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RESEARCH ARTICLE

Finite-Time Adaptive Neural Prescribed Performance Control for High-Order Nonlinearly Parameterized Switched Systems With Unmodeled Dynamics and Input Quantization

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ABSTRACT This study focuses on the adaptive prescribed-time neural control for a class of high-order switched systems with nonlinear parameterization in presence of unmodeled dynamics and quantized input. Different from the existing results on finite-time control on basis of adding a power integrator technique, the controller construction and stability analysis are simplified, and the tracking error remains within a set range over any prescribed time. Under the frame of backstepping design, a state feedback controller is designed. During the controller design procedure, Radial basis function (RBF) neural networks with minimal learning parameters are employed to identify the unknown compounded nonlinear functions, and the control input is quantized. Based on Lyapunov stability theory, the closed-loop system's signals are all assured to be semi-globally uniformly bounded (SGUB), and the tracking error is kept inside a prescribed zone at a finite time. Finally, a numerical simulation is provided to demonstrate the viability and efficacy of the control strategy.

INDEX TERMS High-order nonlinear systems, nonlinear parameterization, switched systems, prescribed-time control, unmodeled dynamics, input quantization.

I. INTRODUCTION

In the past few decades, significant process has been made in the control design and analysis of nonlinear systems since the backstepping technique [1] was proposed. By combining with the powerful approximation ability of neural networks (NNs) or fuzzy logic systems (FLSs), more attention has focused on adaptive control for uncertain nonlinear systems and many remarkable results have been reported. Consequently, remarkable results have been reported in various domains, including systems with strict-feedback forms [2], [3], [4], pure feedback systems [5], [6], [7], as well as time-delay and stochastic systems [8], [9]. When modeling physical systems, especially complexity ones, it is necessary to simply the model for control design and analysis, leading to the occurrence of unmodeled dynamics. It may degrade the system performance or even cause instability in the closed-loop system if ignoring its existence. And naturally, controller design and analysis will bring in more challenge than those systems without unmodeled dynamics and uncertainties. Researchers have investigated state feedback control design for uncertain nonlinear systems with unmodeled dynamics [10], [38], [50], and observer-based control design were discussed in [11], [12], [51], and [52].

Switched systems are made up a collection of continuous or discrete subsystems, as well as a switching law that governs switching for these subsystems [13]. Switched systems have received widespread attention as

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a common type of hybrid system due to their emergence in many practical systems, such as networked systems with a switching mechanism [14] and robot manipulator with variable inertia [15], and so on. Following arbitrary switching, the global probability stabilization for a class of switched stochastic nonlinear systems was examined in [16], tracking control was discussed in [17] for switched uncertain nonlinear systems with pure-feedback form, and in [18] for lower triangular form. In [13] and [20], output feedback control was investigated for a class of uncertain switching nonlinear systems with unmodeled dynamics. The adaptive stabilization problem was investigated for a class of uncertain switched nonlinear systems with linearly parameterized in [21]. High-order nonlinear systems are more general than those with strict-feedback forms and other forms that can be analysized similar to strict-feeback nonlinear systems, and traditional controller design and stability analysis based on quadratic and quadric functions cannot be directly used. For this reason, many efforts have been shifted to explore a novel method for the control design of high-order nonlinear systems, and many remarkable results have been obtained. For example, an analysis of the global finite-time stabilization for a class of switched nonlinear systems with powers of positive odd rational numbers was done in [22], where the nonlinear terms of the systems must meet satisfy the linear growth requirement. Though the linear growth condition was removed in [23], finite-time stabilization and dynamic uncertainties were not taken into consideration.

Recently, some control strategies with prescribed performance for nonlinear systems have been published, which can guarantee the controlled systems' transient performance, and keep the system tracking error within a predetermined range [24], [25]. However, the research results were mostly focused on the infinite cases. Soon afterward, the in-depth studies on finite-time prescribed performance which makes the tracking error convergence in finite-time were developed in [26]. Note that the aforementioned research is conducted by constructing traditional Lyapunov functions for control design targeting the class of strict feedback nonlinear functions. High-order nonlinear systems can provide more accurate descriptions of complex phenomena in the real world, offering a more precise representation of dynamic system behavior. Studying high-order nonlinear systems enables the development of more accurate control strategies to improve system response performance. An investigation on adaptive prescribed performance control for a class of high-order systems with actuator defects was found in [27]. For a class of high-order stochastic nonlinear systems with prescribed performance function of exponential type, the adaptive fuzzy finite-time control problem was addressed in [28].

To reduce the communication rate of control information being transferred over networks, the input is frequently quantized before transmission. The quantization of input signals can be thought of as a map from continuous signals to discrete finite sets. The control signal to the plant in quantized control systems is a piece-wise constant function of time and the system interacts with information quantization [29]. Significant progress has been made in recent years in the field of input quantization control. For instance, adaptive tracking control for uncertain strict-feedback nonlinear systems was addressed in [30]. For high-order nonlinear uncertain systems with input quantization, output tracking control problem was covered in [31]. For stochastic nonlinear nonstrict-feedback systems with full state constraints, the adaptive fuzzy finite-time quantized control problem was examined in [32]. Input quantization-based finite-time tracking control for uncertain interconnected nonlinear systems was researched in [33]. Different from input quantization control, event-driven control triggers and transmits data based on preset conditions. An adaptive backstepping control scheme for nonlinear interconnected systems was presented in [53], aiming to achieve desired system performance by utilizing prespecified performance driven output triggering. The study in [54] introduced event-triggered reference governors that facilitate collisions-free coordination between leaders and followers, even in systems with unreliable communication topologies. [55] addressed the dynamic event-triggered-based adaptive output-feedback tracking control problem in nonlinear multiagent systems with time-varying delay.

According to our knowledge, there are few reports on the outcomes of finite-time prescribed performance control of non-linear parameterized high-order switched non-linear systems in presence of unmodeled dynamics, unknown disturbances together with input quantization. The main contributions of this paper are as follows.

(1) The study is more general compared with the switched systems in strict-feedback form [19], [34], [43] or systems modeled with linear parameterization [21], [27], [46], and the linear growth condition on the unknown parameter is removed during the controller design. Additionally, unmodeled dynamics and unknown disturbances are taken into consideration, which improves the practical values of the study and also the difficulty is improved.

(2) A finite-time prescribed performance controller is constructed to guarantee the output tracking error remains within a predefined region at a finite time and all signals in the closed-loop system are SGUB. Additionally, the communication burden is reduced by introducing hysteresis quantize to decrease the communication rate of the actual input signal, implying that the transmission efficiency is improved and furthermore the chatting phenomenon is avoided.

(3) The computational burden is reduced by estimating the maximal norm of weight vectors of the employed RBFNN basis functions of all subsystems, meaning that only one parameter needs estimating online at each recursive step.

(4) Control design is simpler to achieve the objectives with the sufficient condition $\dot{V} \leq -\alpha_0 V + \mu$ than the traditional finite-time control design method to guarantee $\dot{V} \leq -\alpha_0 V^{\gamma} + \mu$ with $\gamma \in (0, 1)$ [43], [47], [48], which further reduce the computational burden.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are given in Section II. RBF neural networks and controller design are shown in Section III. In Section IV, Stability of the closed-loop system is analysed. Simulation results are given to demonstrate the effectiveness of the proposed control scheme in Section V. Section VI draws the conclusions of this paper.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. SYSTEM DESCRIPTION

Consider the following high-order switched uncertain nonlinear system in presence of unmodeled dynamics and input quantization

$$\begin{cases} \dot{z} = q_{\sigma(t)}(z, y) \\ \dot{x}_i = x_{i+1}^{p_i} + f_{i,\sigma(t)}(\bar{x}_i, \theta) + \Delta_{i,\sigma(t)}(y, z, t), \\ i = 1, 2, \cdots, n-1 \\ \dot{x}_n = q^{p_n}(u(t)) + f_{n,\sigma(t)}(x, \theta) + \Delta_{n,\sigma(t)}(y, z, t) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}} \in R^i$ and $x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in R^n$ are state vectors; $u \in R$ and $y \in R$ are input and output of the system, respectively. For $i = 1, 2, \cdots, n$, $p_i \in R^{\mathrm{odd}}_{\geq 1} \underline{\underline{\Delta}} \{\lambda \in R : \lambda \geq 1$ is a ratio of odd intergers$\}$, the function $\sigma(t) : [0, +\infty) \to M = \{1, 2, \cdots, m\}$ is the switching signal which is assumed to be a piecewise continuous function of time and where m is the number of subsystems; θ is an unknown parameter. For $\forall k \in M$ and $i = 1, 2, \cdots, n$, $f_{i,k}(\bar{x}_i, \theta) : R^{i+1} \to R^r$ are unknown continuous functions with $f_{i,k}(0, \cdots, 0) = 0$, $\Delta_{i,k}(y, z, t)$ are unknown uncertain disturbances, and $z \in R^{n_0}$ stands for unmodeled dynamics.

The goal of the control strategy is to build an adaptive tracking controller based on a finite-time performance function such that all closed-loop system signals are SGUB and the tracking error converges to a predetermined zone at a finite-time.

To achieve the controlled objective, the following assumptions are made.

Assumption 1 [26], [34], [35]: For $i = 1, 2, \cdots, n$ and $\forall k \in M$, there exist uncertain non-negative smooth functions $\varphi_{i1,k}(\cdot)$ and $\varphi_{i2,k}(\cdot)$ such that

$$\left|\Delta_{i,k}(y, z, t)\right| \leq \varphi_{i1,k}(|z|) + \varphi_{i2,k}(y) \tag{2}$$

Assumption 2 [35], [36], [37]: System $\dot{z} = q_k(z, y)$, $\forall k \in M$ is exponentially input-to-state practically stable (Exp-ISpS), if there exists a Lyapunov function $V_0(z)$ satisfying

$$\gamma_1(|z|) \leq V_0(z) \leq \gamma_2(|z|) \tag{3}$$

$$\frac{\partial V_0(z)}{\partial z} q_k(z, y) \leq -aV_0(z) + \gamma_{3,k}(|y|) + b \tag{4}$$

where a and b being positive constants, and $\gamma_1(\cdot)$, $\gamma_2(\cdot)$ and $\gamma_{3,k}(\cdot)$ being κ_∞-functions.

