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RESEARCH ARTICLE

A Nonlinear Adaptive Weight-Based Mutated Whale Optimization Algorithm and Its Application for Solving Engineering Problems

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ABSTRACT Whale optimization algorithm (WOA) is a swarm-based optimization algorithm with exceptional performance and significant originality. In this study, a novel variant of WOA called nonlinear adaptive weight-based mutated WOA (NAWMWOA) is proposed to overcome the shortcomings of original WOA such as easily falling into local optimum and slow convergence speed. In detail, the proposed NAWMWOA includes three novel strategies as comparing with original WOA. Firstly, a nonlinear convergence factor is embedded into the original WOA to balance exploration and exploitation ability. The second improvement is an adaptive weight strategy, which can enhance the exploratory searching trends and improve the solution accuracy. Moreover, the thirdly proposed hybrid mutation strategy has the function of increasing the accuracy and jumping out of the local optimum. The combination of the three strategies significantly improve convergence efficiency and search accuracy of original WOA. To verify the remarkable performance of the proposed NAWMWOA, a series of illustrious WOA variants and state-of-the-art intelligent algorithms is compared with the NAWMWOA on 37 benchmark functions and three typical engineering problems. The details of experimental and statistics results illustrate that the presented NAWMWOA has higher convergence efficiency and better solution accuracy. As a conclusion, the proposed NAWMWOA is a competitive and outstanding algorithm that can effectively solve optimization problems in practical engineering.

INDEX TERMS Whale optimization algorithm, swarm intelligence, adaptive weight, engineering design.

I. INTRODUCTION

With the rapid development of various engineering applications, traditional methods such as gradient descent, which have the disadvantages of low efficiency and narrow application scope, are no longer applicable to engineering problems with increasingly high complexity and quality standards. Therefore, meta-heuristic algorithms with the advantages of high optimization efficiency and simple principle have rapidly developed and widely used in various fields including feature selection [1], convolutional neural networks [2], collaborative robots [3], deep learning [4], extreme learning machines [5], optimization of design [6], multi-threshold color image segmentation [7], and Job-Shop scheduling problem [8]. As meta-heuristic algorithms are required in more and more scenarios, various novel optimization algorithms have been exploited. One of the most remarkable is swarm-based algorithms that mimic the social behavior of natural organisms, such as horse herd optimization algorithm (HOA) [9], marine predators algorithm (MPA) [10] slime mould algorithm (SMA) [11], monarch butterfly optimization (MBO) [12], binary dragonfly algorithm (BDA) [13], harris

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TABLE 1. Several famous meta-heuristic algorithms.

Algorithm	Category
Adaptive multi-objective evolutionary algorithm (AGMOEA) [23] Genetic algorithms (GA) [24] Probability-based incremental learning (PBIL) [25] Genetic programming (GP) [26]	Evolutionary algorithm
Gravitational local search (GLSA) [27] Stochastic fractal Search (SFS) [28] Big bang big crunch (BBBC) [29] Central force optimization (CFO) [30] Gravitational search algorithm (GSA) [31]	Physics-based algorithm
Stock exchange trading optimization (SETO) [32] group search optimizer (GSO) [33] Interactive autodidactic school (IAS) [34] Most valuable player algorithm (MVPA) [35] Teaching-learning-based optimization (TLBO) [36] League championship algorithm (LCA) [37]	Human-based algorithm

hawks optimization (HHO) [14] salp swarm algorithm (SSA) [15], tunicate swarm algorithm (TSA) [16], chimp optimization algorithm (COA) [17], butterfly optimization algorithm (BOA) [18], flower pollination algorithm (FPA) [19], pity beetle algorithm (PBA) [20], ant colony optimization (ACO) [21], and chameleon swarm algorithm (CSA) [22]. In addition, evolutionary algorithms, physics-based algorithms, and human-based algorithms are three meta-heuristic algorithms, and some well-known algorithms among them are shown in Table 1.

The whale optimization algorithm (WOA) [38] is a novel swam-based algorithm proposed by Mirjalili et al. in 2016, which mimics the unique spiral bubble net of humpback whales during hunting to optimize engineering problems. Compared with other algorithms, WOA has advantages of simple structure, few setting parameters, and pure principle. Therefore, WOA is widely applied in various fields including function optimization [39], reservoir optimal operation [40], structure optimization [41], sizing optimization [42] and target assignment [43]. In addition, Brodzicki et al. applied WOA for the optimization task of hyperparameters. Simulations show that the WOA can be successfully used for hyperparameters optimization, achieving an accuracy of 80.60% for Reuters datasets [44]. In order to ensure the safety and reliability of the underwater vehicle, Yan et al. used WOA to plan the three-dimensional path of the underwater vehicle. The results illustrate that the method has the advantages of fast execution speed and high calculation accuracy [45].

However, the simple structure of WOA makes it easy to fall into local optimization and lacks global search ability when solving complex high-dimensional problems. Therefore, some researchers have devoted themselves to developing improvement strategies for the defects of WOA and proposed more advanced WOA, which are applied to solve various complex problems. For example, Niu et al. proposed a combination of oppositely adaptive whale optimization algorithm (AWOA) to predict heat consumption rate of steam turbines. The prediction results show that the prediction model based on AWOA had higher prediction accuracy and stronger generalization ability [46]. Cosine adapted modified whale optimization algorithm (CamWOA), in which cosine function is embedded for the selection of parameters was proposed by Saha et al. to solve a multi-objective engineering problem pertaining to control of switched reluctance motor [47]. Strumberger proposed a hybridized WOA by combining with some other meta-heuristics and applied to perform the resource scheduling problem in cloud environments. Achieved results in all simulations illustrate that the proposed hybrid WOA outperforms other meta-heuristics [48]. An improved WOA (SCA-WOA) combined with the cosine form nonlinear convergence factor was proposed by Yue et al. to obtain the wireless sensor network optimal coverage scheme. The experimental results show that the method has a significant effect in improving the network coverage effect [49]. Lei et al. proposed an improved WOA (IWOA) that embeds Levy flight, opposition-based learning, and nonlinear convergence factor to optimize the mine water optimization scheduling model. The results show that the reuse efficiency of the multi-level scheduling method of mine water reuse is increased by 30.2% and 31.9%, respectively [50]. Jiao and Xiaoqing mutated WOA by introducing the crazy operator and the golden sine algorithm, which improved the global search ability of the WOA, and its outstanding performance was also verified in simulation results [51]. Li et al. proposed DWOA, which embedded the existing discretization methods into WOA, showing an excellent performance in the knapsack problems [52]. Li et al. gave WOA the advantages of the DE, called DE-WOA, which further improves the convergence speed of the model [53]. Zhou et al. introduced a teaching learning-based algorithm and simplex method into the original WOA to achieves a better balance between exploration and exploitation of WOA and successfully applied it to the multi-layer perceptron neural network training [54]. Cai and Du proposed a novel variant of WOA called balanced WOA by combining the dynamic balance strategy and the population reconstruction mechanism, and the feasibility and effectiveness were verified in path planning [55]. A variant of WOA combined with evolution operators named IMOWOA was proposed by Qian et al., and its effectiveness was proved in the deterministic optimization of vehicle structure crashworthiness [56]. In order to perform parameter optimization and feature selection simultaneously for SVM, Wang and Chen proposed a variant of WOA (CMWOA), which combines chaotic and multi-swarm. The experimental results demonstrate that CMWOA-based model significantly outperformed all the other competitors in terms of classification performance and feature subset size [57]. Zhang et al. proposed a novel optimization scheme called EWOA which combined the search updating strategy and the gray correlation mechanism, and it was applied to optimize the SVM random parameters. The experimental results prove that the EWOA-SVM model is superior to other existing methods in terms of fast convergence speed and high prediction accuracy [58]. In order to reasonably allocate mobile conventional missile battle positions, Li et al. optimized the original WOA using the convergence factor and the diversity mutation operation, which provides a reference for the deployment mode of mobile missile positions [59].

Actually, although the cosine function parameters are utilized in CamWOA to improve convergence accuracy, the effect is limited. The strategies such as Levy flight, opposition-based learning, and nonlinear convergence factor are employed in IWOA to improve population diversity of WOA, which reduces the convergence speed of the algorithm. The communication learning mechanism proposed by CMWOA improves the exploration performance of WOA, however, the exploitation ability of population decreases. Moreover, the dominance population used in the WOA variants, have improved convergence speed but greatly reduced population diversity. Thus, the current variants of WOA proposed by researchers always have limitations and drawbacks, and one algorithm cannot always perform as the best on most optimization problems, and there is still some space to improve the overall performance of the algorithm. Therefore, a variant of WOA called nonlinear adaptive weight-based mutated whale optimization algorithm (NAWMWOA) has been proposed in this study, which includes three improvement strategies: nonlinear convergence factor, adaptive weight, and hybrid mutation strategy. The nonlinear convergence factor changes nonlinearly with the iteration time. The global search ability and local development ability are effectively balanced with the aid of the nonlinear convergence factor. The adaptive weight is used to overcome the defect of easily falling into local optimum. Inspired by the SMA and SSA, a hybrid mutation strategy was proposed to increase population diversity and help search agents jump out of local optimum, which can effectively improve the search accuracy.

To verify the performance of proposed NAWMWOA, it is compared with variants of WOA and state-of-the-art algorithms on the basis of CEC2015 [60], CEC2021 [61], and CEC2022 benchmark functions. Moreover, three constraint problems are employed to test the feasibility of NAWMWOA for solving engineering problems. In short, the experimental results of NAWMWOA are better than other comparison algorithms, indicating the superiority of the proposed algorithm.

The content of this study is designed as follows: Section II will introduce the basic principle and composition of the original WOA in detail. Section III will explain the principles and functions of the three improvement strategies. Section IV will conduct a full range of experiments on the proposed NAWMWOA, including comparison with other variant of WOA and advanced algorithms. The application effect of the NAWMWOA in various engineering problems will be shown in Section V. Section VI will summarize this study and discuss the research direction in the next stage.

II. OVERVIEW OF ORIGINAL WOA

In this section, the principles and structure of the WOA will be introduced in detail. WOA is an advanced meta-heuristic algorithm proposed by Australian scientist Mirjalili in 2016, which imitates the unique hunting process of humpback whales. According to the principle of whale hunting, WOA is divided into three stages: encircling prey, bubble-net attacking method, and search for prey. Among them, the bubble net attaching method is also called the exploitation phase, which is mainly applied for depth searches to obtain more accurate solutions. The search for prey is also called the exploration phase, which has the function of widening the search scope and improving the population diversity. Specific details of the WOA are described in the following subsections.

A. ENCIRCLING PREY

Humpback whales can identify and surround their prey. The location of the optimal solution in the search space is uncertain. However, the location of the current optimal individual is known. The whale updates its position to approach the current optimal individual and encircle prey. This behavior is represented by the following equations:

$$D = \left| C \cdot X^*_{(t)} - X_{(t)} \right| \tag{1}$$

where t indicates the current iteration $X^*_{(\mathrm{t})}$ is the optimal solution so far, $X_{(t)}$ represents the whale individual in the current iteration, C is a coefficient vector, $||$ is the absolute value.

The process that individual updates position to the current optimal solution is described as following:

$$X_{(t+1)} = X^*_{(t)} - A \cdot D \tag{2}$$

where A represents a vector described by Eq. (3), $(\cdot)$ represents an element-by-element multiplication.

$$A = 2ar_1 - a \tag{3}$$

$$C = 2r_2 \tag{4}$$

where r_1 and r_2 are random vectors in the interval of [0,1]. a is a convergence factor determined by the iteration time. As iteration time increases, a linearly decreases from 2 to 0.

B. BUBBLE-NET ATTACKING METHOD

At this stage, whales spiral upward from beneath the shoal, creating bubbles to form bubble nets, and then surround

Algorithm 1 Pseudocode of NAWMWOA.

```
Set population size N, maximum iteration times T_max, and dimension number D
Initialize random position of whale X_i(i = 1, 2, ..., N)
Calculate the fitness values of each search agent
Get the best search agent X*
while (t < T) do
  Update h by Eq. (11)
  for each whale
    Update a1, a2, w, A, C, nnR and p
    Update g by Eq. (16)
    if(i < N/2)
      Update B by Eq. (17)
    else if (i ≥ N/2) then
      Update B by Eq. (18)
      if (|A| ≥ 1) then
        Select a random search agent X_rand
        Update the position of current search agent by Eq. (15)
      else if (|A| < 1) then
        Update the position of current search agent by Eq. (14)
      end if
    else if (p ≥ 0.03) then
      Update s, v by Eq. (19) and Eq. (20)
      for j =1 : dim do
        Select a random search agent (X_rand1, X_rand2)
        if(r4 < s)
          Update the position of current search by Eq. (22)
        else if (r4 ≥ s) then
          Update c1 by Eq. (21)
          if c2 ≥ 0
            Update the position of current search by Eq. (23)
          else if (c2 < 0) then
            Update the position of current search by Eq. (24)
          end if
        end if
      end for
    end if
  end for
  Check the space limits
  Calculate the fitness of each search agent
  Update X* if there is a better solution
  t = t + 1
end while
return X*
```

their prey with bubble nets. The mathematical model of this process can be expressed as:

$$D' = \left|X^*_{(t)} - X_{(t)}\right| \tag{5}$$

$$X_{(t+1)} = D' \cdot e^{bl} \cdot cos(2\pi l) + X^*_{(t)} \tag{6}$$

where D' represents the gap between the current whale and the current optimal solution, b is a constant for defining the shape of the logarithmic spiral, and l is a random value in the interval of [0,1].

The two hunting methods mentioned above have a 50% probability of being applied respectively. The mathematical model is presented as follows:

$$X_{(t+1)} = \begin{cases} X^*_{(t)} - A \cdot D & f\ p < 0.5 \\ D' \cdot e^{bl} \cdot cos(2\pi l) + X^*_{(t)} & f\ p \geq 0.5 \end{cases} \tag{7}$$

where p is a random value in the interval of [0, 1].

C. SEARCH FOR PREY

In addition, the whale will change the hunting method based on the change of vector A. When $|A| > 1$, the whale will randomly select another individual as the target to update the position. The mathematical model is as follows:

$$D = |C \cdot X_{rand} - X| \tag{8}$$

$$X_{(t+1)} = X_{rand} - A \cdot D \tag{9}$$

where X_{rand} is a random individual. When $|A| > 1$, global exploration is performed, and when $|A| < 1$, local exploitation is performed.

III. PROPOSED NAWMWOA

Various variants of WOA have been proposed and widely used in various fields. However, with the rapid development of expert systems with applications, the original WOA and some variants of WOA present a tendency to fall into local optimum easily and imprecise convergence results. Therefore, to overcome these shortcomings, three novel improvement mechanisms were embedded into WOA and generated an improved WOA called nonlinear adaptive weight-based mutated whale optimization algorithm (NAWMWOA). The three improvement mechanisms are nonlinear convergence factor, adaptive weight and hybrid mutation strategy.

A. NONLINEAR CONVERGENCE FACTOR

The balance of algorithm between the two main phases (exploration phase and development phase) significantly affects the quality of convergence results, and the decision of which phases to execute made by convergence factor a. However, the convergence factor a decreases linearly from 1 to 0 as the iterations increase, resulting in one type of exploration and exploitation phases overly executed and the other type of exploration and exploitation phases diluted. Then the undesirable result of falling into local optimum or failing to converge will appear. Therefore, the nonlinear convergence factor a_2 is proposed to improve the balance between the two stages, and thus improve the convergence speed and accuracy of the algorithm. Moreover, the a_2 is derived by a formula similar to chaotic random principle. Therefore, the randomness of a_2 improves the possibility of achieving dynamic equilibrium. The mathematical expression of the Nonlinear convergence factor proposed is as follows:

$$R = \frac{t}{T_{max}} \tag{10}$$

$$h = 1 - 2 \cdot h^2 \tag{11}$$

$$a_2 = 2\left(1 + (|h| - 1) \cdot R^{\frac{1}{4}} - |h| \cdot R^2\right) \tag{12}$$

where R represents rate of iteration. T_{max} is maximum iterations. h is a chaotic parameter that controls the fluctuation of a_2. In detail, to make the value of chaos parameter h satisfy the requirement of more uniform random value with chaos mapping, the value range of h is set to [0,1]. Moreover, the randomness of chaotic mapping becomes worse when the value of h is 0.25 according to the rule of chaotic mapping.

