Received 7 November 2023, accepted 13 December 2023, date of publication 18 December 2023, date of current version 4 January 2024.

Digital Object Identifier 10.1109/ACCESS.2023.3344208

RESEARCH ARTICLE

Robust Estimation-Based Control for Generalized One-Sided Lipschitz Nonlinear Systems in the Presence of Output Delays

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This work was supported in part by the Ministry of Science and ICT (MSIT), South Korea, under the ICT Challenge and Advanced Network (ICAN) of HRD Program supervised by the Institute of Information & Communications Technology Planning & Evaluation (IITP) under Grant IITP-2023-RS-2022-00156345; and in part by the National Research Foundation of Korea (NRF) funded by the Korean Government (MSIT) under Grant RS-2023-00219051 and Grant RS-2023-00209107.

ABSTRACT This study presents a robust estimation-based control approach for nonlinear systems with generalized one-sided Lipschitz (OSL) conditions projected for delay measurement. A control scheme was devised for this family of nonlinear systems by deploying the Lyapunov–Krasovskii (LK) method for the delayed system and by encompassing the generalized OSL inequality without using the quadratic inner boundedness (QIB) condition to overcome the conservatism introduced by the QIB condition. The stability of the resulting system is attained by employing a delay-range-dependent technique, while the derivative of the LK functional is scouted using Wirtinger's inequality, which shrinks the conservativeness of typical Jensen's inequality, resulting in stability in the form of linear matrix inequalities (LMIs). Moreover, a sufficient and necessary solution for the main results was achieved through a decoupling mechanism to acquire the controller and estimator gains simultaneously using iterative optimization tools. To attenuate the effects of external disturbances on the system, the L2 gain of the error with reference to the system was computed and incorporated into the dynamics. Furthermore, the LMI-based results are handled using the cone complementary linearization (CCL) method to authenticate the controller and observer gains via convex optimization. Finally, a numerical example demonstrates the success of the presented robust observer control formation for generalized OSL nonlinear systems in the presence of output delays.

INDEX TERMS Robust observer-based control, generalized one-sided Lipschitz nonlinearity, cone complementary linearization, Wirtinger's inequality.

I. INTRODUCTION

Estimating the vision-based motion of unmanned air vehicles, automatic electric throttles of DC motors, PEM fuel cells, electric machines, and various other real-life dynamical systems is challenging in the current era [1], [2], [3], [4]. To design a feedback system, it is always desirable to find the unknown or missing states of the system; however, for several systems, we cannot find system states owing to the absence of a physical sensor. To eliminate this problem, an observer design scheme was used to determine the unavailable states of the system. Consequently, the observer design has garnered considerable attention from researchers in the past few decades owing to its importance in fault diagnosis, energy system analysis, chaos-based secure communications, and synchronization studies [5], [6], [7], [8]. When designing an observer, the stability of the system must be ensured under output delays and resistance to external disturbances.

A considerable amount of studies are available on observer design using Lipschitz and one-sided Lipschitz (OSL)

The associate editor coordinating the review of this manuscript and approving it for publication was Mouquan Shen.

nonlinearities. The Lipschitz nonlinearity is region-based, provides an infeasible solution for larger constant values, and also introduces conservativeness; therefore, the OSL nonlinearity is generally used as it covers a large range of nonlinearities, but it also introduces conservatism owing to the quadratic inner boundedness (QIB) limitations. However, the aforementioned obstacles are addressed using generalized OSL nonlinearity [9], which also does not require the use of QIB conditions and covers the entire range of nonlinearities that the Lipschitz, OSL, and QIB conditions together may have been covered. Over the past few decades, several observer design studies on Lipschitz and OSL nonlinearities have been presented for continuous and discrete-time systems. For instance, the asymptotic stability of a Lipschitz nonlinear observer is determined by having the necessary and sufficient condition for the matrix inequality [10]. A full-order state observer was designed for Lipschitz systems using linear matrix inequalities (LMIs) to ensure asymptotic stability [11]. In [12], the conservatism in the nonlinear observer design for Lipschitz systems is reduced using the Riccati equation in one variable for the existence of a stable solution using the S-procedure. A recent study [13] investigated the observer design for OSL systems in the presence of measurement delays and L2 norm-based perturbations. Moreover, in a study [14], a sufficient condition for the existence of an observer for monotone nonlinearities and one-sided nonlinearities was accomplished. In another study [15], the design of observers for OSL descriptors subject to output delays, delays in state, disturbances, and uncertainties using the Lyapunov–Krasovkii (LK) functional to ensure the stability of such systems was presented. In [16], a full-order nonlinear unknown input observer (UIO) and a reduced-order UIO were designed using a linear matrix inequality (LMI) approach for OSL nonlinear systems. Another work [17] presented a robust estimation-based controller design for OSL nonlinear systems using Young's inequality and a cone complementary linearization (CCL) technique.

The simplest method for investigating the stability is the use of a delay-independent approach. Design conservatism in delay-dependent and delay-range-dependent schemes is reduced by introducing a delay range starting from zero to an upper limit or in a precise range with a lower limit not equal to zero [18], [19], [20], [21]. Controller stabilization analysis independent of delay reduces complexity but introduces conservatism when the lower limit on delay is considered to be zero. Delay-dependent stability schemes are utilized to overcome traditionalism in delays in reference to a delay-independent scheme. Moreover, a more specific condition involves the reflection of the delay starting from a constant lower limit (other than zero) to a definite upper limit [22], [23], [24], [25]. In [26], a delay-range-dependent scheme was studied to determine stability by considering Jensen's inequality. The dynamics involved in the measurement delays are incorporated into the Lyapunov functional, the derivative of which is exploited by involving Jensen's inequality to obtain LMI-based results for determining stability. Because Jensen's inequality is conservative, researchers have been motivated to use Wirtinger's inequality, which provides a more accurate integral than Jensen's inequality. Obtaining the controller and observer gains simultaneously and independently using an output feedback system is challenging because of the dependability of both gains on each other. Therefore, decoupling methods are employed to compute both gains [27].

Motivated by the described factors, this study aims to present observer-based control for generalized OSL nonlinearity under measurement delays and L2 norm-based external disturbances using the LK functional. The derivative of the Lyapunov functional is solved using Wirtinger's inequality, which reduces the conservatism introduced by Jensen's inequality, and provides a more accurate integral than the aforementioned inequality. The stability of a nonlinear system is determined using the Lyapunov functional derivative and resulting LMI. The generalized OSL (GOSL) condition was also added to the resulting LMI. Because the feedback loop dynamics are coupled with system errors, we cannot calculate the observer and controller gains independently. Therefore, we used a decoupling and CCL mechanism to obtain the controller and observer gains. A numerical example was presented to demonstrate the success of the proposed robust observer control formation for generalized OSL nonlinear systems in the presence of output delays.

The main contributions of the proposed study are provided as follows:

1. To the best of our knowledge, the current study is the first to present a robust observer-based control scheme for generalized OSL nonlinear systems under output delays that are subjected to disturbances. Furthermore, the methodology adopts straightforward computations to retrieve the observer and controller gains using the CCL technique.
2. The regional stability of the robust estimation-based control of nonlinear systems with delayed dynamics is ensured by Lyapunov retreatment with Wirtinger's inequality results and local region selection, in contrast to existing techniques [22], [23], [24], [25], [26]. Previously, Jensen's inequality was used to provide a solution of integral terms; therefore, generic treatment was a motivating aspect of the study.
3. The proposed scheme develops a less conservative decoupled condition by corroborating both necessity and sufficiency conditions for the solution of constraints in a main coupled system in the presence of disturbances for generalized OSL nonlinear systems.

The remainder of this paper is organized as follows. The main system under consideration and the augmented system formation are presented in Section II. In Section III, three theorems are proposed, and their proofs are provided. The simulation results for the moving ball simulation example are presented in Section IV. The main results are presented in Section V, and a few future recommendations are provided.

II. SYSTEM DESCRIPTION

Assuming a class of nonlinear dynamic systems in the presence of output delay as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) + f(t,x) + d(t),$$
$$y(t) = Cx(t - \tau(t)), \quad (1)$$

where the system state vector is given by $x(t) \in \Re^n$, the output measurement vector is $y(t) \in \Re^p$, the control input of the system is taken as $u(t) \in \Re^m$, disturbance vector to the system by $d(t) \in \Re^n$, and the nonlinearity is denoted by $f(t,x) \in \Re^n$. A, B, and C are constant system matrices for the linear dynamics. The continuous output function for the delay term is $\tau(t)$, which satisfies:

$$0 \le h_1 \le \tau(t) \le h_2,$$
$$\dot{\tau}(t) \le \mu. \quad (2)$$

The nonlinear function in the system dynamics is assumed to belong to the generalized OSL condition defined below (for reference, see [9]):

Definition 1 ([9]): For positive definite matrices $U, V \in \mathbb{R}^n$ and scalar $\sigma \in \mathbb{R}$, the function $G(t,x)$ is assumed to fulfill the generalized OSL stipulation, such that the inequality provided below holds:

$$(G(z,t) - G(\hat{z},t))^T U(z-\hat{z}) \le \sigma(z-\hat{z})^T V(z-\hat{z}). \quad (3)$$

The idea is to select matrices U and V to satisfy the GOSL condition. Under these conditions, when U = V = I, the GOSL condition becomes the traditional OSL condition, as discussed in [9].

