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RESEARCH ARTICLE

A Global Solution Approach to the Energy-Efficient Ladle Dispatching Problem With Refractory Temperature Control

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ABSTRACT The discussion of energy efficiency in the steel ladle dispatching literature is currently limited to indirectly minimizing waiting and heating times. Not explicitly considering the ladle's thermal balance may lead to sub-optimal solutions and safety concerns regarding the condition of the refractory lining. Hence, this paper studies the energy-efficient ladle dispatching problem with refractory temperature control. A mixed integer linear problem for ladle dispatching that integrates its energy balance is presented. It enables the global solution of the problem using state-of-the-art mixed integer programming solvers. This is achieved by applying piecewise linear models with logarithmic coding to approximate the energy balance. Computational results show that the number of breakpoints employed significantly affects the approximation quality and solution time. However, we show that the error does not affect the feasibility of the problem and yields a negligible difference of 1.4% in the objective function. Hence, this viable approach enables a proper discussion on the energy efficiency of ladle dispatching decisions. For a small but representative production scenario from Tata Steel, IJmuiden, we design and execute an experiment to define a set of operational rules and discuss the potential energy savings. We conclude by presenting the existing compromise between the CO₂ emissions from re-heating the ladles and the reduction in the steel temperature losses from the improved thermal management of the ladles. We show that the average steel temperature losses can be reduced up to 3 °C depending on the refractory temperature requirement. This has the potential to unlock further savings for steelmakers.

INDEX TERMS Energy efficiency, ladle dispatching, mixed integer linear programming (MILP), piecewise linearization, refractory, steelmaking.

I. INTRODUCTION

The iron and steel industry is highly energy-intensive, accounting for 8% of global energy demand [1]. With the transition to green steel production routes, the demand for renewable energy sources is already expected to increase [2]. Hence, improving energy efficiency is critical for steelmakers to remain competitive in a challenging global market. Although much progress has been made in this direction

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over the last fifty years [2], much potential still exists. For example, optimizing the steel ladle logistics in a steelmaking continuous-casting (SCC) plant can be vital to achieving more sustainable operations.

Steel ladles are vessels responsible for moving steel from tapping at primary refining, undergoing secondary metallurgy treatment, and finally reaching the casting machines for producing the steel slabs. After that, the ladles undergo several treatments until ready to transport the steel again. The empty stages involve the removal of the skull, changing sliding plates, adding filler sand, inspecting the refractory

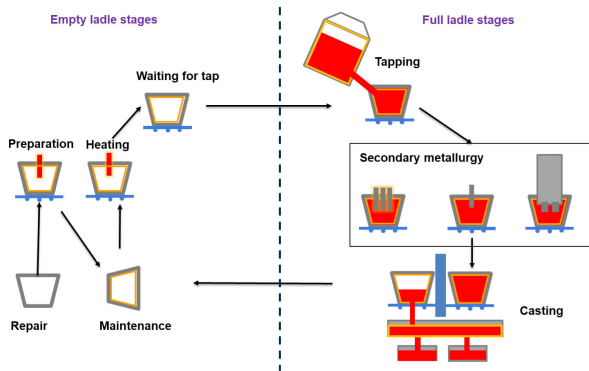


FIGURE 1. A diagram of the steel ladle cycle.

lining, and re-heating the ladles, for example. A ladle remains in the cycle until the end of its refractory lifetime. To be enabled again, ladles must be repaired and undergo a preparation step, which involves a long heating time. These operations are displayed in Fig. 1.

Temperature losses and excessive refractory wear can be avoided by better controlling the ladle's thermal state during its operations, which can be translated into lower emissions and energy savings for steelmakers [3]. This includes defining the sequence of charges assigned to each ladle and for how long it should wait and be heated during each cycle. Therefore, considering the thermal behavior of the ladle becomes an essential aspect of optimized ladle logistics. It allows, for example, determining how to maximize the efficiency of the re-heating applied to ladles in the cycle. Therefore, deciding when and how to assign ladles is vital in the quest for energy efficiency.

The SCC scheduling problem can determine in what sequence, at what time, and on which device molten steel charges should be processed at the various production stages involved [4]. They are represented by the full ladle stages from Fig. 1. The ladle dispatching uses this information as input, being responsible for assigning ladles to charges at the time required [5]. Considering the ladles for the SCC schedule calculation is essential for its applicability [6]. In [7], the authors formulate the ladle dispatching problem as a vehicle routing problem with soft time windows. Reference [8] extends this formulation by adding extra constraints regarding ladle maintenance operations. They highlight that minimizing the number of ladles in operation translates into energy savings from excessive re-heating. Using a different approach, [9] optimizes the empty-ladle stage operations with a heuristic strategy to minimize waiting times, emphasizing how it can be used to enhance the utilization of ladles in the cycle.

In the literature, energy savings are usually implied through the minimization of heating and waiting times, without much discussion of the energy balance of the process. However, the sequence of operations applied to a ladle significantly impacts its thermal condition. Thus, deciding to keep a ladle waiting or re-heating must consider all operations

it undergoes during the cycle. For example, a sequence may appear optimal concerning the overall timing of the operations but not respect a minimum refractory temperature before tapping for some charges. Failing to respect this limit can lead to excessive refractory wear, which causes higher maintenance costs and lower production throughput [3]. Moreover, the condition of the refractory lining also plays an essential role in the thermal capabilities and availability of a ladle. However, it still needs to be adequately addressed. Hence, incorporating more details about the ladle behavior is necessary to improve the discussion on the energy efficiency of ladle dispatching.

Precise models for predicting the ladle's thermal behavior under different operations have been extensively studied. However, their applications are still limited due to the high computational complexity [10], [11], [12]. Recent advances in reduced order modeling (ROM) and machine learning techniques can potentially improve this scenario while not compromising prediction accuracy [13]. This enables the application of accurate models in mathematical programming solutions, especially for integrating the energy balance into ladle dispatching problems.

We present a continuous time mixed integer linear programming (MILP) formulation of the ladle dispatching problem that considers the thermal balance of the ladles. To our knowledge, this is the first attempt to include energy balance in this problem comprehensively. Previous works are focused on minimizing the completion and waiting/heating times to improve energy efficiency with indirect consideration of the thermal state of the ladles [5], [9]. Applying piecewise linear models with logarithmic coding to approximate the nonlinearities arising from the thermal balance makes its solution possible with exact algorithms. This approach is selected to leverage the capabilities of state-of-the-art MILP solvers, which can reach optimality or at least provide solution guarantees [14].

The main outcome of this work lies in providing a detailed discussion of the ladle deployment decisions. It allows us to derive recommended operational practices. This includes, for example, establishing that ladles should only be heated before receiving the next charge of molten steel. We also further understand how practitioners should define limits for the ladle temperature before tapping, aiming to balance process stability and heating efficiency. Finally, we discuss the trade-off between re-heating the ladles and the corresponding benefit from the improved steel temperature control.

The remainder of this paper is organized as follows. Section II briefly reviews the related literature. Section III describes and formulates the energy-efficient ladle dispatching problem with refractory temperature control. Section IV discusses the thermal model characteristics and the global solution approach with piecewise linear models. Section V discusses the solution approach's computational requirements and the proposed problem's energy efficiency. Moreover, managerial insights regarding adopting the proposed model are highlighted based on the results. Finally, the

conclusions and future research directions are available in Section VI.

II. RELATED WORK

Extensive literature is available on the modeling and solution of the SCC scheduling problem. For a more comprehensive analysis of earlier works, the reader should refer to [15]. This paper focuses on the recent literature that explicitly considers ladle dispatching decisions in an offline (or static) setting. In general, ladles are usually considered extra constraints in SCC models [6]. The usual objective is to minimize the total completion time (makespan) for a given set of casting sequences. The available methods differ mainly concerning the mathematical approach to their solution and which ladle dispatching constraints are considered in the model.

Mathematical programming methods employing general MILP solvers have been previously investigated in the literature [4], [16], [17]. Their most significant advantage is providing optimality guarantees for each solution. Fanti et al. [18] proposed a MILP formulation that explicitly considers the assignment of ladles to charges considering cleanliness constraints. They employed a two-step approach to enable its solution with MILP solvers in reasonable CPU time by relaxing the ladle allocation constraints and then using the solution as a warm start for the complete problem.

Directly applying such methods may fail to meet real-time applications' strict computation time requirements. Hence, decomposition approaches have been studied in the SCC literature to find near-optimal solutions quickly. The main idea behind them is to break down a large-scale MILP into smaller tractable sub-problems by exploring the properties of the original problem [19]. Previous attempts explored custom decomposition algorithms [20] and Lagrangian relaxation [21], [22], [23].

