IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received 5 October 2023, accepted 1 November 2023, date of publication 1 December 2023, date of current version 20 December 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3339121

RESEARCH ARTICLE

Transportation Centrality: Quantifying the Relative Importance of Nodes in Transportation Networks Based on Traffic Modeling

MAHENDRA PIRAVEENAN AND NARESSA BELLE SARIPADA
Modelling and Simulation Group, School of Computer Science, Faculty of Engineering, The University of Sydney, Sydney, NSW 2006, Australia
Corresponding author: Mahendra Piraveenan (mahendrarajah.piraveenan@sydney.edu.au)

ABSTRACT The topology of transportation networks such as road and rail networks determines the efficiency and effectiveness of the corresponding transport systems. Quantifying the relative importance of nodes of such networks is vital to understand their dynamics. Centrality metrics which are used in network science often make the assumption that only the shortest paths contribute to the importance of the nodes. In traffic scenarios however, while most traffic would preferentially go though paths of least cost, paths which are costlier are not omitted entirely. In this work, we introduce a new centrality metric, transportation centrality, which considers all paths that go through a node, and uses Logit functions and path lengths to compute the traffic which goes through each path, which in turn is used in centrality calculation. Therefore, this metric can be calculated based on topology alone, while it can also utilise traffic data if this is available. We demonstrate the utility of this new centrality metric by considering the suburban transportation networks of Seoul and Delhi. We also analyse the influence of the sensitivity parameter of the Logit function in the calculation of transportation centrality. We demonstrate that the introduced centrality metric is useful in understanding the relative importance of nodes in transportation networks, including networks for which no traffic data is available.

INDEX TERMS Transportation centrality, transportation networks, centrality, traffic modelling.

I. INTRODUCTION

Designing, modelling and understanding transportation networks is an aspect of great practical importance in urban planning and design [1], [2], [3]. In designing a transportation network, the efficiency of travel, minimisation of travel time, minimisation of travel cost, optimisation of safety, security and comfort, as well as the reliability and robustness of the network against failures and targeted attacks are important considerations. Transportation network topologies need to be designed in such a way that they are reliable and robust against natural and man-made disasters (such as bushfires, floods, earth quakes, and terrorist attacks), because during such events, transportation networks are the conduits of aid, including emergency services and relief supplies, as well as the conduits of escape for the affected population [4], [5], [6]. Therefore it is vital that they are designed to perform well during such disasters. On the other hand, transportation network topologies may need to face pressure due to deliberate choices in urban design. For example, road space reallocation [7], [8] is currently being undertaken in many cities, as these cities make a choice to move away from motor traffic and put more emphasis on public transport, pedestrian traffic and cycling, as well as sustainability initiatives such as parklands. When these processes happen, road space re-allocation takes place, yet the road network should have been designed in such a way that such reallocation does not critically impair the functioning of the road network. Therefore, designing road networks is a challenge which needs to take into account not only considerations of efficiency and cost, but also long term design imperatives and atypical scenarios such as disasters and extreme loads.

Modelling a transportation network at a level of abstraction which is amenable to topological analysis can be challenging due to many reasons. Complex network science often

The associate editor coordinating the review of this manuscript and approving it for publication was Yang Tang.

considers large networks where there are a limited number of node types or link types, or the attributes of nodes and links which are modelled are limited [9], [10]. In particular, a certain level of uniformity among the links is assumed, so that the topological aspects can be focussed upon. In real world transportation networks however, each link is different from others in some way, and it is often difficult to even define a particular link. Indeed, network reliability of real world transportation network has two dimensions [1], [2]: one is connectivity, or the ability for any node to be reached from any other node in the network, which is more amenable to topological analysis from a complex network science perspective. The second one is performance reliability, which measures the ability of a traveller to travel to their destination on schedule, within cost etc, which needs to consider a number of characteristics of nodes and links to be modelled, designed, or measured. Despite these difficulties, complex network science offers some powerful tools to the analysis of large-scale transportation networks, due to its simplicity and rigour.

Complex network science has traditionally used the concept of centrality to capture the relative importance of nodes and links in a system that can be represented as a complex network [9], [11], [12], [13]. A host of centrality measures exist [13], beginning from the classical betweenness centrality [14] (which measures the importance of a node in terms of the relative number of shortest paths which pass through that node) and closeness centrality [14], [15] (which measures the importance of a node in terms of its average geodesic distance form other nodes), to more recent measures such as eigen-vector centrality [16] and percolation centrality [17], which have more mathematically complex definitions, and often take into account quantities such as node attributes or states, link weights, importance of neighboring nodes etc. Nevertheless, it can be observed that those 'medial' centrality measures [18] where the concept of 'path' or 'route' is inherent (namely, betweenness centrality and its many variants), typically only consider shortest paths. Yet, it is obvious that in transportation networks, a passenger has the choice of using all routes, shortest or otherwise, and while passengers do tend to use routes which are less costly, the probability of other routes being used cannot be deemed zero without careful analysis.

Indeed, the concept of 'modal split' [19] in transport modelling is relevant here. Modal split refers to the proportion of traffic from a particular place A to another particular place B which chooses mode i when $i = 1, 2 \ldots M$ modes are available. Classical modal split models use Logit functions, and the underlying assumption is that the more costly a mode is, the less likely for a passenger to use it. However, when the cost of a mode increases, the corresponding probability of it being used decreases exponentially, rather than linearly. The underlying assumption is that people are sensitive to cost variations, and even a slight difference in cost will result in many people choosing the relatively cheaper mode. The same argument could be made for a person choosing between a number of routes, if the costs related to each route is known. This is a more realistic assumption than assuming people only travel through shortest paths, i.e paths which have the lowest cost in terms of distance. Therefore, if route costs are estimated from path lengths, then a centrality measure which captures the likelihood of a passenger travelling through a node based on these path costs is more realistic than a centrality measure which assumes that passengers use only the shortest path. The goal of this paper is to introduce and define such a centrality measure, which we name 'transportation centrality', and demonstrate its utility using a number of real-world road networks.

The contribution of this paper can be clearly elucidated in terms of novelty, necessity, and utility. The novelty will be self-evident because we present a new centrality measure with a clear and straightforward mathematical definition which has not been proposed or defined before. It is also dissimilar to any existing centrality measure, in that it uses Logit functions to estimate traffic flow and uses it in turn in centrality calculation - no other centrality measure has done something similar in any context. The necessity arises from the fact that in transport networks, users use all possible paths between origin and destination, though they are more likely to use the paths which cost less or are quicker. The existing centrality measures i) either focus on shortest paths alone (such as betweenness centrality [14], [20], [21] and its many variants) or ii) need transport data to be computed, such as mobility centrality [41] and DelayFlow centrality [42], [43], and similar transport specific measures which will be discussed in section II-B. Both have limitations because i) it is unrealistic to consider shortest paths alone, when real-world users will consider alternative paths ii) it is not realistic to have accurate transport data available for many cities, and it is impossible in the cases where we are modelling traffic in a new proposed city or suburb. Therefore, there is necessity for a centrality metric which considers all paths, and avoids the need for network data by reasonably estimating traffic based on topological structure and well-known traffic behaviour alone. Finally, the utility of the proposed centrality measure will be demonstrated by applying the metric to a set of real world transport networks.

It is important to note here that the scope of this paper is limited to the introduction of this key concept and preliminary analysis. While we do use topological data from real world transportation networks to illustrate the utility of transportation centrality, we have by no means presented an exhaustive analysis covering an extensive range of real world networks, nor have we focussed on very large transportation networks, deeming these to be beyond the scope of an introductory paper. These are the topics of future work.

The rest of this paper is organised as follows. In section II, we briefly discuss the existing centrality measures and their usage, highlighting that they are unsuitable to be used in the context of transportation network and traffic analysis, which needs to consider all available routes (not merely the shortest paths), volume of traffic and other factors. In section III we introduce and define the concept of 'transportation centrality', and consider some simple examples to illustrate its usage. In section IV we present results of detailed simulation experiments, using data from real world transportation networks from the cities of Seoul and Delhi. Finally, in section V we summarise and present our conclusions, and discuss ways in which this work could be advanced in the future.

II. BACKGROUND

Table 1 summarises the symbols used in this paper to denote various quantities.

TABLE 1. Table of symbols and notations.

Quantity	Symbol
Network size (Number of nodes)	N
Sensitivity Parameter	β
Cost of path i from source node s to target node t	$C^i_{s,t}$
Set of all simple paths from source node s to target node t	$\mathbf{P}_{s,t}$
Set of all simple paths from source node s to target node t which go through the considered node v	$\mathbf{P}^v_{s,t}$
The proportion of traffic between nodes s and t that pass through node v.	$Q^v_{s,t}$
Transportation Centrality of Node v	$TC(v)$
Betweenness Centrality of Node v	$BC(v)$
Closeness Centrality of Node v	$CC(v)$
Straightness Centrality of Node v	$SC(v)$
Percolation Centrality of Node v at time t	$PC^t(v)$
Number of all simple paths from source node s to target node r that pass through node v	$\sigma_{s,r}(v)$
Number of all simple paths from source node s to target node r	$\sigma_{s,r}$
Geodesic distance between nodes v and i	$d_g(v,i)$
Euclidean distance between nodes u and v	$d_E(u,v)$
Shortest path distance between nodes u and v	$d(u,v)$
Adjacency matrix of a network	$\mathbf{A}$
Percolation state of node i at time t	x^t_s
Traffic between nodes i and j	$T_{i.j}$
Traffic between nodes i and j via mode μ	$T^{\mu}_{i.j}$

A. REVIEW OF CENTRALITY MEASURES

Let us first define some fundamental concepts in network science. A network (graph) has N nodes (vertices) connected by L links (edges). The connection structure is called the topology of the network. Each node i has a number of links, k_i, attached to it, which is called the degree of the node i. A shortest path from node s to node t is any minimal set of links which connects s and t. That is, a shortest path is a path between two nodes that has the fewest links if the cost of traveling along each link is the same. If links have varying travel costs, then the shortest path can be defined as a path between s and t for which the associated travel cost is minimal.

A host of centrality measures have been proposed to analyze generic complex networks, especially in the domain of social network analysis. The simplest of these perhaps is the degree centrality, sometimes just called degree, of a node. A node's degree is simply the number of links it has with other nodes in the network, and therefore gives some indication about how important that node is to the network.

A family of betweenness measures have been proposed [14], [20], [21], [22], [23], [24], [25] to measure a node's importance as a conduit of information or traffic flow in a network. The first and perhaps most well-known measure of these is the classical betweenness centrality measure proposed by Freeman [20]. Betweenness centrality measures the fraction of shortest paths that pass through a given node, averaged over all pairs of nodes in a network. It is formally defined, for a directed graph, as

$$BC(v) = \frac{1}{(N-1)(N-2)} \sum_{s \neq v \neq r} \frac{\sigma_{s,r}(v)}{\sigma_{s,r}} \quad (1)$$

where $\sigma_{s,r}$ is the number of shortest paths between source node s and target node r, while $\sigma_{s,r}(v)$ is the number of shortest paths between source node s and target node r that pass through node v. N is the number of nodes (vertices) in the network. A number of weighted betweenness measures, such as the one presented by Wang et al. [25], where weights are given to links, have also been proposed recently.

Closeness centrality [14], [15] is a measure of how close a node is, on average, to the rest of the nodes in the network in terms of shortest paths. It essentially measures the average geodesic distance between a given node and all other nodes in the network. It is defined as

$$CC(v) = \frac{1}{\sum_{i \neq v} d_g(v,i)} \quad (2)$$

where $d_g(v, i)$ is the shortest path (geodesic) distance between nodes v and i. Note that the average is 'inverted' so that the node which is 'closest' to all other nodes will have the highest measure of closeness centrality. There is also the information centrality measure [24] based on closeness centrality, which measures the harmonic mean length of paths ending at a vertex v.

The eigen-vector centrality measure [16] is based on the assumption that a node's centrality is influenced by the centrality scores of its neighbours - that the centrality score of a node is proportional to the sum of the centrality scores of the neighbours. As such, it is defined iteratively. If the centrality scores of nodes are given by the matrix X and the adjacency matrix of the network is A, then we can define x iteratively as

$$\mathbf{x} \propto \mathbf{Ax} \tag{3}$$

i.e

$$\lambda \mathbf{x} = \mathbf{Ax} \tag{4}$$

The centrality scores are obtained by solving this matrix equation. It can be shown that, while there can be many values for λ, only the largest value will result in positive scores for all nodes [26].

