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RESEARCH ARTICLE

Dombi Aggregation Operators for p, q $r-$Spherical Fuzzy Sets: Application in the Stability Assessment of Cryptocurrencies

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ABSTRACT This paper presents novel operational laws for $p, q, r-$spherical fuzzy sets ($p, q, r-$SFSs) by harnessing the Dombi t-norm (DTN) and t-conorm (DTCN). These laws serve as the foundation for a set of aggregation operators (AOs) designed to consolidate $p, q, r-$spherical fuzzy ($p, q, r-$SF) information. Additionally, a multi-criteria decision-making (MCDM) method is outlined for addressing practical decision-making (DM) challenges. To demonstrate the application of the proposed approach, a numerical example is offered. Furthermore, we conducted a comparative study to validate the efficacy of the suggested approach. Finally, we discuss both the advantages and limitations of this innovative approach.

INDEX TERMS $p, q, r-$SFSs, aggregation operators, MDCM, decision making, optimization.

I. INTRODUCTION

DM is the cognitive process of evaluating available information, considering goals and objectives, weighing alternatives, and ultimately choosing a course of action while factoring in personal preferences, risks, and trade-offs. It often involves addressing cognitive biases, ethical considerations, and, in group settings, dynamics and communication. Various decision-making models and frameworks offer structured approaches. The effectiveness of decisions depends on their implementation and subsequent evaluation, making decision-making a critical skill for individuals and organizations, shaping outcomes, and facilitating adaptability in complex and dynamic environments.

Zadeh [1] introduced the concept of fuzzy sets (FSs) as a mapping from a set to the unit interval along the real number line. FSs are employed in DM to create a mathematical framework that accommodates uncertainty and vagueness in the data and preferences. This facilitates the development of decision models capable of handling imprecise information and navigating complex real-word scenarios with greater flexibility and nuance. Numerous academics have employed fuzzy sets theory and its various extensions in their research and practical applications to address a wide array of complex problems and scenarios. For instance, Rodrguez et al. [2] conducted a detailed study of hesitant FSs to elucidate related concepts and developments. Cagman et al. [3] recognized the fundamentals of fuzzy soft set theory and its associated characteristics, and introduced fuzzy soft aggregation operators (AOs) for enhancing DM efficiency.

FSs typically focus solely on assessing an object's membership degree (MD), inadvertently overlooking the significance of considering the non-MD (NMD) in object assessment. It is evident that the absence of an object's

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MD also holds crucial importance in the evaluation process. To address this conceptual gap, Atanassov [4] introduced Intuitionistic Fuzzy Sets (IFSs), a framework that encompasses both MD and NMD for elements. Szmidt et al. [5] introduced a correlation coefficient to measure the relationship strength between IFSs and determine the presence of positive or negative correlation. Zhang et al. [6] introduced a knowledge measure for IFSs, adhering to extended Szmidt-Kacprzyk axioms for intuitionistic fuzzy (IF) entropy, along with a discussion on order property conditions, supported by numerical examples demonstrating the accuracy and superiority of this parametric model over classic models. Zeng et al. [7] familiarized the IF ordered weighted distance operator in response to the need for handling IF information in various scenarios. Wang and Liu [8] extended the range of operators for IFSs by introducing Einstein addition, multiplication, exponentiation, and geometric AOs, enhancing their applicability in situations involving IF values.

In certain practical situations, the total of MD and NMDs may surpass the limit of one, rendering traditional IFS inadequate. In response to these contexts, Yager [9] introduced Pythagorean FSs (PFSs) such that the squared sum of DM and NMD is confined to be less than or equal to 1, offering a versatile framework to manage situations characterized by increased complexity and uncertainty. Li and Zeng [10] a variation of distance metrics for PFSs, aiming to assess the dissimilarity between two Pythagorean fuzzy numbers (PFNs). Thao and Smarandache [11] employed a probabilistic framework to develop the concept of fuzzy entropy for PFSs. Ejegwa [12] addressed the unique properties of Pythagorean fuzzy sets, creating axiomatic definitions for distance and similarity metrics with a focus on resolving challenges in Multiple-Attribute Group DM (MAGDM) using interval-valued Pythagorean fuzzy (PF) data.

Cuong and Kreinovich [13] proposed picture FSs (PiFSs), expanding upon traditional FSs and IFSs. Following this, multiple operations on PiFSs and their corresponding properties were investigated. He and Wang [14] evaluated New Energy Vehicles (NEVs) by analyzing online reviews, employing web scraping to gather reviews, conducting sentiment analysis, and introducing an information transformation mechanism to convert unstructured data into picture fuzzy numbers. Verma and Rohtagi [15] introduced some similarity measures between two PiFSs. Pham et al. [16] introduced an intellectual scientific decision support system based on rule-based methods using picture fuzzy sets. Zhao et al. [17] used PiFSs to enable more efficient and accurate assessments, introducing a Failure Mode and Effect Analysis approach with three essential process enhancements. Kahraman et al. [18] introduced three-dimensional spherical fuzzy sets (SFS) with arithmetic operations and aggregation operators. Haseli and Ghoushchi [19] expanded the application of the Base-Criterion Method (BCM) to decision problems in intricate and uncertain settings, incorporating Three-Dimensional SFSs. Akram et al. [20] proposed a series of AOs for complex spherical fuzzy (SF) data. Mahmood et al. [21] introduced the notions of SFS and T-SFS as a comprehensive extension encompassing FS, IFS, PFS, and PiFSs.

The previously discussed studies are characterized by specific conditions; for instance, IFSs are defined under the condition that the sum of MD (φ) and NMD (ψ) should be less than or equal to 1, i.e., $\varphi + \psi \preceq 1$. PFSs require $\varphi^2 + \psi^2 \preceq 1$, similarly, PiFSs adhere $\varphi + \eta + \psi \preceq 1$, and SFSs follow $\varphi^2 + \eta^2 + \psi^2 \preceq 1$. Also, T-SFSs are bound by the condition that the t^{th} power of φ, η, and ψ should be less than or equal to 1 i.e., $\varphi^t + \eta^t + \psi^t \preceq 1$. However, in T-spherical fuzzy environments, decision-makers are obligated to set the same value of t for all MD, neutral degree (NED), and NMDs, which can influence the overall decision-making process. To address these limitations, $p, q, r-$ spherical fuzzy sets have been introduced. In the realm of $p, q, r-$ SF environments, decision-makers have the flexibility to set different values for MD, NED, and NMD, tailoring their choices to the specific context.

In fuzzy logic applications such as decision-making, control systems, pattern recognition, information fusion, and medical diagnostics, Dombi aggregation operators are used. They are especially beneficial when the information is vague or unknown. The operators aid in the combination and processing of fuzzy data for more accurate outcomes in a variety of disciplines.

A. DOMBI AGGREGATION OPERATORS

The Dombi t-norm (DTN) and Dombi t-conorm (DTCN) offer a way to control the degree of overlap and separation between fuzzy sets, making them highly versatile tools for modelling and analyzing uncertain or imprecise data. One of the distinctive features of Dombi operations is the introduction of a parameter "λ " which allows for fine-tuning the behaviour of the aggregation process. By adjusting this parameter, decision-makers can control the degree of emphasis on the most prominent values within the set, thereby influencing the outcome. Numerous researchers have applied these operations and established various aggregation operators. For example, Seikh and Mandal [22] devised operational laws for IF Numbers (IFNs) using DTN and DTCN, initiating a group of IF Dombi operators and an algorithm for MADM problems in an IF context. Jana et al. [23] used Dombi operations and initiated Pythagorean fuzzy (PF) AOs, effectively addressing DM challenges within the PF context. Seikh and Mandal [24] utilized Dombi operation and proposed a series of AOs for interval-values Fermatean FSs. These operators demonstrate proficiency in managing extensive data sets and capturing interrelationships among decision attributes. Ali and Mahmood [25] introduced a set of complex q-rung orthopair fuzzy Dombi operators and examined their key properties, enabling the development of a MADM technique, which was illustrated with practical examples to evaluate the operators' dominance and consistency. Jana et al. [26] introduced a set of picture fuzzy Dombi operators, including weighted average, order weighted average, and weighted geometric operators. Ashraf et al. [27] established operational laws based on DTN and DTCN and introduced a set of SF Dombi AOs, examining their properties. Gurmani et al. [28] introduced linguistic T-SFSs, defined Dombi operations for linguistic T-spherical fuzzy numbers and developed AOs based on Dombi operations. For further details about Dombi operators, please refer to studies [29], [30], [31], [32], [33], and [34].

The preceding conversation underscores the notable absence of Dombi aggregation operators tailored specifically for the $p, q, r-$spherical fuzzy ($p, q, r-$ SF) context. This indicates an area where further research and development are needed to expand the applicability of Dombi operators to this specific framework.

B. MOTIVATIONS

Dombi aggregation operators offer numerous advantages, including their flexibility to adapt to different decision contexts and their adeptness at handling uncertain and imprecise information. They strike a balance between conservatism and optimism, accommodating varying preferences and risk tolerances. Widely applicable, they find use in fields like multi-criteria decision analysis, data mining, and decision support systems, providing tailored solutions and robust performance even in the presence of outliers and data variations. Dombi operators yield interpretable results and can handle both quantitative and qualitative data, serving as a bridge between numerical and linguistic information. Their versatility and ability to be customized make them valuable tools for addressing a broad spectrum of real-world problems, contributing to more informed and effective decision-making processes.

$p, q-$spherical fuzzy sets offer a robust and versatile approach to representing uncertainty, encompassing both membership neutral and non-membership degrees, thus providing a more comprehensive modelling of ambiguity and imprecision. Their flexibility allows them to handle diverse data types, making them adaptable for various applications where uncertainty prevails. The precision in modelling uncertainty makes them particularly well-suited for capturing and representing ambiguous or vague data accurately. Their robustness against outliers and data variations enhances their reliability in practical applications. Additionally, $p, q, r-$spherical fuzzy sets promote balanced DM by considering both MD and NMDs,leading to more nuanced and rational outcomes. The customizability of parameters p and q allows for tailored solutions to specific problem characteristics, thus accommodating different levels of risk tolerance and uncertainty. Their interpretability, compatibility with quantitative and qualitative data, and effectiveness in multi-criteria decision-making highlight their value in addressing complex and uncertain real-world challenges, offering support for decision-making and data analysis across various domains.

Leveraging the strengths of Dombi operations and harnessing the inherent flexibility of $p, q, r-$spherical fuzzy sets, this research introduces a novel array of aggregation operators tailored to the aggregation of $p, q, r-$spherical fuzzy data. These operators are designed to capitalize on the advantages of Dombi operations while accommodating the nuanced modelling capabilities of $p, q, r-$spherical fuzzy sets, providing a robust and versatile framework for the aggregation and synthesis of complex $p, q-$spherical fuzzy information.

