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RESEARCH ARTICLE

The Weighted Group Consensus for a Kind of Discrete Heterogeneous Multi-Agent Systems With Time Delay and Packet Loss in Cooperative-Competitive Networks Based on Self-Adaptive Controller

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ABSTRACT In this paper, the weighted group consensus for a kind of discrete heterogeneous multi-agent systems (HMASs) with packet loss in cooperative-competitive networks based on self-adaptive controller is studied. Based on self-adaptive controller, packet loss and cooperative-competitive relation, a novel control protocols have been designed for this system without satisfying the in-degree balance of the vertex. Some sufficient conditions have been obtained for the weighted group consensus of this kind of HMASs, by using graph theory, matrix analysis and complex frequency method. Based on weighted parameters, control parameters, packet loss rate and cooperative-competitive relation, the upper bound of the input time delay can be calculated. Finally, some simulation experiments are listed to show the effectiveness of the derived results.

INDEX TERMS Group consensus, heterogeneous, multi-agent systems, self-adaptive controller, time delay, packet loss, cooperative-competitive relation.

I. INTRODUCTION

Multi-agent systems (MASs) have great promising application in many practice systems, such as UAV formation control [1], distributed sensor networks [2], satellite formation control [3], robot formation control [4]. Consensus, as the key problem of MASs, is a hot research topic. Therefore, many scholars have investigated the issue of consensus in the past years [5], [6], [7], [8], [9], [10] and a lot of results have been obtained [8], [11], [12], [13], [14], [15]. In [8], the consensus for a class of discrete-time heterogeneous multi-agent systems is studied. By applying algebraic graph theory and matrix theory, some sufficient conditions for consensus of heterogeneous multi-agent systems are obtained. In [11], the group consensus for HMASs composed of discrete-time first-order and second-order agents is investigated. Some sufficient conditions are derived for consensus of the systems with directed communication topology by applying matrix theory and graph theory. In [12], the leader-following H infinity consensus for discrete-time nonlinear multi-agent

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systems with delay and parameter uncertainty is investigated. On the basis of Lyapunov function technology and the linear matrix inequality method, some new sufficient conditions are derived. In [13], the weighted couple-group consensus of continuous-time heterogeneous multiagent systems with input and communication time delay is investigated. By using graph theory, general Nyquist criterion and Gerschgorin disc theorem, the time delay upper limit that the system may allow is obtained. In [14], the group consensus of heterogeneous multi-agent systems with fixed and switching topologies are investigated. Some sufficient conditions are obtained by using graph theory, matrix theory and Lyapunov theory. In [15], an output sign-consensus of multiagent systems over directed signed graphs are investigates. In [16], the leader-following output consensus for a class of uncertain nonlinear multiagent systems with unknown control directions has been investigated. A novel two-layer distributed hierarchical control scheme is proposed, which can be used to increase the flexibility of controller. Note that the states of all agents only converge to the same consensus value in most of the aforementioned works. However, the consensus values may be different for agents in different subgroups with different environments or tasks. Group consensus is a special case of consensus, which deserves investigation.

However, most existing research on group consensus of MASs is based on homogeneous multi-agent systems [17], [18]. In homogeneous multi-agent systems, all agents can only exchange information in the same subgroup. This is obviously out of line with the actual situation. In fact, almost each agent will be disturbed by the external environment, so it has its own dynamic characteristics. Therefore, there are no identical two multi-agent systems. On the other hand, agents between different subgroups can also exchange information with each other in order to archive consensus. Hence, it is necessary for us to investigate heterogeneous multiagent systems. In [19], the pinning scheme is used to analyze the group consensus of HMASs with first-order agents and second-order agents under fixed and switching topologies. Sufficient conditions are deduced by applying graph theory and the Lyapunov stability approach. In [20], the consensus of HMASs with second-order linear and non-linear agents are investigated. Some sufficient conditions are obtained by using graph theory, Lyapunov technique, Lasalle's invariance principle and other mathematical method. In [21], a robust hierarchical pinning control scheme is used to realize the coordination control for a special nonlinear heterogeneous multi-agent system. In [22], the output formation containment of heterogeneous linear systems with different dimensions and dynamic interaction has been studied. A distributed hybrid active controller is designed by using the discrete-time information of neighbors.

It is worth pointing out that most of the aforementioned works only considered a single cooperation or competition relation between agents. And all agents of the whole complex systems have only the same dynamical behaviors. In many practical systems, cooperative and competitive relation can also coexist. Therefore, it is necessary for us to study the consensus or group consensus of MASs with a cooperative-competitive relation [23], [24], [25]. In [23], the swarming behavior of multiple Euler-Lagrange systems is investigated with cooperation-competition interactions and uncertain parameters, where agents can cooperate or compete with each other. A distributed consensus tracking of the considered multi-agent systems with cooperation-competition interactions and uncertain parameters is studied by using pinning control strategy. In [24], couple-group consensus problems for a class of discrete-time heterogeneous systems consisting of first-order and second-order agents under the influence of communication and input time delays are investigated by utilizing cooperative and competitive interactions among agents. Based on frequency domain analysis and matrix theory, some sufficient conditions are derived and the upper bound of input time delays are consequently estimated. In [25], the bipartite consensus of multi-agent linear systems with cooperative-competitive relation is studied.

On the other hand, there are two kinds of time delay in every real MASs: input and communication time delays. Communication time delays will occur when the agents communicate with each other. Input time delays will occur when the agents are influenced by external disturbances. Both kinds of delays will affect the stability and coordination of the system. Therefore, when studying group consensus of the multiagent system, the input time delay and communication time delay are the main parameters to be considered [26], [27], [28]. In [26], the average-consensus of networked multi-agent systems with heterogeneous time delays is studied. the necessary and sufficient condition for the average consensus is derived. In [27], the leader-following consensus problem of multi-agent systems in discrete-time with time-varying delays is studied. Consensus conditions for multi-agent systems with a delay-dependent cyclic switching signal have been obtained. In [28], the consensus of fractional-order Takagi-Sugeno fuzzy multi-agent systems with time delay is studied.

As we know, packet loss is a common phenomenon of many complex systems. In many real situations, random noise, radio interference, network congestion and other communication failures will all cause packet loss. Packet loss is one of the main factors of system stability. Therefore, it is very necessary for us to investigate the group consensus of MASs with packet loss [29], [30], [31]. In [29], formation tracking for heterogeneous multi-agent systems with loss of multiple communication packets is investigated using the iterative learning control (ILC) method. Convergence conditions are given based on frequency-domain analysis using the general Nyquist stability criterion and Gerschgorin disk theorem. In [30], the consensus problem is studied for a class of multi-agent systems with sampled data and packet losses, where random and deterministic packet losses are considered. A Bernoulli-distributed white sequence and a switched system with stable and unstable subsystems is employed separately to model packet dropouts, such that linear multi-agent systems with sampled data and packet losses can reach consensus. In [31], consensus of nonlinear mixed delay multi-agent systems with random packet losses and time delay is studied. Sufficient conditions are obtained by utilizing the Lyapunov-Krasovskii functional.

However, external interference caused by uncertain factors always exists, they will also lead to the instability of the system. The agent needs to adjust its own behavior to adapt to the changes in the external environment. Therefore, it is reasonable and necessary to design an appropriate self-adaptive controller to achieve group consensus [32], [33].

In the existing literature, researches have been conducted on one or several main factors that can affect the consensus of MASs. However, in many practical applications, due to internal or external reasons, there are many factors which can affect the consensus of MASs, such as heterogeneous, position, velocity, packet loss, time delay, coupling strength and cooperative-competitive relation, etc. All these can lead to slow convergence or make the system malfunction. Therefore, it is necessary for us to study the consensus of MASs under the influence of these factors.

Inspired by above analysis, this paper will investigate weighted group consensus for a kind of discrete HMASs with packet loss and time delay in cooperative-competitive networks based on self-adaptive controller. There are the main triple contributions in this article. First, a novel weighted group consensus protocol and self-adaptive controller are proposed for this discrete HMASs. Second, in this novel protocol, heterogeneous, position, velocity, packet loss, time delay, coupling strength between agents, and cooperative-competitive relation are all considered. Thirdly, graph, matrix, stability and complex frequency theories are used to obtain some sufficient conditions for the group consensus of this HMASs. From the proof process, it is not necessary to demand that the topology of this system is strong connective or contain a spanning tree. Finally, some simulation examples have been given to show the validity of the obtained results.

The rest of this paper are organized as follows. Section II introduces the related symbols, graph theory, definitions, lemmas, and discrete time HMASs model. In Section III, a novel weighted group consensus protocol with self-adaptive controller is proposed and the proof of main results is given. In Section IV, several simulation experiments are given to prove the correctness of the obtained results. In Section V, conclusions are concluded.

II. PRELIMINARY KNOWLEDGE AND MODEL DESCRIPTION

In this part, the relevant theoretical knowledge needed in the process of analyzing the group consensus for multi-agent systems will be introduced first, such as Mathematical symbols, graph theory, Gershgorin disk theory, HMASs model, some definitions and lemmas. The mathematical symbols involved in this article are shown in Table 1.