Straightness centrality [27], [28] measures the efficiency of the routes that begin at a particular node. The efficiency of a route is measured as the ratio between the Euclidian distance and the shortest path distance (along the path) between two nodes. It is assumed that if the Euclidean distance is much shorter than the distance along the path, then that path is less efficient. Therefore, the straightness centrality is defined as

$$SC(v) = \frac{1}{N-1} \sum_{u \neq v} \frac{d_E(u, v)}{d(u, v)} \tag{5}$$

whereby $d_E(u, v)$ is the Euclidian distance and $d(u, v)$ is the the shortest path distance between nodes u and v.

The classical betweenness centrality measure assumes that information flow is through the shortest paths in a network. This is, in many instances, not a realistic assumption [21], [23], [24]. For example, rumours or infections in social networks are likely to follow random paths. Water in a network of canals and electricity in electric circuits will flow through paths of least resistance, not necessarily the shortest ones. A number of centrality measures based on betweenness address this. The flow centrality measure [20] measures the proportion of the 'flow' that goes through a given node, when maximum flow is 'pumped through' a pair of nodes. A random walk-based betweenness measure proposed by Newman [23] considers a network to be like an electric circuit with unit resistance at any link, and measures the 'current' that goes through a node when unit current passes through a pair of nodes. There are a number of other centrality measures based on random walks as well, such as those described in Noh and Rieger [29] or Bonacich [30]. The random-walk centrality introduced by Noh and Rieger [29] measures the average speed with which, a randomly walking message from a node reaches the target node v, averaged over all source nodes. The power centrality [23], [30] of a node v is the number of times a random walk is expected to pass through the node v, averaged over all possible starting points of the random walk.

Percolation centrality (PC) specifically measures the importance of nodes in terms of aiding the percolation through the network [17]. Percolation centrality for a given node, at a given time, is the proportion of 'percolated paths' that go through that node. A 'percolated path' is a shortest path between a pair of nodes, where the source node is fully or partially percolated (e.g., infected). The target node can be percolated or non-percolated, or in a partially percolated state. Formally, percolation centrality of node v at time t is:

$$PC^t(v) = \frac{1}{(N-2)} \sum_{s \neq v \neq r} \frac{\sigma_{s,r}(v)}{\sigma_{s,r}} \frac{x_s^t}{[\sum x_i^t] - x_v^t} \tag{6}$$

where $\sigma_{s,r}$ and $\sigma_{s,r}(v)$ are defined as they are in the definition of betweenness centrality, and the percolation state of node i at time t is denoted by x_i^t. Specifically, $x_i^t = 0$ indicates a non-percolated state at time t, $x_i^t = 1$ indicates a fully percolated state at time t, while a partially percolated state means $0 < x_i^t < 1$ (e.g., for a network of townships used to model infectious disease dynamics, this would be the percentage of people infected in that town). The percolation state associated with a source determines how much importance is given to the potential percolated paths that originate from it. The centrality measure proposed by Berahmand et al. [31], [32] is in this sense similar to percolation centrality, in that it also aims to identify influential spreaders - that is, nodes which are important for the percolation or diffusion of some quantity. Other studies such as Berahmand et al. [33] have looked at the relationship between topological structure and diffusion.

Borgatti and Everett [18] classified the existing centrality measures into radial and medial measures. Radial centralities count walks which start/end from the given vertex. Degree centrality, closeness centrality, eigen-vector centrality, and straightness centrality are examples of this. Meanwhile, medial centralities count walks which pass through the given vertex. Betweenness centrality and its many variants, as well as percolation centrality are examples of this. Centrality measures can also be classified as volume-based or length-based measures. In volume-based centrality measures, the number of paths through / to a particular node is counted. Degree centrality and betweenness centrality are examples of this. In length-based centrality measures, the length of paths through / to a particular node is counted. Closeness centrality is a classical example.

It is also worth to note here briefly that there are avenues of research in complex network science which do not directly measure centrality, but are useful to quantify diffusion, spreading, and traffic in complex networks in a generic sense. For example, Berahmand et al. [34] propose a community detection method which hinges on detecting 'core nodes', which are, in essence, nodes which are important for sustaining community structure. Similarly, Liu et al. [35] discuss the detection of 'controlling nodes' which can exert disproportional influence on the dynamics of the network. In that sense, these nodes also have high importance. However, such methods of quantifying node importance have rarely been used in transportation networks so far, and we will not discuss them further.

B. APPLICATION OF CENTRALITY MEASURES IN TRANSPORTATION NETWORKS

There is indeed a number of research papers which have focussed on using network centrality measures for the purposes of analysing transportation networks. These can be grouped into two categories: 1) papers which use or apply existing generic centrality measures in transportation networks (including papers that review such efforts) 2) papers that introduce a new centrality measure with the express purpose of using it in transportation networks, usually to identify the relative topological importance of nodes in such networks.

Examples of the first category include Jayasinghe et al. [36], who use existing centrality measures to study the importance of nodes in a transportation network in Colombo, Sri Lanka. They use topological metrices such as 'connectivity' (node degree), global intergration and local integration (versions of closeness centrality), and 'choice' (a version of betweenness centrality) in their analysis. Stamos [37] reviews the utility of 17 existing centrality measures, such as betweenness centrality, closeness centrality, and percolation centrality, in transportation networks, particularly in the context of quantifying the importance of nodes during extreme whether events and climate-related disasters. Napitupulu et al. [38] use existing centrality measures such as degree centrality, closeness centrality, betweenness centrality and eigenvector centrality to determine the relative importance of nodes in the road network within the University of Padjadjaran, Jatinangor campus, Indonesia. Wang et al. [39] study the correlation between street (edge) centrality in road networks and land use in Baton Rouge, Louisiana, United States, and they use a combination of existing centrality measures, such as closeness centrality, betweenness centrality and straightness centrality [27], [28] to measure the importance of streets in this town. Finally, Chakrabarti et al. [40] study the correlation between transportation centrality and housing prices in Kolkatta, India, and they consider neighbourhoods as nodes and intra-neighbourhood roads as links in their transportation network. They use betweenness centrality, closeness centrality, and eigenvector centrality to determine the relative importance of these neigbourhoods in intra-neighbourhood road networks. In all these works, well known generic centrality measures are applied on road networks at varying levels of abstraction.

Examples of the second category include Tsiotas and Polizos [41] which introduce 'mobility centrality', a metric that measures the amount of kinetic energy flows through nodes to estimate their related importance. To do so, they need the 'average velocity' of traffic through each link, which they estimate from empirical data from Greece. Hence it is not a measure that can be calculated from topology alone, and needs traffic data in its computation. Cheng et al. [42], [43] introduce 'DelayFlow centrality' which similarly used travel time delay and commuter flow data to quantify the importance of a node in a transportation network. They use data from Singapore in their analysis. Meanwhile, Li et al. [44] propose 'PageRank Algorithm Modified by the Gravity Model (APAMGM)', a measure based on pagerank algorithm and adapted to multiplex networks, and apply it to analyse the urban transportation network of Shenzhen, China, modelled as a multiplex network. Zadeh and Rajabi [45] propose a new centrality measure, which is named as targeted constrained betweenness (TCB) centrality, which, as the name implies, is a version of betweenness centrality, and implemented via an iterative algorithm which computes the Wardrop equilibrium [46] in the considered network and uses flows calculated from it to iteratively calculate edge betweenness of each link in the network. The process terminates when the improvement in edge betweenness calculations become negligible compared to the previous iteration (presumably based on a pre-defined threshold). It could be noted that this centrality measure, which considered edge importance rather than node importance, is computationally expensive to calculate for all but the smallest networks due to its iterative nature.

Even though papers in the second category introduce centrality measures specifically to analyse transportation networks, one key limitation can be observed in all of them. They need traffic data pertaining to a particular network to be computed, or they assume all traffic flows through the shortest paths. These metrics have not proposed a way to consider all possible paths on principle, unless there is pre-existing data that can be used with regards to all paths. In this paper, we propose a relatively simple topology-based centrality measure, which can be computed without traffic data and thus can be applied to any given network, which nevertheless considers all possible paths in a network, and calculates (rather than simulates, or uses pre-existing observations of) the possible traffic flow on all paths. The key novelty is that the proposed centrality measure is defined and calculated in a way that is relevant to transportation networks.

III. METHODOLOGY

A. TRANSPORTATION CENTRALITY–MOTIVATION AND DEFINITION

Betweenness centrality, being a purely topological measure, quantifies the importance of nodes only in terms of their relative placement. However, in road or other transportation networks, the importance of a node (a junction, station, suburb, or region, according to what that transportation network represents) depends not only on its positioning in the network, but also how much traffic, on average, flows through it. If data about traffic through different routes is available, it would be easy to calculate the relative importance of junctions from it, as has been done in several works [41], [42], [43], [45] discussed in section II. However, such data is not always available. What is usually available instead is an estimation of 'costs' related to using a route or mode of transport.

Indeed, in transport science, such costs are often used to calculate 'modal split' - the amount of traffic through different modes [19], [47]. Specifically, if we consider two nodes i, j in a transportation network, and if we assume that a total number of M modes are available for traffic between these pair of nodes, and each of these modes μ is associated with a cost $C^{\mu}_{i,j}$, then the proportion of traffic between i and j that would use the mode μ can be estimated by [47], [48], and [49]:

$$P^{\mu}_{i,j} = \frac{T^{\mu}_{i,j}}{T_{i,j}} = \frac{e^{-\beta C^{\mu}_{i,j}}}{\sum_{\mu=1}^{M} e^{-\beta C^{\mu}_{i,j}}} \quad (7)$$

where M is the number of modes present. Here, β is a parameter of the model which quantifies the sensitivity of the model to mode costs. If $\beta = 0$, the model is insensitive to mode costs, and the traffic would be equally divided between all the available modes. On the other hand, $\beta = \infty$, the model will become 'infinitely' sensitive to mode cost, that is, all traffic will flow through the mode which has the least cost associated to it. Therefore, any realistic model will have a positive finite number as β, which falls within these extremes. The value of β could be calibrated from real world traffic data.

The modal split model assumes that there will not be a linear relationship between the mode costs and the proportion of traffic choosing those modes. Obviously, the smaller the cost of the mode, the more traffic will flow through that mode, but even a small difference in costs may result in a significant portion of traffic switching modes. Therefore, the modal split model uses a Fermi function [50], [51] to model the split of traffic.

Indeed, the modal split method could equally well be applied to estimate the split of traffic between different routes within the same mode. The understanding that people are more likely to choose less costly modes, and this relationship between mode cost and probability of choice is not linear, is applicable when traffic chooses between different routes with varying route costs as well. Our motivation in this work is to use this observation to calculate the importance of nodes to traffic, if costs related to different paths can be estimated. Clearly, if there is more than one route / path between a pair of nodes, then the split of traffic between these nodes will depend on the costs of the paths. Therefore, it makes sense to use the modal split model to estimate the traffic along multiple routes, which may exist between a pair of nodes. Therefore, we propose a centrality measure where each path has a cost associated to it, and the centrality of the nodes depend on path costs in an exponential manner governed by a Fermi function. Therefore, we define 'transportation centrality' of a given node v as:

$$TC(v) = \frac{1}{(N-1)(N-2)}\left[\sum_{s \neq v \neq t} \frac{\sum_{i \in \mathbf{P}^{v}_{s,t}} e^{-\beta C^{i}_{s,t}}}{\sum_{j \in \mathbf{P}_{s,t}} e^{-\beta C^{j}_{s,t}}}\right] \quad (8)$$

where $C^{i}_{s,t}$ is the cost of path i which goes from source node s to target node t, $\mathbf{P}^{v}_{s,t}$ is the set of all simple paths from source node s to target node t which go through the considered node v, and $\mathbf{P}_{s,t}$ is the set of all simple paths from source node s to target node t. β is a tunable parameter of transportation centrality, which represents the sensitivity of transportation centrality to cost differences of paths. A 'simple path' here is defined as 'a path that repeats no vertex' [52].