C. CONTRIBUTIONS

The contribution of the proposed study is as follows:

1. This new framework of aggregation operators provides decision-makers with a powerful tool for synthesizing complex and uncertain $p, q, r-$spherical fuzzy information. It enhances the decision-making process by accommodating a broader range of data types, preferences, and risk tolerances, thus contributing to being more informed and adaptable.
2. The concepts discussed have applications across a range of disciplines, including multi-criteria decision analysis, data mining, decision support systems, and more. This cross-disciplinary relevance makes the contributions valuable not only for researchers in the field of aggregation theory but also for practitioners who seek effective solutions for real-world problems in various domains.
3. The introduced aggregation operators offer parametric properties that enhance the flexibility of the decision-making process. These parameters (p, q, r, and λ) can be fine-tuned by decision-makers to align with the specific requirements and characteristics of the situation, allowing for a more customized and adaptable approach to DM.

   The article is structured as follows: In section II, we introduce fundamental definitions and concepts pertinent to the proposed work. Section III presents novel operational laws, AOs, and their respective properties. Section IV outlines a new MCDM approach built upon the proposed operators. A numerical example illustrating the proposed approach is given in section V. Finally, in section VI, we offer conclusions for the proposed work. The layout of the article is depicted in Figure 1.

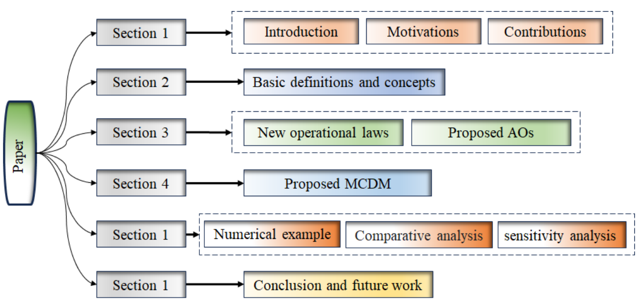

FIGURE 1. Paper layout.

II. PRELIMINARIES

Definition 1 [4]: For any element $h \in H$, we can express the intuitionistic fuzzy set $\mathcal{J}$ as:

$$\mathcal{J} = \left\{\left\langle h, \varphi_{\mathcal{J}}(h), \psi_{\mathcal{J}}(h)\right\rangle | h \in H\right\} \tag{1}$$

where $\varphi_{\mathcal{J}}(h)$, representing the MD of h in $\mathcal{J}$, while $\psi_{\mathcal{J}}(h)$ representing the NMD of h in $\mathcal{J}$, under the condition that $\varphi_{\mathcal{J}}(h) + \psi_{\mathcal{J}}(h) \preceq 1$.

Definition 2 [9]: A PFS $\mathcal{P}$ in the universe of discourse h is defined as follows:

$$\mathcal{P} = \{\langle h, \varphi_{\mathcal{P}}(h), \psi_{\mathcal{P}}(h)\rangle | h \in H\} \tag{2}$$

where, $\varphi_{\mathcal{P}}(h)$: $H \longrightarrow [0,1]$ represents the level of MD and $\psi_{\mathcal{P}}(h) : H \longrightarrow [0,1]$ represents the level NMD of component h in set $\mathcal{H}$ and set $\mathcal{P}$ respectively, under the given condition that $(\varphi_{\mathcal{P}}(h))^2 + (\psi_{\mathcal{P}}(h))^2 \preceq 1$.

A. PICTURE FUZZY SETS

Definition 3 [13]: A PiFS $\mathcal{B}$ on a universe H can be described as:

$$\mathcal{B} = \{\langle h, \varphi_{\mathcal{B}}(h), \eta_{\mathcal{B}}(h), \psi_{\mathcal{B}}(h)\rangle | h \in H\} \tag{3}$$

where $\varphi_{\mathcal{B}}(h)$, $\eta_{\mathcal{B}}(h)$, and $\psi_{\mathcal{B}}(h)$ are the MD, NED, and NMD of an element h in $\mathcal{B}$ respectively such that $\varphi_{\mathcal{B}}, \eta_{\mathcal{B}}, \psi_{\mathcal{B}}$ fulfil the condition $\varphi_{\mathcal{B}}(h) + \eta_{\mathcal{B}}(h) + \psi_{\mathcal{B}}(h) \preceq 1$.

B. SPHERICAL FUZZY SETS

Definition 4 [18]: A SPS $\mathcal{L}$ over the universe of discourse H is defined as follows:

$$\mathcal{L} = \{\langle h, \varphi_{\mathcal{L}}(h), \eta_{\mathcal{L}}(h), \psi_{\mathcal{L}}(h)\rangle | h \in H\} \tag{4}$$

where $\varphi_{\mathcal{L}}(h)$, $\eta_{\mathcal{L}}(h)$, and $\psi_{\mathcal{L}}(h)$ are the MD, NED, and NMD of an element h in $\mathcal{L}$ respectively such that $\varphi_{\mathcal{L}}(h)$, $\eta_{\mathcal{L}}(h)$, and $\psi_{\mathcal{L}}(h)$ fulfil the condition $(\varphi_{\mathcal{L}}(h))^2 + (\eta_{\mathcal{L}}(h))^2 + (\psi_{\mathcal{L}}(h))^2 \preceq 1$.

C. T-SPHERICAL FUZZY SETS

Definition 5 [21]: A $T-$ SFS over a given universe of discourse H takes the form of:

$$\mathcal{T} = \{\langle h, \varphi_{\mathcal{T}}(h), \eta_{\mathcal{T}}(h), \psi_{\mathcal{T}}(h)\rangle | h \in H\} \tag{5}$$

where $\varphi_{\mathcal{T}}(h)$, $\eta_{\mathcal{T}}(h)$, and $\psi_{\mathcal{T}}(h)$ are the MD, NED, and NMD of an element h in $\mathcal{S}$ respectively such that $\varphi_{\mathcal{T}}(h)$, $\eta_{\mathcal{T}}(h)$, and $\psi_{\mathcal{T}}(h)$ satisfy the condition $(\varphi_{\mathcal{T}}(h))^t + (\eta_{\mathcal{T}}(h))^t + (\psi_{\mathcal{T}}(h))^t \preceq 1$ for all $t \succcurlyeq 1$.

D. SCORE AND ACCURACY FUNCTIONS

Definition 6 [21]: Consider $T = (\varphi, \eta, \psi)$ as a $T-$SFS. The score value of $\mathcal{T}$ is defined as follows:

$$Sc\,(\mathcal{T}) = \varphi^t + \eta^t - \psi^t \tag{6}$$

Furthermore, the accuracy function can be expressed as:

$$AC\,(\mathcal{T}) = \varphi^t + \eta^t + \psi^t \tag{7}$$

The $T-$SFN with a higher score is regarded as superior. In instances where the scores of two $T-$SFNs are identical, their ranking is determined by their accuracy values, with the number possessing a greater accuracy value deemed superior. If the accuracy values of two $T-$SFNs are still equal, both numbers are considered equivalent.

E. $p, q, r-$ SPHERICAL FUZZY SETS

Definition 7 [35]: Let H be a finite set. A $p, q, r-$SFS $\mathcal{S}$ over a component $h \in H$ is defined as follows:

$$\mathcal{S} = \{\langle h, \varphi_{\mathcal{S}}(h), \eta_{\mathcal{S}}(h), \psi_{\mathcal{S}}(h)\rangle | h \in H\} \tag{8}$$

where $\varphi_{\mathcal{S}}(h)$, $\eta_{\mathcal{S}}(h)$, and $\psi_{\mathcal{S}}(h)$ are the MD, NED, and NMD of an element h in $\mathcal{S}$ respectively such that $\varphi_{\mathcal{T}}(h)$, $\eta_{\mathcal{T}}(h)$, and $\psi_{\mathcal{T}}(h)$ satisfy the condition $(\varphi_{\mathcal{S}}(h))^p + (\eta_{\mathcal{S}}(h))^r + (\psi_{\mathcal{S}}(h))^q \preceq 1$ for all $p, q \succcurlyeq 1$.

Remark 1: In the condition $\varphi_{\mathcal{S}}(h))^p + (\eta_{\mathcal{S}}(h))^r + (\psi_{\mathcal{S}}(h))^q \preceq 1$, where $p, q \succ 0$ with the relationship $p \prec q$, $p = q$ or $p \succ q$, and r is the least common multiple of p and q, represented as: $r = LCM(p, q)$.

Definition 8 [35]: Consider any three $p, q, r-$SFNs $\mathcal{S} = \langle \varphi, \eta, \psi \rangle$, $\mathcal{S}_1 = \langle \varphi_1, \eta_1, \psi_1 \rangle$ $\mathcal{S}_2 = \langle \varphi_2, \eta_2, \psi_2 \rangle$ and $\zeta > 0$, then

1. $$\mathcal{S}_1 \oplus \mathcal{S}_2 = \left(\begin{array}{c} \sqrt[p]{(\varphi_1)^p + (\varphi_2)^p - (\varphi_1)^p(\varphi_2)^p}, \\ \eta_1\eta_2, \psi_1\psi_2 \end{array} \right),$$
2. $$\mathcal{S}_1 \otimes \mathcal{S}_2 = \left(\begin{array}{c} \varphi_1\varphi_2, \sqrt[r]{(\eta_1)^r + (\eta_2)^r - (\eta_1)^r(\eta_2)^r}, \\ \sqrt[q]{(\psi_1)^q + (\psi_2)^q - (\psi_1)^q(\psi_2)^q} \end{array} \right),$$
3. $$\zeta\mathcal{S} = \left(\sqrt[p]{1 - (1 - (\varphi)^p)^{\zeta}}, \eta^{\zeta}, \psi^{\zeta} \right),$$
4. $$\mathcal{S}^{\zeta} = \left(\varphi^{\zeta}, \sqrt[r]{1 - (1 - (\eta)^r)^{\zeta}}, \sqrt[q]{1 - (1 - (\psi)^q)^{\zeta}} \right).$$

Definition 9 [35]: Let $\mathcal{S} = \langle \varphi, \eta, \psi \rangle$ be a $p, q, r-$SFNs, the score and accuracy functions are defined as:

$$Sc = \frac{1}{3}\left(2 + \varphi^p - \eta^r - \psi^q\right) \tag{9}$$

where $Sc \in [0, 1]$. The accuracy function can be defined as:

$$Ac = \varphi^p + \eta^q \tag{10}$$

where $Ac \in [0, 1]$.

Definition 10 [35]: Let $\mathcal{S}_1 = \langle \varphi_1, \eta_1, \psi_1 \rangle$ and $\mathcal{S}_2 = \langle \varphi_2, \eta_2, \psi_2 \rangle$ represents any pair of $p, q, r-$ SFNs, the comparison rules are defined as:

1. If $Sc\,(\mathcal{S}_1) > Sc\,(\mathcal{S}_2)$, then $\mathcal{S}_1 > \mathcal{S}_2$,
2. If $Sc\,(\mathcal{S}_1) = Sc\,(\mathcal{S}_2)$, and $Ac\,(\mathcal{S}_1) > Ac\,(\mathcal{S}_2)$, $\mathcal{S}_1 > \mathcal{S}_2$;
3. If $Sc\,(\mathcal{S}_1) = Sc\,(\mathcal{S}_2)$, and $Ac\,(\mathcal{S}_1) = Ac\,(\mathcal{S}_2)$, then $\mathcal{S}_1 = \mathcal{S}_2$.