TABLE 1. The mathematical symbols.

symbols	meaning
R	real number set
$R_{N\times N}$	N-dimensional real matrix set
C	complex set
Z	complex
$R_e(Z)$	real part of complex Z
$I_m(Z)$	imaginary part of complex Z
A	matrix
I_N	n-dimension identity matrix
N	dimension
$\lambda_i(A)$	the i^{th} eigenvalue of matrix A
$det(A)$	the determinant of matrix A
L	laplacian matrix
D	in-degree matrix
A	adjacency matrix of G

A. GRAPH THEORY

A weighted directed digraph $G = \left(V(G), E(G), A\right)$ with $m+n$ nodes can be used to represent a discrete HMAS. There is no self-circulation in G. The vertex set $V(G) = \{v_1, v_2, \cdots, v_m, v_{m+1}, \cdots v_{m+n}\}$ denotes the $m+n$ agents of this system. $E(G) \in V \times V$ denotes the edge set of G. $A = (a_{ij}) \in R_{(m+n)\times(m+n)}$ is the adjacency matrix of G. If agent i could receive information from agent j directly, then we have $e_{ij} = (v_i, v_j) \in E(G)$ and $a_{ij} > 0$. Otherwise $(v_i, v_j) \notin E(G)$. $N_i = \left\{v_j \in v \left| e_{ij} \in E(G)\right.\right\}$ denotes the neighbor set of v_i. $d_i = \deg(v_i) = \sum_{j=1} a_{ij}$ denotes the degree of vertex v_i. $D = diag\{d_1, d_2, \cdots, d_m, d_{m+1}, \cdots, d_{m+n}\}$ is the degree matrix of G. $L = D - A$ is the laplacian matrix of G. It can be defined as

$$L = \begin{cases} -a_{ij}, i \neq j \\ \sum_{j=1, j\neq i}^{m+n} a_{ij}, i = j \quad i, j \in \sigma \end{cases}$$

B. HMASs

In this part, a kind of discrete heterogeneous multi-agent system with $m+n$ agents will be introduced. In these $m+n$ agents, m agents have second-order dynamics, and the other n agents have first-order dynamics. The dynamic model of this system can be described as the following equations (1) and (2):

$$\begin{cases} x_i(k+1) = x_i(k) + v_i(k) \\ v_i(k+1) = v_i(k) + u_i(k), \quad i \in \sigma_1 \end{cases} \tag{1}$$

$$x_i(k+1) = x_i(k) + u_i(k), \quad i \in \sigma_2 \tag{2}$$

where $\sigma_1 = \{1,2,\cdots,m\}, \sigma_2 = \{m+1,m+2,\cdots, m+n\}$. $\sigma = \sigma_1 \cup \sigma_2, \sigma_1 \cap \sigma_2 = \phi$. $x_i(k) \in R^N, v_i(k) \in R^N$, $u_i(k) \in R^N$ are the state, velocity and input control of the i^{th} agent, respectively.

Remark 1: To simplify the calculation, this paper only considers one-dimensional systems, that is $N = 1$. It is worth noting that, if $N > 1$, similar results can be obtained by using the Kronecker product of matrix for n-dimensional systems.

This discrete-time heterogeneous multiagent system (1) and (2) composes of first-order and second-order agents. The first-order neighbors of agent can be represented as $N_{i,1}$, the second-order neighbors of agent i can be represented as $N_{i,2}$. Hence, the neighbor node set of agent i can be represented as $N_i = N_{i,1} \cup N_{i,2}$. Then the adjacency matrix G of discrete HMASs (1) and (2) can be described as following matrix:

$$A = \begin{bmatrix} A_{22} & A_{21} \\ A_{12} & A_{11} \end{bmatrix}$$

where $A_{22} \in R_{m\times m}$ represents the adjacency matrix of second-order agents. $A_{11} \in R_{n\times n}$ represents the adjacency matrix of first-order agents. $A_{21} \in R_{m\times n}$ represents the adjacency matrix from second-order agents to first-order agents. $A_{12} \in R_{n\times m}$ represents the adjacency matrix from first-order agents to second-order agents. A_{21} and A_{12} also represents the coupling strength between first-order agents to second-order agents.

The Laplace matrix L of HMASs (1) and (2) can be expressed as the following matrix:

$$\begin{aligned} L &= D - A \\ &= \begin{bmatrix} L_{22} + D_{21} & -A_{21} \\ -A_{12} & L_{11} + D_{12} \end{bmatrix} \\ &= \begin{bmatrix} D_{22} - A_{22} + D_{21} & -A_{21} \\ -A_{12} & D_{11} - A_{11} + D_{12} \end{bmatrix} \end{aligned}$$

where L_{22} and L_{11} represent the Laplace matrix of second-order agents and first-order agents, respectively. D_{22} and D_{11} represent the degree matrix of second-order agents and first-order agents, respectively. D_{21} represents the in-degree weight matrix from second-order agents to first-order agents. D_{12} represents the in-degree weight matrix from first-order agents to second-order agents. D_{22}, D_{21}, D_{11} and D_{12} can be written as the following formula:

$$D_{11} = diag\left\{ \sum_{j\in N_{i,1}} a_{ij}, j \in \sigma_2 \right\},$$

$$D_{12} = diag\left\{ \sum_{j\in N_{i,2}} a_{ij}, j \in \sigma_2 \right\},$$

$$D_{21} = diag\left\{ \sum_{j\in N_{i,1}} a_{ij}, j \in \sigma_1 \right\},$$

$$D_{22} = diag\left\{ \sum_{j\in N_{i,2}} a_{ij}, j \in \sigma_1 \right\}.$$

Definition and lemma:

Definition 1: In graph G, $V(G)$ is the vertex set, $E(G)$ is the edge set. If $V(G)$ can be divided into two mutually disjoint vertex subsets $V_1(G)$ and $V_2(G)$, and the two vertices v_i and v_j associated with edge e_{ij} belong to $V_1(G)$ and $V_2(G)$ separately, G is called a bipartite graph.

(1) The vertex V of graph G can be completely divided into two subsets V_i, V_j:

$$V_1 \cup V_2 = V, V_1 \cap V_2 = \phi.$$

(2) If edge $e_{ij} = (V_i, V_j)$ belongs to E, then V_i belongs to V_1 and V_j belongs to V_2.

Definition 2: The heterogeneous multi-agent systems (1) and (2) is called asymptotically group consensus, if (1) and (2) hold for any initial position and velocity values $\forall i, j \in \sigma_1 \cup \sigma_2$.

$$\begin{aligned} (1) \quad & \lim_{k\to\infty} \left\| x_i(k) - x_j(k) \right\| = 0, \ell_i = \ell_j \\ & \lim_{k\to\infty} \left\| x_i(k) - x_j(k) \right\| \neq 0, \ell_i \neq \ell_j \\ (2) \quad & \lim_{k\to\infty} \left\| v_i(k) - v_j(k) \right\| = 0, \ell_i = \ell_j \\ & \lim_{k\to\infty} \left\| v_i(k) - v_j(k) \right\| \neq 0, \ell_i \neq \ell_j \end{aligned}$$

Definition 3: If a subgraph of a connected graph G is a tree containing all vertices of G, the subgraph is called the spanning tree of G.

Lemma 1: [34] Given a Laplacian matrix $L \in R_{m\times n}$ and vector $k = \left[k_1\ k_2 \cdots k_n \right], k_i \in R$,then the following conditions are equivalent:

(1) For $\lambda_i(L), i \in \left\{1,\ 2,\ \cdots,\ m+n\right\}$, every eigenvalue has a positive real part except the zero eigenvalue;

(2) $L_k = 0$ denotes that $k = \left[k_1\ k_2 \cdots k_n \right]$;

(3) The system asymptotically achieves consensus if the system $\dot{k} = -L_k$ is stable at the zero.

(4) The directed graph with L contains one or more directed spanning trees.

Lemma 2: [34] If $G =< V, E >$ is a connected bipartite graph, then zero is the unique simple eigenvalue of $D + A$, $rank(D + A) = n - 1$, all the nonzero eigenvalues of G have positive real part, D and A are the degree and adjacency matrix of G.

III. RESULTS

In this section, a weighted group consensus of discrete HMASs (1) and (2) will be discussed. Influence on system consensus, such as dynamic control parameter, packet loss, input and communication time delay, interaction between agents and cooperative–competitive relations are all considered. The following content will design and analyze a novel control protocol for the fixed topology. Based on the literatures [32], a novel consensus protocol is proposed as follows:

$$\begin{aligned} & u_i(k) \\ & = \alpha_i \left\{ \begin{array}{l} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij} p \left[x_j(k) - x_i(k) \right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij} p \left[x_j(k) + x_i(k) \right] \end{array} \right\} \end{aligned}$$

$$+\beta_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[v_j(k) - v_i(k)\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[v_j(k) + v_i(k)\right] \end{Bmatrix}, i \in \sigma_1 \tag{3}$$

$$\begin{cases} u_i(k) = \gamma_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(k) - x_i(k)\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(k) + x_i(k)\right] \end{Bmatrix} + w_i(k) \\ w_i(k+1) = \phi_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(k) - x_i(k)\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(k) + x_i(k)\right] \end{Bmatrix} \\ \quad +w_i(k), i \in \sigma_2 \end{cases} \tag{4}$$

where $w_i(k)$ is the novel self-adaptive controller. $\alpha_i > 0$, $\beta_i > 0$, $\gamma_i > 0$, $\phi_i > 0$ represents the variable control parameters. ε_{ij} represents the cooperative or competitive interaction intensity between agent i and agent j. $p \in (0, 1]$ represents packet loss rate. s_i represents the neighbor set of agent i in the same group, they are cooperative with i. d_i represents the neighbor set of agent i in the different group, they are competitive with i. $x_j(k) - x_i(k)$ and $x_j(k) + x_i(k)$ represent the cooperative and competitive relation between agent i and agent j, respectively. $v_j(k) - v_i(k)$ and $v_j(k) + v_i(k)$ have the same meaning of the agents' speeds.

