_i(t))$ are given by Table 1 and Table 2, respectively.

We use a hysteresis quantizer to verify the performance of proposed controller. For the i-th follower, the function relationship between the output of quantizer $q(\chi_i)$ and the

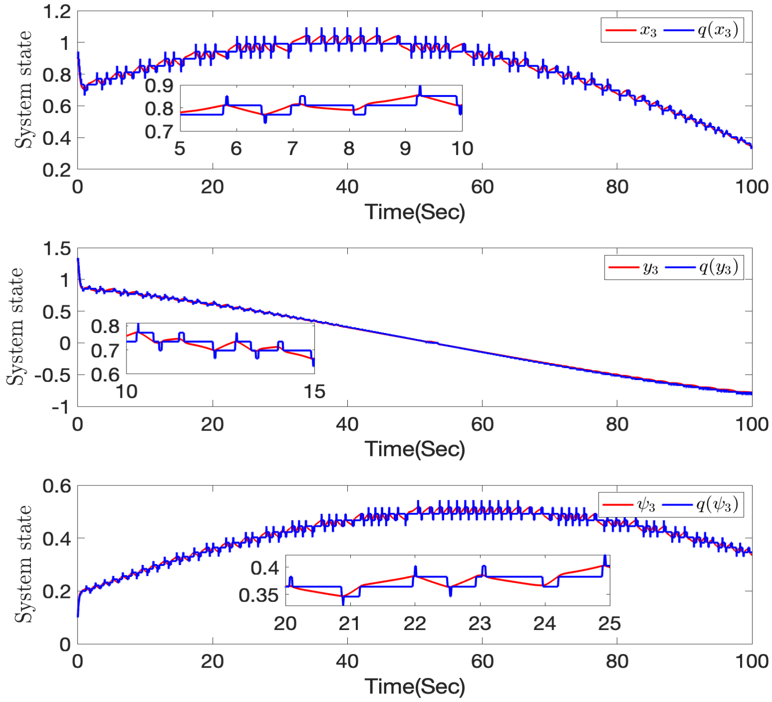

FIGURE 9. The system state η_3 and quantized state $q(\eta_3)$ of the third follower.

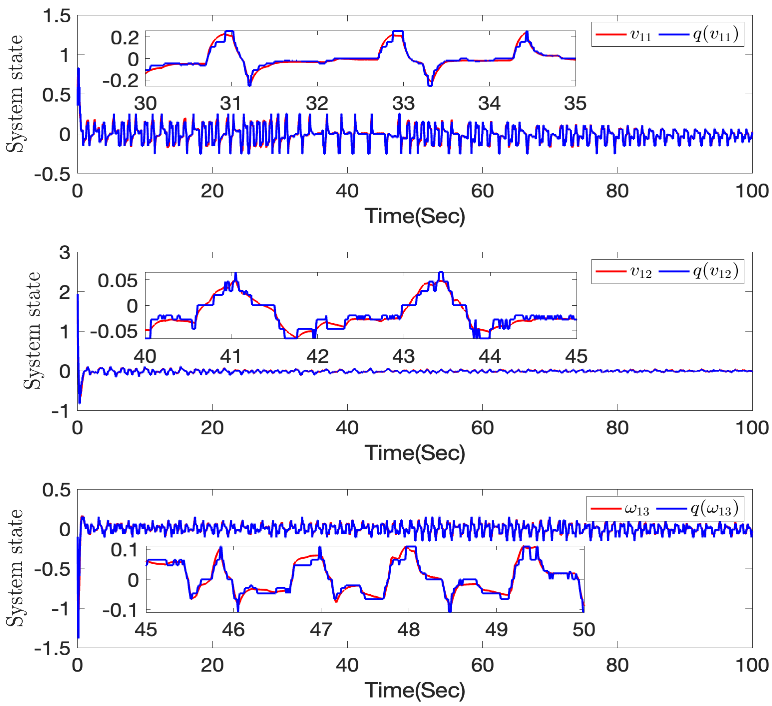

FIGURE 10. The system state v_1 and quantized state $q(v_1)$ of the firrst follower.

TABLE 1. The nonzero elements of every follower's inertia matrix.

	m_{i11}	m_{i22}	m_{i23}	m_{i32}	m_{i33}
$i=1$	14.97	14.95	1.49	1.49	4.8
$i=2$	9.99	9.98	3.95	3.95	4.8
$i=3$	14.99	15.05	2.97	2.97	5.94

TABLE 2. The nonzero elements of every follower's damping matrix.