F. DOMBI T-NORM AND T-CONORM

Definition 11 [36]: Assuming that (m, n) belongs to the real number interval $(0, 1) \times (0, 1)$ with $\gamma \geq 1$, then DTN and DTCN are defined as:

$$\mathcal{T}(m, n) = \frac{1}{1 + \left\{\left(\frac{1-m}{m}\right)^{\gamma} + \left(\frac{1-n}{n}\right)\right\}^{\frac{1}{\gamma}}} \tag{11}$$

$$\mathcal{T}^{*}(m, n) = \frac{1}{1 + \left\{\left(\frac{m}{1-m}\right)^{\gamma} + \left(\frac{n}{1-n}\right)\right\}^{\frac{1}{\gamma}}} \tag{12}$$

III. PROPOSED AGGREGATION OPERATORS

A. $p, q, r-$ SFDWA OPERATOR

Definition 12: Let $\mathcal{S}_k = \langle \varphi_k, \eta_k, \psi_k \rangle$ $(k = 1, 2, \ldots, l)$ be a group of $p, q, r-$SFNs, then the $p, q, r-$SF Dombi weighted averaging ($p, q, r-$ SFDWA) operator is defined as:

$$p, q, r - \text{SFDWA}(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l) = \prod_{k=1}^{l} \xi_k \mathcal{S}_k \tag{13}$$

where $\xi = (\xi_1, \xi_2, \ldots, \xi_l)^T$ is weight vector with $\xi_k \in [0, 1]$ and $\sum_{k=1}^{l} \xi_k = 1$.

Theorem 1: Let $\mathcal{S}_k = \langle \varphi_k, \eta_k, \psi_k \rangle$ be a group of $p, q, r-$SFNs, the $p, q, r-$SFWA operator's structure is explained using Dombi operations with $\gamma > 0$.

$$p, q, r - SFDWA(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l)$$
$$= \left(\sqrt[p]{1 - \frac{1}{1 + \left\{\prod_{k=1}^{l} \xi_k \left(1 - \frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[r]{1 + \left\{\prod_{k=1}^{l} \xi_k \left(1 - \frac{1}{(\eta_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[q]{1 + \left\{\prod_{k=1}^{l} \xi_k \left(1 - \frac{1}{(\psi_k)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right) \tag{14}$$

Proof: We employed mathematical induction to establish the validity of Theorem 1.

Step 1: For $l = 2$, we have

$$p, q, r - \text{SFDWA}(\mathcal{S}_1, \mathcal{S}_2)$$
$$= \left(\sqrt[p]{1 - \frac{1}{1 + \left\{\xi_1 \left(1 - \frac{1}{(\varphi_1)^p}\right)^{p\gamma} \times \xi_2 \left(1 - \frac{1}{(\varphi_2)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[r]{1 + \left\{\xi_1 \left(1 - \frac{1}{(\eta_1)^r}\right)^{r\gamma} \times \xi_2 \left(1 - \frac{1}{(\eta_2)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[q]{1 + \left\{\xi_1 \left(1 - \frac{1}{(\psi_1)^q}\right)^{q\gamma} \times \xi_2 \left(1 - \frac{1}{(\psi_2)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right)$$

$$= \left(\sqrt[p]{1 - \frac{1}{1 + \left\{\prod_{k=1}^{2} \xi_k \left(1 - \frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[r]{1 + \left\{\prod_{k=2}^{2} \xi_k \left(1 - \frac{1}{(\eta_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[q]{1 + \left\{\prod_{k=1}^{2} \xi_k \left(1 - \frac{1}{(\psi_k)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right)$$

Hence, Equation (14) is valid for $l = 2$.

Step 2. Suppose that Equation (14) is valid for $l = n$, i.e.,

$$p, q, r - SFDWA(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l)$$
$$= \left(\sqrt[p]{1 - \frac{1}{1 + \left\{\prod_{k=1}^{n} \xi_k \left(1 - \frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[r]{1 + \left\{\prod_{k=1}^{n} \xi_k \left(1 - \frac{1}{(\eta_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[q]{1 + \left\{\prod_{k=1}^{n} \xi_k \left(1 - \frac{1}{(\psi_k)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right) \tag{15}$$

When $l = n + 1$, we have

$$\left(\sqrt[p]{1 - \frac{1}{1 + \left\{\prod_{k=1}^{n} \xi_k \left(1 - \frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[r]{1 + \left\{\prod_{k=1}^{n} \xi_k \left(1 - \frac{1}{(\eta_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[q]{1 + \left\{\prod_{k=1}^{n} \xi_k \left(1 - \frac{1}{(\psi_k)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right)$$
$$\oplus \left(\sqrt[p]{1 - \frac{1}{1 + \left\{\xi_{k+1} \left(1 - \frac{1}{(\varphi_{k+1})^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[r]{1 + \left\{\xi_{k+1} \left(1 - \frac{1}{(\eta_{k+1})^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}. \frac{1}{\sqrt[q]{1 + \left\{\xi_{k+1} \left(1 - \frac{1}{(\psi_{k+1})^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right)$$
$$= \left(\sqrt[p]{1 - \frac{1}{1 + \left\{\prod_{k=1}^{n+1} \xi_k \left(1 - \frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[r]{1 + \left\{\prod_{k=1}^{n+1} \xi_k \left(1 - \frac{1}{(\eta_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \frac{1}{\sqrt[q]{1 + \left\{\prod_{k=1}^{n+1} \xi_k \left(1 - \frac{1}{(\psi_k)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right)$$

Thus, Equation (14) is true for $l = n + 1$.

Example 1: Let $\mathcal{S}_1 = \langle 0.10, 0.60, 0.40\rangle$, $\mathcal{S}_2 = \langle 0.20, 0.50, 0.40\rangle$, $\mathcal{S}_3 = \langle 0.40, 0.50, 0.50\rangle$, $\mathcal{S}_4 = \langle 0.30, 0.50, 0.60\rangle$ be any four $p, q, r-$SFNs and $\xi = (0.20, 0.30, 0.10, 0.40)^T$ be the weight vector of these $p, q, r-$SFNs. For $\gamma = 2$ and $p = q = r = 1$, the aggregated values can be calculated as:

$$\sqrt[p]{1-\frac{1}{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}$$

$$=\sqrt[1]{1-\frac{1}{1+\left\{\begin{array}{c}(0.2)\left(1-\frac{1}{(0.1)^1}\right)^2\times(0.3)\left(1-\frac{1}{(0.2)^1}\right)^2\\ \times(0.1)\left(1-\frac{1}{(0.4)^1}\right)^2\times(0.4)\left(1-\frac{1}{(0.3)^1}\right)^2\end{array}\right\}^{\frac{1}{2}}}}$$

$$=\sqrt[1]{1-\frac{1}{\{1+(0.2)(81)\times(0.3)(16)\times(0.1)(2.25)\times(0.4)(5.4442)\}^{\frac{1}{2}}}}$$

$$=\sqrt[1]{1-\frac{1}{1+\{38.1006\}^{\frac{1}{2}}}}=\sqrt[1]{1-\frac{1}{7.1725}}=0.8606.$$

$$\frac{1}{\sqrt[r]{1++\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\varphi_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}$$

$$=\frac{1}{\sqrt[1]{1+\left\{\begin{array}{c}(0.2)\left(1-\frac{1}{(0.6)^1}\right)^2\times(0.3)\left(1-\frac{1}{(0.5)^1}\right)^2\\ \times(0.1)\left(1-\frac{1}{(0.5)^1}\right)^2\times(0.4)\left(1-\frac{1}{(0.5)^1}\right)^2\end{array}\right\}^{\frac{1}{2}}}}$$

$$=\frac{1}{\sqrt[3]{1+\left\{\begin{array}{c}(0.2)(0.4443)\times(0.3)(1)\\ \times(0.1)(1)\times(0.4)(1)\end{array}\right\}^{\frac{1}{2}}}}$$

$$=\frac{1}{\sqrt[1]{1+\{0.0888\times 0.3\times 0.1\times 0.4\}^{\frac{1}{2}}}}$$

$$=\frac{1}{\sqrt[1]{1.0326}}=0.9684.$$

$$\frac{1}{\sqrt[q]{1++\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\varphi_k)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}}$$

$$=\frac{1}{\sqrt[1]{1+\left\{\begin{array}{c}(0.2)\left(1-\frac{1}{(0.4)^1}\right)^2\times(0.3)\left(1-\frac{1}{(0.4)^1}\right)^2\\ \times(0.1)\left(1-\frac{1}{(0.5)^1}\right)^2\times(0.4)\left(1-\frac{1}{(0.6)^1}\right)^2\end{array}\right\}^{\frac{1}{2}}}}$$

$$\frac{1}{\sqrt[1]{1+\{(0.2)(2.25)\times(0.3)(2.25)\times(0.1)(1)\times(0.4)(0.4443)\}^{\frac{1}{2}}}}$$

$$=\frac{1}{\sqrt[1]{1+\{0.0053\}^{\frac{1}{2}}}}=0.9321.$$

$p, q, r-SFDWA\,(\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4) = (0.8606, 0.9604, 0.9321).$

B. $p, q, r-$ SFDOWA OPERATOR

Definition 13: Let $\mathcal{S}_k = \langle\varphi_k, \eta_k, \psi_k\rangle$ be a group of $p, q, r-$SFNs, then the $p, q, r-$SF Dombi ordered weighted averaging ($p, q, r-$ SFDOWA) operator is defined as:

$$p, q, r-\text{SFDWA}\,(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l) = \prod_{k=1}^{l}\xi_k\mathcal{S}_{\delta(k)} \tag{16}$$

where $\xi = (\xi_1, \xi_2, \ldots, \xi_l)^T$ is weight vector with $\xi_k \in [0, 1]$ and $\sum_{k=1}^{l}\xi_k = 1$. The highest weight among them is assigned to the k^{th} element represented by $\delta(k)$. This configuration establishes a complete order relationship, in which $\delta(1)$ holds greater precedence over $\delta(2)$, and so on, ultimately leading to $\delta(n)$.

