Received 13 December 2023, accepted 19 December 2023, date of publication 25 December 2023, date of current version 28 December 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3346676

RESEARCH ARTICLE

Robust Control Strategy for Dual-Channel Supply Chain With Free Riding Behavior and Cross-Channel Return

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This work was supported in part by the Natural Science Foundation of Gansu Province under Grant 20JR10RA265 and Grant 20JR10RA269, in part by the Youth Natural Science Foundation of Lanzhou Jiaotong University under Grant 2020043, in part by the Tianyou Youth Talent Lift Program of Lanzhou Jiaotong University, and in part by the Higher Education Innovation Foundation of Gansu Province under Grant 2021B-112.

ABSTRACT In order to obtain customer satisfaction and promote the overall revenue of the supply chain system, offline purchase and online return strategy, called a new kind of cross channel return strategy (hereinafter referred to as ''cross channel return''), has been gradually adopted by more and more enterprises in China. Moreover, free riding behavior is always inevitable in supply chain system. When a supply chain system includes free riding behavior and cross channel return strategy, the system is complex so that its stability can't be ensured easily. Hence, the management of the complex supply chain system mentioned above is a major challenge. In this article, the inventory and profit control strategy for a dual channel supply chain (DCSC) with cross channel return and free riding behavior is developed. Firstly, the mathematical model of the supply chain is set up based on the discrete state space equation. Secondly, utilizing the Lyapunov stability theory and the linear matrix inequality (LMI) method, a robust H_∞ control strategy is designed to suppress the bullwhip effect caused by customer random demand. Thirdly, the impact of consumer channel preference, cross channel return and free riding behavior on inventory, production/ordering and profit in the supply chain system is studied by analyzing four different models, such as N-FC(without free riding behavior or cross channel return), Y-C(with cross channel return only), Y-F(with free riding behavior only), Y-FC(with both free riding behavior and cross channel return). Finally, the simulation tests are carried out to verify the performance of the robust control strategy, and some management insights and conclusions are summarized.

INDEX TERMS Dual-channel supply chain, robust H_∞ control, free riding behavior, offline purchase, online return strategy, bullwhip effect.

I. INTRODUCTION

In recent years, with the rapid development of e-commerce, online shopping has been favored by more and more people. Therefore, more and more manufacturers have been starting to sell products to consumers through traditional retail channels and online channels [1]. Investigation result shows that, in addition to the traditional retail channel, above 50% of manufacturers in China, for instance Lenovo, Xiaomi, and Gree, have established an online retail channel to participate in the market directly [2]. Moreover, many enterprises, like HP, Nike, and others, have been offered direct online purchases channels resemble internet-channels depend on the development of apps on the internet and the numerous network users all over the world [3]. Obviously, when the two channels sell homogeneous products to common consumers, the manufacturer's online channel inevitably faces competition and conflict of interest with the traditional retail channel [4]. Meanwhile, product sale has been gradually showing new characteristics between the two channels, such

The associate editor coordinating the review of this manuscript and approving it for publication was Jinquan Xu.

as cross channel return [5], free riding behavior [6], which have a significant impact on the operation of supply chain.

Among the existing related literatures, it is obviously noted that the vast majority of scholars' researches have focus on the cross-channel return behavior referring to online purchase and offline return [7]. However, due to the emergence of freight insurance and door-to-door pickup services, consumers can complete returns at nearby express delivery outlets or even without leaving their homes [8]. Therefore, online purchase and offline return behavior is gradually not being adopted by consumers. In addition, online purchase and offline return behavior will add additional costs to physical retail stores, such as inventory costs, return costs, etc., which is also not favored by retailers. Offline purchase and online return are a new type of cross channel return strategy. For example, starting from March 15, 2014, Sichuan Province in China officially implemented a 7-day no reason return strategy for offline shopping in the clothing enterprise (such as Raidy Boer and Huihui Hantang), which includes offline purchases and online return strategy. By implementing this strategy, the urgent needs of customers have been met, customer satisfaction has been improved, brand trust has been enhanced, and the overall sales of online and offline channels has been increased, ultimately. Therefore, a win-win situation for manufacturers and retailers has been achieving. Offline purchase and online return strategy is gradually being widely implemented in other provinces of China, such as Guangxi, Jiangsu, Gansu. Therefore, offline purchase and online return will become an important choice in future retail strategies.

Free riding behavior, as a derivative of online channel sales, is inevitable in supply chain system. Consumers always go to stores where they can obtain more product information first, and then purchase products through online channel due to lower prices [9]. This kind of consumption behavior is referred to as free riding behavior in the academic community. In the process of free riding behavior, online channels that do not make promotional efforts indirectly enjoy the increase in sales brought about by retailers' promotional efforts. With the degree of free riding behavior increases, it will inevitably weaken retailers' promotional efforts. Ultimately, it is not conducive to dual channel product sales.

When the supply chain system includes free riding behavior and cross channel return strategy, the system is complex so that its stability can't be ensured easily. Therefore, managing the complex supply chain system mentioned above is a major challenge which has motivated us to conduct the following research.

In summary, the main objective of this study is to address the following research questions:

1) How to guarantee the stability of the supply chain, in which cross channel return, customers' uncertain random demand and free riding behavior is involved?

2) What impact will the two strategies have on inventory and profit of supply chain enterprises?

In order to address the questions mentioned above, motivated by [10] and [11], we established the mathematical model of the supply chain in form of the discrete state space equation, used the robust control method to handle the bullwhip effect caused by customer random demand and analyzed the impact on the inventory and profit of the supply chain when the four strategies adopted.

The main contributions of this article can be summarized as follows:

1) By utilizing the discrete Lyapunov stability theory and LMI method, a robust H_∞ control strategy is adopted to suppress the bullwhip effect caused by customer random demand.

2) The dynamic evolution processes of inventory, production/ordering, and profit are discussed for the supply chain system, in which a new kind of cross channel return strategy (offline purchase and online return) and free riding behavior are considered.

3) A robust state feedback controller is designed to dynamically capture the changing trends of inventory and profit for manufacturer and retailer with the change of system parameters, such as consumer channel preference rate, degree of free riding and cross channel rate.

This article is organized as follows: related literatures are listed and analyzed in section II. In Section III, a novel dynamic uncertain dual-channel supply chain is introduced and its math model is set up. In Section IV, a robust H_∞ control method is designed to suppress the bullwhip effect and give the control strategy for the supply chain system. Three simulation tests are carried out in Section V to demonstrate the performance of the robust control method. The conclusion and management insights are summarized in Section VI.

II. LITERATURE REVIEW

Related literature review for research focus related to supply chain is listed and analyzed in this section. We mainly study three major aspects related to our research.

1) The management strategy by utilizing robust control method

Robust control theory is a new and effective research method which has been applied to DCSC in resent years.

In [12], authors develop an optimal control issue for a remanufacturing system with random customer's demand, in which some scenarios under various conditions is considered. The robust H_∞ control approach is proposed to deal with the issues of dynamic closed-loop supply chain with remanufacture, a third party reverse logistic providers (3PRLP), bullwhip effect and parameter uncertainty in [13]. In [10], a robust H_∞ fuzzy control method is designed to address an uncertain supply chain with customer's random demand, which is modeled as a discrete state space equation. Then, the simulation analysis is conducted to verify the availability of the mentioned method. The disturbance parameters, request of stability, and the performance characteristics are identified qualitatively by confirming their impact on the formulation using a robust optimal method in [14]. Authors in [15] studied the parameter uncertainty and lead time of a closed-loop supply chain by using a fuzzy robust control (FRC) approach. This strategy can be ameliorated and extended to suppress parameter uncertainty, ensure the stability and provide management recommendations for the supply chain. In [16], a multiagent-based supply chain with customer's random demands, which is described as a discrete switching model, is addressed by using a robust H_∞ control method. A novel financing method for a supply chain system with parameter uncertainty is proposed by utilizing fuzzy robust control method in [17]. In addition, many scholars also pay close attention to the robust control method for supply chain system [18], [19].