Heuristic and meta-heuristic approaches are also popular in the literature. They are an efficient way to find good feasible solutions but have the downside of not providing optimality bounds. Armellini et al. [24] generalized [18] and developed an efficient method combining the Constraint Programming and Simulated Annealing methods for its solution. Wei et al. [8] and Tan et al. [7] studied a formulation based on the vehicle routing problem with soft time windows. They considered more detailed empty-ladle operations, such as changing the sliding plate after some charges. [8] developed a fast heuristic and [7] used a scatter search approach for its solution. Tan et al. [25] further investigated a similar formulation, which proposed a hybrid method that combines a scatter search algorithm with a MILP solver for its solution.

Recently, Han et al. [5] introduced the integrated production scheduling and ladle dispatching problem, considering the energy consumption and losses of empty-ladle operations. They proposed an enhanced migrating birds optimization algorithm for its solution. Hong et al. [9] studied empty-ladle operations in more detail. They assume that a production schedule is given and try to optimize the allocation of

ladles only. A genetic algorithm was employed for its solution. Xu et al. [26] attempted to consider explicitly the steel temperature losses between the processing stages using a linear model, aiming to ensure that the casting superheat temperature is respected. A hybrid genetic algorithm combined with a local search heuristic was developed for its solution.

To our knowledge, no previous work incorporates the energy balance of ladle operations in the SCC scheduling context. Hence, more work is necessary in this direction—our closest reference is [26]. However, the authors used a simple linear model that cannot accurately represent the complex processes involved. Our work extends the previous ladle dispatching formulations [5], [9] and proposes a pioneering approach towards energy-efficient refractory temperature control. For that, we introduce the thermal balance of empty-ladle operations and leverage a precise nonlinear model of the ladle's thermal state. Finally, we employ global optimization techniques for its solution using mathematical programming techniques.

III. PROBLEM FORMULATION

In this section, we describe the ladle scheduling problem and the thermal balance of the steel ladles within an SCC plant. After that, it is modeled as a mathematical optimization problem to minimize the weighted sum of heating and idle times of the ladles. Energy efficiency is achieved with this objective by adding a constraint that limits the ladle temperature before tapping the molten steel.

A. LADLE DISPATCHING

Ladle dispatching decisions involve defining the timing of the empty ladle operations and assigning a proper ladle to all charges for a given production schedule. The cycle begins when a ladle is assigned to a charge of molten steel. The ladle undergoes secondary metallurgy treatment and then proceeds to the casting. For simplicity, we consider this entire process the steelmaking (SM) stage. After casting, the remaining slag must be poured before it can be transported to undergo empty ladle operations. Three different operations are commonly applied when a ladle is empty [27]:

- **Maintenance (MT):** The ladle must be repaired during the cycle in a tilting stand. It usually involves removing solidified slag, changing sliding plates, and adding filler sand. At this point, the ladle state can be inspected by taking measurements of the refractory lining thickness and temperature.
- **Heating (HT):** Ladle stands are equipped with burners that can be optionally used to heat the ladles during the cycle. This operation is essential to allow adjusting the refractory temperature before tapping.
- **Waiting (WT):** The ladle can wait before receiving the molten steel from the next assigned charge. The ladle is usually placed in a transfer car and moved to the primary refining equipment.

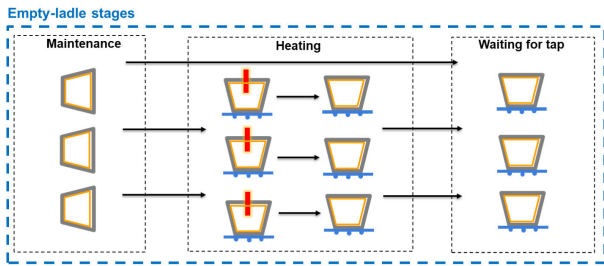


FIGURE 2. The sequence of empty stage operations.

This sequence of operations is depicted in Fig. 2. Naturally, there is a correlation between the ladle operations and the production schedule since the start and end time of the ladle assignment coincides with the tapping event at primary refining. In addition, a ladle can only be reused after it undergoes the empty stages. Each stage has limited resources, called stands, required to apply the respective operations. Finally, cranes or transfer cars are necessary to move the ladles between stages.

B. THERMAL BALANCE OF THE AVERAGE REFRACTORY LINING TEMPERATURE

To transport molten steel at high temperatures, ladles are constructed with particular types of refractory linings. Three layers usually separate the molten steel from the ladle steel shell: the working, safety, and insulation linings. One must choose a proper design, thickness, and material for each layer to achieve a trade-off between ladle capacity and thermal capability [10], [11]. Thermal losses are expected to happen and must be controlled to ensure the energy efficiency and safety of ladle operations [3].

The ladle’s thermal behavior over time depends on the type of operation the ladle undergoes, the refractory’s initial temperature (Tr^I), and lifetime (Lr):

$$Tr = f(\text{OPERATION}, Tr^I, Lr, t) \quad (1)$$

where Tr is the refractory temperature and is used to represent the thermal state of the ladle. Therefore, keeping track of the refractory’s temperature is important for determining the ladle’s thermal balance.

A ladle begins its cycle by being assigned to a molten steel tapping event:

$$Tr^{\text{tap}} = f(\text{TAPPING}, Tr^I, Lr, t_{\text{tap}}) \quad (2)$$

where Tr^I is the average refractory temperature just before tapping. After the tapping finishes, the full ladle will undergo secondary metallurgy operations:

$$Tr^{\text{full}} = f(\text{FULL}, Tr^{\text{tap}}, Lr, t_{\text{full}}) \quad (3)$$

Here, a single operation represents the entire time a ladle is full of molten steel. If necessary, one can expand this definition to consider the effect of different treatments a ladle can undergo during secondary metallurgy. Next, it proceeds

to the casting stage:

$$Tr^{\text{cast}} = f(\text{CASTING}, Tr^{\text{full}}, Lr, t_{\text{cast}}) \quad (4)$$

After that, it is necessary to pour the remaining slag and transport the ladle to the tilting stands for maintenance:

$$Tr_{\text{start}}^{\text{MT}} = f(\text{EMPTY}, Tr^{\text{cast}}, Lr, t_{\text{pouring}} + t_{\text{transport}}) \quad (5)$$

Maintenance requires a minimum amount of processing time, $t_{\text{MT}}^{\text{min}}$, and the ladle can wait a variable amount of time. In the end, it needs to be transported to the next stage:

$$Tr_{\text{end}}^{\text{MT}} = f(\text{EMPTY}, Tr_{\text{start}}^{\text{MT}}, Lr, t_{\text{MT}}^{\text{min}} + t_{\text{MT}}^{\text{idle}} + t_{\text{transport}}) \quad (6)$$

After being positioned in the heating stand, the ladle can start to be heated:

$$Tr_{\text{start}}^{\text{HT}} = f(\text{HEATING}, Tr_{\text{end}}^{\text{MT}}, Lr, t_{\text{HT}}^{\text{heating}}) \quad (7)$$

Furthermore, the ladle can wait without heating before its transportation to the waiting stands:

$$Tr_{\text{end}}^{\text{HT}} = f(\text{EMPTY}, Tr_{\text{start}}^{\text{HT}}, Lr, t_{\text{HT}}^{\text{idle}} + t_{\text{transport}}) \quad (8)$$

Finally, it waits for the tapping of the next charge:

$$Tr^{\text{WT}} = f(\text{EMPTY}, Tr_{\text{end}}^{\text{HT}}, Lr, t_{\text{transport}} + t_{\text{WT}}) \quad (9)$$

Notice that $Tr^I = Tr^{\text{WT}}$ if the ladle is used in the next cycle. This set of calculations can be employed to determine the thermal balance of the steel ladles for the ladle dispatching problem. Moreover, including other ladle operations and extending the thermal balance definition is relatively straightforward.

C. MATHEMATICAL MODELING

The proposed mathematical model includes the assignment of ladles to charges and the timing of the empty ladle operations considering a scheduling horizon of at most 24 hours. Each stage has a limited amount of stands to process the ladles, which are modeled as parallel machines. Repairing a ladle can take several weeks, especially when a complete relining is required. Moreover, preparing a recently repaired ladle can require a few days. Hence, we decided to disregard the ladle repair and preparation because their duration is higher than the scheduling horizon. We also assume that a production schedule is given, similar to [9]. More specifically, it is necessary to specify when a charge is ready for tapping and it must finish casting. Relevant contextual information must also be provided, including the secondary metallurgy and casting duration.

For the proposed energy-efficient ladle dispatching problem, the following assumptions are made:

- All parameters are deterministic.
- The sequence of charges is predefined and provided in order of the end of the casting.
- Ladles have the same remaining lifetime and can process all steel grades.
- There are cranes/transfer cars available to transport ladles between all stages.