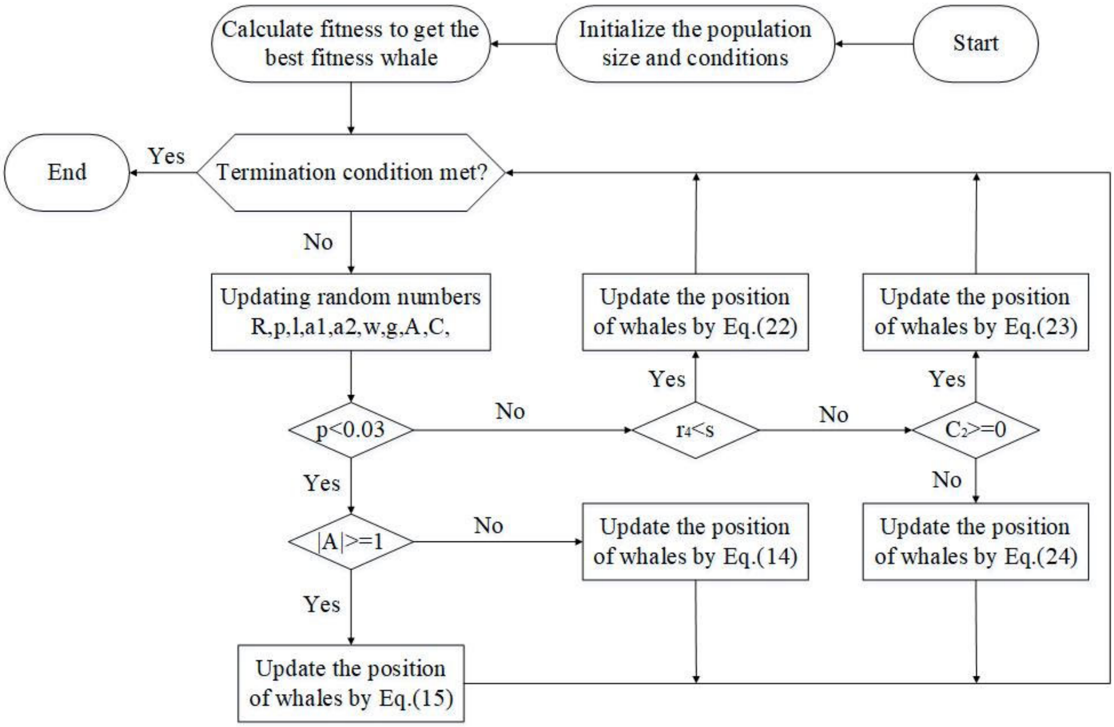

FIGURE 1. Flowchart of NAWMWOA.

Thus, the initial value of h satisfies the following conditions: $1 > h > 0$ and $h \neq 0.25$.

B. ADAPTIVE WEIGHT

To overcome the defect of the original WOA that it is easy to fall into local optimum and not reduce the convergence speed of the algorithm, an adaptive weight strategy that dynamically changes with the number of iterations is proposed.

The adaptive weight is determined by two parts: the first half of the weight plays a major role in regulating the slow convergence rate, and the second half of the weight plays a major role in overcoming the population to fall into the local optimal. The principle is as follows: at the early stage of the iteration, assuming that the whale population falls into small search area, and the difference between the optimal solution and the worst solution is not significant, the value $p_2 \cdot (ub - lb)/t$ is not affected by the population distribution and a large weight value w can still be obtained, thus alleviating the defect of the algorithm falling into the local optimal solution at the early stage of the iteration. With the increase of the iterations, the value $p_2 \cdot (ub - lb)/t$ will gradually decrease, and its influence on the weight w will decrease. If the algorithm does not get the optimal solution at this time, $p_1 \cdot \left(X^*_{(\mathrm{t})} - X^*_{(\mathrm{t})\mathrm{bad}}\right)$ can play a leading role in the adaptive weight, which can enable the algorithm to optimize with a larger step size and thus improve the convergence speed. The detailed descriptions are as follows:

$$w = p_1 \cdot \left(X^*_{(t)} - X^*_{(t)bad}\right) + p_2 \cdot (ub - lb)/t \tag{13}$$

$$X_{(t+1)} = w \cdot X^*_{(t)} - A \cdot D \tag{14}$$

$$X_{(t+1)} = w \cdot X_{rand} - A \cdot D \tag{15}$$

where w indicates the adaptive weight. p_1 and p_2 are two different constants. ub and lb represent upper and lower bounds of the search space, respectively. $X^*_{(\mathrm{t})\mathrm{bad}}$ is the worst solution in the current iteration.

C. HYBRID MUTATION STRATEGY

The structure of the original WOA exploration phase is relatively simple and the form is single, which limits the diversity of individuals and further leads to the imprecision of the algorithm convergence results. Therefore, the hybrid mutation strategy was embedded into WOA to overcome the above defects.

The hybrid mutation strategy is mainly divided into two parts: diffusion mutation mimicking slime mold looking for nutrients (Mutation 1) and chain mutation mimicking salps hunting (Mutation 2). Mutation 1 is based on the globally optimal individual and two random individuals. Mutation 2 is only performed based on globally optimal individuals. The two categories of mutations combined with each other to form a hybrid mutation strategy. Under the action of this strategy, the diversity of individual is further improved, and finally, more accurate and mature convergence results are obtained. The mathematical model of the hybrid mutation strategy is displayed as follows:

$$g = X^*_{(t)} - X^*_{(t)bad} \tag{16}$$

TABLE 2. Descriptions of the 37 benchmark functions.

No.	Functions	$F_i^* = F_i(x^*)$	
CEC2015 Test functions (Search range: $[-100, 100]^D$)			
F1	Rotated Bent Cigar Function	100	Unimodal function
F2	Rotated Discus Function	200	
F3	Shifted and Rotated Weierstrass Function	300	Simple multimodal functions
F4	Shifted and Rotated Schwefel's Function	400	
F5	Shifted and Rotated Katsuura Function	500	
F6	Shifted and Rotated HappyCat Function	600	
F7	Shifted and Rotated HGBat Function	700	
F8	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	800	
F9	Shifted and Rotated Expanded Scaffer's F6 Function	900	
F10	Hybrid Function 1 (N=3)	1000	Hybrid funtions
F11	Hybrid Function 2 (N=4)	1100	
F12	Hybrid Function 3 (N=5)	1200	
F13	Composition Function 1 (N=5)	1300	Composition functions
F14	Composition Function 2 (N=3)	1400	
F15	Composition Function 3 (N=5)	1500	
CEC2021 Test functions (Search range: $[-100, 100]^D$)			
F16	Shifted and Rotated Bent Cigar Function	100	Unimodal function
F17	Shifted and Rotated Schwefel's Function	1100	Basic functions
F18	Shifted and Rotated Lunacek bi-Rasterigin Function		
F19	Expanded Rosenbrock's plus Griewangk's Function	1900	
F20	Hybrid Function 1 (N = 3)	1700	Hybrid functions
F21	Hybrid Function 2 (N = 4)	1600	
F22	Hybrid Function 3 (N = 5)	2100	
F23	Composition Function 1 (N = 3)	2200	Composition functions
F24	Composition Function 2 (N = 4)	2400	
F25	Composition Function 3 (N = 5)	2500	
CEC2022 Test functions (Search range: $[-100, 100]^D$)			
F26	Shifted and full Rotated Zakharov Function	300	Unimodal function
F27	Shifted and full Rotated Rosenbrock's Function	400	Basic functions
F28	Shifted and full Rotated Expanded Schaffer's f6 Function	600	
F29	Shifted and full Rotated Non-Continuous Rastrigin's Function	800	
F30	Shifted and full Rotated Levy Function	900	
F31	Hybrid Function 1 (N = 3)	1800	Hybrid functions
F32	Hybrid Function 2 (N = 6)	2000	
F33	Hybrid Function 3 (N = 5)	2200	
F34	Composition Function 1 (N = 5)	2300	Composition functions
F35	Composition Function 2 (N = 4)	2400	
F36	Composition Function 3 (N = 5)	2600	
F37	Composition Function 4 (N = 6)	2700	

$$B_i = 1 + r_3 \cdot log(\frac{f\left(X_{(t)}^*\right) - f(X_i)}{g} + 1) \tag{17}$$

$$B_i = 1 - r_3 \cdot log(\frac{f\left(X_{(t)}^*\right) - f(X_i)}{g} + 1) \tag{18}$$

$$s = tanh\left|f\,(X_i) - f\left(X_{(t)}^*\right)\right| \tag{19}$$

$$v_j = [-a_2, a_2] \tag{20}$$

$$c_1 = 2 \cdot e^{-(4 \cdot R)^2} \tag{21}$$

$$X_{(t+1),j} = X_{(t)g,j}^* + v_j \cdot (B_{i,j} \cdot (X_{randa,j} - X_{randb,j})) \tag{22}$$

TABLE 3. Various variants of WOA combined with different strategies.

	Nonlinear convergence factor	Adaptive weight	Hybrid mutation strategy
WOA	0	0	0
NWOA	1	0	0
AWWOA	0	1	0
MWOA	0	0	1
NAWMWOA	1	1	1

$$X_{(t+1),j} = X^*_{(t)g,j} + c_1 \cdot ((ub - lb) \cdot r_5 + lb) \quad (23)$$

$$X_{(t+1),j} = X^*_{(t),j} - c_1 \cdot ((ub - lb) \cdot r_5 + lb) \quad (24)$$

The pseudo-code of the NAWMWOA is displayed in Algorithm 1. A readily comprehensible NAWMWOA flowchart is indicated in Fig.1.

D. TIME COMPLEXITY ANALYSIS

Time complexity is one of the important indexes to evaluate the efficiency of algorithm. It can be seen from the pseudo-code of NAWMWOA that the time complexity of the proposed NAWMWOA mainly includes initialization, fitness evaluation, sorting mechanism, adaptive weight update, the hybrid mutation strategy, and position update.

The time complexity of NAWMWOA is determined by the size of the population (N), the dimension of individual whales (D), the number of algorithm iterations (T_{max}). The time complexity of NRMWOA is O (NAWMWOA) = O (Initialize) + O (Fitness evaluation) + T × (O (Update adaptive weight) + O (Perform hybrid mutation strategy) + O (Update the positions of whales)). The time complexity of population initialization is $O(N \times D)$. The time complexity which evaluating the fitness of the initial whale is $O(N)$. The time complexity of the update adaptive weight is $O(D)$. The time complexity which hybrid mutation strategy is $O(N \times D)$. The time complexity of executing the random reuse strategy is $O(N \times D)$. Updating the positions of whales needs a level of $O(N)$. Therefore, the total time complexity is as follows:

$$\begin{aligned} &O\,(NAWMWOA) \\ &\quad = O\,(N \times D) + O\,(N) + O\,(D) \\ &\qquad + T \times (O\,(N \times D) + O\,(N \times D) + O(N)) \end{aligned} \quad (25)$$

IV. EXPERIMENTAL RESULTS AND ANALYSES

In this section, to further demonstrate the superiority and competitiveness of the proposed NAWMWOA, an experiment based on 37 benchmark functions including CEC2015, CEC2021 and CEC2022 was designed and executed. It includes the following parts: In the first place, the effect of each improved strategy is tested separately. Secondly, five variants of WOA are compared with NAWMWOA in performance. Thirdly, five state-of-the-art algorithms are compared with NAWMWOA in performance. Fourthly, the improvement effects of the three strategies on population diversity and the balance between the two stages (exploitation and exploration) are tested.

In order to ensure the fairness of the experiment, all algorithms are executed under the same experimental conditions. Among them, the population size N is 30, the individual dimension D is 30, and the maximum iterations T_{max} is 300,000. At the same time, to reduce experimental errors as much as possible, each algorithm is executed 30 times, and the average value "Mean", standard deviation "Std.", maximum value "Max.", and minimum value "Min." of the experiment are counted.

A. BENCHMARK FUNCTION

In this study, 37 benchmark functions from CEC2015, CEC2021, and CEC2022 are utilized to test the performance of the algorithm such as convergence speed and result accuracy. These benchmark functions are mainly divided into five categories: Basic functions, Unimodal function, Simple multimodal functions, Hybrid functions, and Composition functions. The upper bounds ub and lower bounds lb of the algorithm's search for 37 benchmark functions are 100 and -100 respectively. The source code for all benchmark functions is from the GitHub home page of P-N Suganthan. A detailed description of the 37 benchmark functions is shown in Table 2. Among them, the third and fourth columns represent the theoretical optimal solution and type of corresponding benchmark function, respectively.

B. EFFECT OF THREE STRATEGIES FOR THE WOA

This subsection, variants of single strategy and NAWMWOA are analyzed to investigate the effects of each strategy and their interactions. First of all, different strategy combinations are listed in Table 3, where NWOA, AWWOA and MWOA represent the variant of single strategy that respectively combining WOA with nonlinear convergence factor, adaptive weights, hybrid mutation strategy. 1 and 0 indicate whether the corresponding strategy is embedded, respectively.

According to the experimental requirements in 4.1, the five variants independently perform 30 times experimental for 37 benchmark functions, and the experimental results are displayed in Table 4. At the same time, the "+/=/-" in Table 5 indicates that NAWMWOA is better, worse, or equal compared to other variants of single strategy, respectively. Moreover, the Friedman test [62] is utilized to quantify the

TABLE 4. Comparison results of different variants of WOA.

Functions	Method	NAWMWOA	WOA	NWOA	AWWOA	MWOA
F1	Mean	**5.3868E+03**	7.2228E+09	6.6673E+09	7.4318E+09	6.0277E+03
	Std.	**3.9232E+03**	4.4073E+09	3.9470E+09	3.7479E+09	4.1667E+03
	Min.	2.0909E+02	1.8393E+09	1.5881E+09	1.9881E+09	**1.0003E+02**
	Max.	1.0029E+04	2.2116E+10	1.8107E+10	1.6699E+10	**1.0016E+04**
F2	Mean	2.0000E+02	5.4325E+04	5.1627E+04	5.4649E+04	**2.0000E+02**
	Std.	1.9441E-03	4.1171E+03	5.7170E+03	4.7320E+03	**2.6357E-09**
	Min.	**2.0000E+02**	4.0340E+04	3.8239E+04	4.4172E+04	**2.0000E+02**
	Max.	2.0001E+02	6.0673E+04	5.8977E+04	6.1241E+04	**2.0000E+02**
F3	Mean	**3.1758E+02**	3.3600E+02	3.3513E+02	3.3562E+02	3.2021E+02
	Std.	3.7111E+00	3.6617E+00	2.8730E+00	**2.5596E+00**	4.3209E+00
	Min.	**3.0974E+02**	3.2572E+02	3.2875E+02	3.2986E+02	3.1413E+02
	Max.	**3.2523E+02**	3.4078E+02	3.3958E+02	3.4041E+02	3.3036E+02
F4	Mean	**4.5335E+02**	5.4720E+03	5.6190E+03	5.3149E+03	1.5838E+03
	Std.	**7.6705E+01**	6.8912E+02	6.5053E+02	7.6849E+02	4.4241E+02
	Min.	**4.0643E+02**	4.1877E+03	4.4843E+03	3.9223E+03	5.7585E+02
	Max.	**6.3895E+02**	6.7889E+03	7.0900E+03	6.6273E+03	2.7051E+03
F5	Mean	**5.0010E+02**	5.0136E+02	5.0139E+02	5.0137E+02	5.0026E+02
	Std.	**5.5720E-02**	3.6613E-01	4.5831E-01	5.6550E-01	1.6506E-01
	Min.	**5.0003E+02**	5.0068E+02	5.0064E+02	5.0073E+02	5.0004E+02
	Max.	**5.0023E+02**	5.0218E+02	5.0308E+02	5.0371E+02	5.0059E+02
F6	Mean	6.0025E+02	6.0253E+02	6.0260E+02	6.0227E+02	**6.0023E+02**
	Std.	6.1635E-02	1.1033E+00	9.9451E-01	1.0996E+00	**4.6195E-02**
	Min.	**6.0013E+02**	6.0056E+02	6.0051E+02	6.0055E+02	6.0015E+02
	Max.	6.0043E+02	6.0440E+02	6.0410E+02	6.0399E+02	**6.0035E+02**
F7	Mean	7.0020E+02	7.2465E+02	7.2485E+02	7.2829E+02	**7.0016E+02**
	Std.	1.1997E-01	1.2992E+01	1.1973E+01	1.3716E+01	**6.6933E-02**
	Min.	7.0011E+02	7.0638E+02	7.0022E+02	7.0023E+02	**7.0006E+02**
	Max.	7.0060E+02	7.6118E+02	7.6006E+02	7.5326E+02	**7.0043E+02**
F8	Mean	**8.0560E+02**	1.3309E+05	1.3943E+05	1.4865E+05	8.0917E+02
	Std.	**1.9933E+00**	2.1125E+05	2.7292E+05	1.8721E+05	3.6620E+00
	Min.	**8.0243E+02**	3.4039E+03	3.1927E+03	7.8298E+03	8.0414E+02
	Max.	**8.1130E+02**	8.1786E+05	1.3884E+06	6.7544E+05	8.2271E+02
F9	Mean	**9.1154E+02**	9.1281E+02	9.1269E+02	9.1279E+02	9.1175E+02
	Std.	5.7953E-01	4.2498E-01	4.2125E-01	**3.4082E-01**	5.5119E-01
	Min.	9.0964E+02	9.1153E+02	9.1177E+02	**9.1189E+02**	9.1014E+02
	Max.	9.1248E+02	9.1363E+02	9.1354E+02	**9.1339E+02**	9.1298E+02
F10	Mean	2.8943E+04	1.2718E+07	9.3542E+06	8.6775E+06	**1.9663E+04**
	Std.	1.6696E+04	1.3510E+07	6.1090E+06	6.0587E+06	**1.4117E+04**
	Min.	9.0270E+03	4.9943E+05	2.1346E+05	9.6712E+05	**3.7770E+03**
	Max.	7.9377E+04	4.4746E+07	1.9616E+07	2.6141E+07	**5.6849E+04**
F11	Mean	**1.1160E+03**	1.2005E+03	1.1979E+03	1.2114E+03	1.1222E+03
	Std.	**2.9913E+00**	5.4507E+01	4.3714E+01	3.8900E+01	1.2354E+01
	Min.	**1.1088E+03**	1.1290E+03	1.1395E+03	1.1262E+03	1.1144E+03
	Max.	**1.1226E+03**	1.3172E+03	1.2974E+03	1.2859E+03	1.1873E+03
F12	Mean	**1.4632E+03**	1.9726E+03	1.9640E+03	1.9235E+03	1.5951E+03

TABLE 4. *(Continued.)* Comparison results of different variants of WOA.