Analyzed the above literatures, it can be clearly seen that more scholars have using control theory to solve inventory and profit related problems, indicating that control theory has become a feasible method to solve management problems.

2) the cross-channel return strategy

In the DCSC system, the cross-channel return strategy is one of important interest. Research in the field of returns always focuses more on the handling of returned products, price impact, and other aspects.

In [20], the relationship between the performance of mutilevel inventory system and return processing structure is analyzed, and then how the retailer's return strategy impacts its inventory efficiency. A close loop supply chain consisting of a manufacturer and a retailers in which the sold products can be returned to retailers in two different categories is studied in [21]. In [22], the research is carried out to illustrate the profit condition when the Buy-Online-and-Pick-up-in-Store (BORS) strategy is adopted. Two fiercely competitive dual channel retailers are considered to study the situations who is the optimal one or both of them by offering the BORS strategy. In [23], the authors study the consumer intent of behavior of the DCSC system with BORS strategy, in which the mediating effect for the consumer's satisfaction and offline consumption characteristics. The dynamic strategies are developed for the transfer of the goods in the DCSC with cross-channel return which is regarded as online purchase offline return in [24]. Four different return policies are investigated and compared in [25]. Authors develop a new kind of math models for the DCSC to obtain the optimal order quantities, by which the impact on the resalable return issue is illustrated. In [8], authors develop the dynamic decision process of the DCSC system for manufacturer and customers with freight insurance and cross-channel return in live streaming by using evolutionary game theory.

It is obviously noted that the majority of the literatures focus on the pricing decision problem of online purchase and offline return, however, the case of offline purchase and online return is not considered.

3) The free riding behavior

Baal et al. [26] found that in both offline physical and online channels, the ratio of the free riding behavior has been reached as high as 20%, and this proportion will further increase with the rapid development of e-commerce. Therefore, free riding behavior has received widespread attention from scholars. Authors in [27] focus on the issue of how the free riding behavior influence the decision of a supply chain from the manufacturer's perspective. The impact of free riding behavior on a dual-channel's pricing decisions has been investigated in [28], by which authors find that the consistent pricing by the retailers is decided by the degree of free riding behavior. In [29], the influence of free riding behavior on the price competition between the traditional retailer channel and the online channel is investigated, and then authors find that the free riding behavior can form the price competition between both of the channels and reduce the retailer's profit. How free riding behavior impacts the retailer's effort level is developed in [30], then the authors give a few strategies from a retailer's perspective to deal with free riding behavior. He et al. [31] study the influence of customer's free riding behavior in carbon footprint in a DCSC system. In [32], a new type of DCSC structure with free riding behavior between the retailer and online platform is introduced to study the impact on each other. In addition, online finance services are provided to the retailer who is in the capital constrained situation. How to make decisions on channel differentiation strategy for a manufacturer in the DCSC system with free riding behavior is developed in [33]. How the customer's channel preference rate, free-riding behavior, trade credit financing interest rate, cross-channel return influence the optimal pricing decisions and optimal sales-effort level is investigated in [6]. In [34], a Stackelberg game is used to study the synergistic effect of free riding behavior and reverse return distribution rate on price decision and offline retailer service decisions. In addition, many references [35], [36] have also conducted the research on the free riding behavior in DCSC systems.

According to the literatures mentioned above, it is noted that most of the scholars focus on static issue of supply chain, but the robustness of dynamic process of supply chain is not considered.

III. PROBLEM FORMULATION

Mathematical variables used in the introduced DCSC model are listed in Table 1.

The clothing company named ''Chengdu Huihui Han Tang Culture Communication Co., Ltd'' (hereinafter referred to as ''Huihui Han Tang'') that focuses on the design, manufacturing and retail of Hanfu clothing is chosen as a real example in this article. A novel type of dynamic dual-channel clothing supply chain with free riding behavior and cross channel return (offline purchase and online return) strategy is set up to describe the typical sales model like ''Huihui Han Tang'' in this section. The supply chain consists of a clothing manufacturing and sales enterprise, a retail store, online store's customers and physical store's customers. Clothing manufacturer is regarded as a leader in the supply chain, and the retailer is a subordinate. The Clothing manufacturer produces clothes, sets the price and sells its production both through online channel and offline channel. The retailer places orders from manufacturer. The manufacturer and the retailer both receive the unsatisfied products returned from the consumers gratuitously. Free riding behavior is included in the supply chain which is unavoidable. For the purpose of improving customer's satisfaction, online purchase and offline return strategy has been put into effect in the system.

To set up the mathematical model of the supply chain, the following assumptions is given:

1) Manufacturer and retailer negotiate to adopt cross channel return service which allows consumers to return unsatisfied products purchased from physical stores to the manufacturer through online channels. The strategy provides a barrier free return experience for customers and increases the brand recognition. Assuming the online return rate for offline purchases is: $0 < \alpha < 1$ [37].

2) Consumer demand satisfies an uncertain random distribution [10]. Consumer preference rate for online channels is λ, while consumer preference rate for physical retail stores is $1 - \lambda$.

3) Manufacturers and retailers provide no reason return service and full refund service for customers through both online channel and offline channel. It is assumed that the probability that purchases from offline channel and returns to physical retailer is the same as the probability that purchases from online channel and returns to manufacturers, the ratio is represented as $0 < \mu < 1$. The dissatisfied products returned gratuitously from customers are not damaged which can be sold directly the same as the new products [38].

4) In the case of unreasonable return, the manufacturer shall bear the return processing costs (such as inventory fees and return fees) incurred during the return. Consumers' unsatisfactory products returned to online channel whether purchasing online or offline do not incur any fee of return for them self [25].

5) The process of manufacturer's new production, the retailer's ordering, free riding behavior, cross channel return and customer's unreasonable return could be completed in one of iteration, therefore, the time-delay included in each node of supply chain is not considered in the mathematical model [17].

According to Figure 1, the dynamic mathematical model for the supply chain system, which included free riding behavior and cross channel return, is set up by using the form of state space equation. For instance, Eq. (1) is established by using the following guidelines: 1) the manufacturer's inventory at moment $k + 1$ is equal to the arithmetic sum of manufacturer's inventory, manufacturer's new products and the products sold through offline channel and online channel at moment k, the products sold through the products sold through offline channel and online channel at moment k. 2) the retailer's inventory at moment $k + 1$ is equal to the arithmetic sum of retailer's ordering, the products sold to customers, the gratuitous return from customers and retailer's inventory at moment k. 3) the profit of manufacture and retailer at moment k is equal to the arithmetic sum of the profit sold products, the cost of product return (including the cost of packing and transport), production and storage at moment k.

