IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 10 November 2023, accepted 27 November 2023, date of publication 30 November 2023, date of current version 12 December 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3338024

TOPICAL REVIEW

Advances in Time-Frequency Analysis for Blind Source Separation: Challenges, Contributions, and Emerging Trends

YANGYANG LI AND DZATI ATHIAR RAMLI, (Senior Member, IEEE)
School of Electrical and Electronic Engineering, USM Engineering Campus, Universiti Sains Malaysia, Nibong Tebal 14300, Malaysia
Corresponding author: Dzati Athiar Ramli (dzati@usm.my)

This work was supported by the Ministry of Higher Education Malaysia through the Fundamental Research Grant Scheme under Project FRGS/1/2020/ICT03/USM/02/1.

ABSTRACT Blind source separation (BSS) is a critical task in untangling non-stationary signals without prior information. This paper extensively explores diverse time-frequency analysis (TFA) methods within BSS systems over the past decade. It underscores the pivotal role of TFA in dealing with non-stationary signals by characterizing their attributes across time and frequency domains. This approach provides a comprehensive understanding of signal dynamics that surpasses conventional techniques focusing solely on temporal or spectral domains. The paper delves into various TFA methods, investigating their influencing factors and aiding researchers in selecting relevant techniques aligned with their objectives. Furthermore, it comprehensively reviews contemporary research, categorizing BSS algorithms into three classes. The role of commonly used TFA methods in each class is systematically evaluated, identifying their strengths and limitations during different separation stages. The paper addresses challenges in implementing BSS algorithms, particularly in under-determined systems with fewer mixing channels than source signals. It highlights the central role of TFA in overcoming these challenges and enhancing separation outcomes.

INDEX TERMS Blind source separation (BSS), mixed matrix, source signal separation, suppress noise, time-frequency aggregation, time-frequency analysis (TFA), time-frequency resolution.

I. INTRODUCTION

Non-stationary signals find widespread applications in fields such as medicine, communication, audio processing, geological exploration, machine learning [1], and more. However, these signals are often complex data composed of mixed signals, affected by noise interference, or distorted during transmission. Extracting useful information from these mixed signals poses challenges, including potential similarities between signals, noise interference, and lack of prior information about the signals.

A. BACKGROUND

Blind Source Separation (BSS) technology is a crucial technique specifically designed to address these challenges [2]. It enables the separation of mixed signals using only observed signals, even in the absence of prior information about the signals. This technique is essential in various fields where non-stationary signals are prevalent.

Many real-life problems can be modeled as BSS systems [3], [4], [5], such as the denoising problem in signal processing, which involves removing noise from signals without prior knowledge of the noise characteristics. Another example is the fault detection problem in mechanical vibrations, where identifying faults is essential even in the absence of prior information about the faults. Similar issues arise in electronic surveillance, radar signals with multiple echoes and clutter, and image processing problems involving overlapping multiple images, all of which can be framed as problems related to BSS. Research on these problems and the implementation of improved algorithms are of significant importance for practical applications in human society.

The associate editor coordinating the review of this manuscript and approving it for publication was Lorenzo Mucchi.

However, the study of BSS encounters several challenges [6], including the Non-Gaussian and Non-independent nature of observed signals, the underdetermined nature of mixing systems, ambiguity in mixing processes, and insufficient prior information, among others. All these factors contribute to the complexity of separating observed signals.

To address these challenges, researchers have continuously explored various methods. Time-Frequency Analysis (TFA) is one of the crucial techniques in solving BSS problems. TFA transforms signals into domains with time and frequency bivariate information [7], [8]. By extracting signal features in this time-frequency domain, signals can be effectively analyzed and processed. Specific signal characteristics appear in certain transform domains, aiding in the separation of different signal components and the differentiation between signals and noise. Different TFA methods seek approximate time-frequency features of signals in different transform domains, making these features more accessible for researchers to analyze [9], [10], [11]. However, this process is not straightforward. Over the years, researchers have been exploring how to handle this situation more effectively, leading to significant research outcomes.

In summary, BSS systems are the result of continuous improvements in algorithm performance based on classical TFA methods. This paper is proposed against the backdrop of rapid developments in BSS and TFA. It aims to provide valuable references for the development of BSS technology by comparing and summarizing the advantages and limitations of different TFA techniques in achieving BSS.

B. PAPER CONTRIBUTION

As mentioned earlier, the continuous improvement in the performance of BSS systems largely owes to the advancements in TFA methods. Different time-frequency domains correspond to different TFA methods, and the performance of generated time-frequency data points can also vary. Before delving into the study of algorithms for BSS systems, the primary task is to determine the suitable time-frequency domain. In this paper, the contributions to researchers are as follows:

i. The extensive discussion of common TFA methods, including their features and influencing factors, provides researchers with a clearer understanding of their characteristics and applicable scenarios.
ii. Summarizing the recent applications of TFA in BSS algorithms and conducting comparative simulation experiments on several common TFA methods help researchers comprehend the strengths and limitations of various TFA methods used within different categories of BSS algorithms, offering valuable references for scholars' research.
iii. A comprehensive overview of the challenges faced by BSS and the critical issues that need attention guides researchers in selecting their research directions. The role of TFA in addressing challenges in BSS is explored, and areas requiring further research are identified, providing valuable insights for scholars in their studies.

C. PAPER ORGANIZATION

In this paper, the study conducted a detailed comparative study of various TFA methods applied in BSS systems over the past decade.

In Section II, the BSS system is thoroughly explored, focusing on three key aspects: mixing modes, mixing channels, and algorithms. Part A discusses the characteristics of BSS systems under two different mixing modes, while Part B explores the unique features of BSS systems based on various mixing channels. Part C provides a summary of existing BSS algorithms, categorizing them into four types and emphasizing the significant role of TFA in BSS algorithms.

In Section III, a comparison of different TFAs in BSS implementation is presented. This section is divided into four parts. The first part summarizes the four major influencing factors of TFA in BSS implementation and compares their impacts. The applicable conditions of various TFAs in BSS algorithms are categorized into three types. The B, C, and D parts respectively introduce these three types of algorithms. Part B delves into traditional BSS algorithms, dissecting each step of the algorithm and examining the role of different TFAs in every stage. Experimental results are analyzed to conclude the advantages and limitations of various TFAs in different steps. Part C focuses on the two-step algorithms, exploring the impact of TFAs in estimating the mixing matrix and recovering source signals. Challenges faced and solutions are compared from different perspectives. Part D discusses nonlinear algorithms, reviewing recent research results on implementing BSS using nonlinear methods.

In Section IV, existing research findings are combined to delve deeper into the contributions of TFA in solving BSS problems.

In Section IV, a comprehensive summary of the paper's content is presented, outlining the emerging trends and development directions in the field.

II. BSS SYSTEM

The basic framework of BSS is illustrated in the Figure1. $s_N(t)$, $x_M(t)$ represents the source signal and observation signal, respectively, $n_M(t)$ represents the interference signal in the transmission channel, $\hat{s}_N(t)$ represents the estimated source signal, N is the number of source signals, and M is the number of mixing channels. Variations in mixing modes, the number of mixing channels, and the relationships between the number of sources lead to different blind source systems, each requiring distinct research approaches [12].

Below, we will introduce them separately:

A. BSS MIXING MODELS

BSS systems can be categorized into linear mixing models and nonlinear mixing models based on different mixing modes.

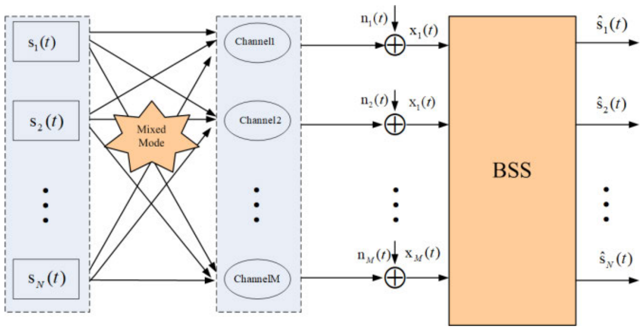

FIGURE 1. The basic framework of BSS.

1) LINEAR MIXING MODELS

In the linear mixing model, the observed signals are linear combinations of source signals and can be represented as:

$$X = AS \tag{1}$$

Here, X is the observed mixed signal matrix, S is the source signal matrix, and A is the mixing matrix. Researchers have further classified linear mixing models according to the characteristics of the mixing process:

The extensive discussion of common TFA methods, including their features and influencing factors, provides researchers with a clearer understanding of their characteristics and applicable scenarios.

i. Linear Instantaneous Mixing Model: The observed signals are linear combinations of source signals at the same moment, indicating an instantaneous mixing process. This model is fundamental, straightforward, and widely applicable, often encountered in various signal processing problems.
ii. Linear Time-Delay Mixing Model: The observed signals result from linear combinations of source signals delayed by different time intervals. This model is commonly used to study delays caused by differences in propagation istances or sensor positions, for instance, in radar signal processing.
iii. Linear Convolution Mixing Model: The observed signals are linear combinations of source signals obtained through convolution operations. This model is employed in situations where signals might undergo convolution due to factors like multipath effects during transmission, often utilized in communication transmission systems.

These classifications facilitate research by providing distinct scenarios, each requiring specific methodologies for analysis and separation.

2) NONLINEAR MIXING MODELS

In nonlinear mixing models, the combination of source signals is not a simple linear weighted sum. In such cases, the mixing process can be described using nonlinear functions. In nonlinear mixing models, the relationship between observed signals and source signals can be represented as:

$$X = f(S) \tag{2}$$

Here, X is the observed signal matrix, S is the source signal matrix, and $f(\cdot)$ is a nonlinear function.

In this category of mixing models, the nonlinear function can be a polynomial, a nonlinearity function, or an uncertain function, making the solving process highly challenging. Current effective research efforts are focused on utilizing deep learning to handle highly nonlinear mixing processes, yet the challenges persist. Consequently, in contemporary algorithms, researchers approximate nonlinear mixing models to linear ones through constraints, such as constraints on the distribution of source signals, to alleviate computational difficulties. Therefore, the subsequent discussions in this paper primarily revolve around linear mixing models.

B. BSS MIXING CHANNELS

BSS techniques can be categorized according to the number of mixing channels, dividing them into single-channel and multi-channel approaches.

1) SINGLE-CHANNEL BLIND SOURCE SEPARATION:

In single-channel BSS, only one channel of observed signals is available, yet this signal may be a complex linear combination of multiple source signals. The primary objective of single-channel BSS is to disentangle the original multiple source signals from this single-channel observed signal [13], [14]. These source signals can encompass diverse types, varying according to the application scenario. Examples include scenarios where the source signals comprise sound and background music, a combination of signals and noise, or multiple individual signals. Single-channel BSS finds significant utility in the field of signal processing, particularly in the separation of multi-component mixed signals. These multi-component mixed signals are compositions of various distinct source signals. Hence, the application of single-channel BSS holds paramount importance in practical contexts.