For simplicity, we may also write

$$TC(v) = \frac{1}{(N-1)(N-2)}\left[\sum_{s \neq v \neq t} Q^{v}_{s,t}\right] \quad (9)$$

where $Q^{v}_{s,t}$ represents the proportion of traffic between nodes s and t that pass through node v. That is,

$$Q^{v}_{s,t} = \frac{\sum_{i \in \mathbf{P}^{v}_{s,t}} e^{-\beta C^{i}_{s,t}}}{\sum_{j \in \mathbf{P}_{s,t}} e^{-\beta C^{j}_{s,t}}} \quad (10)$$

In general, path costs are needed to calculate transportation centrality. The 'cost' can be modelled explicitly if data about path costs is available, but in the absence of such data, the costs can be estimated simply by the path length, or weighted path length. Therefore, transportation centrality in its basic form is a purely topological measure: nevertheless, unlike betweenness centrality, it explicitly captures the behaviour of traffic flow. Indeed, the strength of transportation centrality is that it is able to consider estimated traffic flows through links, even when path costs are not available, while remaining a purely topological measure. It achieves this uniqueness by using Fermi functions to estimate traffic flow through all paths (not just shortest paths) in a realistic manner, and using path lengths as input in these Fermi functions. This is quite realistic as any driver is likely to use the distance involved in each route as an estimate for the cost involved in choosing that route. Therefore, we will use this purely topological interpretation of transportation centrality in the rest of the paper, with the understanding that if more direct estimates of path costs are available, these can be used in the calculation of transportation centrality as well.

If $\beta = \infty$, this means that traffic is infinitely sensitive to path costs, and thus will use only the shortest paths: thus transportation centrality reduces to betweenness centrality. Furthermore, in this case the path costs are immaterial and are not needed for the calculation of transportation centrality. Therefore, transportation centrality is essentially a generic form of betweenness centrality where the cost of paths is explicitly taken into account. On the other hand, if $\beta = 0$ again path costs are immaterial, but transportation centrality reduces to another interesting centrality measure in its own right, one where the proportion of paths going through a particular node is calculated and aggregated for all source-target node pairs, like in the case of betweenness centrality, except that all simple paths are taken into account. Indeed, this centrality measure is relevant to topological analysis of networks from all domains, and we name it 'All-path Betweenness Centrality (ABC)'. However, we will not focus on it specifically in this paper. Finally, let us note that due to the negative exponential function having a range of [0,1], it could be derived from Eq. 8 that for any finite positive value of β, the transportation centrality value of a node will be greater than or equal to its betweenness centrality value. Therefore, the 'raw' values of transportation centrality are not as informative as the rankings of nodes belonging to a particular network which are obtained by sorting them according to the transportation centrality values.

Note here that according to the classification of Borgatti and Everett [18], transportation centrality is a medial centrality: it counts the paths that pass through the considered node, rather than paths that originate or end in the considered node. It is also a volume-based centrality measure, in that it counts the number of paths that pass through a node. Indeed the phrase 'volume' has a very direct interpretation in transportation centrality, as the traffic volume, because essentially, with or without traffic data, the transportation centrality estimates the traffic volume that passes through a given node. What is interesting however is that transportation centrality can also be a length-based centrality measure. If we do not have data for link costs, and estimate route costs by path lengths, then we essentially also count the lengths of the paths that pass trough a given node, though this counting is not done linearly, as would be the case with most other length-based centrality measures such as closeness centrality. Rather, path lengths are used as inputs to Fermi functions which are then used to estimate path costs. Therefore, transportation centrality is an interesting hybrid of volume-based and length-based centrality measures.

B. TRANSPORTATION CENTRALITY–A SIMPLE EXAMPLE

To provide an extremely simple example of the calculation of transportation centrality, let us consider the network shown in Fig. 1. Suppose we want to calculate the transportation centrality of node V_3 in this network. Since V_3 is the node under consideration, the possible source-target pairs not involving this node are [1, 2], [1, 4], [2, 4]. If we consider the pair [1, 2], there are two 'simple paths' between these pairs, namely [1, 2] and [1, 3, 2]. The path [1, 2] has a cost of '1 hop' and does not pass through node V_3, while the path [1, 3, 2] has a cost of 'two hops' and does pass through node V_3. Therefore, the transportation centrality contribution from this pair to node V_3, which is also the estimated proportion of traffic from V_1 to V_2 that passes through V_3, could be calculated as:

$$Q^3_{1,2} = \frac{e^{(-2\beta)}}{e^{(-1\beta)} + e^{(-2\beta)}} \tag{11}$$

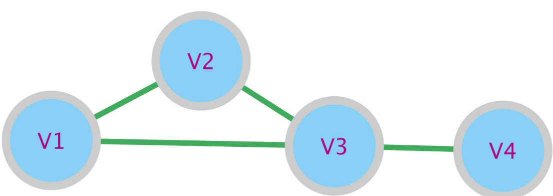


FIGURE 1. An example network for calculating transportation centrality.

Node Pair	List of all simple paths
V_1, V_2	[1,2], [1,3,2]
V_1, V_3	[1,3], [1,2,3]
V_1, V_4	[1,3,4], [1,2,3,4]
V_2, V_3	[2,3], [2,1,3]
V_2, V_4	[2,3,4], [2,1,3,4]
V_3, V_4	[3,4]

FIGURE 2. A simple example for calculating transportation centrality. The list of all simple paths for the network shown in Fig. 1.

Similarly, if we consider the source-target pair [1, 4], there are two simple paths between this pair: [1, 3, 4] and [1, 2, 3, 4]. One of them has a cost of two hops, while the other has a cost of three hops. Therefore, the contribution from the pair [1, 4] to V_3 could be written as:

$$Q^3_{1,4} = \frac{e^{(-2\beta)} + e^{(-3\beta)}}{e^{(-2\beta)} + e^{(-3\beta)}} = 1 \tag{12}$$

which is of course, equal to one as every path which is between this pair of nodes goes through node V_3.

Similarly, if we take the pair of nodes [2, 4], we may also see that there are two simple paths [2, 3, 4], [2, 1, 3, 4], both of which go through node V_3. Therefore, $Q^3_{2,4} = 1$, trivially.

Since the network is non-directed in this example, it is sufficient to consider each pair of nodes once and we may ignore the reverse direction, and simply multiply the summation we obtain by two. The number of nodes $N = 4$. Therefore, the transportation centrality of node V_3 would be given by:

$$TC(3) = \frac{1}{(4-1)(4-2)} 2[\frac{e^{(-2\beta)}}{e^{(-1\beta)} + e^{(-2\beta)}} + 1 + 1] \tag{13}$$

$$= \frac{1}{3}[\frac{3e^{(-2\beta)} + 2e^{(-1\beta)}}{e^{(-1\beta)} + e^{(-2\beta)}}] \tag{14}$$

therefore:

$$TC(3) = \frac{3e^{(-2\beta)} + 2e^{(-1\beta)}}{3\,e^{(-2\beta)} + 3\,e^{(-1\beta)}} \tag{15}$$

$$= \frac{3e^{(-1\beta)} + 2}{3\,e^{(-1\beta)} + 3} \tag{16}$$

Note well that the transportation centrality of a node will always depend on parameter β. If $\beta = \infty$, then $TC(3) = \frac{2}{3}$, which, it can be verified, is the betweenness centrality of node V_3 in this simple network. As the value of β increases towards infinity, the transportation centrality of node V_3 will converge (from above) towards 2/3, since this node provides a 'shortcut' to two out of three pairs of other nodes in it's network, and as traffic which becomes more sensitive towards cost (as the increase of β represents), would prefer to go through this node for traffic from / to these two pairs, and

TABLE 2. TC and BC values for nodes of the network in Fig. 1.

Node	BC	TC	TC ($\beta = 2.0$)
V1	0	$\frac{2e^{(-\beta)}}{3e^{(-\beta)}+3}$	0.08
V2	0	$\frac{2e^{(-\beta)}}{3e^{(-\beta)}+3}$	0.08
V3	2/3 = 0.67	$\frac{3e^{(-\beta)}+2}{3e^{(-\beta)}+3}$	0.706
V4	0	0	0

avoid this node for the third pair. On the other hand, when $\beta = 0$, this represents people randomly choosing routes without consideration for cost, and in this case it can be seen that $TC(3) = \frac{5}{6} = 0.83$.

Therefore the value of $TC(3)$ ranges from 0.83 to $2/3 = 0.67$, depending on β. As we will state later, studies calibrating the sensitivity parameter β to real-world data have shown that β tends to be between 0.1 and 0.2 [53], however in this simple example let us use $\beta = 2.0$. For this value, $TC(3)$ is given by:

$$TC^t(3) = \frac{3e^{(-4)} + 2e^{(-2)}}{3\ e^{(-4)} + 3\ e^{(-2)}} = 0.706 \tag{17}$$

Table 2 shows the betweenness centrality and transportation centrality values of all nodes in Fig. 1. It can be seen that the TC and BC for node V_4 are trivially zero, since this node has only one link and is a peripheral node. For the other three nodes, there are varying TC values. It can be noted that for node V_2, the BC is zero since no shortest path goes though it, though the TC is not zero since some paths do go through it. It can be imagined that in a transportation network of similar topology, some traffic will indeed go through node V_2 especially if there is heavy traffic through all other nodes, since while this is not a short cut to any destination, the travel times might be smaller since most traffic avoids this node. This is what the non-zero value of the TC for this node captures and BC fails to capture. That is, TC makes a distinction between nodes such as V_4 which are strictly peripheral, and nodes such as V_1 and V_2 which are not, while BC fails to make this distinction.

The above-described canonical example demonstrates the utility of transportation centrality, and how it subsumes betweenness centrality as well as how it can be calibrated (by adjusting the parameter β) for real world applications. What is also important to note is that, for all finite values of β, transportation centrality will return a more 'realistic' assessment about the relative importance of nodes by considering non-shortest paths, while we retain the ability to model the sensitivity of traffic to path costs by calibrating β.

C. SOFTWARE AND TOOLS

The simulation experiments were done using software code developed in-house, using the Python language. The NetworkX library was extensively used [54], [55]. Code used in the simulations is made available in GitHub [56]. The Cytoscape software tool [57] was also used to generate the network figures.

IV. SIMULATION RESULTS

A. EXPERIMENTAL SETUP

To provide some real-world examples of the transportation centrality metric, we applied it on two metropolitan transportation networks. The network data was obtained from the data made available by Karduni et al. [58] and the cities of Seoul and Delhi were chosen for the analysis, since these cities had suitable well-defined subnetworks belonging to individual suburbs which are well-suited to transportation centrality analysis. The complete road network of each city is shown in Fig. 3 [58]. We can see from this figure that each city has suburbs where the road network is dense as well as suburbs where the road network is sparse. Each city also has waterways which divide the road network into sections which have relatively very little connectivity with each other. The nodes in this dataset represent junctions where roads actually intersect, and edges represent sections of road which connect these junctions. A crossing of two roads in three-dimensional space where there is no actual intersection, such as when a fly-over, bridge, or tunnel passes over or under another road, is not represented as a node.

In this work we choose transportation networks belonging to four suburbs from each city: For Seoul, we choose (i) Yeomgok (ii) Gwancheol (iii) Jongno2i (iv) Jongno6yuk (it appears that some suburb names are alphanumeral). From Delhi, we choose (i) Dev Nagar (ii) Kucha Pandit (iii) Sangam Vihar East (iv) Sundar Nagari. As figures 4 and 5 show, this choice from several possible suburbs was made so that the selected subnetworks have varying topologies. For example, in Seoul, Jongno2i and Jongno6yuk have road networks which have more or less scale-free topologies [9], [59], while Gwancheol has a grid-like subnetwork, and Yeomgok's subnetwork is dominated by a ring (representing a ring-road). Similarly, in the Delhi road network, Kucha Pandit and Sangam Vihar East have topologies resembling a scale-free structure, Sunder Nagari seems to have a section of a ring attached to a scale-free like structure, while Dev Nagar has a grid-like structure. The deliberate choice of suburbs with prominent topological classes including scale-free, grid, and ring structures was to illustrate how transportation centrality values for nodes differ from other centrality (particularly betweenness) values for them depending on network type. Thus we are able to look at different suburban road network topologies through these sample networks. Thus we could use these to analyse the utility of transportation centrality on differing topologies.