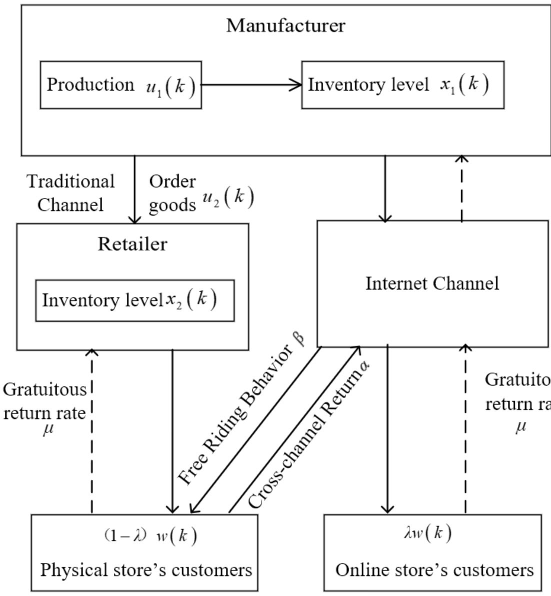

FIGURE 1. The structural sketch of the DCSC system with free riding behavior and cross-channel return.

TABLE 1. Mathematical variables used in the supply chain model.

Variable	Description
$X_1(k)$	The manufacturer's inventory at moment k
$X_2(k)$	The retailer's inventory at moment k
$u_1(k)$	The manufacturer's products at moment k
$u_2(k)$	The retailer's ordering at moment k
$w(k)$	The consumer's uncertain demand at moment k
μ	The return rate with no reason
λ	The consumer preference rate for online channel
$z(k)$	The manufacturer and retailer's current profit
p_m	The unit selling price of online products
p_r	The unit selling price of online products
$c_{m\,I}$	The unit cost of manufacturer's inventory
$c_{m\,O}$	The unit cost of retailer's ordering
$c_{m\,P}$	The unit cost of new production
$c_{m\,R}$	The unit cost of unsatisfactory products returned from customer to manufacturer gratuitously
$c_{r\,R}$	The unit cost of unsatisfactory products returned from customer to retailer gratuitously
$c_{r\,I}$	The unit cost of retailer's inventory

A first order difference equation system is constructed here to describe the relationship between the system states and input variables which represents the internal state changes caused by the input of the system. The discrete system state feedback equation consists of system states, output variables, and control input. The models of four different strategies (N-FC, Y-C, Y-F, Y-FC) are set up, and the block diagram of each model is shown in Figure 2.

1) The inventory model and profit model adopted N-FC strategy.

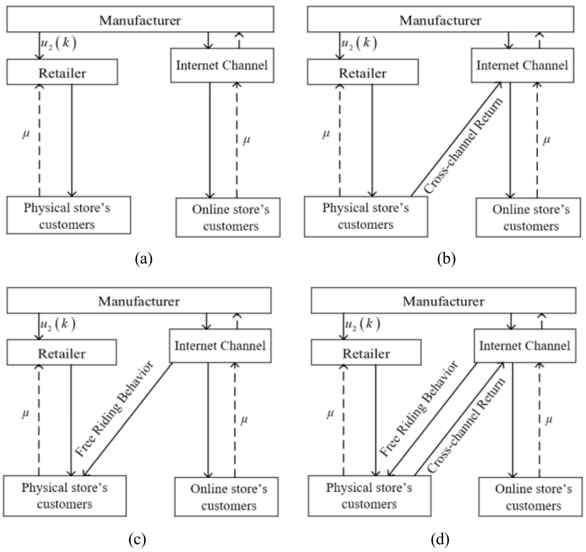

FIGURE 2. The models of four different strategies: (a) N-FC(without free riding behavior or cross channel return), (b) Y-C(with cross channel return only), (c) Y-F(with free riding behavior only), (d) Y-FC(with both free riding behavior and cross channel return).

$$X^{N-FC}(k+1)=\begin{cases}X_1(k+1)=X_1(k)+U_1(k)\\ \quad -U_2(k)+(\mu\lambda-\lambda)w(k)\\ X_2(k+1)=X_2(k)+U_2(k)\\ \quad +(\mu(1-\lambda)-(1-\lambda))w(k)\end{cases} \tag{1}$$

$$Z^{N-FC}(k)=\begin{cases}Z_m=p_m\lambda w(k)+c_{m.O}U_2-c_{m.I}X_1(k)\\ \quad -c_{m.p}U_1-(c_{m.R}\\ \quad +c_{m.I}+p_m)\mu\lambda w(k)\\ Z_r=p_r(1-\lambda)w(k)-c_{m.O}U_2\\ \quad -c_{r.I}X_2(k)-(c_{r.R}+c_{r.I}\\ \quad +p_r)\mu(1-\lambda)w(k)\end{cases} \tag{2}$$

2) The inventory model and profit model adopted Y-C strategy.

$$X^{Y-C}(k+1)=\begin{cases}X_1(k+1)=X_1(k)+U_1(k)-U_2(k)\\ \quad +(\mu\lambda-\lambda+\alpha\mu(1-\lambda))w(k)\\ X_2(k+1)=X_2(k)+U_2(k)+(\mu(1-\lambda)\\ \quad -(1-\lambda)-\alpha\mu(1-\lambda))w(k)\end{cases} \tag{3}$$

$$Z^{Y-C}(k)=\begin{cases}Z_m=p_m\lambda w(k)+c_{m.O}U_2-c_{m.I}X_1(k)\\ \quad -c_{m.p}U_1-(c_{m.R}+c_{m.I}\\ \quad +p_m)(\mu\lambda+\alpha\mu(1-\lambda))w(k)\\ Z_r=p_r(1-\lambda)w(k)-c_{m.O}U_2-c_{r.I}X_2(k)\\ \quad -(c_{r.R}+c_{r.I}+p_r)((1-\alpha)\mu(1-\lambda))w(k)\end{cases} \tag{4}$$

3) The inventory model and profit model adopted Y-F strategy.

$$X^{Y-F}(k+1)=\begin{cases}X_1(k+1)=X_1(k)+U_1(k)-U_2(k)\\ \quad +(\mu\lambda-\lambda-\beta(1-\lambda))w(k)\\ X_2(k+1)=X_2(k)+U_2(k)\\ \quad +(\mu(1-\lambda)-(1-\lambda)+\beta(1-\lambda))w(k)\end{cases} \tag{5}$$

$$Z^{Y-F}(k)=\begin{cases}Z_m=p_m(\lambda+\beta(1-\lambda))w(k)+c_{m.O}U_2\\ \quad -c_{m.I}X_1(k)-c_{m.p}U_1\\ \quad -(c_{m.R}+c_{m.I}+p_m)\mu\lambda w(k)\\ Z_r=p_r(1-\beta)(1-\lambda)w(k)-c_{m.O}U_2\\ \quad -c_{r.I}X_2(k)-(c_{r.R}\\ \quad +c_{r.I}+p_r)\mu(1-\lambda)w(k)\end{cases} \tag{6}$$

4) The inventory model and profit model adopted Y-FC strategy.

$$X^{Y-FC}(k+1)=\begin{cases}X_1(k+1)=X_1(k)+U_1(k)-U_2(k)\\ \quad +(\mu\lambda-\lambda-\beta(1-\lambda)\\ \quad +\alpha\mu(1-\lambda))w(k)\\ X_2(k+1)=X_2(k)+U_2(k)\\ \quad +(\mu(1-\lambda)-(1-\lambda)\\ \quad +\beta(1-\lambda)-\alpha\mu(1-\lambda))w(k)\end{cases} \tag{7}$$

$$Z^{Y-FC}(k)=\begin{cases}Z_m=p_m(\lambda+\beta(1-\lambda))w(k)+c_{m.O}U_2\\ \quad -c_{m.I}X_1(k)-c_{m.p}U_1-(c_{m.R}\\ \quad +c_{m.I}+P_m)(\mu\lambda+\alpha\mu(1-\lambda))w(k)\\ Z_r=p_r(1-\beta)(1-\lambda)w(k)-c_{m.O}U_2\\ \quad -c_{r.I}X_2(k)-(c_{r.R}\\ \quad +c_{r.I}+p_r)((1-\alpha)\mu(1-\lambda))w(k)\end{cases} \tag{8}$$

