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RESEARCH ARTICLE

Gear Shifting and Vehicle Speed Optimization for Eco-Driving on Curved Roads

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ABSTRACT This paper presents a control strategy to optimize both the vehicle speed and gearbox position on curved roads with the aim of maximizing fuel economy. Previous studies have focused only on the vehicle speed optimization during cornering. Combining vehicle speed and gearbox position optimization can significantly maximize the energy-saving for an Internal combustion engine (ICE) vehicle. The problem is formulated as two successive optimization problems. In the first one, based on the road map and traffic data, the vehicle's optimal speed profile is calculated by minimizing the energy spent for the entire curve. In the second one, the gearbox position is optimized for fuel consumption minimization using realistic engine map. The presented control structure's effectiveness was evaluated through co-simulation of Matlab/Simulink and CarSim in various driving scenarios. The results show that approximately 5.25% to 11.44 % of fuel savings can be achieved compared with a typical driver model.

INDEX TERMS Eco-driving, curved roads, optimal control, dynamic programming, fuel economy, gear shifting optimization, speed profile optimization.

I. INTRODUCTION

In 2019, fossil-fuel combustion in road transport contributed to approximately 75% of the total CO2 emissions from transportation [1]. Reducing fuel consumption is crucial due to its impact on the environment, health, and economy. Eco-driving, which involves various fuel-saving decisions such as vehicle maintenance, weight reduction and changing driving behavior, is an effective solution to reduce fuel consumption and CO2 emissions [2], [3]. Optimizing driving behavior, specifically, vehicle speed and gearbox position, is essential for energy efficiency [4]. This can be achieved through an optimization problem under a set of constraints [5].

The optimization problem is often formulated as an Optimal Control Problem (OCP) and solved by the Pontryagin Maximum Principle (PMP) and Dynamic Programming (DP). In [6], PMP was used, and constant speed has been found to be optimal on level roads. The authors of [7] developed an online eco-driving system by combining DP with Model Predictive Control (MPC) for heavy-duty vehicles. Real-world tests showed 3.5% fuel savings over 120 km without increased trip time. In [8] a predictive ecocruise control system has been proposed using slope information for better fuel efficiency. In [9], the authors developed an ecological driving system for up-down slope roads. In [10], the authors proposed an algorithm for reducing fuel consumption using slope and traffic information. In [11], a Relaxed Pontryagin's Minimum Principle (RPMP) is proposed to optimize both the vehicle velocity and gearbox position for a changing road grade scenario. In [12], vehicle speed and gearbox position are simultaneously optimized using Reinforcement Learning (RL) for a car-following scenario.

Overall, many studies aim to enhance vehicle energy efficiency through optimized speed and gear usage based on driving conditions [13], [14], [15], [16]. However, most assume straight-road driving and only consider longitudinal dynamics, neglecting the impact of corners on energy consumption. Road curvature can significantly affect a vehicle's fuel consumption. The authors of [17], [18], and [19] have

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demonstrated that cornering on roads with small radii can lead to additional energy loss for conventional vehicles, electric city buses and electric motorcycles, respectively. Cornering scenarios are common in city driving, yet there is limited research on optimizing speed profiles. In [20], researchers found that maintaining a constant speed on a level road with curved sections can reduce fuel consumption. In [21], a model that considers cornering effects is proposed and sequential quadratic programming is used to find optimal solutions. An energy benefits of 8% are achieved for an electric vehicle considering the urban scenario. In [19], DP and a step-size Newton Iteration algorithm have been used to solve the OCP. The results show an energy saving of up to 6.29% in a real-world urban route for an electric vehicle. The study in [22] shows a fuel savings of up to 17.64% using DP compared with a typical driver model (TDM) for entering and departing curve scenarios. Although the results of [19] and [22] are relevant, more efforts will be needed for real-time application. In fact, they used a dynamic model that depends on the tire characteristics which are often unknown. To the best of our knowledge, no recent study has tried to explore the benefit of optimizing the speed and gearbox position on curved roads.

Co-optimizing vehicle speed and gearbox position can significantly maximize fuel economy for an Internal combustion engine (ICE) vehicle, [23]. Previous works assume fixed gear shifting rules and focus only on determining the optimal speed profile in terms of energy for cornering scenarios. The gear position ratio plays a crucial role in adjusting the operating points of the engine to optimize fuel saving. Combining the optimization of vehicle speed and gear shifting holds great potential for reducing fuel consumption.

Briefly, the existing works only focus on speed optimization for cornering scenarios. The original contribution of this work is to study the potential of introducing the optimization of both the vehicle speed and gearbox position on curved roads using the kinematic bicycle model rather than the dynamic one for fuel efficiency. Based on realistic engine maps, traffic data, and road curvature information, two successive DP algorithms are employed to plan optimal speed and gear position profiles throughout the entire curved roads. The utilization of the kinematic bicycle model, completely independent from the tire characteristics, which can often be uncertain, and dependent on vehicle and road geometry, enhances the suitability of the developed algorithm for real-time eco-driving applications.

The initial findings and analysis of the use of the kinematic bicycle model with DP for curved roads are published in [24]. Speed-only optimization has been taken into account and can be executed in cases where controlling gear shifting is not possible. In this work, these findings are extended in several directions. Firstly, to improve fuel consumption accuracy calculation, a fuel rate model is developed based on real-time simulation data. It consists of a polynomial model which is easy in calculation. Secondly, two sequential optimizations are carried out to optimize both the speed and gearbox position. Thirdly, an in-depth investigation of the effects of the proposed strategy is carried out.

This paper is organized as follows: Section II presents the vehicle dynamics model. Section III is dedicated to describing the problem formulation and the proposed algorithm. IV evaluates the effectiveness of the presented approach by using Matlab/Simulink software and the CarSim simulator. Finally, some conclusions are given in Section V.

II. VEHICLE MODELING

In this section, we introduce the vehicle dynamical model, including both longitudinal and lateral effects. We employ a bicycle model, which treats the two front wheels as one combined front wheel and the two rear wheels as one combined rear wheel. In addition, we consider a gasoline engine vehicle and we assume that the gear shifting can be controlled.