Assumption 1: The function $f(t,x)$ in (1) fulfills the generalized OSL condition defined in (3).

Assumption 2: The system matrix pairs for the observability (A,C) and controllability (A,B) were detectable and stabilizable, respectively.

This study proposes a robust observer-based control strategy for generalized OSL nonlinear systems in the presence of an output delay.

A. OBSERVER DESIGN

The estimation of all unavailable states is required to completely control nonlinear systems; for this purpose, the observer is provided with the following dynamics:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + f(t,\hat{x}) + Bu(t) + K_o(y(t) - \hat{y}(t)),$$
$$\hat{y}(t) = C\hat{x}(t - \tau(t)), \quad (4)$$

where $K_o \in \mathbb{R}^{n\times p}$ is assumed to be the estimation gain, and the estimated state for the original state x is mentioned as $\hat{x}$. To guarantee asymptotic conversion, the estimation error for the original system and the estimated state are required, which is mathematically expressed as follows:

$$e(t) = x(t) - \hat{x}(t). \quad (5)$$

Exploiting the time derivative for error dynamics and including the estimation dynamics from (4), we obtain the following relation:

$$\dot{e}(t) = Ae(t) + \varphi(x,\hat{x}) - K_oCe(t - \tau(t)) + d(t),$$
$$\varphi(x,\hat{x}) = f(t,x) - f(t,\hat{x}). \quad (6)$$

To establish the control algorithm, the observer dynamics are provided by the controller according to the relation:

$$u(t) = K_c\hat{x}(t). \quad (7)$$

where $K_c \in \mathbb{R}^{m\times n}$ represents the gain matrix of the controller. Overall, a closed-loop system is acquired by considering the relationship between (1) and (7) as follows:

$$\dot{x}(t) = Ax(t) + f(t,x) + BK_cx(t) - BK_ce(t) + d(t). \quad (8)$$

Transforming (5) and (8) into an augmented state that encapsulates both the state and error dynamics yields:

$$z(t) = \begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix}^T,$$

where the state equation is obtained as follows:

$$\dot{z}(t) = \bar{A}z(t) + \bar{A}_1 z(t - \tau(t)) + g(t,x,\hat{x}) + D(t),$$
$$\bar{A} = \begin{bmatrix} A + BK_c & -BK_c \\ 0 & A \end{bmatrix}, \bar{A}_1 = \begin{bmatrix} 0 & 0 \\ 0 & -K_oC \end{bmatrix},$$
$$D(t) = \begin{bmatrix} d(t) \\ d(t) \end{bmatrix}, \text{ and } g(t,x,\hat{x}) = \begin{bmatrix} f(t,x) \\ \varphi(x,\widehat{x}) \end{bmatrix}. \quad (9)$$

The convergence of augmented system states and estimation states in the neighborhood of the origin is ensured by involving Wirtinger's inequality, which is integrated into the present study and provided by Lemmas 1 and 2 below (for proofs and results, see [8], [30], and the references therein).

Lemma 1 [8], [30]:

For a given function x, which is continuous and differentiable, such that $[w,v] \to \mathbb{R}^n$. There subsists a matrix $M > 0$ such that it fulfills,

$$\int_w^v \dot{x}^T(s)M\dot{x}(s)ds$$
$$\ge \frac{1}{v-w}[x(v)-x(w)]^T M [x(v)-x(w)]$$
$$+ \frac{3}{v-w}\left[x(v) + x(w) - \frac{2}{v-w}\int_w^v x(s)ds\right]^T$$
$$\times M\left[x(v) + x(w) - \frac{2}{v-w}\int_w^v x(s)ds\right]. \quad (10)$$

Lemma 2 [8], [30]:

For the function $\Theta(\alpha, R)$ defined by

$$\Theta(\alpha,R) = \frac{1}{\alpha}\zeta^T W_1^T R\zeta W_1 + \frac{1}{1-\alpha}\zeta^T W_2^T R\zeta W_2,$$

where $R \in \mathbb{R}^{n\times n}$, $W_1, W_2 \in \mathbb{R}^{n\times m}$, $\zeta \in R^m$, $\alpha \in (0\ 1)$ and n, m are positive scalars. If we select a matrix $T \in \mathbb{R}^{n\times n}$ that satisfies the condition $\begin{bmatrix} R & T \\ * & R \end{bmatrix} > 0$, the inequality must fulfill

$$\min \Theta(\alpha, R) = \begin{bmatrix} W_1\zeta \\ W_2\zeta \end{bmatrix}^T \begin{bmatrix} R & T \\ * & R \end{bmatrix} \begin{bmatrix} W_1\zeta \\ W_2\zeta \end{bmatrix}. \quad (11)$$

This study encapsulated the main conditions for the stability of the estimation-based control systems for generalized OSL nonlinear systems provided in (1). Asymptotic convergence to the origin is provided by involving the augmented system in (9), and the stability condition is derived by assuming the LK functional for delayed dynamics.

III. MAIN RESULTS

The preceding section incorporated the complete derivation and results for the stability of the coupled system and ensured the stability of the system states asymptotically to the origin, provided that the feedback control system was stable.

Theorem 1:

An augmented system (9), introduced by involving a generalized OSL nonlinear system in (1), observer dynamics in (5), and controller in (7), satisfies the condition provided in Assumptions 1 and 2, and converges asymptotically to the neighborhood of origin. For symmetric matrices,

$$P = \begin{bmatrix} P_{1c} & 0 \\ * & P_{1o} \end{bmatrix} > 0, Q_1 = \begin{bmatrix} Q_{1c} & 0 \\ * & Q_{1o} \end{bmatrix} > 0,$$
$$Q_2 = \begin{bmatrix} Q_{2c} & 0 \\ * & Q_{2o} \end{bmatrix} > 0, Q_3 = \begin{bmatrix} Q_{3c} & 0 \\ * & Q_{3o} \end{bmatrix} > 0,$$
$$Z_1 = \begin{bmatrix} Z_{1c} & 0 \\ * & Z_{1o} \end{bmatrix} > 0, Z_2 = \begin{bmatrix} Z_{2c} & 0 \\ * & Z_{2o} \end{bmatrix} > 0$$

and scalars $\varepsilon_1 > 0$, and $\varepsilon_3 > 0$ the $z(t)$ such that the following resultant inequalities exist (12), as shown at the bottom of the next page, where

$$\omega_{1,1} = \bar{A}^T P + P\bar{A} + diag\{\rho\varepsilon_1 U, \rho\varepsilon_3 V\} + \sum_{i=1}^{3} Q_i,$$
$$\omega_{1,8} = P - \frac{1}{2}[diag\{\varepsilon_1 U, \varepsilon_3 V\}],$$
$$\omega_{2,2} = -(1-\mu) Q_3,$$
$$T_1 = \begin{bmatrix} G_3^T & G_4^T & G_5^T & G_6^T \end{bmatrix}^T,$$
$$T_2 = \begin{bmatrix} G_7^T & G_8^T & G_5^T & G_6^T \end{bmatrix}^T,$$
$$G_3 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$G_4 = \begin{bmatrix} I & I & 0 & 0 & -2I & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$G_5 = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$G_6 = \begin{bmatrix} 0 & I & I & 0 & 0 & 0 & -2I & 0 & 0 \end{bmatrix},$$
$$G_7 = \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$G_8 = \begin{bmatrix} 0 & I & 0 & I & 0 & 0 & -2I & 0 & 0 \end{bmatrix},$$
$$\phi_1 = \begin{bmatrix} \tilde{Z}_1 & 0 \\ * & \tilde{Z}_1 \end{bmatrix}, \phi_2 = \begin{bmatrix} \tilde{Z}_2 & 0 \\ * & \tilde{Z}_2 \end{bmatrix},$$
$$\tilde{Z}_1 = \begin{bmatrix} Z_1 & 0 \\ 0 & 3Z_1 \end{bmatrix}, \tilde{Z}_2 = \begin{bmatrix} Z_2 & 0 \\ 0 & 3Z_2 \end{bmatrix}.$$

Proof: Assume an LK functional [13], [26] for the delayed dynamics of the augmented system as follows:

$$V(z) = z^T Pz + \int_{t-\tau(t)}^{t} z^T(\alpha)Q_3 z(\alpha)d\alpha + \sum_{i=1}^{2} \int_{t-h_i}^{t} z^T(\alpha)Q_i z(\alpha)d\alpha + \int_{-h_1}^{0}\int_{t+\theta}^{t} h_1 \dot{z}^T(\alpha)Z_1\dot{z}(\alpha)d\alpha d\theta + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} h_{12}\dot{z}^T(\alpha)Z_2\dot{z}(\alpha)d\alpha d\theta. \quad (13)$$