2) MULTI-CHANNEL BLIND SOURCE SEPARATION:

In the context of multi-channel BSS, there often exists a relationship between the number of observed channels and the number of source signals. This relationship helps define the nature of the problem and falls into three categories: underdetermined BSS, determined BSS, and overdetermined BSS.

i. Overdetermined BSS: In overdetermined situations, the number of observed channels exceeds the number of source signals. Here, information redundancy among the observed channels often leads to multiple possible solutions. Different methods can yield potential solutions, but not all of them are necessarily physically meaningful.
ii. Determined BSS: In determined scenarios, the number of observed channels equals or exceeds the number

of source signals. In such cases, there exists a unique solution theoretically, which can be obtained through suitable algorithms.

iii. Underdetermined BSS: Underdetermined situations occur when the number of observed channels is less than the number of source signals. In other words, more source signals need to be separated than the observed channels can provide information for. Problems in these cases are typically unsolvable with accuracy, as they have an infinite number of solutions.

Underdetermined systems, being more representative of real-world scenarios, are often the focus of intensive research efforts.

C. BSS ALGORITHMS

In recent years, BSS algorithms have emerged one after another [15], [16], [17], [18]. Combined with the separation of BSS in the front, this paper roughly divides BSS algorithms into the following categories:

1) TIME-FREQUENCY ANALYSIS-BASED BSS

This approach leverages the characteristics of signals in both the time and frequency domains [19], [20], [21]. TFA methods, decompose signals in both time and frequency, aiding in the separation of different components within mixed signals. Different TFA methods utilize various mathematical tools to describe the distribution of signals in the time and frequency domains. The methods used in BSS differ accordingly. Below are several common TFA methods and their principles:

i. Short-Time Fourier Transform (STFT):

STFT is based on the idea of Fourier transform. It divides the signal into small segments and computes the Fourier transform for each segment, obtaining the spectrogram of each segment. This approach allows us to capture the frequency spectrum of the signal as it changes over time. The principle behind STFT is to decompose the signal into components with different frequency content while preserving the local characteristics of the signal in time.

ii. Continuous Wavelet Transform (CWT):

CWT employs wavelets, which are localized functions adjustable in both time and frequency, to analyze signals. The signal is convolved with wavelets at various scales, providing a time-frequency representation of the signal. The principle of CWT lies in analyzing different frequency components of the signal and adapting to the signal's characteristics at different time scales.

iii. Wigner-Ville Distribution (WVD):

WVD integrates information about the signal's time and frequency content into a single distribution. It reveals the signal's time-frequency characteristics by computing the instantaneous frequency of the signal. The principle of WVD involves combining the signal's time and frequency information into a unified time-frequency distribution.

iv. Fractional Fourier Transform (FrFT):

FrFT is an extension of the Fourier transform. Unlike the standard Fourier transform, FrFT offers an adaptive rotation in the frequency domain. It allows the observation of information about the signal at different frequency components. FrFT provides flexibility when dealing with non-stationary signals, contributing to improved BSS performance.

2) STATISTICAL PROPERTIES-BASED BSS

This method primarily depends on the statistical properties of signals for separation. Techniques like maximizing independence are applied in Independent Component Analysis (ICA) [22], [23]. Signals are separated by projecting them onto principal components using eigenvalue or singular value decomposition, a process referred to as Principal Component Analysis (PCA) [24]. Moreover, assuming signals are non-negative, separation is accomplished by decomposing them into the product of non-negative matrices, a technique known as Non-negative Matrix Factorization (NMF) [25], and so on.

3) INFORMATION THEORY-BASED BSS

Methods in this category employ information theory concepts like entropy and mutual information to measure the independence between signals, enabling separation. One common approach is minimizing mutual information between signals to achieve separation.

4) NEURAL NETWORK-BASED BSS

Methods based on neural networks, especially deep learning techniques, have made significant progress in BSS. Structures like Convolutional Neural Networks (CNNs) and Recurrent Neural Networks (RNNs) excel in separating time-series and image signals. Deep learning models learn high-level representations of signals, enabling more accurate separation, especially in highly nonlinear mixing scenarios.

List the advantages and disadvantages of these four algorithms through Table 1.

TABLE 1. Comparison of advantages and disadvantages of BSS.

BSS Algorithm	advantages	disadvantages
TFA-Based BSS	Sensitivity to time-frequency characteristics	Easily affected by noise
Statistical Properties-Based BSS	Suitable for various types of signals	Sensitive to the assumption of independence
Information Theory-Based BSS	Capable of effectively handling nonlinear relationships between signals	High computational complexity
Neural Network-Based BSS	Suitable for highly complex mixed signals	Requires a large amount of data for training

In comparison to other algorithms, methods based on TFA for BSS prove to be highly versatile. They offer an extensive range of applications, delivering intricate signal features and showcasing resilience against temporal signal variations.

These characteristics contribute to their prevalent use across various practical applications.

Next, this paper explores various applications of TFA in BSS systems, comparing the strengths and weaknesses found in current research findings for reference.

III. IMPLEMENTATION COMPARISON OF TFA BASED BLIND SOURCE SEPARATION ALGORITHMS

In BSS systems, the primary objective is to extract source signals from mixed signals without prior knowledge of the mixing process or source signal information. In the pursuit of this goal, TFA emerges as particularly crucial. This section will focus on exploring methods for achieving blind source separation and elaborating on the advantages and limitations of time-frequency analysis in each implementation step.

Firstly, this paper categorizes and summarizes the influencing factors of Time-Frequency Analysis (TFA) in blind source separation algorithms, enabling a more comprehensive study of TFA's applications in blind source separation systems.

A. FACTORS AFFECTING TFA

1) TF RESOLUTION

For non-stationary signals, there is no way to perform Fourier Transform (FT) directly, but when they can be regarded as stationary signals in a short time, use this idea to use a window function to truncate the non-stationary signals and perform FT, and realize the analysis of the whole signal by sliding the window function on the time axis, which is the origin of short-time FT(STFT). For signal $x(t) \in L^2(R)$, The definition of STFT is:

$$STFT_x(t,f) = \int_{-\infty}^{\infty} x(\tau)h(\tau - t)e^{-i2\pi f \tau} d\tau \tag{3}$$

$h(t)$ is a window function. Due to the introduction of the window function and the time variable, the research on the signal includes both the frequency characteristics and the time domain characteristics. It is a very practical TFA that is popular with scholars at present. For STFT, the first problem is how to choose the window function [26], [27]. Common window functions include rectangular window, Hanning window, Hamming window, Gaussian window, etc. The principle for selecting the window function is:

i. the main lobe of the window function spectrum should be as narrow as possible, and the energy should be concentrated in the main lobe as much as possible, so as to obtain a higher TF Resolution.
ii. The sidelobe decays with the frequency as soon as possible to reduce the distortion caused by spectrum leakage.

Figure 2 shows the situation of two disjoint chirp signals under different window functions (the window length=63) and the influence of different window lengths when the same window function (Gaussian window) is used. It is obvious from the figure that the window function and window length are very important for two-component signal analysis.

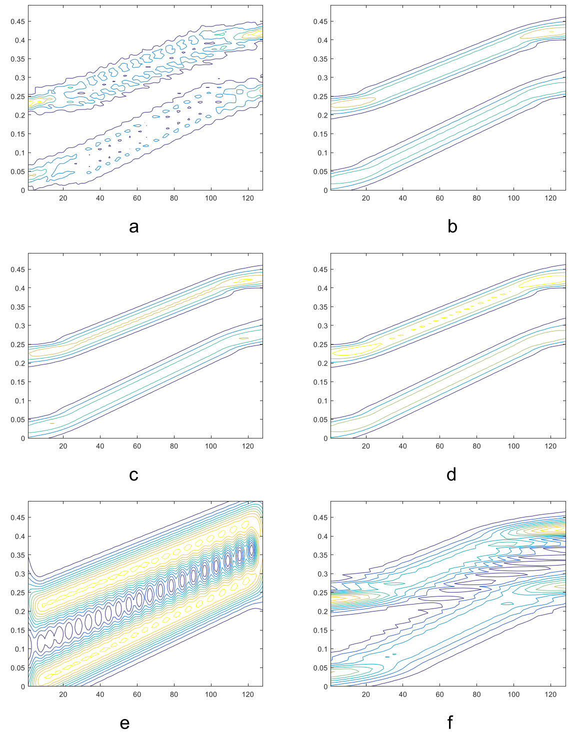

FIGURE 2. Time-frequency diagram of disjoint signals under the influence of different window functions or window lengths (a. rectangular window, b. Hamming window, c. Hanning window, d. Gaussian window, (the window length of a,b,c,d is 63) e. Gaussian window length=13, f. Gaussian window length=127).

Figure 3 shows two intersecting chirp signals, which have different window functions and different window lengths. Because of the intersection, only the improvement of the window is far from enough, because we need to observe local features more accurately, which requires higher TF Resolution.

After the above analysis, once the window function is selected, the window length in the whole signal analysis process is determined. The long window has a higher frequency resolution, and the short window has a higher time resolution [28]. That is to say, to obtain a high frequency resolution, we must pay a low time resolution price. This will lead to no way to find the most suitable window length to achieve the maximum TF Resolution of STFT, which is the biggest problem of STFT.

STFT algorithm has limited TF Resolution due to the unique window function and the fixed window length. In this respect, the performance of Wavelet Transform (WT) is relatively good because the definition formula of WT is:

For a given signal $x(t) \in L^2(R)$, the WT is $WT_x(a,b)$, defined as:

$$WT_x(a,b) = \int_{-\infty}^{\infty} x(t)\Psi(t)dt = \frac{1}{a^{\frac{1}{2}}} \int_{-\infty}^{\infty} x(t)\Psi(\frac{t-b}{a})dt \tag{4}$$

where: a>0 is the scale factor, and b is the time shift factor. $\Psi(\frac{t-b}{a})$ is a family of functions $\Psi(t)$ generated by the

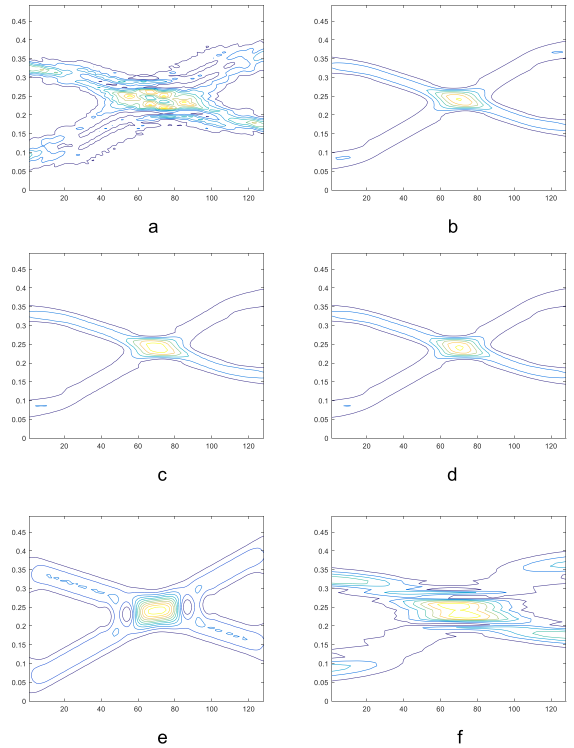

FIGURE 3. Time-frequency diagram of intersection signal under the influence of different window functions or window lengths (a. rectangular window, b. Hamming window, c. Hanning window, d. Gaussian window, (the window length of a,b,c,d is 63) e. Gaussian window length=13, f. Gaussian window length=127).

shift and expansion of the mother wavelet, called wavelet basis. From this definition, WT is essentially the correlation operation between the original signal and the scaled wavelet function family. Although compared with STFT, the TF Resolution of WT is significantly improved due to the existence of wavelet base, the influence of the mother wavelet of WT on the result is too great to make the use of WT very cautious.