The general expression for above equations is described as:

$$\begin{cases}X(k+1)=AX(k)+B_2U(k)+B_1W(k)\\ Z(k)=CX(k)+D_{12}U(k)+D_{11}W(k)\end{cases} \tag{9}$$

where $X(k)=[X_1(k),X_2(k)]^T$ is the system state at moment k, $U(k)=[U_1(k),U_2(k)]^T$ is the control input at moment k, $Z(k)$ is control output at moment k, $W(k)$ is the disturbance of the system at moment k, A, B_2, B_1, C, D_{11}, D_{12} are constant matrices. From Equal. (1) to Equal. (9), $X^m(k+1)$ and $Z^m(k)$, $m\in N-FC, Y-C, Y-F, Y-FC$ describes the inventory state and the profit state of manufacturer and retailer.

Remark 1: The proposed model in Figure 1 is different from game theoretic model which is widely used in the optimal revenue strategy and optimal pricing of supply chain. The game theoretic method is mainly committed to the static problem, in which the customer's demand is considered as

TABLE 2. The difference between game theoretic model and the proposed model.

Model	Customer's demand	Dynamic involution process for inventory, production and ordering	Cross channel return	Model form
Game theoretic model	Deterministic [39,40]	× [33,34]	Online purchase and offline return [22,23]	profit function [39] or cost function [41]
The proposed model	stochastic	√	Offline purchase and online return	state space equation

a deterministic value [39], [40]. Meanwhile, the dynamic involution process for inventory, production and ordering is not considered in most of the existing literature [33], [34], the case of offline purchase and online return is also not paid more attention [22], [23]. Therefore, the advantage of the proposed model is that the customer's random demand is addressed, the dynamic evolution processes for inventory, production and ordering are given, and the new type cross channel return strategy is adopted. Meanwhile, the math model is set up by using state space equation which is different from the existing literatures [39], [41]. The main differences between both of them are listed in Table 2.

Remark 2: The all states of Equal. (9) is expressed by deviation values. In this supply chain system, the actual values of all states are equal to the sum of the nominal value and the deviation value. Therefore, all the system states fluctuate around the nominal value, which can be either positive or negative.

IV. ROBUST CONTROL STRATEGY

The effect of incomplete information transmission in the supply chain, which leads to a gradual increase in demand information, is called the bullwhip effect [42]. The quantitative research on the bullwhip effect is mainly divided into three categories:

1) The bullwhip effect is quantified by using mathematical statistical method, such as, the order volatility variance and inventory volatility variance are described by minimum mean square error (MMSE) [43].

2) By using classical control theory method, the supply chain system is described as a transfer function of orders and inventory. The bullwhip effect is quantitative described by using maximum amplitude and noise bandwidth in frequency domain response [44].

3) By using robust H_∞ control method, the maximum value of the bullwhip effect is described by the ratio of the $l2$ norm of output signal to the $l2$ norm of external demand [16].

To sum up, the third method is applicable to state feedback control systems, but if the bullwhip effect quantification index value is too large, it will lead to insignificant system suppression effect. Therefore, in this article, the third method is chosen to describe the bullwhip effect, but the suppression level cannot be too large when design the controller.

The robust H_∞ control method is suitable for multi-input multi-output (MIMO) control systems. The main purpose is to meet certain performance indicators, improve its anti-interference ability, and ensure the stability of the system by designing output feedback controller. The research on stability issues by using robust H_∞ control method is always to construct discrete Lyapunov functionals and transform it into Riccati equation. However, the solution of Riccati equation is difficult to be obtained. Fortunately, the proposal of the linear matrix method simplifies the difficulty of solving system control problems, compensates adjustment problems for the positive definite symmetric matrix and the large number of parameter when solving the Riccati equation.

In this section, the robust control approach for the DCSC system including uncertain customer demand is designed. The H_∞ criterion is utilized to deal with the bullwhip effect in the DCSC system. If the follow inequality has been satisfied under zero initial condition with $w(k) \equiv 0$, the asymptotically stable of the system described in Eq. (9) is ensured [45].

$$\sum_{k=0}^{\infty} Z^T(k)Z(k) \leq \gamma^2 \sum_{k=0}^{\infty} w^T(k)\, w(k) \tag{10}$$

where γ is the attenuation level for the robust system which is introduced to express the maximum value of the bullwhip effect. The better inhibitory effects can be achieved when the smaller attenuation level γ is set to. In order to clearly describe its specific meaning in the state space equation of inventory and profit, Eq. (10) can be rewritten as follows:

$$\frac{\|Profit\ of\ manufacturing\ and\ retailer\|_2}{\|Customer's\ demand\|_2} \leq \gamma \tag{11}$$

where $\|\cdot\|_2$ is the $l2$ norm in the mathematics. In the meantime, Eq. (11) also demonstrates the control performance of the supply chain system. From the perspective of supply chain operations, the customer's demand can be regarded as a kind of disturbance of the supply chain system. If no measures are taken, the disturbance will be gradually enlarged during the disturbance transmitted to the profit function. By using Eq. (11), the disturbance will be restrained. In general, to effectively suppress the bullwhip effect, it is hoped that the attenuation level γ should be as small as possible, which not only achieves better suppression performance for bullwhip effect, but also faster system control response. Nevertheless, the smaller γ will lead to poor system stability. Therefore, the value γ should be chosen suitably according to specific calculation scenarios. To suppress the impact of this uncertainty disturbance on system stability, a state feedback controller is given as follow parallel distributed compensation (PDC) form:

$$U(k) = KX(k) \tag{12}$$

Then, the new production $u_1(k)$ and ordering $u_2(k)$ can be obtained which can suppress the uncertainty caused by external demand disturbances, guarantee the stability and achieve ideal stable value for the profit of the supply chain.

For the DCSC system described above, Kim et al. [11] provided an LMI solution for the dynamic model, as shown in Theorem 1.

Theorem 1: For given system parameters A, B_2, B_1, C, D_{11}, D_{12}, if exist positive-definite matrix Q, matrix M and constant $\gamma > 0$, such that the following linear matrix inequality holds:

$$\begin{bmatrix} -Q & AQ+B_2M & B_1 & 0 \\ (*) & -Q & 0 & QC^T+M^TD_{12}^T \\ (*) & (*) & -\gamma^2 I & D_{11}^T \\ (*) & (*) & (*) & -I \end{bmatrix} < 0 \tag{13}$$

the asymptotic stability of the supply chain system described in Eq. (9) is ensured. $(*)$ represents the symmetric part in the symmetric matrix.

Proof: We define the Lyapunov function as follows:

$$V(X(k)) = X(k)^T PX(k) \tag{14}$$

where P is positive-definite matrices.

