IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 30 November 2023, accepted 20 December 2023, date of publication 22 December 2023,
date of current version 28 December 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3346197

THEORY

Complete Synthesis Analysis for Direct Data Driven Control

ZENG BOHUA[1] **AND WANG JIANHONG**[2]
[1]Ganzhou Polytechnic, Ganzhou 341000, China
[2]School of Engineering and Sciences, Tecnológico de Monterrey, Monterrey 64849, Mexico

Corresponding author: Zeng Bohua (zeng341000@163.com)

This work was supported in part by the Jiangxi National Science Foundation under Grant 20232BAB201015.

ABSTRACT Due to vast data in information era,direct data driven control is more widely studied all over the world, i.e. designing the unknown controller directly from the measured input-output data,while avoiding the modeling process for the unknown plant. For completeness, after classical model reference control is introduced from our own understanding, i.e. controller design, statistical analysis, algorithm and regularization,we propose a complete synthesis analysis for direct data driven control from some different points, for example, algorithm, statistical analysis and adaptation, so making direct data driven control suit for different environments and paving new roads for further research. To prove our obtained theoretical results, one aircraft flight control system is applied to implement the main essence of direct data driven control strategy.

INDEX TERMS Model reference control, direct data driven control, statistical property, regularization, adaptation.

I. INTRODUCTION

During many engineering applications, an accurate mathematical description of the actual physical plant is needed for latter analysis and controller design. The behavior of the considered plant of interest is modeled from the first principle laws or system identification, meaning a detailed knowledge of the interesting plant is essential to derive a mathematical model from these physical laws. However, many physical plants have complex system structure, so it is very hard to model them using the first principle laws. Instead, a system identification approach can be used to describe the intrinsic behavior of the considered plant. Whatever the physical modeling or system identification approach is used to construct one mathematical model for our considered unknown plant, then the obtained model is deemed as one model basis for latter task mission, for example, controller design, anomaly detection, fault isolation etc. generally, above description means two steps, i.e. the first system modeling and second controller design. Although controller design is our terminated goal, but system model is necessary through spending lots of time and energy resource, being called as model based control.

During our new and convenient lives in new era, information showing in data science blows up with time increases, meaning all our features are contained in the related data, such as figure, face, gene, voice, etc, so the useful feature can be extracted from these data when it is needed, thus making our lives more convenient. By the way, the main essence of above system identification is similar to extract the useful information for that unknown plant. More specifically, after collecting the left and right input-output data with respect to the considered plant, system identification is to plot one continuous or discrete curve, fitting the collected input-output data, i.e. curve fitting problem, that is denoted as one mathematical equation or model. This idea on constructing mathematical model for unknown plant from the input-output data is extended to design controller only through the input-output data directly, being called direct data driven control. Generally, direct data driven control designs the unknown controller to get one original controller from data science directly, while avoiding the first system modeling

The associate editor coordinating the review of this manuscript and approving it for publication was Gyorgy Eigner.

step. As the final goals for system identification and direct data driven control are same with each other, i.e. one for system modeling and the other for controller, so all related methods about how to extract the useful information from our collected data for system identification are benefit for our studied direct data driven control in this new paper. The reason about why direct data driven control is more popular than classical model based control in academy is that more subjects appear to support it, for example, machine learning, deep learning, reinforcement learning, adaptation etc.

Due to the widely studied fact about direct data driven control in academy and engineering respectively, lots of research on it are ongoing. Reference [1] calls it as model free control, while combining the adaptation to analyze the stability property. Data driven dissipativity analysis and its computation approach are done in [2] and [3]. The dual combinations with direct data driven and model predictive control are proposed together in [4], and other innovative statistical properties are analyzed continuously, such as stabilization and optimality [5], data informativity [6]. Data informativity guarantees the enough rich for the measured data,so that the intrinsic principle of the considered plant is excited persistently [7], being applied into one algebraic regulator problem [8]. In recent years, behavior theory is introduced into direct data driven control to yield one new control strategy, i.e. data enable predictive control in [9], where the future outputs are expressed as one linear combination form with respect to the finite inputs and outputs within one finite time interval [10] and [11]. Due to the similar relations for system identification and direct data driven control, many identification methods are suited for direct data driven control, for example, maximum likelihood estimation in data driven control [12], robust parameter estimation for robust data driven controller [13], meaning the designed data driven controller from the noisy environment.Other than above mentioned recent references on direct data driven control strategy, we also study it from different aspects in these years, and achieve some new contributions, being formulated as follows.

(1) optimization algorithm,
(2) statistical analysis,
(3) optimal input signal design,
(4) construct one regularized cost function,
(5) combination with other classical control strategies.

Specifically, [14] gives a complete statistical analysis for one kind of direct data driven control, i.e. virtual reference feedback tuning control, and adaptive idea is combined with direct data driven control together to form adaptive iterative correlation tuning control [15]. Instead direct data driven control is benefit for engineering applications, for example, UAV flight system in [16]. Generally, all of above references tell us research on direct data driven control is worth in both academia and industry, thus bring our new paper. More new contributions are embedded into data driven control from theory and application in [17], for example, differential game, differential geometry, adaptation and persistent excitation. Stability analysis about data driven control is given in [18], which depends on Lyapunov based approach. When idea of data driven strategy is used to design one unknown nonlinear controller, approximate nonlinearity cancellation is introduced in [19], and detectable stage costs are considered in combining data driven and model predictive control [20].