B. RANKING NODES BASED ON BETWEENNESS CENTRALITY AND TRANSPORTATION CENTRALITY

We computed the betweenness centrality and transportation centrality values for all nodes (road intersections or junctions)

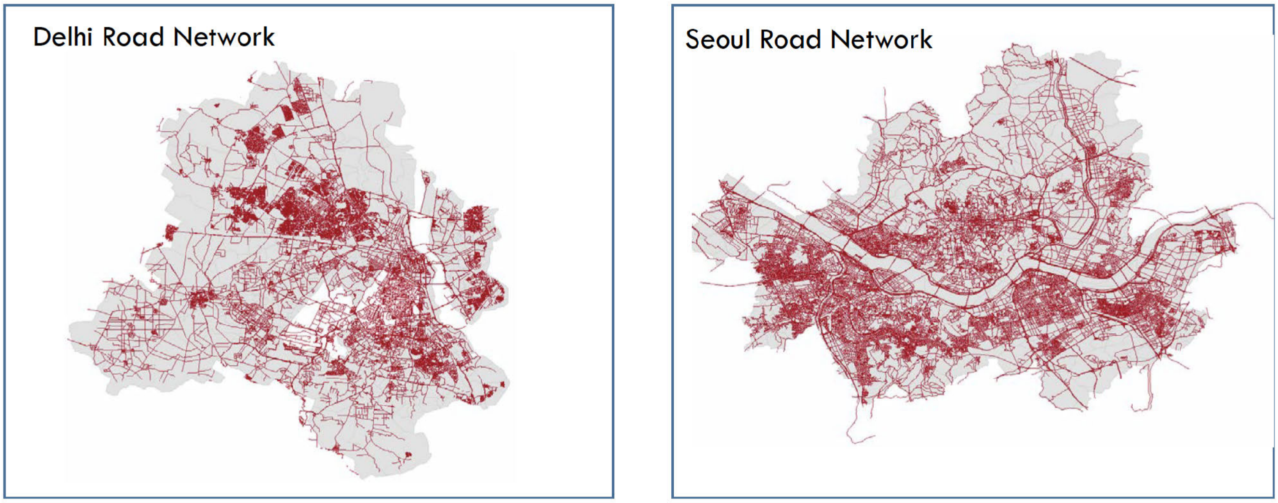


FIGURE 3. Delhi and Seoul Road networks, produced from publicly available data by Karduni et al. [58].

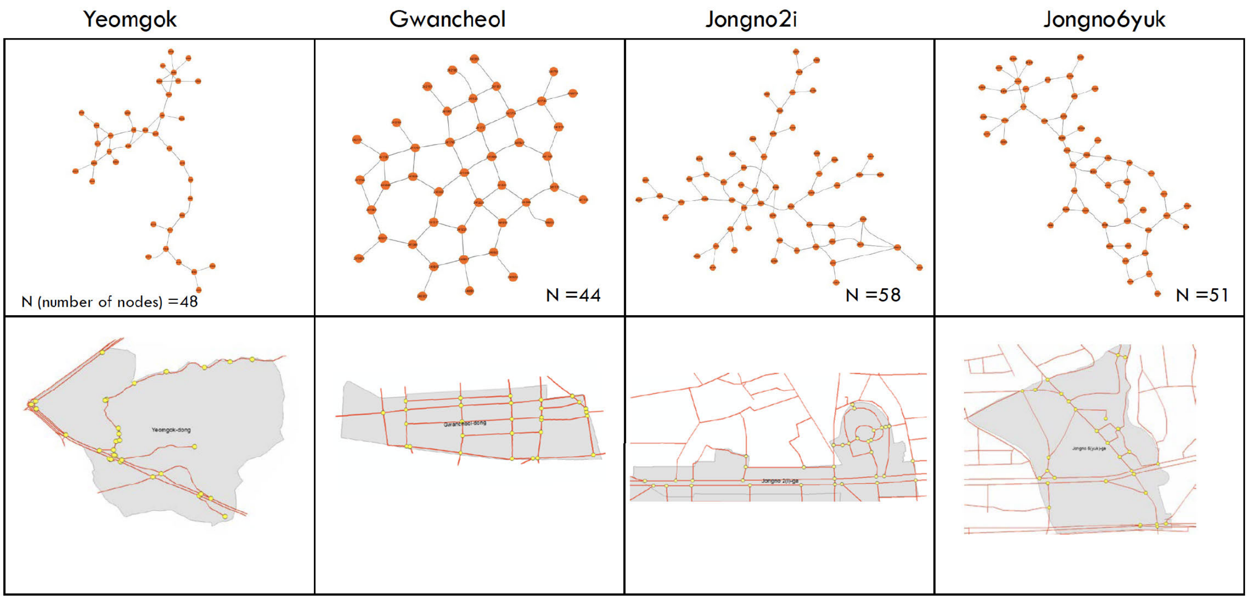


FIGURE 4. Sample subnetworks from Seoul.

in the eight above-mentioned suburbs. Betweenness centrality was chosen for this comparative analysis with transportation centrality because it is computed based on paths, like transportation centrality is, and not based on proximity to other nodes like closeness centrality or eigenvector centrality are. Obviously, the transportation centrality (TC) values are a function of β, and we computed TC for a range of one hundred β values from 0.1 to 10 (thus the step increase was 0.1). It is clear that $\beta = 0$ does not make sense, because this represents no sensitivity to cost, and thus the TC values will reduce to another centrality metric which we have earlier named as 'All-path Betweenness centrality'. This measure is similar to betweenness centrality in that it considers the proportion of paths through a node summed over all possible pairs of nodes, but unlike BC it considers all paths, not shortest paths alone. However, such a measure does not make sense in the transportation context since it has no sensitivity to path lengths, so we do not study this special case ($\beta = 0$) in this paper. Similarly, we also do not focus on the scenario where β value is very high. As we will show, $\beta \geq 10$ tends to result in extreme sensitivity to cost, and beyond this value, the actual change in value of β has negligible effect in the corresponding TC values, and the difference between TC and BC values are minimal. Therefore, we focus on the range where the sensitivity parameter β is from 0.1 to 10.

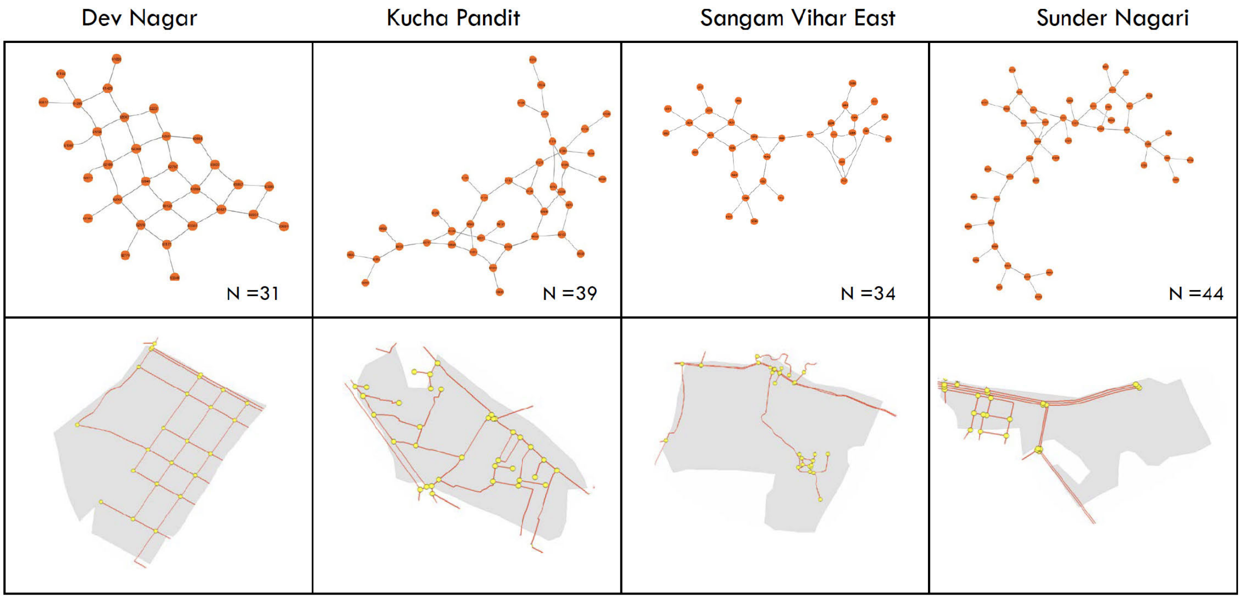


FIGURE 5. Sample subnetworks from Delhi.

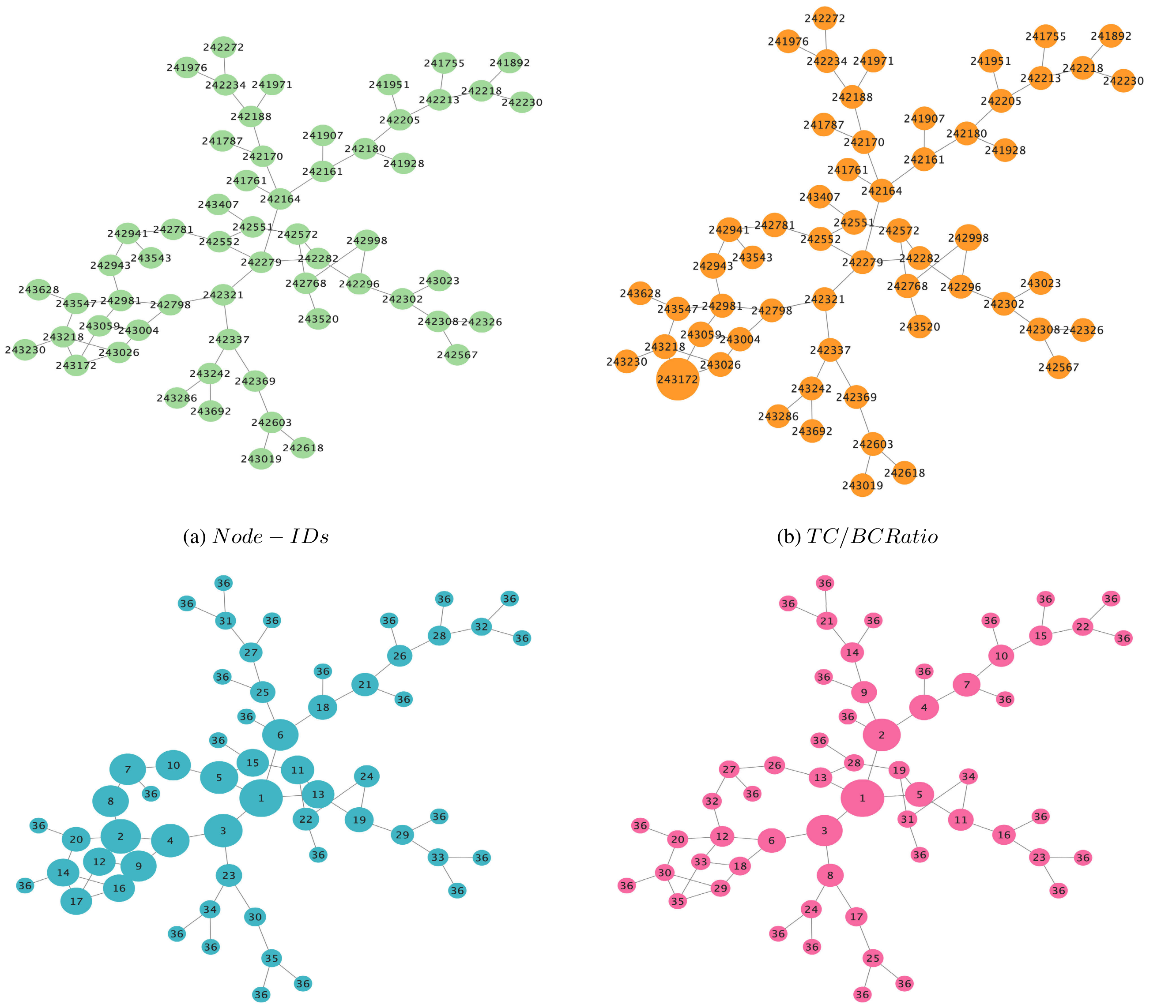
It should be mentioned here that several studies have attempted to 'calibrate' the value of β (the sensitivity of users to differences in cost) in terms of mode choice. For example, Khan et al. [53] models observed travelled behaviour and mode choices in South Eastern Queensland, Australia, and the use a calibrated sensitivity parameter value of $\beta = 0.1234$. Other studies [60], [61] have also suggested a similar low value for β which corresponds to real-world mode choice behaviours of passengers. Here, the sensitivity parameter is used for route (path) choice rather than mode choice, but it is reasonable to assume that real-world β values will be typically low in this scenario too. Therefore, without loss of generality, we choose to focus on $\beta = 0.1$ to illustrate the following example, while emphasising that the transportation centrality metric is relevant for a broad range of β values, as we will show later.

