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RESEARCH ARTICLE

Stability Analysis of Two-Area LFC Power Systems With Two Additive Interval Time-Varying Delays

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ABSTRACT In this paper, the stability problem of load frequency control (LFC) based power systems with two additive interval time-varying delays is studied. Firstly, considering the time delays of transmission from control center to regulator and sensor to control center, the mathematical model of two-area LFC power system based on proportional integral control is established. Secondly, under the condition that the lower bounds of interval time-varying delays are non-zero, an augmented Lyaounov-Krasovskii functional (LKF) is constructed, and a new delay-dependent stability criterion is derived. Since the extension of LKF introduces the nonlinear term of time-varying delay square, a new negative definite integral inequality transformation lemma is used to transform the nonlinear matrix inequality in the stability criterion into linear matrix inequality (LMI) equally-without introducing additional conservatism. Finally, the maximum stability margin of the LFC power systems is obtained by using MATLAB LMI-toolbox, and simulation results based on Simulink-toolbox show the effectiveness of the stability criterion.

INDEX TERMS Interval time-varying delays, load frequency control, power systems, Lyapunov-Krasovskii functional, LMI.

I. INTRODUCTION

Load frequency control (LFC) is an important method to regulate and control the frequency of the power grid, which can keep the frequency stable within the power system area and exchange power with the neighboring areas [1], [2], [3]. The traditional centralized LFC scheme uses a dedicated communication channel to transmit control signals, and the transmission delay is very small which can be ignored [4]. With the continuous development of power grid scale, the open internet has replaced the dedicated networks as the primary communication channels of large-scale data transmission and exchange due to its advantages of low cost and strong flexibility [5], [6]. However, long distance information processing and transmission inevitably result in communication delays, network congestion or failure, and potential network attacks [7]. The researchers show that when the communication delays are large, the dynamic performance of the power system may be reduced or even unstable [8]. Therefore, the effect of time delay on the stability of power system is one of the problems to be solved.

The stability analysis of time-delay systems is always a basic problem in the research of time-delay systems. Time-delay systems have time-delay factors, whose states are not only dependent on the inputs and states of the present time, but also related to the inputs and states of the past time. Therefore, stability analysis of time-delay systems is a challenging task that requires a series of specialized mathematical tools and methods. At present, the most common method is Lyapunov stability theory. The complete Lyapunov-Krasovskii functional (LKF) can provide sufficient and necessary conditions for the stability of linear systems with constant delays, while the simple LKF only provides sufficient conditions for the stability of systems with time-varying delays. Due to the inevitable conservatism of sufficient conditions, the current research focuses on how to

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reduce the conservatism of stability criteria. There are two main methods: one is to construct a novel LKF, and the other is to derive a tight inequality amplification technique. For example, the implicit LKF [9], the time-dependent LKF [10], the vector LKF [11], some other augmented LKFs [12], [13], [14], [15], the delay-fraction theory [16], [17], relaxed quadratic function negative-determination lemmas [18], [19], and so on.

With the continuous development of stability methods for time-delay systems, the analysis method of time-delay systems has been gradually applied to the stability research of time-delay LFC power systems in recent decades. At the beginning, frequency domain method was used to study the delay-dependent stability of LFC schemes, however it can only deal with constant delay [20]. For LFC power system with time-varying delays, Lyapunov stability theory is still used to obtain less conservative stability criteria. Based on Lyapunov stability theory and linear matrix inequality (LMI) method, an approximate method for obtaining delay margin is proposed in [21]. In order to improve the accuracy of delay margin, [22], [23], [24], [25] derived a less conservative random delay-dependent stability criterion. At the same time, the load disturbance is modeled as a bounded uncertainty parameter, and a new inequality technique is used to further reduce the conservatism of the stability criteria for the LFC power systems with time-varying delays [26]. In addition, in order to ensure the stability and anti-interference ability of time-delay LFC power system, researchers have designed many control strategies, such as PI control [27], network predictive control [28], [29], [30], nonlinear control [31], [32], and so on. PI control has good robustness and independent on the exact model of the power system, which is widely used in the industrial field at present. To solve the stability problem of LFC power systems with time-varying delay based on PI controllers, a large number of scholars have given good results [6], [33], [34], [35], [36], [37]. However, these results only consider the case where the lower bound of the time-varying delay is zero. In practice, it is known that the range of delay with non-zero lower bound are often encountered, and such systems are referred to as interval time-delay systems. To the best of the authors' knowledge, most of the existing studies seldom consider the delay of the non-zero bound. Therefore, it is necessary to study the effect of non-zero bound of interval time-varying delay on the performance of LFC power systems in open communication networks.

This paper mainly studies the stability of two-area LFC power system with interval time-varying delays. Not only the transmission delay from the sensor center to the control center, but also the transmission delay from the control center to the regulation center are considered. Using the Lyapunov stability theory and integral inequality technique, a new stability criterion based on LMI is derived. The main contributions are summarized below.

- The stability of two-area LFC power system with interval time-varying delays, whose the lower bounds are not 0, is investigated, which is ignored in the published literature.
- Based on the interval time-varying delays, a novel LKF is constructed, which divides the interval time-varying delay into different time-varying subintervals and contains additional delay-dependent state information.
- In order to obtain the stability criterion with low conservatism, the square term of time-varying delay is introduced in the LKF, resulting in a nonlinear matrix inequality form of the stability criterion. A novel negative definite integral inequality transformation lemma is used to equivalently transform the nonlinear matrix inequalities into LMIs without introducing extra conservatism.

Notation: In this paper, $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$ represent the n-dimensional vector and the $n \times m$ matrix space, respectively. $\mathbb{S}^n$, $\mathbb{S}^n_+$ mean the sets of symmetric and positive definite real matrix spaces. n-order block diagonal matrix diag $\{S_1, S_2, \cdots, S_n\}$ with diagonal partitioned elements $S_1, S_2, \cdots, S_n$. $e_i\,(i = 1, \ldots, m)$ are a column block matrix in which only the $i-$th block is the identity matrix and the others are 0 matrices. Such as, $e_3 = \left[\begin{array}{c} 0\;0\;I\;\underbrace{0\cdots 0}_{m-3}\end{array}\right]$. This symbol $*$ in a block symmetric matrix denotes transpose of the corresponding symmetric element. col$\{\cdot\}$ denotes a column vector. $\Pi(h_1(t), h_2(t))$ denotes Π is the binary function of $h_1(t)$ and $h_2(t)$. $\mathrm{Sym}\{\Xi\} = \Xi + \Xi^T$.

II. PROBLEM FORMULATION AND PRELIMINARY

This section considers the time-delay model of two-area LFC power system based on PI controller, where the basic framework is shown in Figure 1. e^{-sh_i} $(i = 1, 2)$ represent the time delay when the signal is transmitted from the sensor to the control center and from the control center to the governor. There is a contact line model in the power system of two areas. Δf_i, ΔP_{12}, ΔP_{vi}, ΔP_{mi}, ΔP_{di}, ΔP_{ci} $(i = 1, 2)$ are the deviation of frequency, tie-line power exchange, valve position, mechanical output of generator, load disturbance and setpoint, respectively. M_i, D_i, T_{gi}, T_{chi}, β_i, R_i and ACE_i are the moment of inertia of generator unit, generator unit damping coefficient, time constant of the governor, time constant of the turbine, the frequency bias factor of area, speed drop and area control error.

According to Figure 1 and literature [1], the state-space equation of LFC power system is expressed as the following equation (1).

$$\begin{cases} \dot{x}(t) = Ax(t) + B\Delta P_c(t) + D_\omega \Delta P_d \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where the system parameters are as follows:

$$x^T(t) = \Big[\Delta f_1 \;\; \Delta P_{m1} \;\; \Delta P_{v1} \;\; \int \mathrm{ACE}_1 \;\; \Delta P_{12} \;\; \Delta f_2 \;\; \Delta P_{m2} \;\; \Delta P_{v2} \;\; \int \mathrm{ACE}_2\Big],$$

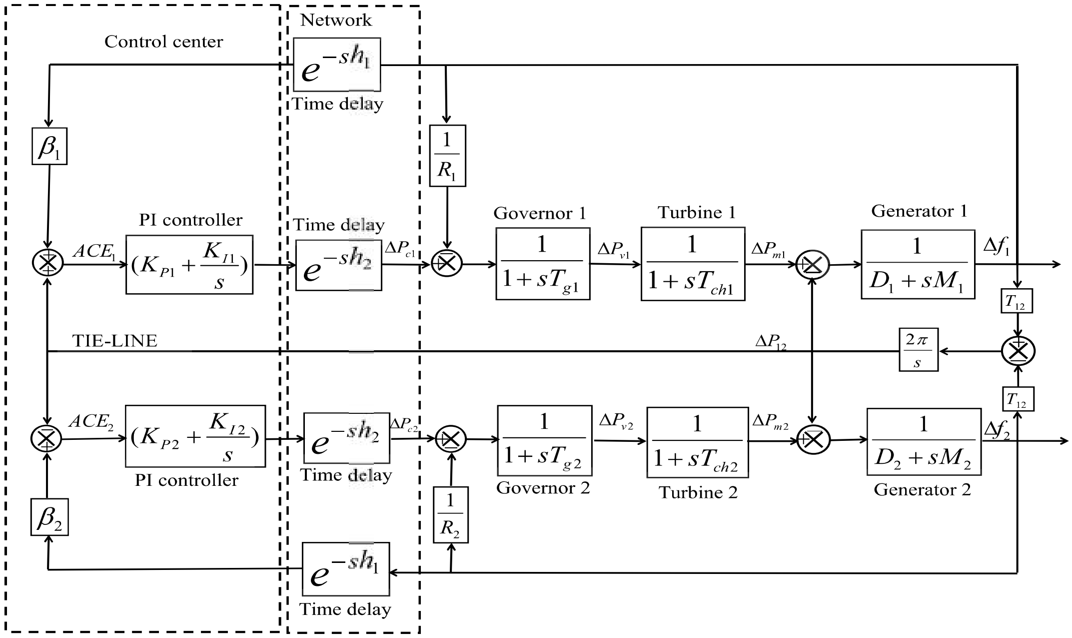

FIGURE 1. The basic diagram of the simplified LFC of two-area power system.