The Lyapunov functional time derivative is exploited to guarantee the stability conditions, and is required to ensure that the resultant is negative definite for asymptotic stability. Thus, the time derivative of (13) is expressed as follows:

$$\dot{V}(z,t) \le 2z^T(t)P\dot{z}(t) + \sum_{i=1}^{2}\left\{z^T(t)Q_i z(t) - z^T(t-h_i)Q_i \times z(t-h_i)\right\} + \dot{z}^T(t)\left\{h_1^2 Z_1 + h_{12}^2 Z_2\right\}\dot{z}(t) + z^T(t)Q_3 z(t) - (1-\mu)z^T(t-\tau(t))Q_3 z(t-\tau(t)) - \int_{t-h_1}^{t} h_1\dot{z}^T(\alpha)Z_1\dot{z}(\alpha)d\alpha - \int_{t-h_2}^{t-h_1} h_{12}\dot{z}^T(\alpha)Z_2\dot{z}(\alpha)d\alpha. \quad (14)$$

Using the dynamics in Equation (9) and substituting the resultant into Equation (14), we obtain:

$$\dot{V}(z,t) \le 2z^T(t)P\left(\bar{A}z(t) + \overline{A_1}z(t-\tau(t)) + g(t,x,\hat{x}) + D(t)\right) + \sum_{i=1}^{3}\left\{z^T(t)Q_i z(t)\right\} - (1-\mu)z^T(t-\tau(t))Q_3 z(t-\tau(t)) - \sum_{i=1}^{2} z^T(t-h_i)Q_i z(t-h_i) + \left(\bar{A}z(t) + \overline{A_1}z(t-\tau(t)) + g(t,x,\hat{x}) + D(t)\right)^T \times \left\{h_1^2 Z_1 + h_{12}^2 Z_2\right\}\left(\bar{A}z(t) + \overline{A_1}z(t-\tau(t)) + g(t,x,\hat{x}) + D(t)\right) - \int_{t-h_1}^{t} h_1\dot{z}^T(\alpha)Z_1\dot{z}(\alpha)d\alpha - \int_{t-h_2}^{t-h_1} h_{12}\dot{z}^T(\alpha)Z_2\dot{z}(\alpha)d\alpha. \quad (15)$$

The integral terms in the inequality in (15) are further treated by dividing the integral as follows:

$$-\int_{t-h_1}^{t} h_1\dot{z}^T(\alpha)Z_1\dot{z}(\alpha)\,d\alpha = -\int_{t-\tau(t)}^{t} h_1\dot{z}^T(\alpha)Z_1\dot{z}(\alpha)\,d\alpha - \int_{t-h_1}^{t-\tau(t)} h_1\dot{z}^T(\alpha)Z_1\dot{z}(\alpha)\,d\alpha.$$

Moreover, involving Wirtinger's inequality provided in Lemma 1, it renders the following:

$$\begin{aligned}
&-\int_{t-\tau(t)}^{t} h_1\dot{z}^T(\alpha)Z_1\dot{z}(\alpha)\,d\alpha - \int_{t-h_1}^{t-\tau(t)} h_1\dot{z}^T(\alpha)Z_1\dot{z}(\alpha)\,d\alpha \\
&\le -\frac{h_1}{\tau(t)}[z(t)-z(t-\tau(t))]^T Z_1[z(t)-z(t-\tau(t))] \\
&\quad -\frac{3h_1}{\tau(t)}\left[z(t)+z(t-\tau)-\frac{2}{\tau(t)}\int_{t-\tau(t)}^{t} z(\alpha)\,d\alpha\right]^T \\
&\quad \times Z_1\left[z(t)+z(t-\tau(t))-\frac{2}{\tau(t)}\int_{t-\tau(t)}^{t} z(\alpha)\,d\alpha\right] \\
&\quad -\frac{h_1}{h_1-\tau(t)}[z(t-\tau(t))-z(t-h_1)]^T Z_1[z(t-\tau(t)) \\
&\quad -z(t-h_1)] \\
&\quad -\frac{3h_1}{h_1-\tau(t)}\left[z(t-\tau(t))+z(t-h_1)-\frac{2}{h_1-\tau(t)}\right. \\
&\quad \left.\times\int_{t-h_1}^{t-\tau(t)} z(\alpha)\,d\alpha\right]^T \\
&\quad \times Z_1\left[z(t-\tau(t))+z(t-h_1)-\frac{2}{h_1-\tau(t)}\int_{t-h_1}^{t-\tau(t)} z(\alpha)\,d\alpha\right].
\end{aligned} \tag{16}$$

By defining σ as follows:

$$\begin{aligned}
\sigma = \Bigg[& z^T(t)\ z^T(t-\tau(t))\ z^T(t-h_1)\ z^T(t-h_2) \\
&\times \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} z^T(\alpha)\,d\alpha \frac{1}{\tau(t)-h_1}\int_{t-h_2}^{t-h_1} z^T(\alpha)\,d\alpha \\
&\times \frac{1}{h_2-\tau(t)}\int_{t-h_2}^{t-\tau(t)} z^T(\alpha)\,d\alpha g^T(t,x,\hat{x})D^T\Bigg]^T,
\end{aligned}$$

G_3, G_4, G_5, and G_6 as follows:

$$\left.\begin{aligned}
&\begin{bmatrix} z(t) & z(t-\tau)\end{bmatrix} = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\sigma \\
&\quad = G_3\sigma = \sigma_1, \\
&\begin{bmatrix} z(t) & z(t-\tau) & \frac{2}{\tau(t)}\int_{t-\tau(t)}^{t} z^T(\alpha)\,d\alpha\end{bmatrix} \\
&\quad = \begin{bmatrix} I & I & 0 & 0 & -2I & 0 & 0 & 0 & 0\end{bmatrix}\sigma = G_4\sigma = \sigma_2, \\
&\begin{bmatrix} z(t-\tau) & z(t-h_1)\end{bmatrix} \\
&\quad = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\sigma = G_5\sigma = \sigma_3, \\
&\begin{bmatrix} z(t-\tau) & z(t-h_1) & -\frac{2}{h_1-\tau(t)}\int_{t-h_1}^{t-\tau} z^T(\alpha)\,d\alpha\end{bmatrix} \\
&\quad = \begin{bmatrix} 0 & I & I & 0 & 0 & -2I & 0 & 0 & 0\end{bmatrix}\sigma = G_6\sigma = \sigma_4.
\end{aligned}\right\} \tag{17}$$

Moreover, from (17), we can obtain (16) as follows:

$$\begin{aligned}
&-\int_{t-h_1}^{t} h_1\dot{z}^T(\alpha)Z_1\dot{z}(\alpha)\,d\alpha \\
&\le -\frac{h_1}{\tau(t)}\sigma_1^T Z_1\sigma_1 - \frac{3h_1}{\tau(t)}\sigma_2^T Z_1\sigma_2 \\
&\quad -\frac{h_1}{h_1-\tau(t)}\sigma_3^T Z_1\sigma_3 - \frac{3h_1}{h_1-\tau(t)}\sigma_4^T Z_1\sigma_4, \\
&= -\frac{h_1}{\tau(t)}\sigma^T G_3^T Z_1 G_3\sigma - \frac{3h_1}{\tau(t)}\sigma^T G_4^T Z_1 G_4\sigma
\end{aligned}$$

$$\begin{bmatrix}
\omega_{1,1} & P\bar{A}_1 & 0 & 0 & 0 & 0 & 0 & \omega_{1,8} & P & h_1\bar{A}^T & h_{12}\bar{A}^T \\
* & \omega_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h_1\bar{A}_1^T & h_{12}\bar{A}_1^T \\
* & * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & 0 & 0 & h_1 & h_{12} \\
* & * & * & * & * & * & * & * & -\gamma I & 0 & 0 \\
* & * & * & * & * & * & * & * & 0 & -h_1Z_1^{-1} & 0 \\
* & * & * & * & * & * & * & * & 0 & * & -h_{12}Z_2^{-1}
\end{bmatrix} - \frac{1}{h_1}T_1^T\phi_1T_1 - \frac{1}{h_{12}}T_2^T\phi_2T_2 < 0, \tag{12}$$

$$-\frac{h_1}{h_1-\tau(t)}\sigma^T G_5^T Z_1 G_5\sigma - \frac{3h_1}{h_1-\tau(t)}\sigma^T G_6^T Z_1 G_6\sigma,$$
$$= -\frac{h_1}{\tau(t)}\sigma^T [G_3^T G_4^T]\begin{bmatrix} Z_1 & 0 \\ 0 & 3Z_1 \end{bmatrix}\begin{bmatrix} G_3 \\ G_4 \end{bmatrix}\sigma - \frac{h_1}{h_1-\tau(t)}\sigma^T [G_5^T\ G_6^T]\begin{bmatrix} Z_1 & 0 \\ 0 & 3Z_1 \end{bmatrix}\begin{bmatrix} G_5 \\ G_6 \end{bmatrix}\sigma. \quad (18)$$