- The ladle’s initial temperature and average remaining lifetime are given.

Because of the simplifications applied, the ladle dispatching model may only partially represent some of the complex decisions involved in an actual steel plant. More detailed ladle maintenance operations [25], cleanliness constraints [24], and transportation equipment availability should be explored in future works.

The indices, sets, parameters, and decision variables are now described.

Indices:

- j : the charge identifier
- i : the stage identifier. Maps indexed numbers to each stage: Steelmaking (SM), Maintenance (MT), Heating (HT), and Waiting (WT).
- m : the machine identifier
- l : the ladle identifier

Sets:

- L : the set of ladles
- I : the set of stages
- M_i : the set of machines at stage i

Parameters:

- UB : a sufficiently large positive number
- LL : the lifetime of all ladles in L
- Tap_j : the tapping start time of charge j
- t_j^{Full} : the secondary metallurgy duration of charge j
- $t_j^{Pouring}$: the pouring duration of charge j
- $t_j^{Casting}$: the casting duration of charge j
- $t_{i,k}^{Transportation}$: the transportation time from stage i to k
- $t_j^{Maintenance}$: the maintenance time for ladle l
- LT_l^I : the initial temperature of ladle l
- LFT^{min} : the minimum value for the final ladle temperature in a cycle

Decision Variables:

Binary variables:

- $Q_{j,l}$: indicates if ladle l is allocated to charge j
- $X_{j,i,m}$: indicates if charge j is processed by machine m at stage i
- K_l : indicates if ladle l is in use
- $Z_{j,v,l}$: indicates if charge j is exactly adjacent to charge v for ladle l

Continuous variables:

- $St_{j,i}$: the starting time of charge j at stage i
- $It_{j,i}$: the idle time of charge j at stage i
- $Ct_{j,i}$: the completion time of charge j at stage i
- Ht_j : the heating time of charge j during the heating stage
- LSt_j : the starting time of charge j in a ladle
- LCT_j : the completion time of charge j in a ladle
- $IT_{j,i}$: the initial temperature of charge j in stage i
- $FT_{j,i}$: the final temperature of charge j in stage i
- HT_j : the temperature of charge j after heating
- LIT_j : the initial temperature of a ladle for charge j
- LST_j : the final temperature of a ladle for charge j after undergoing the secondary metallurgy processes

LCT_j : the final temperature of a ladle for charge j after casting

LFT_j : the final temperature of a ladle for charge j

With the notations above, the ladle dispatching with steel ladle temperature control is now formulated as a continuous time MILP problem. All charges must be allocated to a ladle, which must be processed following the order of the empty stages and respect the thermal balance equations. Moreover, this paper aims to find a schedule that minimizes the weighted idle and heating times, respecting the minimum temperature of the ladle before tapping. An illustration of how the decision variables connect the dispatching decisions to the ladle’s thermal balance is displayed in Fig. 3.

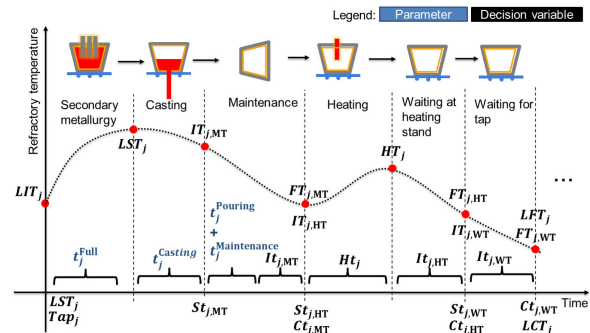


FIGURE 3. The relation between the ladle dispatching decisions and the thermal balance of the steel ladles for a single cycle. The key continuous variables used in the MILP model are represented. Transportation times between stages are omitted from the diagram for simplicity.

1) SCHEDULING CONSTRAINTS

Each charge must be processed by exactly one ladle:

$$\sum_{l \in L} Q_{j,l} = 1, \forall j \in J \tag{10}$$

and by one machine at a time:

$$\sum_{m \in M_i} X_{j,i,m} = 1, \forall j \in J, \forall i \in I \tag{11}$$

For each step, a charge can only start after the preceding one is finished in the same machine:

$$St_{v,i} \geq Ct_{j,i} - UB \cdot (2 - X_{j,i,m} - X_{v,i,m}), \forall i \in I, \forall m \in M_i, \forall j, v \in J, j < v \tag{12}$$

where $j < v$ means that charge j precedes charge v .

In order to follow the production plan, we assume that a charge starts in a ladle when tapping begins. For the current charge j , the initial start time of a charge must match the given tapping start time:

$$LSt_j = Tap_j, \forall j \in J \tag{13}$$

After that, a ladle must undergo maintenance while assigned to charge j :

$$St_{j,MT} = LSt_j + t_j^{Full} + t_j^{Casting} + t_j^{Pouring} + t_{SM,MT}^{Transportation}, \forall j \in J \quad (14)$$

$$Ct_{j,MT} = St_{j,MT} + It_j + \sum_{l \in L} Q_{j,l} \cdot t_l^{Maintenance}, \forall j \in J \quad (15)$$

Once maintenance finishes, the ladle is transported to the heating stands, where it can undergo a variable heating time:

$$St_{j,HT} = Ct_{j,MT} + \sum_{m \in M_{HT}} X_{j,HT,m} \cdot t_{MT,HT}^{Transportation}, \forall j \in J \quad (16)$$

$$Ct_{j,HT} = St_{j,HT} + Ht_j + It_{j,HT}, \forall j \in J \quad (17)$$

After the heating stage, the ladle is transported to the waiting stage before the tapping of the next charge:

$$St_{j,WT} = Ct_{j,HT} + \sum_{m \in M_{HT}} X_{j,WT,m} \cdot t_{HT,WT}^{Transportation}, \forall j \in J \quad (18)$$

$$Ct_{j,WT} = St_{j,WT} + It_{j,WT}, \forall j \in J \quad (19)$$

After that, the ladle is ready to be used for the next charge:

$$LCT_j = Ct_{j,WT} + \sum_{m \in M_{HT}} X_{j,WT,m} \cdot t_{WT,SM}^{Transportation}, \forall j \in J \quad (20)$$

A network-based formulation is used to keep track of the ladle assignment to charges [18]. Two dummy charges are created to represent the assignment of the first and last charges, j_{start} and j_{end} respectively. First, we must ensure that the sequence of ladles in each charge starts and ends with a dummy charge:

$$\sum_{j \in J} Z_{j_{start},j,l} \leq 1, \forall l \in L \quad (21)$$

$$\sum_{j \in J} Z_{j,j_{end},l} \leq 1, \forall l \in L \quad (22)$$

Next, we need to guarantee that all charges are assigned to at most one ladle:

$$\sum_{l \in L} \sum_{v \in J_{v,j} \cup j_{start}} Z_{v,j,l} = 1, \forall j \in J \quad (23)$$

$$\sum_{l \in L} \sum_{v \in J_{j,v} \cup j_{end}} Z_{j,v,l} = 1, \forall j \in J \quad (24)$$

where $J_{v,j}$ is the set of charges where $v < j$. Then, we ensure that in each sequence before and after a charge, there is exactly one charge:

$$\sum_{v \in J_{v,j} \cup j_{start}} Z_{v,j,l} = \sum_{v \in J_{j,v} \cup j_{end}} Z_{j,v,l}, \forall l \in L, \forall j \in J \quad (25)$$

And bind Z and Q to tighten the formulation:

$$\sum_{v \in J_{v,j} \cup j_{start}} Z_{v,j,l} \leq Q_{j,l}, \forall l \in L, \forall j \in J \quad (26)$$

$$\sum_{j \in J_{j,v} \cup j_{end}} Z_{j,v,l} \leq Q_{v,l}, \forall l \in L, \forall v \in J \quad (27)$$

Finally, if a ladle l is assigned to charge j , the next charge v must wait for the previous cycle completion:

$$LSt_v \geq LCT_j - UB \cdot (3 - Q_{j,l} - Q_{v,l} - Z_{j,v,l}), \forall l \in L, \forall j, v \in J, j < v \quad (28)$$

$$LSt_v \leq LCT_j + UB \cdot (3 - Q_{j,l} - Q_{v,l} - Z_{j,v,l}), \forall l \in L, \forall j, v \in J, j < v \quad (29)$$

2) CORRELATION CONSTRAINTS

The initial ladle temperature is the final temperature of the previous charge if the assigned ladle was used before:

$$LIT_v \geq LFT_j - UB \cdot (3 - Q_{v,l} - Q_{j,l} - Z_{j,v,l}), \forall l \in L, \forall j, v \in J, j < v \quad (30)$$

$$LIT_v \leq LFT_j + UB \cdot (3 - Q_{v,l} - Q_{j,l} - Z_{j,v,l}), \forall l \in L, \forall j, v \in J, j < v \quad (31)$$