Based on our previous contributions about direct data driven control from the detailed controller design within an ideal situation, this new paper continues to complete our ongoing research, and formulates some interesting issues. More specifically, our previous contribution does not consider the external noise or disturb, so the yielded controller is an ideal one. This new paper breaks this ideal case, and extends direct data driven control to more general case. For the sake of completeness, the considered closed loop system structure is given with one unknown plant, and unknown controller simultaneously, then our main mission is to design this controller to guarantee the real output track the expected or desired output, i.e. the goal of perfect tracking. Firstly, classical model reference control strategy is applied to be our reviewed model based control, that designs one controller through solving one optimization problem, relating with the unknown plant and expected performance. Consider this optimization problem with the unknown controller as the decision variable, one explicit form about controller and its statistical analysis are all analyzed completely. To improve the computational speed, i.e. applying the existed convex optimization algorithm, we derive one condition to guarantee the constructed optimization problem be one feasible convex optimization problem. Secondly, after describing the dependence on that unknown plant and to avoid the system modeling process, our considered direct data driven control is proposed from different points, such as the explicit form, statistical analysis, optimal input signal. Furthermore, direct data driven control is combined with the adaptive idea, i.e. forming adaptive direct data driven control. Thirdly, these two different control strategies, i.e. classical model reference control and direct data driven control are used in aircraft flight system, while letting aircraft fly according to the orders from the ground station. After comparing the simulation results, the merit of direct data driven control is proven in case of large data set.

This paper is organized as follows. In section II, closed loop system is considered with unknown plant and unknown controller simultaneously and our main work is also described. Section III introduces classical model reference control strategy in more detail, such as algorithm, statistical analysis, convex condition and regularization. To avoid the traditional modeling process, direct data driven control scheme is proposed in section IV, where algorithm, statistical analysis and its adaptive form are all studied. Comparison of these two different control strategies are done in aircraft flight system to guarantee aircraft fly along the desired trajectory. Section VI formulates the main conclusion and points out our next work.

Generally, the main contributions in this continuous paper are listed as follows.

(1) The detailed controller design process are explained for classical model reference control and direct data driven control together.

(2) To improve the readability, some developments are given to obtain one accurate controller estimate. Then this paper generalizes and formulates our previous contributions.

(3) The detailed application of direct data direct control into aircraft flight system is given to combine the theoretic analysis and engineering application.

II. SYSTEM STRUCTURE

Consider the following closed loop system structure with one unknown plant and unknown controller,plotted in Figure 1. It is similar to one aircraft flight control system, being described int latter section V more detailed.

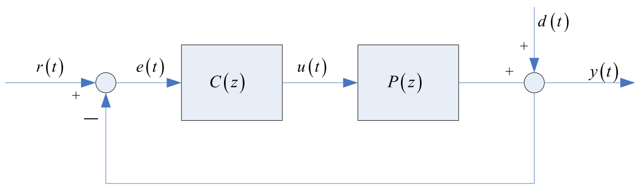


FIGURE 1. Closed loop system structure.

where in above Figure 1, $P(z)$ is the unknown plant, z is the shift operator.$C(z)$ is one unknown feed forward controller, i.e.$\{P(z), C(z)\}$ are unknown. $r(t)$ is one external excitation signal, being used to excite the whole closed loop system. $d(t)$ is also one external disturbance or noise, not being neglected in academy and practice. $\{u(t), y(t)\}$ correspond to the input-output signal for that unknown plant $P(z)$. Error signal $e(t)$ means the deviation between external input $r(t)$ and feedback output $y(t)$,i.e. $e(t) = r(t) - y(t)$.

Observing Figure 1, after simple computations, the following obvious equations are yielded.

$$\begin{cases} cy(t) = P(z)u(t) + d(t) \\ u(t) = C(z)e(t) = C(z)[r(t) - y(t)] \end{cases} \tag{1}$$

i.e.

$$\begin{aligned} y(t) &= P(z)C(z)[r(t) - y(t)] + d(t) \\ y(t) &= \frac{P(z)C(z)}{1 + P(z)C(z)}r(t) + \frac{1}{1 + P(z)C(z)}d(t) \end{aligned} \tag{2}$$

The problem of controller design for Figure 1 is to design that unknown feed forward controller $C(z)$ from different points,i.e. perfect tracking, robustness and adaptation, etc.

III. CLASSICAL MODEL REFERENCE CONTROL

The goal of model reference control guarantees the closed loop output response track one expected or desired reference model $M(z)$, while satisfying other control performances.

A. CONTROLLER DESIGN

Consider the main goal of model reference control, more specifically, we want that closed loop transfer function from external input $P(z)$ to closed loop output $y(t)$ is same with reference model $M(z)$, given in priori, i.e.

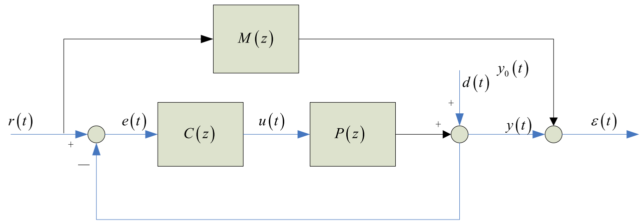


FIGURE 2. Model reference control scheme.

$$\frac{P(z)C(z)}{1 + P(z)C(z)} \rightarrow M(z) \tag{3}$$

Equation (3) is shown in Figure 2.

where from Figure 2, we have

$$\begin{aligned} \varepsilon(t) = y(t) - y_0(t) &= \frac{P(z)C(z)}{1 + P(z)C(z)}r(t) \\ &+ \frac{1}{1 + P(z)C(z)}d(t) - M(z)r(t) \\ &= [\frac{P(z)C(z)}{1 + P(z)C(z)} - M(z)]r(t) \\ &+ \frac{1}{1 + P(z)C(z)}d(t) \end{aligned} \tag{4}$$

Combining equation (3),(4) and the main goal of model reference control, the optimal feed forward controller $C(z)$ is designed from one optimization problem, i.e.

$$C(z) = argmin_{C(z)} \| \frac{P(z)C(z)}{1 + P(z)C(z)} - M(z) \|_2^2 \tag{5}$$

where $\|.\|$ is the commonly used Euclidean norm.