1) COMPARISON BASED ON CENTRALITY VALUES AND RANKS

As an example, Fig. 6 shows the comparison between BC and TC for the Seoul suburb of Jongno2i, for $\beta = 0.1$. Fig. 6 (a) simply shows the network with nodes denoted by node IDs, whereas Fig. 6(b) shows nodes scaled according to their TC/BC Ratio. It could be seen that there are some nodes which have a higher TC/BC ratio compared to others. To understand further how this occurs, in Fig. 6 (c) and Fig. 6 (d) we show the same network where nodes are scaled according to their TC and BC respectively. However, recognising that the 'raw' TC and BC values by themselves do not convey much information, we rank the nodes according to their TC and BC respectively, and show these ranks as labels in Fig. 6 (c) and Fig. 6 (d). Therefore, the 'larger' nodes in the figure have the lower rank according to the considered metric.

We observe that the same node (ID = 242279) has the highest TC and BC Rank. However, from the second rank onwards, this is not the case. Node 242981 is 2^{nd} in rank according to TC, but BC makes it 12^{th} in rank. Similarly, Node 242798 has 4^{th} in rank according to TC, but BC makes it 6^{th} in rank, and node 242552 is 5^{th} in rank according to TC, but BC makes it 13^{th} in rank. In fact, it is easy to see, by comparing Fig. 6 (c) and Fig. 6 (d), that most nodes which form a 'ring' to the left of the main hub, and several nodes which are in the relatively dense cluster attached to this ring, have more significance in terms of TC compared to BC. On the other hand, nodes such as 242164 (TC Rank = 6^{th}, BC Rank = 2^{nd}) which are at topologically significant places of the tree structures that branch from the main hub, have relatively higher significance in terms of BC compared to TC.

It is easy to see why. BC considers only shortest paths, therefore where ever there is a ring structure, only the shorter section of the ring is assumed to be used to reach nodes beyond the ring, while the longer section is ignored, reducing the importance of the nodes in that longer section, though what is the 'shorter' section and what is the 'longer' section changes depending on which pair of nodes are being considered. However, traffic will typically use all available alternative paths, therefore transportation centrality typically gives more importance to nodes which are in such rings. A similar argument could be made to explain why the TC is higher than BC for nodes in dense clusters and grid-like subnetworks. In such structures, there are many alternative paths, and TC considers all such paths, while BC does not. Whereas in tree-like subnetworks the TC value of nodes will be similar to their BC values, but given that the network



(a) $Node-IDs$ (b) $TC/BCRatio$

(c) $Node-TC-based-rank$ (d) $Node-BC-based-rank$

FIGURE 6. The transportation network of Jongno2i suburb of Seoul, Korea. a) Nodes are labeled with Node IDs b) Nodes are labelled with the TC/BC Ratio, and node size is also proportional to the TC/BC Ratio c) Nodes are labelled with TC rank, and the node size is also proportional to TC d) Nodes are labelled with BC rank, and the node size is also proportional to the BC.

also includes rings and dense clusters, the TC based rank of the nodes in tree-like sections of the network will be less significant (numerically higher) compared to their BC based ranks.

2) COMPARISON BASED ON RANK DIFFERENCE

To analyse this further, we also considered the rank difference of nodes - the difference between their TC based ranks and their BC based ranks. Fig. 7 shows such rank differences for the Seoul suburb of Jongno2i. Note that 'rank difference' here means $Rank(TC)-Rank(BC)$, therefore a negative value means the node has a lower numerical value for TC rank and thus more significant in terms of TC. Similarly a positive value means the node is relatively more significant in terms of BC. The nodes in the figure are also colour-coded such that nodes which have a negative rank difference have greenish colours, whereas nodes which have a positive rank difference have reddish colours. We could immediately observe that several nodes in the ring structure and the dense cluster of the rood networks have significant minus values: that is, the TC based rank of these nodes is numerically considerably lower than their BC based ranks. Indeed, there are several nodes where the rank difference is higher than 20, and this is very significant in a network where the total number of nodes is $N = 58$. These are the nodes in the road network whose importance was very significantly underestimated by BC, and

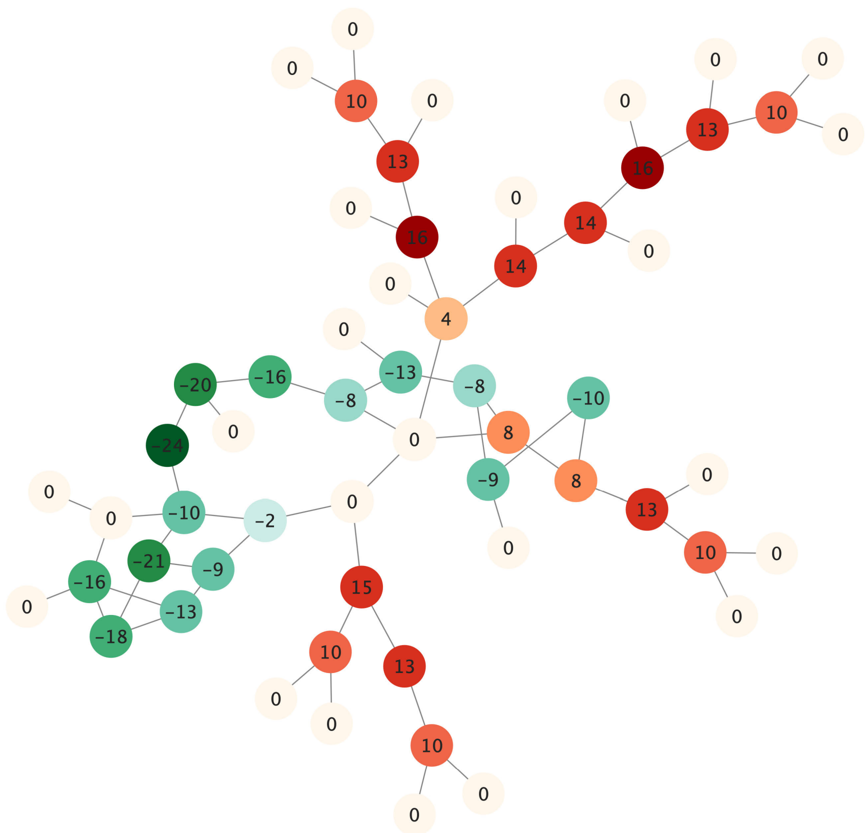


FIGURE 7. The comparison of TC and BC values for Jongno2i suburb, Seoul, Korea. Nodes are labelled with the differences of rank based on TC and BC (TC Rank - BC Rank). Therefore, a positive rank difference means the node is more important based on BC compared to TC, while a negative rank difference means the node is more important based on TC compared to BC. Nodes are colour coded according to rank differences.

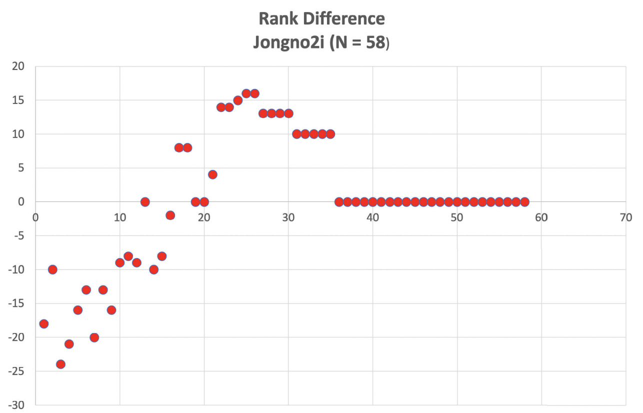


FIGURE 8. The comparison of TC and BC values for Jongno2i suburb, Seoul, Korea. The rank difference based on TC rank - BC rank is shown, as a scatter plot. It can be noted that while the relative ranks of many nodes are the same when calculated based on TC or BC, many nodes also have a significantly lower rank (more significance) based on TC compared to BC, and vice versa.

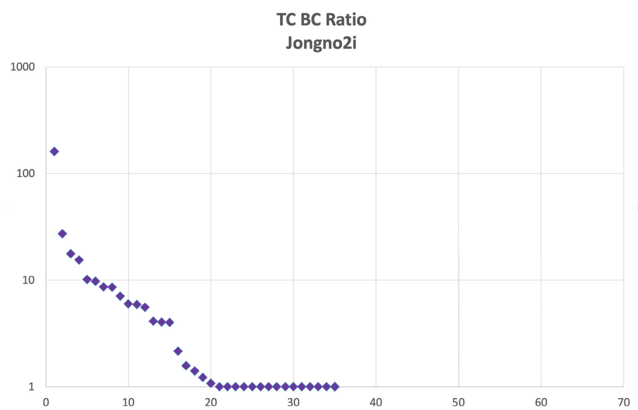
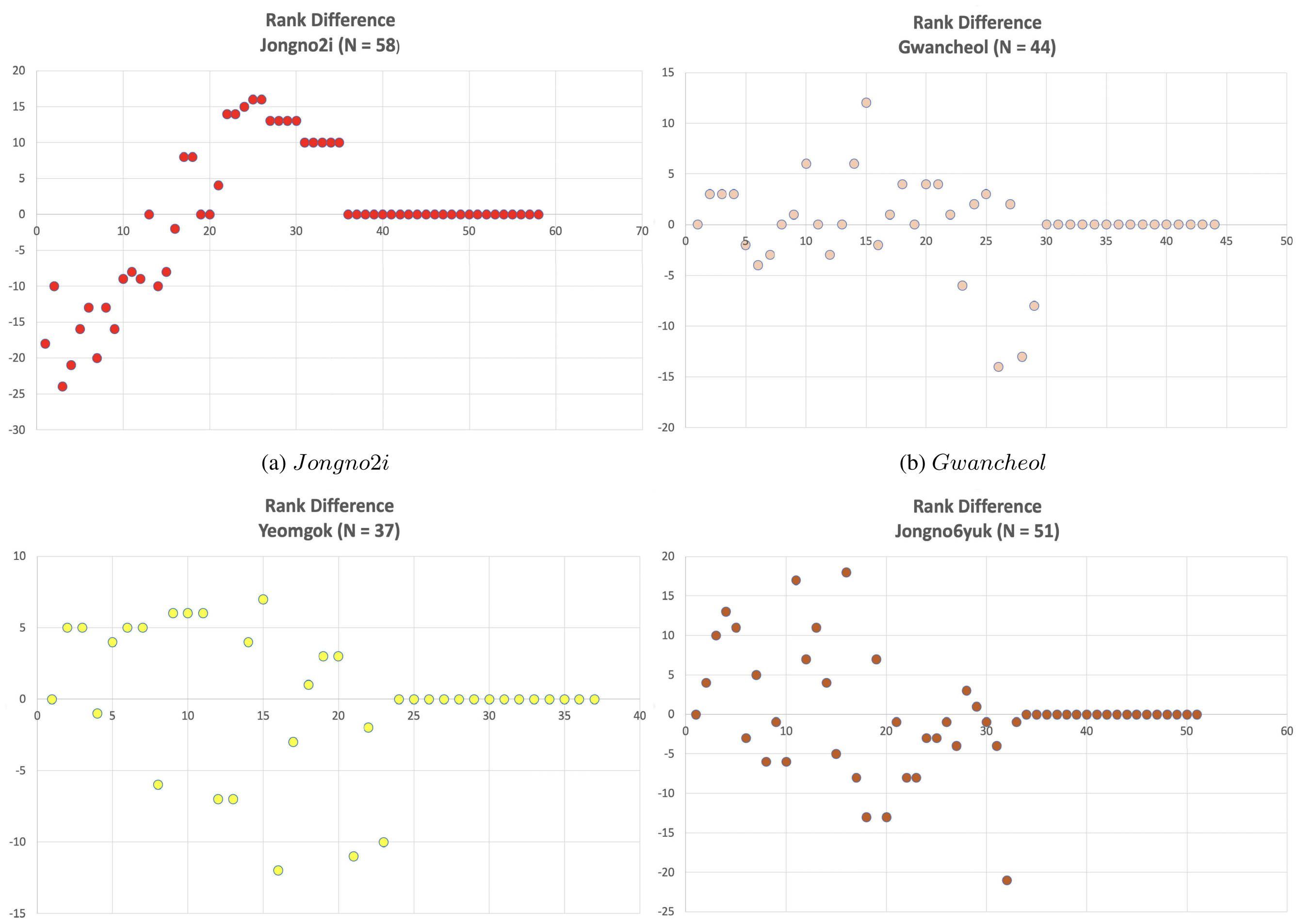


FIGURE 9. The comparison of TC and BC values for Jongno2i suburb, Seoul, Korea. The TC / BC ratio for each node is shown as an ordered scatter plot.