2) EXISTING CROSS-ITEM INTERFERENCE

Both STFT and WT are limited by window function. On this basis, people put forward the joint distribution of energy in TF Domain, namely the famous Wigner-Ville Distribution (WVD). WVD is a joint TF distribution without window function. The definition $WVD_x(t, f)$ for signal $x(t)$ can be expressed as:

$$WVD_x(t,f) = \int_{-\infty}^{\infty} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})e^{-i2\pi f\tau}d\tau \quad (5)$$

In the equation (5), τ is the integral variable, t is the time shift, $*$ represents complex conjugation, $\int_{-\infty}^{\infty} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})$ is the instantaneous correlation function of the signal$x(t)$. Because there is no window function, it is not restricted by the uncertainty principle and can obtain high-resolution TF distribution. However, observing the WVD definition formula, when the signal is composed of multiple components, any two component signals will generate cross terms. that is for example: $x(t) = x_1(t) + x_2(t)$, at this time, the following $WVD_x(t, f)$ formula is brought into (5):

$$WVD_x(t,f) = WVD_{x_1}(t,f) + WVD_{x_2}(t,f) + 2\Re e(WVD_{x_1x_2}(t,f)) \quad (6)$$

In formula (6), $WVD_{x_1}(t,f)$ and $WVD_{x_2}(t,f)$ are the self-WVD of $x_1(t)$ and $x_2(t)$ respectively, $WVD_{x_1x_2}(t,f)$ represent the mutual WVD of $x_1(t)$, $x_2(t)$. It can be seen from formula (6) that although WVD has good TF Resolution, it produces mutual WVD, that is, interference term, which has a great impact on TFA.

Scholars have made a lot of efforts to suppress cross-terms, the most famous one is Pseudo Wigner-Ville Distribution (PWVD), Smooth Pseudo Wigner-Ville Distribution (SPWVD) [29], [30]. PWVD uses the idea of STFT to reduce the correlation between components through windowing and truncation in time domain to achieve cross-term suppression. SPWVD uses the idea of wavelet scaling factor, which is equivalent to adding windows simultaneously in time domain and frequency domain. Figure 4 get WVD, PWVD and SPWVD for two signal components. It can be clearly seen from the figure that the interference of WVD cross terms has a great impact on the analysis. PWVD has inhibition on the cross terms, and the most significant inhibition effect is SPWVD. Table 2 is about the comparison of different traditional TFAs from different aspects.

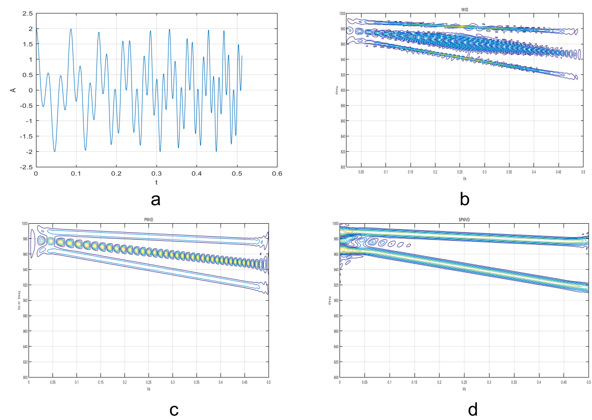

FIGURE 4. Signal quadratic time-frequency distribution (a. signal time-domain waveform b. signal WV time-frequency distribution c. pseudo-WV time-frequency distribution d. smooth pseudo-WV time-frequency distribution).

3) TIME-FREQUENCY MATRIX

TF matrix is a matrix composed of sampling points after TF Transform of the observed signal. For mixed signals $x(t) = A * s(t)$, TF conversion is performed on both sides at the same time $X(t,f) = AS(t,f)$, and the expansion is in matrix form:

$$X(t,f) = \begin{bmatrix} X_1(t,f) \\ X_2(t,f) \\ \vdots \\ X_m(t,f) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} S_1(t,f) \\ S_2(t,f) \\ \vdots \\ S_n(t,f) \end{bmatrix} \quad (7)$$

TABLE 2. Traditional TFA methods.

TFA method	characteristic	advantage	shortcoming	Calculation complexity (N is the signal length, H is the time domain window length, and G is the frequency domain window length)	Improvement direction
STFT	Window function	Simple, TFA	Fixed window length, low time-frequency resolution	$O\left(\frac{NH}{2}\log_2 H + NH\right)$	Time-Frequency aggregation: 1.Window length 2.Renyi entropy
WVD	Introduce energy distribution	No window function, high TF Resolution	Introduction of cross items	$O\left(\frac{N}{2}\log_2 H + N\right)$	1.Suppress cross items
PWVD	Introduce energy distribution	High TF Resolution	Add time window to suppress cross terms and reduce TF Resolution	$O\left(\frac{NH}{2}\log_2 H + 2NH\right)$	1.Suppress cross items 2.Windows improvement
SPWVD	Introduce energy distribution	Less impact of cross items	Time domain, frequency domain windowing, TF Resolution reduction, large amount of computation	$O\left(\frac{NHG}{2}\log_2 H + 2NHG\right)$	1.Time frequency aggregation 2.Window improvement 3.Complexity

$X_m(t,f)$ is the TF conversion coefficient of the observed signal, $S_n(t,f)$ is the TF conversion coefficient of the source signal. For the blind source system, $x(t)$ is the only known information. It is obtained by converting the signal $x(t)$ to the TF Domain $X(t,f)$ through TF conversion, and the effective information is obtained by analyzing the TF matrix composed of the TF Points of the mixed signal, so as to gain the design of the algorithm. In the actual algorithm, the direct use of TF Point analysis has a large amount of computation, and the data points have interference, the algorithm performance is limited.

Therefore, scholars choose to process the TF Points first, such as removing low energy points to reduce the noise ofinterference, or removing redundant data points to reduce the number of TF Points. Then the signal is sparsely represented, which is very important for UBSS, because only in this way can the dimensionality be reduced and the solution be unique. In general, the TF Transform is different, the performance of the obtained TF Points is also different. Therefore, choosing an appropriate TF Transform to obtain high-performance TF Points is crucial for achieving BSS systems. The improved algorithm will be introduced in detail later.

4) TF AGGREGATION

For the performance of TF analysis method, our important criterion is TF aggregation [31], [32], that is, whether the energy reflected by TF analysis is the most concentrated. Only the most concentrated can better reflect the local characteristics of each component signal and have better TF Resolution.

Entropy is a very important tool to describe information uncertainty. The entropy value is used to judge the uncertainty of the signal. Scholars use the general form of representation entropy, namely Renyi entropy, to quantitatively calculate TF aggregation. A large number of results have proved that this is reasonable and very useful. The definition formula is as follows:

$$E_e = \frac{1}{1-\alpha}\log_2\left(\sum_{n=1}^{N}\sum_{k=1}^{K}\left[\frac{TFR[n,k]}{\sum_{n=1}^{N}\sum_{k=1}^{K}TFR[n,k]}\right]^{\alpha}\right) \tag{8}$$

In formula (8), α is an integer greater than 1, representing the order. N is the number of time points, and K is the number of frequency points. $TFR[n, k]$ is the TF representation of the signal.

This formula calculates the TF aggregation of TFA method in a quantitative way. The degree of TF energy accumulation is judged by the size of Renyi entropy. The larger the Renyi entropy value, the more scattered the TF energy is, the worse the TF aggregation is, and the smaller the Renyi entropy value, the more concentrated the TF energy is, the better the TF aggregation is. Therefore, the performance of TFA is judged by Renyi entropy. In addition, scholars will also use Renyi entropy to participate in the parameter determination of TF Transform, so as to adaptively select the optimal TFA. For example, the determination of window functions and parameters in parametric TFA are important applications of Renyi entropy [33], and are also one of the research hotspots in recent years.

Taking into account the influencing factors of TFA, this paper categorizes Time-Frequency Analysis-based blind source separation algorithms into three types:

i. Traditional algorithms that utilize extracted mixed signal features for separation in part B.
ii. Two-step methods involving estimating the mixing matrix based on signal sparsity and then recovering the source signals in part C.
iii. Nonlinear time-frequency analysis methods for separation in part D.

The following will discuss each step within these three major categories of algorithms, emphasizing the significance of each step in BSS, and discussing the advantages and limitations of TFA in each step.

B. TRADITIONAL ALGORITHMS FOR BLIND SOURCE SEPARATION IMPLEMENTATION

The traditional algorithm is only applicable to basic BSS. In basic BSS systems, the number of source signals N is greater than or equal to the number of channels M. Figure 5 illustrates the system overview of basic BSS:

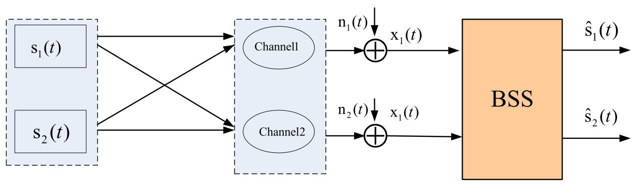

FIGURE 5. The system overview of basic BSS.

For traditional algorithms, the implementation is shown in Figure 6:

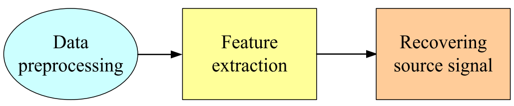

FIGURE 6. Flowchart of traditional algorithm implementation for blind source separation system.

1) DATA PREPROCESSING

Firstly, performing necessary data preprocessing on the received mixed signals is a crucial step in blind source separation systems. These preprocessing operations encompass denoising, normalization, and more, aimed at enhancing the accuracy, separability, and signal quality of the source signals while also helping to reduce computational load for improved separation outcomes. However, the choice of data preprocessing varies due to factors such as the characteristics of the mixed signals, the selected separation algorithm, and the expected separation outcomes.

Equally significant is the selection of time-frequency analysis methods. Different time-frequency analysis methods may require distinct preprocessing steps to ensure an accurate representation of signals in the time-frequency domain [34]. For instance, with STFT, the signal length may need to be adjusted to meet the window function's requirements, while CWT has certain smoothness and sampling rate prerequisites. Furthermore, preprocessing might involve detrending, normalization, and other operations to mitigate noise interference during analysis.

In conclusion, data preprocessing stands as a pivotal component of blind source separation systems, enhancing the effectiveness of source signal separation. The synergy between appropriately chosen time-frequency analysis methods and preprocessing steps contributes to improved source separation, revealing the original information within mixed signals. During data preprocessing, a comprehensive consideration of signal characteristics, separation algorithms, and desired separation outcomes is essential. There are various preprocessing methods, as shown in the following figure:

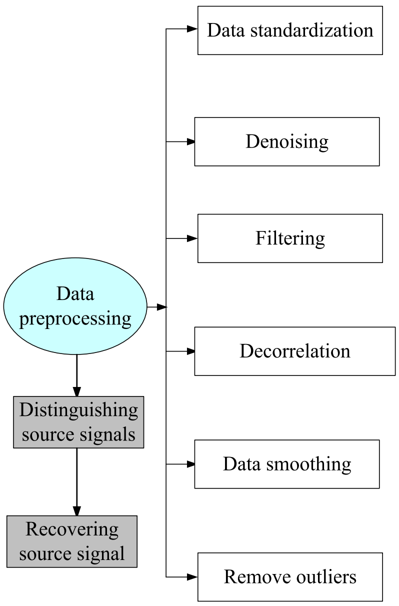

FIGURE 7. Types of data preprocessing.