Assumption 3 [26], [44]: The reference signal $y_d(t)$ and its r-order derivative $y_d^{(r)}$ are known and continuously bounded for $r = 1, \cdots, n$.

Lemma 1: [35], [38] If conditions (3) and (4) are satisfied, indicating that V_0 is an exp-ISpS Lyapunov function for a system $\dot{z} = q_k(z, y)$, then, for any constants $\bar{c} \in (0, a)$ and $r_0 > 0$, any initial condition $z_0 = z(t_0)$ with $\forall t_0 > 0$, any continuous function $\bar{\gamma}$ satisfying $\bar{\gamma}(|y|) \geq \gamma(|y|)$, there exists a finite time $T_0 > \max\left\{0, \ln\left(\frac{V(\xi_0)}{r_0}\right) \Big/ (a - \bar{c})\right\} \geq 0$, a nonnegative function $D(t_0, t)$, defined for all $t \geq t_0$ and a signal described by

$$\dot{v} = -\bar{c}v + \bar{\gamma}(|x_1|) + d,\ v(t_0) = v_0 \tag{5}$$

such that $D(t_0, t) = 0$ for $t \geq t_0 + T_0$, and $V_0(z) \leq v(t) + D(t_0, t)$ with $D(t_0, t) = \max\left\{0, e^{-a(t-t_0)}V_0(z_0) - e^{-\bar{c}(t-t_0)}r_0\right\}$. Without loss of generality, assume that $\bar{\gamma}(s) = s^2\gamma_0(s^2)$, then one has

$$\dot{v} = -\bar{c}v + x_1^2\gamma_0(x_1^2) + d,\ v(t_0) = v_0 \tag{6}$$

where $\gamma_0(\cdot)$ is a non-negative smooth function.

Lemma 2: [39], [40] For the constants $\lambda > 1$ and $\mu > 0$, define the set $\Omega_\delta = \{\delta \in R \,||\delta| < \iota\mu\}$ with $\iota = arc\tanh(\sqrt[\lambda]{1/\lambda})$. Then, for all $\delta \notin \Omega_\delta$, the inequality $1 - \lambda \tanh^\lambda(\delta/\mu) \leq 0$ holds.

Lemma 3 ([49]): Let $p \in R^{odd}_{\geq 1}$ and x, y be any real numbers. For a positive constant c, one has $|x^p - y^p| \leq c\,|x - y| \times \left|(x - y)^{p-1} + y^{p-1}\right|$.

Lemma 4 ([41]): For any positive real numbers m and n, any real-valued function $a(x, y) > 0$, there exists a positive function $c(x, y)$ such that

$$\left|a(x, y)x^m y^n\right| \leq c(x, y)\,|x|^{m+n} + \frac{n}{m+n}\left(\frac{m}{(m+n)c(x, y)}\right)^{\frac{m}{n}} |a(x, y)|^{\frac{m+n}{n}}\,|y|^{m+n} \tag{7}$$

Lemma 5 [42], [43]: For any variable $\eta \in R$ and constant $\pi > 0$, the following inequality holds:

$$0 \leq |\eta| - \eta\tanh\left(\frac{\eta}{\pi}\right) \leq \delta\pi,\ \delta = 0.2785 \tag{8}$$

B. PRESCRIBED PERFORMANCE TRANSFORMATION (PPT)

Definiton 1 [26], [44]: A smooth function $\rho(t)$ is denoted as a performance function when the following properties hold:

a) $\rho(t) > 0$

b) $\dot{\rho}(t) \leq 0$

c) $\lim_{t \to T_f} \rho(t) = \rho_{T_f} > 0$ and $\rho(t) = \rho_{T_f}$ for $\forall t \geq T_f$,

where ρ_{T_f} and T_f are any small constant and the setting time, respectively.

Following Definition 1, we can rewrite the performance function as below

$$\rho(t) = \begin{cases} \left(\rho_0 - \dfrac{t}{T_f}\right) e^{\left(1 - \frac{T_f}{T_f - t}\right)} + \rho_{T_f},\ t \in [0, T_f) \\ \rho_{T_f},\ t \in [T_f, +\infty) \end{cases} \tag{9}$$

where $\rho_0 \geq 1$ and $\rho_{T_f} > 0$ are specified parameters.

To guarantee that the output tracking error $e_1(t) = y(t) - y_d(t)$ always stays inside a specified prescribed performance bound, or to meet the prescribed performance requirements. An error transformation function $S(z_1)$ is designed as

$$S(z_1)=\frac{e^{z_1}-e^{-z_1}}{e^{z_1}+e^{-z_1}} \tag{10}$$

where z_1 is the transformed error.

With the tracking error requirements $-l_1\rho(t) < e_1(t) < l_2\rho(t)$, where $-l_1$ and l_2 being design constants, the tracking error can be rewritten as $e_1(t)=\rho(t)S(z_1)$. And then, the time derivation of $e_1(t)$ becomes

$$\dot{e}_1(t)=\dot{\rho}(t)S(z_1)+\rho(t)\frac{\partial S(z_1)}{\partial z_1}\dot{z}_1(t) \tag{11}$$

In fact,

$$\begin{aligned}\dot{z}_1(t)&=\frac{\dot{e}_1(t)-\dot{\rho}(t)S(z_1)}{\rho(t)\frac{\partial S(z_1)}{\partial z_1}}\\&=-\frac{\dot{\rho}(t)S(z_1)}{\rho(t)\frac{\partial S(z_1)}{\partial z_1}}+\frac{1}{\rho(t)\frac{\partial S(z_1)}{\partial z_1}}(x_2^{p_1}+f_{1,\sigma(t)}(x_1,\theta)\\&\quad+\Delta_{1,\sigma(t)}(y,z,t)-\dot{y}_d)\\&=\Upsilon+\Gamma(x_2^{p_1}+f_{1,\sigma(t)}(x_1,\theta)+\Delta_{1,\sigma(t)}(y,z,t)-\dot{y}_d)\end{aligned} \tag{12}$$

Thus, it can be obtained that

$$\begin{cases}\dot{z}=q_{\sigma(t)}(z,y)\\ \dot{z}_1(t)=\Upsilon+\Gamma(x_2^{p_1}+f_{1,\sigma(t)}(x_1,\theta)+\Delta_{1,\sigma(t)}(y,z,t)-\dot{y}_d)\\ \dot{x}_i=x_{i+1}^{p_i}+f_{i,\sigma(t)}(\bar{x}_i,\theta)+\Delta_{i,\sigma(t)}(y,z,t),\ i=2,\cdots,n-1\\ \dot{x}_n=q^{p_n}(u(t))+f_{n,\sigma(t)}(x,\theta)+\Delta_{n,\sigma(t)}(y,z,t)\\ y=x_1\end{cases} \tag{13}$$

C. QUANTIZED INPUT

According to references [31] and [45], the quantized input $q(u)$ of the controlled system can be expressed as below

$$q(u)=\begin{cases}u_i\mathrm{sgn}(u),\ \frac{u_i}{1+\kappa}<|u|\le u_i,\ \dot{u}<0,\\ \qquad\text{or } u_i<|u|\le\frac{u_i}{1-\kappa},\ \dot{u}>0\\ u_i(1+\kappa)\mathrm{sgn}(u),\ u_i<|u|\le\frac{u_i}{1-\kappa},\ \dot{u}<0,\\ \text{or}\frac{u_i}{1-\kappa}<|u|\le\frac{u_i(1+\kappa)}{1-\kappa},\ \dot{u}>0\\ 0,\ 0\le|u|<\frac{u_{\min}}{1+\kappa},\ \dot{u}<0,\text{ or}\frac{u_{\min}}{1+\kappa}\le|u|\le u_{\min},\\ \dot{u}>0\\ q(u(t^-)),\ \text{othercase}\end{cases} \tag{14}$$

where $u_i=v^{1-i}u_{\min}$ $(i=1,2,\cdots,n)$ and $\kappa=\frac{1-v}{1+v}$ with $u_{\min}>0$ and $0<v<1$. $q(u)$ locates into the set $U=\{0,\pm u_i,\pm u_i(1+\kappa), i=1,2,\cdots,n\}$, $u_{\min}>0$ is a dead-zone range of $q(u)$, and v denotes the measure of quantization density. And $q(u)$ can be divided into two parts as follows

$$q^{p_n}(u(t))=S(u)u^{p_n}(t)+K(t) \tag{15}$$

where $S(u)$ and $K(t)$ satisfy

$$(1-\kappa)^{p_n}\le S(u)\le(1+\kappa)^{p_n},\ |K(t)|\le u_{\min}^{p_n} \tag{16}$$

III. RBF NEURAL NETWORKS AND CONTROLLER DESIGN

In this section, we use RBF Neural Networks to approximate the unknown compounded nonlinear functions arising from the unknown functions and unmodeled dynamics of system (1) during the controller design process. They are of the general form $h(x)=w^{\mathrm{T}}\zeta(x)$, where $\zeta(x)=[\zeta_1(x),\zeta_2(x),\cdots,\zeta_p(x)]^{\mathrm{T}}\in R^p$ being a vector-valued function and $\zeta_i(x)$ as Gaussian functions of the form $\zeta_i(x)=\exp\left(-(x-\mu_i)^{\mathrm{T}}(x-\mu_i)/b_i^2\right)$ with $\mu_i=[\mu_{i1},\mu_{i2},\cdots,\mu_{im}]^{\mathrm{T}}$ being the center of the basis function and b_i the width of the basis function, $i=1,2,\cdots,p$. $w\in R^p$ represents the weight vector. Generally speaking, for any given smooth function $h:\Omega\to R$, where Ω is a compact subset of R^m (m is an appropriate integer) and $\varepsilon>0$, it can be approximated by means of RBF neural networks, that is, there exists a basis function vector $\zeta:R^m\to R^p$ and a weight vector $w^*\in R^p$ such that $\sup\left|h(x)-w^{*T}\zeta(x)\right|\le\varepsilon^*$, $\forall x\in\Omega$. The quantity $h(x)-w^{*T}\zeta(x)=\varepsilon(x)$ is called the network reconstruction error and $|\varepsilon(x)|\le\varepsilon^*$.