	$d_{i11}(v_i)$	$d_{i22}(v_i)=d_{i23}(v_i)=d_{i32}(v_i)=d_{i33}(v_i)$
$i=1$	$2-0.5\|v_{11}(t)\|$	$2-0.5\|v_{12}(t)\|-\|\omega_{13}(t)\|$
$i=2$	$-0.5-2\|v_{21}(t)\|$	$0.5-2\|v_{22}(t)\|+\|\omega_{23}(t)\|$
$i=3$	$1-5\|v_{31}(t)\|$	$1-5\|v_{32}(t)\|-1.5\|\omega_{33}(t)\|$

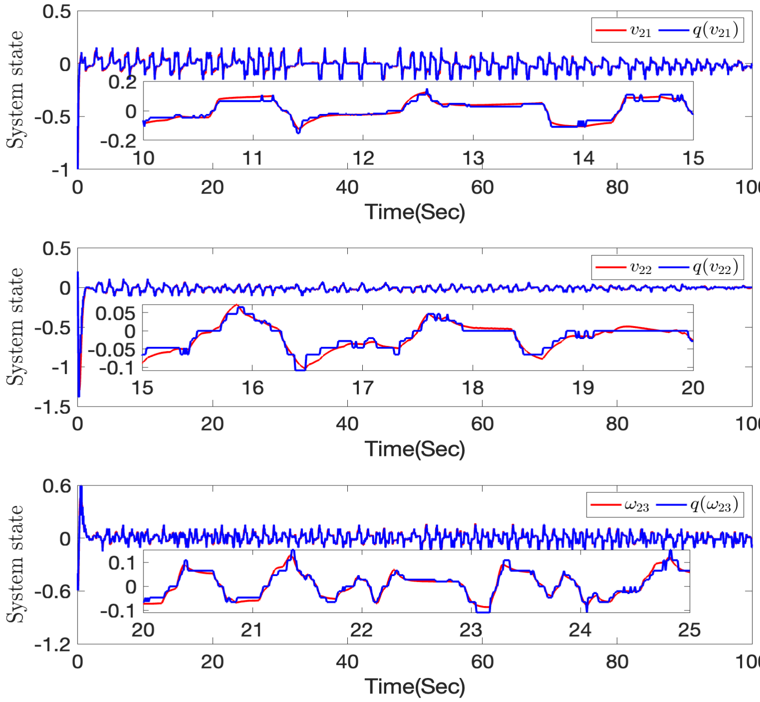

FIGURE 11. The system state v_2 and quantized state $q(v_2)$ of the second follower.

input χ_i is given by (44) and the mapping curve between $q(\chi_i)$ and χ_i is drawn in Figure 4 with $\chi_i \geq 0$, where $\chi_i = (\eta_i^T, \bar{v}_i^T)^T$.

$$
q(\chi_i) = \begin{cases} \chi_{ij} sgn(\chi_i), & \frac{\chi_{ij}}{1+\delta_i} < |\chi_i| \leq \chi_{ij}, \dot{\chi}_i < 0, \\ & or\ \chi_{ij} < |\chi_i| \leq \frac{\chi_{ij}}{1-\delta_i}, \\ & \dot{\chi}_i > 0, \\ \chi_{ij}(1+\delta_i) sgn(\chi_i), & \chi_{ij} < |\chi_i| \leq \frac{\chi_{ij}}{1-\delta_i}, \dot{\chi}_i < 0, \\ & or\ \frac{\chi_{ij}}{1-\delta_i} < |\chi_i| \leq \frac{\chi_{ij}(1+\delta_i)}{1-\delta_i}, \\ & \dot{\chi}_i > 0, \\ 0, & 0 \leq |\chi_i| < \frac{\chi_{i0}}{1+\delta_i}, \dot{\chi}_i < 0, \\ & or\ \frac{\chi_{i0}}{1+\delta_i} \leq |\chi_i| \leq \chi_{i0}, \\ & \dot{\chi}_i > 0, \\ q(\chi_i(t^-)), & \dot{\chi}_i = 0, \end{cases} \tag{44}
$$

where $\chi_{ij} = \varrho_i^{1-j} \chi_{i0}$ with $\chi_{i0} > 0$ being the size of dead-zone in hysteresis quantizer (44), $\varrho_i = (1-\delta_i)/(1+\delta_i) > 0$ being the quantization density and $j = 1, 2, \cdots$. δ_i is a quantization parameter and belongs to $(0, 1)$, and the output of quantizer $q(\chi_i)$ takes the value in $\mathcal{Q}_i = \{0, \pm\chi_{i1}, \pm\chi_{i1}(1+\delta_i), \pm\chi_{i2}, \pm\chi_{i2}(1+\delta_i), \cdots\}$ with $i = 1, 2, 3$.