	Std.	1.5136E+02	3.0163E+02	2.5240E+02	2.5328E+02	**1.0951E+02**
	Min.	**1.2653E+03**	1.5076E+03	1.4896E+03	1.5835E+03	1.3907E+03
	Max.	1.8060E+03	2.8012E+03	2.5752E+03	2.4764E+03	**1.7918E+03**
F13	Mean	**1.5164E+03**	1.5982E+03	1.5947E+03	1.5949E+03	**1.5164E+03**
	Std.	**5.7521E-13**	2.5743E+01	2.5092E+01	2.6116E+01	**5.8118E-13**
	Min.	**1.5164E+03**	1.5453E+03	1.5496E+03	1.5472E+03	1.5164E+03
	Max.	**1.5164E+03**	1.6437E+03	1.6489E+03	1.6417E+03	**1.5164E+03**
F14	Mean	**1.6122E+03**	1.7006E+03	1.7136E+03	1.7220E+03	1.6207E+03
	Std.	**4.6828E+00**	2.4408E+01	4.0272E+01	4.9201E+01	2.1600E+01
	Min.	**1.6046E+03**	1.6346E+03	1.6450E+03	1.6190E+03	1.6070E+03
	Max.	**1.6232E+03**	1.7474E+03	1.7870E+03	1.8512E+03	1.6995E+03
F15	Mean	**2.2679E+03**	2.8026E+03	2.8329E+03	2.8716E+03	2.3410E+03
	Std.	1.2153E+02	1.9907E+02	1.1103E+02	1.1391E+02	1.1785E+02
	Min.	**2.0248E+03**	2.1781E+03	2.6319E+03	2.6884E+03	2.0599E+03
	Max.	**2.5276E+03**	3.1182E+03	3.1182E+03	3.1097E+03	2.5936E+03
F16	Mean	5.4348E+03	1.5686E+10	1.8250E+10	**2.1602E+03**	4.9239E+03
	Std.	6.1724E+03	5.3975E+09	5.6638E+09	**3.0043E+03**	6.1695E+03
	Min.	2.5312E+02	5.7546E+09	5.7521E+09	1.0233E+02	**1.0163E+02**
	Max.	2.0890E+04	2.8786E+10	**3.0153E+10**	**1.3844E+04**	2.0859E+04
F17	Mean	**3.4033E+03**	6.6790E+03	6.5330E+03	5.4465E+03	4.4957E+03
	Std.	**4.4688E+02**	5.7974E+02	6.0461E+02	6.7779E+02	5.8722E+02
	Min.	**2.3963E+03**	5.6809E+03	4.6523E+03	3.8165E+03	3.0040E+03
	Max.	**4.2795E+03**	7.8751E+03	7.4238E+03	6.6797E+03	5.4710E+03
F18	Mean	**8.1194E+02**	1.3229E+03	1.2955E+03	1.2452E+03	8.9937E+02
	Std.	**2.5568E+01**	5.8354E+01	7.1964E+01	7.5040E+01	8.0176E+01
	Min.	**7.8068E+02**	1.2003E+03	1.1215E+03	1.0332E+03	7.9862E+02
	Max.	**9.0057E+02**	1.4293E+03	1.4112E+03	1.3491E+03	1.1035E+03
F19	Mean	**1.9046E+03**	1.2990E+05	2.1656E+05	1.9801E+03	1.9056E+03
	Std.	**1.1034E+00**	1.2528E+05	2.9747E+05	2.0896E+01	1.9863E+00
	Min.	**1.9025E+03**	1.2307E+04	1.0901E+04	1.9412E+03	1.9030E+03
	Max.	**1.9070E+03**	5.9368E+05	1.4953E+06	2.0391E+03	1.9122E+03
F20	Mean	**2.6948E+03**	1.5144E+07	1.5582E+07	2.2256E+06	3.1916E+03
	Std.	**3.0323E+02**	7.8709E+06	1.2331E+07	1.2248E+06	3.6587E+02
	Min.	2.2626E+03	2.7729E+06	1.9587E+06	4.6034E+05	**2.1876E+03**
	Max.	**3.4486E+03**	3.3411E+07	4.9541E+07	5.3970E+06	3.8474E+03
F21	Mean	**2.2975E+03**	3.6509E+03	3.7596E+03	2.9757E+03	2.3497E+03
	Std.	**1.9936E+02**	5.6081E+02	7.6778E+02	3.1748E+02	3.3022E+02
	Min.	**1.9543E+03**	2.7146E+03	2.6858E+03	2.2178E+03	1.7150E+03
	Max.	**2.8462E+03**	5.0153E+03	6.3695E+03	3.4600E+03	3.2208E+03
F22	Mean	1.4491E+04	1.3986E+07	1.0081E+07	2.4629E+05	**1.0982E+04**
	Std.	**9.8249E+03**	1.6237E+07	1.1164E+07	1.5476E+05	5.6403E+03
	Min.	**2.9181E+03**	1.1235E+05	8.0787E+05	2.2853E+04	3.9461E+03
	Max.	**3.3276E+04**	6.0852E+07	5.1156E+07	6.6040E+05	**2.9321E+04**
F23	Mean	**2.3000E+03**	2.3032E+03	2.3033E+03	**2.3000E+03**	**2.3000E+03**
	Std.	1.0799E-07	1.8835E+00	1.9542E+00	**9.9896E-08**	1.0627E-07
	Min.	**2.3000E+03**	2.3015E+03	2.3016E+03	**2.3000E+03**	**2.3000E+03**

TABLE 4. *(Continued.)* Comparison results of different variants of WOA.

	Max.	**2.3000E+03**	2.3113E+03	2.3096E+03	**2.3000E+03**	**2.3000E+03**
F24	Mean	**2.5000E+03**	2.5624E+03	2.5748E+03	**2.5000E+03**	**2.5000E+03**
	Std.	6.4197E-07	3.9118E+01	3.6401E+01	**4.4205E-07**	5.0481E-07
	Min.	**2.5000E+03**	2.5138E+03	2.5169E+03	**2.5000E+03**	**2.5000E+03**
	Max.	**2.5000E+03**	2.6094E+03	2.6074E+03	**2.5000E+03**	**2.5000E+03**
F25	Mean	**3.2327E+03**	4.9979E+03	5.1155E+03	3.3106E+03	3.2691E+03
	Std.	4.8574E+01	6.3778E+02	6.2825E+02	4.6227E+01	**4.1810E+01**
	Min.	**3.1882E+03**	3.6709E+03	4.2043E+03	3.1886E+03	**3.1882E+03**
	Max.	3.3232E+03	6.9953E+03	6.8647E+03	3.4339E+03	**3.3232E+03**
F26	Mean	**3.0000E+02**	7.0579E+04	6.7786E+04	8.1626E+04	**3.0000E+02**
	Std.	**1.2595E-09**	1.0454E+04	9.9124E+03	8.1073E+03	1.3031E-09
	Min.	**3.0000E+02**	3.5716E+04	4.5316E+04	5.6883E+04	**3.0000E+02**
	Max.	**3.0000E+02**	8.6349E+04	8.3564E+04	9.4113E+04	**3.0000E+02**
F27	Mean	8.6473E+02	1.1336E+04	9.5734E+03	1.9293E+04	**8.5594E+02**
	Std.	**4.4300E+01**	6.5407E+03	6.6082E+03	1.0455E+04	5.1723E+01
	Min.	**8.1895E+02**	3.2458E+03	3.0983E+03	2.9880E+03	**8.1895E+02**
	Max.	**9.9667E+02**	2.6834E+04	2.7791E+04	4.5576E+04	**9.9667E+02**
F28	Mean	**6.0004E+02**	6.9026E+02	6.0123E+02	6.0143E+02	6.0026E+02
	Std.	**5.7507E-02**	1.0527E+01	7.9858E-01	1.0081E+00	2.9563E-01
	Min.	**6.0000E+02**	6.7064E+02	6.0019E+02	6.0011E+02	**6.0000E+02**
	Max.	**6.0022E+02**	7.2149E+02	6.0336E+02	6.0443E+02	6.0120E+02
F29	Mean	**8.7381E+02**	9.9747E+02	9.9890E+02	9.8507E+02	9.9577E+02
	Std.	**1.9688E+01**	2.9020E+01	3.6481E+01	3.0490E+01	5.2343E+01
	Min.	**8.4366E+02**	9.3911E+02	9.2879E+02	9.2031E+02	9.1685E+02
	Max.	**9.1992E+02**	1.0847E+03	1.0619E+03	1.0582E+03	1.2064E+03
F30	Mean	**1.1802E+03**	7.5371E+03	6.6871E+03	7.2270E+03	4.0035E+03
	Std.	**2.6360E+02**	1.1674E+03	1.0886E+03	1.1429E+03	2.3990E+03
	Min.	**9.0809E+02**	5.5061E+03	3.8813E+03	5.0091E+03	1.0326E+03
	Max.	**1.8766E+03**	9.7500E+03	8.5002E+03	9.3748E+03	9.2317E+03
F31	Mean	1.2286E+04	3.6332E+08	7.9651E+08	8.9609E+08	**6.2754E+03**
	Std.	8.1102E+03	3.7775E+08	1.3745E+09	1.1362E+09	**5.0325E+03**
	Min.	2.2954E+03	3.2569E+05	1.4946E+06	3.5370E+05	**2.1982E+03**
	Max.	3.1332E+04	1.4612E+09	5.5728E+09	4.2466E+09	**2.1396E+04**
F32	Mean	**2.0327E+03**	2.3185E+03	2.2216E+03	2.2606E+03	2.0519E+03
	Std.	**5.4425E+00**	8.2304E+01	5.3732E+01	8.1316E+01	1.7673E+01
	Min.	**2.0247E+03**	2.1867E+03	2.1252E+03	2.1148E+03	2.0294E+03
	Max.	**2.0421E+03**	2.4662E+03	2.3028E+03	2.5003E+03	2.1053E+03
F33	Mean	**2.2766E+03**	2.4484E+03	2.5362E+03	1.0703E+04	2.2963E+03
	Std.	**4.4559E+01**	1.5683E+02	3.5556E+02	4.2125E+04	5.3841E+01
	Min.	2.2256E+03	2.2473E+03	2.2868E+03	2.2524E+03	**2.2256E+03**
	Max.	**2.3790E+03**	2.9118E+03	4.2697E+03	2.3742E+05	2.4050E+03
F34	Mean	2.8428E+03	**2.6926E+03**	5.4118E+03	6.9973E+03	2.8471E+03
	Std.	2.1029E+02	5.1186E+01	6.1887E+02	1.4582E+03	**2.0774E+02**
	Min.	2.5425E+03	2.6011E+03	4.5060E+03	5.0946E+03	**2.5422E+03**
	Max.	**3.7892E+03**	2.8190E+03	7.1990E+03	1.1342E+04	**3.7892E+03**
F35	Mean	2.8982E+03	7.3852E+03	3.1138E+03	3.0556E+03	**2.8755E+03**

TABLE 4. *(Continued.)* Comparison results of different variants of WOA.

	Std.	5.7016E+01	1.1143E+03	9.1417E+01	**4.1968E+01**	6.3748E+01
	Min.	2.8061E+03	2.8500E+03	2.9693E+03	2.9495E+03	**2.7524E+03**
	Max.	3.0039E+03	9.1934E+03	3.3309E+03	3.1296E+03	**3.0465E+03**
F36	Mean	**3.6510E+03**	7.6416E+03	1.0329E+04	1.1301E+04	3.6902E+03
	Std.	1.6097E+02	2.2901E+03	7.2432E+02	9.1388E+02	**1.5303E+02**
	Min.	3.4214E+03	3.9973E+03	8.6454E+03	9.5077E+03	**3.3759E+03**
	Max.	**4.0601E+03**	1.3876E+04	1.1687E+04	1.3407E+04	4.1517E+03
F37	Mean	2.8002E+03	3.3694E+03	2.8003E+03	2.8010E+03	**2.8001E+03**
	Std.	1.5430E-01	1.8423E+02	1.7726E-01	7.0578E-01	**6.4964E-02**
	Min.	**2.8000E+03**	3.0888E+03	2.8000E+03	2.8003E+03	**2.8000E+03**
	Max.	2.8006E+03	3.7899E+03	2.8007E+03	2.8028E+03	**2.8002E+03**

TABLE 5. Statistic and ARV of NAWMWOA and other variants of WOA.

Method		NAWSWOA	WOA	NWOA	AWWOA	MWOA
+/=/-	Mean		36/0/1	37/0/0	34/2/1	24/3/10
	Std.		35/0/2	35/0/2	29/0/8	22/0/15
	Min.		37/0/0	37/0/0	34/2/1	18/9/10
	Max.		37/0/0	37/0/0	34/2/1	20/7/10
ARV		1.4189	4.0541	3.9730	3.7297	1.8243

average ranking values (ARV) of algorithm and the ARVs are indicated in the last row of Table 5. Smaller ARV represents superior performance. Finally, to quantitatively study the differences between variants and test whether each strategy was statistically significant, Wilcoxon signed-rank test [63] is performed for five variants and the statistical results were shown in Table 6.

Firstly, it can be seen from the ARVs that the ARV of the original WOA is the largest, indicating that each strategy improves the WOA. Secondly, the ARV of MWOA is the smallest among the three variants of single strategy, demonstrating that the improvement effect of hybrid mutation strategy is the most obvious. At the same time, from the Wilkerson test results in Table 5, it can be seen that the difference between MWOA and NAWMWOA is smaller, thus demonstrating that the hybrid mutation strategy plays a dominant role in NAWMWOA. Thirdly, the ARV of NAWMWOA is 1.4189, which is the smallest among all rankings, illustrating that the combination of the three strategies will generate more significant effects than single strategy. In addition, from Table 4 and "+/=/-" in Table 5, it can be seen that NAWMWOA is better than WOA, NWOA, AWWOA and MWOA on 36 (except F34), 37, 34 (except F16, F23, F24) and 24 (except F2, F6, F7, F10, F13, F16, F22, F23, F24, F26, F27, F31, F35, F37) of the 37 benchmark functions, respectively. In conclusion, three strategies complement each other and the most competitive NAWMWOA is proposed.

C. COMPARISON WITH OTHER VARIANTS OF WOA

In order to demonstrate the outstanding competitiveness of NAWMWOA in numerous variants of WOA, five advanced variants of WOA including the enhanced whale optimization algorithm (EWOA) [64], the improved whale optimization algorithm based on nonlinear adaptive weight and golden sine operator (NGS-WOA) [65], the enhanced Whale Optimization Algorithm integrated with salp swarm algorithm (ESSAWOA) [66], the enhanced WOA (WOAAmM) [67], and the chaotic whale optimization algorithm (CWOA) [68] are compared with NAWMWOA for 37 benchmark functions. The main parameters setting of NAWMWOA and other improved WOAs are same as literatures and displayed in Table 7. All experimental conditions were set according to the instructions in 4.1. "Mean.", "Std.", "Min.","Max." of the 30 experimental results are displayed in Table 8. At the same time, the ARVs of six algorithms are shown in Table 9 to intuitively illustrate the performance rankings.

Firstly, it can be seen from Table 9 that NAWMWOA obtained the best "Mean." for 36 benchmark functions except F37 when comparing with five variants of WOA. Secondly, 36 smallest "Min." except F37 and 35 smallest "Max." except F34, F37 are obtained by NAWMWOA for 37 benchmark functions. The above analysis demonstrates that the convergence accuracy of NAWMWOA is the most exceptional among the six algorithms. Thirdly, NAWMWOA generates 28 best "Std." except F3, F6, F7, F9, F15, F18, F25, F29, F35 when solving 37 benchmark functions. It can be

TABLE 6. P-value of Wilcoxon test between NAWMWOA and other variants of WOA.