Next, we will introduce these data preprocessing in detail:

Data Normalization: Standardizing data ensures that each signal channel has a similar scale, preventing one channel from disproportionately affecting separation results.

Denoising: Denoising is a vital preprocessing task. Applying denoising preprocessing to the interfering components in the received mixed signals to obtain relatively clean signals not only enhances the effectiveness of source signal separation but also reduces computational load during the processing.

Filtering: Applying filters helps eliminate high-frequency noise or low-frequency interference in signals. The choice of appropriate filter types (low-pass, high-pass, band-pass, etc.) and cutoff frequencies should be adjusted based on signal characteristics.

Decorrelation: The purpose is to reduce the correlation between mixed signals, enhancing the independence of source signals during separation, thereby improving source signal separation and accurately recovering the original source signals.

Data Smoothing: Data smoothing methods can remove spikes or glitches in data, making the signal more stable and enhancing the stability of separation algorithms.

Outlier Removal: If there are outliers in the data, considering their removal or correction prevents them from affecting separation results.

The preprocessing steps for mixed signals and how to implement them depend on the characteristics of the received

TABLE 3. Comparison of benefits of data preprocessing for different mixed signals.

Mixed signal type	characteristic	preprocessing	Benefits of preprocessing
Biomedical signal separation	Low frequency and high amplitude electrical signals	Denoising Filtering Data Normalization	Highlighting Biomedical Signal Eigenvalues
Speech signal separation	Narrow frequency range, affected by environmental noise and changes in human voice	Denoising Decorrelation,	Improve speech quality and signal separation effect
Sensor array signal separation	Multiple mixing channels receiving multiple source signals may be affected by inaccurate sensor position and signal delay	Filtering Data Normalization Time Delay Estimation	Standardize sensor signals, time delay estimation, and phase alignment to improve the accuracy of signal separation.
Audio separation	The frequency range is wide, including music, vocals, etc. May be affected by environmental noise	Filtering Data Normalization Decorrelation	Remove noise and unnecessary frequency components, improve sound quality and signal separation effect.
Radar signal separation	Usually a time-domain signal, which may be affected by noise and multipath effects	Denoising Filtering Data Normalization Decorrelation Time Delay Estimation Data Smoothing	Remove noise, time delay estimation, and phase alignment to improve the accuracy of signal separation.

data and the desired separation outcomes. In this paper, we summarize the selection of preprocessing and the role of TFA in data preprocessing, providing a reference for researchers with similar needs.

Firstly, we begin by studying the data characteristics of the received mixed signals [35]. BSS systems are applicable in various scenarios, so mixed signals come in diverse forms such as the following introduce.

i. Biomedical Signal Separation: Such as the separation of signals like Electrocardiograms (ECG), heartbeat signals, and Electroencephalograms (EEG), where these mixed signals might result from the combination of various biological signals during measurement.
ii. Speech Signal Separation: In environments with multiple speakers, mixed signals could be a superposition of multiple people's speech sounds received and mixed through a microphone array.
iii. Sensor Array Signal Separation: In sensor array signal processing, mixed signals may be represented as the result of multiple source signals received and mixed by a sensor array.
iv. Audio Separation: In audio separation, mixed signals are typically composed of a linear combination of multiple source audio signals, such as the voices of multiple people speaking or the sounds of multiple instruments playing.
v. Radar Signal Separation: In radar signal processing, mixed signals can be represented as the superposition of echo signals from different targets in the time and frequency domains.

Table 3 presents various characteristics of different mixed signals, indicating suitable preprocessing methods for these characteristics and the expected goals of implementation.

Next, we use two chirp signals to simulate the comparison of results obtained through various TFA methods for data preprocessing in the context of mixed signal analysis.

Figure 8 presents two mixed signals obtained by applying a randomly generated 2×2 mixing matrix in MATLAB to two chirp source signals. Figures 9 to 16 show the comparative results of preprocessing the mixed signals using three different time-frequency transformations: Short-Time Fourier Transform (STFT), Wigner-Ville Distribution (WVD), Fractional Fourier Transform (FrFT) [36].

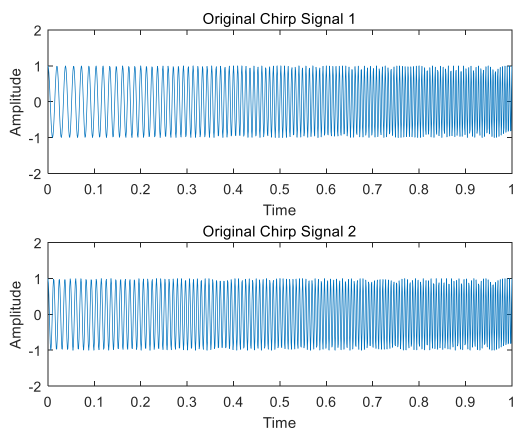

FIGURE 8. Two original chirp signal.

Table 4 presents the conclusions drawn from the above simulations.

In conclusion, when preprocessing mixed signals, researchers can choose suitable time-frequency analysis methods based on the characteristics and needs of the signal. For example, if researchers focus on the instantaneous characteristics of signals, they can try using WVD

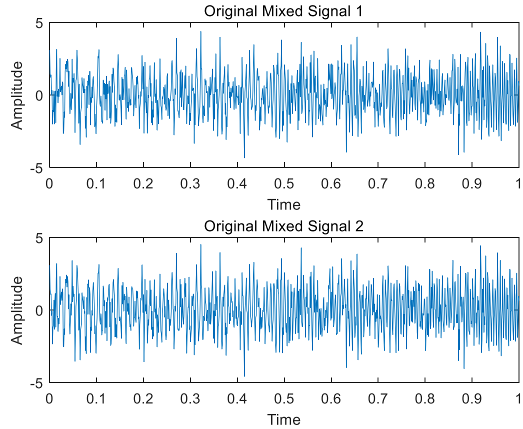

FIGURE 9. Two mixed signal.

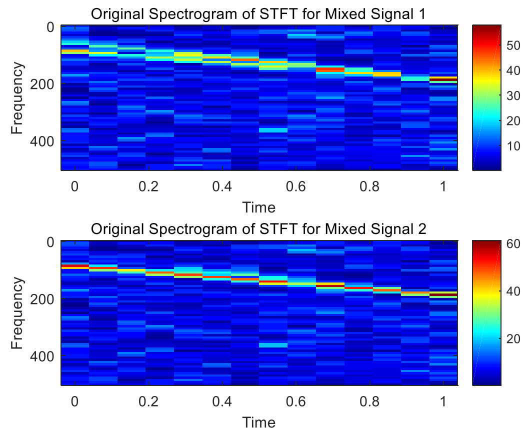

FIGURE 10. Two mixed signal after STFT.

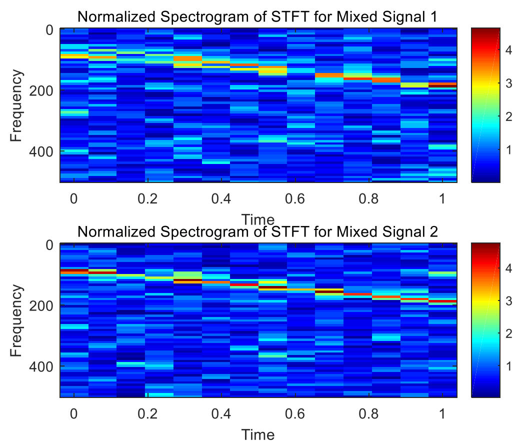

FIGURE 11. Two mixed signal after STFT normalization preprocessing.

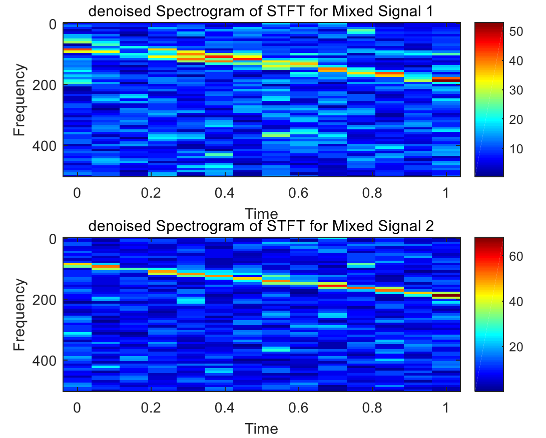

FIGURE 12. Two mixed signal after STFT denoising preprocessing.

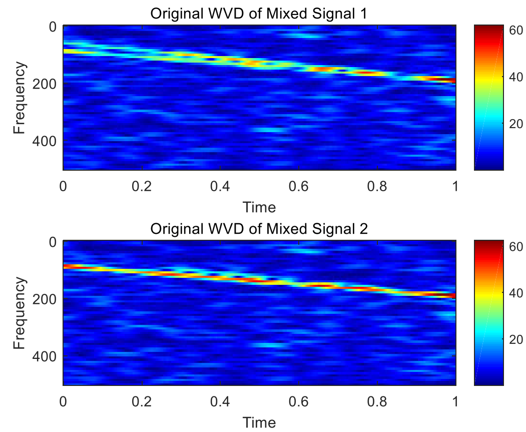

FIGURE 13. Mixed signals after WVD.

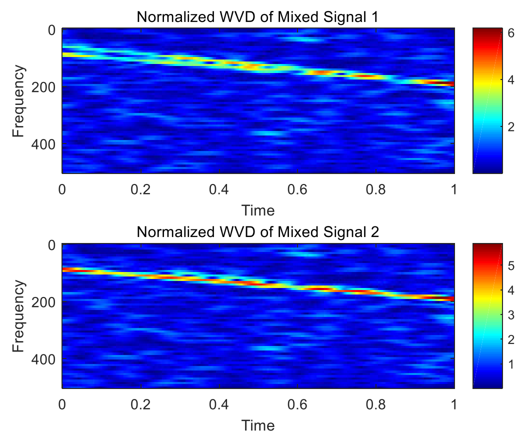

FIGURE 14. Two mixed signals after WVD normalization preprocessing.

or FrFT. If researchers pay more attention to the clear display of frequency components, they can try using STFT. However, for different application scenarios, the effectiveness of different methods may vary. It is recommended to choose the most suitable method through practical experiments.