In simulation, we choose quantization parameters $\chi_{10} = \chi_{20} = \chi_{30} = 0.02$, $\delta_1 = \delta_2 = \delta_3 = 0.05$. Further, it follows from Figure 3 that the leader-weighted matrix $\mathcal{B}_1 = diag(1, 0, 1)$ and the adjacency matrix $\mathcal{A}_1 = (a_{ij})_{3\times 3}$ with $a_{21} = 1$ and other matrix elements being zero. The unknown disturbances are defined by $\xi_i(t) = (0.2\sin(0.05t + 0.125\pi), 0.1\cos(0.01t), 0.01\sin(0.02t) + 0.01\cos(0.01t))^T$, where $i = 1, 2, 3$. The leader's output $p_0(t) = (\sin(0.02t + 0.25\pi), \cos(0.02t + \pi/6), 0.5\sin(0.02t + 0.125\pi))^T$ and three followers' system initial values $\eta_1(0) = (0.8, 1, 0.4)^T$, $v_1(0) = (0.5, 1.8, -0.1)^T$, $\eta_2(0) = (0.8, 1.5, 0)^T$, $v_2(0) = (-1, 0.2, -0.4)^T$, $\eta_3(0) = (0.9, 1.3, 0.1)^T$, $v_3(0) = (0.2, -0.1, 0.7)^T$. The design diagonal matrices $C_{11} = diag(10, 8, 9)$, $C_{21} = diag(7, 7, 6)$, $C_{31} = diag(7, 9, 11)$, $C_{12} = diag(3, 1, 7)$, $C_{22} = diag(3, 2, 7)$, $C_{32} = diag(5, 6, 6)$ and the design parameters $d_K = d_1 = 0.05$, $\varsigma_1 = 1.5$, $\varsigma_2 = 0.8$, $\varsigma_3 = 0.6$, $\gamma_1 = 0.3$, $\gamma_2 = 0.4$, $\gamma_3 = 0.5$. The truth value of $\theta = (\theta_1, \theta_2, \theta_3)^T = (-2, 0.5, -1)^T$ and the initial value of adaptive update law $\hat{\theta}(0) = (\hat{\theta}_1(0), \hat{\theta}_2(0), \hat{\theta}_3(0))^T = (-1.9, 0.6, -0.8)^T$. Let $\bar{e}_{i1}^* = p_i - p_0 = (\bar{e}_{i11}^*, \bar{e}_{i12}^*, \bar{e}_{i13}^*)^T$ represents the tracking error of i-th follower, where $i = 1, 2, 3$.

For the tracking control of multiple surface vessel, the performances of every follower's closed-loop system are given by Figures 5-13. Figure 5 reflects the component of every follower's tracking error and estimation error of $\theta_i (i = 1, 2, 3)$. It also follows from Figure 5 that the component in tracking error and the estimation error of every follower can be made very small. Figure 6 gives the fluctuations of quantization control signals. Figures 7-12 show the system state and quantized state curve of every follower in turn. Moreover, the tracking trajectory (in X-Y) of every follower is given in Figure 13. These Figures show that the cooperative

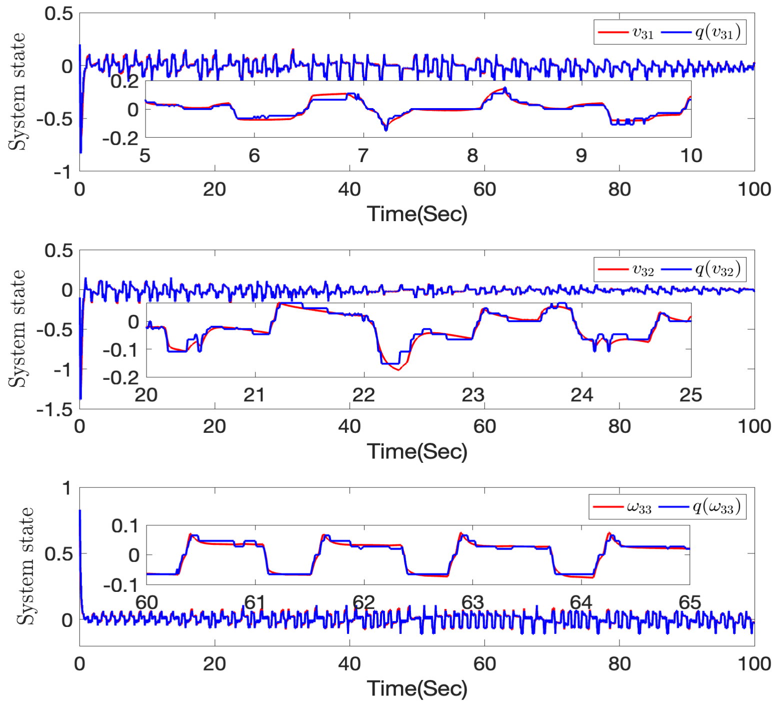

FIGURE 12. The system state v_3 and quantized state $q(v_3)$ of the third follower.

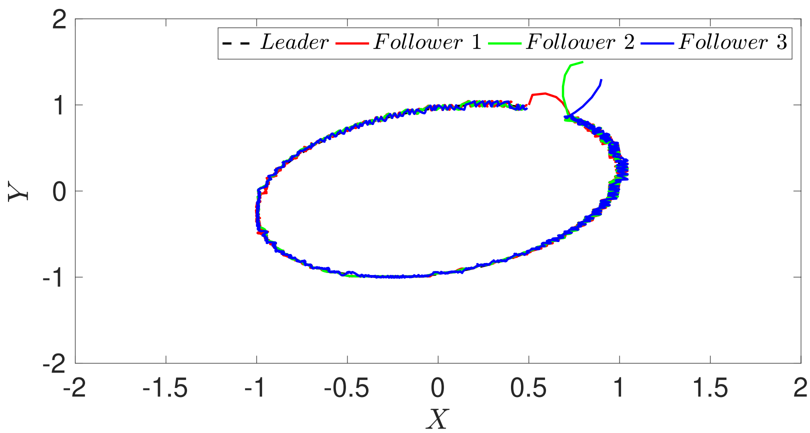

FIGURE 13. The tracking trajectories (in X-Y plane) of three followers.

adaptive control strategy based on quantized states proposed in this paper is feasible.