To get one explicit form for controller $C(z)$, we take the partial derivative with respect to controller $C(z)$ and set the derivative equal to zero, then we have

$$[\frac{P(z)C(z)}{1 + P(z)C(z)} - M(z)]\frac{P(z)}{[1 + P(z)C(z)]^2} = 0 \tag{6}$$

i.e.

$$\begin{aligned} 1 - \frac{1}{1 + P(z)C(z)} - M(z) = 0 \\ P(z)C(z) = \frac{1}{1 - M(z)} - 1 = \frac{M(z)}{1 - M(z)} \end{aligned} \tag{7}$$

so the final controller is that

$$C(z) = \frac{1}{P(z)}\frac{M(z)}{1 - M(z)} \tag{8}$$

where from above equation (8), we see after given that reference model $M(z)$, and identified that considered plant $P(z)$, the optimal controller $C(z)$ is yielded to satisfy the control goal,i.e. equation (3).

Furthermore, due to the dependence of controller $C(z)$ on plant $P(z)$, classical model reference control is also model based control strategy, as plant $P(z)$ appears in that explicit form of controller $C(z)$.

B. STATISTICAL ANALYSIS

For the sake of completeness, the quality of that final controller $C(z)$ is needed to evaluate. Assume one ideal controller $C_0(z)$ exist to satisfy that.

$$\frac{P(z)C_0(z)}{1+P(z)C_0(z)} = M(z) \tag{9}$$

i.e.

$$\begin{aligned} 1-M(z) &= 1-\frac{P(z)C_0(z)}{1+P(z)C_0(z)} \\ &= \frac{1}{1+P(z)C_0(z)} \end{aligned} \tag{10}$$

substituting equation (9) and (10) into equation (8), it holds that

$$\begin{aligned} C(z) &= \frac{P(z)C_0(z)}{1+P(z)C_0(z)}\frac{1+P(z)C_0(z)}{1}\frac{1}{P(z)} \\ &= C_0(z) \end{aligned} \tag{11}$$

Equation (11) means our final controller, showing in equation (8), is equal to that assume ideal controller, while guaranteeing the condition (3), i.e.perfect tracking or matching.

C. ALGORITHM

When to solve that optimization problem (5), lots of classical optimization algorithms can be applied directly, such as Newton algorithm, gradient algorithm, conjugate algorithm etc. To let the final controller be one optimal controller, i.e. $C(z) = C_0(z)$, we can resort to the nice convex optimization algorithm, in case of a convex cost function.

Making use of the property of convex cost function about its second derivative must be positive, we continuously derivative that cost function twice, i.e.

$$\begin{aligned} &\frac{1}{[1+P(z)C(z)]^3} - 3\frac{P(z)C(z)}{[1+P(z)C(z)]^4} \\ &+ \frac{2M(z)}{[1+P(z)C(z)]^3} \geq 0 \end{aligned} \tag{12}$$

Multiplying $[1 + P(z)C(z)]^4$ on both sides of above equation (12) to get.

Making use of the optimality necessary condition to differentiate with respect to $P(z)$ and setting the derivative equal to zero, we have

$$\begin{aligned} &1+P(z)C(z)-3P(z)C(z)+2M(z)(1+P(z)C(z)) \geq 0 \\ &C(z)[2P(z)-2M(z)P(z)] \leq 1+2M(z) \\ &C(z) \leq \frac{1+2M(z)}{2P(z)-2M(z)P(z)} \end{aligned} \tag{13}$$

Equation (13) gives a condition about guaranteeing the constructed cost function (5) be convex, so the final controller, solving by each optimization in form (8), is one optimal controller.

D. REGULARIZATION

Above description about optimal controller design is around that desired condition (3), but disturbance or noise $e(t)$ exists really, so it is necessary to reject the bad effect coming from disturbance $e(t)$.

Combining equation (3) and the bad effect from disturbance $e(t)$, here we consider them together, i.e. designing one optimal controller while satisfying the following two conditions.

$$\begin{aligned} &\frac{P(z)C(z)}{1+P(z)C(z)} \to M(z) \\ &\frac{1}{1+P(z)C(z)} \to 0 \end{aligned} \tag{14}$$

Then one improved optimization problem is constructed to design the optimal controller within the case of above two conditions.

$$\begin{aligned} C(z) &= argmin_{C(z)}\|\frac{P(z)C(z)}{1+P(z)C(z)} - M(z)\|_2^2 \\ &\quad +\lambda\|\frac{1}{1+P(z)C(z)}\|_2^2 \\ &= argmin_{C(z)}[\frac{P(z)C(z)}{1+P(z)C(z)} - M(z)]^2 \\ &\quad +\lambda[\frac{1}{1+P(z)C(z)}]^2 \end{aligned} \tag{15}$$

In equation (15), the second term is called the regularization term, being to restrict the bad effect from disturbance. Regularization parameter λ is chosen by designers.

After simple but tedious calculation, the optimal controller is given by

$$C(z) = \frac{1}{P(z)}\frac{M(z)+\lambda}{1-M(z)} \tag{16}$$

substituting equation (9) into above equation (16), we have.

$$\begin{aligned} C(z) &= \frac{1}{P(z)}\frac{\frac{P(z)C_0(z)}{1+P(z)C_0(z)}+\lambda}{1-\frac{P(z)C_0(z)}{1+P(z)C_0(z)}} \\ &= C_0(z)+\lambda C_0(z)+\frac{\lambda}{P(z)} \end{aligned} \tag{17}$$

Continuing to substitute equation (17) into that first condition, i.e.

$$\begin{aligned} \frac{P(z)C(z)}{1+P(z)C(z)} - M(z) &= 1-\frac{1}{1+P(z)C(z)} \\ &\quad -\frac{P(z)C_0(z)}{1+P(z)C_0(z)} \\ &= -\frac{1}{1+P(z)C(z)}+\frac{1}{1+P(z)C_0(z)} \\ &= \frac{P(z)[\lambda C_0(z)+\frac{\lambda}{P(z)}]}{[1+P(z)C(z)][1+P(z)C_0(z)]} \end{aligned} \tag{18}$$

Equation (18) tells us the goal of perfect tracking is not impossible in case of two conditions, but one minimum value about that improved cost function (15) is obtained, i.e. one trade off between perfect tracking and disturbance rejection.