According to the control protocols (3) and (4), systems (1) and (2) can be rewritten as follows (5) and (6):

$$\begin{cases} x_i(k+1) = x_i(k) + v_i(k) \\ v_i(k+1) = v_i(k) + \alpha_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(k) - x_i(k)\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(k) + x_i(k)\right] \end{Bmatrix} \\ +\beta_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[v_j(k) - v_i(k)\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[v_j(k) + v_i(k)\right] \end{Bmatrix} \\ \quad i \in \sigma_1 \end{cases}, \tag{5}$$

$$\begin{cases} x_i(k+1) = x_i(k) + \gamma_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(k) - x_i(k)\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(k) + x_i(k)\right] \end{Bmatrix} \\ \quad +w_i(k) \\ w_i(k+1) = \phi_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(k) - x_i(k)\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(k) + x_i(k)\right] \end{Bmatrix} \\ \quad +w_i(k), i \in \sigma_2 \end{cases} \tag{6}$$

Furthermore, due to the agent's situation and external interference, almost all multi-agent systems have input and communication time delay inevitably. Therefore, systems (5) and (6) can be described as follows (7) and (8).

$$\begin{cases} x_i(k+1) = x_i(k) + v_i(k) \\ v_i(k+1) = v_i(k) \\ \quad +\alpha_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j\left(k-\tau_{ij}\right) - x_i\left(k - t_i\right)\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j\left(k-\tau_{ij}\right) + x_i\left(k - \tau_i\right)\right] \end{Bmatrix} \\ \quad +\beta_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[v_j\left(k-\tau_{ij}\right) - v_i\left(k - \tau_i\right)\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[v_j\left(k-\tau_{ij}\right) + v_i\left(k - \tau_i\right)\right] \end{Bmatrix}, i \in \sigma_1 \end{cases} \tag{7}$$

$$\begin{cases} x_i(k+1) = x_i(k) \\ \quad +\gamma_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}p\left[x_j\left(k-\tau_{ij}\right) - x_i\left(k - \tau_i\right)\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j\left(k-\tau_{ij}\right) + x_i\left(k - \tau_i\right)\right] \end{Bmatrix} + w_i(k) \\ w_i(k+1) = \phi_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j\left(k-\tau_{ij}\right) - x_i\left(k - \tau_i\right)\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j\left(k-\tau_{ij}\right) + x_i\left(k - \tau_i\right)\right] \end{Bmatrix}, \\ \quad i \in \sigma_2 \end{cases} \tag{8}$$

In systems (7) and (8), the current position and velocity, sampling location and speed are all considered.

Using z-transformation, graph theory and complex frequency-domain method, some sufficient conditions of group consensus for system (5) and (6) with undirected bipartite graph topology have been given in Theorem 1.

Theorem 1: Group consensus of heterogeneous MASs (7) and (8) with undirected bipartite graph topology can be achieved asymptotically under the control protocols (3) and (4) if the input delay τ_i satisfies formula (9), shown at the bottom of the next page, when $i \in \sigma_1$, or the input delay τ_i satisfies formula (10), shown at the bottom of the next page, when $i \in \sigma_2$.

where $d_i = \sum\limits_{v_j \in N_i} a_{ij}$, $i \in \sigma_1 \cup \sigma_2$.

Proof: Do z-transform for equations (7) and (8), we have following equations (11) and (12).

$$\begin{cases} zx_i(z) = x_i(z) + v_i(z) \\ zv_i(z) = \alpha_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} - x_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} + x_i(z)z^{-\tau_i}\right] \end{Bmatrix} \\ \quad +\beta_i \begin{Bmatrix} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[v_j(z)z^{-\tau_{ij}} - v_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[v_j(z)z^{-\tau_{ij}} + v_i(z)z^{-\tau_i}\right] \end{Bmatrix} \\ \quad +v_i(z), i \in \sigma_1 \end{cases} \tag{11}$$

$$\begin{cases} zx_i(z) = x_i(z) \\ +\gamma_i \left\{ \begin{array}{l} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} - x_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} + x_i(z)z^{-\tau_i}\right] \end{array} \right\} + w_i(z) \\ zw_i(z) = \phi_i \left\{ \begin{array}{l} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} - x_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} + x_i(z)z^{-\tau_i}\right] \end{array} \right\} \\ +w_i(z), i \in \sigma_2 \end{cases} \tag{12}$$

where $x_i(z)$, $v_i(z)$ and $w_i(z)$ represent the z-transform of $x_i(k)$, $v_i(x)$ and $w_i(x)$, respectively. Then we have equations (13) and (14).

$$\begin{cases} (z-1)x_i(z) = v_i(z) \\ (z-1)v_i(z) = \alpha_i \left\{ \begin{array}{l} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} - x_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} + x_i(z)z^{-\tau_i}\right] \end{array} \right\} \\ \quad +\beta_i \left\{ \begin{array}{l} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[v_j(z)z^{-\tau_{ij}} - v_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[v_j(z)z^{-\tau_{ij}} + v_i(z)z^{-\tau_i}\right] \end{array} \right\}, i \in \sigma_1 \end{cases} \tag{13}$$

$$\begin{cases} (z-1)x_i(z) = \gamma_i \left\{ \begin{array}{l} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} - x_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} + x_i(z)z^{-\tau_i}\right] \end{array} \right\} \\ \quad +w_i(z) \\ (z-1)w_i(z) = \phi_i \left\{ \begin{array}{l} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} - x_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} + x_i(z)z^{-\tau_i}\right] \end{array} \right\}, \\ \quad i \in \sigma_2 \end{cases} \tag{14}$$

After some calculations, equations (13) and (14) can be rewritten as following equations form (15) and (16).

$$\begin{aligned} &(z-1)^2 x_i(z) \\ &= \alpha_i \left\{ \begin{array}{l} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} - x_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} + x_i(z)z^{-\tau_i}\right] \end{array} \right\} \\ &+(z-1)\beta_i \left\{ \begin{array}{l} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} - x_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} + x_i(z)z^{-\tau_i}\right] \end{array} \right\}, \\ &i \in \sigma_1 \end{aligned} \tag{15}$$

$$\begin{cases} (z-1)^2 x_i(z) = (z-1) \\ \gamma_i \left\{ \begin{array}{l} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} - x_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} + x_i(z)z^{-\tau_i}\right] \end{array} \right\} \\ +(z-1)w_i(z) \\ (z-1)w_i(z) = \phi_i \left\{ \begin{array}{l} \sum\limits_{j\in s_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} - x_i(z)z^{-\tau_i}\right] \\ -\sum\limits_{j\in d_i} a_{ij}\varepsilon_{ij}p\left[x_j(z)z^{-\tau_{ij}} + x_i(z)z^{-\tau_i}\right] \end{array} \right\}, \\ \quad i \in \sigma_2 \end{cases} \tag{16}$$

To simplify the calculation, we define

$$x_{22}(z) = [x_1(z), x_2(z), \cdots, x_n(z)]^T,$$
$$x_{11}(z) = [x_{n+1}(z), x_{n+2}(z), \cdots, x_{n+m}(z)]^T$$

The z-transform L of graph G is defined as follows:

$$\widetilde{L} = z(L) = \begin{cases} -a_{ij}z^{-\tau_{ij}}, i \neq j, \\ \sum\limits_{j\in N_i} a_{ij}z^{-\tau_i}, i = j \quad i, j \in \sigma, \end{cases}$$

where $z(L) \in R_{(m+n)\times(m+n)}$.