In the case of a ladle entering the cycle, LIT_j is given as a parameter if this is the first charge of the ladle:

$$LIT_j \geq LT_j^I - UB \cdot (1 - Z_{j_{start},j,l}), \forall l \in L, \forall j \in J \quad (32)$$

$$LIT_j \leq LT_j^I + UB \cdot (1 - Z_{j_{start},j,l}), \forall l \in L, \forall j \in J \quad (33)$$

Given that the l -th ladle is assigned to charge j , we can define if the ladle was allocated to any charge with [5]:

$$UB \cdot K_l \geq \sum_{j \in J} Q_{j,l}, \forall l \in L \quad (34)$$

3) ENERGY BALANCE CONSTRAINTS

For a given charge j , we start the calculation of the thermal balance estimating the refractory temperature at the end of the secondary metallurgy:

$$LST_j = f(FULL, LIT_j, LL, t_j^{Full}), \forall j \in J \quad (35)$$

As a simplification, we assume that the tapping time is included in t_j^{Full} . Since it has a short duration, we consider the ladle to behave as if it was full during this operation. We can then determine the temperature after casting with:

$$LCT_j = f(CASTING, TS_j, LL, t_j^{Casting}) \quad (36)$$

We now start tracking the charge's temperature throughout the empty ladle stages and connect with the scheduling decision variables. First, let's assign the initial temperature of the maintenance stage:

$$IT_{j,MT} = LCT_j \quad (37)$$

and now calculate its final temperature. The transportation to the next stage is already included to avoid increasing the complexity of the optimization problem:

$$FT_{j,MT} = f(\text{EMPTY}, IT_{j,MT}, LL, Ct_{j,MT} - St_{j,MT} + t_{MT,HT}^{\text{Transportation}}) \quad (38)$$

The initial temperature of the heating stage is the maintenance's final temperature:

$$IT_{j,HT} = FT_{j,MT} \quad (39)$$

We then can calculate the temperature after heating is applied:

$$HT_j = f(\text{HEATING}, IT_{j,HT}, LL, Ht_j) \quad (40)$$

And the final temperature of this stage is set as follows:

$$FT_{j,HT} = f(\text{EMPTY}, HT_j, LL, It_j + t_{HT,WT}^{\text{Transportation}}) \quad (41)$$

Finally, the waiting stage is reached. Its initial temperature is the last stage's final temperature:

$$IT_{j,WT} = FT_{j,HT} \quad (42)$$

and the final temperature is calculated as:

$$FT_{j,WT} = f(\text{EMPTY}, IT_{j,WT}, LL, Ct_{j,MT} - St_{j,MT} + t_{WT,SM}^{\text{Transportation}}) \quad (43)$$

It is now straight-forward to determine the ladle's final temperature at charge j :

$$LFT_j = FT_{j,WT} \quad (44)$$

4) OBJECTIVE FUNCTION

We consider as objective the weighted sum of the heating and idle times:

$$TEC = \lambda_1 \sum_{j \in J} \sum_{i \in I} It_{j,i} + \lambda_2 \sum_{j \in J} Ht_j \quad (45)$$

However, more than this objective is needed to investigate the energy efficiency of the process. Hence, we add a constraint to enforce a minimum ladle temperature at the end of each cycle and leverage the thermal model to control the refractory temperature:

$$LFT_j \geq LFT^{min}, \forall j \in J \quad (46)$$

With this constraint, long idle times will be penalized because it would result in a large temperature decrease and, hence infeasible solutions. Moreover, heating the ladles may be necessary to achieve the desired temperature target.

Increasing the initial temperature of the ladle after casting can only be done by heating the ladles. However, the heating efficiency decreases over time and is a function of the initial temperature [10]. Therefore, the optimizer should choose a heating practice that maximizes the efficiency of the burners with the minimum amount of idle time. It enables us to investigate the implications of the different scheduling decisions in the energy efficiency of the process while considering an accurate control of the ladle temperature.

It is important to note that the heating time is inversely proportional to the waiting time when a fixed production schedule needs to be followed. Therefore, choosing the same weight for the objective function terms can lead to multiple optimal solutions since the total value remains the same. The best solution regarding energy efficiency in this situation is the one with the least heating time, which we will enforce by applying a higher weight to this objective term. This approach is commonly used in the literature [5]. Developing an improved objective function will be the subject of future improvements to the mathematical model.

IV. METHODOLOGY

A. STEEL LADLE THERMAL MODEL

To understand the thermal state of the ladle represented by Eq. 1, analytical and numerical models have been proposed in the literature [11]. They can evaluate the ladle's heat flow given the properties of the refractory material and thermal boundary conditions from the liquid steel and ambient. Using the finite-element method (FEM), high-fidelity models have been developed at Tata Steel, IJmuiden. The material properties are obtained from measurements at Tata Steel's Ceramics Research Center (CRC) or supplier specifications. Boundary conditions were determined using thermocouple measurement campaigns of a ladle in the actual production environment. Operational rules were derived from simulations based on this model and have been applied to improve the thermal management of ladles during operations over the last decade.

As a necessary step towards developing a digital twin of the steel ladles, the CRC applied ROM techniques to derive simplified but highly accurate models with data generated from extensive FEM model simulations. Several physics-based equations were then fitted to this data, providing a reliable prediction of the ladle's thermal state as a function of the operation applied, duration, initial thermal state, and lifetime of the refractory lining. We defined the thermal state of the ladle as the weighted average temperature of the wall slag line, wall wear lining, and safety lining. The bottom temperature is not included because it does not vary significantly [11].

An example of the thermal model outputs is displayed in Fig. 4. At the start of the cycle, the ladle temperature increases with contact with molten steel and slag. After that, the casting operation starts and results in a reduction in the ladle temperature. Next, the temperature decrease becomes more pronounced due to radiation losses as the ladle is empty during maintenance. Heating is then applied, increasing the ladle temperature. Finally, the ladle loses temperature to the environment again until the next tapping event. Note that different dispatching results can be obtained if the initial temperature and lifetime of the ladle are changed.

Hence, integrating this model into a MILP problem enables us to completely understand how the ladle's energy balance affects the energy efficiency of ladle logistics.

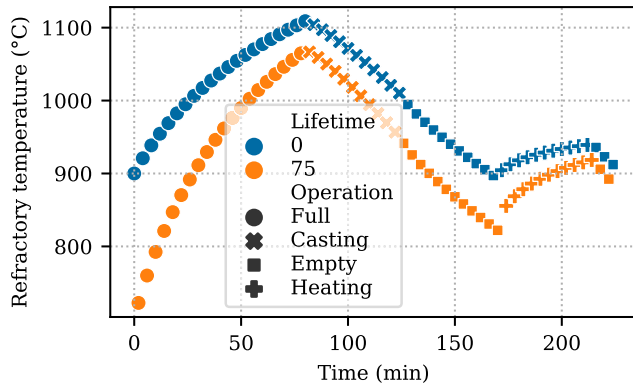


FIGURE 4. Thermal state of the ladle during one cycle for a different set of initial ladle lifetime and temperature specifications. It highlights the impact these parameters can have on the ladle dispatching results.

B. GLOBAL MILP SOLUTION

The mathematical problem described in Section IV is a hard-to-solve optimization problem, with the complexity arising mainly from the energy balance constraints. Hence, we can explore its structure to reduce the computation effort by solving the ladle dispatching and energy balance separately. For that, we propose a two-step solution approach to this problem with a general-purpose MILP solver. First, the problem is simplified by not considering the energy balance constraints and setup with a large number of ladles. We then solve for the objective of minimizing the number of ladles in operation:

$$|L|_{min} := \min \sum_{l \in L} K_l \quad (47)$$

Maximizing the utilization of the steel ladles is essential to increase energy savings [5], [7]. Moreover, the solution found is likely feasible for the complete problem. Hence, it is used as the starting point, and the problem is set with exactly $|L|_{min}$ ladles:

$$\sum_{l \in L} K_l = |L|_{min} \quad (48)$$

This constraint acts as a cut added to the model, allowing the solver to avoid exploring branches with fewer ladles than necessary.

Finally, the problem can be solved with the energy balance constraints. Given the non-linear behavior of the ladle temperature, it can be characterized as a mixed integer non-linear programming (MINLP) problem. One way to solve it is to approximate the nonlinearities with piecewise linear functions. Using this approach enables leveraging the capabilities of state-of-the-art MILP solvers. In addition, these methods can reach the optimal solution or at least provide solution guarantees [14]. Hence, we select it to solve the ladle scheduling problem with energy balance constraints.