Theorem 2: Assume that $\mathcal{S}_k = \langle\varphi_k, \eta_k, \psi_k\rangle$ be a group of $p, q, r-$ SFNs. For $\gamma > 0$, the aggregated value obtained by $p, q, r-$SFDOWA is also a $p, q, r-$ SFNs, and can be defined as:

$$p, q, r-SFDOWA\,(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l)$$

$$=\left(\begin{array}{c}\sqrt[p]{1-\frac{1}{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\varphi_{\delta(k)})^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \frac{1}{\sqrt[r]{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\eta_{\delta(k)})^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}},\\ \frac{1}{\sqrt[q]{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\psi_{\delta(k)})^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}}\end{array}\right) \tag{17}$$

Example 2: Let $\mathcal{S}_1 = \langle 0.10, 0.60, 0.40\rangle$, $\mathcal{S}_2 = \langle 0.20, 0.50, 0.40\rangle$, $\mathcal{S}_3 = \langle 0.40, 0.50, 0.50\rangle$, $\mathcal{S}_4 = \langle 0.30, 0.50, 0.60\rangle$ be any four $p, q, r-$SFNs and $\xi = (0.15, 0.20, 0.40, 0.25)^T$ be the weight vector of these $p, q, r-$SFNs. For $\gamma = 2$ and $p = q = r = 1$, then the aggregated value can calculate as follows:

In the initial phase, we will compute the score values for these using the equation (9) in the following manner:

$$Sc\,(\mathcal{S}_1) = \frac{1}{3}\left(2+(0.10)^1-(0.60)^1-(0.40)^1\right) = 0.3666,$$
$$Sc\,(\mathcal{S}_2) = \frac{1}{3}\left(2+(0.20)^1-(0.50)^1-(0.40)^1\right) = 0.4333,$$
$$Sc\,(\mathcal{S}_3) = \frac{1}{3}\left(2+(0.40)^1-(0.50)^1-(0.50)^1\right) = 0.4666,$$
$$Sc\,(\mathcal{S}_4) = \frac{1}{3}\left(2+(0.30)^1-(0.50)^1-(0.60)^1\right) = 0.4000.$$

Order these $p, q, r-$spherical fuzzy numbers based on their respective score values in the following manner:

$Sc(\mathcal{S}_3) > Sc(\mathcal{S}_2) > Sc(\mathcal{S}_4) > Sc(\mathcal{S}_1)$. Thus, the order of the $p, q, r-$spherical fuzzy numbers according is $\mathcal{S}_{\delta(1)} = (0.40, 0.50, 0.50)$, $\mathcal{S}_{\delta(2)} = (0.20, 0.50, 0.40)$, $\mathcal{S}_{\delta(3)} = (0.30, 0.50, 0.60)$, and $\mathcal{S}_{\delta(4)} = (0.10, 0.60, 0.40)$.

$$p, q, r - SFDOWA(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l) = \begin{pmatrix} \sqrt[p]{1 - \frac{1}{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\varphi_{\delta(k)})^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \frac{1}{\sqrt[r]{1+\left\{\sum_{k=1}^{l}\xi_k\left(1-\frac{1}{(\eta_{\delta(k)})^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \frac{1}{\sqrt[q]{1+\left\{\sum_{k=1}^{l}\xi_k\left(1-\frac{1}{(\psi_{\delta(k)})^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \end{pmatrix}$$

$$\sqrt[p]{1 - \frac{1}{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\varphi_{\delta(k)})^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}} = \sqrt[1]{1-\frac{1}{1+\left\{\begin{matrix}(0.15)\left(1-\frac{1}{(0.4)^1}\right)^2\times(0.2)\left(1-\frac{1}{(0.2)^1}\right)^2\\ \times(0.4)\left(1-\frac{1}{(0.3)^1}\right)^2\times(0.25)\left(1-\frac{1}{(0.1)^1}\right)^2\end{matrix}\right\}^{\frac{1}{2}}}}$$

$$= \sqrt[1]{1-\frac{1}{1+\{0.3375\times 3.2\times 2.17768\times 20.25\}^{\frac{1}{2}}}}$$

$$= \sqrt[2]{1-\frac{1}{1+\{47.6258\}^{\frac{1}{2}}}} = \sqrt[1]{1-\frac{1}{1+6.9011}}$$

$$= \sqrt[2]{1-\frac{1}{7.9011}} = 0.8735.$$

$$\frac{1}{\sqrt[r]{1+\left\{\sum_{k=1}^{l}\xi_k\left(1-\frac{1}{(\eta_{\delta(k)})^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}} = \frac{1}{\sqrt[1]{1+\left\{\begin{matrix}0.15\left(1-\frac{1}{(0.5)^1}\right)^2\times 0.2\left(1-\frac{1}{(0.5)^1}\right)^2\\ \times 0.4\left(1-\frac{1}{(0.5)^1}\right)^2\times 0.25\left(1-\frac{1}{(0.6)^1}\right)^2\end{matrix}\right\}^{\frac{1}{2}}}}$$

$$= \frac{1}{\sqrt[1]{1+\{0.15\times 0.2\times 0.4\times 0.1110\}^{\frac{1}{2}}}}$$

$$= \frac{1}{\sqrt[1]{1+0.01}} = 0.9900.$$

$$\frac{1}{\sqrt[q]{1+\left\{\sum_{k=1}^{l}\xi_k\left(1-\frac{1}{(\eta_{\delta(k)})^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} = \frac{1}{\sqrt[1]{1+\left\{\begin{matrix}0.15\left(1-\frac{1}{(0.5)^1}\right)^2\times 0.2\left(1-\frac{1}{(0.4)^1}\right)^2\\ \times 0.4\left(1-\frac{1}{(0.6)^1}\right)^2\times 0.2\left(1-\frac{1}{(0.4)^1}\right)^2\end{matrix}\right\}^{\frac{1}{2}}}}$$

$$= \frac{1}{\sqrt[1]{1+\{0.15\times 0.45\times 0.1777\times 0.45\}^{\frac{1}{2}}}}$$

$$= \frac{1}{\sqrt[1]{1+\{0.0053\}^{\frac{1}{2}}}}$$

$$= \frac{1}{\sqrt[1]{1+0.0728}} = 0.9321.$$

$$p, q, r - SFDOWA(\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4) = (0.8735, 0.9900, 0.9321).$$

C. $p, q, r-$ SFDWG OPERATOR

Definition 14: Let $\mathcal{S}_k = \langle\varphi_k, \eta_k, \psi_k\rangle$ $(k = 1, 2, \ldots, l)$ be a collection of $p, q, r-$SFNs, then the $p, q, r-$ SF Dombi weighted geometric($p, q, r - SFDWG$) operator is defined as:

$$p, q, r - \text{SFDWG}(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l) = \prod_{k=1}^{l}(\mathcal{S}_l)^{\xi_l} \quad (18)$$

where $(\xi_1, \xi_2, \ldots, \xi_l)^T$ is weight vector with $\xi_k \in [0, 1]$ and $\sum_{k=1}^{l}\xi_k = 1$.

Theorem 3: Let $\mathcal{S}_k = \langle\varphi_k, \eta_k, \psi_k\rangle$ $(k = 1, 2, \ldots, l)$ be a collection of $p, q, r-$SFNs, then the $p, q, r-$ SFWG operator structure is explained using Dombi operation with $\gamma > 0$.

$$p, q, r - SFDWG(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l) = \begin{pmatrix} \frac{1}{\sqrt[p]{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt[r]{1-\frac{1}{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\eta_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt[q]{1-\frac{1}{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\psi_k)^p}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \end{pmatrix} \quad (19)$$

Proof: Mathematical induction was employed to establish the proof of Theorem 3.

Step 1: For $l = 2$, we have

$$p,q,r-\text{SFDWG}(\mathcal{S}_1,\mathcal{S}_2) = \left(\frac{1}{\sqrt[p]{1+\left\{\xi_1\left(1-\frac{1}{(\varphi_1)^r}\right)^{p\gamma}\times\xi_2\left(1-\frac{1}{(\varphi_2)^r}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[r]{1-\frac{1}{1+\left\{\xi_1\left(1-\frac{1}{(\eta_1)^r}\right)^{r\gamma}\times\xi_2\left(1-\frac{1}{(\eta_2)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\xi_1\left(1-\frac{1}{(\psi_1)^r}\right)^{q\gamma}\times\xi_2\left(1-\frac{1}{(\psi_2)^r}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right)$$

$$= \left(\frac{1}{\sqrt[p]{1+\left\{\prod_{k=1}^{2}\xi_k\left(1-\frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[r]{1-\frac{1}{1+\left\{\prod_{k=1}^{2}\xi_k\left(1-\frac{1}{(\eta_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\prod_{k=1}^{2}\xi_k\left(1-\frac{1}{(\psi_k)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right)$$

Hence, Equation (19) holds true when l equals 2.

Step 3. Suppose that Equation (19) holds true for $l = n$, i.e.,

$$p,q,r-SFDWG(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_n) = \left(\frac{1}{\sqrt[p]{1+\left\{\prod_{k=1}^{n}\xi_k\left(1-\frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[r]{1-\frac{1}{1+\left\{\prod_{k=1}^{n}\xi_k\left(1-\frac{1}{(\eta_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\prod_{k=1}^{n}\xi_k\left(1-\frac{1}{(\psi_k)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right)$$

Step 3: When $l = n+1$, we have

$$p,q,r-SFDWG(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_n)\otimes(\mathcal{S}_{n+1})^{\xi_{n+1}} = \left(\frac{1}{\sqrt[p]{1+\left\{\prod_{k=1}^{n}\xi_k\left(1-\frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[r]{1-\frac{1}{1+\left\{\prod_{k=1}^{n}\xi_k\left(1-\frac{1}{(\eta_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\prod_{k=1}^{n}\xi_k\left(1-\frac{1}{(\psi_k)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right)$$

$$\otimes \left(\frac{1}{\sqrt[p]{1+\left\{\xi_{k+1}\left(1-\frac{1}{(\varphi_{k+1})^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[r]{1-\frac{1}{1+\left\{\xi_{k+1}\left(1-\frac{1}{(\eta_{k+1})^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\xi_{k+1}\left(1-\frac{1}{(\psi_{k+1})^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right)$$

$$= \left(\frac{1}{\sqrt[p]{1+\left\{\prod_{k=1}^{n}\xi_k\left(1-\frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[r]{1-\frac{1}{1+\left\{\prod_{k=1}^{n}\xi_k\left(1-\frac{1}{(\eta_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\prod_{k=1}^{n}\xi_k\left(1-\frac{1}{(\psi_k)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \right).$$

Thus, Equation (19) is valid for $l = n + 1$. Therefore, Equation (19) for all positive integers.