VI. CONCLUSION

In this paper, the cooperative adaptive tracking control based on quantized system states is studied for multiple surface vessel systems. Firstly, different from the existing results, the environmental disturbance is introduced into the dynamic equations of surface vessel in the form of disturbance accelerations. Further, quantized system sates and vector backstepping method are used to structure a series of cooperative tracking controllers and adaptive laws, which guarantee that the tracking error between the leader and every follower can be adjusted to be as small as possible.

We only discuss the tracking control of multiple surface vessels systems, and collision avoidance and obstacle avoidance are not considered in this paper. The collision avoidance control and obstacle avoidance control are very practical problems for multiple surface vessel systems, we will try to solve this problem in the future.

APPENDIX

PROOF OF LEMMA 1

Proof: According to the sector boundedness of quantizer (including uniform quantizer and the logarithmic quantizer, hysteresis quantizer) in [26], we have

$$|q(\eta_i) - \eta_i| \leq \delta, \quad |q(\bar{v}_i) - \bar{v}_i| \leq \delta, \quad i = 1, \cdots, \mathcal{N},$$

where $\delta > 0$ is a quantization bound. Then, for (27), we have

$$|\tilde{e}_{i1} - e_{i1}| \le h_i|\eta_i - q(\eta_i)| + \sum_{k=1}^{\mathcal{N}} a_{ik}|\eta_k - q(\eta_k)| \le \delta(h_i + \sum_{i=1}^{\mathcal{N}} a_{ik}) \le \epsilon_{e_1}, \tag{45}$$

where $\epsilon_{e_1} = \max_{1\le i\le\mathcal{N}}\{\delta(h_i + \sum_{k=1}^{\mathcal{N}} a_{ik})\}$ with $h_i = b_i + \sum_{k=1}^{\mathcal{N}} a_{ik}$ being positive constants.

Let $\mathcal{H}^{-1} = (\bar{h}_{ik})_{\mathcal{N}\times\mathcal{N}}$, we can rewrite (7) as $\alpha_i = -\sum_{k=1}^{\mathcal{N}} \bar{h}_{ik}\bar{C}_{k1}e_{k1}$ and (24) as $\tilde{\alpha}_i = -\sum_{k=1}^{\mathcal{N}} \bar{h}_{ik}\bar{C}_{k1}\tilde{e}_{k1}$. For (28), we have

$$|\tilde{\alpha}_i - \alpha_i| = |\sum_{k=1}^{\mathcal{N}} \bar{h}_{ik}\bar{C}_{k1}e_{k1} - \sum_{k=1}^{\mathcal{N}} \bar{h}_{ik}\bar{C}_{k1}\tilde{e}_{k1}| = |\sum_{k=1}^{\mathcal{N}} \bar{h}_{ik}\bar{C}_{k1}|_F \sum_{k=1}^{\mathcal{N}} |e_{k1} - \tilde{e}_{k1}| \le \epsilon_\alpha, \tag{46}$$

where $\epsilon_\alpha = \max_{1\le i\le\mathcal{N}}\{\mathcal{N}\epsilon_{e_1}\sum_{k=1}^{\mathcal{N}} \bar{h}_{ik}|\bar{C}_{k1}|_F\}$. Further, for (29), we also have

$$|\tilde{e}_{i2} - e_{i2}| = |\bar{v}_i - \alpha_i - q(\bar{v}_i) + \tilde{\alpha}_i| \le |\bar{v}_i - q(\bar{v}_i)| + |\tilde{\alpha}_i - \alpha_i| \le \epsilon_{e_2}, \tag{47}$$

where $\epsilon_{e_2} = \delta + \epsilon_\alpha$.

For (30), we can get that

$$\begin{aligned}
&|\tilde{\sigma}_i - \sigma_i| \\
&= |M_i\tilde{J}_i^T(q(\psi_i))\dot{\tilde{\alpha}}_i - M_i\tilde{J}_i^T(q(\psi_i))\bar{C}_{i2}\tilde{e}_{i2} \\
&\quad - h_iM_i\tilde{J}_i^T(q(\psi_i))\tilde{e}_{i1} - \frac{1}{4d_1}M_i\tilde{e}_{i2} - M_iJ_i^T\dot{\alpha}_i \\
&\quad + M_iJ_i^T\bar{C}_{i2}e_{i2} + h_iM_iJ_i^Te_{i1} + \frac{1}{4d_1}M_ie_{i2}| \\
&\le |C_{i2}M_i|_F|J_i^Te_{i2} - \tilde{J}_i^T(q(\psi_i))\tilde{e}_{i2}| \\
&\quad + h_i|M_i|_F|J_i^Te_{i1} - \tilde{J}_i^T(q(\psi_i))\tilde{e}_{i1}| \\
&\quad + \frac{|M_i|_F}{4d_1}|e_{i2} - \tilde{e}_{i2}| + |M_i|_F|\tilde{J}_i^T(q(\eta_i))\dot{\tilde{\alpha}}_i - J_i^T\dot{\alpha}_i|.
\end{aligned} \tag{48}$$