A. VEHICLE DYNAMICS

The longitudinal motion of the controlled vehicle on a curved road [21] can be described by

$$(m + m_\delta)\frac{dv}{dt} = F_w - F_r - F_c \tag{1}$$

where v represents the longitudinal speed, m denotes the vehicle mass, and m_δ reflects the influence of rotating components. Within this context, F_w, F_r, and F_c stand for the available driving force at the wheels, the resistance force considered on a straight road, and the projection of the total centripetal force applied at the center of gravity onto the longitudinal axis, respectively. The traction force on the wheels can be expressed as follows:

$$F_w = \frac{\eta(n) i_d i_t(n) T_e(w_e, \tau)}{r} - F_b \tag{2}$$

with the powertrain efficiency η, the gearbox position n, the final gear ratio i_d, the gearbox ratio i_t, the wheel radius r, the engine torque T_e, the engine throttle τ, the braking force F_b and the engine speed w_e, which depends on v through:

$$w_e = \frac{30 i_d i_t(n) v}{\pi r} \tag{3}$$

The resistance force under consideration on a straight road, which includes aerodynamic resistance, rolling resistance, and slope resistance forces, is expressed as follows:

$$F_r = \frac{1}{2}\rho_a c_d v^2 + c_r mg\cos(\alpha) + mg\sin(\alpha) \tag{4}$$

with the external air density ρ_a, the friction coefficient c_r, the gravitational constant g, the road slope angle α and $c_d = c_f A_f$ where c_f is the drag coefficient and A_f is the frontal area. To account for lateral effects during cornering, we employ the projection of the total centripetal force applied at the center of gravity onto the vehicle's longitudinal axis.

$$F_c(v, s) = m l_r \rho(s)^2 v^2 \tag{5}$$

TABLE 1. Vehicle model parameters.

Symbol	Value	Unit
m	1300	kg
m_δ	33	kg
i_d	3.867	-
n	[1 2 3 4 5]	-
i_t	[3.73 2.048 1.3929, 1.097 0.892]	-
η	[0.85 0.9 0.93, 0.95 0.97]	-
r	0.3	m
ρ_a	1.205	kg/m^3
c_d	0.6138	-
c_r	0.02	-
g	9.8	m/s^2
l_r	1.4	m

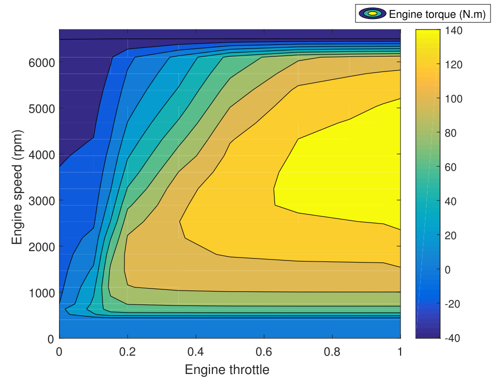

FIGURE 1. Engine torque steady-state map [26].

where l_r is the distance between the center of gravity and the rear axle, ρ is the curvature of the road and s is the traveled distance. The curvature, denoted as ρ can be defined as the reciprocal of the road radius R. The force F_c is computed using the kinematic bicycle model. Despite its simplicity, this model can yield comparable results to the dynamic model for vehicle control applications [21].

Assuming negligible wheel slip and that the force F_r is aligned with the vehicle's longitudinal axis, the vehicle model, utilizing the equations mentioned above, can be expressed as follows:

$$(m+m_\delta)\frac{dv}{dt} = F_w(n, v, \tau, F_b) - \frac{1}{2}\rho_a c_d v^2 - ml_r\rho(s)^2v^2 - mg(c_r\cos(\alpha)+\sin(\alpha)) \quad (6)$$

A vehicle with five gear positions is adopted in this work. Its different parameters are shown in Table 1 [25].

B. TRANSMISSION DYNAMICS

The engaged gearbox position variable n changes with the control variable c_g according to :

$$n_{k+1} = n_k + c_g \quad (7)$$

Control variables $c_g \in [-1, 0, 1]$ include: downshift ($c_g = -1$), maintain current gear ($c_g = 0$), or upshift ($c_g = +1$). In this work, we consider an instantaneous gear-shifting.

C. ENGINE CHARACTERISTICS

1) TORQUE MAP

Fig.1 illustrates the engine torque T_e described as a steady-state map, depending on both engine speed w_e and engine throttle τ. In CarSim, the engine throttle is given according to the acceleration $a_x = \frac{dv}{dt}$ by [26]:

$$\tau = \begin{cases} 0 & a_x < 0 \\ K_\tau \cdot a_x & a_x > 0 \\ \tau_{sat} & (a_x > 0)\cdot(\tau > \tau_{sat}) \end{cases} \quad (8)$$

where K_τ and τ_{sat} equal to 0.1 and 1, respectively.

2) INSTANTANEOUS FUEL RATE MODEL

Accurate fuel rate models are critical to addressing energy efficiency issues. Various fuel rate models have been proposed in the literature [3]. In this work, a polynomial function with seven coefficients is developed based on driver-in-the-loop data.

$$\dot{m}_f = a_1 + a_2w_e + a_3w_e^2 + a_4w_eT_e + a_5T_e + a_6T_e^2 + a_7T_e^3 \quad (9)$$

with engine speed w_e and constant parameters $a_1, a_2, a_3, a_4, a_5, a_6$ and a_7, which equal to -2.5064×10^{-5}, 1.5403×10^{-7}, 2.1191×10^{-11}, 1.5201×10^{-9}, 9.8204×10^{-6}, -1.8863×10^{-7}, 1.7777×10^{-9}, respectively.

The estimation error is less than 0.0001 (kg/s) in a wide operating range, which can thoroughly satisfy the precision criteria necessary for fuel consumption estimation in the subsequent analysis.