The optimal weight vector w^* defined above is a quantity only for analytical purposes. Typically, w^* is chosen as the value of $\mathbf{w}$ that minimizes $\varepsilon(x)$ over Ω, that is

$$w^*=\arg\min_{w\in R^p}\left\{\sup_{x\in\Omega}\left|h(x)-w^{\mathrm{T}}\zeta(x)\right|\right\}.$$

In what follows, an adaptive neural network semi-globally practical finite-time tracking controller is designed for nonlinear system (13). First, the following change of coordinates will be defined

$$\begin{cases}z_1=z_1\\ z_i=x_i-\alpha_{i-1}\end{cases} \tag{17}$$

where $\alpha_{i-1}, i=2,\cdots,n$ is the intermediate control function, the controller u will be designed in the last step.

The detailed control design procedure is given in the following.

Step 1: Differentiating z_1 with respect to time t in the first subsystem yields

$$\dot{z}_1=\Upsilon+\Gamma(x_2^{p_1}+f_{1,\sigma(t)}(x_1,\theta)+\Delta_{1,\sigma(t)}(y,z,t)-\dot{y}_d). \tag{18}$$

Choose the candidate Lyapunov function as $V_1=W_1+\frac{1}{2\delta_1}\tilde{\Theta}_1^2+\frac{v}{\lambda_0}$, where $W_1=\frac{z_1^{p-p_1+2}}{p-p_1+2}$, $\delta_1>0$ and $\lambda_0>0$ are design parameters, $\hat{\Theta}_1$ is the estimations of Θ_1^*, $\tilde{\Theta}_1=\Theta_1^*-\hat{\Theta}_1$ is the estimation error.

By Assumption 1, one has

$$z_1^{p-p_1+1}\Gamma\Delta_{1,k}\le z_1^{p-p_1+1}\Gamma\left(\varphi_{11,k}(|z|)+\varphi_{12,k}(y)\right). \tag{19}$$

From Assumption 2 and Lemma 1, we know that there exists an increasing function $\gamma_1^{-1}(\cdot)$ such that

$$|z|\le\gamma_1^{-1}(v(t)+D(t_0,t)) \tag{20}$$

Let $\bar{\Phi}_{11,k} = \varphi_{11,k} \circ \gamma_1^{-1}(2v) > 0$, then one has

$$\begin{aligned} & z_1^{p-p_1+1}\Gamma\Delta_{1,k} \\ & \quad \le z_1^{p-p_1+1}\Gamma[\varphi_{11,k} \circ \gamma_1^{-1}(v(t)+D(t_0,t))] \\ & \qquad + z_1^{p-p_1+1}\Gamma\varphi_{12,k}(y) \\ & \quad \le z_1^{p-p_1+1}\Gamma\bar{\Phi}_{11,k} + z_1^{p-p_1+1}\Gamma\varphi_{11,k} \circ \gamma_1^{-1}(2D) \\ & \qquad + z_1^{p-p_1+1}\Gamma\varphi_{12,k}(y) \end{aligned} \tag{21}$$

Utilizing Lemmas 4 and 5, one has

$$\begin{aligned} & z_1^{p-p_1+1}\Gamma\Delta_{1,k} \\ & \le z_1^{p+1}\Gamma^{\frac{p+1}{p-p_1+1}}\left(\bar{\Phi}_{11,k}\tanh\left(\frac{z_1^{p-p_1+1}\Gamma\bar{\Phi}_{11,k}}{\bar{d}_{11}}\right)\right)^{\frac{p+1}{p-p_1+1}} \\ & \quad + \frac{2p_1}{p+1} + z_1^{p+1}\Gamma^{\frac{p+1}{p-p_1+1}}(\varphi_{12,k}(\|x\|))^{\frac{p+1}{p-p_1+1}} \\ & \quad + z_1^{p+1}\Gamma^{\frac{p+1}{p-p_1+1}} \\ & \quad + \frac{p_1}{p+1}(\varphi_{11,k} \circ \gamma_1^{-1}(2D))^{\frac{p+1}{p_1}} + 0.2785\bar{d}_{11} \end{aligned} \tag{22}$$

with $\bar{d}_{11}$ being a design parameter.

Denote $\hat{\Phi}_{11,k} = \left(\bar{\Phi}_{11,k}\tanh\left(\frac{z_1^{p-p_1+1}\Gamma\bar{\Phi}_{11,k}}{\bar{d}_{11}}\right)\right)^{\frac{p+1}{p-p_1+1}}$, $d_{12,k}(t_0,t) = \frac{p_1}{p+1}(\varphi_{11,k} \circ \gamma_1^{-1}(2D))^{\frac{p+1}{p_1}}$, $\hat{\Phi}_{12,k} = (\varphi_{12,k}(\|x\|))^{\frac{p+1}{p-p_1+1}}$, $d_{11} = 0.2785\bar{d}_{11}$. Then, one has

$$\begin{aligned} z_1^{p-p_1+1}\Gamma\Delta_{1,k} \le & \; z_1^{p+1}\Gamma^{\frac{p+1}{p-p_1+1}}\left(\hat{\Phi}_{11,k} + \hat{\Phi}_{12,k} + 1\right) \\ & + d_{12,k}(t_0,t) + \frac{2p_1}{p+1} + d_{11} \end{aligned} \tag{23}$$

It should be pointed that $d_{12,k}(t_0,t) = 0$ for any $t \ge t_0 + T_0$, and γ_1^{-1} is the inverse function of γ_1.

Subsequently, it yields (24) from the result of differentiating V_1 as below

$$\begin{aligned} \dot{V}_1 = & \; z_1^{p-p_1+1}\Gamma(x_2^{p_1} - \alpha_1^{p_1}) + z_1^{p-p_1+1}\Gamma\alpha_1^{p_1} \\ & + z_1^{p-p_1+1}\bar{f}_{1,k}(Z_1) + d_{12,k}(t_0,t) \\ & + \frac{2p_1}{p+1} + d_{11} - \frac{1}{\delta_1}\tilde{\Theta}_1\dot{\hat{\Theta}}_1 - \frac{\bar{c}v}{\lambda_0} \\ & + \left(1 - (p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right) \\ & \times \frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0} + \frac{d}{\lambda_0} \end{aligned} \tag{24}$$

where

$$\begin{aligned} \bar{f}_{1,k}(Z_1) = & \; \Upsilon + \Gamma f_{1,k}(x_1,\theta) - \Gamma\dot{y}_d \\ & + \frac{p-p_1+2}{z_1^{p-p_1+1}}\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0} \\ & + z_1^{p_1}\Gamma^{\frac{p+1}{p-p_1+1}}\left(\hat{\Phi}_{11,k} + \hat{\Phi}_{12,k} + 1\right) \end{aligned}$$

is an unknown nonlinear function since $f_{1,k}(x_1,\theta)$ is unknown, and it can be approximated by RBFNN, that is, $\bar{f}_{1,k}(Z_1) = \theta_{1,k}^{*}\varphi_{1,k}(Z_1) + \varepsilon_{1,k}(Z_1)$, where $Z_1 = [x_1, y_d, \dot{y}_d, \rho, \dot{\rho}]$. Thus, one has

$$\begin{aligned} \dot{V}_1 = & \; z_1^{p-p_1+1}\Gamma(x_2^{p_1} - \alpha_1^{p_1}) + z_1^{p-p_1+1}\Gamma\alpha_1^{p_1} \\ & + z_1^{p-p_1+1}(\theta_{1,k}^{*}\varphi_1(Z_1) \\ & + \varepsilon_{1,k}(Z_1)) + d_{12,k}(t_0,t) + \frac{2p_1}{p+1} \\ & + d_{11} - \frac{1}{\delta_1}\tilde{\Theta}_1\dot{\hat{\Theta}}_1 - \frac{\bar{c}v}{\lambda_0} \\ & + \left(1 - (p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0} + \frac{d}{\lambda_0} \end{aligned} \tag{25}$$

Using Lemma 4, one has

$$z_1^{p-p_1+1}\theta_{1,k}^{*}\varphi_1(Z_1) \le z_1^{p+1}\Theta_1^{*}\|\varphi_1(Z_1)\|^{\frac{p+1}{p-p_1+1}} + \frac{p_1}{p+1} \tag{26}$$

and

$$z_1^{p-p_1+1}\varepsilon_{1,k}(Z_1) \le z_1^{p+1} + \frac{p_1}{p+1}\varepsilon_1^{*} \tag{27}$$

where $\Theta_1^{*} = \max_{k\in M}\left\{\left\|\theta_{1,k}^{*}\right\|^{\frac{p+1}{p-p_1+1}}\right\}$ and $\varepsilon_1^{*} = \max_{k\in M}\left\{\varepsilon_{1,k}^{*\frac{p+1}{p_1}}\right\}$.

From the above analysis yields

$$\begin{aligned} \dot{V}_1 \le & \; z_1^{p-p_1+1}\Gamma(x_2^{p_1} - \alpha_1^{p_1}) + z_1^{p-p_1+1}\Gamma\alpha_1^{p_1} \\ & + z_1^{p+1}\Theta_1^{*}\|\varphi_1(Z_1)\|^{\frac{p+1}{p-p_1+1}} \\ & + \frac{3p_1}{p+1} + z_1^{p+1} + \frac{p_1}{p+1}\varepsilon_1^{*} + d_{12,k}(t_0,t) \\ & + d_{11} - \frac{1}{\delta_1}\tilde{\Theta}_1\dot{\hat{\Theta}}_1 \\ & - \frac{\bar{c}v}{\lambda_0} + \left(1 - (p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right) \\ & \frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0} + \frac{d}{\lambda_0} \\ = & \; z_1^{p-p_1+1}\Gamma(x_2^{p_1} - \alpha_1^{p_1}) + z_1^{p-p_1+1} \\ & \left(\Gamma\alpha_1^{p_1} + z_1^{p_1} + z_1^{p_1}\hat{\Theta}_1\|\varphi_1(Z_1)\|^{\frac{p+1}{p-p_1+1}}\right) \\ & - \frac{1}{\delta_1}\tilde{\Theta}_1\left(\dot{\hat{\Theta}}_1 - \delta_1 z_1^{p+1}\|\varphi_1(Z_1)\|^{\frac{p+1}{p-p_1+1}}\right) \\ & - \frac{\bar{c}v}{\lambda_0} + (1 - (p-p_1+2) \\ & \times \tanh^{p-p_1+2}\left(\frac{z_1}{v}\right))\frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0} + d_{1,k}(t_0,t) \end{aligned} \tag{28}$$

where $d_{1,k}(t_0,t) = \frac{p_1}{p+1}\varepsilon_1^{*} + d_{12,k}(t_0,t) + \frac{3p_1}{p+1} + d_{11} + \frac{d}{\lambda_0}$.