Assuming κ and $\tilde{Z}_1$ as follows:

$$\kappa = \frac{\tau(t)}{h_1} \text{ and } \tilde{Z}_1 = \begin{bmatrix} Z_1 & 0 \\ 0 & 3Z_1 \end{bmatrix}. \quad (19)$$

Integrating the relation mentioned in (19), (18) results into:

$$-\int_{t-h_1}^{t} h_1\dot{z}^T(\alpha)Z_1\dot{z}(\alpha)\,d\alpha \le -\frac{1}{\kappa}\sigma^T\left[G_3^T\ G_4^T\right]\tilde{Z}_1\begin{bmatrix} G_3 \\ G_4 \end{bmatrix}\sigma - \frac{1}{1-\kappa}\sigma^T\left[G_5^T\ G_6^T\right]\tilde{Z}_1\begin{bmatrix} G_5 \\ G_6 \end{bmatrix}\sigma. \quad (20)$$

Encompassing Lemma 2 in (20) by defining, $\phi_1 = \begin{bmatrix} \tilde{Z}_1 & 0 \\ * & \tilde{Z}_1 \end{bmatrix} > 0$, we have the following:

$$-\int_{t-h_1}^{t} h_1\dot{z}^T(\alpha)Z_1\dot{z}(\alpha)\,d\alpha \le -\sigma^T\left[G_3^T\ G_4^T\ G_5^T\ G_6^T\right]\phi_1\begin{bmatrix} G_3 \\ G_4 \\ G_5 \\ G_6 \end{bmatrix}\sigma. \quad (21)$$

Defining $T_1 = \left[G_3^T\ G_4^T\ G_5^T\ G_6^T\right]^T$, renders

$$-\int_{t-h_1}^{t} h_1\dot{z}^T(\alpha)Z_1\dot{z}(\alpha)\,d\alpha \le -\sigma^T T_1^T\phi_1 T_1\sigma. \quad (22)$$

Similarly, employing Lemma 1 on the other integral term in (14) provides

$$-\int_{t-h_2}^{t-h_1} h_{12}\dot{z}^T(\alpha)Z_2\dot{z}(\alpha)\,d\alpha = -\int_{t-\tau(t)}^{t-h_1} h_{12}\dot{z}^T(\alpha)Z_2\dot{z}(\alpha)\,d\alpha - \int_{t-h_2}^{t-\tau(t)} h_{12}\dot{z}^T(\alpha)Z_2\dot{z}(\alpha)\,d\alpha \quad (23)$$

Assigning the terms G_7 and G_8 as follows:

$$\begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\sigma = G_7\sigma,$$
$$\begin{bmatrix} 0 & I & 0 & I & 0 & 0 & -2I & 0 & 0 \end{bmatrix}\sigma = G_8\sigma. \quad (24)$$

Using G_5, G_6 defined in (17) and G_7, G_8 defined in (24), (23) becomes

$$-\int_{t-h_2}^{t-h_1} h_{12}\dot{z}^T(\alpha)Z_2\dot{z}(\alpha)\,d\alpha \le -\frac{h_{12}}{\tau(t)-h_1}\sigma^T G_5^T Z_2 G_5\sigma - \frac{3h_{12}}{\tau(t)-h_1}\sigma^T G_6^T Z_2 G_6\sigma - \frac{h_{12}}{h_2-\tau(t)}\sigma^T G_7^T Z_2 G_7\sigma - \frac{3h_{12}}{h_2-\tau(t)}\sigma^T G_8^T Z_2 G_8\sigma$$
$$= -\frac{h_{12}}{\tau(t)-h_1}\sigma^T\left[G_5^T\ G_6^T\right]\begin{bmatrix} Z_2 & 0 \\ 0 & 3Z_2 \end{bmatrix}\begin{bmatrix} G_5 \\ G_6 \end{bmatrix}\sigma - \frac{h_{12}}{h_2-\tau(t)}\sigma^T\left[G_7^T\ G_8^T\right]\begin{bmatrix} Z_2 & 0 \\ 0 & 3Z_2 \end{bmatrix}\begin{bmatrix} G_7 \\ G_8 \end{bmatrix}\sigma. \quad (25)$$

Furthermore, defined α_2 as follows:

$$\alpha_2 = \frac{\tau(t)-h_1}{h_2-h_1}, \quad (26)$$

and utilizing (26) in (25) renders

$$-\int_{t-h_2}^{t-h_1} h_{12}\dot{z}^T(\alpha)Z_2\dot{z}(\alpha)\,d\alpha \le -\frac{1}{\alpha_2}\left[\sigma^T\left[G_5^T\ \ G_6^T\right]\tilde{Z}_2\begin{bmatrix} G_5 \\ G_6 \end{bmatrix}\sigma\right] - \frac{1}{1-\alpha_2}\left[\sigma^T\left[G_7\ \ G_8\right]\tilde{Z}_2\begin{bmatrix} G_7 \\ G_8 \end{bmatrix}\sigma\right]. \quad (27)$$

Implementing Lemma 2 on (27) and by defining $\phi_2 = \begin{bmatrix} \tilde{Z}_2 & 0 \\ * & \tilde{Z}_2 \end{bmatrix} > 0$, we obtain

$$-\int_{t-h_2}^{t-h_1} h_{12}\dot{z}^T(\alpha)Z_2\dot{z}(\alpha)\,d\alpha \le -\sigma^T\left[G_5^T\ \ G_6^T\ \ G_7^T\ \ G_8^T\right]\phi_2\begin{bmatrix} G_5 \\ G_6 \\ G_7 \\ G_8 \end{bmatrix}\sigma. \quad (28)$$

Furthermore, define $T_2 = \left[G_5^T\ G_6^T\ G_7^T\ G_8^T\right]^T$ and use it in (28) to obtain

$$-\int_{t-h_2}^{t-h_1} h_{12}\dot{z}^T(\alpha)Z_2\dot{z}(\alpha)\,d\alpha \le -\sigma^T T_2^T\phi_2 T_2\sigma. \quad (29)$$

By substituting (22) and (29) into (14), we obtain the following relationship for Lyapunov:

$$\dot{V}(z,t) \le z^T(t)P\left(\bar{A}z(t)+\bar{A}_1 z(t-\tau(t))+g\left(t,x,\hat{x}\right)+D\right) + \left(z^T(t)\bar{A}^T+z^T(t-\tau(t))\bar{A}_1^T+g^T\left(t,x,\hat{x}\right)+D\right)Pz(t) + \sum_{i=1}^{3} z^T Q_i z - (1-\mu)z^T(t-\tau(t))Q_3 z(t-\tau) - \sum_{i=1}^{2} z^T(t-h_i) \times Q_i z(t-h_i) + \left[\bar{A}z(t)+\bar{A}_1 z(t-\tau(t))+g\left(t,x,\hat{x}\right)+D\right]^T$$

$$\times\left[h_1^2 Z_1+h_{12}^2 Z_2\right]\left[\bar{A}z(t)+\bar{A}_1 z(t-\tau(t))+g\left(t,x,\hat{x}\right)+D\right]$$
$$-\sigma^T T_1^T\phi_1 T_1\sigma-\sigma^T T_2^T\phi_2 T_2\sigma.$$

Using σ defined in (16), the time-varying terms are separated to obtain (30), as shown at the bottom of the page. From the generalized OSL condition stated in Assumption 1, with the scalars assumed as $\varepsilon_1 > 0$ and $\varepsilon_3 > 0$ resulted into

$$z^T(t)\, diag\{\rho\varepsilon_1 U,\rho\varepsilon_3 V\}\, z(t) -z^T(t)\, diag\{\varepsilon_1 I,\varepsilon_3 I\}\, g\left(t,x,\hat{x}\right) > 0.$$

The above inequality results in the following relation using (16), and can be written as (31), shown at the bottom of the page.

Integrating (31) in (30), renders (31(a)), as shown in the equation at the bottom of next page.

Incorporating the inequality for L_2 gain reduction, the Lyapunov functional will be revised as follows:

$$J(z,t)=\dot{V}(z,t)+\gamma^{-1}z^T z-\gamma D^T D<0,$$
$$=\int_0^t \dot{V}(z,t)dt+\int_0^t \gamma^{-1}z^T z dt-\int_0^t \gamma D^T D dt<0,$$
$$=V(z,t)-V(z,0)+\int_0^t \gamma^{-1}z^T z dt-\int_0^t \gamma D^T D dt<0,$$
$$\sqrt{z^T z dt}=\gamma\sqrt{\int_0^t D^T D dt},\ \|z\|_2<\gamma\,\|D\|_2\,. \quad (32)$$

Using Eq. (31) and Eq. (32) we can write $J(z,t)$, as shown in the equation (31(b)) at the bottom of the next page.