Then the matrix $\widetilde{L}$ can be rewritten as follows:

$$\begin{aligned} \widetilde{L} &= \begin{bmatrix} \widetilde{L}_{22} + \widetilde{D}_{21} & -\widetilde{A}_{21} \\ -\widetilde{A}_{12} & \widetilde{L}_{11} + \widetilde{D}_{12} \end{bmatrix} \\ &= \begin{bmatrix} \widetilde{D}_{22} - \widetilde{A}_{22} + \widetilde{D}_{21} & -\widetilde{A}_{21} \\ -\widetilde{A}_{12} & \widetilde{D}_{11} - \widetilde{A}_{11} + \widetilde{D}_{12} \end{bmatrix} \end{aligned}$$

where

$$\widetilde{A}_{22} = \left(-a_{ij}z^{-\tau_{ij}}\right)_{m\times m}, \quad j \in N_{i,2}, i \in \sigma_1$$

$$\begin{cases} \tau_i < \dfrac{4(\beta_i - \alpha_i)}{\left(\widetilde{d}\varepsilon_{ij}p\right)^2 \beta_i(\beta_i - \alpha_i)^2 + 4\alpha_i + \widetilde{d}\varepsilon_{ij}p(\beta_i - \alpha_i)\sqrt{\left(\widetilde{d}\varepsilon_{ij}p\right)^2 \beta_i^4(\alpha_i - \beta_i)^2 + 4\alpha_i^2}}, & \alpha_i \neq \beta_i \\ \tau_i < \sqrt{\dfrac{1}{p\varepsilon_{ij}\widetilde{d}_i\beta_i}}, & \alpha_i = \beta_i \end{cases} \tag{9}$$

$$\begin{cases} \tau_i < \dfrac{4(\phi_i - \gamma_i)}{\left(\widehat{d}_i\varepsilon_{ij}p\right)^2 \phi_i(\phi_i - \gamma_i)^2 + 4\gamma_i + \widehat{d}_i\varepsilon_{ij}p(\phi_i - \gamma_i)\sqrt{\left(\widehat{d}_i\varepsilon_{ij}p\right)^2 \phi_i^4(\gamma_i - \phi_i)^2 + 4\gamma_i^2}}, & \gamma_i \neq \phi_i \\ \tau_i < \sqrt{\dfrac{1}{p\varepsilon_{ij}\widehat{d}_i\phi_i}}, & \gamma_i = \phi_i \end{cases} \tag{10}$$

$$\widetilde{A}_{21} = \left(-a_{ij}z^{-\tau_{ij}}\right)_{m\times n}, \quad j \in N_{i,1}, i \in \sigma_1$$
$$\widetilde{A}_{12} = \left(-a_{ij}z^{-\tau_{ij}}\right)_{n\times m}, \quad j \in N_{i,2}, i \in \sigma_2$$
$$\widetilde{A}_{11} = \left(-a_{ij}z^{-\tau_{ij}}\right)_{n\times n}, \quad j \in N_{i,1}, i \in \sigma_2$$
$$\widetilde{D}_{22} = diag\left\{\sum_{j\in N_{i,2}} a_{ij}z^{-\tau_i}, i \in \sigma_1\right\}$$
$$\widetilde{D}_{21} = diag\left\{\sum_{j\in N_{i,1}} a_{ij}z^{-\tau_i}, i \in \sigma_1\right\}$$
$$\widetilde{D}_{12} = diag\left\{\sum_{j\in N_{i,2}} a_{ij}z^{-\tau_i}, i \in \sigma_2\right\}$$
$$\widetilde{D}_{11} = diag\left\{\sum_{j\in N_{i,1}} a_{ij}z^{-\tau_i}, i \in \sigma_2\right\}$$

After some calculations, systems (7) and (8) can be written as equation (17):

$$\begin{cases} (z-1)^2 x_{22}(z) = -\left[\alpha_i\varepsilon_{ij}p + \beta_i\varepsilon_{ij}p(z-1)\right] \\ \quad \left(\widetilde{L}_{22}+\widetilde{D}_{21}\right)x_{22}(z) \\ -\left[\alpha_i\varepsilon_{ij}p + \beta_i\varepsilon_{ij}p(z-1)\right]\widetilde{A}_{21}x_{11}(z), i \in \sigma_1 \\ (z-1)^2 x_{11}(z) = -\left[\gamma_i\varepsilon_{ij}p + \phi_i\varepsilon_{ij}p(z-1)\right]\widetilde{A}_{12}x_{11}(z) \\ -\left[\gamma_i\varepsilon_{ij}p + \phi_i\varepsilon_{ij}p(z-1)\right]\left(\widetilde{L}_{11}+\widetilde{D}_{12}\right)x_{22}(z), i \in \sigma_2 \end{cases} \tag{17}$$

Let $x(z) = [x_{22}(z), x_{11}(z)]^T$, equation (17) can be rewritten as matrix form (18)

$$(z-1)^2 x(z) = \eta(z)x(z) \tag{18}$$

where, as shown in the equation at the bottom of the page.

Therefore, the characteristic equation of (17) is

$$C(z) = \det\left((z-1)^2 I - \eta(z)\right) \tag{19}$$

According to the Lyapunov stability principle, if $C(z) = 0$, the roots of equation (19) is $z = 1$ or in the unit circle of the complex plane, then the couple group of HMASs (7) and (8) can be realized.

If $z \neq 1$, let $C(z) = \det(I + Q(z)) = 0$, we can get the following equation (20).

$$Q(z) = -\frac{\eta(z)}{(z-1)^2} = \begin{bmatrix} \frac{[\alpha_i\varepsilon_{ij}p+\beta_i\varepsilon_{ij}p(z-1)](\widetilde{L}_{22}+\widetilde{D}_{21})}{(z-1)^2} & \frac{[\alpha_i\varepsilon_{ij}p+\beta_i\varepsilon_{ij}p(z-1)]\widetilde{A}_{21}}{(z-1)^2} \\ \frac{[\gamma_i\varepsilon_{ij}p+\phi_i\varepsilon_{ij}p(z-1)]\widetilde{A}_{12}}{(z-1)^2} & \frac{[\gamma_i\varepsilon_{ij}p+\phi_i\varepsilon_{ij}p(z-1)](\widetilde{L}_{11}+\widetilde{D}_{12})}{(z-1)^2} \end{bmatrix} \tag{20}$$

If $z = 1$, then $\det\left((z-1)^2 I - \eta(z)\right) = \left(\alpha_i\varepsilon_{ij}P\right)^{m+n} \det\left(\widetilde{L}\right)$. According to Lemma 1, it is obvious that 0 is a characteristic value of L, so z=1 is the roots of equation (17).

Let $z = e^{j\omega}$(j is an imaginary unit). According to Lemma 1 and Nyquist criterion, if the point $(-1, j0)$ is not surrounded by Nyquist curve $Q_i\left(e^{j\omega}\right)$, then the characteristic roots of equation (19) will be located in the unit circle $(Q_i, i \in \sigma_1 \cup \sigma_2)$ of the complex plane. Then the couple groups of HMASs (7) and (8) can be realized. In this case, using Gerschgorin circle theorem, it has

$$\lambda\left(Q\left(e^{j\omega}\right)\right) \in \{Q_i, i \in \sigma_1\} \cup \{Q_i, i \in \sigma_2\}$$

If $i \in \sigma_1$, then

$$Q_i = \left\{\begin{aligned} & x : x \in C \left| x - \frac{p\varepsilon_{ij}[\alpha_i+\beta_i(e^{j\omega}-1)]e^{-j\omega\tau_i}\sum_{j\in N_i} a_{ij}}{(e^{j\omega}-1)^2}\right| \\ & \leq \sum_{j\in N_i}\left|\frac{[\gamma_i+\phi_i(e^{j\omega}-1)]p\varepsilon_{ij}a_{ij}}{(e^{j\omega}-1)^2}e^{-j\omega\tau_{ij}}\right| \end{aligned}\right\} \tag{21}$$

The center of the disk (21) is

$$Q_i\left(e^{j\omega}\right) = \frac{\left[\alpha_i + \beta_i\left(e^{j\omega}-1\right)\right]e^{-j\omega\tau_i}p\varepsilon_{ij}\sum_{j\in N_i} a_{ij}}{\left(e^{j\omega}-1\right)^2} \tag{22}$$

Let $\widetilde{d}_i = \sum_{v_{j\in N_i}} a_{ij}, i \in \sigma_1$, applying the Euler formula to (22), one has

$$Q_i\left(e^{j\omega}\right) = \frac{\begin{aligned}&\widetilde{d}_i\varepsilon_{ij}p\left[(\alpha_i-\beta_i)\cos(\omega\tau_i+\omega)+\beta_i\cos\omega\tau\right]\\ &+j\widetilde{d}_i\varepsilon_{ij}p\left[(\beta_i-\alpha_i)\sin(\omega\tau_i+\omega)-\beta_i\sin\omega\tau_i\right]\end{aligned}}{2(\cos\omega-1)} \tag{23}$$

Suppose that ω_{io} is the first intersection point of $Q_i\left(e^{j\omega}\right)$ on the real axis, equation (24) can be obtained from equation (23)

$$\frac{\widetilde{d}_i\varepsilon_{ij}p\left[(\beta_i-\alpha_i)\sin(\omega_{i0}\tau_i+\omega_{i0})-\beta_i\sin\omega_{i0}\tau_i\right]}{2(\cos\omega_{i0}-1)} = 0 \tag{24}$$

$$\widetilde{d}_i\varepsilon_{ij}p\left[(\beta_i-\alpha_i)\sin(\omega_{i0}\tau_i+\omega_{i0})-\beta_i\sin\omega_{i0}\tau_i\right] = 0 \tag{25}$$

If $\alpha_i \neq \beta_i$, one has

$$\frac{\sin(\omega_{i0}\tau_i+\omega_{i0})}{\sin\omega_{i0}\tau_i} = \frac{\beta_i}{(\beta_i-\alpha_i)} \tag{26}$$