2) FEATURE EXTRACTION

This phase constitutes a pivotal step in realizing the BSS system. As the system operates without any prior knowledge, achieving the separation of received mixed signals to recover the source signals necessitates initiating the process from the mixed signals themselves. Analysis and extraction of features

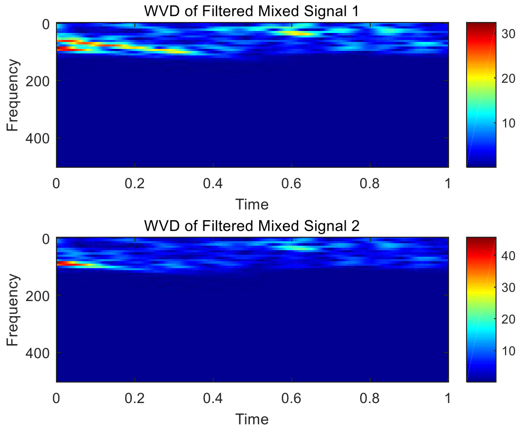

FIGURE 15. **Two mixed signals after WVD denoising preprocessing.**

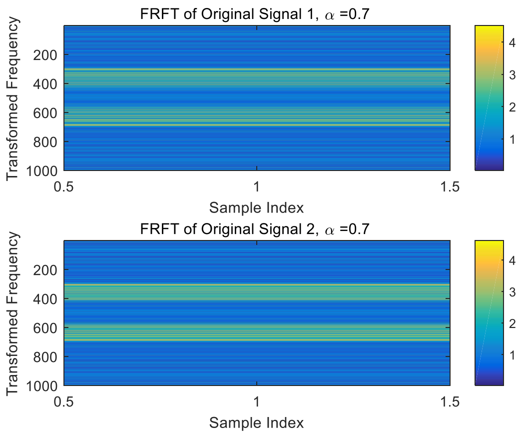

FIGURE 16. **Two mixed signals after FrFT (α =0.7).**

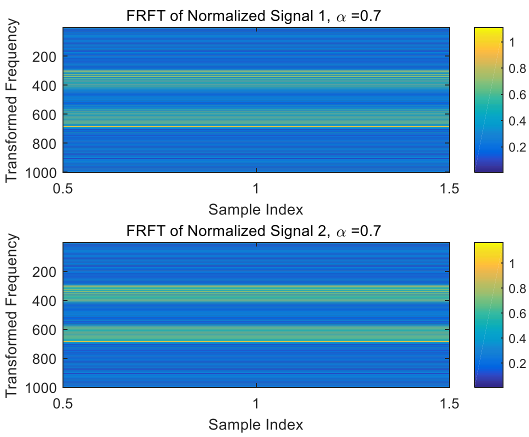

FIGURE 17. **Two mixed signals after FrFT (α =0.7) normalized preprocessing.**

from the mixed signals are conducted to identify methods capable of distinguishing source signals from the mixed ones, thus achieving the separation [37].

The purpose of feature extraction is to transform preprocessed mixed signals into distinctive feature representations, facilitating the separation process. The selection of appropriate feature extraction methods hinges on the data's nature. While time-domain feature extraction methods (such as mean, variance, instantaneous amplitude, and phase) can capture fundamental statistics and instantaneous properties in certain scenarios, they might fall short in capturing temporal variability and frequency changes. Frequency-domain feature extraction methods (such as frequency distribution, power spectral density, frequency correlation) can provide insights into signal frequency components, yet they might not effectively capture instantaneous properties and time-varying frequency characteristics.

TFA amalgamates the strengths of time and frequency domains to more accurately depict a signal's time-varying frequency traits. TFA methods can provide information like energy distribution and variations in instantaneous frequency across various time and frequency intervals, leading to a more comprehensive signal representation. Consequently, TFA enhances the representation of intricate and diverse signals during the feature extraction stage, contributing to a more precise feature representation for blind source separation tasks.

However, different TFA methods yield distinct features and performance outcomes. For instance, time-frequency analysis methods like STFT and Continuous Wavelet Transform (CWT) can capture evolving frequency characteristics over time, while WVD can uncover instantaneous frequency and amplitude traits of signals. FrFT harnesses rotation angles and integrates time and frequency domain information to more accurately capture the characteristics of mixed signals. Moreover, the effectiveness of feature extraction is not only influenced by the chosen TFA method but also by the nature of the signals themselves, the level of noise, and the feature extraction algorithm employed.

During the process of feature extraction, it is common to differentiate different source signals by extracting the instantaneous frequency of mixed signals. This approach is effective because instantaneous frequency can reveal the changing trends of signals, aiding in the discrimination of various source signals.

In addition, many studies will also implement BSS based on other feature extraction methods. As shown in the Figure 18:

Explanation of various signal features is as follows:

i. Instantaneous Frequency Rate of Change: It calculates the rate of change of the signal's instantaneous frequency, revealing the trend of frequency variations.
ii. Spectral Energy Density: By computing the spectral energy of the signal under different time windows, it captures the distribution of signal energy across different frequencies.
iii. Instantaneous Amplitude: Extracted from the amplitude information in time-frequency analysis, it captures the variations in the signal's instantaneous amplitude, useful for distinguishing different source signals.

TABLE 4. Comparison of data preprocessing for different time-frequency analyses.

TFA	Advantages in data preprocessing	Disadvantages in data preprocessing
STFT	1. When analyzing the frequency characteristics of signals within the window, STFT has good frequency resolution for steady-state signals. 2. Easy to implement and widely used.	A trade-off between time resolution and frequency resolution, and it is not possible to obtain both high time and high frequency resolutions simultaneously, resulting in poor preprocessing effects such as denoising.
WVD	very high time-frequency resolution, which can provide accurate information of signals in the time-frequency domain. It is suitable for analyzing the instantaneous frequency changes of signals and is suitable for preprocessing non-stationary signals	1. Affected by cross terms, it is difficult to achieve preprocessing such as denoising; 2. WVD has a high computational complexity and may introduce artifacts. 3. Small scale vibrations present in the signal may cause time-frequency leakage.
FrFT	1. Suitable for analyzing signals with fractional order time-frequency characteristics, which can better capture the time-frequency changes of the signal. 2. Insensitivity to noise	1. The computational complexity is relatively high, especially when achieving fractional order. 2. For different fractional order choices, the results may vary to some extent.

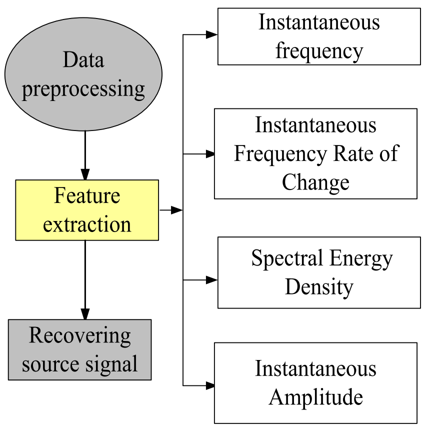

FIGURE 18. Different feature extraction methods.

iv. Group Delay: Describes the signal's propagation speed at different frequencies, providing insight into distinguishing different source signals.
v. Time-Frequency Entropy: Measures the level of disorder of the signal in the time-frequency domain, describing the signal's irregularity.

With the estimated instantaneous frequency obtained, the next step involves distinguishing source signals using methods such as clustering.

3) RECOVERING SOURCE SIGNALS

After extracting features to separate the source signals, we utilize methods such as STFT, CWT, WVD, and FrFT for separating and recovering the source signals from both the original mixed signals and the mixed signals with added noise [38], [39].This allows us to compare the significant role of time-frequency analysis in the process of source signal recovery.

Figure 19-22 illustrate the waveform of the separated signals obtained using the STFT, CWT, WVD, FrFT-based estimation of instantaneous frequency.

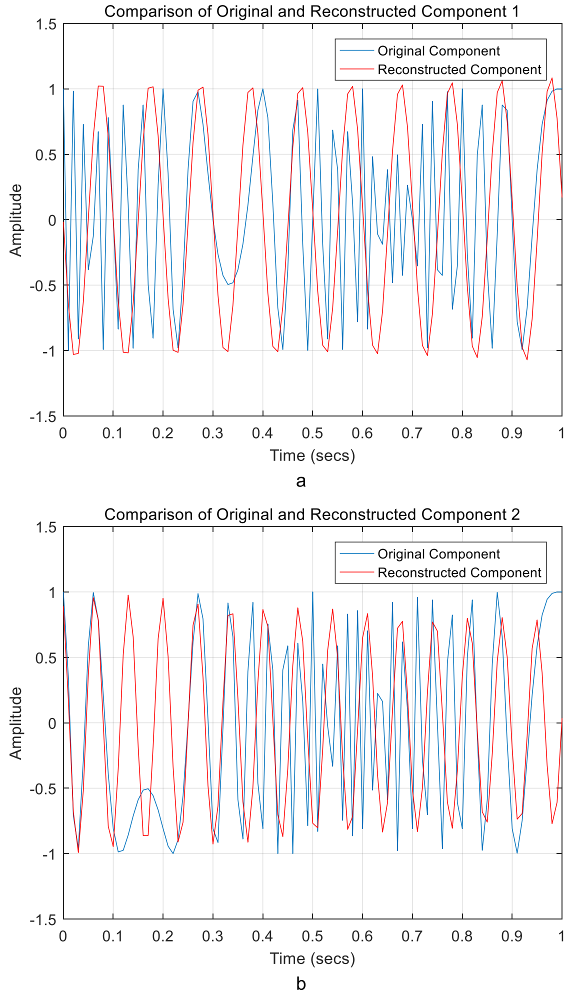

FIGURE 19. Comparison of source signals recovered using STFT (a. Recovered component 1, b. Recovered component 2).

From Figures 19-22, it can be observed that STFT and WVD exhibit relatively better recovery results, whereas CWT shows inferior performance primarily due to the impact of

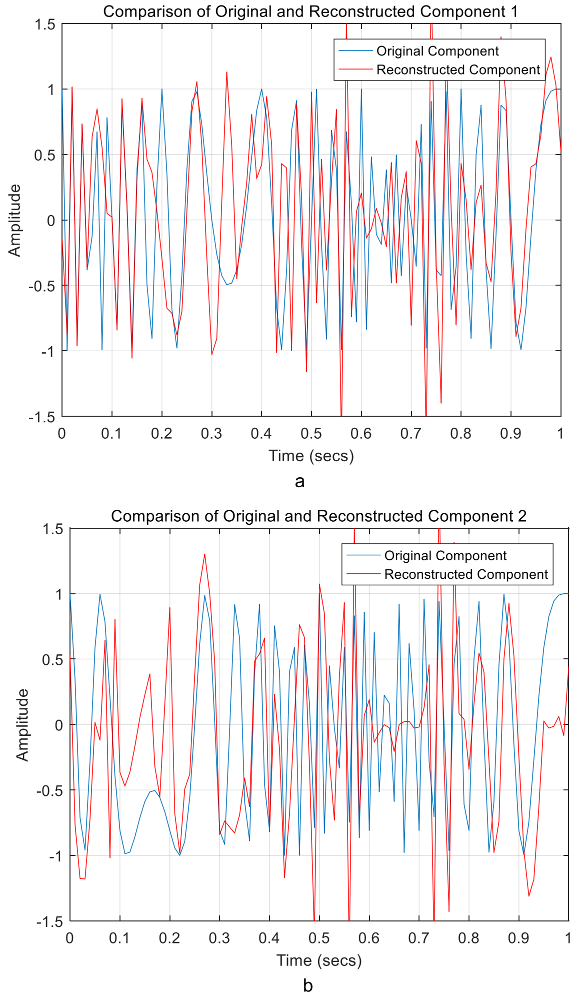

FIGURE 20. **Comparison of source signals recovered using CWT (a. Recovered component 1, b. Recovered component 2).**