III. PROBLEM FORMULATION

Fig.3 shows the considered scenario and the control architecture. After selecting the curved road to traverse, various road parameters such as slope, curvature, speed limits, and engine map are employed to compute optimal profiles for vehicle speed and gearbox position. Starting from the fact that a change in vehicle speed and gearbox position is often inevitable as a driver approaches a curved road, the primary goal is to determine how to plan this change throughout the entire curve with the aim of minimizing fuel consumption. This is accomplished through the use of the kinematic bicycle model and DP. This latter is used both in speed and gear optimization.

Previous studies emphasize that to optimize fuel efficiency, drivers should maintain a constant speed on both flat and circular curved roads [6], [22], [27]. An entire curved road includes straight, clothoid and circular sections as can be seen in Fig.4. For security issues, the flat road speed is usually higher than the circular road speed [22]. Thus, before discussing the details of constituent components of the formulated OCP for speed and gear position planning for the entire curve, let us first study how the vehicle behaves in a

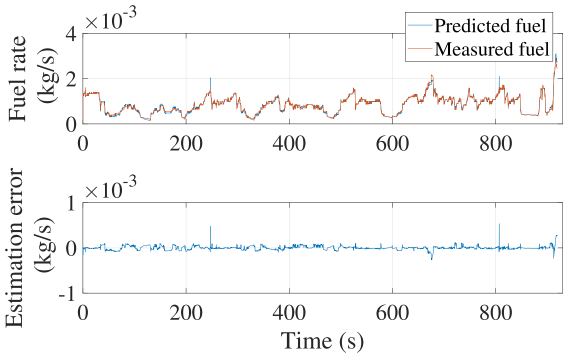

FIGURE 2. Measured and model predicted fuel rate curves.

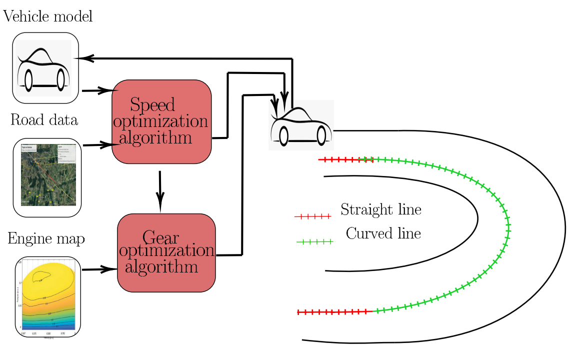

FIGURE 3. Control architecture.

circular section. Contrary to [22], we will focus on identifying the different possible set points of speed and gear position and their fuel consumption for a range of curvature than searching the optimal speed to practice at the circular section. In this work, we suppose that the speed to practice at a circular curve can be received using Vehicle to Infrastructure (V2I) communication based on the traffic state. This could enhance both fuel efficiency and mobility. Moving with constant speed at the circular curve means that the acceleration should be zero, ($a_x = 0$). Using (6), the traction force is given by

$$F_{w,c} = \frac{1}{2}\rho_a c_d v_c^2 + ml_r \rho_c^2 v_c^2 + mg(c_r \cos(\alpha_c) + \sin(\alpha_c)) \tag{10}$$

where $F_{w,c}$, v_c, ρ_c and α_c denote the traction force, the speed, the curvature and the slope at the circular curve. To take the effect of the road curvature into the fuel consumption, the engine torque can be calculated using (2) as

$$T_e = \frac{F_{w,c} r}{\eta(n) i_d i_t(n)} \tag{11}$$

with $F_b = 0$. The engine speed is

$$w_e = \frac{30 i_d i_t(n) v_c}{\pi r}$$

The maximum driver speed in the curve can be given by

$$v \leq \sqrt{\frac{\mu g}{\rho_c}} \tag{12}$$

where $\mu > 0$ is the friction coefficient. Then, the fuel consumption for a given speed with different gear positions within the speed constraints can be obtained using (9). Fig.5 shows the fuel use corresponding to different gearbox positions and curvatures. The used values choice is based on our previous work discussing the limited use of the bicycle model for eco-driving [24]. Clearly, the fuel use varies for different gears. For each gear, the fuel consumption curve initially decreases and then increases as vehicle speed increases. For the same speed and gear, the fuel consumption increases as the curvature increases. This is mainly due to the rapid increase of the lateral force during cornering. In addition, for a given speed, when the gear position increases, the fuel-saving tends to become high. In fact, at higher gears, the engine usually operates in more fuel-efficient regions. However, for velocity $\rho_c = 0.4$ and $v > 43km/h$, we observe that 4-th gear is more efficient than 5-th gear. This is also remarquable for $\rho_c = 0.035$ and $v > 50km/h$. This highlights the importance of identifying the optimal gear at the circular curve.

It is observable that the fuel consumption on a flat road ($\rho = 0$) is always less than the one on a curved road. If we consider a vehicle moving on a flat road with a speed of 50 km/h and the vehicle receives the speed to practice at circular curve $v_c = 25$ km/h, then the vehicle should necessarily decelerate and accelerate. Furthermore, according to Fig.5, the corresponding optimal gears are 5-th and 3-th, respectively. In this case, the vehicle should also successively downshift and upshift in order to operate the engine around its high-efficiency regions. To ensure fuel efficiency of these processes, an OCP is formulated and the following stating is proposed.

IV. OPTIMAL CONTROL DESIGN

The objective of the OCP is to compute the optimal profiles for speed and gear positions, aiming to minimize fuel consumption along the curve over a travel distance denoted as s_f. This optimization must adhere to speed limits and vehicle and engine constraint sets. To accomplish this, the optimization is carried out in the spatial domain, employing the transformation method described by [7]

$$\frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds} \tag{13}$$

where s (traveled distance) is the independent variable.

The current optimization problem will be addressed through numerical solutions using DP. As a result, the equations presented below will be expressed in a discrete format.