Applying the Taylor formula to equation (26), the following equation (27) can be obtained.

$$\begin{aligned} \frac{\sin(\omega_{i0}\tau_i+\omega_{i0})}{\sin\omega_{i0}\tau_i} &= 1 - \frac{\omega_{i0}^2}{2} + \frac{\omega_{i0}}{\omega_{i0}\tau_i} \\ 1 - \frac{\omega_{i0}^2}{2} + \frac{\omega_{i0}}{\omega_{i0}\tau_i} &= \frac{\beta_i}{\beta_i-\alpha_i} \\ \tau_i &= \frac{2(\beta_i-\alpha_i)}{2\alpha_i+\omega_{i0}^2(\beta_i-\alpha_i)} \end{aligned} \tag{27}$$

$$\eta(z) = \begin{bmatrix} -\left[\alpha_i\varepsilon_{ij}p+\beta_i\varepsilon_{ij}p(z-1)\right]\left(\widetilde{L}_{22}+\widetilde{D}_{21}\right) & -\left[\alpha_i\varepsilon_{ij}p+\beta_i\varepsilon_{ij}p(z-1)\right]\widetilde{A}_{21} \\ -\left[\gamma_i\varepsilon_{ij}p+\phi_i\varepsilon_{ij}p(z-1)\right]\widetilde{A}_{12} & -\left[\gamma_i\varepsilon_{ij}p+\phi_i\varepsilon_{ij}p(z-1)\right]\left(\widetilde{L}_{11}+\widetilde{D}_{12}\right) \end{bmatrix}$$

In addition, the first intersection point ω_{i0} of $Q_i\left(e^{j\omega}\right)$ on the real axis is in the unit circle, combining with equation (23), the following equation (28) can be obtained.

$$\left|Q_i\left(e^{j\omega}\right)\right| = \left\{ \begin{array}{l} \left(\frac{\widetilde{d}_i\varepsilon_{ij}p[(\alpha_i-\beta_i)\cos(\omega\tau_i+\omega)+\beta_i\cos\omega\tau]}{2(\cos\omega-1)}\right)^2 \\ +\left(\frac{\widetilde{d}_i\varepsilon_{ij}p[(\beta_i-\alpha_i)\sin(\omega\tau_i+\omega)-\beta_i\sin\omega\tau_i]}{2(\cos\omega-1)}\right)^2 \end{array} \right\}^{\frac{1}{2}} < 1 \tag{28}$$

After some calculations, one has

$$\omega_{i0}^4+(\widetilde{d}_i\varepsilon_{ij}p\beta_i)(\alpha_i-\beta_i)\,\omega_{i0}^2-(\widetilde{d}_i\varepsilon_{ij}p)^2(\alpha_i-\beta_i)^2-\beta_i^2>0 \tag{29}$$

On the basis of equation (29), we get (30), as shown at the bottom of the next page.

According to (27) and (30), we have (31), as shown at the bottom of the next page.

Thus, if $\alpha_i = \beta_i$ and $\gamma_i = \phi_i$, we know that the point $(-\vartheta, j_0)$, $\vartheta \geq 1$ is not be enclosed in Q, $i \in \sigma_1$, one has

$$\left|-\vartheta-\frac{\left[\alpha_i+\beta_i\left(e^{j\omega}-1\right)\right]p\widetilde{d}_i\varepsilon_{ij}e^{-j\omega\tau_i}}{\left(e^{j\omega}-1\right)^2}\right| > \sum_{j\in N_i}\left|\frac{\left[\gamma_i+\phi_i\left(e^{j\omega}-1\right)\right]p\varepsilon_{ij}a_{ij}}{\left(e^{j\omega}-1\right)^2}e^{-j\omega\tau_{ij}}\right| \tag{32}$$

By Applying Euler formula, we obtain (33) and (34), as shown at the bottom of the next page.

After some calculations, we have

$$\vartheta^2+\vartheta\frac{p\varepsilon_{ij}\widetilde{d}_i\beta_i\cos\omega\tau_i}{\cos\omega-1}>0 \tag{35}$$

Because $\vartheta \geq 1$, inequality (35) can be rewritten as inequality (36). Combining with the Taylor formula, we obtain the following form (36).

$$\tau_i^2>\frac{2}{\omega^2}-\frac{1}{p\varepsilon_{ij}\widetilde{d}_i\beta_i} \tag{36}$$

If $\frac{1}{p\varepsilon_{ij}\widetilde{d}_i\beta_i} > \tau_i^2$, inequality (30) holds. Then, the following (37) can be obtained.

$$\tau_i<\sqrt{\frac{1}{p\varepsilon_{ij}\widetilde{d}_i\beta_i}} \tag{37}$$

Note that inequality (37) is one of the necessary insufficient conditions of (30).

Therefore, when $i \in \sigma_1$, we can compute the upper bound of the time delay in the following form (38), as shown at the bottom of the next page.

When $i \in \sigma_2$, we can obtain the following inequality (39).

$$Q_i = \left\{ \begin{array}{l} x : x \in C\left|x-\frac{\left[\gamma_i p\varepsilon_{ij}+\phi_i p\varepsilon_{ij}\left(e^{j\omega}-1\right)\right]\sum\limits_{j\in N_i}a_{ij}e^{-j\omega\tau_i}}{\left(e^{j\omega}-1\right)^2}\right| \\ \leq \sum\limits_{j\in N_i}\left|\frac{\left[\gamma_i p\varepsilon_{ij}+\phi_i p\varepsilon_{ij}\left(e^{j\omega}-1\right)\right]e^{-j\omega\tau_i}}{\left(e^{j\omega}-1\right)^2}a_{ij}e^{-j\omega\tau_{ij}}\right| \end{array} \right\} \tag{39}$$

Thus, the center of the disk (21) is computed as follows.

$$Q_i\left(e^{j\omega}\right)=\frac{\left[\gamma_i p\varepsilon_{ij}+\phi_i p\varepsilon_{ij}\left(e^{j\omega}-1\right)\right]\sum\limits_{j\in N_i}a_{ij}e^{-j\omega\tau_i}}{\left(e^{j\omega}-1\right)^2} \tag{40}$$

Define $\widehat{d} = \sum\limits_{v_j\in N_i} a_{ij}$, applying the Euler formula, the following equation (41), as shown at the bottom of the next page, can be obtained.

Suppose that ω_{io} is the first intersection point of $Q_i\left(e^{j\omega}\right)$ on real axis, combining with (41), we can get equation (42).

$$\frac{\widehat{d}_i\varepsilon_{ij}p\left[(\phi_i-\gamma_i)\sin\left(\omega_{i0}\tau_i+\omega\right)-\phi_i\sin\omega_{i0}\tau_i\right]}{2\left(\cos\omega_{i0}-1\right)}=0 \tag{42}$$

where $\widehat{d}_i\varepsilon_{ij}p\left[(\phi_i-\gamma_i)\sin\left(\omega\tau_i+\omega\right)-\phi_i\sin\omega\tau_i\right]=0$

Similarly, if $\gamma \neq \phi$, after some calculations, we have

$$\tau_i=\frac{2\left(\phi_i-\gamma_i\right)}{2\gamma_i+\omega_{i0}^2\left(\phi_i-\gamma_i\right)} \tag{43}$$

In addition, the first crossover point ω_{i0} of $Q_i\left(e^{j\omega}\right)$ on the real axis is located int the unit circle, the following inequality (44) can be obtained.

$$\left|Q_i\left(e^{j\omega}\right)\right| = \left\{ \begin{array}{l} \left(\frac{\widehat{d}_i\varepsilon_{ij}p[(\gamma_i-\phi_i)\cos(\omega\tau_i+\omega)+\phi_i\cos\omega\tau]}{2(\cos\omega-1)}\right)^2 \\ +\left(\frac{\widehat{d}_i\varepsilon_{ij}p[(\phi_i-\gamma_i)\sin(\omega\tau_i+\omega)-\phi_i\sin\omega\tau_i]}{2(\cos\omega-1)}\right)^2 \end{array} \right\}^{\frac{1}{2}} < 1 \tag{44}$$

After some calculations, the following inequality (45), as shown at the bottom of the next page, can be obtained.

Combining with (45) and (42), one has, (46), as shown at the bottom of the next page.

If $\gamma_i = \phi_i$, the point $(-\vartheta, j_0)$, $\vartheta \geq 1$ is not in the Q, $i \in \sigma_1$, then it cannot be enclosed in the Nyquist curve. Thus, the following inequation (47) can be obtained.

$$\left|-\vartheta-\frac{\left[\gamma_i+\phi_i\left(e^{j\omega}-1\right)\right]p\widehat{d}_i\varepsilon_{ij}e^{-j\omega\tau_i}}{\left(e^{j\omega}-1\right)^2}\right| > \sum_{j\in N_i}\left|\frac{\left[\gamma_i+\phi_i\left(e^{j\omega}-1\right)\right]p\varepsilon_{ij}a_{ij}}{\left(e^{j\omega}-1\right)^2}e^{-j\omega\tau_{ij}}\right| \tag{47}$$

After some calculations, one has

$$\tau_i<\sqrt{\frac{1}{p\varepsilon_{ij}\widehat{d}_i\beta_i}} \tag{48}$$

Note that the inequation (48) is one of the necessary insufficient conditions of inequation (45).