In addition, the difference of the Lyapunov functional is shown:

$$\Delta V_k = V(X(k+1)) - V(X(k)) \tag{15}$$

Substituting Eq. (9) and Eq. (12) into Eq. (15), we have:

$$\Delta V_k = (AX(k) + B_2KX(k) + B_1w(k))^T P(AX(k) + B_2KX(k) + B_1w(k)) - X(k)^T PX(k)$$

In the next step, assuming the zero initial condition and using the Eq. (10), we adopt the follow equation:

$$J = \sum_{K=0}^{\infty} [z^T(k)Z(k) - \gamma^2 w^T(k)w(k)]$$

For any nonzero $w(k) \in l_2[0,\infty)$,

$$J \le \sum_{K=0}^{\infty} [z^T(k)Z(k) - \gamma^2 w^T(k)w(k) + \Delta V_k] \tag{16}$$

and then, substituting Eq. (9) and Eq. (15) into Eq. (16), and let $\delta(k) = [X^T(k), w^T(k)]^T$, $A_K = A + B_2K$, $C_K = C + D_{12}K$,we have:

$$J \le \sum_{K=0}^{\infty} \delta^T(k) Z_K \delta(k)$$

where Z_K is defined as:

$$Z_K = \begin{bmatrix} \Theta & A_K^T PB_1 + C_K^T D_{11} \\ (*) & -\gamma^2 I + B_1^T PB_1 + D_{11}^T D_{11} \end{bmatrix} \tag{17}$$

where $\Theta = A_K^T PA_K - P + C_K^T C_K$. This $Z_K < 0$ in Eq. (17) means $\|Z(k)\|_2 \le \gamma \|w(k)\|_2$ for any nonzero $w(k) \in l_2[0,\infty)$. Therefore, when $Z_K < 0$, Eq. (9) is asymptotically stable with the attenuation level γ. By using Schur complements and let $Q = P^{-1}$, $Z_K < 0$ in Eq. (17) can be rewritten as Eq. (13). The proof is completed now.

Eq. (13) is an LMI form in terms of Q, which can be solved by using LMI Toolbox. The state feedback gain K can be obtained from $K = MQ^{-1}$.

TABLE 3. The value of parameter in the supply chain model.

Parameters	μ	p_m	p_r	$c_{m\,I}$	$c_{m\,O}$
Value	0.1	0.8	0.98	0.015	0.66

Parameters	$c_{m\,P}$	$c_{m\,R}$	$c_{r\,R}$	$c_{r\,I}$
Value	0.55	0.1	0.05	0.045

The design steps of the robust control strategy were shown as follows:

Step 1: Establish the mathematical model for the system.

(1) Establish Eq. (1-8) according to the four different strategies.

(2) Transform Eq. (1-8) into Eq. (9) which is the form of state space equation.

Step 2: Chose the state feedback controller as the PDC form: $U(k) = KX(k)$.

Step 3: Adopt H_∞ criterion (Eq. (10)) to deal with the bullwhip effect. The attenuation level γ is used to describe the strength level of the bullwhip effect.

Step 4: Establish the Lyapunov function (Eq. (14)-(15)) and use Eq. (10), when Eq. (17) < 0, Eq. (9) is asymptotically stable with the attenuation level γ.

Step 5: Transform Eq. (17) < 0 into an LMI form (Eq. (13)), by solving the LMI Toolbox, the state feedback gain K can be obtained from $K = MQ^{-1}$.

V. SIMULATION TEST

The simulation tests are conducted to demonstrate three main performances in this section: 1) The robustness of the supply chain system, 2) Evolution of inventory, production/ordering, and profit for manufacturer and retailer under different sales models, 3) The impact of different system parameters on inventory, production/ordering, and profit of manufacturer and retailer.

In the simulation study, the clothing company named "Huihui Han Tang" mentioned in Section III is chosen as a real case to verify the effectiveness of the proposed method. The actual data is collected by investigating "Huihui Han Tang", and then due to commercial reasons, the collected data is performed dimensionless processing, the parameters are ultimately shown in Table 3. The specific simulations are conducted as follows:

A. SIMULATION ANALYSIS OF BULLWHIP EFFECT SUPPRESSION

Due to the complexity of Y-FC compared to N-FC, Y-C and Y-F, the model Y-FC is used here to verify the performance of the robust control method for the DCSC system. The initial values of inventory are selected as: $x_1(0) = 0$, $x_2(0) = 0$. The nominal values are set as: $\bar{x}_1(k) = 52$, $\bar{x}_2(k) = 42$, , $\bar{u}_1(k) = 30$, $\bar{u}_2(k) = 15$. Based on actual fluctuations, external uncertain demands can be selected as Gaussian distribution, Uniform distribution and sine and cosine mixed disturbance, respectively. Therefore, based on the above three demand distributions, the validation simulation is divided into three cases.

In Case 1, the $d(t)$ is chosen as a Gaussian distribution where the uncertain demand satisfies $d(t) \sim N(30, 0.2^2)$. The simulation results of bullwhip effect suppression are given in Figure 3. Since the values of the inventory of manufacturer and retailer are zero at the initial stage, both manufacturer and retailer are in a state of extreme shortage at this point. In an effort to raise inventory quickly, the manufacturer produces new clothing. In the meantime, the retailer gets orders from manufacturer. After 100 iterations, the inventories of manufacturer and retailer risen speedily and converged to 36 and 29, which can be obtained form Figure 3(a). In Figure 3(b), it is obviously noted that the manufacturer's production and the retailer's ordering are risen firstly and then remained stable at around 35 and 15. In the same way, the manufacturer and retailer's profit also rises rapidly, which is shown in Figure 3(c). The manufacturer's profit remains stable at around 7.5, while the retailer's profit remains stable at around 2.5. Moreover, it can be clearly seen that the bullwhip effect is well restrained by using the robust control method.

The consumer's uncertain demand $d(t)$ is selected as uniform distribution: $d(t) \sim U(30, 31)$ in Case 2 and sine cosine mixed interference: $d(t) = 30 + 0.5\sin(k) + 0.5\cos?(k)$ in Case 3, respectively. After 100 iterations, under uniformly distributed demand, it is obviously noted that the actual inventory of manufacturer remains stable at around 36, while that of retailer remains stable at around 29 in Figure 4(a). Figure 4(b) shows that the actual production of manufacturer remains stable at around 35, while retail orders remain stable at around 15. Manufacturer's profit remains stable at around 7.5, while retailers remain stable at around 2.5, which can be found in Figure 4(c). Similar to Figure 4, under sine cosine mixed demand, it can be seen that the actual inventory of manufacturer converge to 35, while retailer's inventory goes stable at around 29.5 in Figure 5(a). Figure 5(b) shows that the actual production of manufacturer remains stable at around 35, while retail orders remain stable at around 15; Manufacturers' profit remains stable at around 7.8, while retailers' remains stable at around 2.7.

From Figure 3 to Figure 5, it can be easily seen that the bullwhip effect caused by three types of customer uncertain demands has been effectively suppressed by utilizing robust control method. The inventory, ordering, and profit of the system are all within a controllable range even if a small fluctuation deviation exists.

Remark 3: Theoretically, it should be noted that that any kind of unknown but bounded uncertain demand described in Simulation 1 can be suppressed by using the robust H_∞ control approach. In general, the value γ is selected as small as possible to achieve better inhibitory effect. Nevertheless, the smaller γ will lead to poor system stability. Therefore, the value γ should be chosen suitably according to specific calculation scenarios.