A. COST FUNCTION

The cost function is expressed as the cumulative fuel consumption required for a driving mission from $s = s_0$ to $s = s_f$.

$$OF = \sum_{k=0}^{N-1} \dot{m}_{f_k} \left(\omega_{e_k}, T_e(\omega_{e_k}, \tau_k)\right) \frac{\Delta s}{v_k} \tag{14}$$

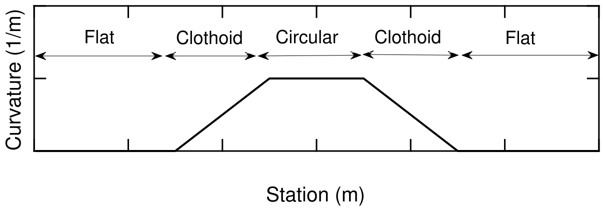

FIGURE 4. Entire curved road.

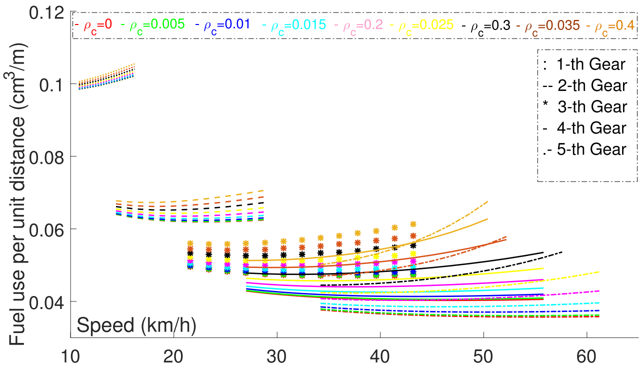

FIGURE 5. Fuel use rate per unit distance for various gears and curvatures.

where $v_k > 0$. $\dot{m}_{f_k}$, w_{e_k} and τ_k denote the fuel rate, the engine speed and the engine throttle at step k, respectively.

B. DISCRET MODEL

$$\begin{cases} v_{k+1} = \dfrac{\Delta s}{v_k}\dfrac{1}{m+m_\delta}\Big(F_{w,k}(n_k, v_k, \tau, F_b) - \dfrac{1}{2}\rho_a c_d v_k^2 \\ \qquad -mg(\sin(\alpha_k) + c_r\cos(\alpha_k)) - m l_r \rho_k(s)^2 v_k^2\Big) \\ \qquad +v_k \\ n_{k+1} = n_k + c_{g,k} \end{cases} \tag{15}$$

where $k = 0, 1, \ldots, N-1$. v_k denotes the vehicle speed at step k, n_k is the gear position at step k, $F_{w,k}$ is the traction force at step k, ρ_k is the road curvature at step k. According to (15), $[v_k,\ n_k]$ is the state variable and $[F_{w,k}, c_{g,k}]$ is the control variable.

The slope of the route, denoted as α_k at step k is defined by (16) and remains unchanged between consecutive steps:

$$\alpha_k = \arctan\left(\frac{z_{k+1} - z_k}{s_{k+1} - s_k}\right) \tag{16}$$

with z_k and s_k are the route altitude and vehicle's travelling distance at step k, respectively.

Utilizing coordinates (x_k, y_k) at step k, derived from three consecutive points on the road map, the horizontal curvature at step k can be determined by [22]

$$\rho_k = \frac{|\sigma'|}{(1+\sigma^2)^{3/2}} \tag{17}$$

where ρ_k denotes the horizontal curvature at the step k, $\sigma = \left(\frac{y_{k-1}-y_k}{x_{k-1}-x_k} + \frac{y_{k+1}-y_k}{x_{k+1}}\right)/2, \sigma' = 2\left(\frac{y_{k-1}-y_k}{x_{k-1}-x_k} - \frac{y_{k+1}-y_k}{x_{k+1}-x_k}\right)/$ $(|x_{k+1} - x_k| + |x_k - x_{k-1}|)$.

C. SET OF CONSTRAINT

The engine speed is always within the range of the idling speed, denoted as w_e^{idle}, and the upper limit of engine speed, indicated as w_e^{max}. Furthermore, the engine torque is bounded, with its maximum determined by the engine speed at full load and its minimum reflecting the maximum engine braking capability. Bounded acceleration serves to prevent aggressive acceleration, which can lead to increased fuel consumption. Vehicle speed is restricted to ensure respect for legal speed limits. In order to prevent the vehicle from experiencing slipping behavior during cornering, the total force applied to the vehicle is constrained not to exceed the maximum friction force, denoted as $F_f = m\mu g$, under normal conditions.

$$T_e^{min}(w_e) \le T_e \le T_e^{\max}(w_e), \quad w_e^{idle} \le w_e \le w_e^{max} \tag{18}$$

$$c_g \in \{-1, 0, +1\}, \quad n \in \{1, 2, 3, 4, 5\} \tag{19}$$

$$a_x^{min} \le a_x \le a_x^{\max}, \quad v^{min} \le v \le v^{\max} \tag{20}$$

$$(ma_x)^2 + \left(mv^2\rho(s)\right)^2 \le (m\mu g)^2 \tag{21}$$

D. OPTIMAL CONTROL PROBLEM

For a given road with a distance length $N\Delta s$, the following hybrid non-linear OCP will be solved:

$$\min\ J = OF$$

subject to:

$$\begin{cases} v_{k+1} = \dfrac{\Delta s}{v_k}\dfrac{1}{m+m_\delta}\Big(F_{w,k}(n_k, v_k, \tau, F_b) - \dfrac{1}{2}\rho_a c_d v_k^2 \\ \qquad -mg(\sin(\alpha_k) + c_r\cos(\alpha_k)) - m l_r \rho_k^2 v_k^2\Big) \\ \qquad +v_k \\ n_{k+1} = n_k + c_{g,k} \end{cases}$$

$$\text{Constraints: (18), (19), (20) and (21)} \tag{22}$$

The energy optimization problem for road vehicles is usually solved using PMP and DP methods [3], [28]. Due to the non-convex and non-linear nature of the formulated OCP with mixed constraints, methods based on PMP need more simplification of the problem and model to be implemented [29]. Thus, the method proposed in this work to search for the global optimal solution is based on DP as it can handle these problems easily. However, the process of controlling the speed and gearbox position will make the computation time of DP too long, which becomes a crucial issue for implementing online eco-driving in real-time, particularly, on curved roads which usually include short distances. The next section presents the proposed approach to handle this problem.