$$
\begin{aligned}
y^T(t) &= \left[\mathrm{ACE}_1 \;\; \int \mathrm{ACE}_1 \;\; \mathrm{ACE}_2 \;\; \int \mathrm{ACE}_2\right], \\
\Delta P_c^T(t) &= \left[\Delta P_{c1}(t) \;\; \Delta P_{c2}(t)\right], \\
\Delta P_d^T &= \left[\Delta P_{d1}(t) \;\; \Delta P_{d2}(t)\right], \\
A &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \\
A_{11} &= \begin{bmatrix} -\dfrac{D_1}{M_1} & \dfrac{1}{M_1} & 0 & 0 & -\dfrac{1}{M_1} \\ 0 & -\dfrac{1}{T_{ch1}} & \dfrac{1}{T_{ch1}} & 0 & 0 \\ -\dfrac{1}{R_1 T_{g1}} & 0 & -\dfrac{1}{T_{g1}} & 0 & 0 \\ \beta_1 & 0 & 0 & 0 & 1 \\ 2\pi T_{12} & 0 & 0 & 0 & 0 \end{bmatrix}, \\
A_{12} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -2\pi T_{12} & 0 & 0 & 0 \end{bmatrix}, \\
A_{21} &= \begin{bmatrix} 0 & 0 & 0 & 0 & \dfrac{1}{M_2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \\
A_{22} &= \begin{bmatrix} -\dfrac{D_2}{M_2} & \dfrac{1}{M_2} & 0 & 0 \\ 0 & -\dfrac{1}{T_{ch2}} & \dfrac{1}{T_{ch2}} & 0 \\ -\dfrac{1}{R_2 T_{g2}} & 0 & -\dfrac{1}{T_{g2}} & 0 \\ \beta_2 & 0 & 0 & 0 \end{bmatrix},
\end{aligned}
$$

$$
\begin{aligned}
B &= \mathrm{diag}\{B_1, B_2\}, \;\; B_1 = \left[0 \; 0 \; \frac{1}{T_{g1}} \; 0 \; 0\right]^T, \\
B_2 &= \left[0 \; 0 \; \frac{1}{T_{g2}} \; 0\right]^T, \\
C &= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, \\
C_{11} &= \begin{bmatrix} \beta_1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \\
C_{12} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
C_{21} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
C_{22} &= \begin{bmatrix} \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
D_\omega &= \mathrm{diag}\{D_{\omega 1}, D_{\omega 2}\}, \\
D_{\omega 1} &= \left[-\frac{1}{M_1} \; 0 \; 0 \; 0 \; 0\right]^T, \\
D_{\omega 2} &= \left[-\frac{1}{M_2} \; 0 \; 0 \; 0\right]^T.
\end{aligned}
$$

LFC is implemented by PI controller with control error ACE as input:

$$\Delta P_{c1}(t) \triangleq u_1(t) = -K_{P1}\mathrm{ACE}_1 - K_{I1}\int \mathrm{ACE}_1, \quad (2)$$

$$\Delta P_{c2}(t) \triangleq u_2(t) = -K_{P2}\mathrm{ACE}_2 - K_{I2}\int \mathrm{ACE}_2, \quad (3)$$

where K_{Pi} and K_{Ii} are the control gain matrices, and ACE_i $(i = 1, 2)$ indicate area control errors. Since there are time-varying delays $h_{1t} \triangleq h_1(t)$ and $h_{2t} \triangleq h_2(t)$ in the feedback channel and the forward channel respectively, we have the following formula

$$\begin{aligned}&\mathrm{ACE}_1=\beta_1\Delta f_1\left(t-h_{1t}\right),\ \ \mathrm{ACE}_2=\beta_2\Delta f_2\left(t-h_{1t}\right),\\&\Delta P_{c1}\left(t\right)=u_1\left(t-h_{2t}\right),\ \ \Delta P_{c2}\left(t\right)=u_2\left(t-h_{2t}\right),\end{aligned}$$

where the frequency bias factors $\beta_1,\ \beta_2>0$, and h_{1t} and h_2t are differentiable and bounded. Given the non-negative constants h_{11}, h_1, h_{21}, h_2 and μ_1, μ_2, the time-varying delays satisfy the following conditions

$$\begin{aligned}&h_{11}\le h_{1t}\le h_1,\ \ h_{21}\le h_{2t}\le h_2,\\&|\dot{h}_{1t}|\le\mu_1,\ \ |\dot{h}_{2t}|\le\mu_2,\quad\forall t>0.\end{aligned}\tag{4}$$

Letting $K=\operatorname{diag}\{K_1,K_2\}$, $K_1=\begin{bmatrix}K_{P1}\ K_{I1}\end{bmatrix}$, $K_2=\begin{bmatrix}K_{P2}\ K_{I2}\end{bmatrix}$, $\omega(t)=\Delta P_d(t)$, $h_t=h_{1t}+h_{2t}$ and $h=h_1+h_2$, the closed-loop LFC power systems can be rewritten in the following form.

$$\begin{cases}\dot{x}(t)=Ax(t)+A_1x(t-h_t)+D_\omega\omega(t)\\y(t)=Cx(t)\\x(t)=\phi(t),\ \ t\in[-h,0],\end{cases}\tag{5}$$

where $\phi(t)$ is a continuous vector function on $[-h,0]$, representing the initial conditions.

$$A_1=\begin{bmatrix}A_{111}&A_{112}\\A_{121}&A_{122}\end{bmatrix},$$

$$A_{111}=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\-\frac{K_{P1}\beta_1}{T_{g1}}&0&0&-\frac{K_{I1}}{T_{g1}}&-\frac{K_{P1}}{T_{g1}}\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix},$$

$$A_{112}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix},$$

$$A_{121}=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&-\frac{K_{P2}}{T_{g2}}\\0&0&0&0&0\end{bmatrix},$$

$$A_{122}=\begin{bmatrix}0&0&0&0\\0&0&0&0\\-\frac{K_{P2}\beta_2}{T_{g2}}&0&0&-\frac{K_{I2}}{T_{g2}}\\0&0&0&0\end{bmatrix}.$$

Definition 1 [22]: For time-delay LFC power systems, the unknown external load disturbances can be described as nonlinear disturbances of current and delay state vectors, which can be expressed as the flowing equation.

$$\hat{D}_w w\left(t\right)=\eta\left(x\left(t\right),x\left(t-h\left(t\right)\right)\right),\tag{6}$$

which satisfies the constraints of the following inequalities

$$\|\eta\left(\cdot\right)\|\le\varpi\|x\left(t\right)\|+\nu\|x\left(t-h\left(t\right)\right)\|.\tag{7}$$

Here, ϖ and ν are known positive scalars. The inequality (7) is further generalized to

$$\begin{aligned}\eta(\,\cdot\,)^T\eta(\,\cdot\,)\le{}&\varpi^2x^T(t)E^TEx(t)\\&+\nu^2x^T(t-h\,(t)\,)N^TNx(t-h\,(t)\,),\end{aligned}\tag{8}$$

where E and N are known positive definite matrices with appropriate dimension.

Lemma 1 [16]: For any matrix $Q\in\mathbb{S}_+^n$ and a vector function $g:[m,n]\longrightarrow\mathbb{R}^n$, the following integral inequality holds

$$\int_m^n\dot{g}^T(\theta)Q\dot{g}(\theta)d\theta\ge\frac{1}{n-m}\varrho^T\bar{Q}\varrho,\tag{9}$$

where

$$\begin{aligned}\bar{Q}&=\operatorname{diag}\{Q,3Q,5Q\},\\\varrho&=\operatorname{col}\{\varrho_1,\varrho_2,\varrho_3\},\\\varrho_1&=g(n)-g(m),\\\varrho_2&=g(n)+g(m)\\&\quad-\frac{2}{n-m}\int_m^n g(\theta)d\theta,\\\varrho_3&=\varrho_1-\frac{6}{n-m}\int_m^n g(\theta)d\theta\\&\quad+\frac{12}{(n-m)^2}\int_m^n(n-\theta)g(\theta)d\theta.\end{aligned}$$

Lemma 2 [15]: For given constant scalar $\alpha\in(0,1]$, matrices $\Lambda_1,\Lambda_2\in\mathbb{S}_+^m$, $\upsilon_1,\upsilon_2\in\mathbb{R}^m$, if there are $\varsigma_1,\varsigma_2\in\mathbb{S}^m$, $\Upsilon_1,\Upsilon_2\in\mathbb{R}^{m\times m}$ such that

$$\begin{bmatrix}\Lambda_1-\varsigma_1&\Upsilon_1\\ *&\Lambda_2\end{bmatrix}>0,$$
$$\begin{bmatrix}\Lambda_1&\Upsilon_2\\ *&\Lambda_2-\varsigma_2\end{bmatrix}>0,$$

then the following inequality is true.

$$\begin{aligned}\frac{1}{\alpha}\upsilon_1^T\Lambda_1\upsilon_1+\frac{1}{1-\alpha}\upsilon_2^T\Lambda_2\upsilon_2\ge{}&2\upsilon_1^T[\alpha\Upsilon_1+(1-\alpha)\Upsilon_2]\upsilon_2\\&+\upsilon_1^T[\Lambda_1+(1-\alpha)\varsigma_1]\upsilon_1\\&+\upsilon_2^T[\Lambda_2+\alpha\varsigma_2]\upsilon_2.\end{aligned}$$

Lemma 3 [19]: For given matrices $P_i\in\mathbb{S}^p$, $(i=0,1,2)$, $\zeta\in\mathbb{R}^p$, the following inequality

$$\zeta^T(\tau_t^2P_2+\tau_tP_1+P_0)\zeta<0$$

holds for all $\tau_t\in[\tau_1,\tau_2]$, if and only if there are a matrix $M\in\mathbb{S}_+^p$ and a skew symmetric matrix $N\in\mathbb{R}^{k\times k}$ such that

$$\begin{bmatrix}P_0&\frac{1}{2}P_1\\ *&P_2\end{bmatrix}-\begin{bmatrix}C\\J\end{bmatrix}^T\begin{bmatrix}-M&N\\ *&M\end{bmatrix}\begin{bmatrix}C\\J\end{bmatrix}<0,$$

where $C=\begin{bmatrix}\frac{\tau_{12}}{2}I&0\end{bmatrix}$, $J=\begin{bmatrix}\frac{\tau_{12}}{2}I&-I\end{bmatrix}$ and $\tau_{12}=\tau_2-\tau_1$.