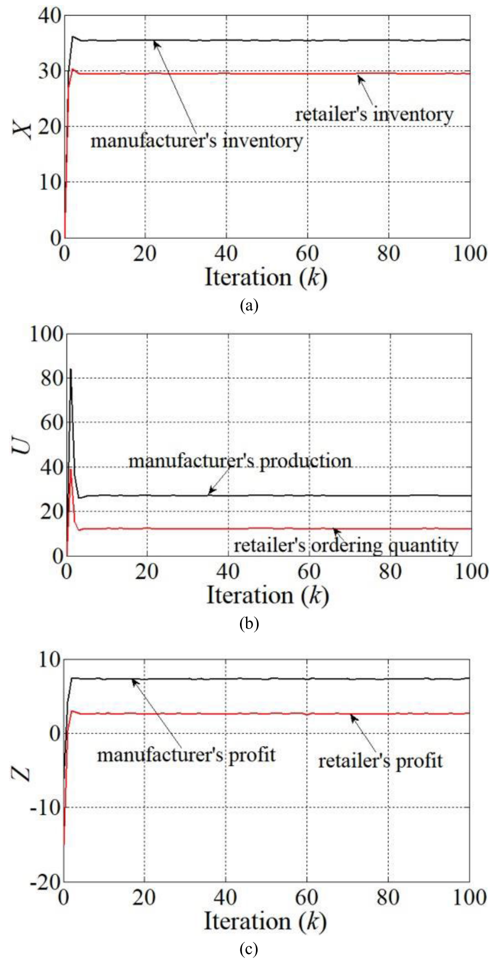

FIGURE 3. The results of simulation test for Bullwhip effect suppression in Case 1; black solid line is manufacturer, red solid line is retailer: (a) inventory, (b) production/ordering, (c) profit.

B. EVOLUTION OF THE SUPPLY CHIAN UNDER FOUR SALES MODELS

In this part, the simulation is carried out to show the evolution process of inventory, production/ordering, and profit for manufacturer and retailer under different sales models. The impact on the supply chain under different channel strategies (N-FC, Y-C, Y-F, and Y-FC) is explored, and the the management mechanism of free riding behavior and cross channel return integration sales strategies is analyzed in this part. Consumer preferences for online and offline channels is set as: $\lambda = 0.5$, means that consumers have the same preference for online and offline sales. Degree of free riding behavior β is chosen to 0.1. The cross-channel return rate α is set to 0.1. By solving the robust control strategy described in Theorem1, the solutions of four models are obtained. The specific evolution process of inventory, production/ordering, and profit changes for manufacturer and retailer are given in Figures 6.

The manufacturer's inventory level X_1 ($X_1^{Y-C} > X_1^{N-FC} > X_1^{Y-FC} > X_1^{Y-F}$) and the retailer's inventory X_2 ($X_2^{Y-F} > X_2^{Y-FC} > X_2^{N-FC} > X_2^{Y-C}$) is presented in Figure 6(a)

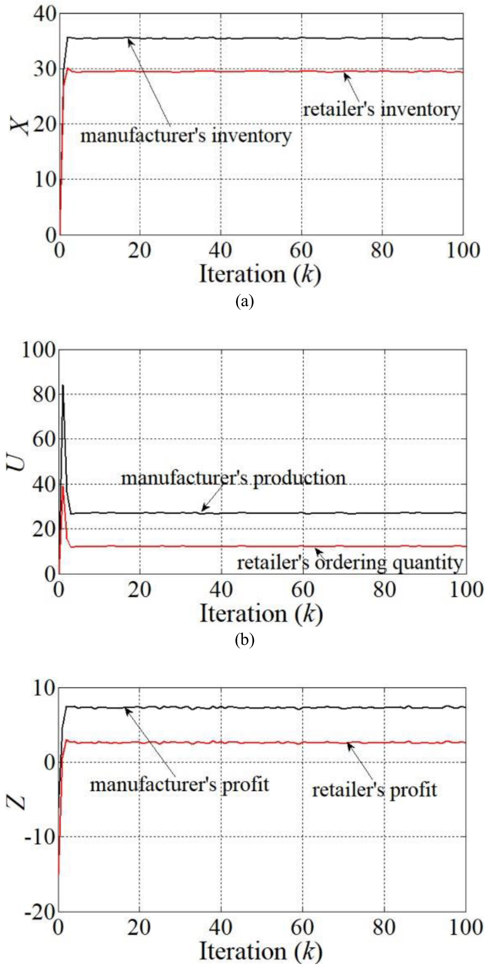

FIGURE 4. **The results of simulation test for Bullwhip effect suppression in Case 2; black solid line is manufacturer, red solid line is retailer: (a) inventory, (b) production/ordering, (c) profit.**

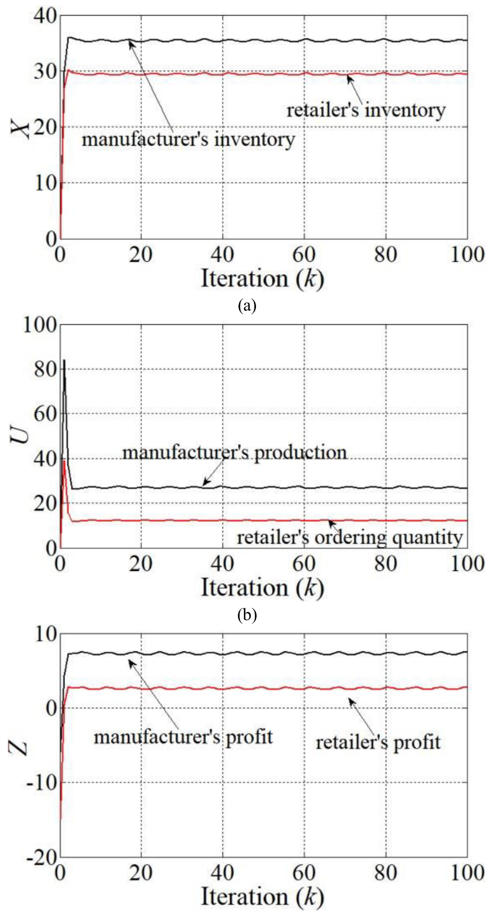

FIGURE 5. **The results of simulation test for Bullwhip effect suppression in Case 3; black solid line is manufacturer, red solid line is retailer: (a) inventory, (b) production/ordering, (c) profit.**

and Figure 6(b). It can be clearly seen that the manufacturer's inventory has not changed hardly, but the retailer's inventory has changed relatively larger. This indicates that different sales strategies have a smaller impact on the inventory of manufacturers, while they have a greater impact on the inventory of retailers. When consumers have the same preferences for both online and offline channels, adopting a cross channel return strategy results in the lowest inventory for retailers. When the supply chain system only includes free riding behavior, manufacturers have the lowest inventory. The reason is that cross-channel return allows offline purchase of goods to be returned through online channels, which alleviates retailer inventory partly, and free riding behavior reduces the retailer's sales, further increasing online channel sales and sharing the manufacturer's inventory backlog problem, therefore, manufacturer's inventory is reduced.

Figure 6(c) and Figure 6(d) show the evolution process of manufacturer's production and retailer's ordering under four different sale strategies. It can be clearly seen that manufacturer's production level U_1 is not changed at all under any strategies, however, the retailer's ordering $U_2(U_2^{Y-C} > U_2^{N-FC} > U_2^{Y-FC} > U_2^{Y-F})$ is converged to different stable value. This indicates that free riding behavior and cross-channel return have no significant impact on manufacturer's production. However, free riding behavior increases the retailer's ordering and the cross-channel return reduces the retailer's ordering. The reason is that a portion of the retailer's sales is transferred to the manufacturer due to the free riding behavior, and the retailer's sales is reduced that resulting in an increase in retailer inventory. Finally, the retailer had to reduce ordering from the manufacturer. Nevertheless, a portion of retailer's inventory is transferred to manufacturer by cross channel return, which reduces the inventory pressure on retailer, and thereby increases the retailer's ordering at last. Therefore, free riding behavior not only increases the inventory of retailer, but also reduces the ordering of physical retail stores.