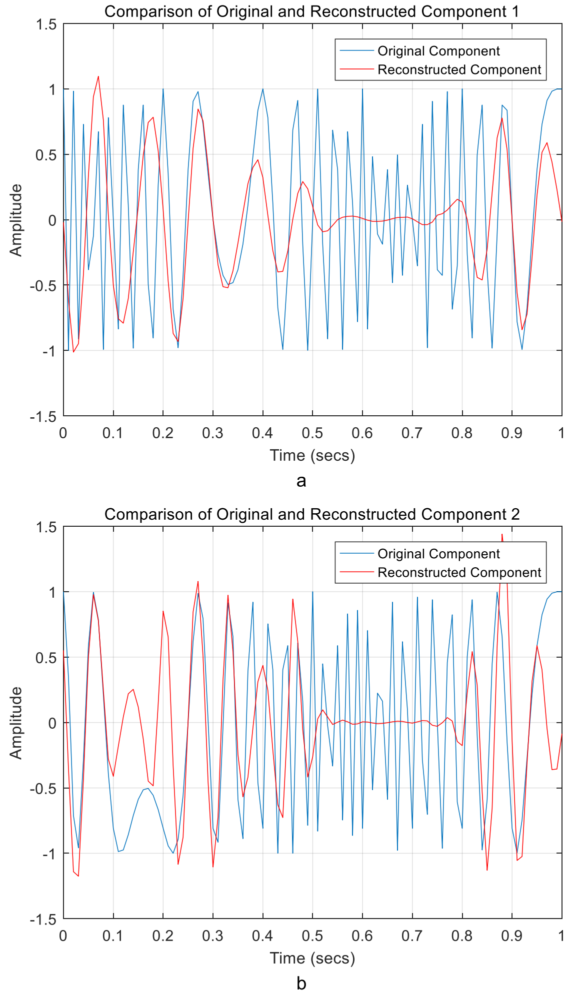

FIGURE 21. **Comparison of source signals recovered using WVD (a. Recovered component 1, b. Recovered component 2).**

cross-terms, leading to significant deviations. FrFT achieves moderate recovery results, but its computational complexity is substantial, requiring further improvements to become practical. Table 5 provides a comparison of the advantages and disadvantages faced in recovering source signals using these four TFAs. Scholars can choose the appropriate time-frequency analysis method based on their specific requirements.

4) PERFORMANCE COMPARISON

The separation performance will be quantified using the Pearson correlation coefficient, represented as r. ranges from -1 to 1, The correlation coefficient between two signals X and Y is calculated using the formula (9):

$$r(\hat{s}, s) = \frac{\sum (s_i - \bar{s})(\hat{s}_i - \bar{\hat{s}})}{\sqrt{\sum (s_i - s)^2 (\hat{s}_i - \bar{\hat{s}})^2}} \tag{9}$$

In the formula (9), s represents the estimated source signals, while $\hat{s}$ represents the actual source signals. and $\bar{s}$ and $\bar{\hat{s}}$ denote the means of s and $\hat{s}$. A higher correlation coefficient, closer to 1, indicates that the separated signals closely resemble the source signals. Conversely, a coefficient closer to 0 suggests a significant disparity between the separated and source signals, while a coefficient closer to -1 indicates a negative correlation, signifying an inverse relationship. In summary, a larger correlation coefficient implies a better separation effect.

Mixed signals with varying signal-to-noise ratios will be separated using the four TFA methods through feature extraction. The estimated source signals will be compared using the Pearson correlation coefficient to evaluate the separation performance, as illustrated in the Figure 23.

Observing Figure 23, it is evident that the separation performance of the WVD is generally poorer. This is due to the quadratic nature of WVD, which generates cross-terms during the transformation process, resulting in weak correlations between the separated signals and the source signals, leading to suboptimal separation performance.

STFT and WT tend to exhibit inferior separation performance in low signal-to-noise ratio conditions. As the

TABLE 5. The advantages and limitations of different TFA in BSS.

TFA	Advantages in BSS	Disadvantages in BSS	Signal types suitable for analysis
STFT	1. The frequency of the signal can be captured over time. 2. can adjust the time and frequency resolution by selecting the window size. 3. Simple implementation and wide application.	Resolution limitation: There is a trade-off between time resolution and frequency resolution, which cannot be achieved at the same time.	The estimation of parts of the signal with rapid frequency changes may not be accurate.
CWT	1. can adapt to frequency changes at different scales 2. can provide high time resolution and frequency resolution.	1. The computational complexity is high and may require a longer computational time. 2. The selection of different scales and wavelet functions is sensitive and needs to be adjusted based on signal characteristics.	Pulse signal
WVD	1. can accurately capture the instantaneous characteristics of the signal. 2. can provide accurate instantaneous frequency information.	1. Easily affected by noise, which may lead to artifacts. 2. The computational complexity is high.	Pulse signal nonlinear signal
FrFT	1. can capture both time-domain and frequency-domain information 2. can capture the rotational characteristics of the signal.	1. The computational complexity is high, especially for large-scale signals. 2. The selection of fractional order may need to be adjusted based on signal characteristics.	nonlinear signal

signal-to-noise ratio increases, their separation performance improves, although they are still significantly affected by the signal-to-noise ratio.

FrFT demonstrates relative insensitivity to noise compared to other methods, ensuring a stable separation performance across various noise environments. It is less influenced by noise during the feature extraction process, providing relatively better performance. However, even in high signal-to-noise ratio situations, there is room for improvement in FrFT's separation performance.

In summary, each method has its suitable scenarios and limitations. In practical applications, selecting an appropriate time-frequency analysis method is crucial based on the specific characteristics of the problem at hand.

To assess the complexity of the algorithms during the blind source separation process, different TFA methods were compared concerning their computational complexity, considering variations in the number of sampled points. This comparison of the complexity of different TFA methods is illustrated in figure 24.

Observing the Figure 24, it is evident that there is a significant disparity in the computational complexity of BSS achieved by the four different TFA methods as the number of sampled points increases.

WVD has a computational complexity of $O(N^3)$ which sharply increases with the number of sampled points. This is due to the extensive involvement of complex number operations in the WVD algorithm implementation, leading to a dramatic surge in computational workload. This poses a substantial challenge to the algorithm's implementation. FrFT's computational complexity falls approximately between $O(N^2)$ and $O(N^3)$, placing it between WVD and other methods in terms of complexity. Although lower than WVD, it still resides within a relatively high complexity range. This high computational complexity significantly impacts the algorithm's implementation, making the optimization of the FrFT algorithm to reduce computational workload a crucial research direction in blind source separation algorithms.

WT has a computational complexity of $O(NlogN)$, rendering it one of the methods with relatively lower computational demands based on wavelet transformation. This characteristic makes it one of the preferred blind source separation methods when computational resources are limited. STFT's computational complexity is $(N^2 logN)$, and it incurs relatively lower complex number operations when the number of sampled points is small. Therefore, in situations where resources are scarce, STFT can be considered an ideal blind source separation method.

In summary, reducing the computational workload, particularly for high-complexity methods such as WVD and FrFT, is a key focus in the research of blind source separation algorithms. When selecting a blind source separation method, it is crucial to balance the pros and cons of various methods based on the practical application requirements and the availability of computational resources.

C. "TWO-STEP" METHODS FOR BLIND SOURCE SEPARATION IMPLEMENTATION

The previously discussed method is the most essential and fundamental approach for achieving BSS. However, its implementation assumes that each source signal is mutually independent, and the number of mixing channels exceeds the number of source signals, thus enabling successful signal separation. Yet, in practical BSS systems, source signals are often not completely independent, and the number of mixing channels may be fewer than the number of source signals.

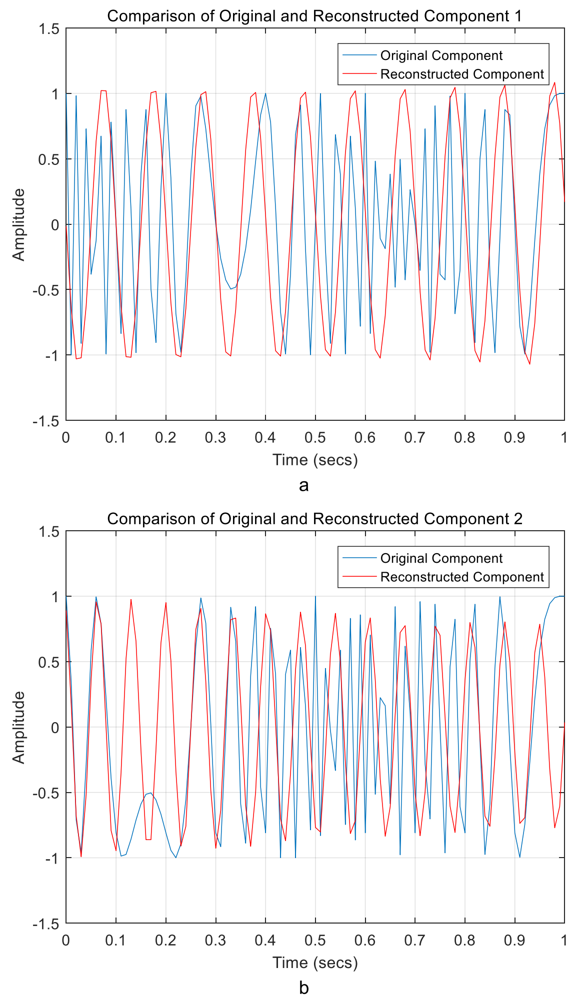

FIGURE 22. Comparison of source signals recovered using FrFT (a. Recovered component 1, b. Recovered component 2).

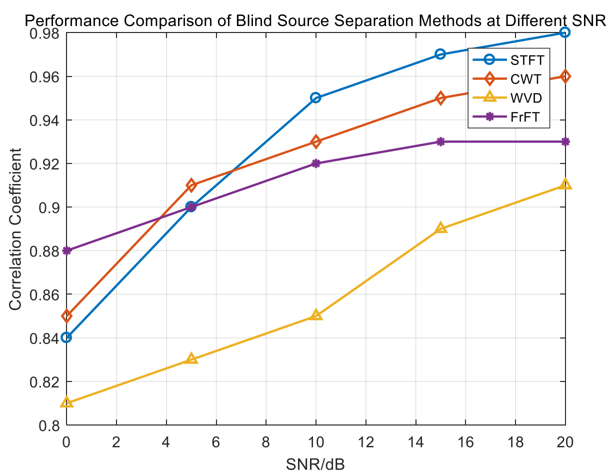

FIGURE 23. Comparison of the separation performance using different TFA.

In such scenarios, the available information about the source signals becomes limited, resulting in an infinite number of possible solutions, making BSS unattainable.

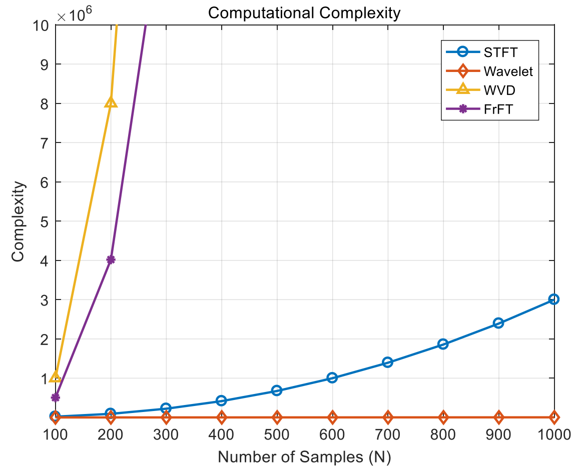

FIGURE 24. Comparison of the complexity of different TFA methods.