III. MAIN RESULTS

In this section, a new stability criterion of two-area LFC power systems with interval time-varying delays is derived. To simplify the derivation, the following symbols and matrices are given.

$$
\begin{aligned}
& h_{t1}=h_{1t}-h_{11},\ \bar{h}_{t1}=h_1-h_{1t},\ h_{t2}=h_{2t}-h_{21},\\
& \bar{h}_{t2}=h_2-h_{2t},\ h_{10}=h_1-h_{11},\ h_{20}=h_2-h_{21},\\
& h_0=h_{11}+h_{21},\ h_{h0}=h-h_0,\ h_{t0}=h_t-h_0,\\
& \bar{h}_{t0}=h-h_t,\ h_{1d}=1-\dot{h}_{1t},\ h_{2d}=1-\dot{h}_{2t},\\
& h_d=1-\dot{h}_t,\ Z_{1t}=h_{1t}Z_{11}+Z_{12},\ Z_{2t}=\bar{h}_{1t}Z_{21}+Z_{22},\\
& Z_{3t}=h_{2t}Z_{31}+Z_{32},\ Z_{4t}=\bar{h}_{2t}Z_{41}+Z_{42},\\
& \rho_{1t}=\int_{t-h_1}^{t-h_{1t}}\frac{x(\theta)}{\bar{h}_{t1}}d\theta,\ \rho_{3t}=\int_{t-h}^{t-h_t}\frac{x(\theta)}{\bar{h}_{t0}}d\theta,\\
& \rho_{5t}=\int_{t-h_{1t}}^{t-h_{11}}\frac{x(\theta)}{h_{t1}}d\theta,\ \rho_{7t}=\int_{t-h_t}^{t-h_0}\frac{x(\theta)}{h_{t0}}d\theta,\\
& \rho_{2t}=\int_{t-h_1}^{t-h_{1t}}\frac{(t-h_{1t}-\theta)x(\theta)}{\bar{h}_{t1}^2}d\theta,\\
& \rho_{4t}=\int_{t-h}^{t-h_t}\frac{(t-h_t-\theta)x(\theta)}{\bar{h}_{t0}^2}d\theta,\\
& \rho_{6t}=\int_{t-h_{1t}}^{t-h_{11}}\frac{(t-h_{11}-\theta)x(\theta)}{h_{t1}^2}d\theta,\\
& \rho_{8t}=\int_{t-h_t}^{t-h_0}\frac{(t-h_0-\theta)x(\theta)}{h_{t0}^2}d\theta,\\
& \varphi_{01t}=\mathrm{col}\{x(t),x(t-h_{11}),x(t-h_{1t}),x(t-h_1)\},\\
& \varphi_{02t}=\mathrm{col}\{x(t),x(t-h_0),x(t-h_t),x(t-h)\},\\
& \varphi_{1t}=\mathrm{col}\{x(t-h_{11}),x(t-h_{1t}),\int_{t-h_{1t}}^{t-h_{11}}x(\theta)d\theta\},\\
& \varphi_{2t}=\mathrm{col}\{x(t-h_{1t}),x(t-h_1),\int_{t-h_1}^{t-h_{1t}}x(\theta)d\theta\},\\
& \varphi_{3t}=\mathrm{col}\{x(t-h_0),x(t-h_t),\int_{t-h_t}^{t-h_0}x(\theta)d\theta\},\\
& \varphi_{4t}=\mathrm{col}\{x(t-h_t),x(t-h),\int_{t-h}^{t-h_t}x(\theta)d\theta\},\\
& \varphi_5(s)=\mathrm{col}\{\dot{x}(s),x(s),\varphi_{01t},\int_s^{t-h_{11}}x(\theta)d\theta\\
&\qquad\times\int_{t-h_{1t}}^{s}x(\theta)d\theta,\int_{t-h_1}^{t-h_{1t}}x(\theta)d\theta\},\\
& \varphi_6(s)=\mathrm{col}\{\dot{x}(s),x(s),\varphi_{01t},\int_s^{t-h_{1t}}x(\theta)d\theta,\\
&\qquad\int_{t-h_{1t}}^{t-h_{11}}x(\theta)d\theta,\int_{t-h_1}^{s}x(\theta)d\theta\},\\
& \varphi_7(s)=\mathrm{col}\{\dot{x}(s),x(s),\varphi_{02t},\int_s^{t-h_0}x(\theta)d\theta,\\
&\qquad\int_{t-h_t}^{s}x(\theta)d\theta,\int_{t-h}^{t-h_t}x(\theta)d\theta\},\\
& \varphi_8(s)=\mathrm{col}\{\dot{x}(s),x(s),\varphi_{02t},\int_s^{t-h_t}x(\theta)d\theta,\\
&\qquad\int_{t-h_t}^{t-h_0}x(\theta)d\theta,\int_{t-h}^{s}x(\theta)d\theta\},\\
& \zeta_t=\mathrm{col}\{x(t),x(t-h_{1t}),x(t-h_t),x(t-h_1),x(t-h),\\
&\qquad x(t-h_{11}),x(t-h_0),\dot{x}(t),\dot{x}(t-h_{1t}),\dot{x}(t-h_t),\\
&\qquad \dot{x}(t-h_1),\dot{x}(t-h),\dot{x}(t-h_{11}),\dot{x}(t-h_{11}),\\
&\qquad \dot{x}(t-h_0),\rho_{1t},\rho_{2t},\rho_{3t},\rho_{4t},\\
&\qquad \rho_{5t},\rho_{6t},\rho_{7t},\rho_{8t},D_\omega\omega(t)\}.
\end{aligned}
$$

Theorem 1: For given scalars h_{11}, h_{21}, h_1, h_2, μ_1, μ_2, ϖ, ν, positive definite matrices $E=\mathrm{diag}\{\varepsilon_1,\cdots,\varepsilon_n\}$ and $N=\mathrm{diag}\{n_1,\cdots,n_n\}$, if there are $Z_{r1}, X_r\in\mathbb{S}^{3n}$, $Z_{r2}\in\mathbb{S}_+^{3n}$, $Y_r\in\mathbb{R}^{3n\times 3n}$, $H_r\in\mathbb{S}_+^{n}$, $D_r\in\mathbb{S}_+^{14n}$, $Q_r\in\mathbb{S}_+^{8n}$, $G_r\in\mathbb{R}^{14n\times 14n}$ and $U_r\in\mathbb{R}^{4n\times n}$, such that

$$h_{11}Z_{11}+Z_{12}>0,\quad h_{10}Z_{21}+Z_{22}>0, \tag{10}$$

$$h_0Z_{31}+Z_{32}>0,\quad h_{h0}Z_{41}+Z_{42}>0, \tag{11}$$

$$\begin{bmatrix}\bar{H}_1-X_1 & Y_1\\ * & \bar{H}_2\end{bmatrix}>0,\quad \begin{bmatrix}\bar{H}_1 & Y_2\\ * & \bar{H}_2-X_2\end{bmatrix}>0, \tag{12}$$

$$\begin{bmatrix}\bar{H}_3-X_3 & Y_3\\ * & \bar{H}_4\end{bmatrix}>0,\quad \begin{bmatrix}\bar{H}_3 & Y_4\\ * & \bar{H}_4-X_4\end{bmatrix}>0, \tag{13}$$

$$\begin{bmatrix}\Omega_{10}(\gamma_i) & \frac{1}{2}\Omega_{11}(\gamma_i)\\ * & \Omega_{21}(\gamma_i)\end{bmatrix}-\begin{bmatrix}C_1\\ J_1\end{bmatrix}^T\begin{bmatrix}-D_i & G_i\\ * & D_i\end{bmatrix}\begin{bmatrix}C_1\\ J_1\end{bmatrix}0, \tag{14}$$

$$\begin{bmatrix}\Omega_0(\delta_k) & \frac{1}{2}\Omega_1(\delta_k)\\ * & \Omega_2(\delta_k)\end{bmatrix}-\begin{bmatrix}C_2\\ J_2\end{bmatrix}^T\begin{bmatrix}-D_k & G_k\\ * & D_k\end{bmatrix}\begin{bmatrix}C_2\\ J_2\end{bmatrix}<0. \tag{15}$$

Then, the system (3) is stable for $h_{it}\in[h_{i1},h_i]$, $\dot{h}_{1t}\triangleq\gamma_i\in\{-\mu_1,\mu_1\}$ and $\dot{h}_t\triangleq\delta_k\in\{-\mu,\mu\}$, $(i=1,2;\ k=3,4;\ r=1,2,3,4)$.