Choose

$$\alpha_1 = -z_1\left(\Gamma^{-1}c_1 + 1 + \hat{\Theta}_1\|\varphi_1(Z_1)\|^{\frac{p+1}{p-p_1+1}}\right)^{\frac{1}{p_1}} = -z_1\beta_1 \tag{29}$$

$$\dot{\hat{\Theta}}_1 = \delta_1 z_1^{p+1} \|\varphi_1(Z_1)\|^{\frac{p+1}{p-p_1+1}} - \sigma_1 \hat{\Theta}_1 \tag{30}$$

where $\beta_1 = \left(\Gamma^{-1} c_1 + 1 + \hat{\Theta}_1 \|\varphi_1(Z_1)\|^{\frac{p+1}{p-p_1+1}}\right)^{\frac{1}{p_1}}$.

Then, one has

$$\begin{aligned} \dot{V}_1 \le\ & z_1^{p-p_1+1}\Gamma(x_2^{p_1} - \alpha_1^{p_1}) - c_1 z_1^{p+1} + \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 - \frac{\bar{c}v}{\lambda_0} \\ & + \left(1 - (p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} \\ & + d_1(t_0,t) \end{aligned} \tag{31}$$

Combining with Lemmas 3 and 5, one has

$$\begin{aligned} & z_1^{p-p_1+1}\Gamma(x_2^{p_1} - \alpha_1^{p_1}) \\ & \quad \le c\,|\Gamma| \left|z_1^{p-p_1+1}\right| |z_2| \left|z_2^{p_1-1} + \alpha_1^{p_1-1}\right| \\ & \quad = c\,|\Gamma|\,|z_1|^{p-p_1+1}|z_2|^{p_1} + c\left|\beta_1^{p_1-1}\right| |\Gamma|\,|z_1|^{p}\,|z_2| \\ & \quad \le z_1^{p+1} + h_{10} z_2^{p+1} \end{aligned} \tag{32}$$

where $h_{10} = \frac{p_1}{p+1}\left(\frac{2(p-p_1+1)}{(p+1)}\right)^{\frac{p-p_1+1}{p_1}}(c\,|\Gamma|)^{\frac{p+1}{p_1}} + \frac{1}{p+1}\left(\frac{2p}{p+1}\right)^{p} \times \left(c\left|\beta_1^{p_1-1}\right||\Gamma|\right)^{p+1}$ is a continuous function. Then, one has

$$\begin{aligned} \dot{V}_1 \le\ & -(c_1-1)z_1^{p+1} + h_{10}z_2^{p+1} + \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 - \frac{\bar{c}}{\lambda_0}v \\ & + \left(1 - (p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} \\ & + d_1(t_0,t) \end{aligned} \tag{33}$$

Step 2: Choose the Lyapunov function $V_2 = V_1 + W_2 + \frac{1}{2\delta_2}\tilde{\Theta}_2^2$, where $\delta_2 > 0$ is a design parameter, $\tilde{\Theta}_2 = \Theta_2^* - \hat{\Theta}_2$, $W_2 = \frac{z_2^{p-p_2+2}}{p-p_2+2}$ is the continuous-differential function about $\bar{x}_2$, $\hat{\Theta}_1$ and $\hat{\lambda}$.

According to the definition of z_2, one has $z_2 = x_2 - \alpha_1$, and then

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 \\ &= x_3^{p_2} + f_{2,k}(\bar{x}_2, d) + \Delta_{2,k}(z,x) - \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 - \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d \\ &\quad - \frac{\partial\alpha_1}{\partial\dot{y}_d}\ddot{y}_d - \frac{\partial\alpha_1}{\partial\rho}\dot{\rho} - \frac{\partial\alpha_1}{\partial\dot{\rho}}\ddot{\rho} - \frac{\partial\alpha_1}{\partial\hat{\Theta}_1}\dot{\hat{\Theta}}_1 \end{aligned} \tag{34}$$

and

$$\begin{aligned} z_2^{p-p_2+1}\Delta_{2,k} \le\ & z_2^{p+1}\left(\hat{\Phi}_{21,k} + \hat{\Phi}_{22,k} + 1\right) \\ & + d_{22,k}(t_0,t) + \frac{2p_2}{p+1} + d_{21} \end{aligned} \tag{35}$$

where $\hat{\Phi}_{21,k} = \left(\bar{\Phi}_{21,k}\tanh\left(\frac{z_2^{p-p_2+1}\bar{\Phi}_{21,k}}{\bar{d}_{21}}\right)\right)^{\frac{p+1}{p-p_2+1}}$, $d_{21} = 0.2785\bar{d}_{21}$, $\hat{\Phi}_{22,k} = (\varphi_{22,k}(\|x\|))^{\frac{p+1}{p-p_2+1}}$, $d_{22,k}(t_0,t) = \frac{p_2}{p+1}(\varphi_{21,k}\circ\gamma_1^{-1}(2D))^{\frac{p+1}{p_2}}$, $\bar{d}_{21}$ is a design parameter.

From the above analysis, we have

$$\begin{aligned} \dot{V}_2 =\ & \dot{V}_1 + z_2^{p-p_2+1}\dot{z}_2 - \frac{1}{\delta_2}\tilde{\Theta}_2\dot{\hat{\Theta}}_2 \\ \le\ & -(c_1-1)z_1^{p+1} + \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 - \frac{\bar{c}}{\lambda_0}v \\ & + \left(1 - (p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right) \\ & \times \frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} + d_1(t_0,t) + z_2^{p-p_2+1}(x_3^{p_2} - \alpha_2^{p_2}) \\ & + z_2^{p-p_2+1}\alpha_2^{p_2} + z_2^{p-p_2+1} \\ & \times \bar{f}_{2,k}(\bar{x}_2,d) + d_{22,k}(t_0,t) + \frac{2p_2}{p+1} + d_{21} - \frac{1}{\delta_2}\tilde{\Theta}_2\dot{\hat{\Theta}}_2 \end{aligned} \tag{36}$$

where

$$\begin{aligned} \bar{f}_{2,k}(Z_2) =\ & f_{2,k}(\bar{x}_2,d) - \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 - \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d - \frac{\partial\alpha_1}{\partial\dot{y}_d}\ddot{y}_d \\ & - \frac{\partial\alpha_1}{\partial\rho}\dot{\rho} - \frac{\partial\alpha_1}{\partial\dot{\rho}}\ddot{\rho} - \frac{\partial\alpha_1}{\partial\hat{\Theta}_1}\dot{\hat{\Theta}}_1 \\ & + z_2^{p_2}\left(\hat{\Phi}_{21,k} + \hat{\Phi}_{22,k} + 1\right) + h_{10}z_2^{p_2} \end{aligned}$$

and $Z_2 = [x_1, x_2, y_d, \dot{y}_d, \ddot{y}_d, \rho, \dot{\rho}, \ddot{\rho}, \hat{\Theta}_1]$. Similar to the previous analysis, it can be obtained that $\bar{f}_{2,k}(Z_2)$ can be approximated by RBFNN, that is, $\bar{f}_{2,k}(Z_2) = \theta_{2,k}^*\varphi_{2,k}(Z_2) + \varepsilon_{2,k}(Z_2)$.

Using Lemma 4, one has

$$z_2^{p-p_2+1}\theta_{2,k}^*\varphi_2(Z_2) \le z_2^{p+1}\Theta_2^*\|\varphi_2(Z_2)\|^{\frac{p+1}{p-p_2+1}} + \frac{p_2}{p+1} \tag{37}$$

$$z_2^{p-p_2+1}\varepsilon_{2,k}(Z_2) \le z_2^{p+1} + \frac{p_2}{p+1}\varepsilon_2^* \tag{38}$$

where $\Theta_2^* = \max_{k\in M}\left\{\left\|\theta_{2,k}^*\right\|^{\frac{p+1}{p-p_2+1}}\right\}$ and $\varepsilon_2^* = \max_{k\in M}\{\varepsilon_{2,k}^{*\frac{p+1}{p_2}}\}$.

Thus, one has

$$\begin{aligned} \dot{V}_2 \le\ & -(c_1-1)z_1^{p+1} + z_2^{p-p_2+1}(x_3^{p_2} - \alpha_2^{p_2}) + \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 - \frac{\bar{c}}{\lambda_0}v \\ & + \left(1 - (p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} \\ & - \frac{1}{\delta_2}\tilde{\Theta}_2\left(\dot{\hat{\Theta}}_2 - \delta_2 z_2^{p+1}\|\varphi_2(Z_2)\|^{\frac{p+1}{p-p_2+1}}\right) \\ & + z_2^{p-p_2+1}\left(\alpha_2^{p_2} + h_{10}z_2^{p_2} + z_2^{p_2} + z_2^{p_2}\hat{\Theta}_2\|\varphi_2(Z_2)\|^{\frac{p+1}{p-p_2+1}}\right) \\ & + d_{1,k}(t_0,t) + \frac{p_2}{p+1}\varepsilon_2^* + d_{22,k}(t_0,t) + \frac{3p_2}{p+1} + d_{21} \end{aligned} \tag{39}$$

Choose

$$\alpha_2 = -z_2\left(c_2 + \hat{\Theta}_2\|\varphi_2(Z_2)\|^{\frac{p+1}{p-p_2+1}} + 1\right)^{\frac{1}{p_2}} = -z_2\beta_2 \tag{40}$$

$$\dot{\hat{\Theta}}_2 = \delta_2 z_2^{p+1} \|\varphi_2(Z_2)\|^{\frac{p+1}{p-p_2+1}} - \sigma_2\hat{\Theta}_2 \tag{41}$$

where $\beta_2 = \left(c_2 + \hat{\Theta}_2 \|\varphi_2(Z_2)\|^{\frac{p+1}{p-p_2+1}} + 1\right)^{\frac{1}{p_2}}$.