In 1999, Lee et al. [40] proposed the characterization of super complete basis. In 2008, B, Tan et al. [41] proposed an UBSS method based on the estimation of the number of source signals. Firstly, the observed signals were united and symmetrically processed to map them to the upper half of the unit circumference, Then the number of source signals is estimated by statistical knowledge, and the estimated mixing matrix is obtained. Finally, the source signal is recovered by the shortest path method. In 2019, J, Pei et al. [42] applied the rational ICA method to the UBSS problem with unknown number of source signals. Firstly, the observed signals are divided into multiple parts according to time, and then the rational ICA algorithm is used to estimate the mixing matrix and source signal in each time interval, and then the adaptive K-C-means clustering algorithm is used to determine the number and mixing matrix of source signals, Finally, the source signals are registered according to the classification label, and then all the source signals are recovered. However, the algorithm requires that the number of active source signals at any time cannot exceed the number of observed signals, resulting in some limitations of the algorithm.

In recent years, sparse representation has successfully solved the problem of UBSS. It was originally proposed by Bofill and Zibulivsky [43] using "two-step" method. The algorithm first estimates the mixing matrix by clustering algorithm, and then restores the source signal by shortest path method. Since then, there have been many UBSS two-step methods for sparse source signals [44], called Sparse Component Analysis (SCA).

The implementation schematic is shown in the Figure 25. In general, SCA uses a "two-step" approach

i. To enhance the sparsity of signals, some clustering algorithms such as k-means clustering and fuzzy c-means clustering are used to cluster the signals that have shown directivity in the transform domain, so as to estimate the mixing matrix.

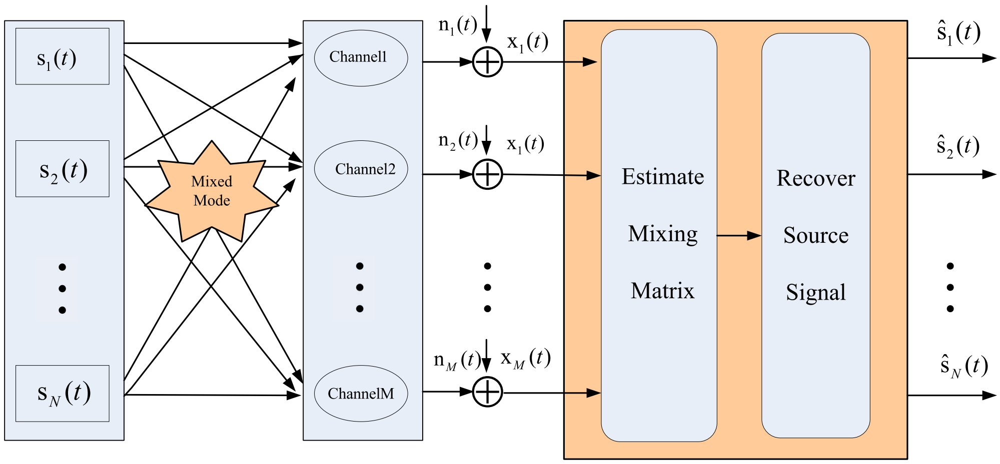

FIGURE 25. The implementation schematic of "Two-step" methods.

ii. After estimating the mixing matrix, some reconstruction algorithms such as compressed sensing are used to reconstruct the source signal. Finally, the obtained source signal is transformed back to the time domain through the inverse transformation of the transform domain.

1) MIXED MATRIX

For the first step, the mixed matrix estimation method: Bofill proposed the potential function method to find the straight-line trend in the signal scatter diagram and determine the mixed matrix, which belongs to the estimation method of statistical clustering [44]. Li uses the K-means [45] statistical clustering method to estimate the mixed matrix. Theis and Lang proposed winner takes all algorithm to estimate mixed matrix [46]. In order to improve the estimation accuracy, He et al. [47] Proposed weighted K-means clustering blind signal separation algorithm. These methods use all data samples, and there are large errors in clustering estimation. In 2005, Abrard and Deville proposed the time-frequency ratio algorithm [48], in 2013, Zhang et al. [49] proposed the mixing matrix based on subspace projection as well as clustering methods, in 2017, Xiang et al. [50] proposed an algorithm estimation using time-frequency independent complex argument points detection and adaptive hierarchical clustering. Yilmaz and Rickard proposed degenerate unmixing Estimation Technology (DUET) [51], which assumes that the signal is very sparse and forms a time-frequency mask. Therefore, Li [52] also proposed a mixed matrix estimation method based on wavelet packet transform. In 2020, Xu et al [53] used the law of large numbers to obtain the estimation of the mixing matrix and graphically display the number of source signals.

In references [54], in order to overcome the limitations of the previous algorithms, the concept and retrieval method of single source interval of signal are proposed. The method of estimating mixed matrix by using samples of single source interval is fast and effective, and has higher accuracy than clustering method. Single source interval means that in some intervals, only one signal is large or non-zero, and other signals are zero or small. In the time-frequency domain, the single source interval is used to estimate the mixed matrix, which is called the single source interval mixed matrix estimation method [54]. For enhance the clarity of observed signal directional vectors, researchers have made various attempts. Among these, Single Source Point Detection (SSP) is pivotal. By isolating data points dominated by a single source signal and reducing interference points, the validity of the data is improved significantly. The different directions obtained from sparse data points contribute to the different column vectors of the mixing matrix. This process represents the first step in the two-step method, wherein the mixing relations of the signals are solved based on the observed signals, leading to the estimation of the mixing matrix.

To understand the improvement idea of SSP detection, the mixed signal observation matrix in TF Domain is:

$$X(t,f)=\begin{bmatrix}X_1(t,f)\\X_2(t,f)\\\vdots\\X_m(t,f)\end{bmatrix}=\begin{bmatrix}a_{11}\,a_{12}&\cdots&a_{1n}\\a_{21}\,a_{22}&\cdots&a_{2n}\\\vdots\;\vdots&\ddots&\vdots\\a_{m1}\,a_{m2}&\cdots&a_{mn}\end{bmatrix}\begin{bmatrix}S_1(t,f)\\S_2(t,f)\\\vdots\\S_n(t,f)\end{bmatrix}\tag{10}$$

For the TF ratio of the observed signal of each channel to the M-th channel, the TF ratio of the mixed signal $X(t,f)$ at the SSP is a constant. Using this relationship, as long as all TF SSPs are found, the mixed vector corresponding to the source signal $S_k(t,f)$ can be estimated by using formula (11).

$$\hat{a}_k=\left[\frac{1}{L_k}\sum_{i=1}^{L_k}\frac{X_1(t_{k_i},f_{k_i})}{X_m(t_{k_i},f_{k_i})},\cdots,\frac{1}{L_k}\sum_{i=1}^{L_k}\frac{X_M(t_{k_i},f_{k_i})}{X_m(t_{k_i},f_{k_i})}\right]^T\tag{11}$$

L_k indicates the number of TF SSPs. It can be seen that by achieving effective SSP detection in the sparse TF Domain, and the mixed matrix can be estimated through a series of methods such as clustering, then the separation of source signals can be achieved. The performance of SSP detection directly determines the estimation accuracy of the mixed matrix.

For the improvement of SSP detection algorithm, the general idea is divided into two categories:

i. After the TF Transform of the observed signal, the TF points obtained are filtered to improve the performance of the subsequent SSP detection algorithm;

ii. A more suitable TF transform is performed on the observed signal to obtain TF points (TF intervals) that are easier to detect SSPs or single source areas.

For the first improvement idea, the general algorithm will be divided into four steps to achieve [55], [56]. The first step is to perform TF Transform on the observed signal, the second step is to delete the low energy points of the TF points to remove the interference noise, or to remove the redundant data points to reduce the number of TF points. In the third step, for the use of quadratic TF analysis, we should also focus on removing the interference of cross terms. In the fourth step, we should gain SSP detection for the filtered TF points. In 2018, Y, Q, Chen et al. [57] proposed using local station and distribution symmetry to detect unit interval and mixing matrix estimation is obtained through clustering algorithm. The proposed method does not require region division of hypersphere and is easy to operate, so as to effectively eliminate pseudo SSPs and improve the clustering features of observed signals. In 2019, Y, B, Li et al. [58] proposed remove the low-energy TF points to avoid the effect of noise and reduce the amount of calculation. In 2020, based on the idea of mean clustering, Li et al. [59] processed a single source point near each initial clustering center to obtain the final estimation result of the mixing matrix.

The second way is to improve the traditional TFA method. The commonly used method is to carry out STFT on the observed signal to obtain the corresponding TF point. For a certain time, only one non-zero source point is screened. The quality of the SSP obtained is different with different screening methods [57], [60], [61]. Because the physical meaning of STFT is clear and the calculation is relatively simple, many of the current estimation of mixing matrix and the separation of source signals are based on the TF Domain of STFT.

There are many other papers that try to achieve SSP detection using other TFA methods [62], [63], [64], [65]. The SPWVD of the observed signal is obtained by using the improved WVD. In this TF Domain, it is very obvious that there are several TF intervals that may exist in the single source domain of the signal. In this way, the corresponding single source domain can be found by comparing the variances in several TF intervals, instead of calculating the entire TF Domain through search and comparison, and the computational complexity is significantly reduced.

In [62], Tang, et al. proposed the TFA method of Gabor-SPWVD combination to solve the problem that the existence of the cross-term of the quadratic TF distribution leads to the inaccurate selection of the self-term TF points. The main idea is to obtain the TF diagram of suppressing the cross-term interference through the sum operation of the Gabor transform results and the SPWVD results. After truncation, it is the self-term TF point area. By combining the TF point region with the SPWVD result again, the effective TF point map of the mixed signal is obtained, and the estimation of the separation matrix and the blind separation of multiple overlapping frequency hopping signals are got.

Lei et al. [63] proposed an improved single source point detection using SPWVD, and obtained a clear and robust TF map, in which effective data were extracted [64] combines STFT with SPWVD. First, the received signal undergoes STFT, cluster it in the added window, select the desired approximate region, then segment FFT the signal, find the corresponding time point, and finally SPWVD the signal in the selected approximate region, compare the energy of each region, compare the points obtained in the second step, and select the required position information point.

Peng et al. [65] also chose to combine the STFT with the SPWVD, but the idea is quite different. Peng first used the WVD to realize the spatial TF distribution of the received signal, and then covered a layer of STFT in the TF distribution domain to extract the data points under the cover. After simulation verification, the data points extracted by this idea meet the TF points of the single source signal. In short, the estimation method of mixed matrix using single source interval requires that the source signal has enough single source interval data to ensure the estimation of mixed matrix. When the mixed signal is not sufficiently sparse, the mixed matrix cannot be completely estimated, that is, if a source signal does not have enough single source interval, the column vector of the corresponding mixed matrix will not be estimated.

Table 6 is a summary of the TFA comparison of the above improvements.