By applying Schur's complement to (32), the form of (10) is obtained. This completes the proof of Theorem 1:

Remark 1: The feedback control of Lipschitz and OSL nonlinear systems has been debated for the past decade [8], [9], [10], [11], [12]. The Lipchitz condition offers less generic stability, as it encapsulates a small class of nonlinear systems. For OSL, nonlinearity integrates the QIB inequality to provide a feasible solution. To incorporate the lower complexity offered by QIB and broaden the scope, a generalized one-sided Lipchitz condition is assumed in this study. This condition can be redefined as a special case of nonlinear Lipschitz and one-sided nonlinear Lipschitz systems. Additionally, a generalized OSL constant may acquire more features and scalar selection through Monte Carlo simulations, as in [9], to ensure the stability of the error dynamics for various systems.

Remark 2: The scheme proposed in Theorem 1 involves a robust delay range-dependent stability criterion for solving the feedback stability problem. Previous approaches offer the stability condition of the system without considering the impact of delayed terms, as proposed in [22], [23], [24], and [25]. This study involves a more generic delay-range-dependent approach with nonzero upper and lower bounds. This aids in its effective application to real-time problems.

Remark 3: The estimation-based delay-range-dependent approach has been studied previously [28]. Jensen's inequality was involved in treating complex integral terms that offered a certain conservatism. To eradicate the problem

$$\dot{V}(z)\le\sigma^T\begin{bmatrix} \bar{A}^T P+PA+\sum_{i=1}^{3}Q_i & P\bar{A}_1 & 0 & 0 & 0 & 0 & 0 & P & P\\ * & -(1-\mu)Q_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & 0 & 0 & 0\\ * & * & * & * & * & * & * & 0 & 0\\ * & * & * & * & * & * & * & * & 0 \end{bmatrix}\sigma$$
$$+\sigma^T[\bar{A}\quad \bar{A}_1\quad 0\quad 0\quad 0\quad 0\quad 0\quad I\quad I]^T[h_1^2 Z_1+h_{12}^2 Z_2][\bar{A}\quad \bar{A}_1\quad 0\quad 0\quad 0\quad 0\quad 0\quad I\quad I]\sigma$$
$$-\sigma^T T_1^T\phi_1 T_1\sigma-\sigma^T T_2^T\phi_2 T_2\sigma. \quad (30)$$

$$\sigma^T(t)\begin{bmatrix} diag\begin{Bmatrix}\rho\varepsilon_1 U\\ -\rho\varepsilon_3 V\end{Bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}diag\begin{Bmatrix}\varepsilon_1 U\\ -\varepsilon_3 V\end{Bmatrix} & 0\\ * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & 0 & 0 & 0\\ * & * & * & * & * & * & * & 0 & 0\\ * & * & * & * & * & * & * & * & 0 \end{bmatrix}\sigma(t)>0 \quad (31)$$

and enhance stability, the present study proposed in Theorem 1 involves Wirtinger's inequality, which extends to traditional Jensen's inequality but offers less conservatism. To the best of our knowledge, this is the first study that covers a robust delay-range-dependent approach for the estimation-based control of a generalized OSL nonlinear system with Wirtinger's inequality.

Theorem 1 ensures the stability of the robust estimation-based controller scheme. The observer and controller gains are identified by solving the matrix inequalities for the dynamics mentioned in (1). However, the observer and controller are not retrieved simultaneously from Theorem 1 matrix inequalities. Hence, the decoupling approach is used in the forthcoming theorem to compute both gains simultaneously and independently using simulation tools.

Theorem 2: For the existence of a solution to the constraints provided in Theorem 1, the following sufficient and necessary condition is provided by the matrix inequalities, (33) and (34), as shown at the bottom of the next page, where,

$$\psi_{1,1} = \bar{P}_{1c}A^T + A\bar{P}_{1c} + B\bar{K}_c + \bar{K}_c^T B^T + \bar{P}_{1c}(\rho\varepsilon_2 V)\bar{P}_{1c} + \sum_{i=1}^{3}\Omega_i,$$

$$\psi_{1,8} = I - \frac{1}{2}\varepsilon_1 U\bar{P}_{1c},$$

$$\psi_{2,2} = -(1-\mu)\Omega_3,$$

$$\psi_{1,9} = I,$$

$$\psi_{1,10} = h_1\left[k_c^T B^T + \bar{P}_{1c}A^T\right],$$

$$\psi_{1,11} = h_{12}\left[k_c^T B^T + \bar{P}_{1c}A^T\right],$$

$$\Omega_1 = \bar{P}_{1c}Q_{1c}\bar{P}_{1c},$$

$$\Omega_2 = \bar{P}_{1c}Q_{2c}\bar{P}_{1c},$$

$$\Omega_3 = \bar{P}_{1c}Q_{3c}\bar{P}_{1c},$$

$$\phi_{1C} = \begin{bmatrix} \Phi_1 & 0 & 0 & 0 \\ 0 & 3\Phi_1 & 0 & 0 \\ 0 & 0 & \Phi_1 & 0 \\ 0 & 0 & 0 & 3\Phi_1 \end{bmatrix},$$

$$\phi_{1C} = \begin{bmatrix} \Phi_2 & 0 & 0 & 0 \\ 0 & 3\Phi_2 & 0 & 0 \\ 0 & 0 & \Phi_2 & 0 \\ 0 & 0 & 0 & 3\Phi_2 \end{bmatrix},$$

$$\Phi_1 = \bar{P}_{1c}Z_{1c}\bar{P}_{1c},\ \Phi_2 = \bar{P}_{1c}Z_{2c}\bar{P}_{1c},\ \Xi_1 = \bar{P}_{1c}\bar{\Phi}_1\bar{P}_{1c},$$

$$\Xi_2 = \bar{P}_{1c}\bar{\Phi}_2\bar{P}_{1c},\ \lambda_{1,1} = A^T P_{1o} + P_{1o}A + \rho\varepsilon_4 V + \sum_{i=1}^{3} Q_{io},$$

$$\lambda_{1,2} = -\bar{K}_o C,\ \lambda_{1,8} = P_{1o} + \frac{1}{2}(\alpha\varepsilon_4 U),$$

$$\lambda_{1,9} = P_{1o},\ \lambda_{1,10} = h_1 A^T P_{1o},$$

$$\lambda_{1,11} = h_{12}A^T P_{1o},\ \lambda_{2,10} = -h_1 C^T \bar{K}_o^T,$$

$$\lambda_{2,11} = -h_{12}C^T\bar{K}_o^T,\ \bar{K}_o = P_{1o}K_o,$$

$$\zeta_1 = P_{1o}Z_{1o}^{-1}P_{1o},\ \zeta_2 = P_{1o}Z_{2o}^{-1}P_{1o},$$

$$\phi_{1o} = \begin{bmatrix} Z_{1o} & 0 & 0 & 0 \\ 0 & 3Z_{1o} & 0 & 0 \\ 0 & 0 & Z_{1o} & 0 \\ 0 & 0 & 0 & 3Z_{1o} \end{bmatrix},$$

$$\dot{V}(z,t) \le \sigma^T \begin{bmatrix} \omega_{1,1} & P\bar{A}_1 & 0 & 0 & 0 & 0 & 0 & \omega_{1,8} & P \\ * & \omega_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & * & 0 \end{bmatrix} \sigma + \sigma^T \begin{bmatrix} \bar{A} & \bar{A}_1 & 0 & 0 & 0 & 0 & 0 & I & I \end{bmatrix}^T$$
$$\times \left[h_1^2 Z_1 + h_{12}^2 Z_2\right] \begin{bmatrix} \bar{A} & \bar{A}_1 & 0 & 0 & 0 & 0 & 0 & I & I \end{bmatrix} \sigma - \sigma^T T_1^T \phi_1 T_1 \sigma - \sigma^T T_2^T \phi_2 T_2 \sigma. \quad (31a)$$

$$J(z,t) \le \sigma_d^T \begin{bmatrix} W & P\bar{A}_1 & 0 & 0 & 0 & 0 & 0 & \omega_{1,8} & P \\ * & \omega_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -Q_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & * & * & -\gamma I \end{bmatrix} \times \sigma_d + \sigma_d^T \begin{bmatrix} \bar{A} & \bar{A}_1 & 0 & 0 & 0 & 0 & 0 & I & I \end{bmatrix}$$
$$\times (h_1^2 Z_1 + h_{12}^2 Z_2) \times \begin{bmatrix} \bar{A} \ \bar{A}_1 \ 0\ 0\ 0\ 0\ 0\ I\ I \end{bmatrix} \sigma_d - \sigma_d^T T_3^T \phi_1 T_3 \sigma_d - \sigma_d^T T_4^T \phi_2 T_4 \sigma_d, \quad (31b)$$

$$\phi_{2o}=\begin{bmatrix} Z_{2o} & 0 & 0 & 0\\ 0 & 3Z_{2o} & 0 & 0\\ 0 & 0 & Z_{2o} & 0\\ 0 & 0 & 0 & 3Z_{2o}\end{bmatrix},$$

are satisfied for the real matrices $P_{1c}^{-1}=\bar{P}_{1c}>0$, and $P_{1o}^{-1}=\bar{P}_{1o}>0$. The controller gain is given by $K_c=P_{1c}\bar{K}_c$, and the observer gain is given by $K_o=\bar{P}_{1o}\bar{K}_o$.