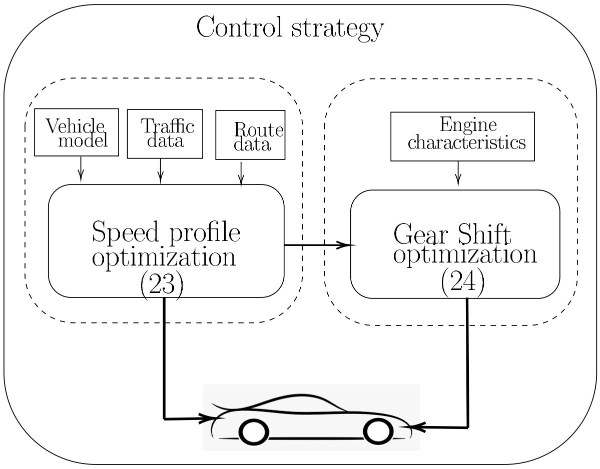

FIGURE 6. Control strategy.

E. PROPOSED CONTROL STRATEGY

To reduce the computational burden, we divide the formulated optimal control problem into two optimization subproblems. Their solutions are computed using two consecutive steps as shown in Fig. 6. This technique has been successfully employed for a car-following scenario in Bentaleb, al [30]. The first optimization problem calculates the optimal speed based on the vehicle model, route data (curvature and slope) and traffic data (speed at circular curve and speed limit). This first step consists of controlling the driving force F_w with the objective of reducing the energy spent for driving on the curved road. The second optimization problem aims to find the optimal gear-shifting while taking into consideration the engine characteristics and speed and acceleration profile, provided by the first subproblem, to be followed along the curve. The goal of the second subproblem is identifying the optimal gearbox positions that enable the engine to function in more efficient regions, minimizing fuel consumption.

1) SPEED OPTIMIZATION

Thus, the following speed optimization problem

$$\begin{aligned}
&\min\ J_1 = \sum_{k=0}^{N-1} F_{w,k}^2 + w_1(v_k - v_c)^2 \\
&\text{subject to:} \\
&\quad v_{k+1} = \frac{\Delta s}{v_k}\frac{1}{m+m_\delta}\Big(F_{w,k} - \frac{1}{2}\rho_a c_d v_k^2 \\
&\quad -mg(\sin(\alpha_k) + c_r\cos(\alpha_k)) - ml_r\rho_k^2 v_k^2\Big) + v_k \\
&\quad \text{Constraints: (20), (21)}
\end{aligned} \tag{23}$$

is adopted in this paper where v_c is the speed at circular curve and w_1 equals 1 if ρ equals $\rho_c = 1/R$, otherwise w_1 is taking equal to 0. The primary objective of this problem is to minimize the energy expended during the specified curve while assuring a constant speed at the circular section. The vehicle speed is the state variable, while the driving force, which encompasses the braking force, serves as the control variable. The optimization process provides the speed and the corresponding acceleration profiles to the following second optimization problem.

2) GEAR OPTIMIZATION

This second sub-problem identifies the most efficient gear-shifting schedule to minimize fuel consumption using the following

$$\begin{aligned}
&\min\ J_2 = \sum_{k=0}^{N-1} \dot{m}_{f_k}\left(\omega_{e_k}, T_e(\omega_{e_k}, \tau_k)\right)\frac{\Delta s}{v_k} \\
&\text{subject to:} \\
&\quad n_{k+1} = n_k + c_{g,k} \\
&\quad \text{Constraints: (8), (18), (19)}
\end{aligned} \tag{24}$$

where engine speed and engine throttle are calculated using (3) and (8). The gear position is the state variable, and c_g is the control variable.

F. DP ALGORITHM

DP can solve a problem of the form [31]

$$\begin{aligned}
&\min_{\pi^*=\{\xi_0,\ldots\xi_{N-1}\}} p(x_N) + \sum_{k=0}^{N-1} q(x_k, \xi_k(x_k)) \\
&\text{s.t.}\quad x_{k+1} = f(x_k, \xi_k(x_k)) \\
&\qquad x_k \in \mathcal{X}_k \\
&\qquad \xi_k(x_k) \in \mathcal{U}_k(x_k).
\end{aligned} \tag{25}$$

where q represents the cost incurred per step, p stands for the terminal cost, and π denotes an admissible policy. Meanwhile, $\mathcal{X}_k$ and $\mathcal{U}_k$ define the feasible sets within the state space and control input, respectively. The optimal costs, denoted as $\mathfrak{J}_k^*$, associated with the state x_k, can be computed iteratively, starting with the costs at the terminal state, $\mathfrak{J}_k^*(x_N) = h(x_N)$, and evolving as follows

$$\mathfrak{J}_k^*(x_k) = \min_{u_k \in \mathcal{U}_k(x_k)} \left\{\mathfrak{J}_{k+1}^*(x_{k+1}) + q_k(x_k, u_k)\right\} \qquad k = N-1, \ldots, 0 \tag{26}$$