Example 3: Let $\mathcal{S}_1 = \langle 0.80, 0.10, 0.10\rangle$, $\mathcal{S}_2 = \langle 0.70, 0.20, 0.30\rangle$, $\mathcal{S}_3 = \langle 0.90, 0.20, 0.20\rangle$, $\mathcal{S}_4 = \langle 0.50, 0.30, 0.40\rangle$ be any four $p, q, r-$SFNs and $\xi = (0.25, 0.5, 0.10, 0.15)^T$ be the weight vector of these $p, q, r-$SFNs. We have $\gamma = 2. p = q = r = 1$. The aggregated value by using $p, q, r-$SFDWG operator can be calculated as follows:

$$\frac{1}{\sqrt[p]{1++\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\varphi_k)^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}} = \frac{1}{\sqrt[1]{1+\left\{\begin{array}{l}(0.25)\left(1-\frac{1}{(0.8)^1}\right)^2\times(0.5)\left(1-\frac{1}{(0.7)^1}\right)^2\\ \times(0.10)\left(1-\frac{1}{(0.9)^1}\right)^2\times(0.15)\left(1-\frac{1}{(0.5)^1}\right)^2\end{array}\right\}^{\frac{1}{2}}}}$$

$$= \frac{1}{\sqrt[1]{1+\{(0.0156)\times(0.0918)\times(0.00123)\times(0.15)\}^{\frac{1}{2}}}}$$

$$= \frac{1}{\sqrt[1]{1+\{(0.0000002)\}^{\frac{1}{2}}}} = \frac{1}{\sqrt[1]{1+0.0004}} = 0.9996.$$

$$\sqrt[r]{1-\frac{1}{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\varphi_k)^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}} = \sqrt[1]{1-\frac{1}{1+\left\{\begin{array}{l}(0.25)\left(1-\frac{1}{(0.1)^1}\right)^2\times(0.5)\left(1-\frac{1}{(0.2)^1}\right)^2\\ \times(0.10)\left(1-\frac{1}{(0.2)^1}\right)^2\times(0.15)\left(1-\frac{1}{(0.3)^1}\right)^2\end{array}\right\}^{\frac{1}{2}}}}$$

$$= \sqrt[1]{1-\frac{1}{1+\{20.25\times 8\times 1.6\times 0.8166\}^{\frac{1}{2}}}}$$

$$= \sqrt[1]{1-\frac{1}{1+211.6627}}$$

$$= \sqrt[1]{1 - \frac{1}{212.6627}} = 0.9953.$$

$$\sqrt[q]{1 - \frac{1}{1 + \left\{\prod_{k=1}^{l} \xi_k \left(1 - \frac{1}{(\varphi_k)^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}}$$

$$= \sqrt[1]{1 - \frac{1}{1 + \left\{ \begin{array}{l} (0.25)\left(1 - \frac{1}{(0.1)^1}\right)^2 \times (0.5)\left(1 - \frac{1}{(0.3)^1}\right)^2 \\ \times (0.10)\left(1 - \frac{1}{(0.2)^1}\right)^2 \times (0.15)\left(1 - \frac{1}{(0.4)^1}\right)^2 \end{array} \right\}^{\frac{1}{2}}}}$$

$$= \sqrt[1]{1 - \frac{1}{1 + \{20.25 \times 2.7221 \times 1.6 \times 0.3375\}^{\frac{1}{2}}}}$$

$$= \sqrt[1]{1 - \frac{1}{1 + \{29.7661\}^{\frac{1}{2}}}}$$

$$\sqrt[1]{1 - \frac{1}{1 + 5.4558}} = 0.8452.$$

Thus, $p, q, r-$SFDWG$(\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4)$

$$= (0.9996, 0.9953, 0.8452).$$

D. $p, q, r-$ SFDOWG OPERATOR

Definition 15: For any set $p, q, r-$SFNs represented as $\mathcal{S}_k = \langle \varphi_k, \eta_k, \psi_k \rangle$ and a weighted vector $\xi = (\xi_1, \xi_2, \ldots, \xi_l)^T$ with each element $\xi_k \in [0, 1]$ such that the sum of all ξ_k for k from 1 to n, an $p, q-$ SFDOWG operator can be define or this collection of $p, q-$SFNs. This operator is a mapping denotes as $p, q-$SFDOWG is expressed as follows:

$$p, q, r - \text{SFDOWG}(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l) = \prod_{k=1}^{l} \left(\mathcal{S}_{\delta(l)}\right)^{\xi_l} \tag{20}$$

The highest weight among them is assigned to the l^{th} element represented by $\delta(l)$. This configuration establishes a complete order relationship, in which $\delta(1)$ holds greater precedence over $\delta(2)$, and so on, ultimately leading to $\delta(n)$.

Theorem 4: Let $\mathcal{S}_k = \langle \varphi_k, \eta_k, \psi_k \rangle \, (k = 1, 2, \ldots, l)$ be a collection of $p, q, r-$SFNs and $\gamma > 0$, then the aggregated value obtained by $p, q, r-$SFOWG operator is also $p, q, r-$SFN and can written as follows:

$$p, q, r - SFDOWG\left(\mathcal{S}_{\delta(1)}, \mathcal{S}_{\delta(2)}, \ldots, \mathcal{S}_{\delta(l)}\right) = \left(\begin{array}{c} \dfrac{1}{\sqrt[p]{1 + \left\{\prod_{k=1}^{l} \xi_k \left(1 - \frac{1}{(\varphi_{\delta(k)})^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt[r]{1 - \dfrac{1}{1 + \left\{\prod_{k=1}^{l} \xi_k \left(1 - \frac{1}{(\eta_{\delta(k)})^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt[q]{1 - \dfrac{1}{1 + \left\{\prod_{k=1}^{l} \xi_k \left(1 - \frac{1}{(\psi_{\delta(k)})^q}\right)^{\psi\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right) \tag{21}$$

Example 4: Let $\mathcal{S}_1 = \langle 0.40, 0.50, 0.50 \rangle$, $\mathcal{S}_2 = \langle 0.20, 0.60, 0.30 \rangle$, $\mathcal{S}_3 = \langle 0.40, 0.30, 0.30 \rangle$, $\mathcal{S}_4 = \langle 0.60, 0.20, 0.30 \rangle$ be any four $p, q, r-$SFNs and $\xi = (0.25, 0.15, 0.30, 0.30)^T$ be the weight vector of these $p, q, r-$SFNs. We have $\gamma = 2$. $p = q = r = 2$, then the aggregated value can calculate as follows:

In the initial phase, we will compute the score values for these using the equation (9) in the following manner:

$$Sc(\mathcal{S}_1) = \frac{1}{3}\left(2 + (0.40)^2 - (0.50)^2 - (0.50)^2\right) = 0.6333,$$
$$Sc(\mathcal{S}_2) = \frac{1}{3}\left(2 + (0.20)^2 - (0.60)^2 - (0.30)^2\right) = 0.5833,$$
$$Sc(\mathcal{S}_3) = \frac{1}{3}\left(2 + (0.40)^2 - (0.30)^2 - (0.30)^2\right) = 0.7400,$$
$$Sc(\mathcal{S}_4) = \frac{1}{3}\left(2 + (0.60)^2 - (0.20)^2 - (0.30)^2\right) = 0.8233.$$

Order these $p, q, r-$spherical fuzzy numbers based on their respective score values in the following manner:

$Sc(\mathcal{S}_4) > Sc(\mathcal{S}_3) > Sc(\mathcal{S}_1) > Sc(\mathcal{S}_2)$. Thus, the order of the $p, q, r-$spherical fuzzy numbers according is $\mathcal{S}_1 = (0.60, 0.20, 0.30)$, $\mathcal{S}_2 = (0.40, 0.30, 0.30)$, $\mathcal{S}_3 = (0.40, 0.50, 0.50)$, and $\mathcal{S}_4 = (0.20, 0.60, 0.30)$.

$$p, q, r - SFDOWG(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_l) = \left(\begin{array}{c} \dfrac{1}{\sqrt[p]{1 + \left\{\prod_{k=1}^{l} \xi_k \left(1 - \frac{1}{(\varphi_{\delta(k)})^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \dfrac{1}{\sqrt[r]{1 + \left\{\prod_{k=1}^{l} \xi_k \left(1 - \frac{1}{(\varphi_{\delta(k)})^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}, \\ \sqrt[q]{1 - \dfrac{1}{1 + \left\{\prod_{k=1}^{l} \xi_k \left(1 - \frac{1}{(\psi_{\delta(k)})^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}} \end{array} \right)$$

$$\frac{1}{\sqrt[p]{1 + \left\{\prod_{k=1}^{l} \xi_k \left(1 - \frac{1}{(\varphi_{\delta(k)})^p}\right)^{p\gamma}\right\}^{\frac{1}{\gamma}}}}$$

$$= \frac{1}{\sqrt[1]{1 + \left\{ \begin{array}{l} 0.25\left(1 - \frac{1}{(0.6)^1}\right)^2 \times 0.15\left(1 - \frac{1}{(0.4)^1}\right)^2 \\ \times 0.3\left(1 - \frac{1}{(0.4)^1}\right)^2 \times 0.3\left(1 - \frac{1}{(0.2)^1}\right)^2 \end{array} \right\}^{\frac{1}{2}}}}$$

$$= \frac{1}{\sqrt[1]{1 + \{0.25(0.4443) \times 0.15(2.25) \times 0.3(2.25) \times 0.3(16)\}^{\frac{1}{2}}}}$$

$$= \frac{1}{\sqrt[1]{1 + \{0.1110 \times 0.3375 \times 0.6750 \times 4.8\}^{\frac{1}{2}}}}$$

$$= \frac{1}{\sqrt[1]{1 + \{0.1213\}^{\frac{1}{2}}}}$$

$$= \frac{1}{\sqrt[1]{1 + 0.3482}} 0.7417.$$

$$\sqrt[r]{1-\frac{1}{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\eta_{\delta(k)})^r}\right)^{r\gamma}\right\}^{\frac{1}{\gamma}}}}$$
$$=\sqrt[2]{1-\frac{1}{1+\left\{\begin{matrix}0.25\left(1-\frac{1}{(0.2)^2}\right)^4\times 0.15\left(1-\frac{1}{(0.3)^2}\right)^4\\ \times 0.3\left(1-\frac{1}{(0.5)^2}\right)^4\times 0.3\left(1-\frac{1}{(0.6)^2}\right)^4\end{matrix}\right\}^{\frac{1}{2}}}}$$
$$=\sqrt[2]{1-\frac{1}{1+\left\{\begin{matrix}0.25(331776)\times 0.15(10451.86)\\ \times 0.3(81)\times 0.3(9.9869)\end{matrix}\right\}^{\frac{1}{2}}}}$$
$$=\sqrt[2]{1-\frac{1}{1+\{82944\times 1567.779\times 24.3\times 2.9960\}^{\frac{1}{2}}}}$$
$$=\sqrt[2]{1-\frac{1}{1+\{9467120414.18\}^{\frac{1}{2}}}}$$
$$=\sqrt[2]{1-\frac{1}{1+97299.1285}}=0.9999.$$
$$\sqrt[q]{1-\frac{1}{1+\left\{\prod_{k=1}^{l}\xi_k\left(1-\frac{1}{(\eta_{\delta(k)})^q}\right)^{q\gamma}\right\}^{\frac{1}{\gamma}}}}$$
$$=\sqrt[2]{1-\frac{1}{1+\left\{\begin{matrix}0.25\left(1-\frac{1}{(0.3)^2}\right)^4\times 0.15\left(1-\frac{1}{(0.3)^2}\right)^4\\ \times 0.3\left(1-\frac{1}{(0.5)^2}\right)^4\times 0.3\left(1-\frac{1}{(0.3)^2}\right)^4\end{matrix}\right\}^{\frac{1}{2}}}}$$
$$=\sqrt[2]{1-\frac{1}{1+\left\{\begin{matrix}0.25(10451.86)\times 0.15(10451.86)\\ \times 0.3(81)\times 0.3(10451.86)\end{matrix}\right\}^{\frac{1}{2}}}}$$
$$=\sqrt[2]{1-\frac{1}{1+\{2612.965\times 1567.779\times 24.3\times 3135.558\}^{\frac{1}{2}}}}$$
$$=\sqrt[2]{1-\frac{1}{1+\{312132900116.04\}^{\frac{1}{2}}}}$$
$$=\sqrt[2]{1-\frac{1}{1+558688.5537}}$$
$$=\sqrt[2]{1-\frac{1}{558689.5537}}0.9999.$$