Functions	NAWMWOA	WOA	NWOA	AWWOA	MWOA
F1	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.658330524
F2	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.517047891
F3	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.106394173
F4	N/A	1.7344E-06	1.7344E-06	1.7344E-06	1.7344E-06
F5	N/A	1.7344E-06	1.7344E-06	1.7344E-06	6.31976E-05
F6	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.165026566
F7	N/A	1.7344E-06	1.7344E-06	1.92092E-06	0.125438239
F8	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.000358884
F9	N/A	2.35342E-06	2.8786E-06	1.7344E-06	0.289477072
F10	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.031603382
F11	N/A	1.7344E-06	1.7344E-06	1.7344E-06	4.44934E-05
F12	N/A	3.51524E-06	5.75165E-06	2.8786E-06	0.00210526
F13	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.123134655
F14	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.036826128
F15	N/A	1.92092E-06	1.7344E-06	1.7344E-06	0.01319417
F16	N/A	1.7344E-06	1.7344E-06	0.014795424	0.643516595
F17	N/A	1.7344E-06	1.7344E-06	1.92092E-06	2.35342E-06
F18	N/A	1.7344E-06	1.7344E-06	1.7344E-06	3.88218E-06
F19	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.020671114
F20	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.000125057
F21	N/A	1.7344E-06	1.7344E-06	2.35342E-06	0.517047891
F22	N/A	1.7344E-06	1.7344E-06	1.92092E-06	0.158855499
F23	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.033268897
F24	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.428430029
F25	N/A	1.7344E-06	1.7344E-06	1.02463E-05	0.019569215
F26	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.002255124
F27	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.734325291
F28	N/A	1.7344E-06	1.7344E-06	1.7344E-06	4.44934E-05
F29	N/A	1.7344E-06	1.7344E-06	1.7344E-06	1.7344E-06
F30	N/A	1.7344E-06	1.7344E-06	1.7344E-06	5.21649E-06
F31	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.002584559
F32	N/A	1.7344E-06	1.7344E-06	1.7344E-06	5.21649E-06
F33	N/A	3.88218E-06	2.35342E-06	3.51524E-06	0.191522107
F34	N/A	8.91873E-05	1.7344E-06	1.7344E-06	0.599935928
F35	N/A	1.92092E-06	1.7344E-06	2.35342E-06	0.213358073
F36	N/A	1.7344E-06	1.7344E-06	1.7344E-06	0.349345562
F37	N/A	1.7344E-06	0.177907383	1.7344E-06	0.000205153

seen that the stability of NAWMWOA is the most significant among the six algorithms.

The results in Table 9 further illustrate the significant competitiveness of NAWMWOA. According to detailed statistical results of "Mean.", the NAWMWOA is superior to EWOA, NGS-WOA, ESSAWOA, WOAAmM, and CWOA on 36, 37, 37, 37, 36 out of 37 benchmark functions. In terms of "Min.", the NAWMWOA is more outstanding than the above five algorithms on 36, 37, 37, 37, 37 out of 37 benchmark functions. It can be seen from the statistics in row "Max." that the NAWMWOA is superior to 36, 37, 37, 37, 36 out of 37 benchmark functions. At the same

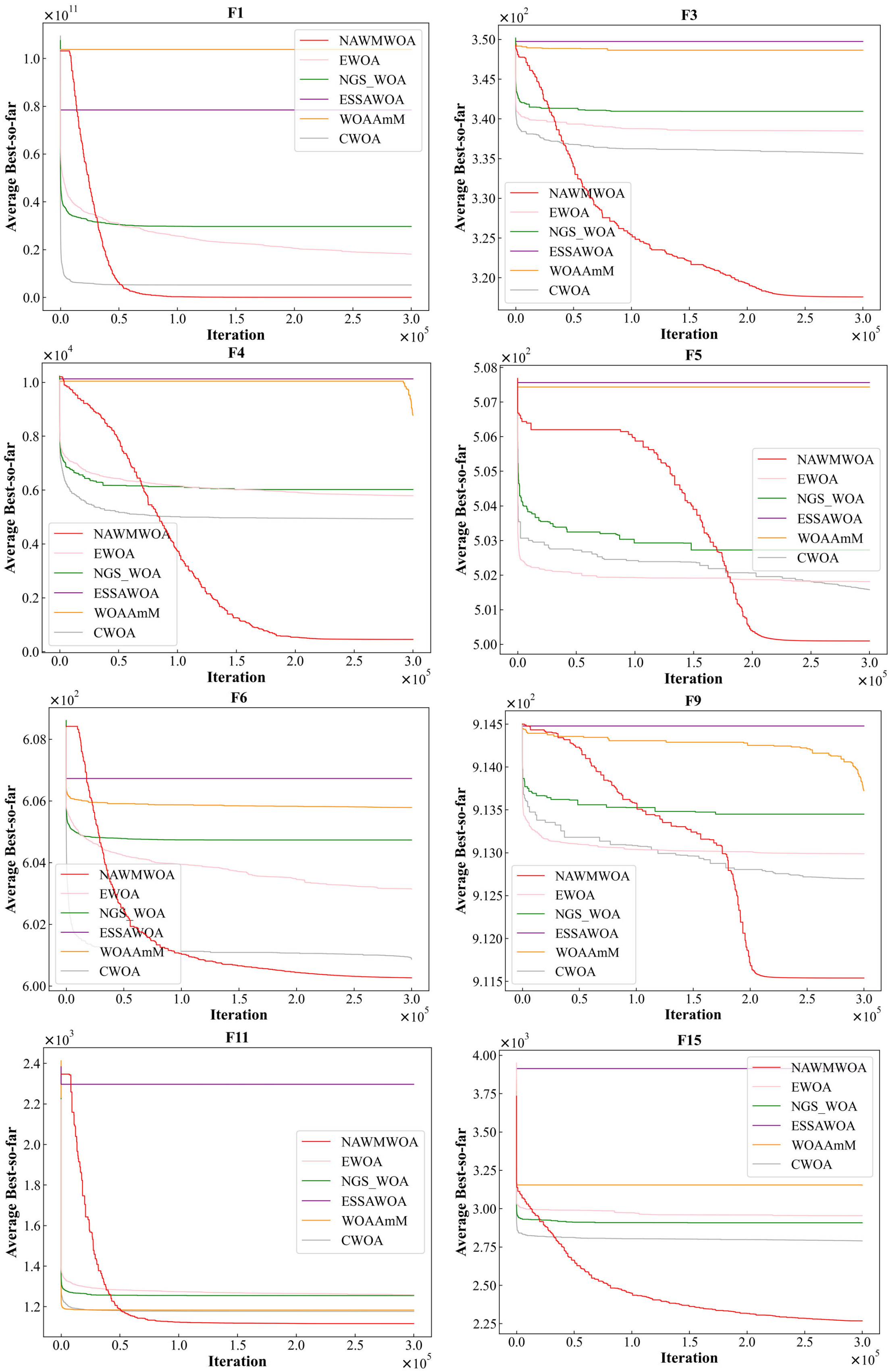

FIGURE 2. Convergence curves of NAWMWOA and other improved WOAs.

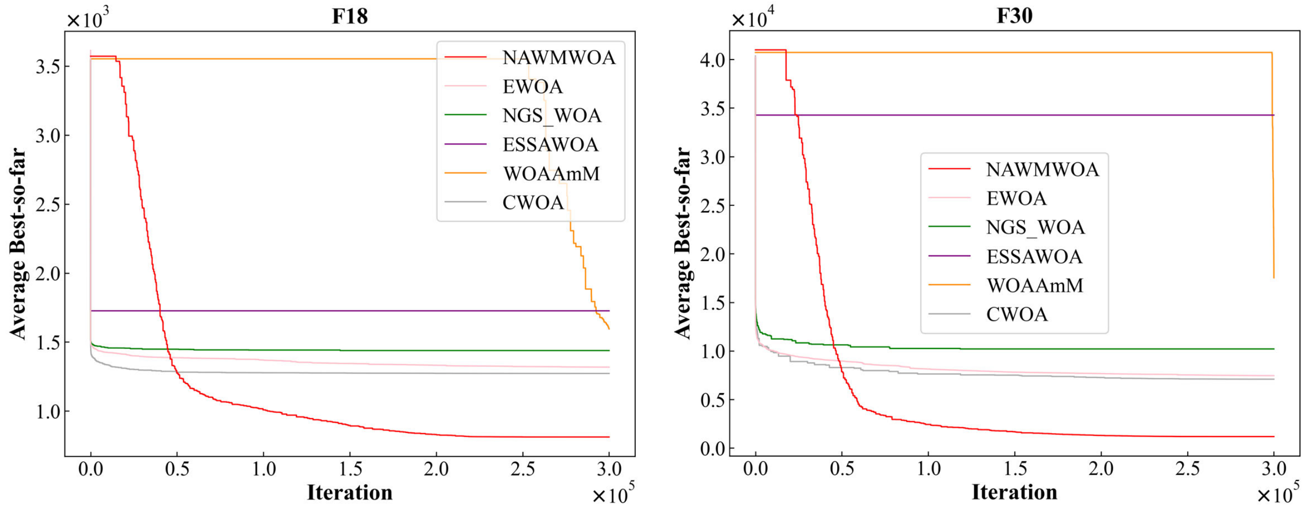

FIGURE 2. *(Continued.)* **Convergence curves of NAWMWOA and other improved WOAs.**

TABLE 7. **Main parameters of NAWMWOA and other improved WOAs.**

Algorithms	Parameters values
EWOA	$a2 = [-1, -2]$
NGS-WOA	$k = 1/2$, $r_1 = [0, 2\pi]$, $r_2 = [0, \pi]$
ESSAWOA	$C_1 = [2/e, 2]$, $k = 10000$
WOAAmM	$BF^1 = 1$ or 2, $BF^2 = 1$ or 2
CWOA	$M = 1.07$, $x_{initial} = 0.7$
NAWMWOA	$p_1 = 0.6$, $p_2 = 0.4$, $h = [0.1]$, $h \neq 0.25$

time, the NAWMWOA is better than EWOA, NGS-WOA, ESSAWOA, WOAAmM, and CWOA on 36, 37, 37, 37, 36 out of 37 benchmark functions for "Std.". Moreover, from the ARVs shown in Table 9, the ARV of NAWMWOA is 1.0541 which is significantly lower than the other five variants of WOA. The above statistics show that the NAWMWOA has a significant advantage compared with other variants of WOA.

In order to intuitively exhibit the convergence characteristics for variants of WOA, the convergence curves of 10 benchmark functions (F1, F3, F4, F5, F6, F9, F11, F15, F18, F30) are displayed in Fig. 2. Firstly, it can be seen from Fig. 2 that the convergence results of NAWMWOA are the most accurate among the 10 benchmark functions, and the advantages of NAWMWOA's results on F3, F4, F9 and F15 are particularly significant. Secondly, it can be seen from Fig. 2 that the convergence speed of NAWMWOA in the early stage is relatively fast except in F5 and F9.

As a conclusion, both experimental statistics and convergence curves demonstrate that the NAWMWOA is the best choice among all the compared variants of WOA.

D. COMPARISON WITH OTHER STATE-OF-THE-ART ALGORITHMS

In order to further demonstrate that the proposed NAWMWOA is superior to other algorithms, five state-of-the-art algorithms (nonlinear based chaotic harris hawks optimization (NCHHO) [69], grey wolf optimizer algorithm with a two-phase mutation (TMGWO) [70], dispersed foraging slime mould algorithm (DFSMA) [71], incremental grey wolf optimizer (I-GWO) [72], enhanced arithmetic optimization algorithm (EAOA) [73]) are adapted to compare with NAWMWOA, which are variants of the corresponding original algorithm and have better performance than the original algorithm. Therefore, it is a difficult challenge for NAWMWOA. The configuration of the experiment is the same as the settings in the previous section, in which the number of iterations is set to 300000, the individual dimension is set to 30, and the number of populations is set to 30. The experiment is performed 30 times with the above configuration and calculated "Mean", "Std.", "Min." and "Max.". The details of the experimental results are presented in Table 10, Table 11 and Fig.3.

The NCHHO is a variant based on HHO proposed in 2021, which performs excellently among many novel algorithms in the field of meta-heuristics. Experimental data in Table 10 indicate that the proposed NAWMWOA generates 36 better "Mean", 32 better "Std.", 36 better "Min." and 36 better "Max." for all the 37 functions as compared with NCHHO. Compared with TMGWO and I-GWO, the better performance of NAWMWOA can be verified by the obtained 37 better "Mean", 27 better "Std.", 37 better "Min." and 36 better "Max." out of the 37 functions. DFSMA is an advanced algorithm for improving SMA proposed in 2021, which has an excellent ability to solve multimodal and hybrid functions. In terms of the comparison results between the proposed NAWMWOA and DFSMA, 25 better "Mean",

TABLE 8. Comparison results of NAWMWOA with other improved WOAs.

Functions	Method	NAWMWOA	EWOA	NGS-WOA	ESSAWOA	WOAAmM	CWOA
F1	Mean	**5.3868E+03**	1.8078E+10	2.9222E+10	7.8014E+10	1.0153E+11	5.1768E+09
	Std.	**3.9232E+03**	7.6747E+09	9.0041E+09	1.4304E+09	1.5504E+10	3.9340E+09
	Min.	**2.0909E+02**	7.2345E+09	1.0811E+10	7.2313E+10	7.1619E+10	4.2768E+08
	Max.	**1.0029E+04**	4.0543E+10	4.8670E+10	7.8398E+10	1.2979E+11	1.5181E+10
F2	Mean	**2.0000E+02**	5.9023E+04	5.6811E+04	7.7122E+06	1.6198E+07	4.7945E+04
	Std.	**1.9441E-03**	3.7620E+03	4.5510E+03	1.3381E+07	2.6502E+07	7.4910E+03
	Min.	**2.0000E+02**	4.3712E+04	4.4753E+04	1.3728E+05	1.4959E+05	3.0957E+04
	Max.	**2.0001E+02**	6.1255E+04	6.2136E+04	6.5437E+07	1.2138E+08	6.1244E+04
F3	Mean	**3.1758E+02**	3.3848E+02	3.3871E+02	3.4974E+02	3.4227E+02	3.3561E+02
	Std.	3.7111E+00	2.8227E+00	3.0205E+00	**1.8922E+00**	3.6274E+00	2.6973E+00
	Min.	**3.0974E+02**	3.3251E+02	3.3178E+02	3.4578E+02	3.3286E+02	3.2897E+02
	Max.	**3.2523E+02**	3.4433E+02	3.4522E+02	3.5246E+02	3.4982E+02	3.4025E+02
F4	Mean	**4.5335E+02**	5.7881E+03	5.8734E+03	1.0126E+04	8.6936E+03	4.9298E+03
	Std.	**7.6705E+01**	8.1126E+02	7.3492E+02	4.8902E+02	4.2610E+02	7.0204E+02
	Min.	**4.0643E+02**	4.2548E+03	3.6677E+03	8.7139E+03	7.3129E+03	3.4856E+03
	Max.	**6.3895E+02**	8.2384E+03	7.0143E+03	1.0930E+04	8.8509E+03	6.8017E+03
F5	Mean	**5.0010E+02**	5.0181E+02	5.0188E+02	5.0756E+02	5.0621E+02	5.0158E+02
	Std.	**5.5720E-02**	4.3641E-01	7.0170E-01	1.0780E+00	1.2899E+00	4.9201E-01
	Min.	**5.0003E+02**	5.0103E+02	5.0111E+02	5.0489E+02	5.0343E+02	5.0077E+02
	Max.	**5.0023E+02**	5.0295E+02	5.0423E+02	5.0925E+02	5.0754E+02	5.0262E+02
F6	Mean	**6.0025E+02**	6.0314E+02	6.0470E+02	6.0672E+02	6.0578E+02	6.0086E+02
	Std.	6.1635E-02	1.1158E+00	5.9591E-01	**1.4627E-02**	6.8633E-01	6.4836E-01
	Min.	**6.0013E+02**	6.0039E+02	6.0358E+02	6.0664E+02	6.0451E+02	6.0034E+02
	Max.	**6.0043E+02**	6.0526E+02	6.0594E+02	6.0672E+02	6.0671E+02	6.0263E+02
F7	Mean	**7.0020E+02**	7.4518E+02	7.6837E+02	8.4669E+02	8.3693E+02	7.0505E+02
	Std.	1.1997E-01	1.8213E+01	1.7948E+01	**4.0519E-03**	1.6653E+01	7.3943E+00
	Min.	**7.0011E+02**	7.2206E+02	7.4181E+02	8.4668E+02	7.8067E+02	7.0022E+02
	Max.	**7.0060E+02**	7.9600E+02	8.0233E+02	8.4669E+02	8.4532E+02	7.3276E+02
F8	Mean	**8.0560E+02**	2.5287E+05	2.0307E+06	5.2997E+07	1.3494E+08	2.4313E+04
	Std.	**1.9933E+00**	2.4207E+05	1.9601E+06	4.8026E+06	1.1344E+08	5.3643E+04
	Min.	**8.0243E+02**	1.3456E+04	2.7359E+05	2.7546E+07	2.3184E+07	8.9345E+02
	Max.	**8.1130E+02**	1.0746E+06	7.4089E+06	5.4053E+07	4.1706E+08	2.6220E+05
F9	Mean	**9.1154E+02**	9.1299E+02	9.1297E+02	9.1448E+02	9.1372E+02	9.1270E+02
	Std.	5.7953E-01	5.0796E-01	3.7837E-01	**1.6519E-01**	2.3166E-01	4.6885E-01
	Min.	**9.0964E+02**	9.1192E+02	9.1213E+02	9.1409E+02	9.1302E+02	9.1168E+02
	Max.	**9.1248E+02**	9.1381E+02	9.1372E+02	9.1472E+02	9.1384E+02	9.1361E+02
F10	Mean	**2.8943E+04**	1.6654E+07	2.6138E+07	4.5831E+08	4.8069E+08	6.4696E+06
	Std.	**1.6696E+04**	1.7826E+07	2.1379E+07	2.0077E+08	2.0943E+08	4.4551E+06
	Min.	**9.0270E+03**	1.9628E+06	8.6157E+05	1.3365E+08	1.2348E+08	8.5831E+05
	Max.	**7.9377E+04**	7.6502E+07	9.1932E+07	1.0716E+09	9.6548E+08	2.0127E+07
F11	Mean	**1.1160E+03**	1.2580E+03	1.2493E+03	2.1603E+03	1.1782E+03	1.1762E+03
	Std.	**2.9913E+00**	1.0845E+02	3.3478E+01	1.8503E+02	3.9351E+01	4.0542E+01
	Min.	**1.1088E+03**	1.1354E+03	1.1917E+03	1.6648E+03	1.1265E+03	1.1269E+03
	Max.	**1.1226E+03**	1.5333E+03	1.3212E+03	2.2953E+03	1.2742E+03	1.2586E+03
F12	Mean	**1.4632E+03**	2.0772E+03	2.1096E+03	6.4953E+05	2.6818E+05	1.8561E+03