The cost-to-go function, denoted as $\mathfrak{J}_k^*(x_k)$, represents the optimal solution for the subproblem with (N-k) steps, starting at state x_k and ending at state x_N. DP's solution yields an optimal control policy, denoted as π^*. In situations where the transition between states at step k and step $k+1$ is not feasible, the associated cost q_k is set to a significantly large value Q. To use DP, we need complete data about the expected road which is defined by a set of points. Especially, the road is made up of equidistant points $P = \{P_0, P_1, P_2, \ldots, P_N\}$. For speed optimization, every point $p_k \in P$ for $k = 0, 1, \ldots, N$ has its own characteristic settings; $p_k = [S_k, h_k, \rho_k, \alpha_k, v_k^{max}]$ where S_k is the distance, and h_k is the altitude, ρ_k is the curvature, α_k is the slope, v_k^{max} is the maximum speed. It is assumed that $s_k, h_k, \rho_k, v_k^{max}$ are known for $k = 0, 1, ..., N$. The function $f(x_k, u_k)$ is the dynamic model constraint. q_k is calculated by the sum of the traction force needed and the deviation from the circular curve between steps k and k+1. Having these data and taking $x_k = v_k$ and $u_k = F_{w,k}$ as the state and control variables, respectively, the solution to OCP (23) can be computed with DP (26). For gear opitimization, $p_k = [v_k^*, a_{x,k}^*], f(x_k, u_k) = (n_k + c_{g,k})$, $x_k = n_k$ and $u_k = c_{g,k}$ are used where v_k^* and $a_{x,k}^*$ are the first sub-problem solutions. q_k is calculated by the fuel consumption needed between steps k and k+1. The optimal gear position n_k^* is obtained using (26).

TABLE 2. DP algorithm parameters.

Symbol	Meaning	Value	Unit
Δs	distance discretization	1	m
Δv	Speed discretization	0.855	km/h
v^{min}	minimum velocity	5.55	km/h
v^{max}	maximum velocity	16.66	km/h
ΔF_w	force discretization	4.5	N
F_w^{max}	maximum traction force	2000	N
F_w^{min}	minimum traction force	-266	N
a_x^{min}	minimum acceleration	-1.6	m/s²
a_x^{max}	maximum acceleration	0.75	m/s²
w_e^{min}	minimum engine speed	1000	rpm
w_e^{max}	maximum engine speed	2100	rpm

V. SIMULATION RESULTS

As mentioned above, our main goal is to determine the optimal strategy for managing changes in vehicle speed and gear position across the entire curve while minimizing fuel consumption, relying on the kinematic bicycle model. In this section, after presenting the simulation setup, we will start analyzing the vehicle behavior for the entire curve. Then, a discussion concerning the comparison with a typical driver model during cornering is presented.

A. SIMULATION SETUP

The proposed eco-driving strategy is implemented and solved using the MATLAB toolbox. The desktop computer employed for this task features an Intel(R) Core(TM) i5-4300U CPU running at 1.90-2.5 GHz, with 4GB of RAM. To address the optimization problem (26), we make use of open-source DP software provided by ETH Zurich [32]. The DP parameters used in the simulation are detailed in Table 2. All simulations are conducted within the context of a continuous-curved road scenario covering a total distance of 300 meters. This road comprises two straight sections, each measuring 100 meters, two clothoid sections, each measuring 25 meters, and a circular section covering 50 meters. Additionally, it is assumed that the vehicle drives from the straight sections into two different circular curves, one with a large curvature of 0.025 1/m (equivalent to a small radius of 40 meters) and the other with a small curvature of 0.01 m (corresponding to a large radius of 100 meters). Furthermore, we consider that the vehicle travels at 50 km/h on a straight road and receives speeds of 25 km/h and 30 km/h to practice at the circular curve for the large and small curvature, respectively.

B. VEHICLE BEHAVIOUR FOR THE ENTIRE CURVE

Fig.7 illustrates the DP-speed algorithm solutions for both the large and small curvature. It is observable that when the curvature increases (corresponding to less circular speed), the rate of deceleration remarkably tends to become high. Furthermore, As depicted in Fig 7, for minimum energy spent, the vehicle needs to smoothly decelerate before approaching the first clothoid section while increase lightly its deceleration rate within it before the circular one. Then, it should maintain the received circular speed within the circular section. The vehicle starts directly the acceleration process after the circular curve and keeps a constant acceleration until reaching the speed of the straight section. For the small curvature, the vehicle reduces its acceleration just after the second clothoid than for the large curvature for energy saving.

Fig.8 shows the DP-gear algorithm solutions based on the above results. It is clear that the vehicle adapts its gear shifting before and after the circular curve both in the two cases. The engine speed and torque constraints are well respected. For minimum fuel consumption, the vehicle stays as can as possible at higher gear and then downshifts and upshifts according to the deceleration and acceleration process, respectively. For the large curvature, the vehicle downshifts until the 3-th gear. In the other case, it downshifts until the 4-th gear. This matches well with the results discussed in Section III where the optimal gears corresponding to speeds of 25 km/h and 30 km/h are 3-th and 4-th, respectively.

To highlight the importance of optimizing both speed and gear position, a case where the controlled vehicle is only driven with the optimal speed without controlling the gearbox position was tested. Combining the optimization of vehicle speed and gearbox position yields 3.36% and 7.37% of fuel savings compared with optimizing only the vehicle speed for large and small curvature, respectively. This shows the benefit of optimizing both the vehicle speed and gearbox position.

C. CARSIM VALIDATION

This section assesses the presented algorithm through co-simulation conducted in Matlab/Simulink and CarSim environment (as depicted in Fig. 9). For the sake of comparison, we opt for the typical driver model as presented in [33] to represent the experimental control group. This model emulates the average driving behavior observed during cornering based on the experiences of drivers in India, illustrated in Figure 10.

The driver model decelerates at a rate denoted as d_r before entering the curve and accelerates at a rate denoted as a_r upon exiting the curve. Moreover, the deceleration maneuver is sustained for a specific duration after the onset of the curve, a period determined by ΔV_d and Δv_d. As for the acceleration

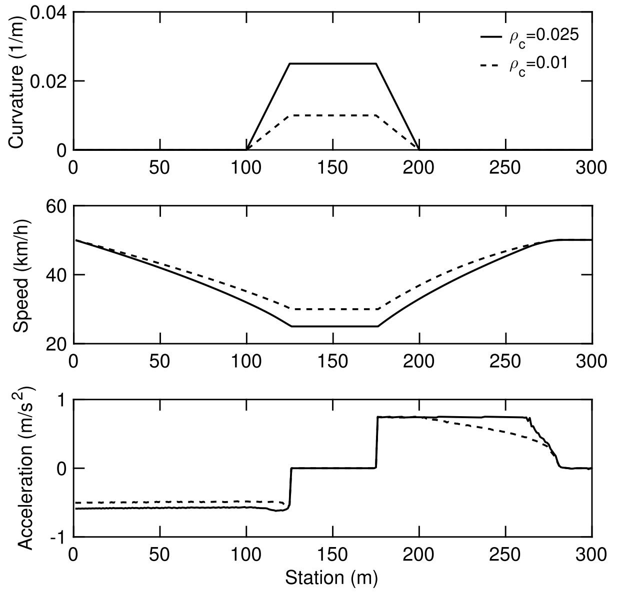

FIGURE 7. Vehicle simulation results for both small and large curvature.