Thus,

$$p,q,r-\mathrm{SFDOWG}(\mathcal{S}_1,\mathcal{S}_2,\mathcal{S}_3,\mathcal{S}_4)=(0.7417,0.9998,0.9999).$$

E. SOME PROPERTIES OF THE PROPOSED AGGREGATION OPERATORS

Property 1: If $\mathcal{S}_k=\langle\varphi_k,\eta_k,\psi_k\rangle$ $(k=1,2,3,\ldots,l)$ are equal, that is $\mathcal{S}_k=\mathcal{S}$ for all k, then

1. $$p,q,r-SFDWA(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)=\mathcal{S},$$
2. $$p,q,r-SFDOWA(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)=\mathcal{S},$$
3. $$p,q,r-SFDWG(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)=\mathcal{S},$$
4. $$p,q,r-SFDOWG(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)=\mathcal{S}.$$

Property 2: If $\mathcal{S}_k=\langle\varphi_h,\eta_h,\psi_h\rangle$ $(k=1,2,3,\ldots,l)$ be a set of $p,q,r-SFNs$, and let $\mathcal{S}^-=\frac{\min\mathcal{S}_k}{k}$, $\mathcal{S}^+=\frac{\max\mathcal{S}_k}{k}$ then

1. $$\mathcal{S}^-\leq p,q,r-SFDWA(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)\leq\mathcal{S}^+,$$
2. $$\mathcal{S}^-\leq p,q,r-SFDOWA(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)\leq\mathcal{S}^+,$$
3. $$\mathcal{S}^-\leq p,q,r-SFDWG(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)\leq\mathcal{S}^+,$$
4. $$\mathcal{S}^-\leq p,q,r-SFDOWG(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)\leq\mathcal{S}^+.$$

Property 3: Let $\mathcal{S}_k=\langle\varphi_k,\eta_k,\psi_k\rangle$ $(k=1.2,\ldots,l)$ and $\mathcal{S}_k{}^*=\langle\varphi^*{}_k,\eta^*{}_k,\psi^*{}_k\rangle$ $(k=1,2,\ldots,l)$ be the two sets of $p,q,r-SFNs$, if $\mathcal{S}_k\leq\mathcal{S}_k{}^*$ for all k, then

1. $$p,q,r-SFDWA(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)\leq p,q,r-SFDWA(\mathcal{S}^*{}_1,\mathcal{S}^*{}_2,\ldots,\mathcal{S}^*{}_l),$$
2. $$p,q,r-SFDOWA(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)\leq p,q,r-SFDOWA(\mathcal{S}^*{}_1,\mathcal{S}^*{}_2,\ldots,\mathcal{S}^*{}_l),$$
3. $$p,q,r-SFDWG(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)\leq p,q,r-SFDWG(\mathcal{S}^*{}_1,\mathcal{S}^*{}_2,\ldots,\mathcal{S}^*{}_l),$$
4. $$p,q,r-SFDOWG(\mathcal{S}_1,\mathcal{S}_2,\ldots,\mathcal{S}_l)\leq p,q,r-SFDOWG(\mathcal{S}^*{}_1,\mathcal{S}^*{}_2,\ldots,\mathcal{S}^*{}_l).$$

Definition 16: Suppose $\mathcal{S}_1=\langle\varphi_1,\eta_1,\psi_1\rangle$ and $\mathcal{S}_2=\langle\varphi_2,\eta_2,\psi_2\rangle$ be any pair of SFNs, then the Euclidean distance between $\mathcal{S}_1$ and $\mathcal{S}_2$ defined as:

$$D(\mathcal{S}_1,\mathcal{S}_2)=\left(\sqrt[r]{\frac{1}{2}\left(\begin{matrix}(\varphi_1-\varphi_2)^p+(\eta_1-\eta_2)^r\\+(\psi_1-\psi_2)^q\end{matrix}\right)}\right)\quad(22)$$

IV. MODEL FOR MAGDM WITH SPHERICAL FUZZY INFORMATION

In this section, we aim to tackle the challenges associated with MCDM in the context of $p, q, r-$ SFSs. We will employ $p, q, r-$SF aggregation operators that have been introduced for this purpose. To facilitate the discussion, we establish a set of assumptions and notations that will be utilized to define and evaluate MCDM problems when dealing with $p, q, r-$SFSs. Through the application of this method, our objective is to enhance the DM process and provide a comprehensive assessment framework within this sitting.

Suppose we denote the set alternatives as $\mathcal{X} = \{\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_m\}$ and the set of attributes (criteria) be denoted as $= \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_n\}$. Let the importance degree of the attributes be represented as $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^T$ Such that $\xi_j \in [0, 1]$ and $\sum_{j=1}^{n} \xi_j = 1.0$ there is a group of t experts denoted by $\mathcal{E}x = \{\mathcal{E}x_1, \mathcal{E}x_2, \ldots, \mathcal{E}x_t\}$, who are investigate to give the evaluation information, and importance degree of experts is denoted as $w = (w_1, w_2, \ldots, w_t)^T$ such that $w_e \in [0, 1]$,$(e = (1, 2, \ldots, t)$ $\sum_{e=1}^{t} w_t = 1$. The experts $\mathcal{E}x_t$ assesses each attribute $\mathcal{C}_j$ of each alternative $\mathcal{X}_i$ by the form $\mathcal{S}_{ij} = \langle \varphi_{ij}, \eta_{ij}, \psi_{ij} \rangle$.

Step 1: Gather the evaluation scores corresponding to each criterion and amalgamate them into a decision matrix as follows:

$$Y = \begin{pmatrix} \mathcal{C}_{11} & \cdots & \mathcal{C}_{1n} \\ \vdots & \ddots & \vdots \\ \mathcal{C}_{m1} & \cdots & \mathcal{C}_{mn} \end{pmatrix} \tag{23}$$

Step 1. Transform the cost-type attributes ($\mathfrak{C}$) into benefit-type attributes ($\mathfrak{B}$) using the following formula.

$$r_{ij} = \begin{cases} \mathcal{S}_{ij} = \langle \varphi_{ij}, \eta_{ij}, \psi_{ij} \rangle & \text{for } \mathfrak{B} \\ (\mathcal{S}_{ij})^{\mathcal{C}} = \langle \varphi_{ij}, \eta_{ij}, \psi_{ij} \rangle & \text{for } \mathfrak{C} \\ \text{where } i = 1, 2, \ldots, m, j = 1, 2, \ldots n \end{cases}$$

Step 2. Calculate the weights of each attribute by the following formula:

$$\xi_j = \frac{1 + D(r_{ij})}{\sum_{i=1}^{m} (1 + (r_{ij}))} \tag{24}$$

where

$$D(r_{ij}) = \left(\sqrt[r]{\frac{1}{2} \left(\begin{matrix} (\varphi_{ij} - \varphi_{ij})^p + (\eta_{ij} - \eta_{ij})^r \\ +(ij - \psi_{ij})^q \end{matrix} \right)} \right).$$

Step 4. Utilize the $p, q, r-$SFDWA or $p, q, r-$SFDWGA operator to integrate the rating values of alternatives.

Step 5. Calculate the score value of each alternative using equation (9) The graphical layout is presented in figure 2.

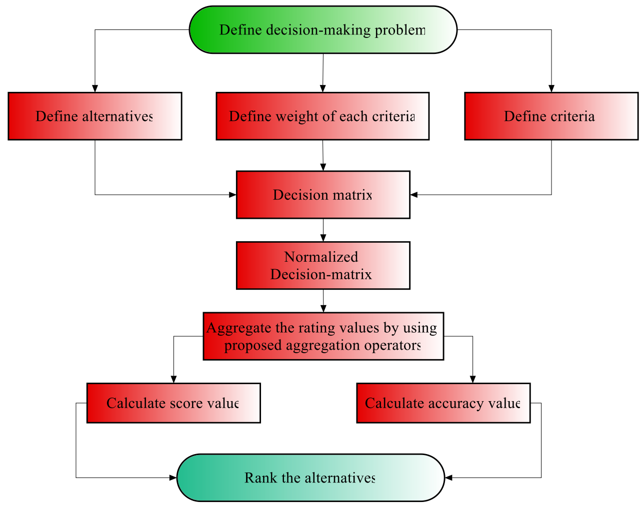

FIGURE 2. Layout of the proposed MCGDM approach.

TABLE 1. The rating values of alternatives concerning criterion $\mathcal{C}_1$.

	$\mathcal{C}_1$ Security
$\mathcal{X}_1$	(0.50,0.30,0.40)
$\mathcal{X}_2$	(0.55,0.65,0.45)
$\mathcal{X}_3$	(0.35,0.45,0.60)
$\mathcal{X}_4$	(0.30,0.40,0.55)
$\mathcal{X}_5$	(0.70,0.45,0.15)

V. NUMERICAL EXAMPLE

More people are embracing cryptocurrencies as a way to independently facilitate international money transfers, free from conventional financial institutions or government oversight. Cryptocurrencies have many benefits because of their decentralized architecture, such as lower costs and faster transaction processing when single points of vulnerability are eliminated. However, it's critical to recognize the negative aspects of cryptocurrencies. Particularly, price volatility is a problem because the values of cryptocurrencies can change dramatically over time.

Moreover, the substantial energy consumption associated with bitcoin mining operations has raised concerns about their energy-intensive nature. This dual nature of cryptocurrencies exemplifies both positive characteristics and potential drawbacks that users and other participants in the cryptocurrency ecosystem should consider. In this context, our objectives are to scrutinize the price stability of the top five cryptocurrencies, $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{X}_3$, $\mathcal{X}_4$ and $\mathcal{X}_5$.