TC is able to give a much more accurate indication of their relative significance in the road network. On the other hand, there are many nodes in the tree-like substructures which have a positive rank difference, and many have a difference of higher than ten. These are the nodes that the BC recognises 'fairly', but their BC rank is numerically lower than their TC rank because other nodes are recognised by TC as relatively more important.

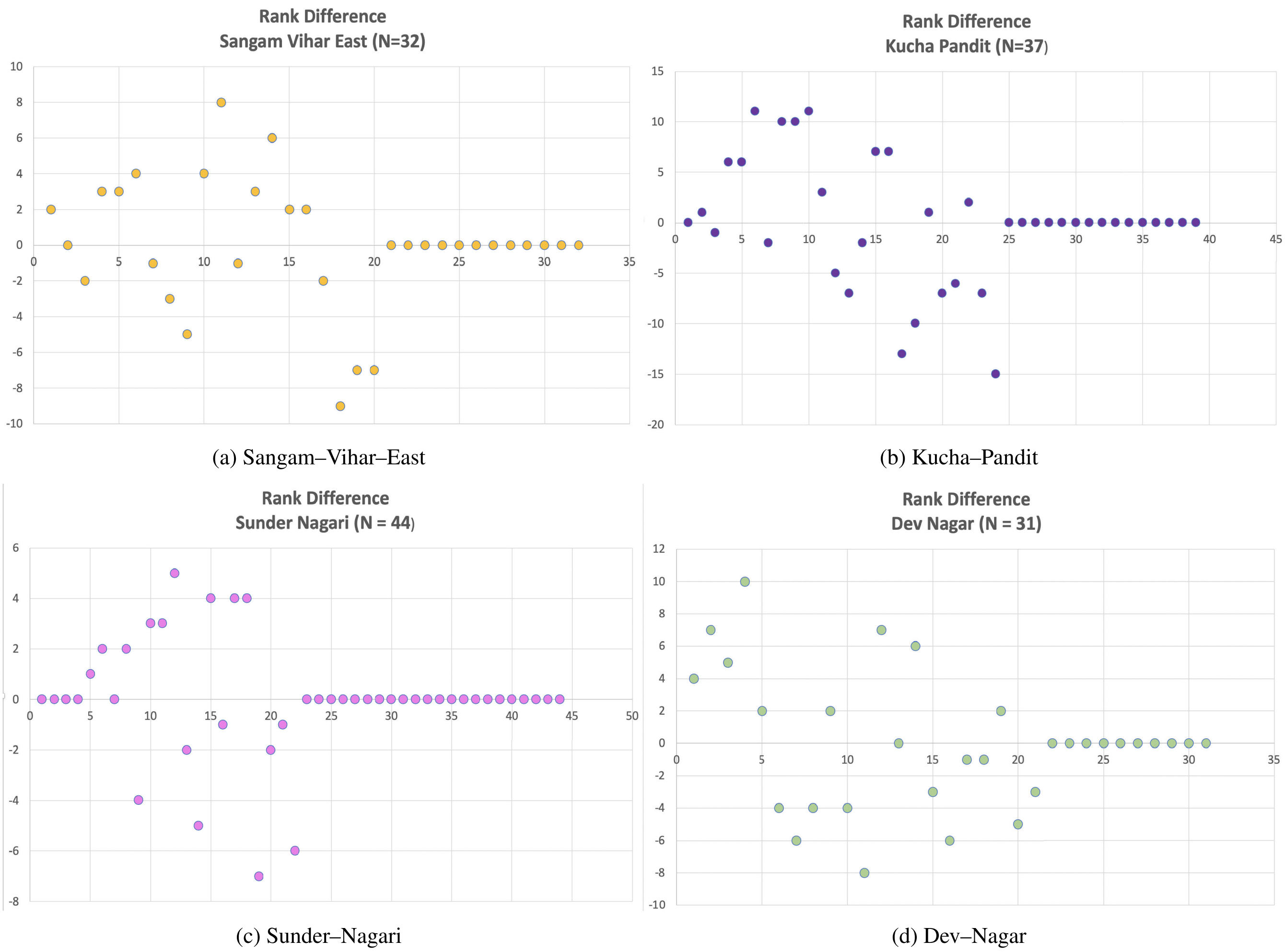


(a) *Jongno2i* (b) *Gwancheol* (c) *Yeomgok* (d) *Jongno6yuk*

FIGURE 10. TC - BC rank difference scatterplots for the four considered suburbs from Seoul. The values shown are $Rank(TC) - Rank(BC)$, so a negative value indicates the node is more important in terms of TC, while a positive value indicates the node is more important in terms of BC. The network sizes for each suburb are also indicated.

The overall rank-difference distribution for Jongno2 is shown in Fig. 8, which corresponds to Fig. 7. It can be seen that nearly half the nodes in the network have a TC rank that is different from their BC rank. These are the nodes whose importance is either underestimated or overestimated by BC because BC considers only the shortest paths, while TC considers all paths while taking into account path costs and the sensitivity of customers to it. The other half of nodes, where the TC rank is not different from the BC rank, are mostly peripheral nodes, which have BC and TC both equal to zero, as well as a few hub nodes which happen to have the same TC and BC rank. For comparison, Fig. 9 shows the ratio of the numerical values of TC and BC for the suburb Jongno2i. Unsurprisingly, the ratio is always higher than 1, since TC considers all paths while BC considers only shortest paths, and the value ranges from 161 to 1 (we assume $0/0 = 1$ in this case because peripheral nodes clearly have the same TC and BC value, though this value is zero). This figure illustrates that while there are nodes which have a numerically much higher TC than BC, comparing raw values does not make much sense, and this is the reason we compared ranks based on TC and BC.

We undertook a similar analysis of all the eight considered suburbs. To be succinct, we choose not to show the relevant values or ranks based on TC and BC, but summarise the rank difference as calculated above for Seoul suburbs in Fig. 10 and Delhi suburbs in Fig. 11 respectively. We can note, in general, that the rank difference is most significant in suburbs with a scale-free structure, such as Jongno2i, Jongno6yuk, and Kulcha Pandit. It is comparatively less significant in suburbs with a grid-like structure, like Dev Nagar and Gwancheol, and the least significant in suburbs with chain-like road networks, such as Yeomgok and Sunder Nagari. This is despite the fact that Sunder Nagari is the largest Delhi suburb we have considered in terms of number of nodes, so this result it clearly not simply a product of network size. Therefore we could conclude that the more heterogeneity there is in terms of degree distribution, the more useful TC is as a metric, since there are more opportunities for significant alternative paths to the shortest



(a) Sangam–Vihar–East

(b) Kucha–Pandit

(c) Sunder–Nagari

(d) Dev–Nagar

FIGURE 11. TC - BC rank difference scatterplots for the four considered suburbs from Delhi. The values shown are *Rank*(*TC*) − *Rank*(*BC*), so a negative value indicates the node is more important in terms of TC, while a positive value indicates the node is more important in terms of BC. The network sizes for each suburb are also indicated.

paths. In terms of grid-like structures there are plenty of alternative paths, but the length difference between them is insignificant, where as in terms of networks with long chain-like structures, alternative paths are few. However, in all cases there are significant proportion of nodes which have a 'negative' rank difference, and thus their importance had been underestimated by measures like BC which consider only the shortest paths.

C. THE INFLUENCE OF THE SENSITIVITY PARAMETER ON THE UTILITY OF TRANSPORTATION CENTRALITY

Of course, the above analysis depended, to some extent, on the value of the sensitivity parameter β. If the value of β is higher, then the difference between TC and BC ranks will be less in general. However, we chose to illustrate the case of $\beta = 0.1$ as a realistic scenario based on studies such as Khan et al. [53] which suggest a β value around 0.1. Now let us focus on the sensitivity parameter β in more detail, and analyse for what range of β is the transportation centrality metric distinctly meaningful. To do so, for each suburb mentioned above, we computed the ratio of the average TC and average BC in that suburb for a particular value of the sensitivity parameter β. We considered values of β from 0.1 to 10.0 as mentioned above. The premise was that if this ratio approaches unity, then TC is converging towards BC and thus losing relevance. The results are shown in Fig. 12 and Fig. 13.

From these figures, it is apparent that as the value of β increases (that is, the sensitivity of the model to path costs increase), the transportation centrality, as expected, quickly converges towards betweenness centrality. This is because, as the sensitivity to path cost increases, the paths with least cost (in this case, the shortest paths) are increasingly chosen, and all other simple paths are omitted, so transportation centrality reduces to betweenness centrality. We may observe from these figures that if $\beta = 2.0$ or higher, the ratio between average TC and average BC converges to unity. Therefore, we may conclude that, even though in theory β could take any value from zero to infinity, for real world cities, transportation centrality is useful only when a β value between 0.0 and 2.0 is used.

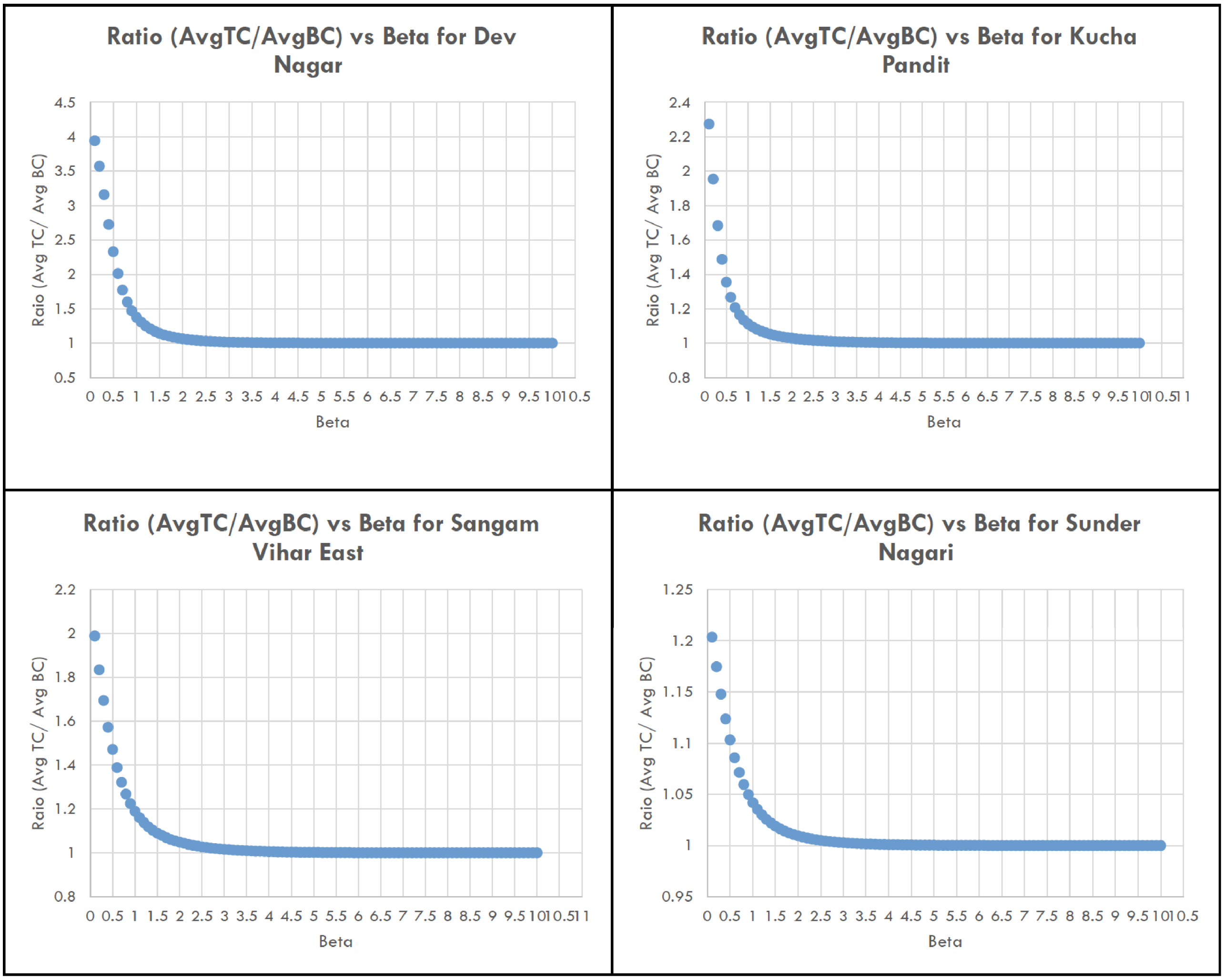


FIGURE 12. Average TC / Average BC Ratio against sensitivity parameter β for Delhi suburbs.