The technique's successful application involves choosing a grid and an interpolation approach. Since the thermal models used in the work consist of well-behaved equations, a simple

equidistant fixed-grid is applied as in [28]. For interpolation, several representations to incorporate piecewise linear models into a MILP problem from a grid are available [28]. Due to the high number of expressions involved in the thermal balance, we employ the logarithmic coding. Reference [28] show that it performs better for large-scale problems. We then construct a J1 triangulation over the grid points and obtain the final output through a convex combination of the vertices of the active simplex [28].

Piecewise linear models must be applied for each charge in the production schedule. In addition, the solver must also determine binary and linear decision variables to use the thermal balance correctly for the entire production schedule. Hence, we expect this problem to be more challenging than related formulations in the literature, which do not include the energy balance [6]. Although logarithmic encoding is applied, we expected longer solution times when the number of breakpoints and charges in the production schedule increases. Therefore, the number of breakpoints must be chosen wisely to achieve an adequate approximation quality and prevent the problem from becoming intractable [14].

V. RESULTS & ANALYSIS

In this section, we analyze the computational aspects and energy savings potential of the proposed optimization problem. The results are evaluated for the Tata Steel, IJmuiden, use case [29]. We first discuss the relation between the approximation quality and solution time. Moreover, we evaluate the effect of the approximation error on the problem's optimality. Finally, we design an experiment that enables the definition of operation rules and to understand their energy efficiency implications. The optimization problem was implemented in Python 3.10 with Pyomo 6.5.0 modeling language [30]. All experiments were executed using Gurobi 10.0.0 [31] on a 128-core system (AMD EPYC 7702P) with 256GB RAM.

A. COMPUTATIONAL EXPERIMENTS

1) APPROXIMATION ERROR AND OPTIMALITY ANALYSIS

As discussed in Section IV-B, we used an equidistant grid with a fixed number of breakpoints per dimension to approximate the thermal models. To understand the quality of the approximation, we calculate the root mean squared error (RMSE) as a function of the number of breakpoints. It is calculated for a rectangular grid constructed with fifty evenly-spaced breakpoints. The results per ladle operation are displayed in Fig. 5. As expected, the error decreases with a higher number of breakpoints. Moreover, the approximation is inadequate with four breakpoints, and minor improvements are obtained when more than twenty are used.

Notice from Fig. 5 that the error is consistently higher for the empty heating operations, even with the increase in the number of breakpoints. This is mainly caused by faster heat exchange at the beginning of the heating operation. The increased curvature in the function that describes the

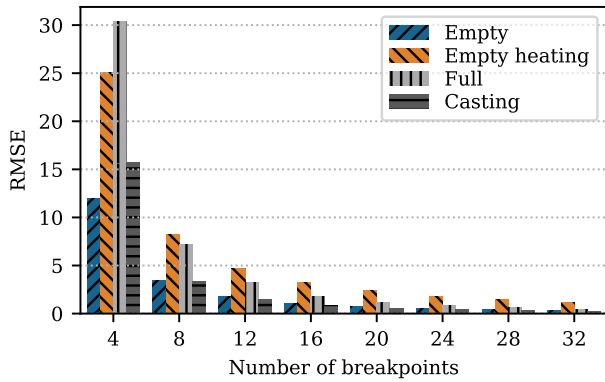


FIGURE 5. Approximation error of the linearized model.

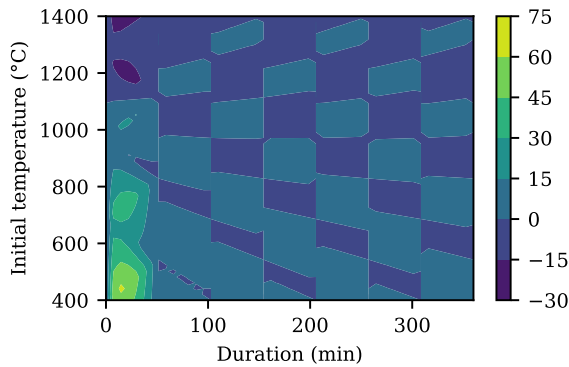


FIGURE 6. Contour plot of the approximation error for the heating operation considering eight breakpoints. A positive value means the approximation is underestimating the actual value.

thermal state then compromises the quality of the linear approximation. This is highlighted in Fig. 6. Thus, future works could explore more intelligent ways to construct the triangulation depending on the operation. It includes, for example, using a different number of breakpoints with a variable distance between them.

The solution’s feasibility is unaffected since the approximation underestimates the actual value. However, this can introduce a deviation from the real optimum. We illustrate this for a single cycle and eight breakpoints in Fig. 7, where around ten extra minutes of heating must be applied to ensure feasibility.

2) PERFORMANCE EVALUATION

Increasing the number of breakpoints is the most straightforward way to control the approximation error. However, this considerably increases the problem size and the time required to solve it. To evaluate this compromise, we solve an example instance of the problem for a different number of breakpoints. We consider a typical production schedule for one casting machine based on production data from Tata Steel, IJmuiden. It consists of three casts with a maintenance break between each and requires a minimum of five ladles for its execution. Each ladle has a different initial temperature (Table 1), a lifetime of 0, and a temperature limit of 700 °C

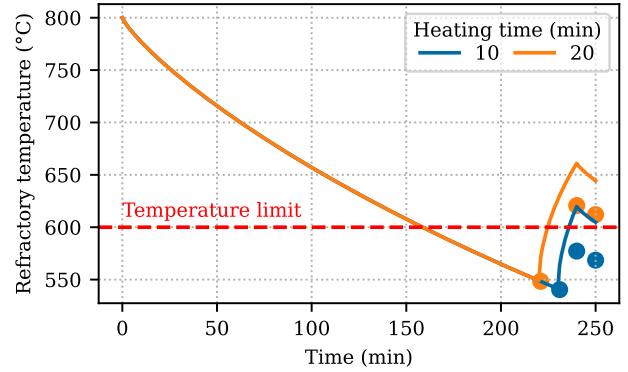


FIGURE 7. The temperature predictions from the end of casting until the tapping of the next charge. Solid lines are the actual model predictions, while dots are the linear approximations for eight breakpoints. A ladle lifetime of 0 is used.

TABLE 1. Ladle configuration used for the instance.

Ladle	Initial temperature (°C)
1	1100
2	1000
3	900
4	800
5	700

is considered. Appendix A-A shows the complete set of parameters. The objective weights are $\lambda_1 = 1$ and $\lambda_2 = 2$, and the solver is allowed to run for a maximum of 10000 seconds or a 0.1% optimality gap.

The results are displayed in Table 2. Observe that a maximum of 20 breakpoints can be used to solve this small instance in a reasonable amount of time. This increase in solution time agrees with results reported in the literature [28], highlighting limitations for the solution of the current model with MILP solvers. Note that each instance has a different solution due to the approximation error. Furthermore, the increase in the optimality gap shows that the problem becomes more difficult as we increase its size. Notice that a different objective value is reached, and more specifically, that the solution with eight breakpoints is around 1.4% higher than the others, even though it has a lower optimality gap.

TABLE 2. Computational results considering a different number of breakpoints and model inputs. The constraint value is set to 700 °C.

Grid size	Gap (%)	Objective	Runtime (s)
8	0.09	1422.89	6235
12	0.52	1396.85	10000
16	3.53	1401.10	10000
20	2.44	1396.32	10000
24	-	-	-
28	-	-	-
32	-	-	-

Each solution’s heating times and ladle allocation are displayed in Table 3. Notice that a different ladle assignment was obtained for the last charge because of a difference in

TABLE 3. Heating times and ladle allocation for different number of breakpoints and a constraint value of 700 °C.

Breakpoints	8	12	16	20	8	12	16	20
Charge	Heating (min)				Ladle			
1	0	0	0	0	5	5	5	5
2	0	0	0	0	4	4	4	4
3	0	0	0	0	3	3	3	3
4	4	1	2	3	2	2	2	2
5	3	1	1	2	1	1	1	1
6	0	0	0	0	5	5	5	5
7	0	0	0	0	4	4	4	4
8	47	44	45	14	3	3	3	3
9	48	10	26	24	2	2	2	2
10	0	15	0	26	1	1	1	1
11	0	6	7	7	5	5	5	5
12	0	0	0	0	4	4	4	4
13	0	0	0	0	1	2	1	3
14	0	0	0	0	5	1	2	2
15	0	0	0	0	3	5	5	5
16	0	0	0	0	2	3	3	1
17	0	0	0	0	4	4	4	4
18	0	0	0	0	1	2	1	3
19	0	0	0	0	5	1	2	2
20	0	0	0	0	3	5	5	5
21	0	0	0	0	2	3	3	1

the heating and waiting times. This also shows that some flexibility exists regarding the ladle dispatching decisions. Moreover, the difference in the heating time can be considered negligible in practice. For example, the setup time of the heating stand can be longer than the effective heating duration and thus becomes ineffective. More details for the empty heating operation are currently not present in the model and can be included in future developments.