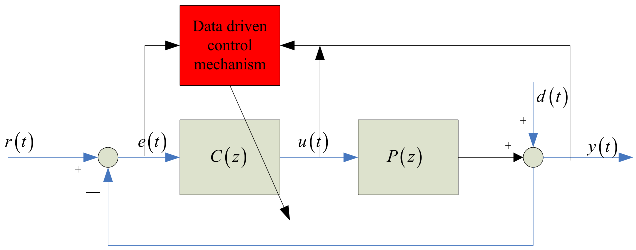


FIGURE 3. Direct data driven control scheme.

IV. DIRECT DATA DRIVEN CONTROL

Observing Figure 1 or cost functions (5), (15), plant $P(z)$ exists in all above mathematical derivation. Moreover, those two final controllers (8) and (16) are dependent on unknown plant $P(z)$, meaning we must identify that unknown plant $P(z)$ firstly, and substitute its explicit or implicit forms in the final controller.

A. ALGORITHM

To avoid the identification process for the unknown plant $P(z)$ and achieve our terminate goal of designing unknown controller $C(z)$, here this section proposes our considered direct data driven control, whose idea is plotted in Figure 3.

To show the main essence of direct data driven control and avoid the identification process for the unknown plant, only observed input-output, corresponding to the unknown controller are applied to design the controller directly, i.e. extracting some useful information from the observed input-output data. Two sensors are placed on the left and right side of controller $C(z)$, whose output is $u(t)$ and input is $e(t) = r(t) - y(t)$. Furthermore, reference model $M(z)$ means $y(t) = M(z)r(t)$,then the input signal $r(t)$ satisfies $r(t) = M^{-1}(z)y(t)$, so the input-output data with respect to controller $C(z)$ are formulated as follows.

$$\begin{aligned} &\{e(t), u(t)\} \to \{r(t) - y(t), u(t)\} \\ &\to \{M^{-1}(z)y(t) - y(t), u(t)\} \\ &\to \{(M^{-1}(z) - 1)y(t), u(t)\} \end{aligned} \tag{19}$$

Based on above input-output data, direct data driven control works to design one optimal controller only through data $\{u(t), y(t)\}$, while considering the property of perfect tacking, i.e.

$$\begin{aligned} &C(z) = argmin_{C(z)} \frac{1}{N}\sum_{t=1}^{N}[u(t) - C(z)y_1(t)]^2; \\ &y_1(t) = (M^{-1}(z) - 1)y(t) \end{aligned} \tag{20}$$

where N is the total number of observed data.

Observing equation (20),$\{u(t), y(t)\}_{t=1}^{N}$ are collected by some physical sensors, $M(z)$ is the given reference model, only $C(z)$ is unknown, and unknown plant $P(z)$ does not exist. When to obtain one explicit form for optimal controller, by differentiating with respect to $C(z)$ and setting the derivative equal to zero, we have

$$\begin{aligned} &\frac{2}{N}\sum_{t=1}^{N}[u(t) - C(z)y_1(t)]y_1(t) = 0; \\ &y_1(t) = (M^{-1}(z) - 1)y(t) \end{aligned} \tag{21}$$

i.e.

$$\begin{aligned} \sum_{t=1}^{N} u(t)y_1(t) &= C(z)\sum_{t=1}^{N} y_1(t)y_1(t); \\ C(z) &= [\sum_{t=1}^{N} y_1(t)y_1(t)]^{-1}[\sum_{t=1}^{N} u(t)y_1(t)] \\ &= \frac{\phi_{uy_1}(w)}{\phi_{y_1}(w)} = \frac{(M^{-1}(z) - 1)\phi_{uy}(w)}{(M^{-1}(z) - 1)^2\phi_y(w)} \\ &= \frac{\phi_{uy}(w)}{(M^{-1}(z) - 1)\phi_y(w)} \end{aligned} \tag{22}$$

where $\phi_y(w)$, $\phi_{uy}(w)$ are auto spectrum and cross spectrum between input-output $\{u(t), y(t)\}_{t=1}^{N}$, i.e.

$$\phi_y(w) = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} y^T(t)y(t)$$

$$\phi_{yu}(w) = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} y^T(t)u(t)$$

without loss of generality, the detailed algorithm for direct data driven control is listed as follows.

Algorithm 1
Step 1: Given one reference model $M(z)$ in priori.
Step 2: Collect the input-output data $\{u(t), y(t)\}_{t=1}^{N}$ with respect to that unknown controller $C(z)$.
Step 3: Compute two kinds of power spectral $\{\phi_y(w), \phi_{uy}(w)\}$.
Step 4: Set the designed controller be that

$$C(z) = \frac{\phi_{uy}(w)}{(M^{-1}(z) - 1)\phi_y(w)}$$

Step 5: Testify whether above controller $C(z)$ is good, through satisfying the following condition.

$$e(t) = u(t) - C(z)(M^{-1}(z) - 1)y(t) \to 0$$

or return to step 2.

Above algorithm gives a rough controller by using the idea of direct data driven control, that only depends on input-output data $\{u(t), y(t)\}_{t=1}^{N}$ and reference model $M(z)$.