TABLE 8. *(Continued.)* **Comparison results of NAWMWOA with other improved WOAs.**

	Std.	**1.5136E+02**	2.9238E+02	2.5355E+02	7.4224E+05	3.7611E+05	1.9228E+02
	Min.	**1.2653E+03**	1.4482E+03	1.6102E+03	1.3714E+04	3.6802E+03	1.5323E+03
	Max.	**1.8060E+03**	2.9826E+03	2.6650E+03	2.2006E+06	1.5298E+06	2.3170E+03
F13	Mean	**1.5164E+03**	1.6166E+03	1.6078E+03	1.6650E+03	1.6638E+03	1.5859E+03
	Std.	**5.7521E-13**	1.6472E+01	2.5129E+01	2.1497E+01	1.7431E+01	2.9209E+01
	Min.	**1.5164E+03**	1.5695E+03	1.5595E+03	1.6255E+03	1.6408E+03	1.5367E+03
	Max.	**1.5164E+03**	1.6448E+03	1.6420E+03	1.7149E+03	1.7092E+03	1.6395E+03
F14	Mean	**1.6122E+03**	1.7478E+03	1.7596E+03	2.2669E+03	2.1261E+03	1.6903E+03
	Std.	**4.6828E+00**	6.2266E+01	4.1715E+01	1.0972E+02	1.3996E+02	3.9297E+01
	Min.	**1.6046E+03**	1.6536E+03	1.6846E+03	2.0010E+03	1.8927E+03	1.6359E+03
	Max.	**1.6232E+03**	1.8888E+03	1.8387E+03	2.3614E+03	2.2859E+03	1.7719E+03
F15	Mean	**2.2679E+03**	2.9539E+03	2.8861E+03	3.9129E+03	3.1384E+03	2.7891E+03
	Std.	1.2153E+02	1.7875E+02	9.4271E+01	5.2513E+02	**3.8907E+01**	1.3205E+02
	Min.	**2.0248E+03**	2.3725E+03	2.6772E+03	3.2430E+03	2.9696E+03	2.6049E+03
	Max.	**2.5276E+03**	3.1513E+03	3.0590E+03	5.5383E+03	3.1513E+03	3.1182E+03
F16	Mean	**5.4348E+03**	1.6708E+10	3.3845E+10	8.4721E+10	1.2088E+11	5.5428E+09
	Std.	**6.1724E+03**	7.9612E+09	9.0265E+09	2.2016E+06	1.6764E+10	3.9158E+09
	Min.	**2.5312E+02**	5.1071E+09	1.8478E+10	8.4717E+10	8.9684E+10	1.6152E+09
	Max.	**2.0890E+04**	3.7569E+10	5.0892E+10	8.4726E+10	1.6265E+11	2.2269E+10
F17	Mean	**3.4033E+03**	6.9869E+03	7.4593E+03	1.1089E+04	1.0026E+04	6.8347E+03
	Std.	**4.4688E+02**	8.8741E+02	5.3137E+02	4.6558E+02	9.7259E+02	8.4959E+02
	Min.	**2.3963E+03**	4.5902E+03	6.1541E+03	1.0197E+04	8.2513E+03	5.1333E+03
	Max.	**4.2795E+03**	8.3727E+03	8.6952E+03	1.1957E+04	1.0982E+04	8.7741E+03
F18	Mean	**8.1194E+02**	1.3180E+03	1.4081E+03	1.7248E+03	1.5946E+03	1.2724E+03
	Std.	2.5568E+01	7.5898E+01	4.8608E+01	1.7556E-01	**5.6972E-07**	6.8593E+01
	Min.	**7.8068E+02**	1.1684E+03	1.2891E+03	1.7242E+03	1.5946E+03	1.0807E+03
	Max.	**9.0057E+02**	1.4673E+03	1.4972E+03	1.7250E+03	1.5946E+03	1.4191E+03
F19	Mean	**1.9046E+03**	2.6562E+05	5.9329E+05	4.2810E+07	7.4851E+07	1.4864E+04
	Std.	**1.1034E+00**	4.2048E+05	7.4387E+05	5.9525E+06	5.5141E+07	1.0991E+04
	Min.	**1.9025E+03**	1.6125E+04	2.9117E+04	2.1468E+07	1.3591E+07	3.6370E+03
	Max.	**1.9070E+03**	1.8505E+06	4.0880E+06	4.5587E+07	2.6057E+08	4.3351E+04
F20	Mean	**2.6948E+03**	4.3902E+07	2.3874E+07	9.5742E+08	1.0656E+09	9.4382E+06
	Std.	**3.0323E+02**	3.2325E+07	1.4931E+07	4.6134E+08	6.2351E+08	5.3930E+06
	Min.	**2.2626E+03**	2.0796E+06	7.0661E+06	1.9211E+08	1.8609E+08	1.2644E+06
	Max.	**3.4486E+03**	1.4972E+08	8.2690E+07	1.8151E+09	2.7649E+09	2.3772E+07
F21	Mean	**2.2975E+03**	3.5681E+03	4.6587E+03	9.5107E+03	3.1215E+03	3.5532E+03
	Std.	**1.9936E+02**	5.3540E+02	9.6474E+02	2.0584E+03	3.7151E+02	4.0939E+02
	Min.	**1.9543E+03**	2.7288E+03	3.1710E+03	5.3046E+03	2.4166E+03	2.8305E+03
	Max.	**2.8462E+03**	4.8721E+03	7.3654E+03	1.3049E+04	3.9756E+03	4.5439E+03
F22	Mean	**1.4491E+04**	7.8713E+07	4.1458E+07	9.3466E+08	1.6298E+07	2.1570E+07
	Std.	**9.8249E+03**	1.3622E+08	6.5056E+07	4.3636E+08	2.3204E+07	3.2494E+07
	Min.	**2.9181E+03**	1.2849E+06	1.2744E+06	2.3712E+08	2.7210E+05	5.6391E+05
	Max.	**3.3276E+04**	6.5347E+08	3.5152E+08	2.0445E+09	8.7099E+07	1.6616E+08
F23	Mean	**2.3000E+03**	2.3069E+03	2.3167E+03	2.3727E+03	2.3021E+03	2.3027E+03
	Std.	**1.0799E-07**	4.7245E+00	9.4477E+00	1.1416E+01	4.8729E-01	9.5346E-01
	Min.	**2.3000E+03**	2.3023E+03	2.3040E+03	2.3464E+03	2.3016E+03	2.3015E+03

TABLE 8. *(Continued.)* **Comparison results of NAWMWOA with other improved WOAs.**

	Max.	**2.3000E+03**	2.3211E+03	2.3458E+03	2.3855E+03	2.3038E+03	2.3049E+03
F24	Mean	**2.5000E+03**	2.5810E+03	2.5945E+03	2.6457E+03	2.5373E+03	2.5799E+03
	Std.	**6.4197E-07**	2.9338E+01	1.8873E+01	8.0230E+00	4.0445E+01	3.8391E+01
	Min.	**2.5000E+03**	2.5224E+03	2.5557E+03	2.6341E+03	2.5054E+03	2.5089E+03
	Max.	**2.5000E+03**	2.6168E+03	2.6188E+03	2.6617E+03	2.6069E+03	2.6113E+03
F25	Mean	**3.2327E+03**	6.0144E+03	7.2298E+03	1.3791E+04	4.1554E+03	4.1932E+03
	Std.	4.8574E+01	7.9630E+02	9.6476E+02	**9.5320E-01**	2.6020E+02	2.3828E+02
	Min.	**3.1882E+03**	4.5586E+03	5.3289E+03	1.3787E+04	3.7936E+03	3.8199E+03
	Max.	**3.3232E+03**	7.3338E+03	9.3136E+03	1.3792E+04	4.8572E+03	4.6288E+03
F26	Mean	**3.0000E+02**	8.3394E+04	8.1796E+04	3.1660E+08	5.4458E+09	5.4295E+04
	Std.	**1.2595E-09**	7.6529E+03	7.8188E+03	4.0441E+08	1.8661E+10	9.6507E+03
	Min.	**3.0000E+02**	5.4993E+04	6.4930E+04	2.1924E+05	2.4924E+05	2.8392E+04
	Max.	**3.0000E+02**	9.4229E+04	9.4229E+04	9.2557E+08	9.8826E+10	7.2381E+04
F27	Mean	**8.6473E+02**	1.7382E+04	3.4542E+04	1.0874E+05	8.7314E+04	5.6606E+03
	Std.	**4.4300E+01**	1.0870E+04	1.1872E+04	9.4656E+03	6.5881E+03	2.3207E+03
	Min.	**8.1895E+02**	5.1150E+03	1.2910E+04	7.7066E+04	5.3690E+04	2.5943E+03
	Max.	**9.9667E+02**	5.4336E+04	5.9294E+04	1.1320E+05	8.9122E+04	1.1304E+04
F28	Mean	**6.0004E+02**	6.0194E+02	6.0219E+02	6.1390E+02	6.0471E+02	6.8523E+02
	Std.	**5.7507E-02**	1.0176E+00	9.8597E-01	3.3375E+00	2.6078E+00	1.0835E+01
	Min.	**6.0000E+02**	6.0048E+02	6.0031E+02	6.0592E+02	6.0082E+02	6.5590E+02
	Max.	**6.0022E+02**	6.0410E+02	6.0428E+02	6.2059E+02	6.0828E+02	7.0335E+02
F29	Mean	**8.7381E+02**	9.6508E+02	1.0071E+03	1.3187E+03	1.0809E+03	9.8491E+02
	Std.	1.9688E+01	3.0497E+01	3.2520E+01	**3.1420E-03**	3.8918E+01	3.3217E+01
	Min.	**8.4366E+02**	9.1823E+02	9.6218E+02	1.3187E+03	9.7174E+02	9.2220E+02
	Max.	**9.1992E+02**	1.0430E+03	1.0748E+03	1.3187E+03	1.1322E+03	1.0556E+03
F30	Mean	**1.1802E+03**	7.4494E+03	8.8561E+03	3.3346E+04	1.7408E+04	7.0906E+03
	Std.	**2.6360E+02**	1.3044E+03	1.1625E+03	2.4528E+03	4.9105E+02	9.7621E+02
	Min.	**9.0809E+02**	5.0149E+03	7.2682E+03	2.1968E+04	1.4764E+04	4.2831E+03
	Max.	**1.8766E+03**	9.8193E+03	1.1602E+04	3.4145E+04	1.7499E+04	8.8019E+03
F31	Mean	**1.2286E+04**	8.3971E+08	1.6423E+09	2.4570E+10	2.1977E+10	3.6395E+08
	Std.	**8.1102E+03**	1.5321E+09	1.4146E+09	4.2470E+09	5.1287E+09	8.4899E+08
	Min.	**2.2954E+03**	1.6297E+06	1.7031E+07	1.2035E+10	9.1656E+09	1.1197E+06
	Max.	**3.1332E+04**	7.1410E+09	6.2598E+09	2.8273E+10	3.3636E+10	3.6161E+09
F32	Mean	**2.0327E+03**	2.2336E+03	2.2643E+03	2.8405E+03	2.4287E+03	2.3173E+03
	Std.	**5.4425E+00**	8.3008E+01	4.9448E+01	1.4245E+02	2.0399E+02	8.0037E+01
	Min.	**2.0247E+03**	2.0856E+03	2.1215E+03	2.5455E+03	2.1954E+03	2.1323E+03
	Max.	**2.0421E+03**	2.3602E+03	2.3703E+03	3.1188E+03	2.8166E+03	2.4588E+03
F33	Mean	**2.2766E+03**	2.7007E+03	2.8243E+03	2.8121E+05	2.6923E+05	2.4741E+03
	Std.	**4.4559E+01**	4.7823E+02	7.7155E+02	3.4899E+05	4.5405E+05	3.2480E+02
	Min.	**2.2256E+03**	2.2587E+03	2.2625E+03	3.1990E+03	1.0803E+04	2.2546E+03
	Max.	**2.3790E+03**	4.8371E+03	6.6505E+03	1.5242E+06	2.2006E+06	3.9297E+03
F34	Mean	2.8428E+03	6.6413E+03	7.0789E+03	3.4382E+04	1.3419E+04	**2.6188E+03**
	Std.	2.1029E+02	2.1841E+03	9.3931E+02	9.8539E+03	**1.1796E-04**	3.2204E+01
	Min.	**2.5425E+03**	4.5502E+03	5.7177E+03	1.8719E+04	1.3419E+04	2.5745E+03
	Max.	3.7892E+03	1.3419E+04	9.4258E+03	4.8353E+04	1.3419E+04	**2.7282E+03**
F35	Mean	**2.8982E+03**	3.0888E+03	3.0669E+03	5.5281E+03	3.1548E+03	7.5573E+03

TABLE 8. *(Continued.)* **Comparison results of NAWMWOA with other improved WOAs.**

	Std.	5.7016E+01	4.3435E+01	3.8083E+01	1.8010E+03	**1.5916E+01**	8.3804E+02
	Min.	**2.8061E+03**	2.9756E+03	2.9914E+03	3.5326E+03	3.1287E+03	6.0644E+03
	Max.	**3.0039E+03**	3.1920E+03	3.1341E+03	1.4395E+04	3.1968E+03	9.3194E+03
F36	Mean	**3.6510E+03**	1.0663E+04	1.1332E+04	2.5691E+04	8.6199E+03	5.7411E+03
	Std.	**1.6097E+02**	1.3509E+03	6.4217E+02	5.6460E+03	1.2891E+03	1.7660E+03
	Min.	**3.4214E+03**	7.6216E+03	9.9983E+03	1.7901E+04	5.4207E+03	3.9837E+03
	Max.	**4.0601E+03**	1.2938E+04	1.2627E+04	4.0619E+04	1.0056E+04	1.0559E+04
F37	Mean	2.8002E+03	**2.8000E+03**	2.8147E+03	1.0227E+04	2.8016E+03	3.3304E+03
	Std.	1.5430E-01	**2.0863E-04**	7.8124E+00	1.5264E+03	3.4222E+00	1.5106E+02
	Min.	2.8000E+03	**2.8000E+03**	2.8017E+03	5.3954E+03	2.8001E+03	3.1249E+03
	Max.	2.8006E+03	**2.8000E+03**	2.8301E+03	1.3223E+04	2.8192E+03	3.7372E+03

TABLE 9. **Statistical results of NAWMWOA and other improved WOAs.**

Method		NAWSWOA	EWOA	NGS-WOA	ESSAWOA	WOAAmM	CWOA
+/=/-	Mean		36/0/1	37/0/0	37/0/0	37/0/0	36/0/1
	Std.		33/0/4	33/0/4	30/0/7	31/0/5	34/0/3
	Min.		36/0/1	37/0/0	37/0/0	37/0/0	37/0/0
	Max.		36/0/1	37/0/0	37/0/0	37/0/0	36/0/1
ARV		**1.0541**	3.2703	3.8649	5.7297	4.5946	2.4865

24 better "Std.", 25 better "Min." and 23 better "Max." out of the 37 functions are obtained by NAWMWOA. Moreover, the EAOA is a famous state-of-the-art algorithm. Compared with the EAOA, the proposed NAWMWOA products 37 better "Mean", 34 better "Std.", 37 better "Min." and 37 better "Max." out of the 37 functions. The results displayed in Table 10 verify the excellent performance of NAWMWOA.

The outstanding performance of NAWMWOA is further demonstrated by the statistics in Table 11. According to detailed statistical results of "Mean.", the NAWMWOA is superior to NCHHO, TMGWO, DFSMA, I-GWO, and EAOA on 36, 37, 24, 37, 37 out of 37 benchmark functions. It can be seen from the statistics in row "Min." that the NAWMWOA is superior to 36, 37, 25, 37, 37 out of 37 benchmark functions. In terms of "Max.", the NAWMWOA is more outstanding than the above five algorithms on 36, 36, 23, 37, 37 out of 37 benchmark functions. At the same time, the NAWMWOA is better than NCHHO, TMGWO, DFSMA, I-GWO, and EAOA on 31, 27, 24, 35, 34 out of 37 benchmark functions for "Std.". Moreover, the ARV of NAWMWOA shown in Table 11 is 1.3243, which is the smallest ARV among all state-of-the-art algorithms. The above statistics demonstrate that the NAWMWOA has a significant advantage compared with other state-of-the-art algorithms.