Proof: Sufficiency: Applying congruence transformations with $diag\{P_{1c},P_{1c},P_{1c},P_{1c},P_{1c},P_{1c},P_{1c},I,I,I,I\}$ and $diag\{I,I,I,I,I,I,I,\bar{P}_{1o},\bar{P}_{1o},\bar{P}_{1o}\}$ to inequalities (33) and (34), respectively, results in (35), as shown at the bottom of the next page, where

$$\tilde{\psi}_{1,1}=A^TP_{1c}+P_{1c}A+P_{1c}\bar{K}_c+\bar{K}_c^TP_{1c}+(\rho\varepsilon_2U)+\sum_{i=1}^{3}Q_{i,1},$$
$$\tilde{\psi}_{2,2}=-(1-\mu)Q_{3c},$$
$$\tilde{\psi}_{1,8}=P_{1c}-\frac{1}{2}\varepsilon_2UI,$$
$$\tilde{\psi}_{1,9}=P_{1c},$$
$$\tilde{\psi}_{1,10}=h_1\left[k_c^TB^T+A^T\right],$$
$$\tilde{\psi}_{1,11}=h_{12}\left[k_c^TB^T+A^T\right],$$

$$\tilde{\phi}_{1C}=\begin{bmatrix} Z_1 & 0 & 0 & 0\\ 0 & 3Z_1 & 0 & 0\\ 0 & 0 & Z_1 & 0\\ 0 & 0 & 0 & 3Z_1\end{bmatrix},$$
$$\tilde{\phi}_{2C}=\begin{bmatrix} Z_2 & 0 & 0 & 0\\ 0 & 3Z_2 & 0 & 0\\ 0 & 0 & Z_2 & 0\\ 0 & 0 & 0 & 3Z_2\end{bmatrix}$$

and (36), as shown at the bottom of the next page, where

$$\tilde{\lambda}_{1,9}=I,\ \tilde{\lambda}_{1,10}=h_1A^T,\ \tilde{\lambda}_{1,11}=h_{12}A^T,$$
$$\tilde{\lambda}_{2,10}=-h_1C^TK_o^T,\ \tilde{\lambda}_{2,11}=-h_{12}C^TK_o^T.$$

Furthermore assigning

$$\Gamma_3=\begin{bmatrix} -BK_cP_{1c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \quad (37)$$

$$\Gamma_1=\left\{\begin{bmatrix} \psi_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{1,8} & \psi_{1,9} & \psi_{1,10} & \psi_{1,11}\\ * & \psi_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & -\Omega_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -\Omega_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & * & 0 & 0 & h_1I & h_{12}I\\ * & * & * & * & * & * & * & * & -\gamma I & 0 & 0\\ * & * & * & * & * & * & * & * & * & -h_1\Xi_1 & 0\\ * & * & * & * & * & * & * & * & * & * & -h_{12}\Xi_2\end{bmatrix}\right\}-T_1^T\phi_{1c}T_1-T_2^T\phi_{2c}T_2<0 \quad (33)$$

$$\Gamma_2=\left\{\begin{bmatrix} \lambda_{1,1} & \lambda_{1,2} & 0 & 0 & 0 & 0 & 0 & \lambda_{1,8} & \lambda_{1,9} & \lambda_{1,10} & \lambda_{1,11}\\ * & \lambda_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{2,10} & \lambda_{2,11}\\ * & * & -Q_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -Q_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & * & 0 & 0 & h_1P_{1o} & h_{12}P_{1o}\\ * & * & * & * & * & * & * & * & -\gamma I & 0 & 0\\ * & * & * & * & * & * & * & * & * & -h_1\zeta_1 & 0\\ * & * & * & * & * & * & * & * & * & * & -h_{12}\zeta_2\end{bmatrix}\right\}-T_1^T\phi_{1o}T_1-T_2^T\phi_{2o}T_2<0 \quad (34)$$

and using (36)-(38), we have

$$\begin{bmatrix} \Gamma_1 & \Gamma_3^T \\ * & \varsigma\Gamma_2 \end{bmatrix} < 0, \tag{38}$$

where $\varsigma > 0$ represents a relatively large number of cases. By interchanging the columns and rows in (38) and using (20) and (21), we obtain the form in (9) using $P = diag\{P_{1c}, P_{1o}\}$.:

Necessity: Assume a positive-definite symmetric matrix P such that it fulfills the conditions provided in Theorem 1 and a matrix P is defined as follows:

$$P = \begin{bmatrix} \widehat{P}_{1c} & \varpi \\ * & \varpi \end{bmatrix} \text{ and } P = \begin{bmatrix} \varpi & \varpi \\ * & \widehat{P}_{1o} \end{bmatrix},$$

where ϖ is any entry having no impact overall on mathematics. Integrating the relation P for both controller and observer matrix inequalities in (9), we have (39) and (40), as shown at the bottom of the next page.

Multiplying the following inequalities with (39) and (40) as per the congruence transformation rules, as shown in the equation at the bottom of page 12.

Further, employing $\hat{P}_{1c}^{-1} = \bar{P}_{1c}$ and $\hat{P}_{1o} = P_{1o}$,, the forms of inequalities (33) and (34) are obtained.

Remark 4: The presented results of Theorem 2 provide sophisticated circumstances for identifying the observer gain K_o and controller gain K_c for the system in (8). Decoupling schemes are explored in literature for various augmented matrices for linear and nonlinear systems as provided in ([26], [29] and references therein). The constraints in Theorem 2 ensure less complex computations compared to Theorem 1 and ensure simultaneous and independent constraints to compute the gains for the controller and observer using the simulation tools.

Remark 5: The results of Theorem 2 offer more generic, sufficient, and necessary conditions for solving the matrix inequality for the robust estimation-based design in Theorem 1. The decoupled methodology provides less-tedious conditions for attaining observer and controller gains for a robust design involving measurement delays. The resultant matrices obtained in Theorem 2 are more convenient for providing a solution because the bilinear matrix inequality terms in Γ_3 [29] are avoided.

A solution to Theorem 2 that treats the bilinear terms and provides a convenient solution is required. This is provided in Theorem 3 with the most probable cases for the nonlinear terms.

Theorem 3: For the solution of inequalities in Theorem 2 in (21), symmetric matrices $P > 0$, $Q_{1,1} > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$ and $\bar{K}_1$ of proper dimension and anyone of the following result exists.

(i) If $\rho\varepsilon_2 V + \bar{\gamma} > 0$, (41), as shown at the bottom of page 12, where $\hat{\psi}_{1,1} = \bar{P}_{1c}A^T + A\bar{P}_{1c} + B\bar{K}_1 + \bar{K}_1^T B^T + \sum_{i=1}^{3} \Omega_i$.

$$\Gamma_1 = \begin{bmatrix} \tilde{\psi}_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\psi}_{1,8} & \tilde{\psi}_{1,9} & \tilde{\psi}_{1,10} & \tilde{\psi}_{1,11} \\ * & \tilde{\psi}_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & Q_{1c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & Q_{2c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\gamma I & h_1 I & h_{12} I \\ * & * & * & * & * & * & * & * & * & -h_1 Z_{1c}^{-1} & 0 \\ * & * & * & * & * & * & * & * & * & * & -h_{12} Z_{2c}^{-1} \end{bmatrix} - \sigma^T T_1^T \tilde{\phi}_{1c} T_1 \sigma - \sigma^T T_2^T \tilde{\phi}_{2c} T_2 \sigma < 0, \tag{35}$$

$$\Gamma_2 = \begin{bmatrix} \lambda_{1,1} & \lambda_{1,2} & 0 & 0 & 0 & 0 & 0 & \lambda_{1,8} & \tilde{\lambda}_{1,9} & \tilde{\lambda}_{1,10} & \tilde{\lambda}_{1,11} \\ * & \lambda_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\lambda}_{2,10} & \tilde{\lambda}_{2,11} \\ * & * & -Q_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & 0 & h_1 I & h_{12} I \\ * & * & * & * & * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -h_1 Z_{1o}^{-1} & 0 \\ * & * & * & * & * & * & * & * & * & * & -h_{12} Z_{2o}^{-1} \end{bmatrix} - \sigma^T T_1^T \phi_{1o} T_1 \sigma$$
$$- \sigma^T T_2^T \phi_{2o} T_2 \sigma < 0 \tag{36}$$

(ii) If $\rho\varepsilon_2 V + \bar{\gamma} = 0$, (42), as shown at the bottom of the next page, where $\hat{\psi}_{1,1} = \bar{P}_{1c}A^T + A\bar{P}_{1c} + B\bar{K}_1 + \bar{K}_1^T B^T + \sum_{i=1}^{3} \Omega_i$.