B. STATISTICAL ANALYSIS

To testify the statistical property about optimal controller $C(z)$ in equation (22),we take the expectation operator on both sides of equation (22) to get.

$$E[C(z)] = \frac{E[\phi_{uy}(w)]}{(M^{-1}(z)-1)E[\phi_y(w)]} \tag{23}$$

where the following equation are used in deriving equation (23) and assume the covariance of disturbance $d(t)$ be σ^2.

$$\begin{aligned}
y(t) &= \frac{P(z)C(z)}{1+P(z)C(z)}r(t) + \frac{1}{1+P(z)C(z)}d(t);\\
u(t) &= \frac{C(z)}{1+P(z)C(z)}r(t) - \frac{C(z)}{1+P(z)C(z)}d(t);\\
\phi_{uy}(w) &= \frac{P(z)C^2(z)}{[1+P(z)C(z)]^2}\phi_r(w) - \frac{C(z)}{[1+P(z)C(z)]^2}\sigma^2;\\
\phi_y(w) &= \frac{P^2(z)C^2(z)}{[1+P(z)C(z)]^2}\phi_r(w) + \frac{1}{[1+P(z)C(z)]^2}\sigma^2;\\
E[r(t)d(t)] &= 0;\\
M(z) &= \frac{P(z)C_0(z)}{1+P(z)C_0(z)};\\
M^{-1}(z)-1 &= \frac{1}{1+P(z)C_0(z)}
\end{aligned}$$

Then after simple computations, equation (23) is simplified as that

$$E[C(z)] = C_0(z) - \frac{[P(z)C_0(z)+1]\sigma^2}{P^2(z)C_0^2(z)\phi_r(w)+\sigma^2} \tag{24}$$

Here $\phi_r(w)$ denotes the input power spectral. The second term is the bias term, meaning that optimal controller is a biased controller. By the way, external input $r(t)$ and disturbance $e(t)$ are uncorrelated.

An interesting problem about reducing this bias term is solved through choosing an approximate input power spectral $\phi_r(w)$, so that our designed controller $C(z)$ is one unbiased controller, i.e. $E[C(z)] = C_0(z)$. The process of choosing an approximated input power spectral corresponds to the subject of optimal input design, i.e.

$$\begin{aligned}
&argmin_{\phi_r(w)} \frac{1}{\phi_r(w)}\\
&subject\ \ to\ \ \phi_1(w) \le \phi_r(w) \le \phi_2(w)
\end{aligned} \tag{25}$$

or its equivalent form as

$$\begin{aligned}
&argmin_{\phi_r(w)} \frac{[P(z)C_0(z)+1]\sigma^2}{P^2(z)C_0^2(z)\phi_r(w)+\sigma^2}\\
&subject\ \ to\ \ \phi_1(w) \le \phi_r(w) \le \phi_2(w)
\end{aligned} \tag{26}$$

where $\{\phi_1(w), \phi_2(w)\}$ are upper and lower bound for input power spectral. Based on our previous contribution, one explicit form for input power spectral $\phi_r(w)$ is derived through our own derivations.

C. ADAPTATION

Here this last section proposes to combine the adaptive idea, i.e. adaptation, into solving that optimal controller $C(z)$ recursively in real time way.

As

$$\begin{aligned}
C(z) &= [\sum_{t=1}^{N} y_1(t)y_1(t)]^{-1}[\sum_{t=1}^{N} u(t)y_1(t)];\\
y_1(t) &= (M^{-1}(z)-1)y(t)
\end{aligned} \tag{27}$$

In case of the total number or observed input-output data be t, regressor variable $\varphi_1(t)$ and unknown parameter vector θ are defined as follows

$$\begin{aligned}
C_t(z) &= [\sum_{i=1}^{t} y_1(i)y_1(i)]^{-1}[\sum_{t=1}^{t} u(i)y_1(i)]\\
&= F(t)[\sum_{i=1}^{t} u(i)y_1(i)]\\
F^{-1}(t) &= \sum_{i=1}^{t} y_1(i)y_1(i)
\end{aligned} \tag{28}$$

$C_t(z)$ means the designed controller $C(z)$ is related with the total number of observed input-output data t, using one recursive form as that

$$\begin{aligned}
C_{t+1}(z) &= F(t+1)[\sum_{i=1}^{t+1} u(i)y_1(i)];\\
F^{-1}(t+1) &= \sum_{i=1}^{t+1} y_1(i)y_1(i)\\
&= \sum_{i=1}^{t} y_1(i)y_1(i) + y_1(t+1)y_1(t+1)\\
&= F^{-1}(t) + y_1(t+1)y_1(t+1)
\end{aligned} \tag{29}$$

Expressing $C_{t+1}(z)$ as a function of $C_t(z)$ to get

$$C_{t+1}(z) = C_t(z) + \Delta C_{t+1}(z) \tag{30}$$

i.e.

$$\begin{aligned}
C_{t+1}(z) &= F(t+1)[\sum_{i=1}^{t} u(i)y_1(i) + u(t+1)y_1(t+1)]\\
&= F(t+1)[F^{-1}(t)C_t(z) + u(t+1)y_1(t+1)];\\
F^{-1}(t+1)C_t(z) &= F^{-1}(t)C_t(z)\\
&\quad + y_1(t+1)y_1(t+1)C_t(z);\\
F^{-1}(t)C_t(z) &= F^{-1}(t+1)C_t(z)\\
&\quad - y_1(t+1)y_1(t+1)C_t(z)
\end{aligned} \tag{31}$$

Then

$$\begin{aligned}
C_{t+1}(z) &= F(t+1)[F^{-1}(t+1)C_t(z)\\
&\quad - y_1(t+1)y_1(t+1)C_t(z) + u(t+1)y_1(t+1)]\\
&= C_t(z) + F(t+1)y_1(t+1)\\
&\quad \times (u(t+1) - C_t(z)y_1(t+1))
\end{aligned} \tag{32}$$

This result is that

$$C_{t+1}(z) = C_t(z) + F(t+1)y_1(t+1)\varepsilon(t+1);$$
$$\varepsilon(t+1) = u(t+1) - C_t(z)y_1(t+1);$$
$$F(t+1) = F(t) - \frac{F(t)y_1(t+1)y_1(t+1)F(t)}{1 + y_1(t+1)F(t)y_1(t+1)} \quad (33)$$

where equation (33) shows one recursive expression in deriving that optimal controller with time increases. The merit of recursive form is to implement the recursive algorithm in real time way.