The convergence characteristics of six algorithms for 10 benchmark functions (F4, F5, F6, F8, F17, F24, F29, F30, F32, F36) are visually displayed in the form of curves in Fig. 3. Firstly, it can be seen from Fig. 3 that the convergence results of NAWMWOA are the smallest among the six state-of-the-art algorithms and its advantages are particularly significant on F4, F9 and F30. Secondly, in all convergence curves except F8, the NAWMWOA is distinctly faster than the other five competitive algorithms in terms of the convergence speed. The mature convergence results and faster convergence speed of NAWMWOA are mainly due to three improvement strategies. The nonlinear convergence factor has the function of improving the balance between the two phases (exploration phase and development phase), thereby improving the convergence speed of the algorithm. The adaptive weight and the hybrid mutation strategy can improve population diversity and avoid falling into local optimum, Therefore, the accuracy of the algorithm convergence results is improved.

In summary, based on the experimental results and convergence curves, the NAWMWOA is the best choice compared with other five state-of-the-art algorithms.

E. ANALYSIS OF BALANCE AND DIVERSITY

To further examine the improvement effect of the proposed strategies for NAWMWOA, the balance and population diversity of two algorithms are experimented with out of 4 categories of benchmark functions including Unimodal functions (F26), Basic functions (F27, F28), Hybrid functions (F31, F32), and Composition functions (F35, F36, F37). The experimental results are visually displayed in Fig. 4. The (a), (b) and (c) in Fig. 4 represent the balance analysis of WOA, the balance analysis of NAWMWOA and the diversity comparison of the two algorithms, respectively. In (a), (b),

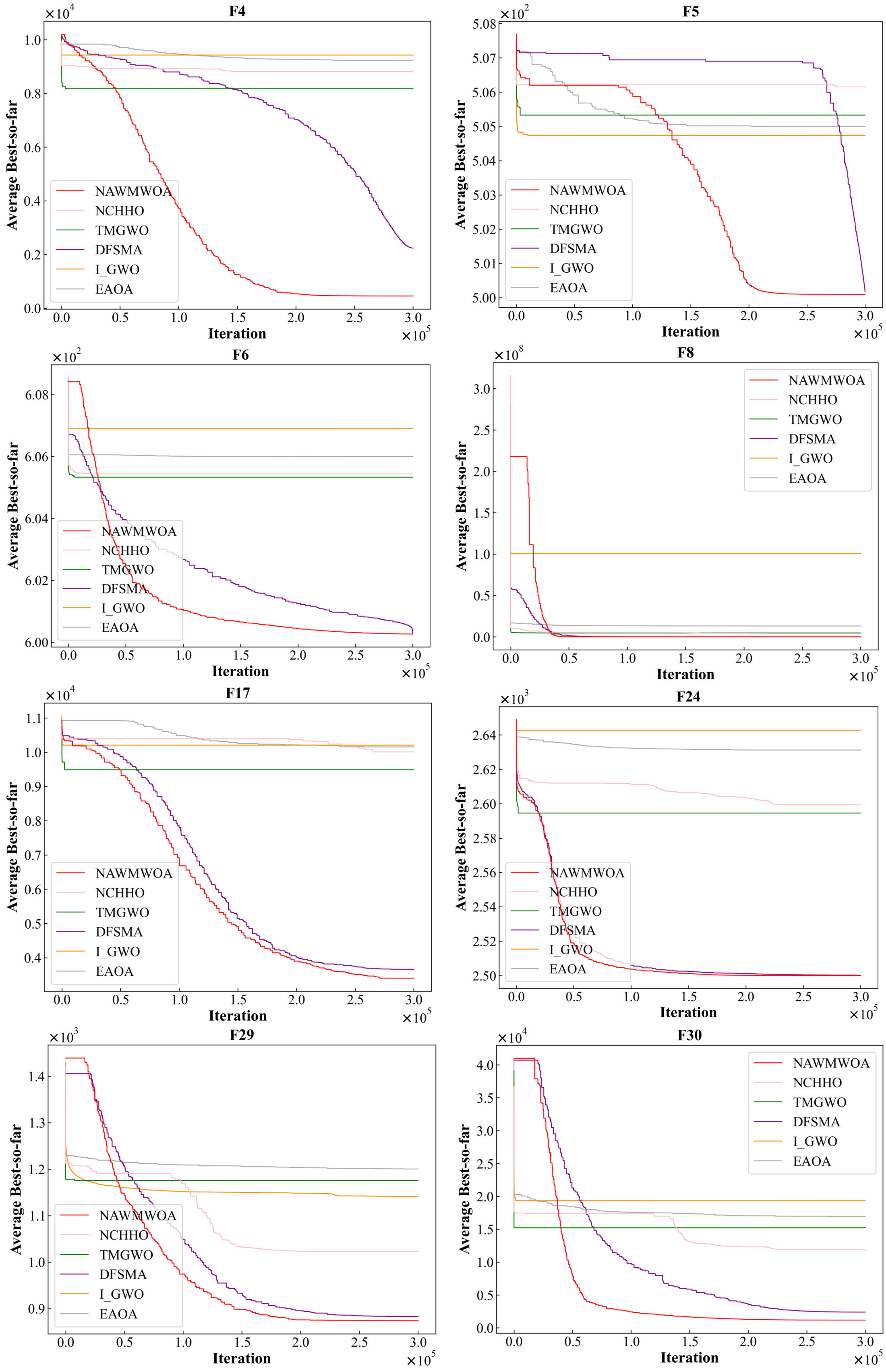

FIGURE 3. Convergence curves of NAWMWOA and other variants of WOA.

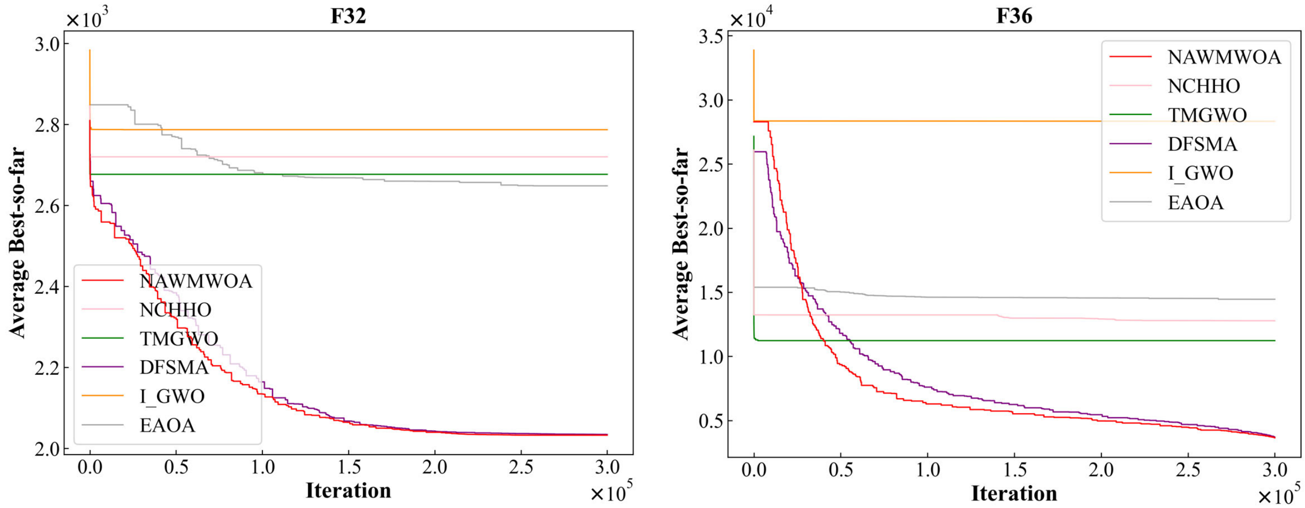

FIGURE 3. ***(Continued.)*** **Convergence curves of NAWMWOA and other variants of WOA.**

the exploration curve and the exploitation curve are used to measure the balance of the algorithm.

First of all, it can be seen from (a) and (b) that both WOA and NAWMWOA have more exploration than exploitation in the early stage of iteration. NAWMWOA has improved its exploration ability on F26 and F28. However, the proportion of development is large in NAWMWOA out of Hybrid functions (F31, F32) and Composition functions (F35, F36, F37).

As can be seen from Fig.4 (c), the population diversity of both WOA and NAWMWOA showed a declining trend. However, compared with WOA, the population diversity of NAWMWOA is higher in the early iteration, which is conducive to improving the accuracy of the solution. In addition, the population diversity of NAWMWOA declines faster than that of WOA, indicating that NAWMWOA has a faster convergence rate than WOA, which further indicates that NAWMWOA considers both convergence rate and convergence accuracy.

V. NAWMWOA FOR SOLVING ENGINEERING BENCHMARKS

In this section, three classical engineering benchmarks are adopted to test the feasibility of the proposed NAWMWOA in engineering problems, which include three-bar truss design [74] tension-compression spring design (TCSD) [75], and welded beam design (WBD) [76]. Penalty functions for solving constraint problems mainly include static, dynamic, annealing, adaptive, co-evolutionary, and death penalty [77]. Although the death penalty abandons infeasible solutions, it has the characteristics of a small amount of calculation and simple method. Therefore, under the premise of ensuring the experimental requirements, the death penalty is used to deal with the three constraint problems in this study.

A. THREE-BAR TRUSS DESIGN PROBLEM

In this engineering problem, the design objective is to achieve the minimum weight of the three-bar truss design based on three constraints. The design parameters include the length of the rods on both sides ($A_1 = A_3$) and the length of the middle rod (A_2). The three-dimensional diagram of the three-bar truss design is shown in Fig. 5. The constraints and formula are explained as follows:

$$
\begin{aligned}
&\text{Consider}: \\
&\text{Minimize}: \vec{x} = [x_1 x_2] = [A_1 A_2] \\
&\text{Subject to}: \ f(x_1, x_2) = (2\sqrt{2x_1} + x_2) \times l \\
&\qquad\qquad g_1 = \frac{\sqrt{2}\,x_1 + x_2}{\sqrt{2}\,x_1^2 + 2x_1x_2}P - \sigma \leq 0 \\
&\qquad\qquad g_2 = \frac{x_2}{\sqrt{2}\,x_1^2 + 2x_1x_2}P - \sigma \leq 0 \\
&\qquad\qquad g_3 = \frac{1}{x_1 + \sqrt{2}\,x_2}P - \sigma \leq 0 \\
&\qquad\qquad 0 \leq A_1 \leq 1 \ and \ 0 \leq A_2 \leq 1 \\
&\qquad\qquad l = 100cm, P = 2KN/cm^2, \ and \ \sigma = 2KN/cm^2
\end{aligned}
$$

In order to ensure the accuracy of the experiment, the results of all the compared algorithms are based on the death penalty function. Then, the proposed NAWMWOA and seven advanced meta-heuristics including African vultures optimization algorithm (AVOA) [78], arithmetic optimization algorithm (AOA) [79], SSA, moth flame optimization (MFO) [80]. wild horse optimizer (WHO) [81], HHO, and grasshopper optimization algorithm (GOA) [82] are utilized to experiment with three-bar truss design problem. The results obtained by NAWMWOA and other algorithms are provided in Table 12. From Table 12, it can be observed that the optimal solution obtained by NAWMWOA is 263.895833, which is superior to generated results of all the other approaches.

TABLE 10. *(Continued.)* **Comparison results of NAWMWOA and other improved algorithms.**

Functions	Method	NAWMWOA	NCHHO	TMGWO	DFSMA	I-GWO	EAOA
F1	Mean	**5.3868E+03**	2.8224E+10	2.0413E+10	1.8084E+04	6.8438E+10	6.2620E+10
	Std.	**3.9232E+03**	7.8315E+09	1.4112E+09	2.2279E+04	6.7726E+09	5.3547E+09
	Min.	2.0909E+02	1.3610E+10	1.7741E+10	**2.0287E+02**	5.4098E+10	5.1786E+10
	Max.	**1.0029E+04**	4.3556E+10	2.3308E+10	9.6563E+04	8.2269E+10	7.2272E+10
F2	Mean	**2.0000E+02**	5.6737E+04	5.1811E+04	2.0335E+02	1.7562E+05	1.8892E+05
	Std.	**1.9441E-03**	2.6561E+03	1.1366E+03	1.6984E+00	2.8664E+04	2.3071E+05
	Min.	**2.0000E+02**	5.0791E+04	4.8356E+04	2.0090E+02	1.3561E+05	5.8667E+04
	Max.	**2.0001E+02**	5.9907E+04	5.3977E+04	2.0829E+02	2.4603E+05	1.1677E+06
F3	Mean	3.1758E+02	3.3874E+02	3.3487E+02	**3.1409E+02**	3.4634E+02	3.4465E+02
	Std.	3.7111E+00	1.8332E+00	**8.8290E-01**	2.8609E+00	2.6334E+00	2.3598E+00
	Min.	3.0974E+02	3.3361E+02	3.3166E+02	**3.0934E+02**	3.4111E+02	3.3849E+02
	Max.	3.2523E+02	3.4203E+02	3.3669E+02	**3.2226E+02**	3.5123E+02	3.4875E+02
F4	Mean	**4.5335E+02**	7.7605E+03	6.1355E+03	2.2371E+03	9.4303E+03	9.2185E+03
	Std.	**7.6705E+01**	5.4918E+02	1.6655E+02	5.6391E+02	5.7772E+02	5.2848E+02
	Min.	**4.0643E+02**	6.4625E+03	5.7429E+03	1.3566E+03	8.2599E+03	8.2511E+03
	Max.	**6.3895E+02**	8.6267E+03	6.4035E+03	3.4965E+03	1.0447E+04	1.0257E+04
F5	Mean	**5.0010E+02**	5.0366E+02	5.0170E+02	5.0018E+02	5.0473E+02	5.0500E+02
	Std.	**5.5720E-02**	9.3588E-01	1.2577E-01	9.9631E-02	7.7157E-01	9.2104E-01
	Min.	**5.0003E+02**	5.0204E+02	5.0127E+02	5.0004E+02	5.0258E+02	5.0248E+02
	Max.	**5.0023E+02**	5.0572E+02	5.0187E+02	5.0040E+02	5.0632E+02	5.0662E+02
F6	Mean	**6.0025E+02**	6.0506E+02	6.0487E+02	6.0026E+02	6.0690E+02	6.0600E+02
	Std.	6.1635E-02	4.7640E-01	7.1124E-02	**5.1982E-02**	6.6611E-01	1.8077E-01
	Min.	**6.0013E+02**	6.0393E+02	6.0472E+02	6.0018E+02	6.0561E+02	6.0558E+02
	Max.	**6.0043E+02**	6.0590E+02	6.0496E+02	6.0038E+02	6.0896E+02	6.0649E+02
F7	Mean	**7.0020E+02**	7.7313E+02	7.6819E+02	7.0029E+02	8.6796E+02	8.1813E+02
	Std.	**1.1997E-01**	1.0258E+01	7.2739E+00	1.9763E-01	3.9568E+01	1.0480E+01
	Min.	**7.0011E+02**	7.4522E+02	7.5782E+02	7.0012E+02	7.8897E+02	7.9414E+02
	Max.	**7.0060E+02**	7.8925E+02	7.9485E+02	7.0070E+02	9.5643E+02	8.3376E+02
F8	Mean	8.0560E+02	1.6174E+06	8.2806E+05	**8.0520E+02**	1.0063E+08	1.3053E+07
	Std.	1.9933E+00	2.4442E+06	5.1470E+05	**1.5702E+00**	1.3762E+08	5.4467E+06
	Min.	**8.0243E+02**	5.9820E+04	2.3559E+05	8.0352E+02	1.3220E+07	3.1282E+06
	Max.	**8.1130E+02**	1.0968E+07	1.5967E+06	8.1193E+02	7.0412E+08	2.3527E+07
F9	Mean	9.1154E+02	9.1362E+02	9.1305E+02	**9.1096E+02**	9.1407E+02	9.1399E+02
	Std.	5.7953E-01	**9.9102E-02**	2.2045E-01	7.8282E-01	2.1433E-01	1.7556E-01
	Min.	9.0964E+02	9.1337E+02	9.1263E+02	**9.0870E+02**	9.1352E+02	9.1355E+02
	Max.	9.1248E+02	9.1373E+02	9.1342E+02	**9.1234E+02**	9.1446E+02	9.1431E+02
F10	Mean	**2.8943E+04**	2.2976E+07	1.0496E+07	7.3883E+04	3.1955E+08	1.5395E+08
	Std.	**1.6696E+04**	1.4145E+07	3.2219E+06	4.4986E+04	1.8761E+08	5.9460E+07
	Min.	**9.0270E+03**	3.8207E+06	3.1820E+06	1.4144E+04	1.0922E+08	5.0838E+07
	Max.	**7.9377E+04**	6.9732E+07	1.5549E+07	2.0230E+05	7.8651E+08	3.3007E+08
F11	Mean	**1.1160E+03**	1.2488E+03	1.1824E+03	1.1158E+03	1.6677E+03	1.4606E+03
	Std.	**2.9913E+00**	3.2250E+01	9.4665E+00	3.3238E+00	2.2204E+02	8.8403E+01
	Min.	1.1088E+03	1.1798E+03	1.1539E+03	**1.1079E+03**	1.3100E+03	1.3391E+03
	Max.	1.1226E+03	1.3141E+03	1.1958E+03	**1.1208E+03**	2.1464E+03	1.6333E+03
F12	Mean	1.4632E+03	2.1240E+03	1.6243E+03	**1.3863E+03**	2.8310E+05	3.9173E+04

TABLE 10. *(Continued.)* Comparison results of NAWMWOA and other improved algorithms.