(iii) If $\rho\varepsilon_2 V + \bar{\gamma} < 0$, (43), as shown at the bottom of page 13, where $\psi_{1,1} = \bar{P}_{1c}A^T + A\bar{P}_{1c} + B\bar{K}_1 + \bar{K}_1^T B^T + \sum_{i=1}^{3} \Omega_i - J_1$ and $J = -P_{1c}(\rho\varepsilon_2 V + \bar{\gamma})P_{1c}$.

$$
\left[\begin{array}{cccccccccccccccccccccc}
\omega_{1,1} & \varpi & \hat{P}_{1c}A_1 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & \omega_{1,8} & \varpi & \hat{P}_{1c} & \varpi & h_1A^T & \varpi & h_{12}A^T & \varpi \\
* & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & \omega_{2,2} & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & h_1A_1^T & \varpi & h_{12}A_1^T & \varpi \\
* & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & -Q_1 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi \\
* & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & -Q_2 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi \\
* & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi \\
* & * & * & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & * & * & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & 0 & \varpi & 0 & \varpi & h_1 & \varpi & h_{12} & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & -\gamma I & \varpi & 0 & \varpi & 0 & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & -h_1Z_{1c}^{-1} & \varpi & 0 & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & \varpi & \varpi & \varpi \\
* & -h_{12}Z_{2c}^{-1} & \varpi \\
* & \varpi
\end{array}\right]
- T_1^T\phi_1T_1 - T_2^T\phi_2T_2 < 0 \tag{39}
$$

$$
\left[\begin{array}{cccccccccccccccccccccc}
\varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & \hat{\omega}_{1,1} & \varpi & \hat{P}_{1o}A_1 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & \hat{\omega}_{1,8} & \varpi & \hat{P}_{1o} & \varpi & h_1A^T & \varpi & h_{12}A^T \\
* & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & \hat{\omega}_{2,2} & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & h_1A_1^T & \varpi & h_{12}A_1^T \\
* & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & -\hat{Q}_1 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 \\
* & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & -\hat{Q}_2 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 \\
* & * & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & * & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 \\
* & * & * & * & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & * & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 & \varpi & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & 0 & \varpi & 0 & \varpi & -h_1 & \varpi & -h_{12} \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & -\gamma I & \varpi & 0 & \varpi & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & \varpi & \varpi & \varpi & \varpi \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & -h_1\hat{Z}_{1o}^{-1} & \varpi & 0 \\
* & \varpi & \varpi \\
* & -h_{12}\hat{Z}_{2o}^{-1}
\end{array}\right]
- T_1^T\phi_1T_1 - T_2^T\phi_2T_2 < 0. \tag{40}
$$

Proof: For the case of $\rho\varepsilon_2 V + \bar{\gamma} > 0$, by applying Shur's compliment on the inequality in (33), we obtain (41); thus, both these inequalities are equivalent to $|\rho\varepsilon_2 V + \bar{\gamma}| \geq \rho\varepsilon_2 V + \bar{\gamma}$. Therefore, if (41) is satisfied, then (33) is also satisfied. In the case of $\rho\varepsilon_2 V = 0$, the inequality in (33) is reduced to (42) and both are equivalent. By removing the last row and column of (41), (42) is obtained. In the case that $\rho\varepsilon_2 V < 0$,(43) is obtained by setting $J = -P_{1c}(\rho\varepsilon_2 V +$

$$
\begin{bmatrix}
\hat{P}_{1c}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \hat{P}_{1c}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \hat{P}_{1c}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \hat{P}_{1c}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{P}_{1c}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{P}_{1c}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{P}_{1c}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0
\end{bmatrix}
$$

$$
\begin{bmatrix}
0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{P}_{1o} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{P}_{1o}
\end{bmatrix}
$$

$$
\Gamma_1 = \begin{bmatrix}
\hat{\psi}_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{1,8} & \psi_{1,9} & \psi_{1,10} & \bar{P}_{1c}\sqrt{|\rho\varepsilon_2 V + \bar{\gamma}|} \\
* & \psi_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -\Omega_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\Omega_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & 0 & h_1 I & h_{12} I & 0 \\
* & * & * & * & * & * & * & * & -h_1\Xi_1 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -h_{12}\Xi_2 & 0 \\
* & * & * & * & * & * & * & * & * & * & -V
\end{bmatrix} - \sigma^T T_1^T \phi_{1c} T_1 \sigma
$$
$$
- \sigma^T T_2^T \phi_{2c} T_2 \sigma < 0 \tag{41}
$$

$$
\Gamma_1 = \begin{bmatrix}
\hat{\psi}_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{1,8} & \psi_{1,9} & \psi_{1,10} \\
* & \psi_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & -\Omega_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\Omega_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & 0 & h_1 I & h_{12} I \\
* & * & * & * & * & * & * & * & -h_1\Xi_1 & 0 \\
* & * & * & * & * & * & * & * & * & -h_{12}\Xi_2
\end{bmatrix} - T_1^T \phi_{1c} T_1 - T_2^T \phi_{2c} T_2 < 0 \tag{42}
$$

$\bar{\gamma})P_{1c}$ to (33). This implies that if $\rho\varepsilon_2 V + \bar{\gamma} < |\rho\varepsilon_2 V + \bar{\gamma}|$, which also results in $-J < P_1 |\rho\varepsilon_2 V + \bar{\gamma}| P_1$ as a result if the inequality in (41) is satisfied, then the inequality in (43) is also satisfied.

The resulting nonlinear terms in the matrix inequalities for Theorems 2 and 3 require treatment using the CCL algorithm (as provided in [8], [26], [28]) for which the controller gain K_c matrix is rendered by optimizing

$$\min Trace \times \begin{pmatrix} P_{1c}\bar{P}_{1c} + J\bar{J} + \bar{P}_{1c}\theta\bar{P}_{1c}\bar{J} \\ + \sum_{i=1}^{2} \Phi_i\bar{\Phi}_i + \Xi\bar{\Xi}_i + \bar{P}_{1c}Z_{io}\bar{P}_{1c}\bar{\Phi}_i + \bar{P}_{1c}\bar{\Phi}_i\bar{P}_{1c}\bar{\Xi}_i \end{pmatrix}, \tag{44}$$

subject to

$$\begin{bmatrix} P_{1c} & I \\ I & \bar{P}_{1c} \end{bmatrix} \geq 0, \begin{bmatrix} J & I \\ I & \bar{J} \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} \theta & P_{1c}\bar{J} \\ P_{1c} & \bar{T} \end{bmatrix} \geq 0, \begin{bmatrix} \Xi_k & I \\ I & \bar{\Xi}_k \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} \Phi_k & I \\ I & \bar{\Phi}_k \end{bmatrix} \geq 0, \begin{bmatrix} Z_{kc} & P_{1c} \\ P_{1c} & \Phi_k \end{bmatrix} \geq 0, \begin{bmatrix} \Phi_k & P_{1c} \\ P_{1c} & \Xi_k \end{bmatrix} \geq 0, \tag{45}$$

for $k = 1, 2$, where $\theta = \rho\varepsilon_1 + \beta\varepsilon_2$, $J = \bar{P}_{1c}\theta\bar{P}_{1c}$, $\Phi_i = \bar{P}_{1c}Z_{io}\bar{P}_{1c}$ and $\Xi_i = \bar{P}_{1c}\bar{\Phi}_i\bar{P}_{1c}$.

Similarly, for the observer gain matrix, the optimization of

$$\min Trace\left(P_{1o}\bar{P}_{1o} + \sum_{i=1}^{2} S_i\bar{S}_i + Z_{io}\bar{Z}_{io} + P_{1o}\bar{Z}_{io}P_{1o}\bar{S}_i\right) \tag{46}$$

subject to

$$\begin{bmatrix} P_{1o} & I \\ I & \bar{P}_{1o} \end{bmatrix} \geq 0, \begin{bmatrix} S_k & I \\ I & S_K \end{bmatrix} \geq 0, \begin{bmatrix} \bar{Z}_{ko} & \bar{P}_{1o} \\ \bar{P}_{1o} & \bar{S}_K \end{bmatrix} \geq 0,$$

for $k = 1, 2$, where $S_i = P_{1o}\bar{Z}_{io}P_{1o}$, is required to be performed.

Remark 6: This Theorem offers less conservative results than Theorem 1 and provides a solution for nonlinear constraints using various simulation tools. For this purpose, multiple nonlinear terms were incorporated to obtain the results. This approach provides less conservatism, as it may acquire positive, negative, or zero values, in contrast to the approach of taking an upper bound in [8]. The linearization algorithm (as proposed in [26], [28]) treats the aforementioned nonlinear terms.