3) LIMITATIONS

Piecewise linear models are a powerful tool to convert a MINLP into a MILP. However, it may result in intractable MILPs depending on the quality required for the approximation [14]. Improving the approximation quality in the solution approach proposed in this paper entails increasing the number of linear pieces. This is necessary because prediction errors can propagate between consecutive ladle cycles, leading to an inaccurate thermal balance. It can be seen in Table 2 that this leads to a high solution time and even intractable instances with a low amount of charges.

Hence, future works should address two aspects of the current solution approach: (1) leverage optimal piecewise linear fitting to construct error-bound approximations with the minimum number of linear pieces [32], [33]; (2) improve the MILP solution with, for example, decomposition methods [20], heuristics and meta-heuristics. This will expand the potential applications of the proposed model, especially for real-time ladle dispatching applications.

B. ENERGY EFFICIENCY UNDER CONSTRAINED MINIMUM TEMPERATURE BEFORE TAPPING

We decided to consider the same production scenario used in the previous section. It provides enough complexity to represent common ladle dispatching decisions, thus enabling

a discussion on energy efficiency. Even though slight deviations from optimality are expected, we can analyze the trends in the solutions to discuss operational rules and the energy efficiency of the ladle dispatching problem. Managerial insights are then discussed to complement the results.

It is important to note that the results presented show the primary trend for the steel ladles from Tata Steel, IJmuiden. A comprehensive analysis must consider several variables that can lead to discrepancies in real-life scenarios. These variables include differences in ladle design and variations in the wear rate of the ladles during operation. Additionally, the results are based on model predictions, which can have approximation errors compared to actual measurements.

The experiment designed to achieve this goal involves solving several instances of the problem with a different value for the refractory temperature before tapping constraint and the lifetime of the ladles. The constraint assumes values between 600 °C and 800 °C at 25 °C steps. This interval is chosen after preliminary tests found that this production scenario is unfeasible for higher values, and the constraint does not affect the solution below the lower value. The ladle lifetime is evaluated at 0, 15, 30, 45, 60, and 75, within the current practice of Tata Steel, IJmuiden [3]. They are included in the analysis to quantify the variation of the results as a function of the refractory wear level. We select 12 breakpoints to achieve a trade-off between approximation quality and execution time. The objective weights are $\lambda_1 = 1$ and $\lambda_2 = 2$, and the solver is allowed to run for a maximum of 10000 seconds or a 0.1% optimality gap.

After solving each instance, we store the decision variables for further analysis. It includes the timing and machine allocation for each stage and the respective ladle allocation for each charge. After that, the non-linear thermal model is applied to the resulting schedule to calculate the thermal profile for all ladles deployed. It results in refractory temperature profiles for each cycle, similar to the examples in Fig. 4. Post-processing routines are further applied to extract timing and refractory temperature statistics grouped by charge, ladle, and stage. Furthermore, we also store the optimality gap, objective function, and solution time results for each instance. Their statistics are reported in Appendix B.

1) ANALYSIS OF THE OPERATIONAL DECISIONS

Analyzing the experimental results enables us to derive operational rules from the optimization model. For a better organization, each rule is discussed separately below.

a: LADLES SHOULD ONLY BE RE-HEATED CLOSE TO TAPPING

The average relative idle time per stage is displayed in Fig. 8, which shows that the idle time before tapping reduces as more heating is required. When the ladle is empty at a higher temperature, one can expect the refractory temperature losses to be higher. This is explained by radiation losses, which are proportional to the fourth power of the material's

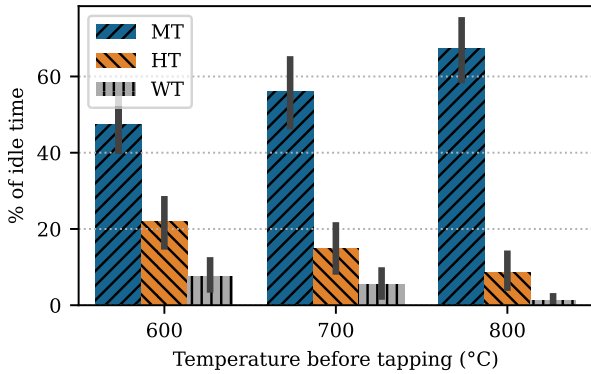


FIGURE 8. Percentage of total idle time per charge for the 600 °C, 700 °C and 800 °C constraint values. The error bars display the 95 % confidence interval calculated with the bootstrap technique.

temperature [10]. Hence, it is recommended to heat the ladles right before receiving molten steel.

This can be further seen in Fig. 9, which displays the schedule for an 800 °C temperature requirement and ladles with a lifetime of 45 charges. Notice that most idle time happens at the maintenance stage, with the ladles being directly transported to the next charge after heating. Thus, practitioners should ensure that the plant layout allows quick movement of ladles from the heating stands to the primary refining equipment. Moreover, it is well known that covering the ladles with a lid reduces radiation losses [10], [34]. Hence, keeping the ladles lidded may also be evaluated if long waiting times are expected after heating them during the cycle. Adding this decision can be further explored to improve the proposed model and investigate if it can reduce the required re-heating.

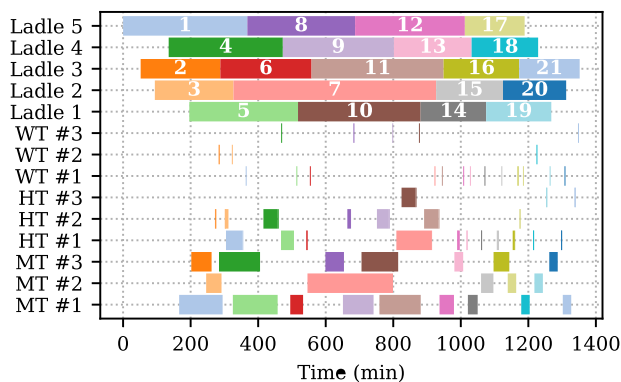


FIGURE 9. Gantt chart of the optimal schedule for a constraint value of 800 °C and ladle lifetime of 45 charges. Each charge number is displayed on the top bars.

b: AVOID LADLES AT HIGH TEMPERATURES

Fig. 10 displays the average idle time per ladle. Notice that different results were obtained depending on the temperature requirement. However, the idle time is consistently lower for ladle 5. From Table 1, this is the ladle with the lowest temperature. A closer look shows that the difference is more pronounced for the more relaxed temperature requirement of 600 °C. Since the objective function prioritizes reducing

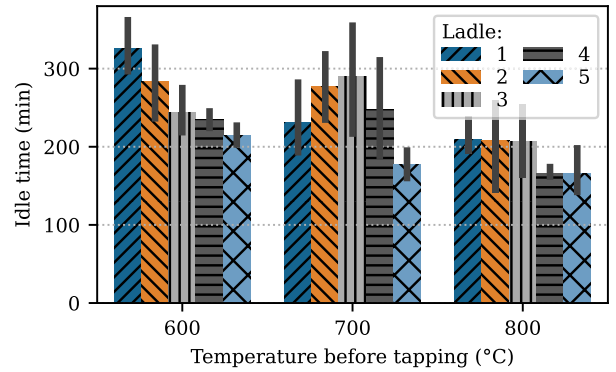


FIGURE 10. Average idle time per ladle for the 600 °C, 700 °C and 800 °C constraint values. The error bars display the 95 % confidence interval calculated with the bootstrap technique.

the heating time, the ladles with the highest temperature are idler on average and waste energy considerably in this scenario. It achieves feasibility with minimum heating applied but with significant overall temperature variations. This is undesired as it can lead to a higher refractory wear rate from thermomechanical stresses [35].

In the case of a strict temperature requirement (800 °C), Fig. 10 also shows that the difference in the idle time becomes lower between the ladles. Since heating is applied, the idle time is automatically reduced. The average heating times per ladle are available in Fig. 11. Notice that even though the initial temperature of the ladle is already high, it requires more heating. Therefore, using ladles at a high temperature should be avoided if the optimization target is to minimize the overall heating time.

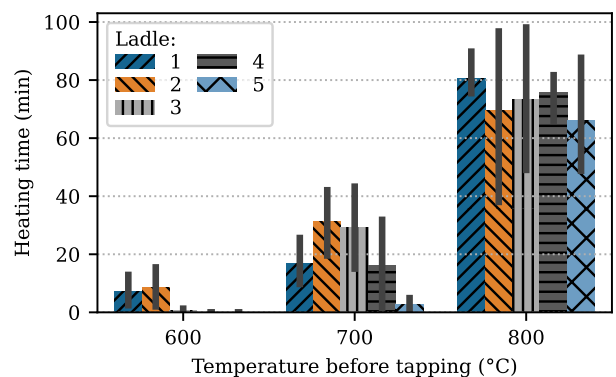


FIGURE 11. Average heating time per ladle for the 600 °C, 700 °C and 800 °C constraint values. The error bars display the 95 % confidence interval calculated with the bootstrap technique.