Then, one has

$$\begin{aligned}\dot{V}_2 \le& -(c_1-1)z_1^{p+1} - c_2 z_2^{p+1} + z_2^{p-p_2+1}(x_3^{p_2} - \alpha_2^{p_2}) \\ &+ \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 + \frac{\sigma_2}{\delta_2}\tilde{\Theta}_2\hat{\Theta}_2 \\ &- \frac{\bar{c}}{\lambda_0}v + \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right) \\ &\frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} + d_2(t_0,t)\end{aligned} \tag{42}$$

where $d_{2,k}(t_0,t) = d_{1,k}(t_0,t) + \frac{p_2}{p+1}\varepsilon_2^{*\frac{p+1}{p_2}} + d_{22,k}(t_0,t) + \frac{3p_2}{p+1} + d_{21}$.

Also by means of lemmas 3 and 4, one has

$$z_2^{p-p_2+1}(x_3^{p_2} - \alpha_2^{p_2}) \le z_2^{p+1} + h_{20} z_2^{p+1} \tag{43}$$

with $h_{20} = \frac{p_2}{p+1}\left(\frac{2(p-p_2+1)}{p+1}\right)^{\frac{p-p_2+1}{p_2}} c^{\frac{p+1}{p_2}} + \frac{1}{p+1}\left(\frac{2p}{p+1}\right)^{p} \left(c\left|\beta_2^{p_2-1}\right|\right)^{p+1}$ being a continuous function.

Then, one has

$$\begin{aligned}\dot{V}_2 \le& -(c_1-1)z_1^{p+1} - (c_2-1)z_2^{p+1} + h_{20}z_3^{p+1} \\ &+ \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 + \frac{\sigma_2}{\delta_2}\tilde{\Theta}_2\hat{\Theta}_2 - \frac{\bar{c}}{\lambda_0}v \\ &+ \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} \\ &+ d_{2,k}(t_0,t)\end{aligned} \tag{44}$$

Step i $(3 \le i \le n-1)$: Choose Lyapunov function $V_i = V_{i-1} + W_i + \frac{1}{2\delta_i}\tilde{\Theta}_i^2$, Where $\delta_i > 0$ is a design parameter, $\tilde{\Theta}_i = \Theta_i^* - \hat{\Theta}_i$ is the estimation error and $\hat{\Theta}_i$ is the estimation of Θ_i^*. $W_i = \frac{z_i^{p-p_i+2}}{p-p_i+2}$ is the continuous-differential function about $\bar{x}_i$, $\bar{\hat{\Theta}}_{i-1}$ and $\hat{\lambda}$.

From the above analysis, one has

$$\begin{aligned}\dot{V}_i \le& -(c_1-1)z_1^{p+1} - (c_2-1)z_2^{p+1} - \cdots - (c_{i-1}-1)z_{i-1}^{p+1} \\ &+ h_{i-1,0}z_i^{p+1} \\ &+ \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 + \frac{\sigma_2}{\delta_2}\tilde{\Theta}_2\hat{\Theta}_2 + \cdots + \frac{\sigma_{i-1}}{\delta_{i-1}}\tilde{\Theta}_{i-1}\hat{\Theta}_{i-1} - \frac{\bar{c}}{\lambda_0}v \\ &+ \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} \\ &+ d_{i-1,k}(t_0,t) \\ &+ z_i^{p-p_i+1}(x_{i+1}^{p_i} + f_{i,k}(\bar{x}_i,d) - \dot{\alpha}_{i-1} + \Delta_{i,k}) - \frac{1}{\delta_i}\tilde{\Theta}_i\dot{\hat{\Theta}}_i\end{aligned} \tag{45}$$

and

$$\begin{aligned}z_i^{p-p_i+1}\Delta_{i,k} \le& z_i^{p+1}\left(\hat{\Phi}_{i1,k} + \hat{\Phi}_{i2,k} + 1\right) + d_{i2,k}(t_0,t) \\ &+ \frac{2p_i}{p+1} + d_{i1}\end{aligned} \tag{46}$$

where $\hat{\Phi}_{i1,k} = \left(\bar{\Phi}_{i1,k}\tanh\left(\frac{z_i^{p-p_i+1}\bar{\Phi}_{i1,k}}{\bar{d}_{i1}}\right)\right)^{\frac{p+1}{p-p_i+1}}$, $\hat{\Phi}_{i2,k} = (\varphi_{i2,k}(\|x\|))^{\frac{p+1}{p-p_i+1}}$, $d_{i1} = 0.2785\bar{d}_{i1}$, $d_{i2,k}(t_0,t) = \frac{p_i}{p+1}(\varphi_{i1,k}\circ\gamma_1^{-1}(2D))^{\frac{p+1}{p_i}}$, $\bar{d}_{i1}$ is a design parameter.

$$\begin{aligned}\dot{V}_i \le& -(c_1-1)z_1^{p+1} - (c_2-1)z_2^{p+1} - \cdots - (c_{i-1}-1)z_{i-1}^{p+1} \\ &+ h_{i-1,0}z_i^{p+1} \\ &+ \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 + \frac{\sigma_2}{\delta_2}\tilde{\Theta}_2\hat{\Theta}_2 + \cdots + \frac{\sigma_{i-1}}{\delta_{i-1}}\tilde{\Theta}_{i-1}\hat{\Theta}_{i-1} - \frac{\bar{c}}{\lambda_0}v \\ &+ \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} \\ &+ d_{i-1,k}(t_0,t) \\ &+ z_i^{p-p_i+1}(x_{i+1}^{p_i} - \alpha_i^{p_i}) + z_i^{p-p_i+1}\alpha_i^{p_i} + z_i^{p-p_i+1}\bar{f}_{i,k}(Z_i) \\ &+ d_{i2,k}(t_0,t) \\ &+ \frac{2p_i}{p+1} + d_{i1} - \frac{1}{\delta_i}\tilde{\Theta}_i\dot{\hat{\Theta}}_i\end{aligned} \tag{47}$$

where $\bar{f}_{i,k}(Z_i) = f_{i,k}(\bar{x}_i,d) - \dot{\alpha}_{i-1} + z_i^{p_i}\left(\hat{\Phi}_{i1,k} + \hat{\Phi}_{i2,k} + 1\right) + h_{i-1,0}z_i^{p_i}$, and $Z_i = [x_1, x_2, \cdots x_i, y_d, \dot{y}_d, \cdots, y_d^{(i)}, \rho, \dot{\rho}, \cdots, \rho^{(i)}, \hat{\Theta}_1, \hat{\Theta}_2, \cdots, \hat{\Theta}_{i-1}]$.

Similar to the previous analysis, it can be obtained that $\bar{f}_{i,k}(Z_i)$ can be approximated by RBFNN, that is, $\bar{f}_{i,k}(\cdot) = \theta_{i,k}^*\varphi_{i,k}(Z_i) + \varepsilon_{i,k}(Z_i)$.

Using Lemma 4, one has

$$z_i^{p-p_i+1}\theta_{i,k}^*\varphi_i(Z_i) \le z_i^{p+1}\Theta_i^*\|\varphi_i(Z_i)\|^{\frac{p+1}{p-p_i+1}} + \frac{p_i}{p+1} \tag{48}$$

$$z_i^{p-p_i+1}\varepsilon_{i,k}(Z_i) \le z_i^{p+1} + \frac{p_i}{p+1}\varepsilon_i^* \tag{49}$$

where $\Theta_i^* = \max_{k\in M}\left\{\left\|\theta_{i,k}^*\right\|^{\frac{p+1}{p-p_i+1}}\right\}$ and $\varepsilon_i^* = \max_{k\in M}\left\{\varepsilon_{i,k}^{*\frac{p+1}{p_i}}\right\}$.

And then, one has

$$\begin{aligned}\dot{V}_i \le& -(c_1-1)z_1^{p+1} - (c_2-1)z_2^{p+1} - \cdots - (c_{i-1}-1)z_{i-1}^{p+1} \\ &+ \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 + \frac{\sigma_2}{\delta_2}\tilde{\Theta}_2\hat{\Theta}_2 \\ &+ \cdots + \frac{\sigma_{i-1}}{\delta_{i-1}}\tilde{\Theta}_{i-1}\hat{\Theta}_{i-1} - \frac{\bar{c}}{\lambda_0}v \\ &+ \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} \\ &+ z_i^{p-p_i+1}(x_{i+1}^{p_i} - \alpha_i^{p_i}) \\ &- \frac{1}{\delta_i}\tilde{\Theta}_i\left(\dot{\hat{\Theta}}_i - \delta_i z_i^{p+1}\|\varphi_i(Z_i)\|^{\frac{p+1}{p-p_i+1}}\right) + z_i^{p-p_i+1}\left(\alpha_i^{p_i} + \right. \\ &\left. z_i^{p_i}\hat{\Theta}_i\|\varphi_i(Z_i)\|^{\frac{p+1}{p-p_i+1}} + h_{i-1,0}z_i^{p_i} + z_i^{p_i}\right) + d_{i,k}(t_0,t)\end{aligned} \tag{50}$$

where $d_{i,k}(t_0,t) = d_{i-1,k}(t_0,t) + \frac{p_i}{p+1}\varepsilon_{i,k}^* + d_{i2,k}(t_0,t) + \frac{3p_i}{p+1} + d_{i1}$.

Choose

$$\alpha_i = -z_i\left(c_i + 1 + \hat{\Theta}_i \|\varphi_i(Z_i)\|^{\frac{p+1}{p-p_i+1}}\right)^{\frac{1}{p_i}} = -z_i\beta_i \quad (51)$$

$$\dot{\hat{\Theta}}_i = \delta_i z_i^{p+1} \|\varphi_i(Z_i)\|^{\frac{p+1}{p-p_i+1}} - \sigma_i\hat{\Theta}_i \quad (52)$$

with $\beta_i = \left(c_i + 1 + \hat{\Theta}_i \|\varphi_i(Z_i)\|^{\frac{p+1}{p-p_i+1}}\right)^{\frac{1}{p_i}}$.