Consider a DM system that incorporates five factors (criteria), empowering qualified experts to evaluate stability and select the most stable coin. The evaluation criteria are determined by the weight vector $\xi = (0.25, 0.21, 0.3, 0.34)$. the considered factors for evaluation are listed below:

1. $\mathcal{C}_1$: Security,
2. $\mathcal{C}_2$: Price limits,
3. $\mathcal{C}_3$: Demand and supply,
4. $\mathcal{C}_4$: Decentralization.

The rating values of alternatives with respect to each criterion is presented in Tables 1 to 4.

TABLE 2. The rating values of alternatives concerning criterion $\mathcal{C}_2$.

$\mathcal{C}_2$ Price limits	
$\mathcal{X}_1$	(0.60,0.20,0.30)
$\mathcal{X}_2$	(0.20,0.15,0.25)
$\mathcal{X}_3$	(0.25,0.30,0.35)
$\mathcal{X}_4$	(0.35,0.40,0.50)
$\mathcal{X}_5$	(0.50,0.20,0.10)

TABLE 3. The rating values of alternatives concerning criterion $\mathcal{C}_1$.

$\mathcal{C}_3$ Demand and supply	
$\mathcal{X}_1$	(0.50,0.30,0.40)
$\mathcal{X}_2$	(0.45,0.35,0.45)
$\mathcal{X}_3$	(0.25,0.35,0.30)
$\mathcal{X}_4$	(0.40,0.30,0.10)
$\mathcal{X}_5$	(0.65,0.25,0.20)

TABLE 4. The rating values of alternatives concerning criterion $\mathcal{C}_1$.

$\mathcal{C}_4$ Decentralization	
$\mathcal{X}_1$	(0.40,0.20,0.30)
$\mathcal{X}_2$	(0.25,0.25,0.45)
$\mathcal{X}_3$	(0.35,0.45,0.60)
$\mathcal{X}_4$	(0.30,0.40,0.55)
$\mathcal{X}_5$	(0.80,0.25,0.45)

TABLE 5. Aggregated values by using p, q, r − SFDWA operator.

Alternatives	Aggregated values
$\mathcal{X}_1$	(0.3418,0.2283,0.3209)
$\mathcal{X}_2$	(0.3645,0.5163,0.3822)
$\mathcal{X}_3$	(0.1952,0.3887,0.4869)
$\mathcal{X}_4$	(0.1628,0.2133,0.4746)
$\mathcal{X}_5$	(0.5994,0.4373,0.1564)

TABLE 6. Aggregated values by using p, q, r − SFDOWA operator.

Alternatives	Aggregated values
$\mathcal{X}_1$	(0.4132,0.2036,0.2789)
$\mathcal{X}_2$	(0.4534,0.4918,0.3396)
$\mathcal{X}_3$	(0.2658,0.3616,0.4357)
$\mathcal{X}_4$	(0.2365,0.1911,0.4197)
$\mathcal{X}_5$	(0.6617,0.4152,0.1018)

The more stable coin is chosen from the list of considered alternatives using the proposed methodology. Tables 5, 6, 7, and 8 present the cumulative values obtained using the proposed aggregation operators for the situation of $\gamma = 1$.

Figure 3 provides a systematic layout of the proposed.

Table 9 presents a summary of score values and corresponding rankings for alternatives.

Table 8 provides us with valuable insights into the relative ranking of alternatives when employing various aggregation methods, namely, p, q, r − SFDWA and p, q, r − SFDOWA, p, q, r − SFDWG, and p, q, r − SFDOWG. We observe that, under p, q, r − SFDWA and p, q, r − SFDOWG, the ranking order of alternatives is $\mathcal{X}_5 \succ \mathcal{X}_1 \succ \mathcal{X}_4 \succ \mathcal{X}_2 \succ \mathcal{X}_3$. In contrast, when applying p, q, r − SFDOWA and p, q, r − SFDWG, the ranking order shifts slightly to $\mathcal{X}_5 \succ \mathcal{X}_1 \succ \mathcal{X}_2 \succ \mathcal{X}_4 \succ \mathcal{X}_3$. The examination indicates a remarkable consistency in the ranking order of alternatives concerning the proposed AOs. Consequently, we can confidently choose any of the proposed operators for the aggregation process with similar outcomes. The graphical view of the score values of alternatives is presented in Figure 4.

TABLE 7. Aggregated values by using p, q, r − SFDWG operator.

Alternatives	Aggregated values
$\mathcal{X}_1$	(0.3872,0.2081,0.3815)
$\mathcal{X}_2$	(0.4045,0.4907,0.3428)
$\mathcal{X}_3$	(0.2367,0.3690,0.4411)
$\mathcal{X}_4$	(0.1628,0.2133,0.4746)
$\mathcal{X}_5$	(0.5994,0.4373,0.1564)

TABLE 8. Aggregated values by using p, q, r − SFDOWG operator.

Alternatives	Aggregated values
$\mathcal{X}_1$	(0.3012,0.2616,0.3488)
$\mathcal{X}_2$	(0.3261,0.5563,0.4101)
$\mathcal{X}_3$	(0.1593,0.4270,0.5217)
$\mathcal{X}_4$	(0.1227,0.2539,0.5140)
$\mathcal{X}_5$	(0.5598,0.4712,0.1985)

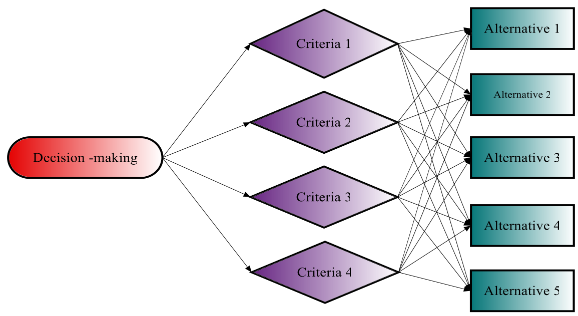

FIGURE 3. Model for the numerical example.

TABLE 9. The score values of alternatives derived from the proposed AOs.

Operator	Score values				
	$\mathcal{X}_1$	$\mathcal{X}_2$	$\mathcal{X}_3$	$\mathcal{X}_4$	$\mathcal{X}_5$
p, q, r − SFDWA	0.6539	0.5739	0.5500	0.5858	0.7145
p, q, r − SFDOWA	0.6838	0.6161	0.5834	0.6144	0.7517
p, q, r − SFDWG	0.6573	0.6005	0.5751	0.5853	0.7062
p, q, r − SFDOWG	0.6335	0.5429	0.5236	0.5621	0.6840

A. SENSITIVITY ANALYSIS

Within this section, we have delved into the examination of the influence of the Dombi parameter γ as well as the parameters embedded within the proposed aggregation operators (p,

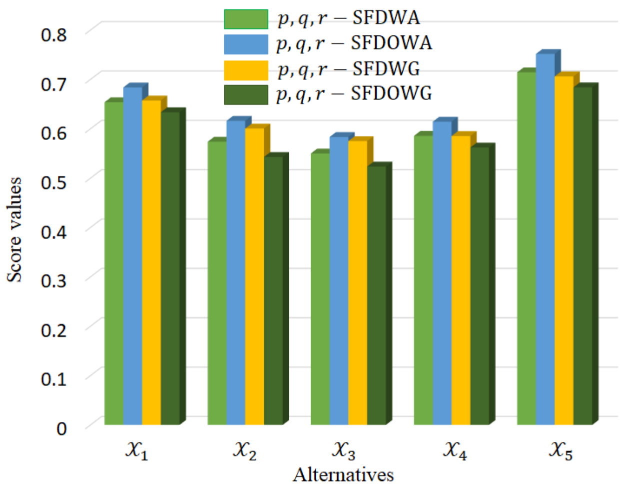

FIGURE 4. **Score values of alternatives obtained by proposed aggregation operators.**

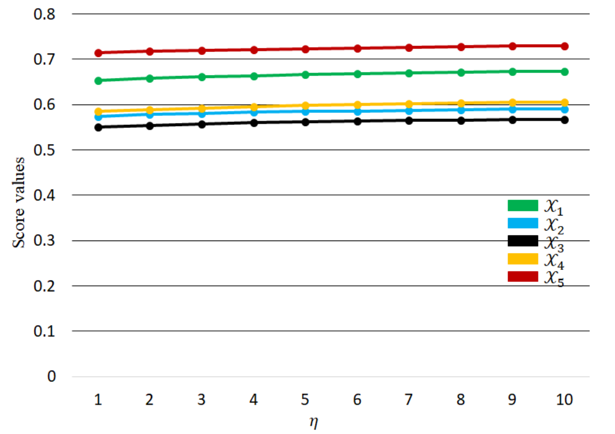

FIGURE 5. **The trend in score values across various values of parameter γ.**

TABLE 10. **The score values of alternatives under varying values of parameter γ.**

η	Score values				
	$\mathcal{X}_1$	$\mathcal{X}_2$	$\mathcal{X}_3$	$\mathcal{X}_4$	$\mathcal{X}_5$
1	0.6539	0.5739	0.5500	0.5858	0.7145
2	0.6590	0.5782	0.5541	0.5893	0.7173
3	0.6612	0.5809	0.5578	0.5926	0.7196
4	0.6631	0.5832	0.5598	0.5953	0.7221
5	0.6659	0.5848	0.5617	0.5985	0.7238
6	0.6687	0.5863	0.5633	0.6008	0.7254
7	0.6704	0.5875	0.5651	0.6022	0.7267
8	0.6719	0.5886	0.5662	0.6040	0.7281
9	0.6728	0.5898	0.5669	0.6049	0.7291
10	0.6731	0.5906	0.5674	0.6057	0.7297

TABLE 11. **The impact of parameter p over score values for $\gamma = 1$, $q = 2$.**

p	q	r	Score values				
			$\mathcal{X}_1$	$\mathcal{X}_2$	$\mathcal{X}_3$	$\mathcal{X}_4$	$\mathcal{X}_5$
1	2	2	0.6576	0.5835	0.5591	0.6014	0.7257
2	2	2	0.6682	0.5942	0.5683	0.6124	0.7318
3	2	6	0.6783	0.6013	0.5776	0.6216	0.7385
4	2	4	0.6863	0.6075	0.5845	0.6302	0.7423
5	2	10	0.6930	0.6120	0.5923	0.6387	0.7484
6	2	6	0.6996	0.6274	0.6001	0.6462	0.7536
7	2	14	0.7041	0.6308	0.6053	0.6531	0.7591
8	2	8	0.7097	0.6352	0.6091	0.6603	0.7628
9	2	18	0.7138	0.6399	0.6132	0.6670	0.7647
10	2	10	0.7169	0.6427	0.6168	0.6739	0.7686

r, and q). We have thoroughly explored the effects of these variables and their interactions on the outcomes.

B. THE IMPACT OF DOMBI PARAMETER η OVER RANKING ORDER

To assess how the Dombi parameter γ affects the score values and ranking order, we employed the p, q, r − SFDWA operator and explored a range of γ values spanning from 1 to 10. The findings from this analysis are succinctly summarized in Table 10.