Here,

$$
\begin{aligned}
& C_1=\begin{bmatrix}\frac{h_{10}}{2}I & 0\end{bmatrix},\quad J_1=\begin{bmatrix}\frac{h_{10}}{2}I & -I\end{bmatrix},\\
& C_2=\begin{bmatrix}\frac{h_{h0}}{2}I & 0\end{bmatrix},\quad J_2=\begin{bmatrix}\frac{h_{h0}}{2}I & -I\end{bmatrix},\\
& \bar{H}_r=\mathrm{diag}\{H_r,3H_r,5H_r\},\\
\Omega_{10}(\gamma_i)=\ & \dot{h}_{1t}\Delta_{11}^TZ_{11}\Delta_{11}+\dot{\bar{h}}_{1t}\Delta_{21}^TZ_{21}\Delta_{21}\\
& +\mathrm{Sym}\left\{\Delta_{13}^T\left(Z_{12}-h_{11}Z_{11}\right)\Delta_{11}\right.\\
& \left.+\Delta_{23}^T\left(h_1Z_{21}+Z_{22}\right)\Delta_{21}\right\}\\
& +\Delta_{51}^TQ_1\Delta_{51}-\Delta_{63}^TQ_2\Delta_{63}\\
& +h_{1d}\left(\Delta_{61}^TQ_2\Delta_{61}-\Delta_{53}^TQ_1\Delta_{53}\right)\\
& +\mathrm{Sym}\left\{\Lambda_{10}^TQ_1\Lambda_1+\Lambda_{20}^TQ_2\Lambda_2\right\}\\
& +h_{10}h_{1d}h_1e_9\left(H_1-H_2\right)e_9^T\\
& +h_{10}^2e_{13}H_2e_{13}^T-\Gamma_1^T\left(\bar{H}_1+X_1\right)\Gamma_1\\
& +\Gamma_2^T\bar{H}_2\Gamma_2+\mathrm{Sym}\left\{\Gamma_1^TY_2\Gamma_2\right\},
\end{aligned}
$$

$$
\begin{aligned}
\Omega_{11}(\gamma_i) = {} & \mathrm{Sym}\Big\{\dot{h}_{1t}\Delta_{11}^T Z_{11}\Delta_{12} + \dot{\bar{h}}_{1t}\Delta_{21}^T Z_{21}\Delta_{22} \\
& +\Delta_{13}^T Z_{11}\Delta_{11} + \Delta_{13}^T (Z_{12} - h_{11}Z_{11})\Delta_{12} \\
& +\Delta_{23}^T (h_1 Z_{21} + Z_{22})\Delta_{22} - \Delta_{23}^T Z_{21}\Delta_{21}\Big\} \\
& + \mathrm{Sym}\Big\{\Delta_{52}^T Q_1\Delta_{51} - \Delta_{64}^T Q_2\Delta_{63} \\
& +\Lambda_{11}^T Q_1\Lambda_1 + \Lambda_{21}^T Q_2\Lambda_2\Big\} \\
& + h_{1d}\mathrm{Sym}\Big\{\Delta_{62}^T Q_2\Delta_{61} - \Delta_{54}^T Q_1\Delta_{53}\Big\} \\
& - h_{10}h_{1d}e_9 (H_1 - H_2) e_9^T \\
& + \frac{1}{h_{10}}\left(\Gamma_1^T X_1\Gamma_1 + \Gamma_2^T X_2\Gamma_2\right) \\
& + \frac{1}{h_{10}}\mathrm{Sym}\Big\{\Gamma_1^T Y_1\Gamma_2 - \Gamma_1^T Y_2\Gamma_2\Big\}, \\
\Omega_{21}(\gamma_i) = {} & \mathrm{Sym}\Big\{\Delta_{13}^T Z_{11}\Delta_{12} - \Delta_{23}^T Z_{21}\Delta_{22}\Big\} \\
& + \dot{h}_{t1}\Delta_{12}^T Z_{11}\Delta_{12} + \dot{\bar{h}}_{1t}\Delta_{22}^T Z_{21}\Delta_{22} \\
& + \Delta_{52}^T Q_1\Delta_{52} - \Delta_{64}^T Q_2\Delta_{64} \\
& + h_{1d}\left(\Delta_{62}^T Q_2\Delta_{62} - \Delta_{54}^T Q_1\Delta_{54}\right) \\
& \mathrm{Sym}\Big\{\Lambda_{12}^T Q_1\Lambda_1 + \Lambda_{22}^T Q_2\Lambda_2\Big\}, \\
\Omega_0(\delta_k) = {} & \dot{h}_{t0}\Delta_{31}^T Z_{31}\Delta_{31} + \dot{\bar{h}}_{t0}\Delta_{41}^T Z_{41}\Delta_{41} \\
& + \mathrm{Sym}\Big\{\Delta_{33}^T (Z_{32} - h_0 Z_{31})\Delta_{31} \\
& +\Delta_{43}^T (hZ_{41} + Z_{42})\Delta_{41}\Big\} + \Delta_{71}^T Q_3\Delta_{71} \\
& - \Delta_{83}^T Q_4\Delta_{83} + h_d\Delta_{81}^T Q_4\Delta_{81} - h_d\Delta_{73}^T Q_3\Delta_{73} \\
& + \mathrm{Sym}\Big\{\Lambda_{30}^T Q_3\Lambda_3 + \Lambda_{40}^T Q_4\Lambda_4\Big\} \\
& + h_{h0}h_d he_{10}(H_3 - H_4)e_{10}^T + h_{h0}^2 e_{14}H_4 e_{14}^T \\
& - \Gamma_3^T\bar{H}_3\Gamma_3 - \Gamma_3^T X_3\Gamma_3 \\
& + \Gamma_4^T\bar{H}_4\Gamma_4 + \mathrm{Sym}\Big\{\Gamma_3^T Y_4\Gamma_4 + \Delta_0^T U\Pi_0\Big\} \\
& - e_{24}Ie_{24}^T + \varpi^2 e_1 E^T E e_1^T + \nu^2 e_3 N^T N e_3^T, \\
\Omega_1(\delta_k) = {} & \mathrm{Sym}\Big\{\dot{h}_{t0}\Delta_{31}^T Z_{31}\Delta_{32} + \dot{\bar{h}}_{t0}\Delta_{41}^T Z_{41}\Delta_{42} \\
& + \Delta_{33}^T Z_{31}\Delta_{31} + \Delta_{33}^T (Z_{32} - h_0 Z_{31})\Delta_{32} \\
& + \Delta_{43}^T (Z_{42} + hZ_{41})\Delta_{42} - \Delta_{43}^T Z_{41}\Delta_{41}\Big\} \\
& + \mathrm{Sym}\Big\{\Delta_{72}^T Q_3\Delta_{71} - \Delta_{84}^T Q_4\Delta_{83}\Big\} \\
& + h_d\mathrm{Sym}\Big\{\Delta_{82}^T Q_4\Delta_{81} - \Delta_{74}^T Q_3\Delta_{73}\Big\} \\
& + \mathrm{Sym}\Big\{\Lambda_{31}^T Q_3\Lambda_3 + \Lambda_{41}^T Q_4\Lambda_4\Big\} \\
& - h_{h0}h_d e_{10}(H_3 - H_4)e_{10}^T \\
& + \frac{1}{h_{h0}}\left(\Gamma_3^T X_3\Gamma_3 + \Gamma_3^T X_3\Gamma_3\right) \\
& + \frac{1}{h_{h0}}\mathrm{Sym}\Big\{\Gamma_3^T Y_3\Gamma_4 - \Gamma_3^T Y_4\Gamma_4\Big\}, \\
\Omega_2(\delta_k) = {} & \dot{h}_{t0}\Delta_{32}^T Z_{31}\Delta_{32} + \dot{\bar{h}}_{t0}\Delta_{42}^T Z_{41}\Delta_{42} \\
& + \mathrm{Sym}\Big\{\Delta_{33}^T Z_{31}\Delta_{32} - \Delta_{43}^T Z_{41}\Delta_{42}\Big\} \\
& + \Delta_{72}^T Q_3\Delta_{72} - \Delta_{84}^T Q_4\Delta_{84} \\
& + h_d\left(\Delta_{82}^T Q_4\Delta_{82} - \Delta_{74}^T Q_3\Delta_{74}\right) \\
& + \mathrm{Sym}\Big\{\Lambda_{32}^T Q_3\Lambda_3 + \Lambda_{42}^T Q_4\Lambda_4\Big\}.
\end{aligned}
$$