IV. SIMULATION RESULTS

In this section, we describe the effectiveness of the proposed method. Assume a system with the state equations provided in (1) and constant matrices and nonlinearities provided as follows:

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T \text{ and}$$
$$f(t, x) = -(x_1^2 + x_2^2)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{47}$$

respectively. The system in (46) identifies the motion of any object. For instance, a robot whose motion can be reactionary or progressive is exposed to a measurement delay of t - 0.18 + 0.05 * sin (0.01 * t). The CCL technique is implemented on the results of Theorem 3 along with the constraints mentioned in (44) and (46). The bounds on the delays for the controller and observer were assumed to be $h_1 = 0.5s$ and $h_2 = 0.6s$, $h_1 = 300ms$, and $h_2 = 0.01s$, respectively. The values for μ for the controller and observer are taken as 0.1 and 0.13, respectively. The system is a globally OSL system for $\rho = 0$ and $\rho = -20$, of the controller and observer. The constraints for the generalized one-sided U and V, were adopted from [9] as follows:

$$U = V = \begin{bmatrix} 157.8364 & -10.5208 \\ -10.5208 & 144.1245 \end{bmatrix}.$$

After simulation, the controller and the observer gain matrix are computed as follows:

$$K_c = \begin{bmatrix} 3.5964 & 0 \\ 2.1044 & 0 \end{bmatrix} \text{and} K_o = \begin{bmatrix} \text{-4.0965 -0.1232} \\ \text{-0.1229 -4.2571} \end{bmatrix}, \tag{48}$$

for $\gamma = 40$ and 200, respectively. The system states, estimated states, and estimation results are presented in Figs. 1, 2, and 3, respectively, which depict the tracking of the estimated states to the original states, and the estimation error converges asymptotically in the neighborhood of the origin.

To evaluate the performance of the proposed methodology in the presence of disturbances,

$$d = \begin{bmatrix} 0.2\sin(12t) \\ 0.15\cos(8t) \end{bmatrix}.$$

$$\Gamma_1 = \begin{bmatrix} \psi_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{1,8} & \psi_{1,9} & \psi_{1,10} \\ * & \psi_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\Omega_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\Omega_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & 0 & -h_1 I & -h_{12} I \\ * & * & * & * & * & * & * & * & -h_1\Xi_1 & 0 \\ * & * & * & * & * & * & * & * & * & -h_{12}\Xi_2 \end{bmatrix} - T_1^T\phi_{1c}T_1 - T_2^T\phi_{2c}T_2 < 0, \tag{43}$$

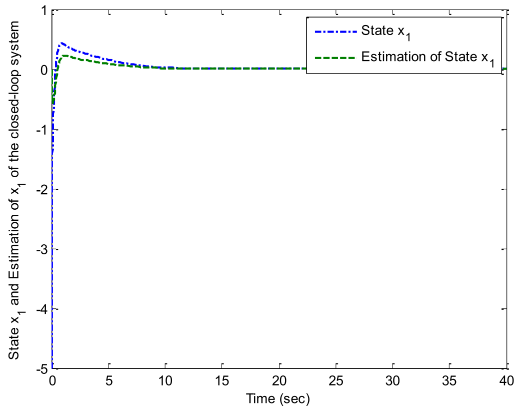

FIGURE 1. Original state x_1 and its estimation error for the closed-loop system in the presence of output delay.

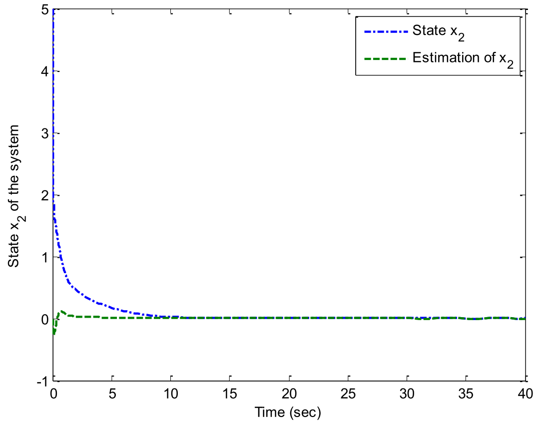

FIGURE 2. Original state x_2 and its estimation error for the closed-loop system in the presence of output delay.

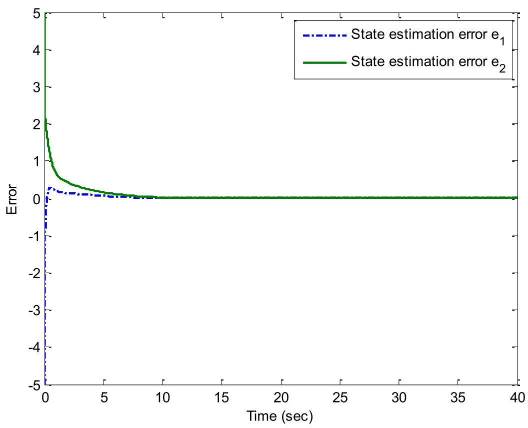

FIGURE 3. Observer error for original states of plant and the required estimated states.

Figures 4, 5, and 6 show the plots of the states and estimation errors for the proposed robust observer-based control in the presence of perturbations. The estimation error converges in the neighborhood or origin, even in the presence of a disturbance.

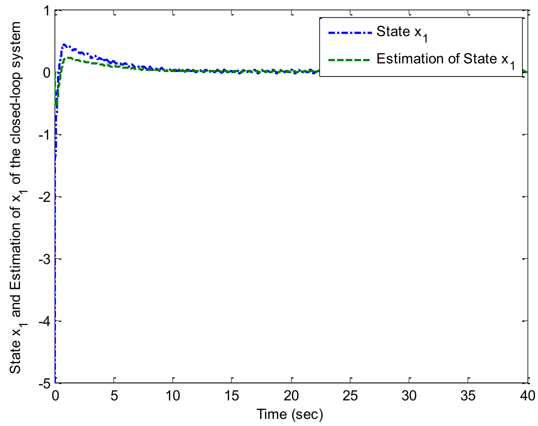

FIGURE 4. Original state x_1 and its estimated state of the closed-loop system in the presence of disturbance and output delay.

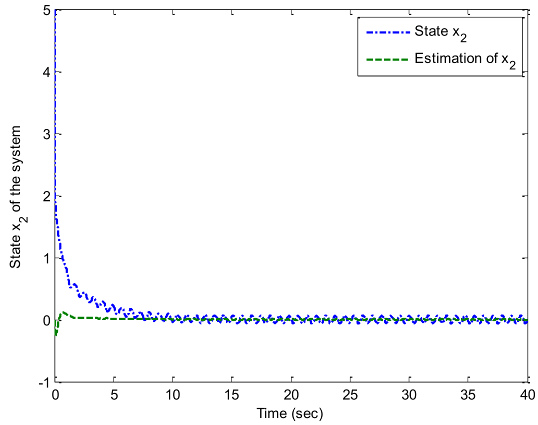

FIGURE 5. Original state x_2 and its estimated state of the closed-loop system in the presence of disturbance and output delay.

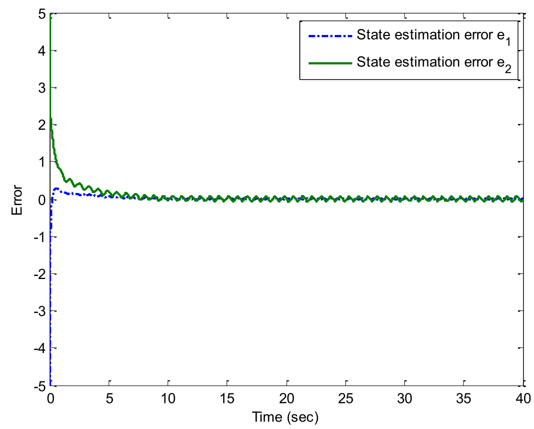

FIGURE 6. Observer error for original states of plant and the required estimated states in the presence of disturbances and delay.

A comparison of the computed upper bounds of delay to ensure stability is summarized in Table. 1 for h_2, for $h_1 = 10$. This shows that the proposed methodology is applicable to broader ranges of delay, in contrast to [8]. Another notable

TABLE 1. Allowable upper limit of h_2, assuming $h_1 = 10$s, $\rho = -2.9$ and $\rho = -20$.

Methods	Estimation technique in [8]	Proposed methodology
h_2 (for observer)	552 s	$\approx 10^{10}$ s
h_2 (for controller)	572 s	$\approx 10^{11}$ s

aspect of the proposed technique is that it can be employed for a range of time delays, even with $h_1 \neq 0$.

V. CONCLUSION

This study explores a robust observation-based control scheme in the presence of output time-delayed dynamics by assuming generalized OSL nonlinearity. The estimation technique for GOSL nonlinear systems under measurement delays and external disturbances was exploited using the LK functional, Wirtinger's inequality, LMIs, decoupling, and CCL methods. The proposed controller was based on an observer that exhibits asymptotic convergence and is robust against external disturbances. The LK stability method was deployed, the derivative of which is exploited by considering Wirtinger's inequality, which resulted in nonlinear inequalities being converted into LMIs to guarantee stability. Disturbance rejection was achieved through the concept of L2-gain. A decoupling mechanism was applied to the LMI-based results deduced from the derivative of the Lyapunov functional to extricate the observer and controller gains. A sufficient and necessary condition for a robust estimation-based controller for generalized OSL nonlinear systems in the presence of a measurement delay was presented. A simulation result assuming moving ball practical systems is provided to demonstrate that the proposed approach is applicable to a wider class of electrical and electronic systems extended to delayed dynamics.

ACKNOWLEDGMENT

(Muhammad Sohaib Mirza and Amad Zafar are co-first authors.)

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