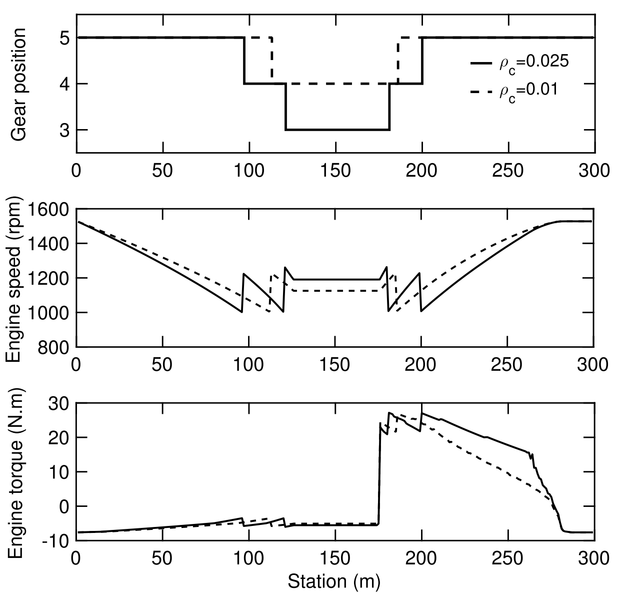

FIGURE 8. Simulation results for both small and large curvature.

maneuver, it starts for a certain duration during the curve, which can be determined by ΔV_a and Δv_a. The gear position of the typical driver model is determined by an Automatic Transmission (AT) in CarSim, where each gear is controlled within a defined range of engine speed and engine throttle settings.

The proportional Integral (PI) controller is utilized within the CarSim environment to track the optimal and typical driver speeds by controlling both throttle and brake inputs. The typical driver parameters used for this evaluation are as follows: $d_r = 0.0.697$, $a_r = 0.73$, $t_d = 0.3124$, and $t_a = 0.316$. Here, $t_d = \Delta v_d/\Delta V_d$ and $t_d = \Delta v_a/\Delta V_a$ represent the proportions of deceleration and acceleration transition lengths, respectively. In the simulation results, the optimal

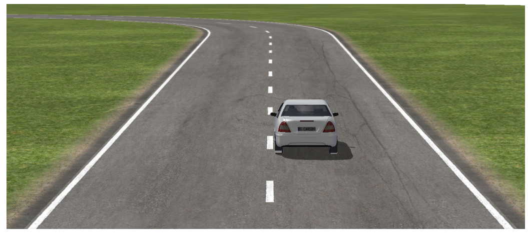

FIGURE 9. Carsim environnement.

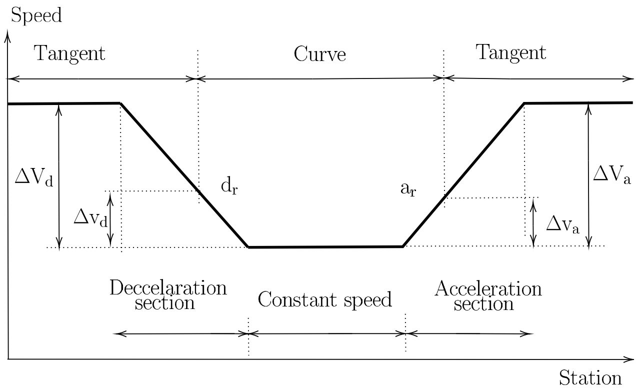

FIGURE 10. Typical driver model.

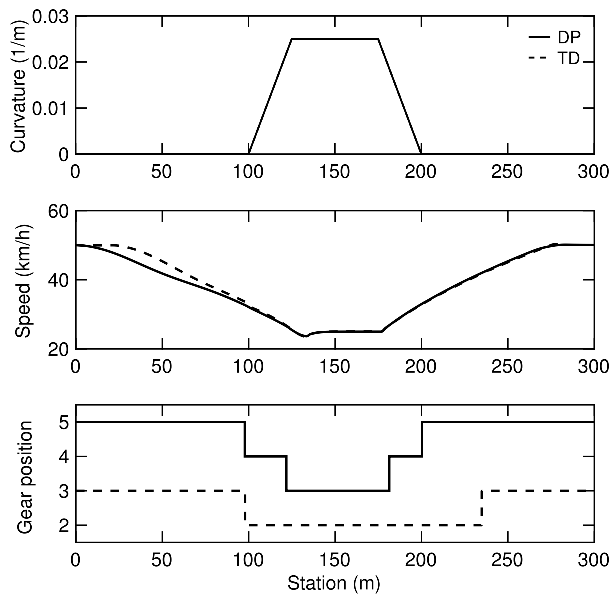

FIGURE 11. CarSim simulation results for large curvature.

solutions, denoted as DP, are depicted with solid lines, while those associated with the typical driver, denoted as TD, are represented by dotted lines.