	Std.	1.5136E+02	1.7366E+02	**7.4763E+01**	9.9833E+01	6.0705E+05	3.8047E+04
	Min.	1.2653E+03	1.7075E+03	1.4710E+03	**1.2390E+03**	3.6926E+03	4.6290E+03
	Max.	1.8060E+03	2.4633E+03	1.7321E+03	**1.6190E+03**	3.0853E+06	1.6132E+05
F13	Mean	**1.5164E+03**	1.6212E+03	1.6196E+03	**1.5164E+03**	1.6521E+03	1.6490E+03
	Std.	**5.7521E-13**	1.0116E+01	2.8056E+00	7.5650E-07	1.9004E+01	1.9066E+01
	Min.	**1.5164E+03**	1.5966E+03	1.6101E+03	1.5164E+03	1.6235E+03	1.5959E+03
	Max.	**1.5164E+03**	1.6439E+03	1.6241E+03	1.5164E+03	1.7135E+03	1.7006E+03
F14	Mean	**1.6122E+03**	1.7385E+03	1.6800E+03	**1.6093E+03**	2.1040E+03	1.9235E+03
	Std.	4.6828E+00	4.1312E+01	4.2027E+00	**2.4032E+00**	1.3928E+02	7.5809E+01
	Min.	**1.6046E+03**	1.6804E+03	1.6723E+03	1.6052E+03	1.8498E+03	1.7957E+03
	Max.	1.6232E+03	1.8315E+03	1.6866E+03	**1.6149E+03**	2.4780E+03	2.1104E+03
F15	Mean	2.2679E+03	3.0456E+03	2.6413E+03	**2.2468E+03**	3.2464E+03	3.4409E+03
	Std.	1.2153E+02	9.6418E+01	1.0901E+02	**7.4398E+01**	1.9923E+02	2.7028E+02
	Min.	**2.0248E+03**	2.7944E+03	2.3644E+03	2.0787E+03	3.0082E+03	3.0219E+03
	Max.	2.5276E+03	3.1510E+03	2.7564E+03	**2.3631E+03**	3.9358E+03	4.0647E+03
F16	Mean	5.4348E+03	3.6049E+10	3.7860E+10	**3.0711E+03**	8.8945E+10	6.2317E+10
	Std.	6.1724E+03	4.9694E+09	1.9826E+09	**3.2072E+03**	2.2748E+10	7.3425E+09
	Min.	2.5312E+02	2.6496E+10	3.5518E+10	**1.0040E+02**	5.6912E+10	4.6213E+10
	Max.	2.0890E+04	4.6036E+10	4.7182E+10	**1.6706E+04**	1.4701E+11	7.7122E+10
F17	Mean	**3.4033E+03**	8.7960E+03	6.9176E+03	3.6604E+03	1.0206E+04	1.0156E+04
	Std.	4.4688E+02	6.0894E+02	**2.6243E+02**	5.7958E+02	5.6619E+02	4.2361E+02
	Min.	**2.3963E+03**	7.2071E+03	6.0797E+03	2.4094E+03	8.6632E+03	9.2505E+03
	Max.	**4.2795E+03**	9.9141E+03	7.3155E+03	4.6512E+03	1.1112E+04	1.1213E+04
F18	Mean	8.1194E+02	1.3138E+03	1.3329E+03	**7.9504E+02**	1.9464E+03	1.5576E+03
	Std.	2.5568E+01	4.0238E+01	3.7397E+01	**1.8050E+01**	5.1422E+02	6.7886E+01
	Min.	7.8068E+02	1.2340E+03	1.2518E+03	**7.6223E+02**	1.4146E+03	1.4137E+03
	Max.	9.0057E+02	1.4005E+03	1.3935E+03	**8.3767E+02**	3.2501E+03	1.6999E+03
F19	Mean	**1.9046E+03**	9.4628E+05	5.1307E+05	1.9048E+03	3.9451E+07	7.8285E+06
	Std.	**1.1034E+00**	7.5528E+05	3.5817E+04	1.4645E+00	3.6200E+07	5.5597E+06
	Min.	**1.9025E+03**	1.4215E+05	4.3411E+05	1.9025E+03	1.6943E+06	2.0858E+06
	Max.	**1.9070E+03**	3.0161E+06	5.8980E+05	1.9077E+03	1.5674E+08	2.6322E+07
F20	Mean	**2.6948E+03**	1.7328E+07	1.2178E+07	3.5239E+03	7.0609E+08	2.3362E+08
	Std.	**3.0323E+02**	7.5327E+06	1.0375E+06	3.4695E+02	4.1819E+08	1.3459E+08
	Min.	**2.2626E+03**	8.6651E+06	1.0681E+07	2.7963E+03	7.1530E+07	2.4923E+07
	Max.	**3.4486E+03**	4.5121E+07	1.5245E+07	4.2431E+03	1.9127E+09	6.2322E+08
F21	Mean	2.2975E+03	4.1168E+03	4.3350E+03	**2.2909E+03**	8.8019E+03	6.8637E+03
	Std.	**1.9936E+02**	3.8556E+02	2.0555E+02	2.8076E+02	2.6283E+03	9.5824E+02
	Min.	1.9543E+03	3.2095E+03	3.7919E+03	**1.6255E+03**	4.9170E+03	5.3957E+03
	Max.	2.8462E+03	5.1698E+03	4.6100E+03	**2.8440E+03**	1.4504E+04	9.1647E+03
F22	Mean	**1.4491E+04**	4.3888E+07	3.9020E+07	1.6266E+04	7.3784E+08	4.8860E+08
	Std.	9.8249E+03	4.9552E+07	8.1518E+06	**6.2862E+03**	3.6909E+08	2.8480E+08
	Min.	**2.9181E+03**	4.3349E+06	2.3773E+07	3.2972E+03	1.6682E+08	6.4935E+07
	Max.	**3.3276E+04**	2.2932E+08	5.5032E+07	2.7118E+04	1.4607E+09	1.1871E+09
F23	Mean	**2.3000E+03**	2.3133E+03	2.3112E+03	2.3000E+03	2.3646E+03	2.3578E+03
	Std.	**1.0799E-07**	7.2334E+00	1.4044E+00	1.0183E-04	1.1271E+01	8.1893E+00
	Min.	**2.3000E+03**	2.3042E+03	2.3085E+03	2.3000E+03	2.3349E+03	2.3360E+03

TABLE 10. *(Continued.)* Comparison results of NAWMWOA and other improved algorithms.

	Max.	**2.3000E+03**	2.3307E+03	2.3134E+03	2.3000E+03	2.3845E+03	2.3733E+03
F24	Mean	**2.5000E+03**	2.5952E+03	2.5572E+03	2.5000E+03	2.6427E+03	2.6312E+03
	Std.	**6.4197E-07**	1.6538E+01	4.9469E+00	4.1978E-04	1.0874E+01	8.2425E+00
	Min.	**2.5000E+03**	2.5569E+03	2.5447E+03	2.5000E+03	2.6149E+03	2.6122E+03
	Max.	**2.5000E+03**	2.6167E+03	2.5644E+03	2.5000E+03	2.6587E+03	2.6446E+03
F25	Mean	**3.2327E+03**	6.9198E+03	6.0670E+03	3.2435E+03	2.1726E+04	1.0650E+04
	Std.	4.8574E+01	8.7839E+02	9.6925E+01	**3.0478E+01**	8.5190E+03	9.6257E+02
	Min.	**3.1882E+03**	5.7003E+03	5.6659E+03	3.1882E+03	9.9377E+03	8.9424E+03
	Max.	**3.3232E+03**	8.8792E+03	6.1598E+03	3.2604E+03	4.2275E+04	1.3388E+04
F26	Mean	**3.0000E+02**	7.8479E+04	7.9936E+04	3.0000E+02	2.8428E+05	1.2812E+05
	Std.	**1.2595E-09**	5.3567E+03	2.5205E+03	3.4946E-06	7.3352E+04	1.4756E+05
	Min.	**3.0000E+02**	6.1996E+04	7.0722E+04	3.0000E+02	1.2522E+05	8.4006E+04
	Max.	**3.0000E+02**	8.5630E+04	8.3105E+04	3.0000E+02	4.7314E+05	9.1954E+05
F27	Mean	**8.6473E+02**	2.9792E+04	2.9154E+04	8.6999E+02	1.0210E+05	7.3269E+04
	Std.	**4.4300E+01**	7.4021E+03	1.2681E+03	6.7769E+01	2.7778E+04	1.2553E+04
	Min.	**8.1895E+02**	1.7675E+04	2.5523E+04	8.1895E+02	6.4401E+04	3.7709E+04
	Max.	**9.9667E+02**	4.1513E+04	3.1040E+04	1.0211E+03	1.8212E+05	9.6980E+04
F28	Mean	**6.0004E+02**	6.0571E+02	6.0163E+02	6.0008E+02	7.3872E+02	6.0837E+02
	Std.	**5.7507E-02**	9.1113E-01	2.0686E-01	8.8510E-02	1.6942E+01	1.9024E+00
	Min.	**6.0000E+02**	6.0357E+02	6.0109E+02	6.0000E+02	7.0477E+02	6.0498E+02
	Max.	**6.0022E+02**	6.0745E+02	6.0199E+02	6.0040E+02	7.9316E+02	6.1179E+02
F29	Mean	**8.7381E+02**	1.0176E+03	1.0838E+03	8.8272E+02	1.1410E+03	1.2003E+03
	Std.	1.9688E+01	1.7085E+01	**9.5270E+00**	1.9033E+01	6.3153E+01	2.7159E+01
	Min.	**8.4366E+02**	9.7176E+02	1.0546E+03	8.4563E+02	1.0352E+03	1.1445E+03
	Max.	**9.1992E+02**	1.0331E+03	1.1005E+03	9.3172E+02	1.3167E+03	1.2533E+03
F30	Mean	**1.1802E+03**	9.7374E+03	8.5557E+03	2.3958E+03	1.9338E+04	1.6932E+04
	Std.	**2.6360E+02**	1.1238E+03	4.5009E+02	1.8143E+03	7.5183E+03	2.1753E+03
	Min.	9.0809E+02	6.3472E+03	7.4705E+03	**9.0163E+02**	1.1365E+04	1.1325E+04
	Max.	**1.8766E+03**	1.1313E+04	9.1951E+03	6.5885E+03	4.3383E+04	2.1272E+04
F31	Mean	**1.2286E+04**	2.8366E+09	1.3490E+09	1.4839E+04	1.9733E+10	1.1758E+10
	Std.	**8.1102E+03**	1.5249E+09	1.3892E+08	1.1609E+04	6.0466E+09	3.2649E+09
	Min.	2.2954E+03	8.6762E+08	1.0156E+09	**2.2784E+03**	6.0760E+09	5.4025E+09
	Max.	**3.1332E+04**	7.4876E+09	1.6338E+09	4.8310E+04	3.0185E+10	1.8493E+10
F32	Mean	**2.0327E+03**	2.4768E+03	2.2799E+03	2.0344E+03	2.7872E+03	2.6485E+03
	Std.	**5.4425E+00**	9.0204E+01	1.3773E+01	8.4748E+00	1.7524E+02	1.6028E+02
	Min.	2.0247E+03	2.3055E+03	2.2492E+03	**2.0233E+03**	2.4409E+03	2.2900E+03
	Max.	**2.0421E+03**	2.6360E+03	2.3099E+03	2.0680E+03	3.0652E+03	2.9765E+03
F33	Mean	**2.2766E+03**	3.2283E+03	2.5138E+03	2.2802E+03	1.3485E+05	9.3817E+04
	Std.	**4.4559E+01**	1.0575E+03	7.2873E+01	4.6355E+01	2.0607E+05	9.6666E+04
	Min.	**2.2256E+03**	2.3052E+03	2.3819E+03	2.2291E+03	3.2451E+03	3.0768E+03
	Max.	**2.3790E+03**	7.2781E+03	2.6910E+03	2.4486E+03	1.0325E+06	3.6540E+05
F34	Mean	2.8428E+03	6.7202E+03	5.4831E+03	**2.6994E+03**	4.2501E+03	2.0840E+04
	Std.	2.1029E+02	8.0791E+02	**8.9247E+01**	1.1517E+02	7.2052E+02	6.9530E+03
	Min.	**2.5425E+03**	5.1758E+03	5.2911E+03	2.5525E+03	3.0416E+03	1.2397E+04
	Max.	3.7892E+03	8.5639E+03	5.6249E+03	**3.0209E+03**	5.7977E+03	3.5812E+04
F35	Mean	**2.8982E+03**	3.1216E+03	3.0284E+03	2.9237E+03	1.1286E+04	4.0573E+03

TABLE 10. *(Continued.)* **Comparison results of NAWMWOA and other improved algorithms.**

	Std.	5.7016E+01	**8.7335E+00**	2.1266E+01	7.6179E+01	5.4841E+02	4.4854E+02
	Min.	2.8061E+03	3.0746E+03	2.9794E+03	**2.7856E+03**	1.0214E+04	3.1597E+03
	Max.	**3.0039E+03**	3.1235E+03	3.0648E+03	3.1923E+03	1.2343E+04	4.8843E+03
F36	Mean	**3.6510E+03**	1.1588E+04	9.9661E+03	3.7028E+03	2.8323E+04	1.4453E+04
	Std.	1.6097E+02	9.1898E+02	**7.9788E+01**	1.2047E+02	6.7182E+03	1.0771E+03
	Min.	**3.4214E+03**	9.9955E+03	9.7972E+03	3.4616E+03	1.7315E+04	1.2698E+04
	Max.	4.0601E+03	1.3852E+04	1.0112E+04	**3.9489E+03**	4.1993E+04	1.7070E+04
F37	Mean	2.8002E+03	**2.8000E+03**	2.8243E+03	2.8003E+03	4.9243E+03	5.8715E+03
	Std.	1.5430E-01	**1.9261E-04**	1.4597E+01	1.9041E-01	4.4482E+02	1.2028E+03
	Min.	2.8000E+03	**2.8000E+03**	2.8040E+03	2.8001E+03	3.9397E+03	3.3300E+03
	Max.	2.8006E+03	**2.8000E+03**	2.8612E+03	2.8008E+03	5.6352E+03	8.3329E+03

TABLE 11. **Comparison results of NAWMWOA and other improved algorithms in terms of "Mean."**

Method		NAWMWOA	NCHHO	TMGWO	DFSMA	I-GWO	EAOA
+/=/-	Mean		36/0/1	37/0/0	24/1/12	37/0/0	37/0/0
	Std.		31/0/6	27/0/10	24/0/13	35/0/2	34/0/3
	Min.		36/0/1	37/0/0	25/0/12	37/0/0	37/0/0
	Max.		36/0/1	36/0/1	23/0/14	37/0/0	37/0/0
ARV		1.3243	3.8108	3.1892	1.7297	5.7838	5.1622

TABLE 12. **Comparison results for three-bar truss design problem.**

Algorithms	Optimal values for variables		Optimum weight
	A1=A3	A2	
NAWMWOA	0.78861147	0.40842825	**263.895833**
AVOA	0.78868039	0.40823341	263.895843
AOA	0.79369	0.39426	263.9154
SSA	0.78866541	0.40827578	263.895843
MFO	0.78824477	0.40946690	263.895979
WHO	0.798	0.3816	263.9181
HHO	0.78866281	0.40828313	263.895843
GOA	0.78889755	0.40761957	263.895881

TABLE 13. **Comparison results for the tension-compression spring design problem.**

Algorithms	Optimal values for variables			Optimum weight
	D	D	N	
NAWMWOA	0.0619193	0.635203	4.12079	**0.0121181**
AVOA	0.0516698	0.3562553	11.3161	0.0126652
AOA	0.05	0.349809	11.8637	0.012124
TSA	0.05108	0.34289	12.089	0.0126555
SSA	0.051207	0.345215	12.004032	0.0126763
MFO	0.05	0.30950	6.3793	0.013970
EPO	0.051087	0.342908	12.0898	0.0126569
MVO	0.05	0.315956	14.22623	0.0128169
GWO	0.050178	0.341541	12.07349	0.0126783

The NAWMWOA ultimately provides the most effective design arrangement. Therefore, the proposed NAWMWOA is highly competitive in solving the three-bar truss design problem.