Note that

$$|e_{ij}|^2 \le \frac{1}{r_1}e^{-c(t-t_0)}V(t_0) + \frac{d}{cr_1} \le \frac{1}{r_1}V(t_0) + \frac{d}{cr_1} \triangleq M_0^2, \tag{49}$$

with $M_0 = (V(t_0)/r_1 + d/cr_1)^{1/2}$ and $j = 1, 2$, then we have

$$\begin{aligned}
&|C_{i2}M_i|_F|J_i^Te_{i2} - \tilde{J}_i^T(q(\psi_i))\tilde{e}_{i2}| \\
&\quad \le |C_{i2}M_i|_F|J_i^T - \tilde{J}_i^T(q(\psi_i))|_F|e_{i2}| \\
&\quad + |C_{i2}M_i|_F|e_{i2} - \tilde{e}_{i2}||\tilde{J}_i^T(q(\psi_i))|_F \\
&\quad \le 2|C_{i2}M_i|_FM_0 + |C_{i2}M_i|_F\epsilon_{e_2} \\
&\quad \le \epsilon_{\sigma 1},
\end{aligned} \tag{50}$$

where $\epsilon_{\sigma 1} = \max_{1\le i\le\mathcal{N}}\{2|C_{i2}M_i|_FM_0 + |C_{i2}M_i|_F\epsilon_{e_2}\}$. In similar way, we have

$$\begin{aligned}
&h_i|M_i|_F|J_i^Te_{i1} - \tilde{J}_i^T(q(\psi_i))\tilde{e}_{i1}| \\
&\quad = h_i|M_i|_F|J_i^Te_{i1} - \tilde{J}_i^T(q(\psi_i))e_{i1} \\
&\quad\quad - \tilde{J}_i^T(q(\psi_i))\tilde{e}_{i1} + \tilde{J}_i^T(q(\psi_i))e_{i1}| \\
&\quad \le h_i|M_i|_F|J_i^T - \tilde{J}_i^T(q(\psi_i))|_F|e_{i1}| \\
&\quad\quad + h_i|M_i|_F|e_{i1} - \tilde{e}_{i1}||\tilde{J}_i^T(q(\psi_i))|_F \\
&\quad \le 2h_i|M_i|_FM_0 + h_i|M_i|_F\epsilon_{e_1} \\
&\quad \le \epsilon_{\sigma 2},
\end{aligned} \tag{51}$$

where $\epsilon_{\sigma 2} = \max_{1\le i\le\mathcal{N}}\{2h_i|M_i|_FM_0 + h_i|M_i|_F\epsilon_{e_1}\}$, and

$$\begin{aligned}
&|M_i|_F|\tilde{J}_i^T(q(\eta_i))\dot{\tilde{\alpha}}_i - J_i^T\dot{\alpha}_i| \\
&= |M_i|_F|J_i^T\dot{\alpha}_i - \tilde{J}_i^T(q(\eta_i))\dot{\alpha}_i + \tilde{J}_i^T(q(\eta_i))\dot{\alpha}_i - \tilde{J}_i^T(q(\psi_i))\dot{\tilde{\alpha}}_i| \\
&\le |M_i|_F|J_i^T - \tilde{J}_i^T(q(\psi_i))|_F|\dot{\alpha}_i| + |M_i|_F|\dot{\alpha}_i - \dot{\tilde{\alpha}}_i||\tilde{J}_i^T(q(\psi_i))|_F \\
&\le 2|M_i|_F|\sum_{k=1}^{\mathcal{N}}\frac{\partial\alpha_i}{\partial\eta_k}\bar{v}_k + \frac{\partial\alpha_i}{\partial p_0}\dot{p}_0| + |\sum_{k=1}^{\mathcal{N}}\frac{\partial\alpha_i}{\partial\eta_k}\bar{v}_k + \frac{\partial\alpha_i}{\partial p_0}\dot{p}_0 \\
&\quad - \sum_{k=1}^{\mathcal{N}}\frac{\partial\alpha_i}{\partial\eta_k}q(\bar{v}_k) - \frac{\partial\alpha_i}{\partial p_0}\dot{p}_0||M_i|_F \\
&\quad + \mathcal{N}\delta|M_i|_F|\sum_{k=1}^{\mathcal{N}}\frac{\partial\alpha_i}{\partial\eta_k}|_F \\
&\le 2|M_i|_F|\sum_{k=1}^{\mathcal{N}}\frac{\partial\alpha_i}{\partial\eta_k}|_F(|e_{k2}| + |\alpha_k|) + 2K|M_i|_F|\frac{\partial\alpha_i}{\partial p_0}|_F \\
&\quad + \mathcal{N}\delta|M_i|_F|\sum_{k=1}^{\mathcal{N}}\frac{\partial\alpha_i}{\partial\eta_k}|_F \\
&\le 2|M_i|_F|\sum_{k=1}^{\mathcal{N}}\frac{\partial\alpha_i}{\partial\eta_k}|_F(\mathcal{N}M_0 + \mathcal{N}M_0|\sum_{k=1}^{\mathcal{N}}\bar{h}_{ik}\bar{C}_{k1}|_F) \\
&\quad + 2K|M_i|_F|\frac{\partial\alpha_i}{\partial p_0}|_F + \mathcal{N}\delta|M_i|_F|\sum_{k=1}^{\mathcal{N}}\frac{\partial\alpha_i}{\partial\eta_k}|_F \\
&\le \epsilon_{\sigma 3},
\end{aligned} \tag{52}$$