Generally, adaptive direct data driven control scheme is plotted in Figure 4.

Here the adaptive idea is introduced with our considered direct data driven control strategy, so that the designed controller can be adjusted with the varying environment, for example, varying control structure, varying plant and others.

V. AIRCRAFT FLIGHT CONTROL SYSTEM

This section gives some simulations to prove our proposed theories in this paper.

The leader-follower flight mode is shown in Figure 5, where according to the desired formation in priori, UAV group need to keep the given formation structure online, then the followers continuously obtain the leader's speed, heading angle and altitude information, so the followers will adjust their own positions to keep the same with the leader.

Generally, the control principle of UAV formation flight is summarized as follows.

Control principle of UAV formation flight
Step 1: the leader sends some flight orders to the followers through data communication, such as heading angle, speed or velocity, and formation commands or orders.
Step 2: the followers calculate the formation error signal, while considering the previous results.
Step 3: the followers calculate the current distance between the two adjacent UAVs, according to the information, sent from the leader.
Step 4: calculate the deviation signals about velocity and heading angle between the leader and followers, then generate the corresponding control rules and send the flight orders to the followers, making the followers fly along the desired trajectory.

During the aircraft formation simulation, three aircrafts exist, i.e. one leader and two followers, and the simulation time interval is 160 second. The original position, horizontal velocity, and heading angle are denoted respectively. The whole flight stages are divided into two stages. The first stage is in [0, 80], and the second stage is in [80, 140].More specifically, within the first stage, the leader fly in a straight and level flight with a constant velocity, and its heading angle is 0. Similarly, in the second stage, the leader will rotate around 10^0 at a constant angular velocity, but its flight velocity remains the same.

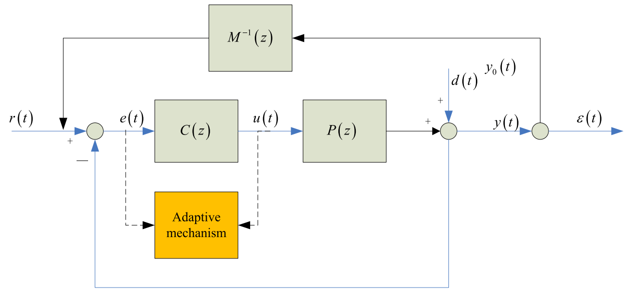


FIGURE 4. Adaptive direct data driven control scheme.

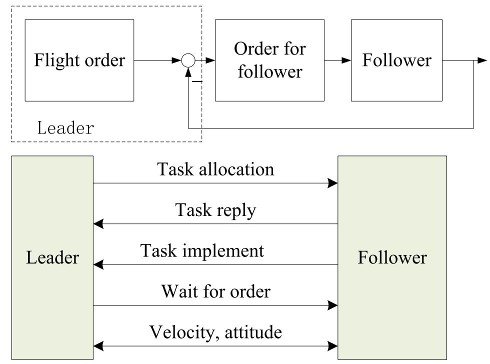


FIGURE 5. Leader-follower mode.

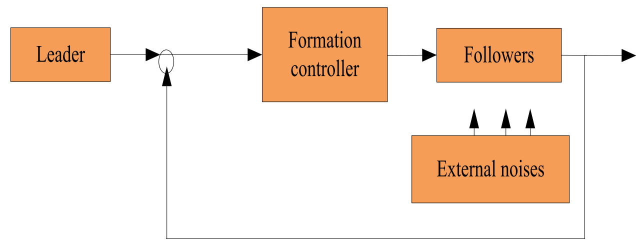


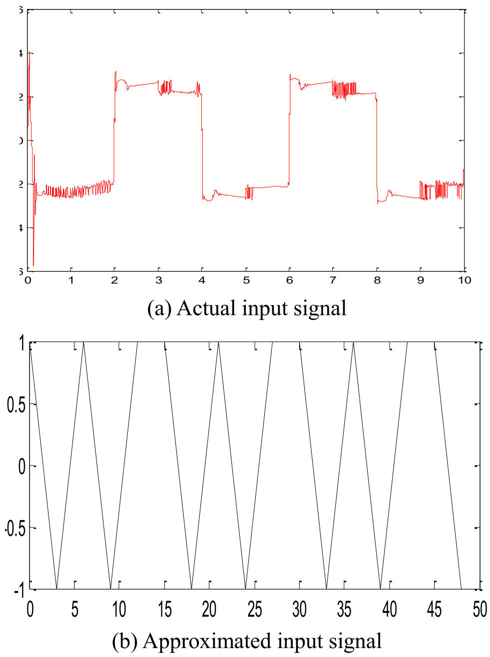
FIGURE 6. Flight control structure.

The above four steps are the following formation flight control structure in Figure 6.

To avoid the identification process for that follower, two sides of data are collected around the formation controller, after one order from leader is sent or excite the nonlinear closed loop system. This order from leader is chosen a constant or one special signal, i.e. square wave, showing in Figure 7. The corresponding output signal is measured and plotted in Figure 8. Based on the input signal and output signal in Figure 7 and Figure 8, our mission is to design one nonlinear form for that formation controller. Through using our proposed direct data driven control strategy, one approximated linear controller is applied to replace that nonlinear formation controller.

Specifically, the commonly used linear affine form is used here, i.e.

$$u(t) = a_0 + \sum_{i=1}^{4} a_i e(t) \quad (34)$$



(a) Actual input signal

(b) Approximated input signal

FIGURE 7. The applied input signal.