To illustrate it, Fig. 12 displays the thermal profiles for different temperature requirements. Notice that each temperature requirement leads to different ladle allocation decisions, but a similar thermal behavior is reached at the final cast. Hence, most of the initial energy of the hot ladles is wasted as they eventually converge to the same temperature. Fig. 12 also suggests that the most challenging decisions during the ladle dispatching come from shorter casting

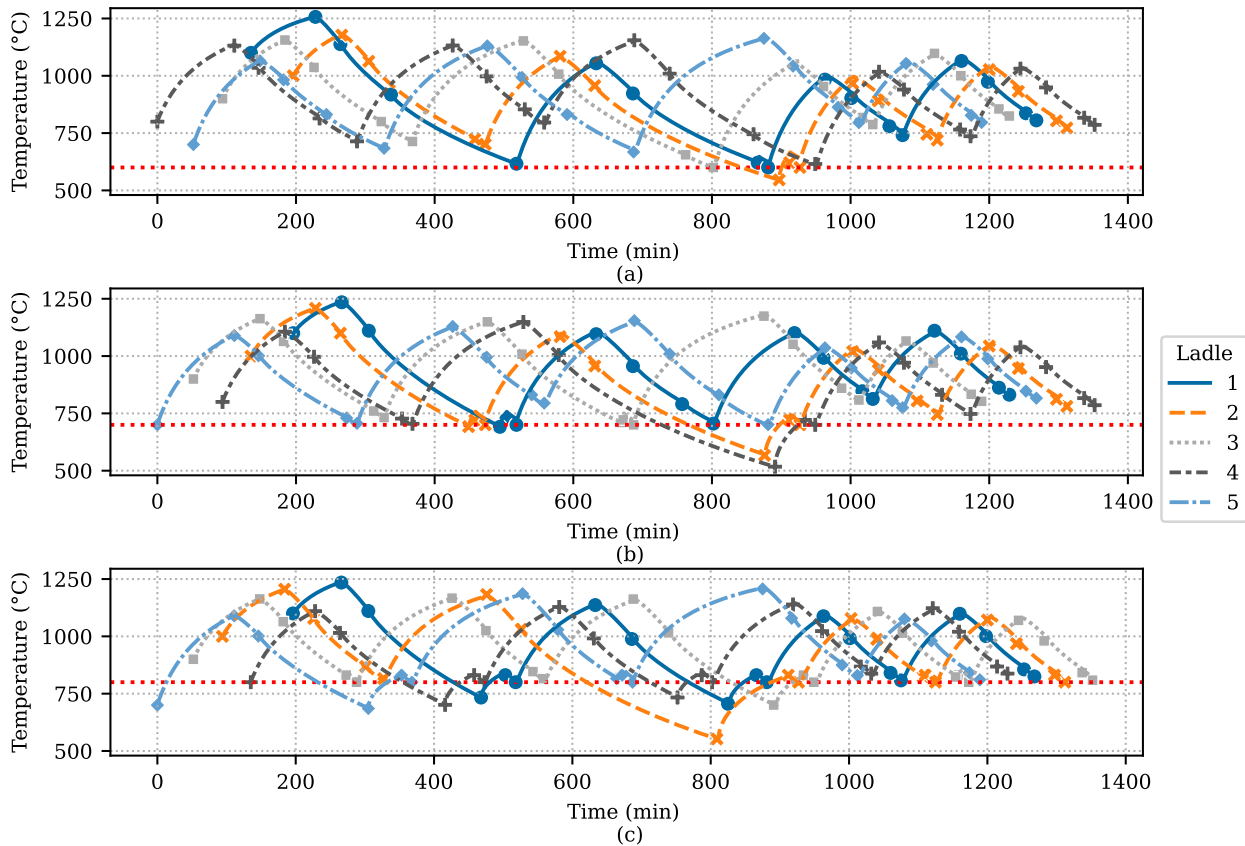


FIGURE 12. Optimal temperature profile for different temperature limits and a ladle lifetime of 45. (a) 600 °C, (b) 700 °C and (c) 800 °C. The markers are the approximations from the piecewise linear model and the continuous lines are the thermal model predictions.

sequences. Notice that the optimal solution is mainly driven by ensuring that ladles are the least idle at a long casting sequence. Hence, practitioners should then experience more instability in the thermal state of the ladles in such a situation.

c: A STRICT TEMPERATURE REQUIREMENT DEMANDS MORE LADLES IN CIRCULATION

In all examples displayed in Fig. 12, the ladle temperature before tapping is close to 750 °C at the end of the scheduling horizon. This is valid even without any heating being applied previously. Moreover, we found that five ladles are insufficient when the temperature requirement is over 850 °C when solving this instance. Hence, more ladles in circulation are required to allow sufficient heating time and respect the production schedule.

Having more ladles in circulation is a common practice to allow production continuity during an unexpected ladle breakdown. This can be done in our model by initializing it with $|L|_{min} + 1$ ladles. Practitioners can leverage this to increase the ladle temperature before heating. Of course, this will significantly increase energy losses and energy consumption from re-heating [7], [8]. In this case, energy efficiency is achieved by heating the ladles at the lowest possible temperature and ensuring they wait the minimum

time until being charged with molten steel. It will maximize the efficiency of the heating applied.

2) ANALYSIS OF THE ENERGY EFFICIENCY

The discussion from the previous section highlighted the reasoning behind the main operational decisions derived from the model. It is also possible to notice that they are a direct consequence of the thermal balance application. Hence, the solution is likely the most energy-efficient regarding the heating required. To start the potential savings discussion, we display the total amount of heating in Fig. 13. It shows that the amount of heating grows as we increase the temperature requirement. Thus, considering the thermal behavior of the ladles in scheduling decisions is a crucial enabler in understanding how to maximize the heating efficiency, especially under different temperature requirements.

In Fig. 14, we highlight that the heating efficiency decreases as more time is spent on this operation. It happens because the working lining can absorb a maximum amount of energy from the burners. Natural gas combustion will generate a maximum amount of energy, which over time causes the refractory temperature to reach a steady-state condition [10]. Hence, it is essential to have longer heating times only when the heat transfer is the most effective. From Fig. 12, notice that extended heating periods are usually

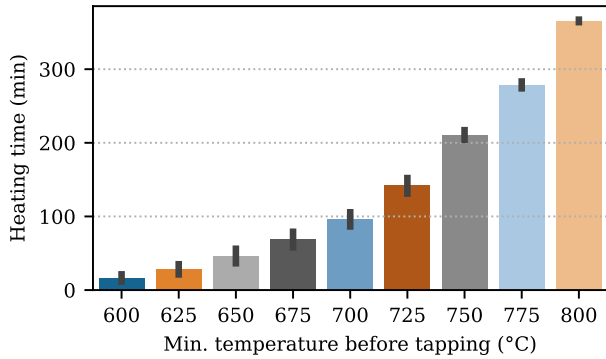


FIGURE 13. Heating times per constraint value. The error bars display the 95 % confidence interval calculated with the bootstrap technique.

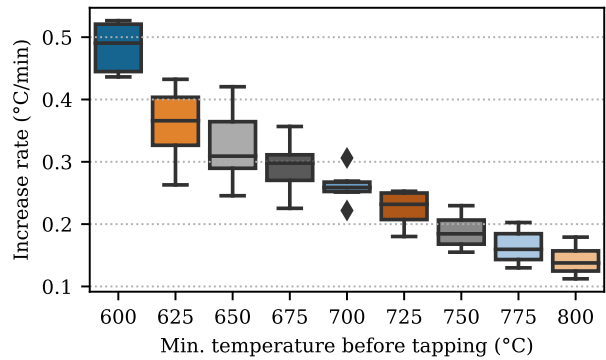


FIGURE 14. Boxplot of the temperature increase rate per constraint value. It is calculated as the difference between the initial and final temperature of the heating stage divided by its duration.

applied in ladles at lower temperatures, maximizing the refractory’s energy absorption.