where $|\partial\alpha_i/\partial\eta_k|_F$ and $|\partial\alpha_i/\partial p_0|_F$ are known constans from (7), $\bar{C}_0 = \max_{1\le i\le\mathcal{N}}\{(1 + |\sum_{k=1}^{\mathcal{N}}\bar{h}_{ik}\bar{C}_{k1}|_F)\}$, and $\epsilon_{\sigma 3} = \max_{1\le i\le\mathcal{N}}\{|M_i|_F(2|\sum_{k=1}^{\mathcal{N}}\partial\alpha_i/\partial\eta_k|_F\mathcal{N}M_0\bar{C}_0 + 2K|\partial\alpha_i/\partial p_0|_F + \mathcal{N}\delta|\sum_{k=1}^{\mathcal{N}}\partial\alpha_i/\partial\eta_k|_F)\}$. Further, we also have

$$\frac{1}{4d_1}|M_i|_F|e_{i2} - \tilde{e}_{i2}| \le \frac{1}{4d_1}|M_i|_F\epsilon_{e_2} \le \epsilon_{\sigma 4}, \tag{53}$$

where $\epsilon_{\sigma 4} = \max_{1\le i\le\mathcal{N}}\{\epsilon_{e_2}|M_i|_F/4d_1\}$. Substituting (50)-(53) into (48) yields that

$$|\tilde{\sigma}_i - \sigma_i| \le \epsilon_{\sigma 1} + \epsilon_{\sigma 2} + \epsilon_{\sigma 3} + \epsilon_{\sigma 4} \triangleq \epsilon_\sigma, \tag{54}$$

where $\epsilon_\sigma = \epsilon_{\sigma 1} + \epsilon_{\sigma 2} + \epsilon_{\sigma 3} + \epsilon_{\sigma 4}$.

For (31), we have

$$
\begin{aligned}
&|\tilde{H}_i - H_i| \\
&= |\omega_{i3} M_i S^T J_i^T(\psi_i)\bar{v}_i + \bar{C}_i\bar{v}_i - q(\omega_{i3}) M_i S^T \tilde{J}_i^T(q(\psi_i))q(\bar{v}_i) \\
&\quad - \tilde{C}_i(q(\bar{v}_i))q(\bar{v}_i)| \\
&\leq |\omega_{i3} M_i S^T J_i^T(\psi_i)\bar{v}_i - q(\omega_{i3}) M_i S^T \tilde{J}_i^T(q(\psi_i))q(\bar{v}_i)| \\
&\quad + |\bar{C}_i\bar{v}_i - \tilde{C}_i(q(\bar{v}_i))q(\bar{v}_i)|, \qquad (55)
\end{aligned}
$$

which leads to

$$
\begin{aligned}
&|\omega_{i3} M_i S^T J_i^T(\psi_i)\bar{v}_i - q(\omega_{i3}) M_i S^T \tilde{J}_i^T(q(\psi_i))q(\bar{v}_i)| \\
&= |M_i|_F |\omega_{i3} S^T J_i^T(\psi_i)\bar{v}_i - \omega_{i3} S^T \tilde{J}_i^T(q(\psi_i))\bar{v}_i \\
&\quad - q(\omega_{i3}) S^T \tilde{J}_i^T(q(\psi_i))q(\bar{v}_i) + \omega_{i3} S^T \tilde{J}_i^T(q(\psi_i))\bar{v}_i| \\
&\leq |M_i|_F ||\omega_{i3}\bar{v}_i||S^T J_i^T(\psi_i) - S^T \tilde{J}_i^T(q(\psi_i))|_F \\
&\quad + |M_i|_F |\omega_{i3}\bar{v}_i - q(\omega_{i3})q(\bar{v}_i)||S^T \tilde{J}_i^T(q(\psi_i))|_F \\
&\leq 2|M_i|_F |M_0^2 \bar{C}_0^2 + |M_i|_F |\bar{v}_i||w_{i3} - q(\omega_{i3})| \\
&\quad + |M_i|_F |\delta + \omega_{i3}||\bar{v}_i - q(\bar{v}_i)| \\
&\leq 2|M_i|_F M_0^2 \bar{C}_0^2 + \delta |M_i|_F M_0 \bar{C}_0 + \delta |M_i|_F (\delta + M_0 \bar{C}_0) \\
&\leq \epsilon_{H1}, \qquad (56)
\end{aligned}
$$

with $\epsilon_{H1} = \max_{1\leq i\leq \mathcal{N}}\{|M_i|_F\big(2M_0^2\bar{C}_0^2 + \delta M_0\bar{C}_0 + \delta(\delta + M_0\bar{C}_0)\big)\}$, and

$$
\begin{aligned}
&|\bar{C}_i\bar{v}_i - \tilde{C}_i(q(\bar{v}_i))q(\bar{v}_i)| \\
&= |\bar{C}_i\bar{v}_i - \tilde{C}_i(q(\bar{v}_i))\bar{v}_i + \tilde{C}_i(q(\bar{v}_i))\bar{v}_i - \tilde{C}_i(q(\bar{v}_i))q(\bar{v}_i)| \\
&\leq |\bar{C}_i - \tilde{C}_i(q(\bar{v}_i))|_F |\bar{v}_i| + |\bar{v}_i - q(\bar{v}_i)||\delta I_3 + \bar{C}_i|_F \\
&\leq |\bar{C}_i - \tilde{C}_i(q(\bar{v}_i))|_F M_0\bar{C}_0 + \delta(\delta + |\bar{C}_i|_F). \qquad (57)
\end{aligned}
$$