$$
\begin{aligned}
\Delta_0 &= \mathrm{col}\{e_1, e_3, e_8, e_{24}\}, \\
\Pi_0 &= \mathrm{col}\{Ae_1, A_1e_3, e_{24}, -e_8\}, \\
\Delta_{11} &= \mathrm{col}\{e_6, e_2, -h_{11}e_{19}\}, \quad \Delta_{12} = \mathrm{col}\{0, 0, e_{19}\}, \\
\Delta_{13} &= \mathrm{col}\{e_{13}, e_9, e_6 - h_{1d}e_2\}, \\
\Delta_{21} &= \mathrm{col}\{e_2, e_4, h_1e_{15}\}, \Delta_{22} = \mathrm{col}\{0, 0, -e_{15}\}, \\
\Delta_{23} &= \mathrm{col}\{e_9, e_{11}, e_2 - h_{1d}e_4\}, \\
\Delta_{31} &= \mathrm{col}\{e_7, e_3, -h_{10}e_{21}\}, \Delta_{32} = \mathrm{col}\{0, 0, e_{21}\}, \\
\Delta_{33} &= \mathrm{col}\{e_{14}, e_{10}, e_7 - h_de_3\}, \\
\Delta_{41} &= \mathrm{col}\{e_3, e_5, he_{17}\}, \Delta_{42} = \mathrm{col}\{0, 0, -e_{17}\}, \\
\Delta_{43} &= \mathrm{col}\{e_{10}, e_{12}, e_3 - h_de_5\}, \\
\Delta_{51} &= \mathrm{col}\{e_{13}, e_6, e_1, e_6, e_2, e_4, 0, -h_{11}e_{19}, h_1e_{15}\}, \\
\Delta_{52} &= \mathrm{col}\{0, 0, 0, 0, 0, 0, 0, e_{19}, -e_{15}\}, \\
\Delta_{53} &= \mathrm{col}\{e_9, e_2, e_1, e_6, e_2, e_4, -h_{11}e_{19}, 0, h_1e_{15}\}, \\
\Delta_{54} &= \mathrm{col}\{0, 0, 0, 0, 0, 0, e_{19}, 0, -e_{15}\}, \\
\Delta_{61} &= \mathrm{col}\{e_9, e_2, e_1, e_6, e_2, e_4, 0, -h_{11}e_{19}, h_1e_{15}\}, \\
\Delta_{62} &= \mathrm{col}\{0, 0, 0, 0, 0, 0, 0, e_{19}, -e_{15}\}, \\
\Delta_{63} &= \mathrm{col}\{e_{11}, e_4, e_1, e_6, e_2, e_4, h_1e_{15}, -h_{11}e_{19}, 0\}, \\
\Delta_{64} &= \mathrm{col}\{0, 0, 0, 0, 0, 0, -e_{15}, e_{19}, 0\}, \\
\Delta_{71} &= \mathrm{col}\{e_{14}, e_7, e_1, e_7, e_3, e_5, 0, -h_0e_{21}, he_{17}\}, \\
\Delta_{72} &= \mathrm{col}\{0, 0, 0, 0, 0, 0, 0, e_{21}, -e_{17}\}, \\
\Delta_{73} &= \mathrm{col}\{e_{10}, e_3, e_1, e_7, e_3, e_5, -h_0e_{21}, 0, he_{17}\}, \\
\Delta_{74} &= \mathrm{col}\{0, 0, 0, 0, 0, 0, e_{21}, 0, -e_{17}\}, \\
\Delta_{81} &= \mathrm{col}\{e_{10}, e_3, e_1, e_7, e_3, e_5, 0, -h_0e_{21}, he_{17}\}, \\
\Delta_{82} &= \mathrm{col}\{0, 0, 0, 0, 0, 0, 0, e_{21}, -e_{17}\}, \\
\Delta_{83} &= \mathrm{col}\{e_{12}, e_5, e_1, e_7, e_3, e_5, he_{17}, -h_0e_{21}, 0\}, \\
\Delta_{84} &= \mathrm{col}\{0, 0, 0, 0, 0, 0, -e_{17}, e_{21}, 0\}, \\
\Lambda_{10} &= \mathrm{col}\{e_6 - e_2, -h_{11}e_{19}, -h_{11}e_1, -h_{11}e_6, -h_{11}e_2, \\
&\qquad -h_{11}e_4, h_{11}^2(e_{19} - e_{20}), h_{11}^2e_{20}, h_{11}h_1e_{15}\Big\}, \\
\Lambda_{11} &= \mathrm{col}\{0, e_{19}, e_1, e_6, e_2, e_4, -2h_{11}e_{20}, (h_{11} + h_1)e_{15}\}, \\
\Lambda_{12} &= \mathrm{col}\{0, 0, 0, 0, 0, 0, e_{19} - e_{20}, e_{20}, -e_{15}\}, \\
\Lambda_{20} &= \mathrm{col}\{e_2 - e_4, h_1e_{15}, h_1e_1, h_1e_6, h_1e_2, h_1e_4, \\
&\qquad h_1^2(e_{15} - e_{16}), -h_{11}h_1e_{19}, h_1^2e_{16}\Big\}, \\
\Lambda_{21} &= \mathrm{col}\{0, -e_{15}, -e_1, -e_6, -e_2, -e_4, \\
&\qquad -2h_1(e_{15} - e_{16}), (h_{11} + h_1)e_{19}, -2h_1e_{16}\}, \\
\Lambda_{22} &= \mathrm{col}\{0, 0, 0, 0, 0, 0, e_{15} - e_{16}, -e_{19}, e_{16}\}, \\
\Lambda_{30} &= \mathrm{col}\{e_7 - e_3, h_1e_{15}, -h_0e_{21}, -h_0e_1, -h_0e_7, \\
&\qquad -h_0e_3, -h_0e_5, h_0^2(e_{21} - e_{22}), h_0^2e_{22}, -h_0he_{17}\Big\}, \\
\Lambda_{31} &= \mathrm{col}\{0, e_{21}, e_1, e_7, e_3, e_5, -2h_2(e_{21} - e_{22}),
\end{aligned}
$$

$$-2h_0e_{22}, (h+h_0)e_{17}\},$$
$$\Lambda_{32} = \mathrm{col}\{0, 0, 0, 0, 0, 0, e_{21}-e_{22}, e_{22}, -e_{17}\},$$
$$\Lambda_{40} = \mathrm{col}\Big\{e_3-e_5, he_{17}, he_1, he_7, he_3, he_5, h^2(e_{17}-e_{18}), -h_0he_{21}, h^2e_{18}\Big\},$$
$$\Lambda_{41} = \mathrm{col}\{0, e_{17}, -e_1, -e_7, -e_3, -e_5, -2h(e_{17}-e_{18}), (h+h_0)e_{21}, -2he_{18}\},$$
$$\Lambda_{42} = \mathrm{col}\{0, 0, 0, 0, 0, 0, e_{17}-e_{18}, -e_{21}, e_{18}\},$$
$$\Lambda_1 = \mathrm{col}\{0, 0, e_8, e_{13}, h_{1d}e_9, e_{11}, e_6, -h_{1d}e_2, h_{1d}e_2-e_4\},$$
$$\Lambda_2 = \mathrm{col}\{0, 0, e_8, e_{13}, h_{1d}e_9, e_{11}, h_{1d}e_2, e_6-h_{1d}e_2, -e_4\},$$
$$\Lambda_3 = \mathrm{col}\{0, 0, e_8, e_{14}, e_{10}, e_{12}, e_1, -h_de_3, h_de_3-e_5\},$$
$$\Lambda_4 = \mathrm{col}\{0, 0, e_8, e_{14}, e_{10}, e_5, h_de_3, e_7-h_de_3, -e_5\},$$
$$\Gamma_1 = \mathrm{col}\{e_2-e_4, e_2+e_4-2e_{15}, e_2-e_4-6e_{15}+12e_{16}\},$$
$$\Gamma_2 = \mathrm{col}\{e_6-e_2, e_6+e_2-2e_{19}, e_6-e_2-6e_{19}+12e_{20}\},$$
$$\Gamma_3 = \mathrm{col}\{e_3-e_5, e_3+e_5-2e_{17}, e_2-e_3-6e_{17}+12e_{18}\},$$
$$\Gamma_4 = \mathrm{col}\{e_7-e_3, e_7+e_3-2e_{21}, e_2-e_7-6e_{21}+12e_{22}\}.$$

Proof: Construct the following LKF.

$$V(t) = \sum_{u=1}^{3} V_u(t) \tag{16}$$

with

$$V_1(t) = \varphi_{1t}^T Z_{1t}\varphi_{1t} + \varphi_{2t}^T Z_{2t}\varphi_{2t} + \varphi_{3t}^T Z_{3t}\varphi_{3t} + \varphi_{4t}^T Z_{4t}\varphi_{4t},$$
$$V_2(t) = \int_{t-h_{1t}}^{t-h_{11}} \varphi_5^T(\theta)Q_1\varphi_5(\theta)d\theta + \int_{t-h_1}^{t-h_{1t}} \varphi_6^T(\theta)Q_2\varphi_6(\theta)d\theta + \int_{t-h_t}^{t-h_0} \varphi_7^T(\theta)Q_3\varphi_7(\theta)d\theta + \int_{t-h}^{t-h_t} \varphi_8^T(\theta)Q_4\varphi_8(\theta)d\theta,$$
$$V_3(t) = h_{10}\int_{t-h_1}^{t-h_{1t}} (h_1-t+\theta)\dot{x}^T(\theta)H_1\dot{x}(\theta)d\theta + h_{10}\int_{t-h_{1t}}^{t-h_{11}} (h_1-t+\theta)\dot{x}^T(\theta)H_2\dot{x}(\theta)d\theta + h_{h0}\int_{t-h}^{t-h_t} (h-t+\theta)\dot{x}^T(\theta)H_3\dot{x}(\theta)d\theta + h_{h0}\int_{t-h_t}^{t-h_0} (h-t+\theta)\dot{x}^T(\theta)H_4\dot{x}(\theta)d\theta.$$