Fig.11 and Fig.12 show the obtained results for the case of large curvature. By optimizing both speed and gearbox position, the vehicle consumes 0.0632 kg of fuel, whereas the vehicle employing the typical driver model consumes 0.0667 kg of fuel. Using the DP solutions, the vehicle applies an appropriate rate of deceleration before entering the clothoid section by utilizing the engine brake. This is

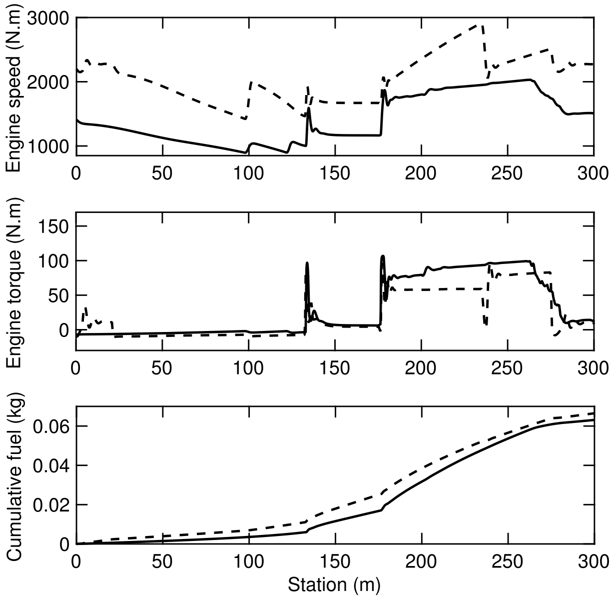

FIGURE 12. CarSim simulation results for large curvature.

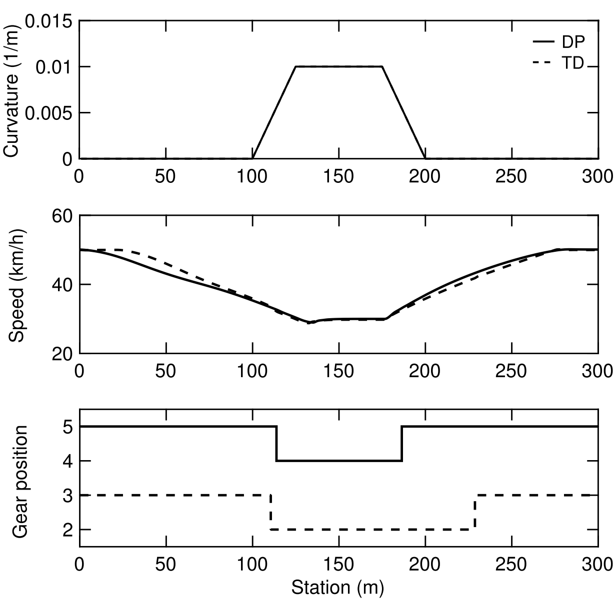

FIGURE 13. CarSim simulation results for small curvature.

particularly important in eco-driving applications because coasting down before taking corners can significantly impact fuel consumption. Then, both vehicles keep the optimal speed (25 km/h) for the overall circular curve. After, the vehicles speed up smoothly to reach and maintain the straight-line speed of 50 km/h. For gear position, the vehicle with DP stays longer at 5-th gear to operate the engine in more efficient operating regions. In addition, it shifts to slower gears late and to higher gears earlier as the vehicle should decelerate and accelerate, respectively. With the typical driver model, the vehicle runs with low gears. In fact, the fixed gear shift schedule of the typical driver vehicle consistently readies the engine for potential accelerations to meet performance

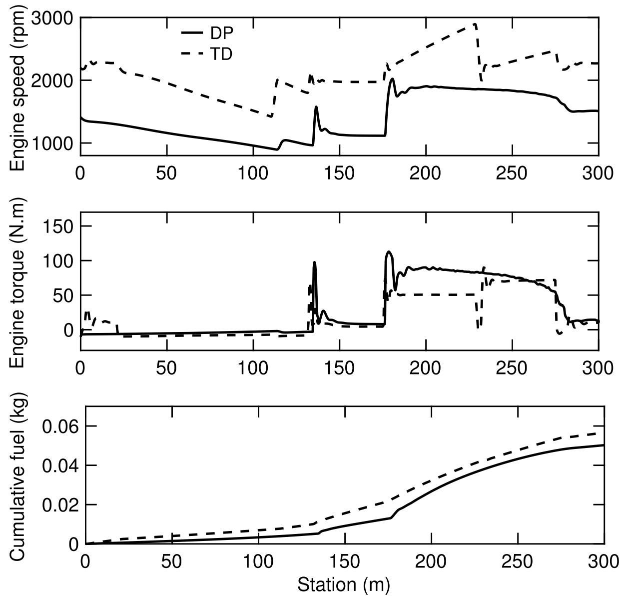

FIGURE 14. CarSim simulation results for small curvature.

demands. Compared with the TD model, the vehicle can realize a fuel savings of 5.25% through the utilization of the proposed approach.

The simulation results of small curvature are shown in Fig.13 and Fig.14. The vehicle using the DP algorithm and TD model consumes 0.0503 kg and 0.0568 kg of fuel, respectively. We can draw the same conclusion that decelerating to the circular speed using the appropriate deceleration rate before entering the circular section and accelerating with smoother acceleration when leaving the curve is a good practice for energy saving. In addition, staying as can as possible at the highest gear is an optimal strategy on curved roads for eco-driving. In comparison to the vehicle driven by the typical driver, the vehicle achieves fuel savings amounting to 11.44%.

VI. CONCLUSION

This paper discussed a control strategy for optimizing vehicle speed and gear position profiles on curved roads to enhance fuel efficiency. Upon receiving the required speed for the circular section, the algorithm calculates optimal vehicle speed and gear position profiles for curves of varying radii, employing realistic engine maps and route information. The simulation results have demonstrated notable fuel economy improvements, ranging from 5.25% to 11.44%, as compared to a typical driver model, for both small and large curves. To achieve energy savings on curved roads, the vehicle needs to decelerate and accelerate at suitable rates before and after entering the circular section, respectively. Furthermore, it's beneficial to maintain the highest possible gear position whenever possible. Future research may involve assessing the controller's performance in more complex traffic scenarios, such as involving the presence of preceding vehicles in a lane-changing scenario.

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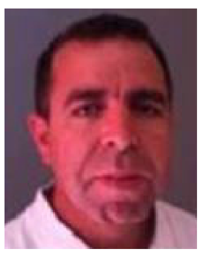

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