For $|\bar{C}_i - \tilde{C}_i(q(\bar{v}_i))|_F$, we also have

$$
\begin{aligned}
&|\bar{C}_i - \tilde{C}_i(q(\bar{v}_i))|_F \\
&\leq |C_i(v_i)J_i^T(\psi_i) - \tilde{C}_i(q(v_i))\tilde{J}_i^T(q(\psi_i))|_F \\
&\leq |J_i^T(\psi_i) - \tilde{J}_i^T(q(\psi_i))|_F |C_i(v_i)|_F \\
&\quad + |C_i(v_i) - \tilde{C}_i(q(v_i))|_F |\tilde{J}_i^T(q(\psi_i))|_F \\
&\leq 2|C_i(v_i)|_F + |C_i(v_i) - \tilde{C}_i(q(v_i))|_F. \qquad (58)
\end{aligned}
$$

From the definition of $C_i(v_i)$ in [28], we know that $c_{i1}(v_i) = \ell_{i1} v_{i2} + \ell_{i2}\omega_{i3}$ and $c_{i2}(v_i) = \ell_{i3} v_{i1}$ with ℓ_{i1}, ℓ_{i2} and ℓ_{i3} being known positive constants, then we can get that

$$
\begin{aligned}
&|q(c_{i1}(v_i)) - c_{i1}(v_i)| \\
&\leq |\ell_{i1} q(v_{i2}) + \ell_{i2} q(\omega_{i3}) - \ell_{i1} v_{i2} - \ell_{i2}\omega_{i3}| \\
&\leq \ell_{i1}|q(v_{i2}) - v_{i2}| + \ell_{i2}|q(\omega_{i3}) - \omega_{i3}| \\
&\leq \delta(\ell_{i1} + \ell_{i2}), \qquad (59)
\end{aligned}
$$

and in similar way, we can easly obtain that other elements are also bounded in $C_i(v_i) - \tilde{C}_i(q(v_i))$, which means that $|C_i(v_i) - \tilde{C}_i(q(v_i))|_F$ is bounded. From the definitions of $C_i(v_i)$ and $\bar{C}_i = C_i(v_i)J_i^T$, combining with

$$
|v_i| \leq |J_i(\psi_i)|_F |\bar{v}_i| \leq |e_{i2} + \alpha_i| \leq M_0\bar{C}_0, \qquad (60)
$$

the boundedness of $|C_i(v_i)|_F$ and $|\bar{C}_i|_F$ can be obtained in (57) and (58), respectively. Then, we have

$$
\begin{aligned}
|\bar{C}_i\bar{v}_i - \tilde{C}_i(q(\bar{v}_i))q(\bar{v}_i)| &\leq \epsilon_{i1} M_0\bar{C}_0 + \delta(\delta + \epsilon_{i2}) \\
&\leq \epsilon_{H2}, \qquad (61)
\end{aligned}
$$

where $\epsilon_{H2} = \max_{1\leq i\leq \mathcal{N}}\{\epsilon_{i1} M_0\bar{C}_0 + \delta^2 + \delta\epsilon_{i2})\}$ with ϵ_{i1} being the boundary of $|\bar{C}_i - \tilde{C}_i(q(\bar{v}_i))|_F$ and ϵ_{i2} being the boundary of $|\bar{C}_i|_F$. Substituting (56) and (61) into (55) yields that

$$
|\tilde{H}_i - H_i| \leq \epsilon_{H1} + \epsilon_{H2} \triangleq \epsilon_H, \qquad (62)
$$

where $\epsilon_H = \epsilon_{H1} + \epsilon_{H2}$.

For (32), we have

$$
\begin{aligned}
&|\bar{\Gamma}_i J_i^T(\psi_i)\bar{v}_i - \tilde{\Gamma}_i \tilde{J}_i^T(q(\psi_i))q(\bar{v}_i)| \\
&\leq |\bar{\Gamma}_i J_i^T(\psi_i)\bar{v}_i - \bar{\Gamma}_i \tilde{J}_i^T(q(\psi_i))q(\bar{v}_i)| \\
&\quad + |\bar{\Gamma}_i \tilde{J}_i^T(q(\psi_i))q(\bar{v}_i) - \tilde{\Gamma}_i \tilde{J}_i^T(q(\psi_i))q(\bar{v}_i)| \\
&\leq |\bar{\Gamma}_i|_F |J_i^T(\psi_i)\bar{v}_i - \tilde{J}_i^T(q(\psi_i))\bar{v}_i + \tilde{J}_i^T(q(\psi_i))\bar{v}_i \\
&\quad - \tilde{J}_i^T(q(\psi_i))q(\bar{v}_i)| + |\bar{\Gamma}_i - \tilde{\Gamma}_i|_F |\tilde{J}_i^T(q(\psi_i))|_F |q(\bar{v}_i)| \\
&\leq |\bar{\Gamma}_i|_F |J_i^T(\psi_i) - \tilde{J}_i^T(q(\psi_i))|_F |\bar{v}_i| + |\bar{\Gamma}_i - \tilde{\Gamma}_i|_F |\delta I_3 + \bar{v}_i| \\
&\quad + |\bar{\Gamma}_i|_F |\bar{v}_i - q(\bar{v}_i)||\tilde{J}_i^T(q(\psi_i))|_F \\
&\leq 2|\bar{\Gamma}_i|_F M_0\bar{C}_0 + \delta|\bar{\Gamma}_i|_F + \delta|\bar{\Gamma}_i - \tilde{\Gamma}_i|_F + |\bar{\Gamma}_i - \tilde{\Gamma}_i|_F M_0\bar{C}_0 \\
&\leq \epsilon_{\bar{\Gamma}}, \qquad (63)
\end{aligned}
$$