$Z_i(t)$, $(i=1,2,3,4)$ are affine functions of $h_{1t} \in [h_{11}, h_1]$ and $h_t \in [h_0, h]$. Obviously, $V(t) > 0$ according to $Z_{i2} > 0$ and the inequalities (10) and (11). Calculating the derivative of $V(t)$, we get

$$\begin{aligned}
\dot{V}_1 &= \varphi_{1t}^T\dot{Z}_{1t}\varphi_{1t} + 2\dot{\varphi}_{1t}^T Z_{1t}\varphi_{1t} + \varphi_{2t}^T\dot{Z}_{2t}\varphi_{2t} + 2\dot{\varphi}_{2t}^T Z_{2t}\varphi_{2t} \\
&\quad + \varphi_{3t}^T\dot{Z}_{3t}\varphi_{3t} + 2\dot{\varphi}_{3t}^T Z_{3t}\varphi_{3t} + \varphi_{4t}^T\dot{Z}_{4t}\varphi_{4t} + 2\dot{\varphi}_{4t}^T Z_{4t}\varphi_{4t} \\
&= \zeta^T(t)(\Delta_{11}+h_{1t}\Delta_{12})^T\dot{h}_{t1}Z_{11}(\Delta_{11}+h_{1t}\Delta_{12})\zeta(t) \\
&\quad + \zeta^T(t)(\Delta_{21}+h_{1t}\Delta_{22})^T\dot{\bar{h}}_{t1}Z_{21}(\Delta_{21}+h_{1t}\Delta_{22})\zeta(t) \\
&\quad + \zeta^T(t)(\Delta_{31}+h_t\Delta_{32})^T\dot{h}_{t0}Z_{31}(\Delta_{31}+h_t\Delta_{32})\zeta(t) \\
&\quad + \zeta^T(t)(\Delta_{41}+h_t\Delta_{42})^T\dot{\bar{h}}_{t0}Z_{41}(\Delta_{41}+h_t\Delta_{42})\zeta(t) \\
&\quad + 2\zeta^T(t)\Delta_{13}^T(h_{t1}Z_{11}+Z_{12})(\Delta_{11}+h_{1t}\Delta_{12})\zeta(t) \\
&\quad + 2\zeta^T(t)\Delta_{23}^T(\bar{h}_{t1}Z_{21}+Z_{22})(\Delta_{21}+h_{1t}\Delta_{22})\zeta(t) \\
&\quad + 2\zeta^T(t)\Delta_{33}^T(h_{t0}Z_{31}+Z_{32})(\Delta_{31}+h_t\Delta_{32})\zeta(t) \\
&\quad + 2\zeta^T(t)\Delta_{43}^T(\bar{h}_{t0}Z_{41}+Z_{42})(\Delta_{41}+h_t\Delta_{42})\zeta(t),
\end{aligned} \tag{17}$$

$$\begin{aligned}
\dot{V}_2 &= \varphi_5^T(t-h_{11})Q_1\varphi_5(t-h_{11}) - \varphi_6^T(t-h_1)Q_2\varphi_6(t-h_1) \\
&\quad + \varphi_7^T(t-h_0)Q_3\varphi_7(t-h_0) - \varphi_8^T(t-h)Q_4\varphi_8(t-h) \\
&\quad - h_{1d}\varphi_5^T(t-h_{1t})Q_1\varphi_5(t-h_{1t}) + h_{1d}\varphi_6^T(t-h_{1t})Q_2\varphi_6(t-h_{1t}) \\
&\quad - h_d\varphi_7^T(t-h_t)Q_3\varphi_7(t-h_t) + h_d\varphi_8^T(t-h_t)Q_4\varphi_8(t-h_t) \\
&\quad + 2\int_{t-h_{1t}}^{t-h_{11}} \varphi_5^T(\theta)\,d\theta Q_1\frac{\partial}{\partial t}\varphi_5(\theta) + 2\int_{t-h_1}^{t-h_{1t}} \varphi_6^T(\theta)\,d\theta Q_2\frac{\partial}{\partial t}\varphi_6(\theta) \\
&\quad + 2\int_{t-h_t}^{t-h_0} \varphi_7^T(\theta)\,d\theta Q_3\frac{\partial}{\partial t}\varphi_7(\theta) + 2\int_{t-h}^{t-h_t} \varphi_8^T(\theta)\,d\theta Q_4\frac{\partial}{\partial t}\varphi_8(\theta) \\
&= \zeta^T(t)\Big((\Delta_{51}+h_{1t}\Delta_{52})^T Q_1(\Delta_{51}+h_{1t}\Delta_{52}) - h_{1d}(\Delta_{53}+h_{1t}\Delta_{54})^T Q_1(\Delta_{53}+h_{1t}\Delta_{54})\Big)\zeta(t) \\
&\quad + \zeta^T(t)\Big(h_{1d}(\Delta_{61}+h_{1t}\Delta_{62})^T Q_2(\Delta_{61}+h_{1t}\Delta_{62}) - (\Delta_{63}+h_{1t}\Delta_{64})^T Q_2(\Delta_{63}+h_{1t}\Delta_{64})\Big)\zeta(t) \\
&\quad + \zeta^T(t)\Big((\Delta_{71}+h_t\Delta_{72})^T Q_3(\Delta_{71}+h_t\Delta_{72}) - h_d(\Delta_{73}+h_t\Delta_{74})^T Q_3(\Delta_{73}+h_t\Delta_{74})\Big)\zeta(t) \\
&\quad + \zeta^T(t)\Big(h_d(\Delta_{81}+h_t\Delta_{82})^T Q_4(\Delta_{81}+h_t\Delta_{82}) - (\Delta_{83}+h_t\Delta_{84})^T Q_4(\Delta_{83}+h_t\Delta_{84})\Big)\zeta(t) \\
&\quad + 2\zeta^T(t)\Big(\Big(\Lambda_{10}+h_{1t}\Lambda_{11}+h_{1t}^2\Lambda_{12}\Big)^T Q_1\Lambda_1
\end{aligned}$$

$$
\begin{aligned}
&+\left(\Lambda_{20}+h_{1t}\Lambda_{21}+h_{1t}^{2}\Lambda_{22}\right)^{T}Q_{2}\Lambda_{2}\\
&+\left(\Lambda_{30}+h_{t}\Lambda_{31}+h_{t}^{2}\Lambda_{32}\right)^{T}Q_{3}\Lambda_{3}\\
&+\left(\Lambda_{40}+h_{t}\Lambda_{41}+h_{t}^{2}\Lambda_{42}\right)^{T}Q_{4}\Lambda_{4}\Big)\zeta(t), \qquad (18)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_3 =\ & h_{10}^{2}\dot{x}^{T}(t-h_{11})H_2\dot{x}(t-h_{11})\\
&+h_{h0}^{2}\dot{x}^{T}(t-h_0)H_4\dot{x}(t-h_0)\\
&+h_{10}h_{1d}\bar{h}_{t1}\dot{x}^{T}(t-h_{1t})(H_1-H_2)\dot{x}(t-h_{1t})\\
&+h_{h0}h_{d}\bar{h}_{t0}\dot{x}^{T}(t-h_{t})(H_3-H_4)\dot{x}(t-h_{t})\\
&-h_{10}\int_{t-h_1}^{t-h_{1t}}\dot{x}^{T}(\theta)H_1\dot{x}(\theta)d\theta\\
&-h_{10}\int_{t-h_{1t}}^{t-h_{11}}\dot{x}^{T}(\theta)H_2\dot{x}(\theta)d\theta\\
&-h_{h0}\int_{t-h}^{t-h_{t}}\dot{x}^{T}(\theta)H_3\dot{x}(\theta)d\theta\\
&-h_{h0}\int_{t-h_t}^{t-h_{0}}\dot{x}^{T}(\theta)H_4\dot{x}(\theta)d\theta, \qquad (19)
\end{aligned}
$$

Letting $\alpha=\frac{h_{1t}}{h_{10}}$, $\alpha=\frac{h_t}{h_{h0}}$, the integral terms in $\dot{V}_3(t)$ can be rewritten and simplified to the following inequalities according to lemmas 1 and 2.

$$
\begin{aligned}
&-h_{10}\int_{t-h_1}^{t-h_{1t}}\dot{x}^{T}(\theta)H_1\dot{x}(\theta)d\theta\\
&\quad-h_{10}\int_{t-h_{1t}}^{t}\dot{x}^{T}(\theta)H_2\dot{x}(\theta)d\theta\\
&\le-\frac{1}{\alpha}\zeta^{T}(t)\Gamma_1^{T}\overline{H_1}\Gamma_1\zeta(t)\\
&\quad-\frac{1}{1-\alpha}\zeta^{T}(t)\Gamma_2^{T}\overline{H_2}\Gamma_2\zeta(t)\\
&\le-\zeta^{T}(t)\Big\{\Gamma_1^{T}\left[(1-\alpha)X_1+\bar{H}_1\right]\Gamma_1\\
&\quad+2\Gamma_1^{T}\left[\alpha Y_1+(1-\alpha)Y_2\right]\Gamma_2\\
&\quad+\Gamma_2^{T}\left[\bar{H}_2+\alpha X_2\right]\Gamma_2\Big\}\zeta(t), \qquad (20)\\
&\quad-h_{h0}\int_{t-h}^{t-h_t}\dot{x}^{T}(\theta)H_3\dot{x}(\theta)d\theta\\
&\quad-h_{h0}\int_{t-h_t}^{t-h_0}\dot{x}^{T}(\theta)H_4\dot{x}(\theta)d\theta\\
&\le-\frac{1}{\beta}\zeta^{T}(t)\Gamma_3^{T}\overline{H_3}\Gamma_3\zeta(t)\\
&\quad-\frac{1}{1-\beta}\zeta^{T}(t)\Gamma_4^{T}\overline{H_4}\Gamma_4\zeta(t)\\
&\le-\zeta^{T}(t)\Big\{\Gamma_3^{T}\left[(1-\beta)X_3+\overline{H_3}\right]\Gamma_3\\
&\quad+2\Gamma_3^{T}\left[\beta Y_3+(1-\beta)Y_4\right]\Gamma_4\\
&\quad+\Gamma_4^{T}\left[\bar{H}_4+\beta X_4\right]\Gamma_4\Big\}\zeta(t) \qquad (21)
\end{aligned}
$$

For an any matrix $U=\mathrm{col}\{U_1,U_2,U_3,U_4\}$, the following equation holds.