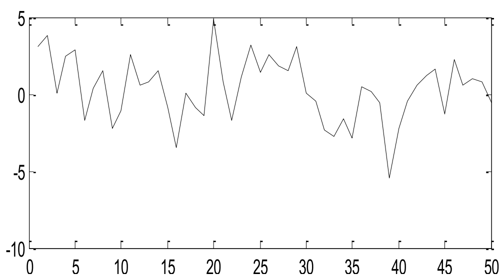


FIGURE 8. The observed output signal.

i.e. the controller $C(z)$ is expressed as that.

$$C(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + a_4 z^4 \tag{35}$$

and the given transfer function $M(z)$ is that

$$M(z) = \frac{0.5z^2 + 2z + 0.5}{z^3 - 1.6z^2 + 0.8z + 1} \tag{36}$$

Then our mission is to change to design these four above unknown parameters $\{a_0, a_1, a_2, a_3, a_4\}$ from the input-output measured data. It corresponds to one data fitting problem, i.e. designing three parameters to guarantee the real output be same with the output in Figure 8.

In practice, leader sends one order to those two followers, i.e. telling two followers to fly around leader. The flight trajectory of leader is the desired or expected flight path. After two followers receive the order from the leader, then two followers will modify their flight situations, and track the leader as soon as possible. According to the leaderąŕs flight trajectory, the formation controllers will design the above four parameters, so that the two followers will fly near to the leader. Figure 7 shows the whole flight trajectories for the leader and two followers, and the horizontal velocity varying curve and heading angle varying curve are all plotted in Figure 10 and Figure 11. From these three Figures, we see after two followers receive the order from the leader, they will fly near to the leader, then achieving the tracking goal perfectly. This tracking performance can be proven during the later 40 second, where three flight trajectories are closed to each other.

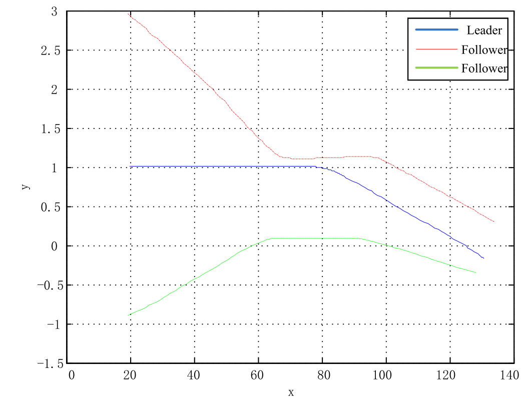


FIGURE 9. The whole flight trajectories for UAV group.

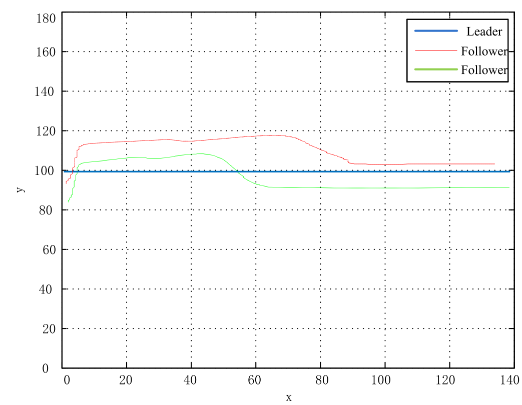


FIGURE 10. The horizontal velocity varying curve.

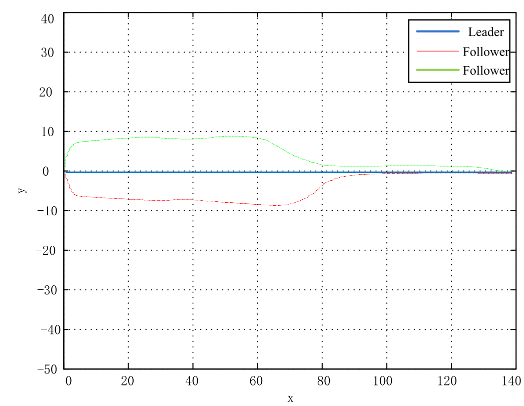


FIGURE 11. The heading angle varying curve.

Furthermore, to make the simulation more efficient, Figure 12 shows two flight trajectories. One is the flight trajectory for the leader, and the other corresponds to one followerąŕs trajectory. To connect it with our theoretical

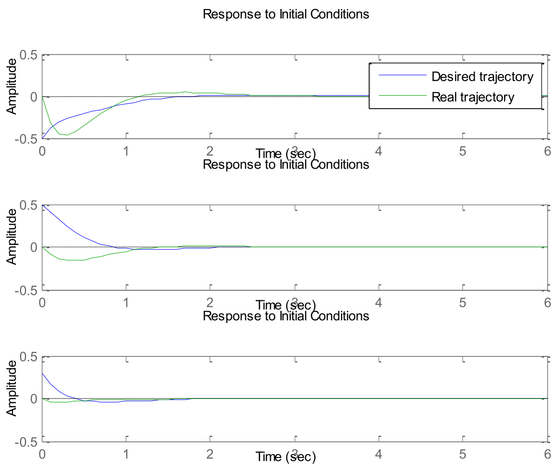


FIGURE 12. **Two flight trajectories for leader and follower.**

analysis, we call the leaderąŕs trajectory as the desired trajectory, and the followerąŕs trajectory is the real trajectory. As these two trajectories approach closely after some seconds, during which the leader and follower communicate in data link.

VI. CONCLUSION

Although lots of research on direct data driven control exist, our new paper gives a complete synthesis analysis about it and proposes some new directions for latter considerations, for example, statistical analysis, real time optimization algorithm, regularization and adaptation. Based on our own contributions, we find that regularization is suited for perfect tracking, and adaptation is benefit for time varying controller, i.e. guaranteeing the designed controller varies with the different environments. In future, our main work will concern on nonlinear data driven control.