The improved thermal management of the steel ladles leads to a reduction in process variability. To illustrate this, we display the distribution of the refractory temperature during the steelmaking (full) stage in Fig. 15. Notice that the temperature becomes more stable as the constraint value increases. In the context of steel temperature control, this could lead to lower deviations in the casting temperature

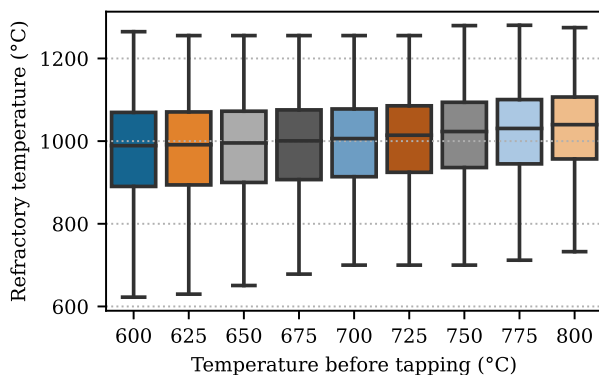


FIGURE 15. Boxplot of the ladle temperature during the steelmaking (full) stage. The average temperature increases and is followed by a reduction in the variability as the constraint value becomes more strict.

and allow steelmakers to increase the production throughput. Moreover, increasing the initial refractory temperature will reduce the heat exchange rate with the steel. Hence, it is inversely proportional to the steel temperature losses [34].

Heating the ladles comes with the downside of an increase in the CO₂ emissions, as it needs to burn natural gas. We can estimate the Scope 1 emissions from heating the ladles as described in [36]. A comparison between the predicted emissions and steel temperature losses is displayed in Fig. 16. Note that we can reduce the average steel temperature losses by around 3 °C depending on the refractory temperature requirement. Reducing steel temperature losses can result in further savings in emissions during primary and secondary refining [36], [37]. Hence, there is a trade-off between emissions from heating the ladles and steel temperature losses that should be further explored to understand the process’s energy efficiency thoroughly.

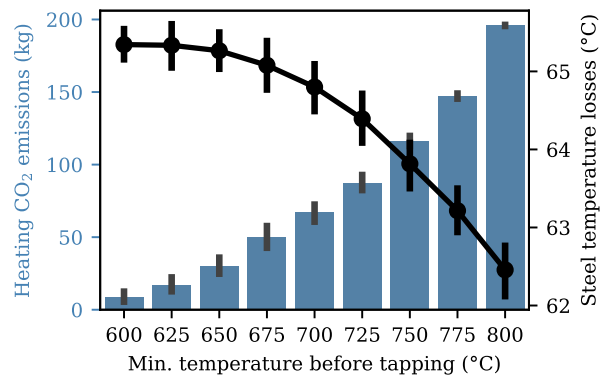


FIGURE 16. The compromise between the emissions and steel temperature losses. The bars display the average CO₂ emissions and the line the average steel temperature losses. The error bars display the 95 % confidence interval calculated with the bootstrap technique. All values displayed are per charge.

C. MANAGERIAL INSIGHTS

The proposed solution can be used for training to initiate discussions regarding the optimal thermal management of steel ladles. However, the solution method proposed in this paper must be improved to enable the model’s application in real time.

As a prerequisite, accurate models of the thermal behavior of steel ladles must be developed. They should be validated through dedicated campaigns using thermocouple measurements. Moreover, it is essential to ensure that ladle tracking systems are in place to collect ladle logistics data reliably. Investing in laser scanning technology to measure the wear level and thermal state of the ladles as frequently as possible is also crucial to accurately estimate the thermal state of the ladle in the eventual real-time application of the model.

Deploying a production scheduling system is a known organizational challenge [38]. Because of the complexity involved in the models, involving stakeholders as early as possible in the development process is essential for its success. Hence, change management methods are recommended to ensure a smoother adoption of the technology [39].

Finally, ladles will still be required in every steel plant even after the industry’s energy transition. Hence, evaluating ladle re-heating technologies with cleaner fuels, such as electrical and hydrogen-based burners instead of natural gas, is advisable to reduce CO₂ emissions further. We highlight that hydrogen-based technology is currently under development and should be carefully investigated before its adoption [40].

VI. CONCLUSION

A novel ladle dispatching with temperature control problem using non-linear refractory temperature models was proposed and solved with piecewise linear models with logarithmic coding and state-of-the-art MILP solvers. The objective is to minimize the sum of waiting and heating times during empty-ladle operations, constrained by a minimum ladle temperature requirement for tapping. The benefits of the model were evaluated for the Tata Steel, IJmuiden, use case.

The energy savings potential was analyzed for a small but representative production scenario. We showed that the heating applied during the cycle increases with the minimum temperature requirement. Based on the results, it is possible to derive operation rules for ladle dispatching. For example, cold ladles should be heated right before use. Some of the rules derived from our analysis are already current practices in steel plants. Our analysis provides a validation/legitimization for these established rules.

With the control of the ladle temperature before tapping, a reduction of the variability of the process was also observed. We also showed how the ladle’s energy efficiency decreases once more heating is applied, varying significantly with the ladle’s lifetime. It allows the definition of a range of values that balance process stability and heating efficiency. It should be noted that heating the ladles during the cycle yields CO₂ emissions. Hence, we highlight the trade-off between heating and steel temperature losses. We showed that the average steel temperature losses can be reduced by up to 3 °C depending on the refractory temperature requirement.

Our results demonstrated the importance of considering the energy balance in the ladle dispatching problem. However, future works must address open issues to unlock the model’s full potential. First, improvements to the solution approach should be investigated to allow the use of the lifetime as an input to the model and its application to large production scenarios and in real-time. It can include, for example, developing heuristic or machine learning approaches or an enhanced triangulation method. Moreover, model improvements should be applied to consider decisions that involve ladles entering or leaving the cycle. For example, adding the cost of pre-heating the ladles before the cycle can help practitioners synchronize this operation to unlock further savings. Finally, evaluating the CO₂ emissions and steel temperature losses trade-off in a multi-objective solution to the problem can also be the subject of future works. It can provide more insights into the CO₂ footprint of ladle operations and contribute even further to the sustainability of the steel industry.

**APPENDIX A
INSTANCE CONFIGURATION**

A. PARAMETERS

The interested reader can find a detailed description of the Tata Steel, IJmuiden, SCC plant in [29]. In this work, we consider that each stage has three machines operating in parallel. The transportation times are displayed in Table 4. In addition, ladles have a minimum maintenance time of 25 minutes, and each charge’s pouring time is defined as 20 minutes. The production schedule is available in Table 5. The steelmaking duration is calculated as the difference between the casting start and tapping start values.

TABLE 4. Transportation time between stages.

Origin	Destination	Duration (min)
SM	MT	10
MT	HT	10
HT	WT	10
WT	SM	5

TABLE 5. The production schedule used for the problem instance.

Cast	Charge	Tapping start (min)	Casting start (min)	Casting duration (min)
1	1	0	146	35
1	2	52	182	34
1	3	94	226	42
1	4	135	264	36
1	5	196	305	39
2	6	288	475	49
2	7	327	526	50
2	8	368	579	51
2	9	473	631	50
2	10	518	686	53
2	11	558	739	51
3	12	687	917	42
3	13	802	961	42
3	14	881	1001	38
3	15	927	1040	37
3	16	949	1077	36
3	17	1012	1119	39
3	18	1032	1159	38
3	19	1075	1198	38
3	20	1125	1242	42
3	21	1173	1282	37

B. BOUNDS DEFINITION

The definition of bounds to the input variables is required to apply the piecewise linear models [28]. Hence, we established a maximum idle and heating duration of 500 minutes for each stage. Naturally, the minimum stage duration is zero. Moreover, the refractory temperature is allowed to be between 400 °C and 1350 °C. These limits are considered for generating the approximation grid and as bounds for each decision variable connected to a piecewise linear model. They were chosen based on the data used to fit the thermal models described in Section IV-A.

In addition, the mathematical problem defined in Section III employs the big-M technique for modeling discrete

choices. Although they are simple and do not require additional constraints and continuous variables, they can compromise the computational performance. Their application requires the definition of a sufficiently large number (UB) that directly affects the tightness of the optimization problem. We define it as the sum of the casting end time of the last charge and the maximum possible duration of all empty stages (1500 minutes). This value is not expected to be the tightest possible. However, it is simple to calculate for other instances and good enough for the objectives of this paper.

APPENDIX B COMPUTATIONAL RESULTS

See Table 6.

TABLE 6. Optimality gap statistics considering a maximum runtime of 10000 seconds.

Temperature requirement (°C)	Mean	St.Dev.	Min	Max
600	0.43	0.22	0.15	0.77
625	0.17	0.12	0.09	0.35
650	0.40	0.69	0.09	1.82
675	0.30	0.40	0.09	1.11
700	0.42	0.45	0.09	1.26
725	0.17	0.18	0.09	0.54
750	0.70	0.54	0.09	1.58
775	0.74	0.50	0.09	1.38
800	2.19	0.56	1.37	2.76

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