Therefore, when $i \in \sigma_2$ the upper bound of the delay is (49), as shown at the bottom of the next page.

where $d_i = \widetilde{d}_i \cup \widehat{d}_i = \sum\limits_{v_j\in N_i} a_{ij}$, $i \in \sigma_1 \cup \sigma_2$

To sum up, the proof of Theorem 1 is completed.

Remark 1: The variable weighting coefficient α_i, β_i, γ_i, ϕ_i of the control protocols (7) and (8) are designed to make the state of agents converge to any target state. The controller designed in this paper has strong flexibility, adaptability, and can speed up the grouping consensus of the system.

Remark 2: When $i \in \sigma_1$, according to equation (24) and Gerschgorin disc theory, we can get equation (22). If $\alpha_i \neq \beta_i$, according to the Euler formula, equation (24) and (27), we can get equation (31). If $\alpha_i = \beta_i$ according to equation (32) and Euler formula, we can get (37). According to the analysis of the above results, the formula (9) can be obtained. When $i \in \sigma_2$, according to similar methods, formula (10) in can be obtained.

Remark 3: From Theorem 1, we can know that the control parameters $(\alpha_i, \beta_i, \gamma_i, \phi_i)$, ε_{ij} and p are the key factors for the multi-agent systems achieving weighted group consensus. The system tolerance can be improved and control costs can

$$\omega_{i0}^2 > \frac{(\widetilde{d}_i\varepsilon_{ij}p\beta_i)(\beta_i-\alpha_i)+\sqrt{(\widetilde{d}_i\varepsilon_{ij}p\beta_i)^2(\alpha_i-\beta_i)^2+4((\widetilde{d}_i\varepsilon_{ij}p)^2(\alpha_i-\beta_i)^2+\beta_i^2)}}{2} \tag{30}$$

$$\tau_i < \frac{4(\beta_i-\alpha_i)}{4\alpha_i+((\widetilde{d}_i\varepsilon_{ij}p\beta_i)(\beta_i-\alpha_i)+\sqrt{(\widetilde{d}_i\varepsilon_{ij}p\beta_i)^2(\alpha_i-\beta_i)^2+4((\widetilde{d}_i\varepsilon_{ij}p)^2(\alpha_i-\beta_i)^2+\beta_i^2)})(\beta_i-\alpha_i)} \tag{31}$$

$$\begin{aligned}&\left|-\vartheta-p\varepsilon_{ij}\widetilde{d}_i\frac{[\alpha_i+\beta_i(\cos\omega+j\sin\omega-1)](\cos\omega\tau_i-j\sin\omega\tau_i)}{(\cos\omega+j\sin\omega-1)^2}\right|\\&\quad>\sum_{j\in N_i}\left|p\varepsilon_{ij}a_{ij}\frac{[\gamma_i+\phi_i(\cos\omega+j\sin\omega-1)]}{(\cos\omega+j\sin\omega-1)^2}(\cos\omega\tau_{ij}-j\sin\omega\tau_{ij})\right|\end{aligned} \tag{33}$$

$$\left|-\vartheta-\frac{p\varepsilon_{ij}\widetilde{d}_i\beta_i\cos\omega\tau_i}{2(\cos\omega-1)}+j\frac{\beta_ip\varepsilon_{ij}\widetilde{d}_i\sin\omega\tau_i}{2(\cos\omega-1)}\right|>\sum_{j\in N_i}\left|\frac{\phi_ip\varepsilon_{ij}a_{ij}}{2(\cos\omega-1)}\right| \tag{34}$$

$$\begin{cases}\tau_i<\dfrac{4(\beta_i-\alpha_i)}{(\widetilde{d}_i\varepsilon_{ij}p)^2\beta_i(\beta_i-\alpha_i)^2+4\alpha_i+\widetilde{d}\varepsilon_{ij}p(\beta_i-\alpha_i)\sqrt{(\widetilde{d}_i\varepsilon_{ij}p)^2\beta_i^4(\alpha_i-\beta_i)^2+4\alpha_i^2}}, & \alpha_i\neq\beta_i\\ \tau_i<\sqrt{\dfrac{1}{p\varepsilon_{ij}\widetilde{d}_i\beta_i}}, & \alpha_i=\beta_i\end{cases} \tag{38}$$

$$Q_i\left(e^{j\omega}\right)=\frac{\widehat{d}_i\varepsilon_{ij}p[(\gamma_i-\phi_i)\cos(\omega\tau_i+\omega)+\gamma_i\cos\omega\tau]+j\widehat{d}_i\varepsilon_{ij}p[(\phi_i-\gamma_i)\sin(\omega\tau_i+\omega)-\phi_i\sin\omega\tau_i]}{2(\cos\omega-1)} \tag{41}$$

$$\omega_{i0}^2>\frac{(\widehat{d}_i\varepsilon_{ij}p)^2\phi_i(\gamma_i-\phi_i)+\widehat{d}_i\varepsilon_{ij}p\sqrt{(\widehat{d}_i\varepsilon_{ij}p)^2\phi_i^4(\gamma_i-\phi_i)^2+4\gamma_i^2}}{2} \tag{45}$$

$$\tau_i<\frac{4(\phi_i-\gamma_i)}{(\widehat{d}_i\varepsilon_{ij}p)^2\phi_i(\gamma_i-\phi_i)^2+4\gamma_i+\widehat{d}_i\varepsilon_{ij}p(\phi_i-\gamma_i)\sqrt{(\widehat{d}_i\varepsilon_{ij}p)^2\phi_i^4(\gamma_i-\phi_i)^2+4\gamma_i^2}} \tag{46}$$

$$\begin{cases}\tau_i<\dfrac{4(\phi_i-\gamma_i)}{(\widehat{d}_i\varepsilon_{ij}p)^2\phi_i(\phi_i-\gamma_i)^2+4\gamma_i+\widehat{d}_i\varepsilon_{ij}p(\phi_i-\gamma_i)\sqrt{(\widehat{d}_i\varepsilon_{ij}p)^2\phi_i^4(\gamma_i-\phi_i)^2+4\gamma_i^2}}, & \gamma_i\neq\phi_i\\ \tau_i<\sqrt{\dfrac{1}{p\varepsilon_{ij}\widehat{d}_i\phi_i}}, & \gamma_i=\phi_i\end{cases} \tag{49}$$

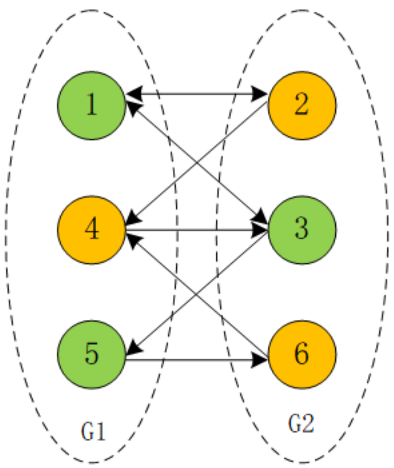
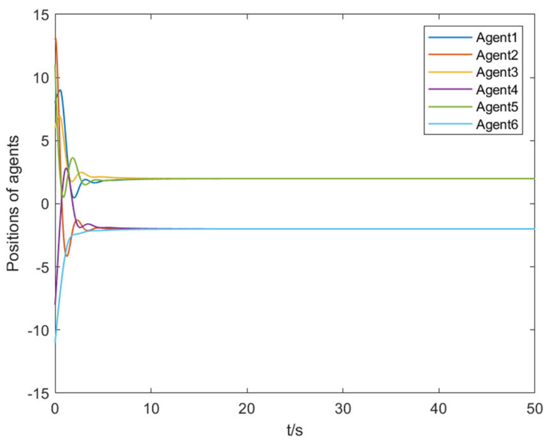
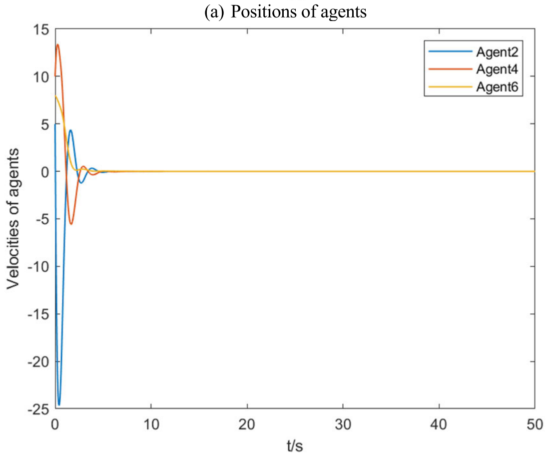


FIGURE 1. **Bipartite digraph topology of the heterogeneous multi-agent system.**