ACKNOWLEDGMENT

Furthermore, Wang Jianhong would like to thank MIT for giving a Professor position for him.

AUTHOR CONTRIBUTIONS

Two authors exist in this paper. Zeng Bohua writes this paper and finishes all mathematical derivations. Wang Jianhong revises all English grammar during this second revised form.

REFERENCES

[1] X. Zhang, S. Grammatico, G. Schildbach, P. Goulart, and J. Lygeros, "On the sample size of random convex programs with structured dependence on the uncertainty," *Automatica*, vol. 60, pp. 182–188, Oct. 2015.

[2] X. Zhang, M. Kamgarpour, A. Georghiou, P. Goulart, and J. Lygeros, "Robust optimal control with adjustable uncertainty sets," *Automatica*, vol. 75, no. 1, pp. 249–259, Jan. 2017.

[3] T. Alamo, R. Tempo, and E. F. Camacho, "Randomized strategies for probabilistic solutions of uncertain feasibility and optimization problems," *IEEE Trans. Autom. Control*, vol. 54, no. 11, pp. 2545–2559, Nov. 2009.

[4] D. P. Bertsekas, "Affine monotonic and risk-sensitive models in dynamic programming," *IEEE Trans. Autom. Control*, vol. 64, no. 8, pp. 3117–3128, Aug. 2019.

[5] D. Bertsimas and V. Goyal, "On the power and limitations of affine policies in two-stage adaptive optimization," *Math. Program.*, vol. 134, no. 2, pp. 491–531, Sep. 2012.

[6] L. Blackmore, M. Ono, and B. C. Williams, "Chance-constrained optimal path planning with obstacles," *IEEE Trans. Robot.*, vol. 27, no. 6, pp. 1080–1094, Dec. 2011.

[7] G. C. Calafiore, "Random convex programs," *SIAM J. Optim.*, vol. 20, no. 6, pp. 3427–3464, Jan. 2010.

[8] M. C. Campi and S. Garatti, "Wait and judge scenario optimization," *Automatica*, vol. 50, no. 12, pp. 3019–3029, 2016.

[9] M. C. Campi, S. Garatti, and M. Prandini, "The scenario approach for systems and control design," *Annu. Rev. Control*, vol. 33, no. 2, pp. 149–157, Dec. 2009.

[10] D. S. Callaway and I. A. Hiskens, "Achieving controllability of electric loads," *Proc. IEEE*, vol. 99, no. 1, pp. 184–199, Jan. 2011.

[11] M. Farina, L. Giulioni, and R. Scattolini, "Stochastic linear model predictive control with chance constraints—A review," *J. Process Control*, vol. 44, no. 2, pp. 53–67, Aug. 2016.

[12] S. Garatti and M. C. Campi, "Modulating robustness in control design: Principles and algorithms," *IEEE Control Syst. Mag.*, vol. 33, no. 2, pp. 36–51, Apr. 2013.

[13] U. Rosolia and F. Borrelli, "Learning model predictive control for iterative tasks. A data-driven control framework," *IEEE Trans. Autom. Control*, vol. 63, no. 7, pp. 1883–1896, Jul. 2018.

[14] W. Jianhong and R. A. Ramirez-Mendoza, "Finite sample properties of virtual reference feedback tuning with two degrees of freedom controllers," *ISA Trans.*, vol. 99, pp. 37–49, Apr. 2020.

[15] W. Jianhong, Z. Ying, and R. A. Ramirez-Mendoza, "Direct data driven scheme for UAV flight control," *IEEE Access*, vol. 10, pp. 108241–108250, 2022.

[16] W. Jianhong, W. Yanxiang, and R. A. Ramirez-Mendoza, "Adaptive iterative correlation tuning for closed loop system with two parametrised controllers," *Int. J. Syst. Sci.*, vol. 52, no. 9, pp. 1835–1849, 2020.

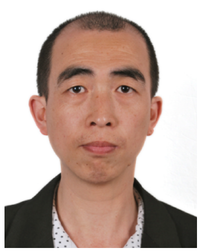
[17] W. Jianhong, R. A. Ramirez-Mendoza, and R. Morales-Menendez, *Data Driven Strategies: Theory and Application*. Boca Raton, FL, USA: CRC Press, 2023.

[18] J. Wang, M. Fečkan, and Y. Guan, "Local and global analysis for discontinuous atmospheric Ekman equations," *J. Dyn. Differ. Equ.*, vol. 35, no. 1, pp. 663–677, Mar. 2023.

[19] C. De Persis, M. Rotulo, and P. Tesi, "Learning controllers from data via approximate nonlinearity cancellation," *IEEE Trans. Autom. Control*, vol. 68, no. 10, pp. 6082–6097, Jan. 2023.

[20] J. Köhler, M. N. Zeilinger, and L. Grüne, "Stability and performance analysis of NMPC: Detectable stage costs and general terminal costs," *IEEE Trans. Autom. Control*, vol. 68, no. 10, pp. 6114–6129, Oct. 2023.

ZENG BOHUA received the master's degree in control engineering from the Jiangxi University of Science and Technology, Ganzhou, Jiangxi, China, in 2021. He is currently teaching with the Ganzhou Polytechnic. His research interest includes advanced control strategy.



WANG JIANHONG received the Diploma degree in engineering cybernetics from Yunnan University, China, in 2007, and the Dr.Sc. degree from the College of Automation Engineer, Nanjing University of Aeronautics and Astronautics, China, in 2011. From 2013 to 2015, he was a Postdoctoral Fellow with Informazione Politecnico di Milano. From 2016 to 2018, he was a Professor with the University of Seville. Since 2019, he has been a Professor with Tecnológico de Monterrey. His current research interests include real-time and distributed control and optimization and system identification.