However, it could be argued that the average of transportation centrality converging with the average of betweenness centrality does not necessarily imply that the transportation centrality distribution, in general, converges with the betweenness centrality distribution. For example, the transportation centrality values could become more extreme, either much higher or much lower than betweenness centrality, while the average of TC appears to converge with the average of BC. Therefore, we also considered the Pearson correlation between TC distribution and BC distribution for the sets of suburbs considered above from Delhi or Seoul. These results are presented in Fig. 14 and Fig. 15. We could see from these figures that the Pearson correlations between TC and BC are always quite high for all β values for any suburb. This is not surprising as transportation centrality gives higher weights to paths with low costs, i.e., shortest paths, even though it does consider other paths also, unlike BC. However, we note that as the parameter β increases, this correlation increases further, and converges towards unity. This indicates that the TC and BC distributions do become identical (or shifted from one another by a constant value), as the sensitivity to the cost, β, increases. For all suburbs considered, $\beta \geq 4.0$ results in the correlation approaching unity. For some suburbs however, the convergence happens slower than others. Consider Dev Nagar, for example. As shown in Fig. 5, the road network of this suburb has a grid-like structure, and a fairly homogenous degree distribution. We could observe from Fig. 14 that the convergence between TC and BC for this suburb happens at a higher value of β compared to the other suburbs of Delhi. Similarly, it could be observed from Fig. 15 that for Gwancheol, which also has a grid-like structure, the convergence happens at a higher value of β compared to other Seoul suburbs. It is clear that in such a suburb which has a grid-like road network, there would be several alternative paths with equal path lengths with similar costs, therefore it is comparatively more important to consider non-shortest paths. Therefore, TC retains its relevance for a bigger range of β values.

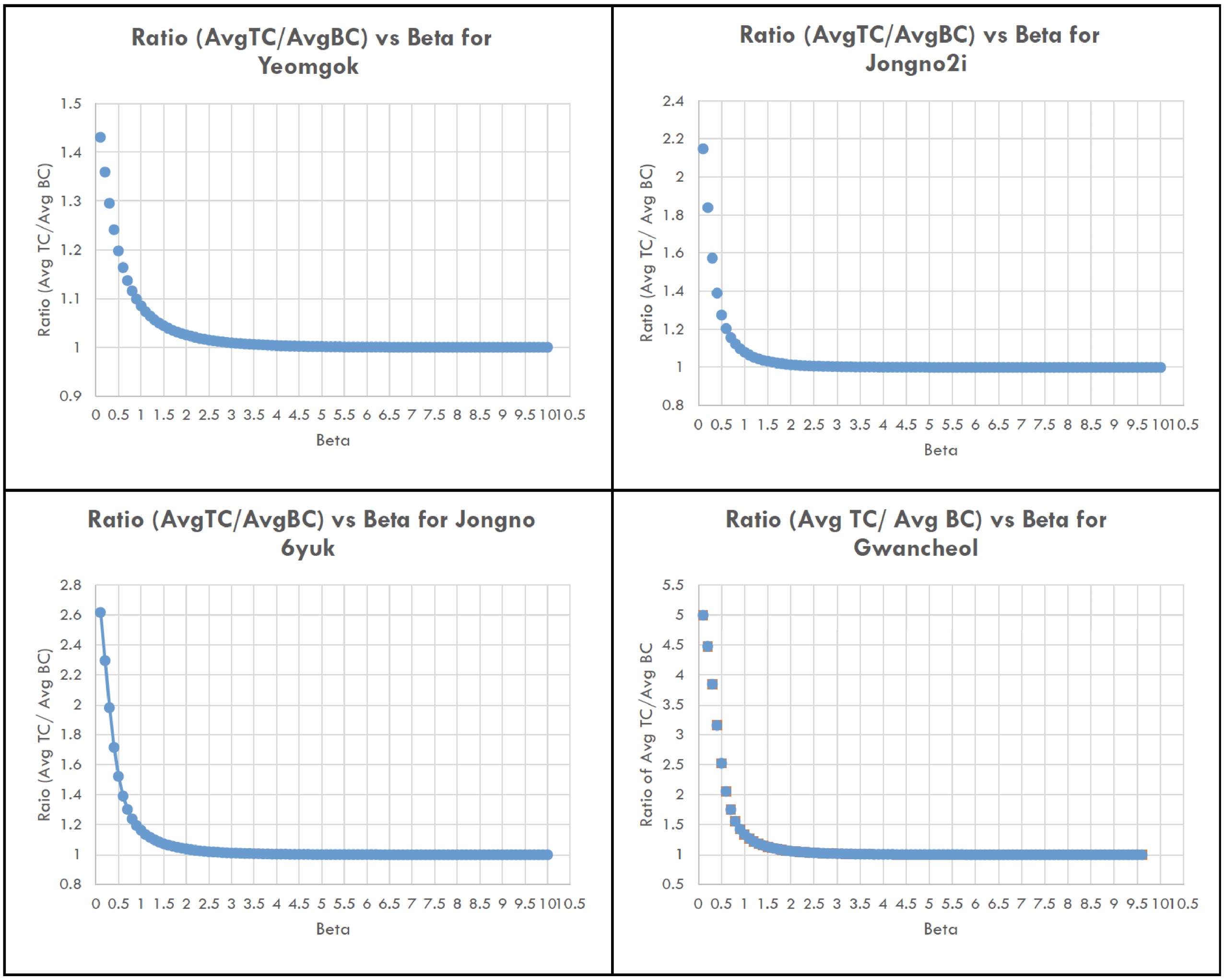


FIGURE 13. **Average TC / Average BC Ratio against sensitivity parameter β for Seoul suburbs.**

It is also interesting to note a similarity between the transportation centrality computation and bounded rationality calculations made in game theory [62], [63], [64]. It is that the sensitivity parameter β essentially functions like the rationality parameter λ used in deriving quantal response equilibria (QRE) solutions [62], [63], [64] for games involving players with bounded rationality [65], [66], [67], [68], [69]. The higher the β, the higher the sensitivity to costs of alternative paths, so the non-shortest paths are chosen with increasingly less likelihood. Therefore as β increases, the TC distribution converges towards the BC distribution for any network. However, it is only rational that the path costs be considered in calculating available paths, therefore, the increased sensitivity to β corresponds to increased rationality in a system with boundedly rational agents. However, this is exactly what the rationality parameter λ captures. It is therefore no surprise that the QRE model uses Logit (Fermi) functions similar to those that are used in modal split in transportation science and those we have introduced here in calculating transportation centrality. The novelty of the transportation centrality formulation lies in the fact that it has brought into a single framework the hitherto unconnected concepts of centrality calculation, and sensitivity parameters and Fermi functions used in calculations of boundedly rational equilibrium solutions such as the QRE calculation process, to present a centrality metric suitable for transportation that mimics the behaviour of boundedly rational people, who stochastically and non-linearly choose alternative paths by taking path costs into consideration.

D. DISCUSSION

Betweenness centrality and its many variants have played a crucial role in our ability to identify important nodes in a complex network based on the number of paths that go through them. However, these metrics have made the assumption that movements or traffic of whatever quantity that traverses between nodes in such networks always happens through the shortest paths alone. While this is a

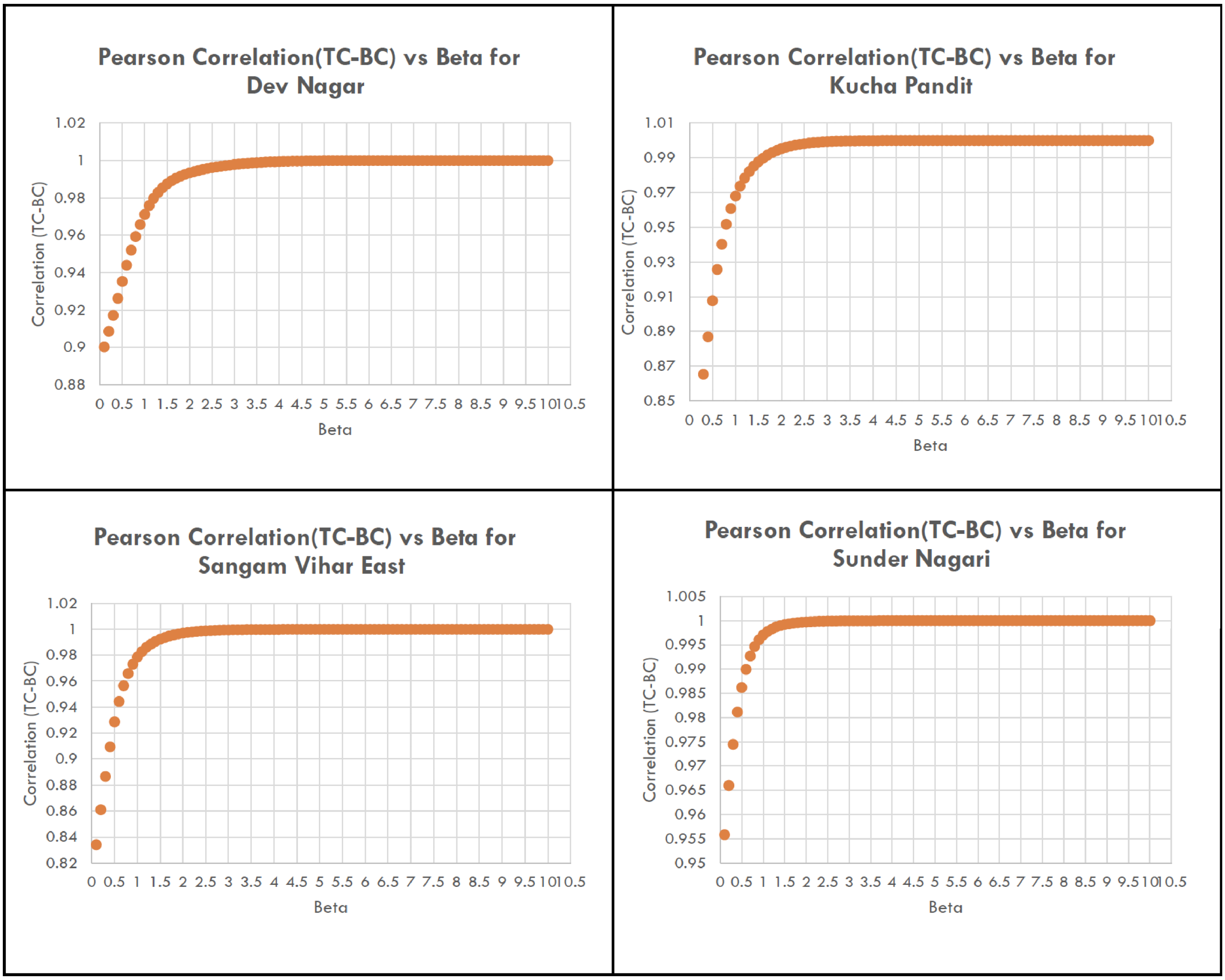


FIGURE 14. TC-BC Pearson correlation against sensitivity parameter β for Delhi suburbs.

perfectly valid assumption in some contexts, and a useful starting point for estimation of node importance in others, it is vital to recognise that in transportation networks, not all traffic moves through the shortest paths. Despite the fact that there is an incentive to move through shortest or less costly paths, the path selection is stochastic. Therefore, in a city where the rationality of drivers and commuters is bounded, or it is difficult to calculate or predict path costs, a proportion of traffic always moves through non-shortest paths, or non-optimal paths in terms of cost. A centrality measure which is applied in such a setting will be useful if it takes all paths into account.