It can be obtained that

$$\begin{aligned}\dot{V}_i \le& -(c_1-1)z_1^{p+1} - (c_2-1)z_2^{p+1} - \cdots - (c_{i-1}-1)z_{i-1}^{p+1} \\ &- c_i z_i^{p+1} + z_i^{p-p_i+1}(x_{i+1}^{p_i} \\ &- \alpha_i^{p_i}) + \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 + \frac{\sigma_2}{\delta_2}\tilde{\Theta}_2\hat{\Theta}_2 + \cdots \\ &+ \frac{\sigma_{i-1}}{\delta_{i-1}}\tilde{\Theta}_{i-1}\hat{\Theta}_{i-1} + \frac{\sigma_i}{\delta_i}\tilde{\Theta}_i\hat{\Theta}_i - \frac{\bar{c}}{\lambda_0}v \\ &+ \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} \\ &+ d_{i,k}(t_0,t) \end{aligned} \quad (53)$$

Due to

$$z_i^{p-p_i+1}(x_{i+1}^{p_i} - \alpha_i^{p_i}) \le z_i^{p+1} + h_{i0}z_{i+1}^{p+1} \quad (54)$$

with $h_{i0} = \frac{p_i}{p+1}\left(\frac{2(p-p_i+1)}{p+1}\right)^{\frac{p-p_i+1}{p_i}} c^{\frac{p+1}{p_i}} + \frac{1}{p+1}\left(\frac{2p}{p+1}\right)^p \left(c\left|\beta_i^{p_i-1}\right|\right)^{p+1}$ being a continuous function. Then, one has

$$\begin{aligned}\dot{V}_i \le& -(c_1-1)z_1^{p+1} - (c_2-1)z_2^{p+1} - \cdots - (c_{i-1}-1)z_{i-1}^{p+1} \\ &- (c_i-1)z_i^{p+1} \\ &+ h_{i0}z_{i+1}^{p+1} + \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 + \frac{\sigma_2}{\delta_2}\tilde{\Theta}_2\hat{\Theta}_2 + \cdots \\ &+ \frac{\sigma_{i-1}}{\delta_{i-1}}\tilde{\Theta}_{i-1}\hat{\Theta}_{i-1} + \frac{\sigma_i}{\delta_i}\tilde{\Theta}_i\hat{\Theta}_i \\ &- \frac{\bar{c}}{\lambda_0}v + \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right) \\ &\frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} + d_i(t_0,t) \end{aligned} \quad (55)$$

Stepn : Combining with the above analysis, one has

$$\begin{aligned}\dot{V}_{n-1} \le& -(c_1-1)z_1^{p+1} - (c_2-1)z_2^{p+1} - \cdots \\ &- (c_{i-1}-1)z_{i-1}^{p+1} - (c_{n-1}-1)z_{n-1}^{p+1} \\ &+ h_{n-1,0}z_n^{p+1} + \frac{\sigma_1}{\delta_1}\tilde{\Theta}_1\hat{\Theta}_1 + \frac{\sigma_2}{\delta_2}\tilde{\Theta}_2\hat{\Theta}_2 + \cdots \\ &+ \frac{\sigma_{i-1}}{\delta_{i-1}}\tilde{\Theta}_{i-1}\hat{\Theta}_{i-1} + \frac{\sigma_{n-1}}{\delta_{n-1}}\tilde{\Theta}_{n-1}\hat{\Theta}_{n-1} \\ &- \frac{\bar{c}}{\lambda_0}v + \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right) \\ &\frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} + d_{n-1,k}(t_0,t) \end{aligned} \quad (56)$$

According to the definition of z_n, one has

$$\begin{aligned}\dot{z}_n &= \dot{x}_n - \dot{\alpha}_{n-1} \\ &= q^{p_n}(u(t)) + f_{n,k}(x,d) + \Delta_{n,k}(z,x) - \dot{\alpha}_{n-1} \end{aligned} \quad (57)$$

Define the candidate Lyapunov function as $V_n = V_{n-1}+W_n+\frac{(1-\delta)^{p_n}}{2\delta_n}\tilde{\Theta}_n^2$ with $W_n = \frac{z_n^{p-p_n+2}}{p-p_n+2}$ being a continuous differential function, and $\delta_n > 0$ is a design parameter, $\tilde{\Theta}_n = \Theta_n^* - \hat{\Theta}_n$ is the estimation error and $\hat{\Theta}_n$ is the estimation of Θ_n^*.

From the above analysis, one has

$$\begin{aligned}\dot{V}_n \le& -\sum_{i=1}^{n-1}(c_i-1)z_i^{p+1} + h_{n-1,0}z_n^{p+1} + \sum_{i=1}^{n-1}\frac{\sigma_i}{\delta_i}\tilde{\Theta}_i\hat{\Theta}_i \\ &- \frac{\bar{c}}{\lambda_0}v + (1-(p-p_1 \\ &+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)) \frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} + d_{n-1}(t_0,t) \\ &+ z_n^{p-p_n+1}(q^{p_n}(u(t)) \\ &+ f_{n,k}(x,d) - \dot{\alpha}_{n-1} + \Delta_{n,k}) - \frac{(1-\delta)^{p_n}}{\delta_n}\tilde{\Theta}_n\dot{\hat{\Theta}}_n \end{aligned} \quad (58)$$

And

$$\begin{aligned} z_n^{p-p_n+1}\Delta_{n,k} \le& z_n^{p+1}\left(\hat{\Phi}_{n1,k} + \hat{\Phi}_{n2,k} + 1\right) + d_{n2,k}(t_0,t) \\ &+ \frac{2p_n}{p+1} + d_{n1} \end{aligned} \quad (59)$$

where $\hat{\Phi}_{n1,k} = \left(\bar{\Phi}_{n1,k}\tanh\left(\frac{z_n^{p-p_n+1}\bar{\Phi}_{n1,k}}{\bar{d}_{n1}}\right)\right)^{\frac{p+1}{p-p_n+1}}$, $d_{n2,k}(t_0,t) = \frac{p_n}{p+1}(\varphi_{n1,k} \circ \gamma_1^{-1}(2D))^{\frac{p+1}{p_n}}$, $\hat{\Phi}_{n2,k} = (\varphi_{n2,k}(y))^{\frac{p+1}{p-p_n+1}}$, $d_{n1} = 0.2785\bar{d}_{n1}$, $\bar{d}_{n1}$ is a design parameter.

$$\begin{aligned}\dot{V}_n \le& -\sum_{i=1}^{n-1}(c_i-1)z_i^{p+1} + h_{n-1,0}z_n^{p+1} + \sum_{i=1}^{n-1}\frac{\sigma_i}{\delta_i}\tilde{\Theta}_i\hat{\Theta}_i \\ &- \frac{\bar{c}}{\lambda_0}v + (1-(p-p_1 \\ &+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)) \frac{x_1^2\bar{\gamma}\left(\left|x_1^2\right|\right)}{\lambda_0} + d_{n-1,k}(t_0,t) \\ &+ z_n^{p-p_n+1}q^{p_n}(u(t)) \\ &+ z_n^{p-p_n+1}\bar{f}_{n,k}(Z_n) - \frac{(1-\delta)^{p_n}}{\delta_n}\tilde{\Theta}_n\dot{\hat{\Theta}}_n \\ &+ d_{n2,k}(t_0,t) + \frac{2p_n}{p+1} + d_{n1} \end{aligned} \quad (60)$$

where $\bar{f}_{n,k}(Z_n) = f_{n,k}(x,d) - \dot{\alpha}_{n-1} + z_n^{p_n}\left(\hat{\Phi}_{n1,k} + \hat{\Phi}_{n2,k} + 1\right) + \frac{1}{2}z_n^{p-p_n+1} + h_{n-1,0}z_n^{p+1} + z_n^{p_n}$ and $Z_n = [x_1, x_2, \cdots x_n, y_d, \dot{y}_d, \cdots, y_d^{(n)}, \rho, \dot{\rho}, \cdots, \rho^{(n)}, \hat{\Theta}_1,$

$$\hat{\Theta}_2, \cdots, \hat{\Theta}_{n-1}].$$

Similar to the previous analysis, it can be obtained that $\bar{f}_{n,k}(Z_n)$ can be approximated by RBFNN, that is, $\bar{f}_{n,k}(Z_n) = \theta_{n,k}^*\varphi_{n,k}(Z_n) + \varepsilon_{n,k}(Z_n)$.

Using Lemma 4, one has

$$z_n^{p-p_n+1}\theta^*_{n,k}\varphi_n(Z_n) \le (1-\delta)^{p_n} z_n^{p+1}\Theta_n^* \|\varphi_n(Z_n)\|^{\frac{p+1}{p-p_n+1}} + \frac{p_n}{p+1} \tag{61}$$

$$z_n^{p-p_n+1}\varepsilon_{n,k}(Z_n) \le z_n^{p+1} + \frac{p_n}{p+1}\varepsilon^*_{n,k} \tag{62}$$

Where $\Theta_n^* = \max_{k\in M}\left\{\frac{\|\theta^*_{n,k}\|^{\frac{p+1}{p-p_n+1}}}{(1-\delta)^{p_n}}\right\}$ and $\varepsilon^*_{n,k} = \max_{k\in M}\left\{\varepsilon^{*\frac{p+1}{p_n}}_{n,k}\right\}$. Then, one has

$$\begin{aligned}\dot{V}_n \le& -\sum_{i=1}^{n-1}(c_i-1)z_i^{p+1} + \sum_{i=1}^{n-1}\frac{\sigma_i}{\delta_i}\tilde{\Theta}_i\hat{\Theta}_i - \frac{\bar{c}}{\lambda_0}v \\ &+ \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right) \\ &\times \frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0} - \frac{(1-\delta)^{p_n}}{\delta_n}\tilde{\Theta}_n \\ &\left(\dot{\hat{\Theta}}_n - \delta_n z_n^{p+1}\|\varphi_n(Z_n)\|^{\frac{p+1}{p-p_n+1}}\right) + z_n^{p-p_n+1} \\ &\times\left(q^{p_n}(u(t)) + h_{n-1,0}z_n^{p+1} + z_n^{p_n} + (1-\delta)^{p_n}z_n^{p_n}\hat{\Theta}_n\right. \\ &\left.\|\varphi_n(Z_n)\|^{\frac{p+1}{p-p_n+1}}\right) + d_{n,k}(t_0,t)\end{aligned} \tag{63}$$

where $d_{n,k}(t_0,t) = d_{n-1,k}(t_0,t) + \frac{p_n}{p+1}\varepsilon^*_{n,k} + d_{n2,k}(t_0,t) + \frac{3p_n}{p+1} + d_{n1}$. Due to $q^{p_n}(u(t)) = g(u)u^{p_n}(t) + K(t)$ with $|K(t)| \le u_{\min}^{p_n}$, then one has $z_n^{p-p_n+1}K(t) \le \frac{1}{2}z_n^{2(p-p_n+1)} + \frac{1}{2}u_{\min}^{2p_n}$, thus

$$\begin{aligned}\dot{V}_n \le& -\sum_{i=1}^{n-1}(c_i-1)z_i^{p+1} + \sum_{i=1}^{n-1}\frac{\sigma_i}{\delta_i}\tilde{\Theta}_i\hat{\Theta}_i - \frac{\bar{c}}{\lambda_0}v \\ &+ \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right) \\ &\times \frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0} - \frac{(1-\delta)^{p_n}}{\delta_n}\tilde{\Theta}_n \\ &\left(\dot{\hat{\Theta}}_n - \delta_n z_n^{p+1}\|\varphi_n(Z_n)\|^{\frac{p+1}{p-p_n+1}}\right) + z_n^{p-p_n+1} \\ &\times\left(g(u)u^{p_n}(t) + (1-\delta)^{p_n}z_n^{p_n}\hat{\Theta}_n\|\varphi_n(Z_n)\|^{\frac{p+1}{p-p_n+1}}\right) \\ &+ d_{n,\sigma(t)}(t_0,t) + \frac{1}{2}u_{\min}^{2p_n}\end{aligned} \tag{64}$$