where $\epsilon_{\bar{\Gamma}} = \max_{1\leq i\leq \mathcal{N}}\{2\epsilon_{i3} M_0\bar{C}_0 + \delta\epsilon_{i3} + \delta\epsilon_{i4} + \epsilon_{i4} M_0\bar{C}_0\}$ with ϵ_{i3} being the boundary of $|\bar{\Gamma}_i|_F$ and ϵ_{i2} being the boundary of $|\bar{\Gamma}_i - \tilde{\Gamma}_i|_F$. According to the definition for damping matrix in [28], the proof for the boundedness of $|\bar{\Gamma}_i|_F$ and $|\bar{\Gamma}_i - \tilde{\Gamma}_i|_F$ are similar to (59), then the calculation process is omitted.

For (33), from (63), we can get that

$$
\begin{aligned}
&|\tilde{\kappa}_i - \kappa_i| \\
&\leq |\tilde{e}_{i2}^T M_i^{-1} \tilde{\Gamma}_i(q(\bar{v}_i))\tilde{J}_i^T(q(\psi_i))q(\bar{v}_i) - \tilde{e}_{i2}^T M_i^{-1} \bar{\Gamma}_i(\bar{v}_i) J_i^T(\psi_i)\bar{v}_i| \\
&\quad + |\tilde{e}_{i2}^T M_i^{-1} \bar{\Gamma}_i(\bar{v}_i) J_i^T(\psi_i)\bar{v}_i - e_{i2}^T M_i^{-1} \bar{\Gamma}_i(\bar{v}_i) J_i^T(\psi_i)\bar{v}_i| \\
&\leq \epsilon_{\bar{\Gamma}} |q(\bar{v}_i) + \sum_{k=1}^{\mathcal{N}} \bar{h}_{ik}\bar{C}_{k1}\tilde{e}_{k1}| + \epsilon_{e_2} |M_i^{-1}|_F |J_i^T(\psi_i)|_F |\bar{\Gamma}_i(\bar{v}_i)\bar{v}_i| \\
&\leq \epsilon_{\bar{\Gamma}} |\delta I_3 + \bar{v}_i| + \epsilon_{\bar{\Gamma}} |\sum_{k=1}^{\mathcal{N}} \bar{h}_{ik}\bar{C}_{k1}(\delta I_3 + e_{k1})| \\
&\quad + \epsilon_{e_2} |M_i^{-1}|_F |\bar{\Gamma}_i(\bar{v}_i)\bar{v}_i| \\
&\leq \delta\epsilon_{\bar{\Gamma}} + \delta\epsilon_{\bar{\Gamma}} M_0\bar{C}_0 + \delta\epsilon_{\bar{\Gamma}} |\sum_{k=1}^{\mathcal{N}} \bar{h}_{ik}\bar{C}_{k1}|_F \\
&\quad + \mathcal{N} M_0 \epsilon_{\bar{\Gamma}} |\sum_{k=1}^{\mathcal{N}} \bar{h}_{ik}\bar{C}_{k1}|_F + \epsilon_{e_2}\epsilon_{i5} |M_i^{-1}|_F \\
&\leq \epsilon_\kappa, \qquad (64)
\end{aligned}
$$

where $\epsilon_\kappa = \max_{1\leq i\leq \mathcal{N}}\{\epsilon_{\bar{\Gamma}} |\sum_{k=1}^{\mathcal{N}} \bar{h}_{ik}\bar{C}_{k1}|_F(\delta + \mathcal{N} M_0) + \epsilon_{e_2}\epsilon_{i5}|M_i^{-1}|_F + \delta\epsilon_{\bar{\Gamma}}(1 + M_0\bar{C}_0)\}$ with the boundary ϵ_{i5} of $|\bar{\Gamma}_i(\bar{v}_i)\bar{v}_i|$.

So far, we have completed the proof of Lemma 1. ■

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QINXUE XU received the B.S. degree from Yantai University, Yantai, China, in 2021, where he is currently pursuing the M.S. degree. His research interests include multi-agent systems and cooperative control.

LIKANG FENG received the Ph.D. degree in control theory and control engineering from the Shandong University of Science and Technology, Qingdao, China, in 2021. He is currently a Lecturer with the School of Mathematics and Information Sciences, Yantai University, Yantai, China. His research interests include stochastic nonlinear systems and impulsive systems.

LIQIANG YAO received the Ph.D. degree in control theory and control engineering from the Shandong University of Science and Technology, Qingdao, China, in 2020. He is currently an Associate Professor with the School of Mathematics and Information Sciences, Yantai University, Yantai, China. His research interests include stochastic nonlinear systems and multi-agent systems.