Figure 6(e) and Figure 6(f) show the manufacturer's profit Z_m ($Z_m^{Y-F} > Z_m^{Y-FC} > Z_m^{N-FC} > Z_m^{Y-C}$) and the retailer's profit Z_r ($Z_r^{Y-C} > Z_r^{N-FC} > Z_r^{Y-FC} > Z_r^{Y-F}$), respectively. It is noted that the profit of manufacturer in various policies are higher than those of retailer. When the retailer adopts Y-C strategy, the retailer's profit significantly increases. When the supply chain system includes Y-F

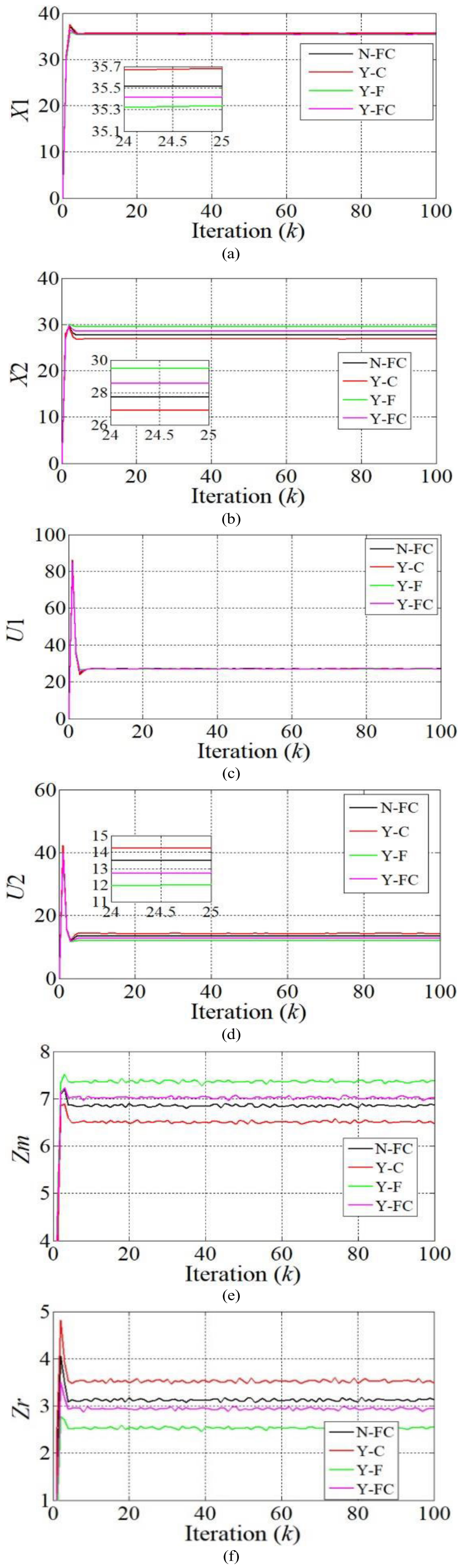

FIGURE 6. **The simulation results of four sales strategies; black solid line is N-FC strategy, red solid line is Y-C strategy, green solid line represents Y-F strategy, magenta solid line represents Y-FC strategy: (a),(b) inventory, (c),(d) production/ordering, (e),(f) profit.**

strategy only, the manufacturer's profit remarkably rises. Due to free riding behavior is not a proactive sales strategy, it often spontaneously exists in the supply chain system. Therefore, when there is free riding behavior in the DCSC system, the largest beneficiary of adopting the cross-channel return policy is the retailer, and the indirect beneficiary is the manufacturer. The reason is that the cross-channel return strategy not only reduces the retailer's inventory and improves customer satisfaction, but also indirectly enhances consumers purchasing desire in physical stores, thereby increasing the overall sales volume of online and offline channel and achieving a win-win case for both manufacturer and retailer.

The customer's uncertain demand and the parameters uncertainty result in the changes of inventory, production/order quantity, and profit which are the main reason to form the bullwhip effect. From Figure 5, it can be noted that dynamic evolution processes of new production volume and ordering quantities are given by utilizing the proposed robust controller, meanwhile the bullwhip effect is suppressed and the robust stability of the supply chain system is achieved.

C. THE IMPACT OF SYSTEM PARAMETERS ON Y-FC MODEL

The impact of free riding behavior rate β, cross channel return rate α and the consumer preference rate for online channels λ on inventory, production/ordering and profit of the supply chain adopted the Y-FC model is studied in this part. The initial inventory values and nominal values are the same as Part A.

1) When $\lambda = 0.1$, $\beta = 0.1$, the simulation is conducted with α changing from 0.1 to 0.9. The simulation result is shown in Figure 7. It is obviously noted that the manufacturer's inventory and production remain unchanged along with the cross-channel return rate α increases. However, the retailer's inventory has significantly decreased which leads to a shortage of goods, then resulting in a significant increase in their ordering. On the one hand, the reason for this is that, when retailer implements cross channel return strategy, more and more inventory generated by returns is transferred to manufacturer, thereby reducing the inventory level of retailer. On the other hand, cross channel return strategy increases consumer's confidence, and the increased sales have led to retailer out of stock, thereby increasing ordering to replenish inventory. Manufacturer's inventory should be increased due to accepting cross channel return inventory, but their inventory has not increased sharply as we imagine because of the increase of retailer's ordering. This indicates that when a cross channel return strategy is implemented in the DCSC system, it is necessary to appropriately increase the manufacturer's production and the retailer's ordering to address the market problem of under-supply.

2) When $\lambda = 0.1$, $\alpha = 0.1$, the simulation is conducted with β changing from 0.1 to 0.9. The simulation result is shown in Figure 8. With the increase of free riding behavior β, the difference in inventory between manufacturers and retailers is not significant, fluctuating around a stable value when $\beta < 0.4$. However, when $\beta > 0.4$, manufacturer's inventory significantly decreases and retailer's inventory significantly

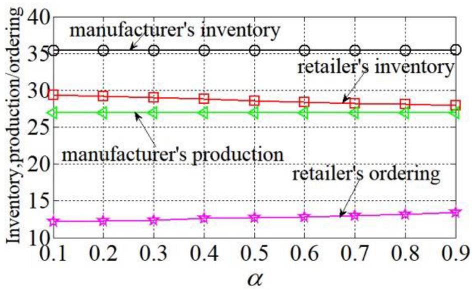

FIGURE 7. The trends of inventory, production and ordering of manufacturer and retailer with changes in α; black solid line is manufacturer's inventory, red solid line is retailer's inventory, green solid line is manufacturer's production, magenta solid line is retailer's ordering.

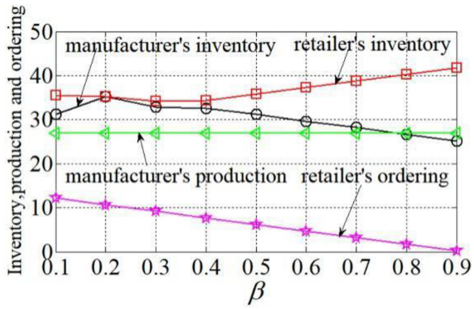

FIGURE 8. The trends of inventory, production and ordering of manufacturer and retailer with changes in β; black solid line is manufacturer's inventory, red solid line is retailer's inventory, green solid line is manufacturer's production, magenta solid line is retailer's ordering.