Choose

$$u = -z_n\left(c_n + \hat{\Theta}_n\|\varphi_n(Z_n)\|^{\frac{p+1}{p-p_n+1}}\right)^{\frac{1}{p_n}} \tag{65}$$

$$\dot{\hat{\Theta}}_n = \delta_n z_n^{p+1}\|\varphi_n(Z_n)\|^{\frac{p+1}{p-p_n+1}} - \bar{\sigma}_n\hat{\Theta}_n \tag{66}$$

then, one has

$$\begin{aligned}\dot{V}_n \le& -\sum_{i=1}^{n-1}(c_i-1)z_i^{p+1} - c_n z_n^{p+1} + \sum_{i=1}^{n-1}\frac{\sigma_i}{\delta_i}\tilde{\Theta}_i\hat{\Theta}_i \\ &+ \frac{\sigma_n(1-\delta)^{p_n}}{\delta_n}\tilde{\Theta}_n\hat{\Theta}_n - \frac{\bar{c}}{\lambda_0}v \\ &+ \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0} \\ &+ \frac{1}{2}u_{\min}^{2p_n} + d_{n,\sigma(t)}(t_0,t)\end{aligned} \tag{67}$$

That is,

$$\begin{aligned}\dot{V}_n \le& -\sum_{i=1}^{n}(c_i-1)z_i^{p+1} + \sum_{i=1}^{n-1}\frac{\sigma_i}{\delta_i}\tilde{\Theta}_i\hat{\Theta}_i \\ &+ \frac{\sigma_n(1-\delta)^{p_n}}{\delta_n}\tilde{\Theta}_n\hat{\Theta}_n - \frac{\bar{c}}{\lambda_0}v \\ &+ \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0} \\ &+ \frac{1}{2}u_{\min}^{2p_n} + d_{n,k}(t_0,t)\end{aligned} \tag{68}$$

Note that $\frac{\bar{\sigma}_i}{\delta_i}\tilde{\Theta}_i\hat{\Theta}_i \le -\frac{\bar{\sigma}_i}{2\delta_i}\tilde{\Theta}_i^2 + \frac{\bar{\sigma}_i}{2\delta_i}\Theta_i^{*2}, i = 1, 2, \cdots, n$, which can be obtained by using Young's inequality. Then, one has

$$\begin{aligned}\dot{V}_n \le& -\sum_{i=1}^{n}(c_i-1)z_i^{p+1} - \sum_{i=1}^{n-1}\frac{\bar{\sigma}_i}{2\delta_i}\tilde{\Theta}_i^2 - \frac{\bar{\sigma}_n(1-\delta)^{p_n}}{2\delta_n}\tilde{\Theta}_n^2 \\ &- \frac{\bar{c}}{\lambda_0}v + \sum_{i=1}^{n-1}\frac{\bar{\sigma}_i}{2\delta_i}\Theta_i^{*2} + \frac{\bar{\sigma}_n(1-\delta)^{p_n}}{2\delta_n}\Theta_n^{*2} \\ &+ \left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0} \\ &+ \frac{1}{2}u_{\min}^{2p_n} + d_{n,k}(t_0,t)\end{aligned} \tag{69}$$

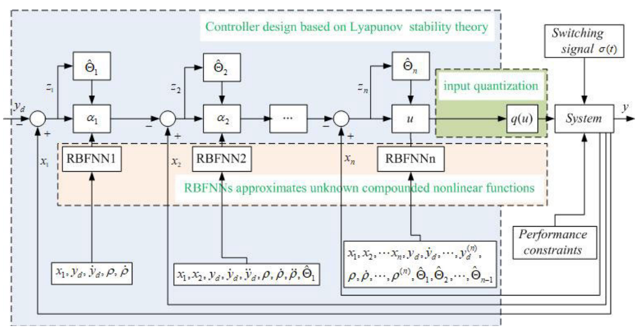

FIGURE 1. Control principle and flow structure diagram.

IV. STABILITY ANALYSIS

The main result of this work is summarized in the following theorem.

Theorem 1: Consider the closed-loop system consisting of system (1) under Assumptions 1 and 2, the virtual control laws (29) and (40), the controller (65), and the adaptive

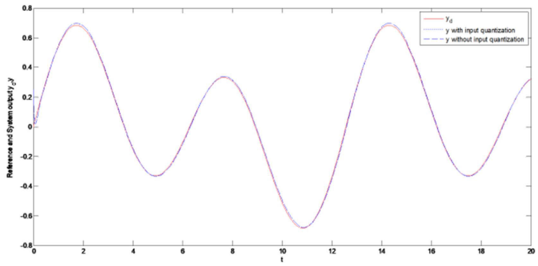

FIGURE 2. **Tracking result.**

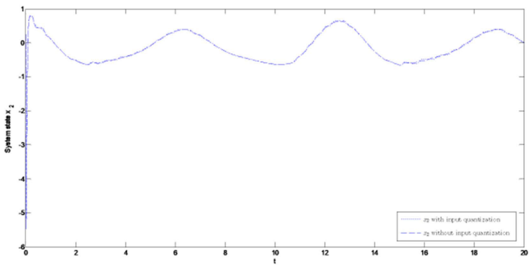

FIGURE 3. **System state x_2.**

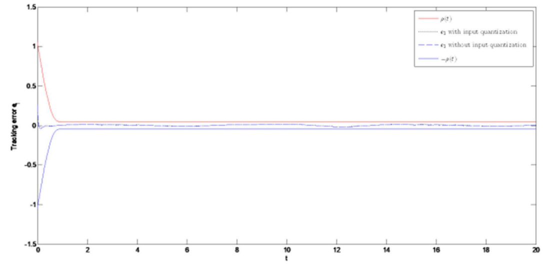

FIGURE 4. **Tracking error.**

laws (30), (41) and (66), all signal of the closed-loop system are SGUB and the tracking error converges to a prescribed zone at a finite-time.

Proof: Let us consider the following Lyapunov function candidate

$$V=\sum_{i=1}^{n}W_i+\sum_{i=1}^{n-1}\frac{1}{2\delta_i}\tilde{\Theta}_i^2+\frac{(1-\delta)^n}{2\delta_n}\tilde{\Theta}_n^2+\frac{v}{\lambda_0} \tag{70}$$

From Young's inequality, it can be seen that $a^{p_i-1}z_i^{p-p_i+2}\leq a^{p+1}+z_i^{p+1}$, implying $-z_i^{p+1}\leq -a^{p_i-1}z_i^{p-p_i+2}+a^{p+1}$. When $p_i>1$, After choosing $a=\left(\frac{1}{p-p_i+2}\right)^{\frac{1}{p_i-1}}$, one has $-z_i^{p+1}\leq -\frac{z_i^{p-p_i+2}}{p-p_i+2}+\left(\frac{1}{p-p_i+2}\right)^{\frac{p+1}{p_i-1}}<-\frac{z_i^{p-p_i+2}}{p-p_i+2}+1$; when $p_i=1$, choose $a=1$, one has $-z_i^{p+1}\leq -z_i^{p-p_i+2}+1\leq -\frac{z_i^{p-p_i+2}}{p-p_i+2}+1$.

Let $C=\min\{c_i-1\}$, yields

$$\begin{aligned}\dot{V}\leq&-C\sum_{i=1}^{n}z_i^{p+1}-\sum_{i=1}^{n}\frac{\bar{\sigma}_i}{2\delta_i}\tilde{\Theta}_i^2-\frac{\bar{\sigma}_n(1-\delta)^{p_n}}{2\delta_n}\tilde{\Theta}_n^2\\&-\frac{\bar{c}}{\lambda_0}v+\sum_{i=1}^{n-1}\frac{\bar{\sigma}_i}{2\delta_i}\Theta_i^{*2}\\&+\frac{\bar{\sigma}_n(1-\delta)^{p_n}}{2\delta_n}\Theta_n^{*2}+(1-(p-p_1+2)\\&\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\Big)\frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0}+\frac{1}{2}u_{\min}^{2p_n}+d_n(t_0,t)\\\leq&-C\sum_{i=1}^{n}\frac{z_i^{p-p_i+2}}{p-p_i+2}-\sum_{i=1}^{n-1}\frac{\bar{\sigma}_i}{2\delta_i}\tilde{\Theta}_i^2-\frac{\bar{\sigma}_n(1-\delta)^{p_n}}{2\delta_n}\tilde{\Theta}_n^2\\&-\frac{\bar{c}}{\lambda_0}v+\sum_{i=1}^{n-1}\frac{\bar{\sigma}_i}{2\delta_i}\Theta_i^{*2}+\frac{\bar{\sigma}_n(1-\delta)^{p_n}}{2\delta_n}\Theta_n^{*2}\\&+(1-(p-p_1+2)\ \tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\Big)\\&\times\frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0}+\frac{1}{2}u_{\min}^{2p_n}+nC+d_n(t_0,t)\end{aligned} \tag{71}$$

Denote $\alpha_0=\min\{C,\sigma_i,\bar{\sigma}_1,\bar{c}\}$ and $\mu=\sum_{i=1}^{n-1}\frac{\bar{\sigma}_i}{2\delta_i}\Theta_i^{*2}+\frac{\bar{\sigma}_n(1-\delta)^{p_n}}{2\delta_n}\times\Theta_n^{*2}+\frac{\bar{\sigma}_1}{2\delta_1}\lambda^{*2}+\frac{\bar{\gamma}(|x_1|)}{\lambda_0}+\frac{1}{2}u_{\min}^{2p_n}+nC+d_n(t_0,t)$, one has

$$\dot{V}\leq-\alpha_0V+\mu+\left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}\left(|x_1^2|\right)}{\lambda_0} \tag{72}$$

Two cases are considered as follows.