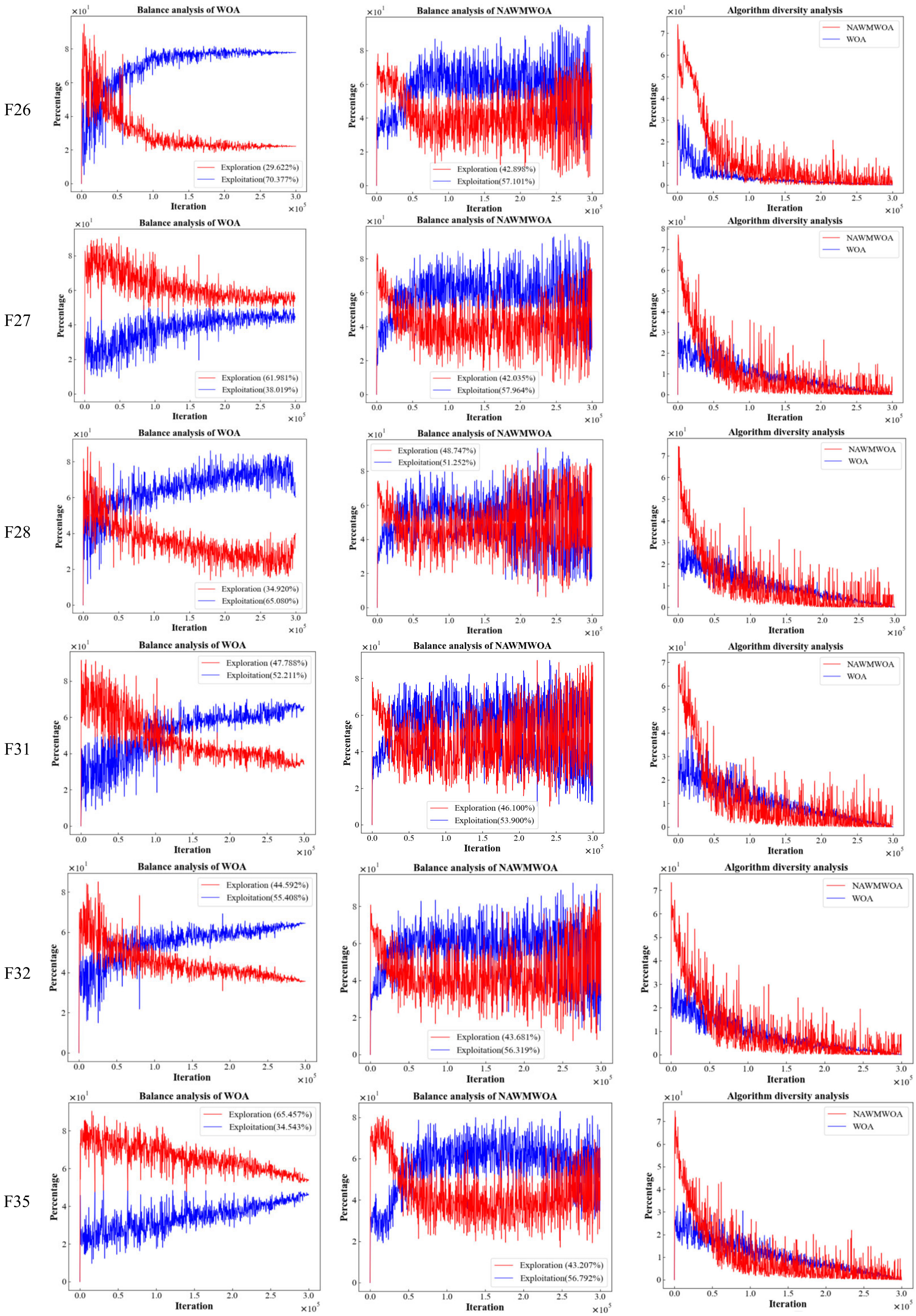

FIGURE 4. **Balance analysis and diversity analysis performed by NAWMWOA and WOA.**

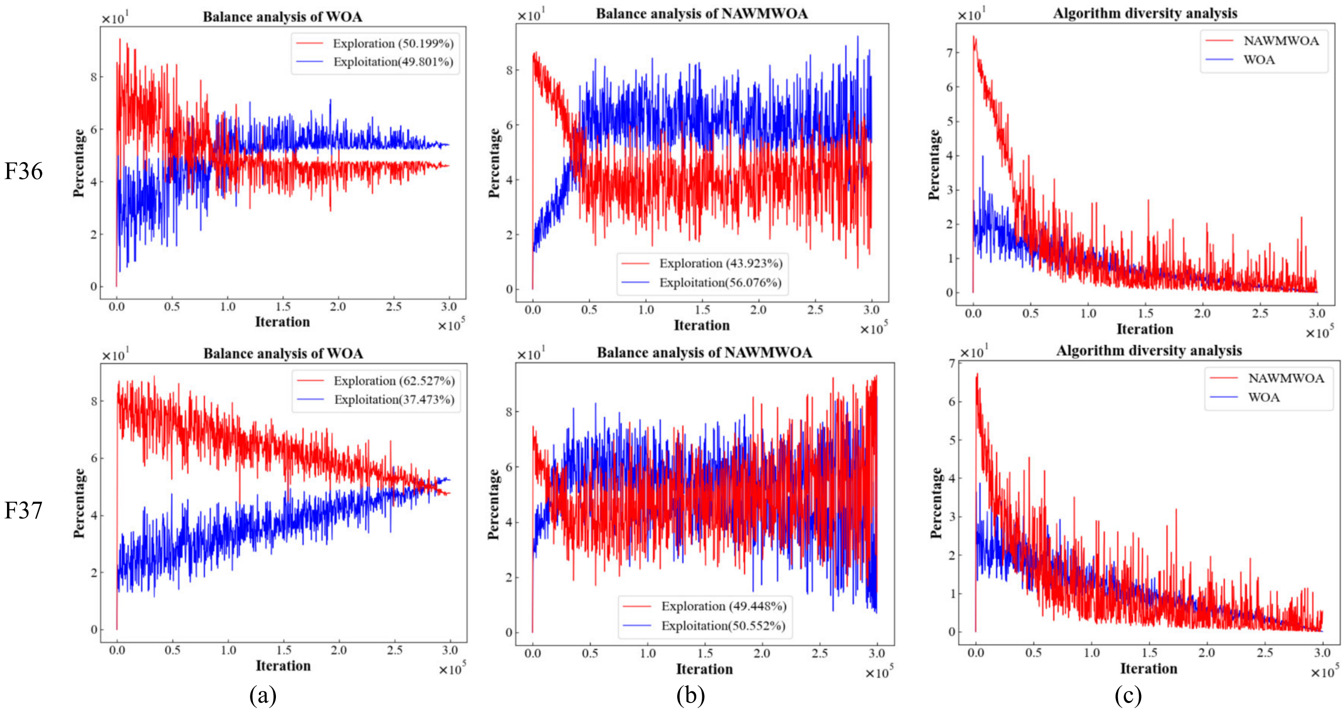

FIGURE 4. ***(Continued.)*** **Balance analysis and diversity analysis performed by NAWMWOA and WOA.**

TABLE 14. **Comparison results for welded beam design problem.**

Algorithms	Optimal values for variables				Optimum weight
	h	l	t	b	
NAWMWOA	0.2057297	3.4704863	9.0366236	0.2057296	**1.724852**
WOA	0.205396	3.484293	9.037426	0.206276	1.730499
WEMFO	0.1	2.4074	5.8609	0.1	1.84670
MFO	0.19869	4.3017	10	0.20009	2.0100
MBA	0.20573	3.4705	9.0366	0.20573	1.7249
GSA	0.182129	3.856979	10.00000	0.202376	1.879952
SHO	0.205563	3.474846	9.035799	0.205811	1.725661
GWO	0.205678	3.475403	9.036964	0.206229	1.726995
PSO	0.197411	3.315061	10	0.201395	1.820395

B. TENSION-COMPRESSION SPRING DESIGN

Tension-compression spring design is a classical engineering design problem. The purpose of the spring design is to minimize the weight of the spring under the premise of constraints. The three design variables are wire diameter (d), mean coil diameter (D), and number of active coils (N). The three constraints are shear stress, surge frequency and deflection. Fig. 6 illustrates the 3D diagram of the spring. The formulation of the Tension/compression spring design problem is explained as follows:

$$\text{Consider}: $$
$$\text{Minimize}: \vec{x} = [x_1 \quad x_2 \quad x_3] = [d\ D\ N]$$
$$\text{Subject to}: f(\vec{x}) = (x_3+2)\, x_2 x_1^2$$
$$g_1(\vec{x}) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0$$
$$g_2(\vec{x}) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} \le 0$$
$$g_3(\vec{x}) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \le 0$$
$$g_4(\vec{x}) = \frac{x_1 + x_2}{1.5} - 1 \le 0$$

In order to verify the superiority of the proposed NAWMWOA in the tension-compression spring design problem, eight advanced meta-heuristics are employed to solve the problem and the experimental results are displayed in Table 13, which includes AVOA, AOA, TSA, SSA, MFO, emperor penguin optimizer (EPO) [83], multi-verse optimizer (MVO) [84], and GWO. As can be seen in Table 13, In order to ensure fairness, all the algorithms employ similar penalty functions and the tension-compression spring design problem is executed 10 times. It can be seen from

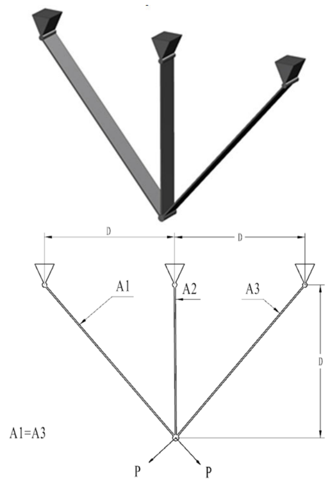

FIGURE 5. 3D model diagram and structural parameters of three-bar truss design problem.

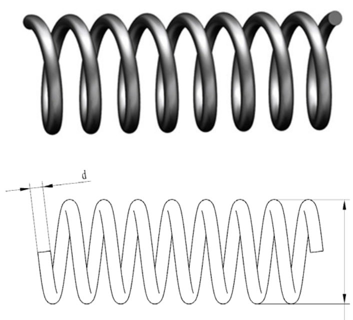

FIGURE 6. 3D model diagram and structural parameters of tension-compression spring design problem.

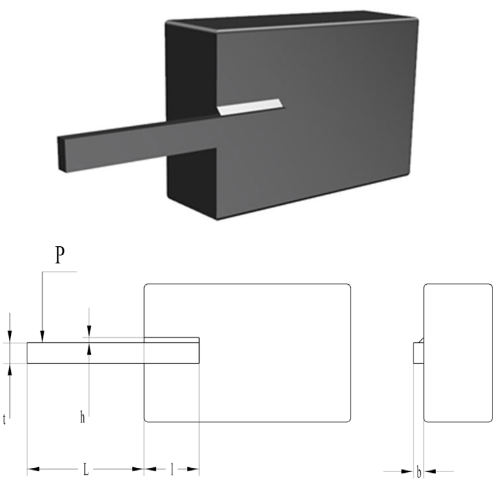

FIGURE 7. 3D model diagram and structural parameters of welded beam design problem.

Table 13, that the optimal objective function value obtained by NAWMWOA is 0.0121181, which is better than the results of other 8 algorithms. The enhanced efficacy of the proposed NAWMWOA is verified by the tension-compression spring design problem compared with other well-established methods, which indicates that NAWMWOA has an excellent performance in solving problem Tension-compression spring design problem.

C. WELDED BEAM DESIGN PROBLEM

In order to further prove the feasibility and superiority of the proposed NAWMWOA in engineering design problems, WBD problems are adopted for experiments. The design objective of the WBD is to minimize the cost of the welded beam while satisfying the more conditions. the engineering benchmark constraints are thickness of the weld (h), length of the clamped bar (l), height of the bar (t), and thickness of the bar (b). The schematic diagram is shown in Fig. 7. The mathematical model is explained as follows:

$$
\begin{aligned}
&\text{Consider :}\\
&\text{Minimize : } \vec{x} = [x_1 \quad x_2 \quad x_3 \quad x_4] = [h\ l\ t\ b]\\
&\text{Subject to : } f(\vec{x}) = 1.10471x_1^2 + 0.04811x_3x_4\,(14.0 + x_4)\\
&\qquad g_1(\vec{x}) = \tau(\vec{x}) - \tau_{max} \leq 0\\
&\qquad g_2(\vec{x}) = \sigma(\vec{x}) - \sigma_{max} \leq 0\\
&\qquad g_3(\vec{x}) = \delta(\vec{x}) - \delta_{max} \leq 0\\
&\qquad g_4(\vec{x}) = x_1 - x_4 \leq 0\\
&\qquad g_5(\vec{x}) = P - P_C(\vec{x}) \leq 0\\
&\qquad g_6(\vec{x}) = 0.125 - x_1 \leq 0\\
&\qquad g_7(\vec{x}) = 1.10471x_1^2 + 0.04811x_3x_4\,(14.0 + x_2)\\
&\qquad - 5.0 \leq 0
\end{aligned}
$$

Variable range:

$$0.1 \leq x_1 \leq 2, 0.1 \leq x_2 \leq 10, 0.1 \leq x_3 \leq 10, 0.1 \leq x_4 \leq 2$$

where

$$
\begin{aligned}
\tau(\vec{x}) &= \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}\tau' = \frac{P}{\sqrt{2}\,x_1x_2}\tau''\\
\tau(\vec{x}) &= \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + \left(\tau''\right)^2}\tau'\\
&= \frac{P}{\sqrt{2}\,x_1x_2}\tau'' = \frac{MR}{J}M = P(L + \frac{x_2}{2})\\
R &= \sqrt{\frac{x_2^2}{4} + (\frac{x_1 + x_3}{2})^2}\\
J &= 2\left\{\sqrt{2}\,x_1x_2\left[\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}\\
\sigma(\vec{x}) &= \frac{6PL}{x_4x_3^2}, \quad \delta(\vec{x}) = \frac{6PL^3}{Ex_3^2x_4}\\
P_C(\vec{x}) &= \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}})\\
P &= 60001b, \quad L = 14in \ldots max = 0.25in..\\
E &= 30 \times 1^6psi, \quad G = 12 \times 10^6psi\\
\tau_{max} &= 13600psi, \quad \sigma_{max} = 30000psi
\end{aligned}
$$

In this experiment, the NAWMWOA and eight meta-heuristic algorithms including WOA, double adaptive weights for stabilization of moth flame optimizer (WEMFO) [85], MFO, mine blast algorithm (MBA) [86], GSA, spotted hyena optimizer (SHO) [87], GWO and particle swarm optimization (PSO) [88] are utilized to solve the welded beam design problem with certain constraints. The experimental results are listed in Table 14, in which the NAWMWOA obtains the optimization results with a minimum value of 1.724852, indicating the best performance among all the comparison algorithms. It can be seen from the experimental results in Table 14 that NAWMWOA has the smallest experimental results, indicating that NAWMWOA has a prominent ability in solving welded beam design problem.

VI. CONCLUSION AND FUTURE DIRECTIONS

In this study, the nonlinear convergence factor, adaptive weight and hybrid mutation strategy are introduced into the original WOA and a novel variant of WOA called NAWMWOA is proposed. The nonlinear convergence factor improves the balance between exploration phase and development phase of the WOA. The adaptive weight has the function of overcoming the defect of original WOA that is easy to fall into local optimum in the early stage of iteration. The hybrid mutation strategy is introduced to improve the convergence accuracy of the algorithm. Two sets of advanced meta-heuristics are adopted to perform experiments with the proposed NAWMWOA on 37 benchmark functions (CEC2015, CEC2021, CEC2022), which includes five variants of WOA (EWOA, NGS-WOA, ESSAWOA, WOAAmM, CWOA), and five state-of-the-art algorithms (NCHHO, TMGWO, DFSMA, I-GWO, EAOA). The experimental results composed of "Mean", "Std.", "Max.", and "Min." conclude that the NAWMWOA performed significantly better than the other algorithms. In detail, the NAWMWOA outperforms EWOA, NGS-WOA, ESSAWOA, WOAAmM, and CWOA on 36, 37, 37, 37, and 36 out of 37 functions, respectively. In addition, the overall performance of NAWMWOA is generally superior to NCHHO, TMGWO, DFSMA, I-GWO, and EAOA on 36, 37, 24, 37, and 37 out of 37 functions. In order to demonstrate the effectiveness of the proposed NAWMWOA to solve practical problems, three kinds of classical engineering benchmarks including three-bar truss design problem, tension-compression spring design problem, and welded beam design problem are used for experiments. The experimental results show that NAWMWOA has a better ability to solve practical problems than other algorithms in three kinds of engineering design problems.

The research and application of intelligent algorithms should have great improvement room. Since the introduction of new mechanisms increases the complexity of algorithms and reduces the convergence efficiency of algorithms, improving the convergence speed of algorithms is the main direction of future research. Besides, it may be applied to intelligent medical and health protection systems [89], sustainable transport systems [90] and sustainable supplier selection [91] by combining machine learning algorithms and decision models. Since the introduction of algorithm models increases the complexity of algorithms and reduces the convergence efficiency, improving the convergence speed of algorithms is the main direction of future research.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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