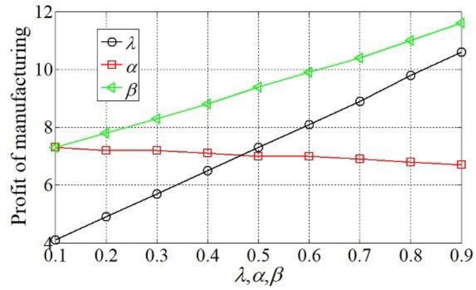

FIGURE 9. The profit trends of manufacturer with changes of α, β, γ; black solid line is the profit trends with changes of γ, red solid line is the profit trends with changes of α, green solid line is the profit trends with changes of β.

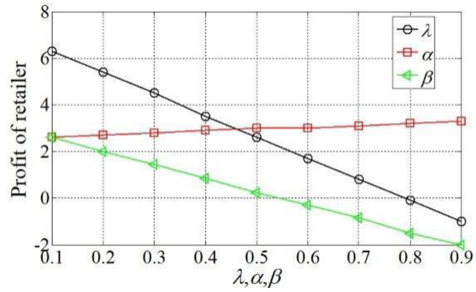

FIGURE 10. The profit trends of retailer with changes of α, β, γ; black solid line is the profit trends with changes of γ, red solid line is the profit trends with changes of α, green solid line is the profit trends with changes of β.

increases. Regardless of the value of β, the manufacturer's production remains unchanged, but the retailer's ordering continues to decline. The reason for this is firstly that when free riding behavior is included in the supply chain, more and more retailer's sales are shifted to manufacturer, resulting in a significant decrease in retail sales and a rapid decrease in their ordering. Secondly, the portion of sales that originally owned by retailer transfers to online channel which will increase manufacturer sales and reduce manufacturer inventory. Nevertheless, in fact, the decrease of retailer's ordering has not led to an overall increase in manufacturer's production. Finally, free riding behavior can reduce the sales efforts of retailer, thereby reducing product sales throughout the entire supply chain, which is detrimental to both manufacturer and retailer. This indicates that when free riding behavior is included in the supply chain system, corresponding measures should be taken to reduce the degree of free riding behavior to below 0.4, thus improving supply chain revenue.

3) The simulation is conducted to analyze the tends of profit for manufacturer and retailer with α, β, λ changing from 0.1 to 0.9, respectively. The simulation result is shown in Figure 9 and Figure 10. From a profit perspective, manufacturer's profit increases with the increase of λ and β, and decrease with the increase of α. Retailer's profit increases with the increase of α, and decreases with the increase of λ and β. On the one hand, this conclusion indicates that the most direct beneficiary of the implementation of cross channel return strategy is retailer. On the other hand, it indicates that the implementation of cross channel return strategy enhances consumer's confidence, thereby enhancing the overall revenue of the supply chain. However, when $\beta > 0.5$, the retailer's profit drops below zero. Therefore, in order to achieve a win-win situation for manufacturer and retailer, effective method should be adopted to control the degree of free riding behavior. Although the increase of α will lead to a decrease in manufacturer's profit, cross channel returns can increase the total profit of the supply chain in the long run.

Remark 4: Game theoretic method is a common method for decision-making problem in many existing articles which have achieved a lot of research results [46], [47]. However, the dynamic issues, such as customer random demand and the evolution process of inventory and so on, can't be addressed by using the game theoretic method. Therefore, the proposed robust control method in this paper is adopted to deal with the dynamic issues of the supply chain. Meanwhile, the robustness of the supply chain can be ensured. In fact, in terms of decision-making, the results from the proposed method will be the same as the results from the game theoretic method, the proposed method don't have advantages compared to the game theoretic method. Nevertheless, this article provides a fresh perspective for the operation of supply chain, which is an important supplementary method for the game theoretic method.

D. MANAGEMENT INSIGHTS

According to the simulation results mentioned above, we have summarized the management insights as follows:

1) Because free riding behavior in the chain system is always inevitable, retails and manufacturers need to balance their profits based on consumers channel preferences. When the consumers prefer online channel, the manufacturers should increase their new production. In addition, manufacturers and retailers should actively promote cross channel return strategies to improve consumer satisfaction, enhance consumers' purchasing desire in physical stores, suppress manufacturers' undersupply and retailers' oversupply, alleviate online shortage pressure and achieve a win-win situation for both manufacturers and retailers. Specifically, when consumers prefer online consumption channels, retailers should actively introduce low-priced and precise marketing models to increase sales in physical stores. At the same time, manufacturers should expand production to avoid the risk of out of stock in online consumption channels.

2) When free riding behavior is included in the channel supply chain system, manufacturers and retailers should actively promote cross channel return strategies and take corresponding measures to control the degree of free riding behavior. The degree of free riding behavior should be controlled below 0.5. At this point, the advantage of online and offline omnichannel supply can be achieved by improving customer satisfaction and reducing the negative impact of customer demand uncertainty, thereby expanding the customer flow and revenue of manufacturers and retailers.

3) The retailer is direct beneficiary of implementing the cross-channel return strategy which enhances consumers' purchasing desire in physical stores, reduces inventory in physical stores and ultimately achieves a win-win situation for both manufacturers and retailers. When facing fierce market competition, with consumer channel preferences and degree of free riding increase, manufacturers' profits continue to increase. However, retailers' profits are the opposite. If the degree of free riding behavior is too high, it will cause serious losses for retailers' profits.

VI. CONCLUSION

In this article, the robust control method for a DCSC system with free riding behavior and cross-channel return is studied. Firstly, the dynamic mathematical models of a dual-channel supply chain system with different sale strategies (N-FC, Y-F, Y-C, Y-FC) are created based on the state space equation which include inventory equations and profit equations. Secondly, based on the discrete Lyapunov function and the LMI method, a robust H_∞ control strategy is designed to suppress the bullwhip effect caused by customer random demand. Finally, the simulation analysis is introduced to demonstrate the performance of the robust control method, and then some conclusions are summarized as follows:

1) By using the robust control method, the bullwhip effect can be effectively restrained under different kinds of disturbances and the inventory, production/ordering and profit of the supply chain have converged to stable values.

2) The robust control method can provide inventory management strategies and profit control strategies for supply chain managers, such as the robust control theory can give specific real-time inventory dynamic evolution process (i.e. the evolution process of inventory state X), meanwhile give specific operational references for implementing this dynamic process (i.e. the evolution process of control input U).

3) Free riding behavior can increase the retailer's inventory, decrease the retailer's sales effort and reduce the sales volume of whole supply chain which is unfavorable to the entire supply chain system. Meanwhile, cross-channel return can increase retailer's profit and improve the customer's satisfaction which is advantageous to the supply chain.

The recycling and remanufacturing are not be considered in the dual-channel supply chain, however, recycling and remanufacturing is a topic of general interest. In addition, the parameter uncertainty is not included in the supply model which is exited in each node of the supply chain in practice. T-S fuzzy model can be also considered to deal with the operation issue of the supply chain. In our future work, the recycling and remanufacturing should be thought over firstly. Secondly, future study may focus on the the parameter uncertainty which is another main influence factor of bullwhip effect. Finally, future research may investigate the T-S fuzzy model which can be used to deal with the operation issue with different initial inventories and different operational strategies.

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