With this motivation, in this paper we introduced transportation centrality, a centrality measure which considers all paths that go through a node in calculating that node's centrality. This metric could be implemented in two ways: 1) where path cost data is available and used directly 2) where path cost data is not available and transportation centrality makes an estimation of path costs based on topology, thus becoming a purely topological measure. We primarily focussed on the second method in this paper, since that can be applied to any transportation network regardless of the availability of traffic data, including newly conceptualised or planned transportation systems for which such data cannot possibly exist. To overcome the challenge of estimating the relative importance of paths without directly using data from the given city or area, transportation centrality uses a model similar to the modal split model, parameterised by a sensitivity parameter, which estimates the relative amount of traffic through each alternative path between a source and a destination, based on path costs. Though path costs, if available, can directly be used in this estimation, in the absence of path cost data, transportation centrality uses path lengths as the estimator for path costs, thus becoming a purely topological measure. Importantly, transportation centrality does not use path lengths in a linear fashion to serve as a proxy for path weights. Instead, inspired by traffic flow modelling in transportation science, it uses Logit (Fermi) functions which

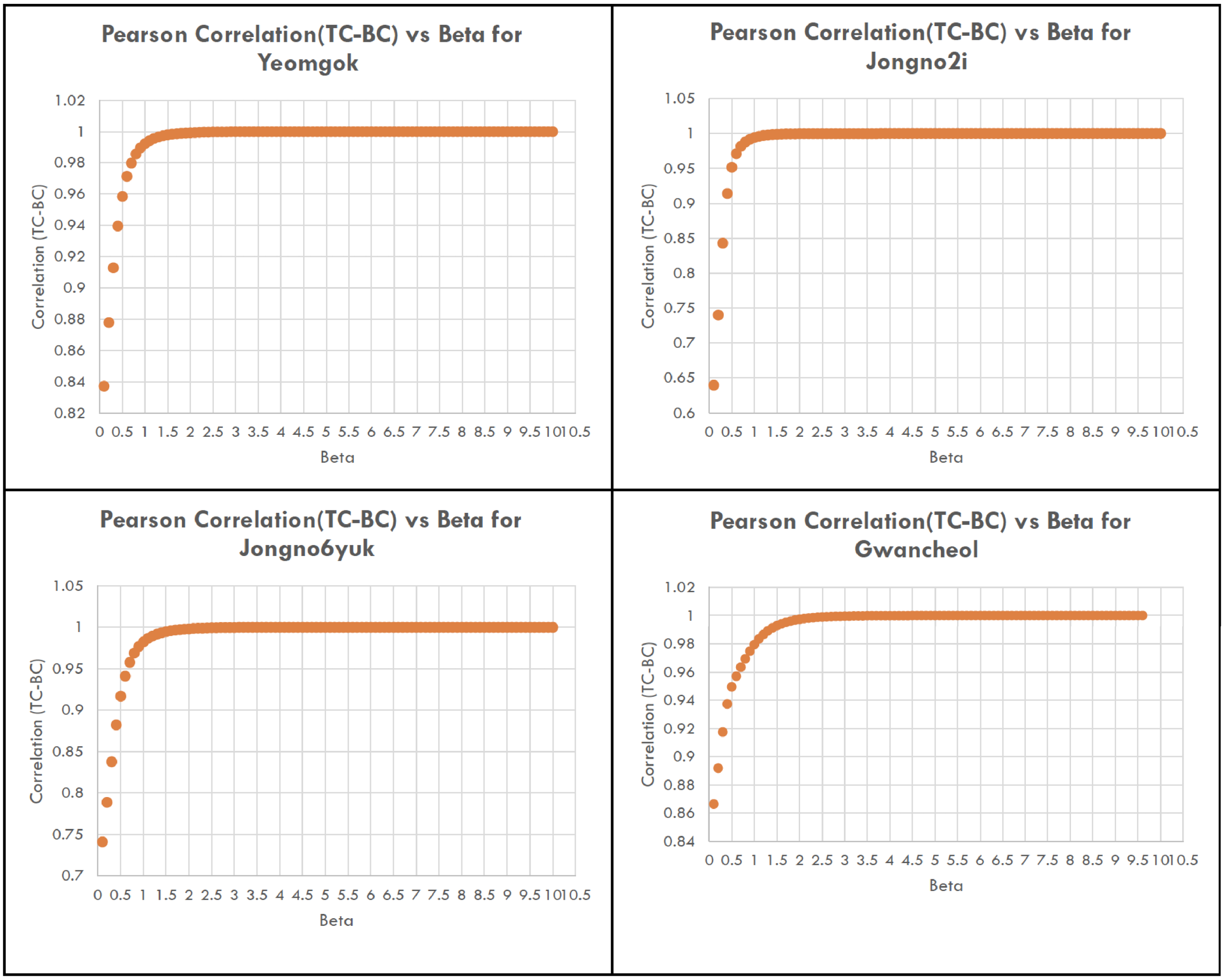


FIGURE 15. TC-BC Pearson correlation against sensitivity parameter β for Seoul suburbs.

estimate the traffic flow through each path based on path lengths, which in turn are used to estimate the centrality of each node.

It is useful here, to provide a brief comparison between transport centrality (TC) and the existing centrality metrics that we have discussed in section II. We have compared TC with betweenness centrality (BC) extensively in the previous section. In short, BC takes into consideration only shortest paths in determining the importance of nodes, while TC takes into account all paths. Thus, it more realistically represents the behaviour of traffic, and thus more accurately determines the importance of junctions. However, compared to an implementation of betweenness centrality using the Brandes algorithm [22], which can be calculated in $O(NL)$ time, TC is more computationally intensive to calculate at present, though future work may find more efficient implementations of TC. Percolation Centrality has the same disadvantage as betweenness centrality compared to TC. It only considers shortest paths, though it takes into account the node states. It also has an implementation which can be calculated in $O(NL)$ time [17], like BC using Brandes algorithm, so in that sense it also can be less computationally intensive to compute compared to TC at the moment. Closeness centrality and eigenvector centrality are not defined in terms of paths, instead focussing on relative location of a node within the topology, and therefore cannot be used to compare importance of nodes in terms of traffic, therefore TC need not be compared with them. Straightness centrality measures the efficiency of routes that begin at a particular node, in a geographical sense. As the name implies, a node will have high straightness centrality if routes originating from it are relatively 'straight': while that is a useful measure, it does not capture importance in terms of traffic flow like TC does.

On the other hand, several centrality measures discussed in section II-B, including mobility centrality and DelayFlow centrality, do address the importance of nodes in transport networks directly, by considering all paths. However, they need traffic data from a particular city as well as topological information to be calculated, and such data is often not available and impossible to get for new suburbs. Compared to these, the strength of TC is that is has a principled way of estimating the traffic from topology alone, using existing concepts in traffic modelling, and it then uses these estimates in the computation of centrality. Therefore, it can be computed using topological data alone, and therefore can be computed on any transport network.

It should also be emphasized that the sensitivity parameter β used in the definition of transportation centrality is a tunable parameter, and has been used in other contexts in transportation science previously, such as modal split. In other words, while the transportation centrality introduced here is a novel concept, the sensitivity parameter used in its definition is not new. When undertaking transportation centrality analysis, the choice of this parameter should be based on observed behaviour of traffic in that city, where possible, and not necessarily to obtain results which are distinct from another centrality measure. As mentioned earlier, several studies ([53], [60], [61]) have calibrated the sensitivity parameter and found values less than one, and we have used a value of 0.1234 in several examples in this paper, following Khan et al. [53]. If a more suitable value can be obtained by calibration to a different city, that value should be used when applying transportation centrality analysis for that city.

V. CONCLUSION

A. SUMMARY

In this paper, we first mathematically defined the new transportation centrality metric. We showed that the metric is governed by a sensitivity parameter, and when the value of this sensitivity parameter is very large, the transportation centrality reduces to betweenness centrality. Therefore, betweenness centrality is a special case of transportation centrality as far as transportation networks are concerned, whereby all drivers / commuters display infinite rationality in their decision regarding path / route choices. We observed the similarity with game theory, where Nash equilibrium is a special case of quantal response equilibrium, and is obtained when the rationality parameter used in the quantal response equilibrium assumes the value of infinity. On the other hand, when the sensitivity parameter in transportation centrality has a value of zero, we showed that transportation centrality reduces to another centrality metric, which we have named All-path Betweenness centrality.

We demonstrated the utility of the new transportation centrality metric by calculating it for the nodes of a number of transportation networks from Seoul, Korea and Delhi, India. We showed that for each of the networks, there were a number of nodes which had a lower numerical rank based on transportation centrality compared to that based on betweenness centrality. This indicated that the relative importance of such nodes was not properly represented by betweenness centrality, and transportation centrality highlighted their importance better. The difference between TC based ranks and BC based ranks was especially significant in heterogenous network topologies resembling scale-free networks. We used a sensitivity parameter value of 0.1234 in these experiments, based on the value calculated by a previous study.

We also showed that, in general, a sensitivity parameter value of less than 2.0 results in transportation centrality values, and the rankings of nodes based on it, being significantly different from betweenness centrality values, and rankings based on them. Therefore, we showed that, as long as the value of sensitivity parameter is less than 2.0, transportation centrality is a better measure to quantify the importance of nodes in transportation networks compared to betweenness centrality. Since the observed sensitivity parameter values for real-world transportation networks are well-within this range (based on modal split modelling), transportation centrality is distinctly useful in most real world scenarios in quantifying the importance of nodes in transportation networks. However, it should be acknowledged that the value of sensitivity parameter could vary considerably in transportation networks, especially if the level of abstraction is 'higher' than junctions and roads / railways (for example, if we analyse a transportation network where nodes are regions or townships).

B. LIMITATIONS AND FUTURE WORK

The introduced metric has certain limitations at present, though some of these could be overcome by further research. The main limitation is regarding its implementation in software, and the time complexity associated with it. It should be noted that betweenness centrality nominally has a time complexity of $O(N^3)$, which makes it expensive in terms of time. The Brandes algorithm [22] of betweenness centrality is able to achieve a subcubic time complexity of $O(NL)$, which is particularly efficient for sparse networks. Percolation centrality [17] similarly has an implementation which has a time complexity of $O(NL)$ [70]. Transportation centrality is currently implemented as an algorithm which has $O(N^3)$ time complexity. To make transportation centrality calculations more viable for large networks, a more efficient algorithm needs to be implemented, and this is one of the main focuses of future research in this area. It could be noted however that transportation networks are typically very sparse, and direct links between far-flung nodes are extremely rare. Therefore, the performance of any transportation centrality calculation algorithm which is implemented with $O(NL)$ complexity, if this can be achieved, is expected to be quite good.

Obviously, this paper presented the concept of transportation centrality with a number of preliminary examples of its use, though the examples used real-world transportation network data. A lot more research needs to be done in demonstrating the utility of transportation centrality in a broad range of networks. These include railway networks, airport networks and networks from other transportation domains. We also need to study the utility of transportation centrality where the transportation networks are modelled at a higher level of abstraction, for example networks of townships or cities connected by road or railway links. As mentioned earlier, we also need to analyse the challenges of applying transportation centrality in very large transportation networks. Furthermore, the utility of the metric in transportation networks with different topologies, such as scale-free networks, small-world networks, hierarchical networks, random networks, and grids need to be understood, though the presented examples already give some useful indications about this aspect. Nevertheless, the definition of the transportation centrality metric with some preliminary analysis is presented in this paper so that other researchers could begin to use it, and/or pursue some of the research directions mentioned here.

It is our expectation that transportation centrality will provide very useful insights about the relative importance of nodes in a range of transportation networks and systems.

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MAHENDRA PIRAVEENAN received the B.E. degree (Hons.) in computer systems from the Department of Electrical and Electronic Engineering, The University of Adelaide, in 2004, and the Ph.D. degree in complex systems from The University of Sydney, in 2010. He was with the software industry. He is currently an Associate Professor with the School of Computer Science, Faculty of Engineering, The University of Sydney. He has published more than 35 journal articles and more than 30 peer-reviewed international conference proceedings. His research interests include complex networks, game theory, epidemics modeling, and multi-agent systems. He serves on the editorial board of *Mathematics* and *Applied Mathematics* journals. He is an active Assessor of the Australian Research Council.

NARESSA BELLE SARIPADA received the bachelor's degree in computer science from the University of the Philippines Cebu and the master's degree in complex systems (transport specialization) from The University of Sydney, in 2019. She is currently a Senior Transport Specialist with Clean Air Asia.