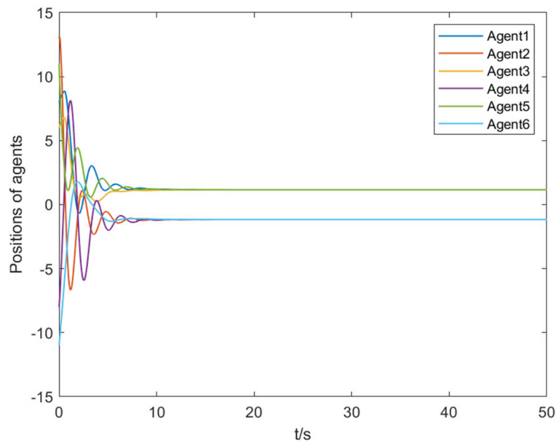
(a) Positions of agents



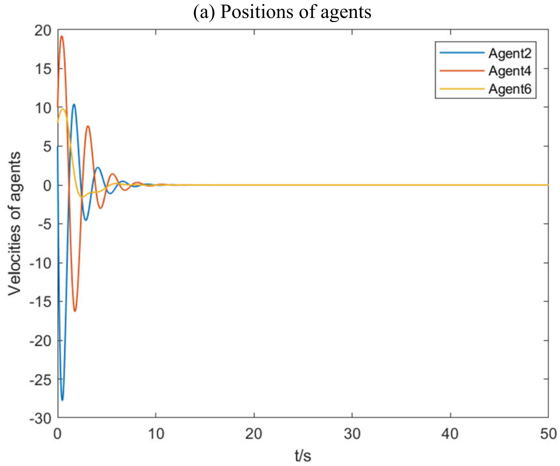
(b) Velocities of agents

FIGURE 2. **The state trajectories of the agents under undirected topology in Figure 1 with different input time delays where $p = 0.5$, $\varepsilon_{ij} = 1$, $\tau_1 = 0.6$, $\tau_2 = 0.2$, $\tau_3 = 1.6$, $\tau_4 = 0.1$, $\tau_5 = 0.05$, $\tau_6 = 0.1$.**

be reduced if we have set control parameters reasonably for information interactions between agents. Meanwhile, the communication time delay τ_{ij} has no fundamental impact on weighted group consensus of the multi-agent systems.



(a) Positions of agents



(b) Velocities of agents

FIGURE 3. **The state trajectories of the agents under undirected topology in Figure 1 with different input time delays where $p = 1$, $\varepsilon_{ij} = 1$, $\tau_1 = 0.3$, $\tau_2 = 0.2$, $\tau_3 = 0.5$, $\tau_4 = 0.2$, $\tau_5 = 0.1$, $\tau_6 = 0.15$.**

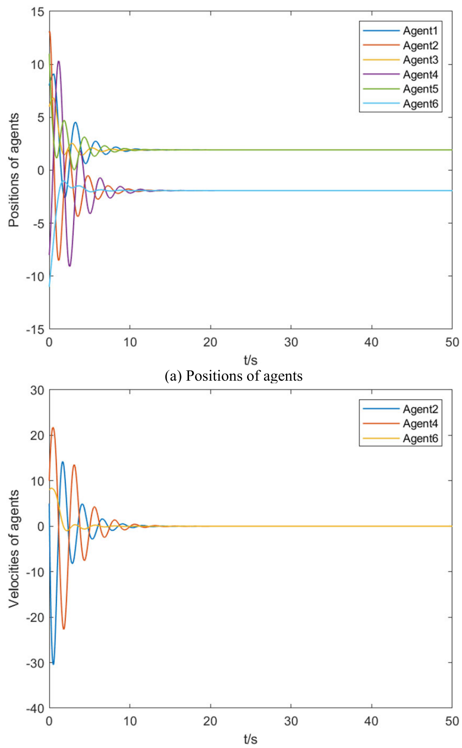
Yet, the input time delay τ_i has impact on weighted group consensus of the multi-agent systems.

Remark 4: In order to achieve weighted group consensus of discrete heterogeneous multi-agent systems, many control protocols based on cooperation or competition relation between agents have been proposed, while the protocols based on competition-cooperative relation are relatively rare. At the same time, the control parameters $(\alpha_i, \beta_i, \gamma_i, \phi_i)$, ε_{ij} and p setting under the novel protocol are strict, and the setting values also affect the group consensus of the systems. If the above problems can be handled effectively, the system can achieve group consensus.

IV. SIMULATIONS

In this section, we will verify the correctness and effective of the weighted group consensus for discrete-time heterogeneous multi-agent systems in the cooperative-competitive network with time delays.

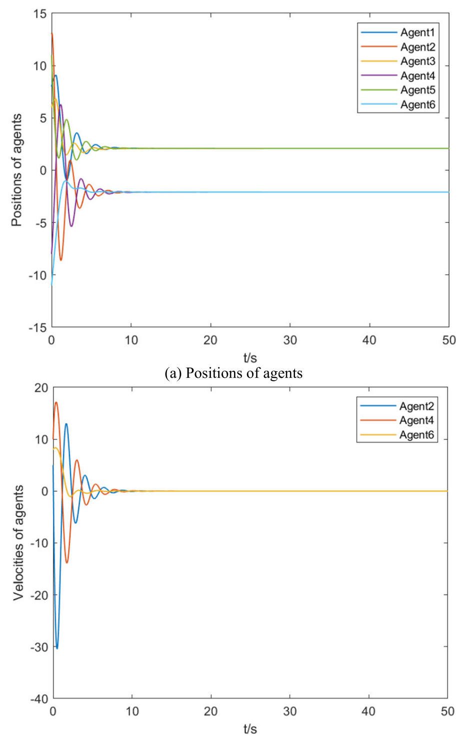
A heterogeneous multi-agent system under the bipartite digraph topology which has a spanning tree is designed as Figure 1. The system has eight agents which are divided into two subgroups G_1 and G_2. Agents 2,4 and 6 are second-order



(a) Positions of agents

(b) Velocities of agents

FIGURE 4. The state trajectories of the agents under undirected topology in Figure 1 with different input time delays where $p = 1$, $\varepsilon_{ij} = 0.1$, $\tau_1 = 1.1$, $\tau_2 = 0.45$, $\tau_3 = 2$, $\tau_4 = 0.2$, $\tau_5 = 0.15$, $\tau_6 = 0.3$.

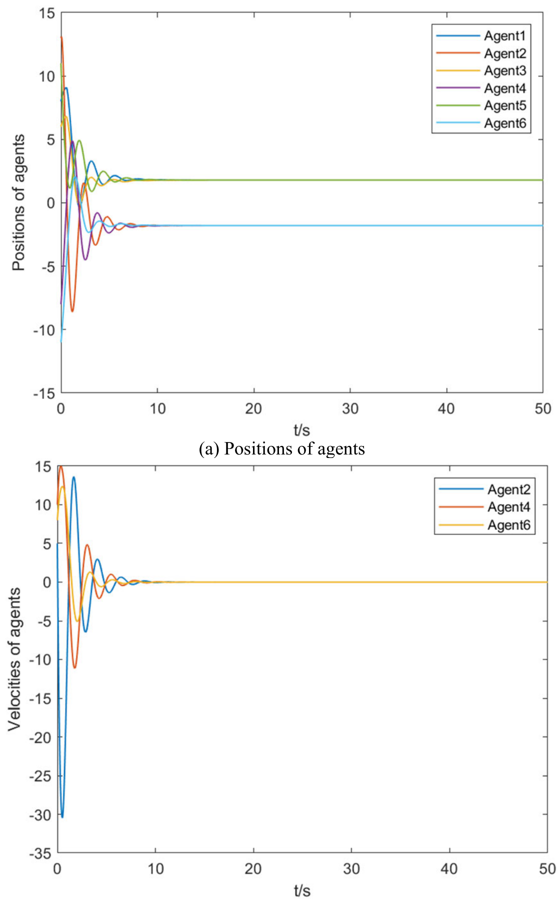


(a) Positions of agents

(b) Velocities of agents

FIGURE 5. The state trajectories of the agents under undirected topology in Figure 1 with different input time delays where $p = 0.5$, $\varepsilon_{ij} = 1$, $\alpha_i = \beta_i = 1.6$, $\gamma_i = \phi_i = 0.7$ $\tau = 1$.

agents. Agents 1,3 and 5 are first-order agents. Thus, the system is a heterogeneous multi-agent system.



(a) Positions of agents

(b) Velocities of agents

FIGURE 6. The state trajectories of the agents under undirected topology in Figure 1 with different input time delays where $p = 0.5$, $\varepsilon_{ij} = 1$, $\alpha_i = \beta_i = 0.7$, $\gamma_i = \phi_i = 1.6$ $\tau = 1$.

For simplicity, we assume each edge in the bipartite digraph to be $a_{ij} = 1, i, j = (1, 2, 3, 4, 5, 6)$. The initiate state of the system (7) and (8) is $x_i(0) = \left[8,\ 13,\ 6,\ -8,\ 11,\ -11\right]^T$. Considering that the control parameters are dynamic, we set different values for them.