$$
\begin{aligned}
0=&\begin{bmatrix}x(t)\\ x(t-h_t)\\ \dot{x}(t)\\ D_\omega w(t)\end{bmatrix}^{T}U\Big[Ax(t)+A_1x(t-h_t)\\
&+D_\omega w(t)-\dot{x}(t)\Big]\\
=&\ \zeta^{T}(t)\Delta_0^{T}U\Pi_0\zeta(t). \qquad (22)
\end{aligned}
$$

It is obvious that the following inequality holds from the constraint condition (8).

$$
\begin{aligned}
&(D_\omega w)^{T}D_\omega w-\varpi^{2}x^{T}(t)E^{T}Ex(t)\\
&\quad-\nu^{2}x^{T}(t-h(t))N^{T}Nx(t-h(t))\le 0. \qquad (23)
\end{aligned}
$$

Finally, the equations (16)–(23) in the derivation process are summarized and sorted into the following inequalities.

$$
\begin{aligned}
\dot{V}(t)\le\ &\zeta_t^{T}\left[h_t^{2}\Psi_2\left(\dot{h}_t\right)+h_t\Psi_1\left(\dot{h}_t\right)+\Psi_0\left(\dot{h}_t\right)\right]\zeta_t\\
&+\zeta_t^{T}\Big[h_{1t}^{2}\Psi_{21}\left(\dot{h}_{1t}\right)+h_{1t}\Psi_{11}\left(\dot{h}_{1t}\right)\\
&+\Psi_{10}\left(\dot{h}_{1t}\right)\Big]\zeta_t \qquad (24)
\end{aligned}
$$

According to Lemma 3, if and only if there exist $D_r\in\mathbb{S}_{+}^{10n}$ and skew-symmetric matrices $G_r\in\mathbb{R}^{14n\times 14n}$, $(r=1,2,3,4)$, the non-linear inequality (24) can be converted to the LMIs (14) and (15) in Theorem 1. And then $\dot{V}(t)<0$ holds. Therefore, by Lyapunov stability theorem, it can guarantee that two-area LFC power system with interval time-varying delay (4) is asymptotically stable. The proof is completed. ■

TABLE 1. **The two-area LFC power system parameters.**

Areas\ parameters	T_{gi}	T_{chi}	R_i	β_i	M_i	D_i
Area-1	0.1	0.3	0.05	21	10	1
Area-2	0.17	0.4	0.05	21.5	12	1.5

Remark 1: For nominal LFC systems where there is no extra load disturbance, the delay-dependent stability criterion for ascertaining internal stability of the system can be deduced from the main result by letting $\varpi=\nu=0$.

IV. NUMERICAL EXAMPLES

In this section, we will directly show the effectiveness of the stability criterion in this paper by using examples of commonly used two-region nominal LFC power systems. Given $\varpi=\nu=0$, different $K_{Pi}=K_P$ and $K_{Ii}=K_I$, $(i=1,2)$ values, the maximum allowable delay upper bound (MADUBs) of two-area LFC power systems can be obtained by solving the LMIs of the stability criterion through MATLAB LMI-toolbox. The two-area LFC power system parameters are shown in Table 1 with $T_{12}=0.1986$ pu/rad.

TABLE 2. The MADUBs h_2 for different h_{21}, μ_1 and μ_2.

h_{21}	0.5			1			1.5		
$\mu_1\backslash\mu_2$	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8
0.1	2.439	2.437	2.433	2.656	2.650	2.543	2.785	2.668	2.548
0.2	2.324	2.331	2.339	2.417	2.410	2.408	2.683	2.564	2.443
0.5	2.255	2.261	2.276	2.329	2.320	2.315	2.337	2.331	2.323
0.8	1.935	1.924	1.917	2.025	2.019	2.017	2.046	2.031	2.022

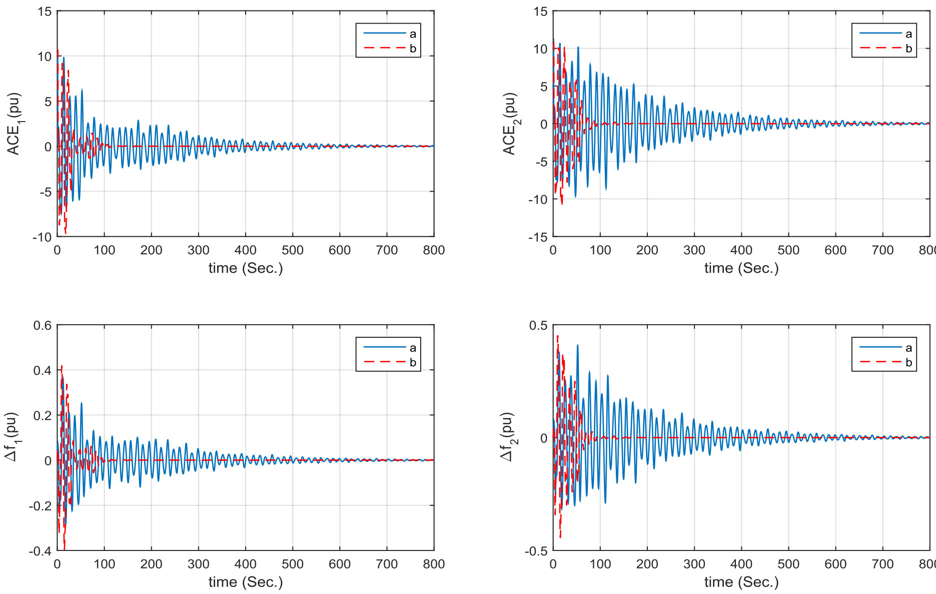

FIGURE 2. Frequency deviation and control error responses of two-area LFC power system with $(K_P, K_I) = (0.05, 0.4)$ and $h_{11} = 0.5$, $h_1 = 1.5$.

TABLE 3. The MADUBs h_1 for different h_{11}, μ_1 and μ_2.

h_{11}	0.2			1			1.5		
$\mu_1\backslash\mu_2$	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8
0.1	1.689	1.683	1.681	1.701	1.695	1.690	1.714	1.708	1.700
0.2	1.685	1.679	1.675	1.694	1.691	1.683	1.711	1.703	1.694
0.5	1.680	1.673	1.669	1.688	1.681	1.675	1.705	1.695	1.685
0.8	1.671	1.668	1.661	1.681	1.676	1.672	1.688	1.682	1.678

A. MADUBS SOLUTION

Case I: $K_P = 0.05$, $K_I = 0.4$: In Table 2, for given $h_{11} = 0.5$ and $h_1 = 1.5$, the MADUBs of h_2 with different h_{21}, μ_1, μ_2 are obtained by solving LMIs in theorem 1 through Matlab LMI-toolbox. In Table 3, for given $h_{21} = 0.8$ and $h_2 = 2$, the MADUBs of h_1 with different h_{11}, μ_1, μ_2 are obtained by solving LMIs in theorem 1. From the tables, we can find that the MADUBs increase as the lower bound h_{11} or h_{21} of the delays increases. These corresponding results cannot be obtained using the methods in the references, because the cases where the lower bounds of the delays are non-zero are ignored.

Case II: Different K_P and K_I values: For given $h_{11} = 0.5$, $h_1 = 1.5$ and $h_{21} = 1.5$, the MADUBs h_2 are calculated under different μ_1, μ_2 in Table 4. For given $h_{21} = 0.5$, $h_2 = 2$ and $h_{11} = 1$, Table 4 give the corresponding MADUBs h_1 with different μ_1, μ_2. It's obvious from these tables that the MADUBs decreases as μ_1, μ_2 increases under fixed h_{11}, h_1 or h_{21}, h_2 and (K_P, K_I). For given h_{11}, h_1 or h_{21}, h_2 and μ_1, μ_2, K_P, the MADUBs increase as K_I decreases, whereas the MADUBs increases as K_P increases for given K_I.

B. SIMULATION

In order to verify that the MADUBs are within the range of the actual values of the LFC power systems, some system state response curves under some MADUBs are given in this section. Assume that the initial condition of the simulation is the area load increases by 0.1 step load at $t = 20$s. Time-varying delays and (K_P, K_I) assumptions are as follows:

- In Figure 2, $(K_P, K_I) = (0.05, 0.4)$ and $h_{11} = 0.5$, $h_1 = 1.5$:
  a. $h_{1t} = 1 - 0.5sin0.2t$,
  $h_{2t} = \frac{4.285}{2} - \frac{1.285}{2}sin\frac{0.4t}{1.285}$;

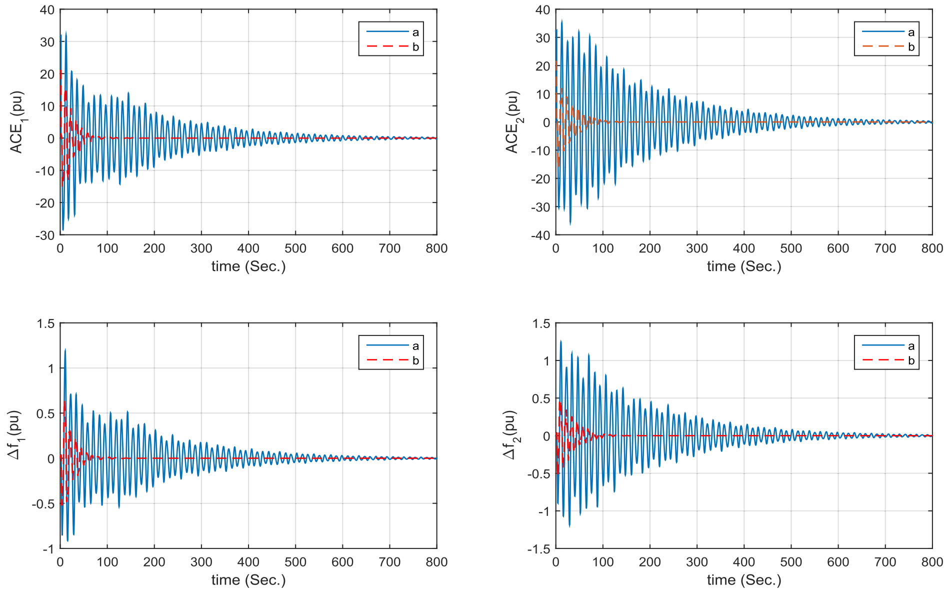

FIGURE 3. Frequency deviation and control error responses of two-area LFC power system with $(K_P, K_I) = (0.05, 0.4)$ and $h_{21} = 0.8$, $h_2 = 2$.