From the data presented in Table 10, it is evident that as we increment the Dombi parameter γ, the score values also increase. In simpler terms, there exists a positive correlation between the parameter γ and the corresponding score values. This indicates that a higher γ value results in higher score values, which may imply a stronger or more pronounced impact of the Dombi parameter on the given context. Figure 5 provides a visual representation of the score values across various γ values. This graphical depiction offers a clear and intuitive way to observe how score values change in response of different γ values.

C. THE IMPACT OF PARAMETER p, q, AND r OVER SCORE VALUES

To evaluate the impact of the parameter on both score values and ranking order, we utilized the p, q, r − SFDWA and investigated a range of p, q, and r values from 1 to 10. The outcomes of this examination are concisely outlined in Tables 11 and 12.

From the data presented in Table 11, it becomes evident that as we keep the parameters γ and q fixed and progressively increase the value of p from 1 to 10, the score values of the alternatives show a consistent increase. Notably, the ranking order remains unaltered during this process. This attribute of the proposed operators holds significant importance in practical decision-making scenarios.

To elaborate, consider a scenario where we see that as we raise parameter p, the score values for the alternatives increase. This observation provides an optimistic perspective for decision-makers. In other words, it suggests that decision-makers can be more optimistic by assigning higher values to the parameter p during the aggregation process. Conversely, if decision-makers lean towards a more

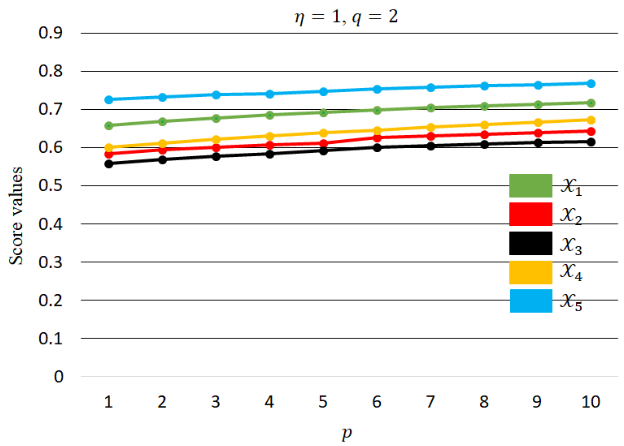

FIGURE 6. The score values of alternatives corresponding to different values of the parameter p.

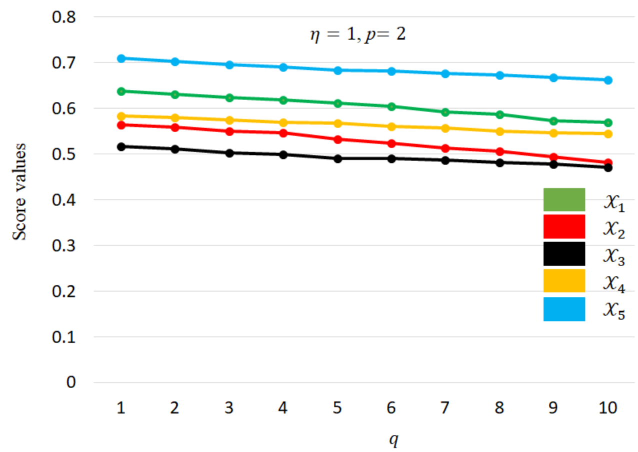

FIGURE 7. The behavior of alternatives for different values of q.

TABLE 12. The influence of p, q, and r.

q	p	r	Score values				
			x_1	x_2	x_3	x_4	x_5
1	2	2	0.6383	0.5642	0.5167	0.5835	0.7097
2	2	2	0.6304	0.5589	0.5124	0.5796	0.7026
3	2	6	0.6232	0.5502	0.5034	0.5744	0.6954
4	2	4	0.6191	0.5474	0.5001	0.5704	0.6902
5	2	10	0.6117	0.5326	0.4913	0.5671	0.6845
6	2	6	0.6055	0.5247	0.4902	0.5611	0.6811
7	2	14	0.5923	0.5125	0.4874	0.5565	0.6761
8	2	8	0.5869	0.5071	0.4815	0.5506	0.6725
9	2	18	0.5726	0.4947	0.4786	0.5473	0.6684
10	2	10	0.5701	0.4820	0.4709	0.5451	0.6619

pessimistic stance, they can opt for lower values of the parameter p, resulting in deceasing score values for the overall evaluation. In essence, this flexibility in parameter selection allows decision-makers to tailor the aggregation process to their specific outlook, accommodating both optimistic and pessimistic viewpoints in the decision-making process. The influence of parameter p over score values of alternatives is presented in Figure 6.

The impact of q over score values is presented in Table 12.

By examining Table 12, it becomes apparent that when we elevate the value of the parameter q the corresponding score values exhibit a decrease. This insight is crucial in understanding how adjustments in the parameter q can influence the overall evaluation of alternatives, providing valuable information for decision makers. Figure 7 illustrates the influence of parameter q on score values.

D. COMPARTIVE ANALYSIS

Our objective is to highlight the effectiveness of our proposed operators in comparison to existing methods in a Pythagorean fuzzy environment [37], [38], [39], [40]. We consider neutral judgments of $p, q, r-$ spherical fuzzy set as zero and employ a weight vector of (0.20, 0.15, 0.10, 0.25, 0.30). The summarized results, including optimal scores and ordering of choices, are presented in Table 13.

TABLE 13. The obtained score values of alternatives using various existing approaches.

Approaches	Score values				
	x_1	x_2	x_3	x_4	x_5
Garg [37]	0.3817	0.3452	0.3113	0.3691	0.4332
Garg [38]	0.4172	0.4585	0.4030	0.4683	0.4854
Rani et al. [39]	0.2131	0.1724	0.1643	0.1985	0.2546
Wei and Lu [40]	0.4733	0.4137	0.3892	0.4356	0.5214

The table distinctly illustrates that the top-ranked option aligns with the conclusions derived from the proposed approach, underscoring the reliability of our approach when juxtaposed with the latest advancements in the field. This comparison serves to showcase the superiority of our proposed operators in decision-making scenarios within the Pythagorean fuzzy framework.

Additionally, we conducted a comparison between the proposed approach and certain approaches developed within a T-spherical fuzzy environment. The outcomes of this comparison are detailed in Table 14.

TABLE 14. Comparative study with some existing approaches.

Approaches	Score values				
	x_1	x_2	x_3	x_4	x_5
Jun et al. [41]	0.3423	0.2993	0.2675	0.3284	0.4163
Munir et al. [42]	0.4651	0.4561	0.4121	0.4506	0.5005
Rahim et al. [35]	0.5344	0.4835	0.4472	0.5062	0.5537
Garg et al. [43]	0.4415	0.4342	0.3917	0.4394	0.4760
Mahood et al. [29]	0.3254	0.3114	0.3075	0.3266	0.3320

Table 14 illustrates that the ranking order of alternatives aligns with the proposed order, signifying the sustainability of our approach as a viable alternative. The key strength of our method lies in its ability to enhance a flexible decision-making environment by incorporating four parameters: γ, p, q, and r, which play integral roles throughout the evaluation of alternatives.

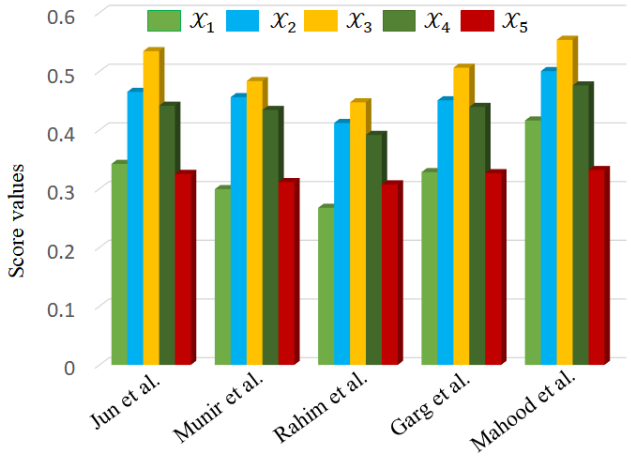

FIGURE 8. The score values obtained by existing approaches.

In contrast, existing methods within the spherical fuzzy and T-spherical fuzzy contexts face limitations. Our proposed approach effectively circumvents these constrains, presenting a more adaptable and unconstrained structure. This flexibility empowers decision-makers to navigate the complexities of decision-making processes more realistically. By leveraging the factors γ, p, q, and r, our approach offers a versatile and comprehensive solution, addressing the shortcoming of the existing methods and providing a nuanced perspective on overcoming their limitations. The score values of alternatives obtained by different existing approaches is presented in Figure 8.

E. ADVANTAGES

1. The proposed methodology appears to greater versatility compared to the existing methods. Its extensive applicability makes it suitable for various situations or issue domains. The suggested strategy surpasses the constraints of conventional approaches and, by incorporating more comprehensive and flexible systems, provides a more adaptive solution for DM tasks. The broad application of this framework gives it a more general and adaptable structure that can address diverse DM scenarios, yielding precise and consistent results.
2. The flexibility of the proposed DM approach is heightened, emphasizing the limitations inherent in conventional methods by leveraging the parameters γ, p, q, and r. These parameters contribute to the enhanced flexibility of the proposed approach, allowing decision-makers to personalize and adjust the DM process to meet their requirements and preferences. As a results, the proposed approach establishes a more tailored and adaptable foundation.

F. LIMITATIONS

While the proposed work exhibits several strengths, it is important to acknowledge some restrictions.

1. The performance of the proposed approach may be sensitive to the chosen values of parameters such as γ, p, q, and r. The robustness of the method across a wide range of parameter values needs to be thoroughly investigated.
2. The introduction of parameters such as γ, p, q, and r may introduce a level of subjectivity in DM process. Different decision-makers might assign different values to these parameters, potentially leading to varying outcomes.
3. The incorporation of additional parameters and operational laws may introduce complexity to the DM process.

VI. CONCLUSION AND FUTURE WORK

Aggregation operators are critical to the DM process. This paper offers innovative operational laws, aggregation operators, and their accompanying attributes by leveraging the capabilities of Dombi operations and the flexibility inherent in $p, q, r-$SFSs. These operators form the basis of the MCDM method, which handles actual DM challenges. A real-life scenario involving the selection of a stable cryptocurrency demonstrates the proposed technique's efficacy. A comparative study is conducted to validate the suggested approach. Furthermore, the benefits and drawbacks of the suggested strategy are extensively evaluated. In the future, the proposed framework has the potential to be applied to real-life scenarios [44], [45], [46] for the evaluate its effectiveness and applicability in practical DM contexts.

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CONTRIBUTIONS OF THE AUTHORS

All contributors to this research project have played an equal role in designing, implementing, analyzing results, and writing the manuscript. Each author has made an equivalent contribution to every facet of the study, including conceptualizing the research plan, assessing outcomes, and crafting the final manuscript.

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