Case 1: $x_1\in\Omega_{x_1}=\{x_1\,||x_1|<0.8841v\}$ for any positive constant v. It yields e_1 is bounded because x_1 and y_d are bounded. As $\bar{\gamma}\left(|x_1^2|\right)$ is non-negative continuous function, $\left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}(|x_1^2|)}{\lambda_0}$ is bounded and μ_0 is assumed to be its upper bound. Then, one has

$$\dot{V}\leq-\alpha_0V+\mu+\mu_0. \tag{73}$$

Furthermore, the following result is true:

$$0\leq V\leq\left(V(0)-\frac{\mu+\mu_0}{\alpha_0}\right)e^{-\alpha_0t}+\frac{\mu+\mu_0}{\alpha_0} \tag{74}$$

Case 2: $x_1\notin\Omega_{x_1}$. Based on the fact that $\frac{x_1^2\bar{\gamma}(|x_1^2|)}{\lambda_0}\geq 0$ and Lemma 2, it follows $\left(1-(p-p_1+2)\tanh^{p-p_1+2}\left(\frac{z_1}{v}\right)\right)\frac{x_1^2\bar{\gamma}(|x_1^2|)}{\lambda_0}\leq 0$. Then, one has $\dot{V}\leq-\alpha_0V+\mu$. Furthermore, the following result holds

$$0\leq V\leq\left(V(0)-\frac{\mu}{\alpha_0}\right)e^{-\alpha_0t}+\frac{\mu}{\alpha_0} \tag{75}$$

which implies

$$0\leq V\leq V(0)+\frac{\mu+\mu_0}{\alpha_0},\ t>0. \tag{76}$$

With μ_0 being mentioned above.

Summarize the above two situations with the definition of V in (70), we obtain the conclusion that all signals of the closed-loop system are SGUB.

Additionally, one can obtain that $0\leq V(t)\leq V(0)+\frac{\mu}{\alpha_0}=\frac{1}{2}l^2$, with l being a positive constant. By the transformation of

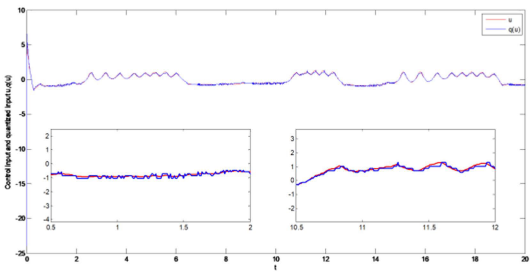

FIGURE 5. Control input u and quantized input q(u).

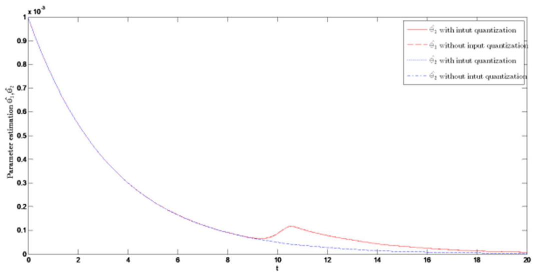

FIGURE 6. Estimated parameters $\hat{\Theta}_1$ and $\hat{\Theta}_2$.

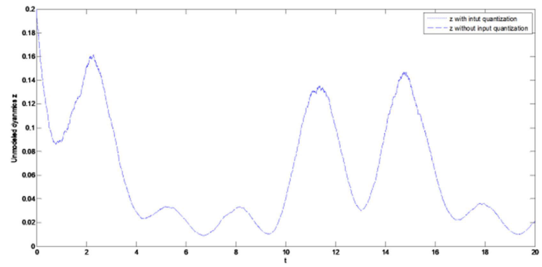

FIGURE 7. Unmodeled dynamics z.

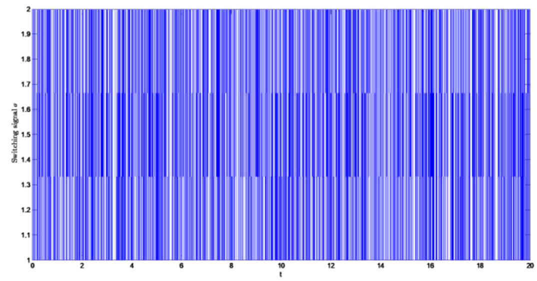

FIGURE 8. Switching signal $\sigma(t)$.

$S(z_1) = \frac{e_1(t)}{\rho(t)} = \frac{e^{z_1} - e^{-z_1}}{e^{z_1} + e^{-z_1}}$, it can be obtained that $-l \le z_1 = \frac{1}{2} \ln\left(\frac{1+\delta(t)}{1-\delta(t)}\right) \le l$, where $\delta(t) = \frac{e_1(t)}{\rho(t)}$, which implies that

$$-\rho(t) < \left(\frac{e^{-2l} - 1}{e^{-2l} + 1}\right)\rho(t) \le e_1 = \left(\frac{e^{2l} - 1}{e^{2l} + 1}\right)\rho(t) < \rho(t) \tag{77}$$

So the following inequality can be proved that $|e_1(t)| < \rho(t)$, $t > 0$.

From the definition of $\rho(t)$, it can be seen that the tracking error $e_1(t)$ converges to the prescribed invariant region $\Omega_{e_1} = \left\{e_1 \left||e_1| < \rho_{T_f}, t \ge T_f\right.\right\}$ in finite time $T = T_f$, which means that the tracking error $e(t) = y(t) - y_d(t)$ always remains in the prescribed performance bound.

That is the proof of Theorem 1.

V. SIMULATION RESULTS

In this section, we will provide a numerical example to demonstrate the effectiveness of the proposed control scheme. Consider the following nonlinear system:

$$\begin{cases} \dot{z} = q_{\sigma(t)}(z, y), \\ \dot{x}_1 = x_2^{p_1} + f_{1,\sigma(t)}(x_1, \theta) + \Delta_{1,\sigma(t)}, \\ \dot{x}_2 = q^{p_2}(u) + f_{2,\sigma(t)}(x_1, \theta) + \Delta_{2,\sigma(t)}, \\ y = x_1 \end{cases} \tag{78}$$

where $p_1 = 1, p_2 = 3, f_{1,1} = x_1 \sin x_1, f_{2,1} = \theta x_1 x_2^2, f_{1,2} = x_1^2, f_{2,1} = x_1 x_2, q_1(z, x) = -z + 0.6x_1^2, q_2(z, x) = -2z + 0.5x_1^2, \Delta_{11} = \Delta_{12} = z x_1 \sin x_2$ and $\Delta_{21} = \Delta_{22} = z^2 x_1 \sin x_2$. To satisfy Assumptions 1-2, we choose $V_0(z) = z^2$, thus $\dot{V}_0(z) \le -1.5z^2 + 2.5x_1^4 + 0.625$. Select $a = 1.5, b = 0.625$, $\gamma_3(|x_1|) = 2.5x_1^4$. Then the assumption 3 is fulfilled. Based on Lemma 1, let $\bar{c} = 1.2 \in (0, a)$, then dynamical signal function r is $\dot{r} = -1.2r + 2.5x_1^4 + 0.625$.

The unknown parameter is chosen as $\theta = 0.1$. The initial conditions of variables are chosen as: $x_1(0) = x_2(0) = 0.25$, $z(0) = 0.2$ and $r(0) = 0.2$. The initial values of the weight parameters are set as $\hat{\Theta}_1(0) = \hat{\Theta}_2(0) = 0.001$.

The FTPF is

$$\rho(t) = \begin{cases} \left(\rho_0 - \frac{t}{T_f}\right) e^{\left(1 - \frac{T_f}{T_f - t}\right)} + \rho_{T_f}, t \in [0, T_f) \\ \rho_{T_f}, t \in [T_f, +\infty) \end{cases}$$

where $\rho_0 = 1$ and $\rho_{T_f} = 0.05$, $T_f = 1$, $\rho(0) = 1.05$.

The virtual control law and controller are designed as shown in (29) and (65), and the adaptive laws are designed as shown in (30) and (66) with $n = 2$. The parameters are chosen as $n_1 = n_2 = 20$, $b_1 = b_2 = 0.2$, $\mu_{1ij} = 0.1(i - n_1), i = 1, 2, \cdots 5, j = 1, 2, \cdots, n_1$, $\mu_{2ij} = 0.1(i - n_2), i = 1, 2, \cdots, 9, j = 1, 2, \cdots, n_2$, $c_1 = c_2 = 30$, $\delta_1 = \delta_2 = 0.8$, $\sigma_1 = \sigma_2 = 0.3$. Fig. 2 shows the tracking result of the system (1), Fig. 3 shows the system state x_2, Fig. 4 depicts the tracking error, the control input u and quantized input $q(u)$ are shown in Fig. 5, Fig. 6 shows the estimated parameters $\hat{\Theta}_1$ and $\hat{\Theta}_2$, and the unmodeled dynamics z and switching signal $\sigma(t)$ are shown in Figs. 7 and 8.

From simulation results, it can be observed that all signals of the closed-loop are bounded, and the tracking error remain within a predefined region in a finite time, which shows the effectiveness of the control scheme.

VI. CONCLUSION

In this study, we investigate the finite-time adaptive neural prescribed performance control for high-order nonlinearly parameterized switching systems in the presence of unmodeled dynamics and quantized input. Combining with RBF neural networks with minimal learning parameters to identify the unknown compounded nonlinear functions, the computational burden is further lessen by introducing a hysteresis quantizer to reduce the communication burden. In the framework of backstepping technique, a simple control design scheme is investigated by introducing an innovative prescribed performance function that makes the tracking error remain within a predefined region in a finite time and also simplify the stability analysis of the closed-loop system. Based on Lyapunov stability theory, all signals of the closed-loop system are SGUB. Finally, the effectiveness of the developed control scheme is illustrated through a numerical simulation.

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