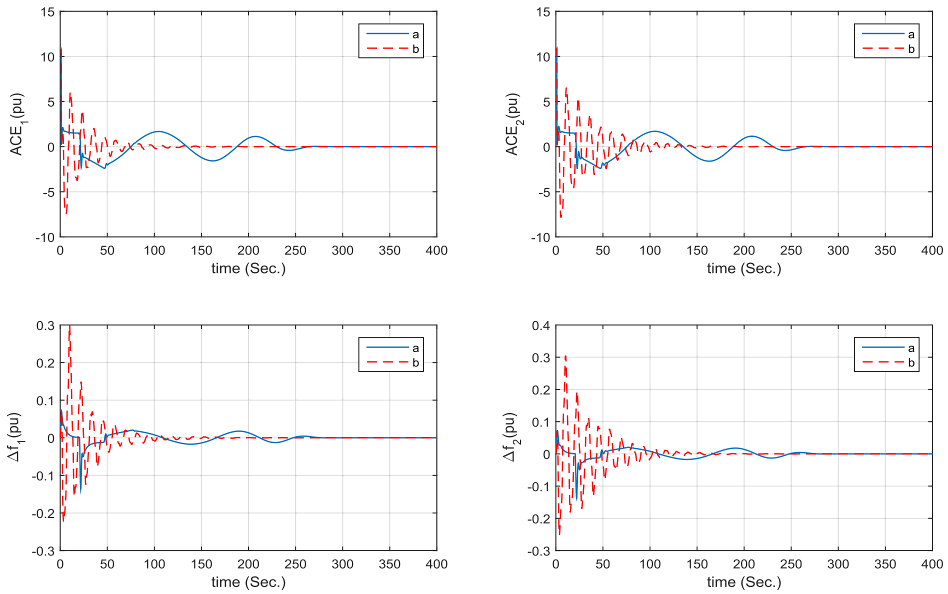

FIGURE 4. Frequency deviation and control error responses of two-area LFC power system with $h_{11} = 0.5$, $h_1 = 1.5$ and $h_{21} = 1.5$.

b. $h_{1t} = 1 - 0.5sint$,

$h_{2t} = \frac{3.315}{2} - \frac{1.315}{2} sin \frac{1.6t}{1.315}$;

- In Figure 3, $(K_P, K_I) = (0.05, 0.4)$ and $h_{21} = 0.8$, $h_2 = 2$:

a. $h_{1t} = \frac{3.214}{2} - \frac{0.214}{2} sin \frac{0.2t}{0.214}$,

$h_{2t} = 1.4 - 0.6sin\frac{t}{3}$;

b. $h_{1t} = \frac{2.672}{2} - \frac{0.672}{2} sin \frac{1.6t}{0.672}$,

$h_{1t} = 1.4 - 0.6sin\frac{4t}{3}$;

- In Figure 4, $h_{11} = 0.5$, $h_1 = 1.5$ and $h_{21} = 1.5$:

a. $(K_P, K_I) = (0.1, 0.05)$, $h_{1t} = 1 - 0.5sin0.2t$,

$h_{2t} = \frac{32.724}{2} - \frac{29.724}{2} sin \frac{0.4t}{29.724}$;

b. $(K_P, K_I) = (0.1, 0.4)$, $h_{1t} = 1 - 0.5sin0.4t$,

$h_{2t} = \frac{4.325}{2} - \frac{1.325}{2} sin \frac{1.6t}{1.325}$;

- In Figure 5, $h_{21} = 0.5$, $h_2 = 2$ and $h_{11} = 1$:

a. $(K_P, K_I) = (0, 0.15)$,

$h_{1t} = \frac{14.873}{2} - \frac{12.873}{2} sin \frac{0.2t}{12.873}$,

$h_{2t} = \frac{2.5}{2} - \frac{1.5}{2} sin \frac{0.4t}{1.5}$;

TABLE 4. The MADUBs h_2 for different μ_1, μ_2 and (K_P, K_I).

gain matrices	K_I	0.05			0.15			0.4		
K_P	$\mu_1\backslash\mu_2$	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8
0	0.1	28.932	28.927	28.918	11.887	11.884	11.881	2.525	2.517	2.513
	0.2	28.928	28.922	28.919	11.879	11.875	11.872	2.521	2.515	2.509
	0.5	28.915	28.911	28.904	11.871	11.868	11.862	2.498	2.493	2.490
0.05	0.1	29.654	29.635	29.627	12.157	12.152	12.148	2.785	2.781	2.778
	0.2	29.638	29.621	29.619	12.155	12.149	12.144	2.780	2.777	2.771
	0.5	29.619	29.605	29.588	12.143	12.137	12.130	2.732	2.709	2.700
0.1	0.1	31.224	31.217	31.211	12.177	12.174	12.171	2.829	2.827	2.827
	0.2	31.218	31.207	31.200	12.172	12.168	12.160	2.825	2.825	2.825
	0.5	31.210	31.198	31.191	12.162	12.157	12.151	2.744	2.714	2.701

TABLE 5. The MADUBs h_1 for different μ_1, μ_2 and (K_P, K_I).

gain matrices	K_I	0.05			0.15			0.4		
K_P	$\mu_1\backslash\mu_2$	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8
0	0.1	31.773	31.767	31.759	13.873	13.867	13.861	1.128	1.128	1.127
	0.2	31.766	31.758	31.747	13.869	13.863	13.857	1.128	1.126	1.125
	0.5	31.755	31.749	31.666	13.855	13.847	13.840	1.126	1.124	1.121
0.05	0.1	33.785	33.781	33.773	14.555	14.543	14.539	2.879	2.868	2.799
	0.2	33.777	33.765	33.759	14.551	14.538	14.531	2.800	2.769	2.701
	0.5	33.766	33.759	33.744	14.513	14.508	14.511	2.284	2.251	2.198
0.1	0.1	35.998	35.989	35.981	14.577	14.569	14.559	2.888	2.876	2.805
	0.2	35.980	35.977	35.971	14.570	14.561	14.550	2.810	2.777	2.707
	0.5	35.963	35.957	35.944	14.562	14.557	14.547	2.293	2.261	2.205

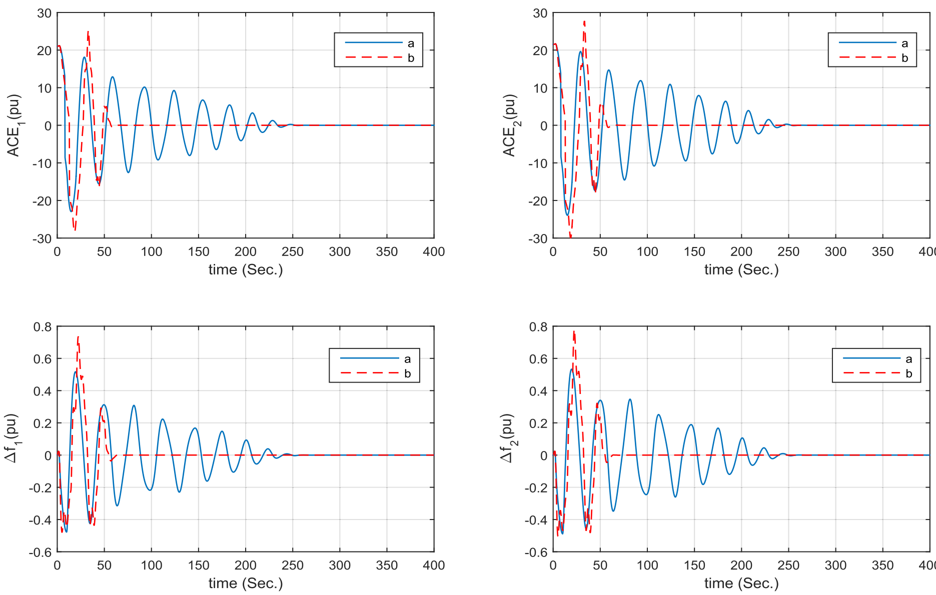

FIGURE 5. Frequency deviation and control error responses of two-area LFC power system with $h_{21} = 0.5$, $h_2 = 2$ and $h_{11} = 1$.

b. $(K_P, K_I) = (0.05, 0.4)$,

$$h_{1t} = \frac{14.840}{2} - \frac{12.840}{2} sin \frac{t}{12.840},$$

$$h_{2t} = \frac{2.5}{2} - \frac{1.5}{2} sin \frac{1.6t}{1.5}.$$

V. CONCLUSION

This paper deals with the stability problem of PI-type interval time-varying delay two-area LFC power systems. The lower bounds of the considered interval time-varying delays are non-zero, which are ignored in the literature. The relevant LKFs are augmented with some additional state variables with lower-bound non-zero time delay information. Combined with the free weight matrix integral inequality technique, a low conservative stability criterion based on LMI approach is obtained. At the same time, a negative-definite inequality equivalent transformation lemma is used to transform the contained nonlinear matrix inequalities into the corresponding LMIs to avoid introducing additional conservatism. Finally, in the numerical simulation examples, the Matlab LMI-toolbox is used to solve the LMIs in the theorem, and the MADUBs within the stability margin of the two-area LFC power system are obtained.

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