Example 1: Assume $\alpha \neq \beta, \gamma \neq \phi$,

$$\left\{\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4,\ \alpha_5,\ \alpha_6\right\} = \left\{0.9,\ 0.2,\ 0.25,\ 0.6,\ 2.9,\ 0.4\right\}$$
$$\left\{\beta_1,\ \beta_2,\ \beta_3,\ \beta_4,\ \beta_5,\ \beta_6\right\} = \left\{2,\ 0.8,\ 1.7,\ 0.8,\ 3.5,\ 0.6\right\}$$
$$\left\{\gamma_1,\ \gamma_2,\ \gamma_3,\ \gamma_4,\ \gamma_5,\ \gamma_6\right\} = \left\{0.35,\ 0.8,\ 0.2,\ 1.5,\ 1.9,\ 2.3\right\}$$
$$\left\{\phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5,\ \phi_6\right\} = \left\{1,\ 1.2,\ 0.8,\ 2,\ 2.4,\ 3.3\right\}$$

(1) Let $p = 0.5$ and $\varepsilon_{ij} = 1$, according to Theorem 1, we can get

$$0 < \tau_1 < \min\left\{0.766,\ 1.62\right\}, 0 < \tau_2 < \min\left\{0.36,\ 2.58\right\},$$
$$0 < \tau_3 < \min\left\{1.7,\ 2.28\right\}, 0 < \tau_4 < \min\left\{0.29,\ 0.32\right\},$$
$$0 < \tau_5 < \min\left\{0.17,\ 0.229\right\}, 0 < \tau_6 < \min\left\{0.27,\ 0.47\right\}.$$

Let

$$\left\{\tau_1,\ \tau_2,\ \tau_3,\ \tau_4,\ \tau_5,\ \tau_6\right\} = \left\{0.6,\ 0.2,\ 1.6,\ 0.1,\ 0.05,\ 0.1\right\}.$$

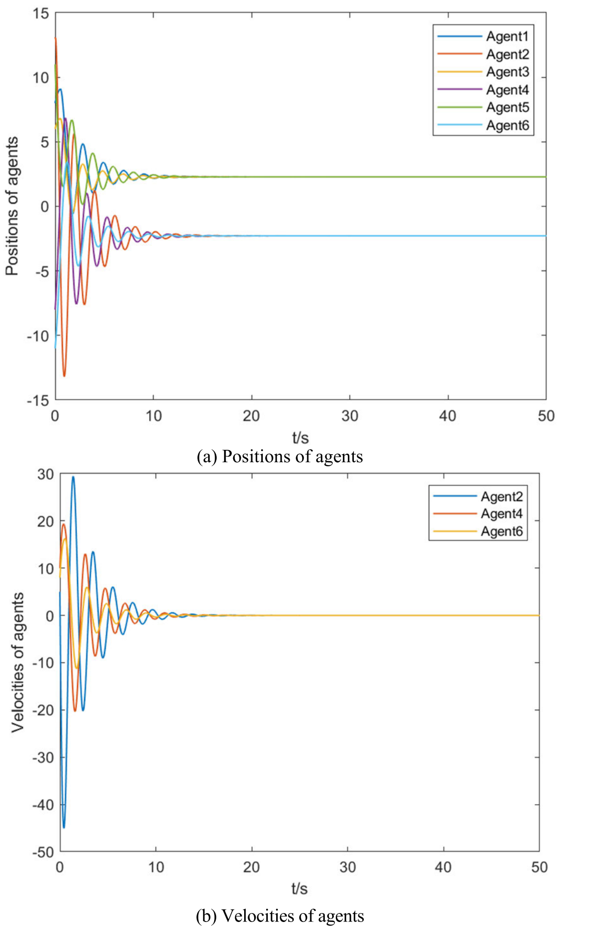


FIGURE 7. The state trajectories of the agents under undirected topology in Figure 1 with different input time delays where $p = 1, \varepsilon_{ij} = 1, \alpha_i = \beta_i = 1.6, \gamma_i = \phi_i = 0.7$ $\tau = 0.5$.

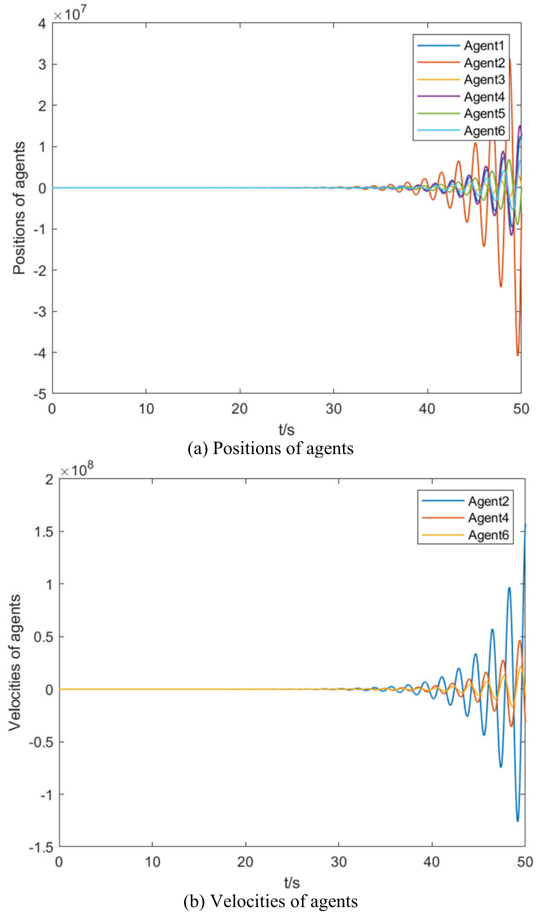


FIGURE 8. The state trajectories of the agents under undirected topology in Figure 1 with different input time delays where $p = 1, \varepsilon_{ij} = 0.1, \alpha_i = \beta_i = 1.6, \gamma_i = \phi_i = 0.7$ $\tau = 2$.

The simulation results are shown as follows:

(2) Let $p = 1$ and $\varepsilon_{ij} = 1$, we can get

$$0 < \tau_1 < \min\{0.397,\ 0.97\},$$
$$0 < \tau_2 < \min\{0.39,\ 1.63\}, 0 < \tau_3 < \min\{0.54,\ 1.89\},$$
$$0 < \tau_4 < \min\{0.24,\ 1.55\}, 0 < \tau_5 < \min\{0.13,\ 0.19\} \text{ and}$$
$$0 < \tau_6 < \min\{0.16,\ 0.45\}.$$

Let

$$\{\tau_1,\ \tau_2,\ \tau_3,\ \tau_4,\ \tau_5,\ \tau_6\} = \{0.3,\ 0.2,\ 0.5,\ 0.2,\ 0.1,\ 0.15\}.$$

The simulation results are shown as follows:

(3) Let $p = 1$ and $\varepsilon_{ij} = 0.1$, we can get

$$0 < \tau_1 < \min\{1.15,\ 1.79\}, 0 < \tau_2 < \min\{0.49,\ 2.89\},$$
$$0 < \tau_3 < \min\{2.3,\ 5.18\}, 0 < \tau_4 < \min\{0.33,\ 3.03\},$$
$$0 < \tau_5 < \min\{0.20,\ 0.26\} \text{ and } 0 < \tau_6 < \min\{0.40,\ 0.49\}.$$

Let

$$\{\tau_1,\ \tau_2,\ \tau_3,\ \tau_4,\ \tau_5,\ \tau_6\} = \{1.1,\ 0.45,\ 2,\ 0.2,\ 0.15,\ 0.3\}.$$

The simulation results are shown as follows:

Example 2: Assume $\alpha = \beta, \gamma = \phi$.

(1) Let $p = 0.5, \varepsilon_{ij} = 1, \alpha_i = \beta_i = 1.6, \gamma_i = \phi_i = 0.7$, we can get $0 < \tau < \min\{1.12,\ 1.68\}$. Let $\tau = 1$. The simulation results are shown as follows:

(2) Let $p = 0.5, \varepsilon_{ij} = 1, \alpha_i = \beta_i = 0.7, \gamma_i = \phi_i = 1.6$, we can get $0 < \tau < \min\{1.12,\ 1.69\}$. Let $\tau = 1$. The simulation results are shown as follows:

(3) Let $p = 1, \varepsilon_{ij} = 1, \alpha_i = \beta_i = 1.6, \gamma_i = \phi_i = 0.7$, we can get $0 < \tau < \min\{0.79,\ 1.2\}$. Let $\tau = 0.5$.The simulation results are shown as follows:

(4) Let $p = 1, \varepsilon_{ij} = 0.1, \alpha_i = \beta_i = 1.6, \gamma_i = \phi_i = 0.7$, we can get $0 < \tau < \min\{2.5,\ 3.78\}$. according to the upper bound calculated, we assume $\tau = 2$. As can be seen from Figure 8, the system is divergent at this time.

V. CONCLUSION

This paper has studied the weighted group consensus for discrete-time heterogeneous multi-agent systems. An adaptive controller is designed for this system and a grouping consensus protocol is proposed. For the systems composed of first-order and second-order agents with cooperative-competitive relation, a novel weighted group consensus protocol is designed to promote the multi-agent systems to achieve weighted group consensus. The effects of time delays, packet loss, cooperative-competitive relation and coupling strength between agents are considered. We derived the sufficient conditions for the group consensus by using Graph theory, Matrix theory and complex frequency domain methods. Finally, simulation examples have been presented to demonstrate the performance of the proposed protocol. Our future work will extend to more complex group consensus issues for heterogeneous multi-agent systems. For example, we will consider group consensus under switching topologies.

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