To estimate the mixing matrix based on TFA, it is essential to establish a sparse model in the transform domain of the observed signals. This model facilitates dimensionality reduction, allowing solutions for underdetermined systems. Different levels of sparsity in observed signals exhibit distinct characteristics in various transform domains. The scatter plot below provides a visual representation of the sparsity of observed signals in different transform domains. A stronger sparsity implies fewer non-zero points in the signal, with most data points having zero amplitude. In such cases, the scatter plot of the observed signal points demonstrates clear directionality. Each direction corresponds to a column vector of the mixing matrix. Therefore, the clearer the directionality presented in the scatter plot, the higher the accuracy of the estimated mixing matrix. Figure 26 illustrates the scatter plots of signals in different transform domains.

Comparing the scatter plots of figures a to f in Figure 26, it is evident that the number of time-frequency points in scatter plot f, obtained after Single Source Point (SSP) processing, is indeed fewer than the number of time-frequency points in scatter plot obtained without SSP. Additionally, the

TABLE 6. Improvement of TFA in BSS.

	Improvement direction	paper	Improvement points	advantage	shortcoming
Single source point improvement	Suppress interference time-frequency point	[57] [58][59]	Remove noise points, redundant data points and cross-term interference	Improve the quality of time-frequency points and reduce the amount of calculation	The superiority is not high, and it does not improve the time-frequency clustering characteristics essentially
	Improvement of traditional TFA method	[60][57][61]	After STFT, remove data points that do not meet the characteristics of single source point	Improve the clustering characteristics of single source points	The clustering effect is not obvious
		[62][63][64][65]	STFT+SPWVD superimposed time-frequency diagram	The clustering effect of time-frequency points obtained after superposition is obvious	Large amount of calculation and unstable performance

directional vectors in scatter plot f are significantly clearer. f shows the scatter plot of time-frequency points obtained from STFT after removing non-single source points by setting a threshold, achieving SSP.

From the Figure 26, it is evident that different TFA methods exhibit varying levels of sparsity and directionality in observed signals. Therefore, the choice of TFA method is crucial. Only with an appropriate TFA method can clear directional vectors be obtained. These distinct directional vectors are essential for achieving a more accurate mixing matrix.

To analyze the impact of different TFA methods on the accuracy of mixed matrix estimation, this paper employs the Normalized Mean Square Error (NMSE) to evaluate the estimation accuracy of the mixed matrix. The mathematical expression for NMSE is provided as follows [12]:

$$NMSE = -10log_{10}\left(\frac{\sum_{i=1}^{M}\sum_{j=1}^{N}a_{ij}^2}{\sum_{i=1}^{M}\sum_{j=1}^{N}(\hat{a}_{ij}-a_{ij})^2}\right) \tag{12}$$

a_{ij} and $\hat{a}_{ij}$ represent the values of the true source mixing matrix and the estimated mixing matrix, respectively. Equation (12) illustrates that NMSE changes with the deviation of the estimated mixing matrix values. The greater the deviation, the higher the NMSE, indicating better estimation accuracy of the measurement mixing matrix, resulting in a smaller NMSE value.

The performance comparison of the estimated mixing matrices in different environments under the scenario of M=2, N=3 is shown in Figure 27.

As depicted in the Figure 27, with the increase in SNR, the impact of noise in the mixed signal diminishes, leading to smaller NMSE values. This indicates an enhancement in the accuracy of estimated mixing matrices. Directly estimating the mixing matrix based on frequency-domain characteristics yields relatively low precision, regardless of low or high SNR conditions. Both STFT and CWT are significantly affected by noise. At low SNR levels, the accuracy of the estimated mixing matrix is low. However, with the increase in SNR, the accuracy of the mixing matrix estimation shows the most significant improvement for these methods. On the other hand, the mixing matrix estimated using FrFT exhibits the highest accuracy at low SNR levels. However, its accuracy remains relatively stable with increasing SNR, suggesting that FrFT is less influenced by noise. Nonetheless, there is room for improvement in the estimation accuracy of FrFT.

2) SOURCE SIGNAL SEPARATION

For the second step of source signal recovery, there are mainly the shortest path method to find the L1 norm solution. The L1 norm solution solved by linear programming is used as the sparse representation of the signal, and the sparse representation of the signal is usually approximate to the estimated source signal. Yilmaz et al. [51] pointed out that the L0 norm solution as the sparse representation of the source signal is the most sparsity, but there is no uniqueness and is sensitive to noise; The L1 norm solution as the sparse representation of the source signal is unique and robust to noise. At the same time, if the source signal is sufficiently sparse, the L1 norm solution is equivalent to the L0 norm solution with high probability and is similar to the source signal. Takigawa et al. [66] have also analyzed the performance of L1 norm solution, which shows that when the number of non-zero source signals at t time is less than the number of perceptron and the source signal distribution is very steep, the L1 norm solution has good performance, otherwise it cannot get good separation effect.

Since then, Georgiev, Theis and Cichocki have considered a more relaxed K-SCA condition. The K-SCA condition refers to a condition that the number of non-zero source signals is less than the number of observation signals at some sparse points. They have proved that the K-SCA condition is a sufficient condition for the estimation of source signals and mixed matrix [67], [68]. This method constructs the normal vector in polar coordinates, accumulates the data orthogonal to each vector in m-dimensional space, and then determines its normal vector, but there are also some problems, such as errors caused by angular resolution and complex calculation.

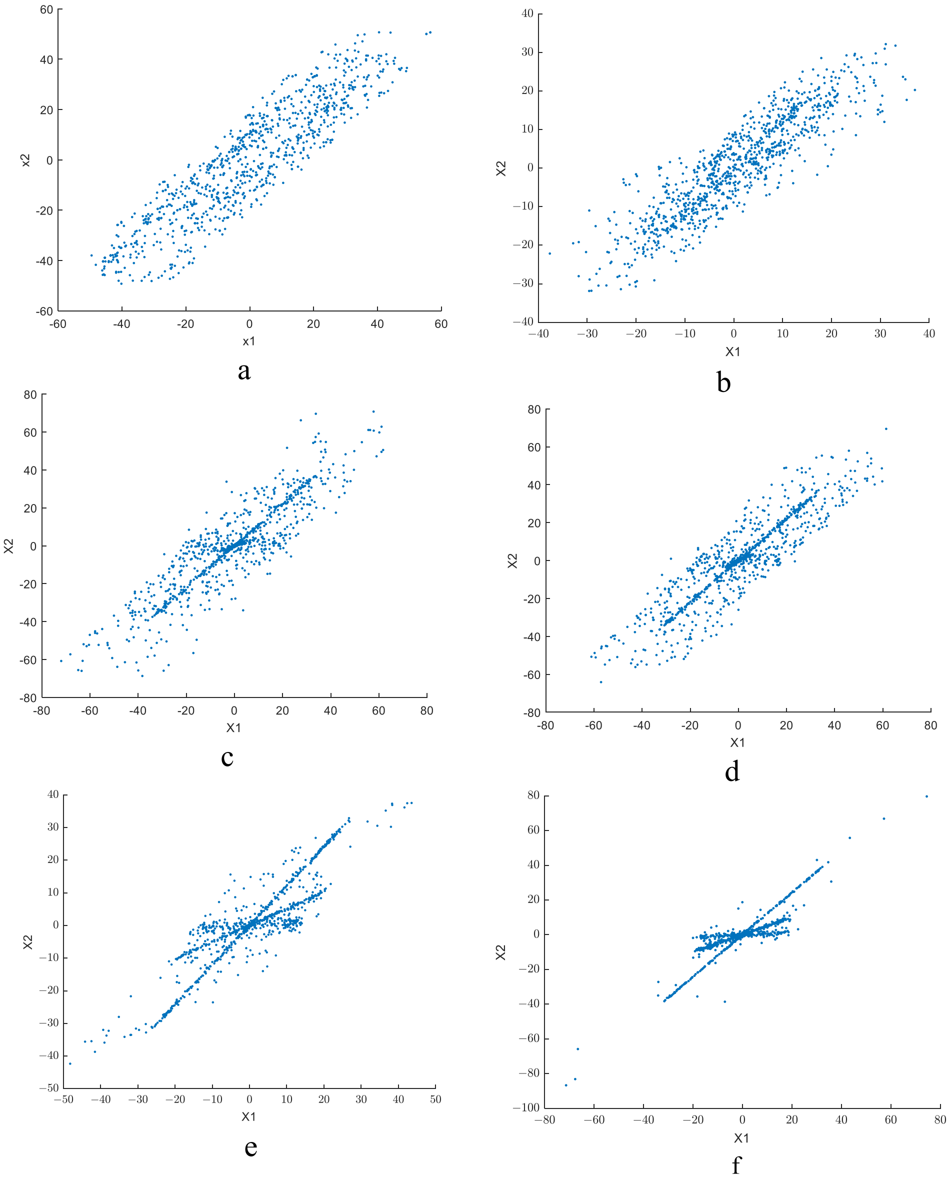

FIGURE 26. Scatter plot comparison of mixed signals. (a) time domain; (b) frequency domain; (c) wavelet domain; (d)STFT domain (e) fractional domain(p=0.65) (f) STFT with SSP.

At 2015, Wang et al. [69] proposed estimating the source signals using both time frequency technology and temporary structure so as to improve the separation performance. In 2020, Xu et al. [52] Proposed a method to estimate the source signal using the minimum intersection angle criterion.

In addition to the improvement of the above steps, the algorithm is improved from other angles. For example, in 2021, Haddad et al. [70] expressed the mixed signal as a linear combination of delay components selected from the over complete dictionary by using the spatial constraints between different channels, which can greatly reduce the impact of delay on separation.

It can be seen from the above steps that the first premise of all algorithms is that the signal is sparse. The strength of sparsity directly determines the performance of blind source separation system. Generally, ideal results can be obtained only when the source signal is sufficiently sparse [71], [72], [73]. The underdetermined blind source separation algorithm based on sparsity has gradually developed into the mainstream algorithm. It is the first thing to determine the sparsity of the signal before the algorithm.

Although most signals are naturally sparse, it is difficult to estimate the mixing matrix accurately due to the weak signal sparsity in the time domain [74], [75]. Therefore, many algorithms transform the observed signals to the frequency domain through time-frequency transformation, and then solve the problem of underdetermined blind source separation in the time-frequency domain. The above classical algorithms and improved algorithms use STFT or CWT to improve the signal sparsity, but the STFT is an investigation of the global characteristics of the signal, and the amount of calculation and algorithm performance need to be improved.

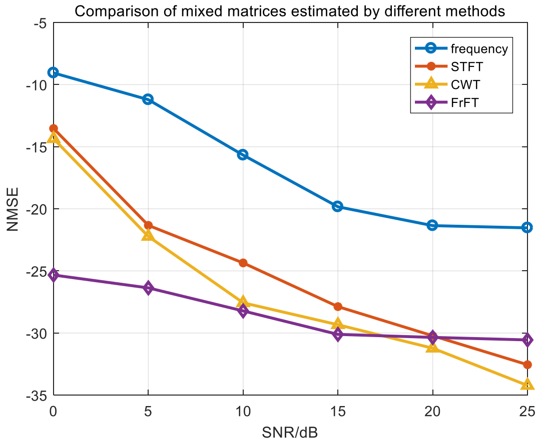

FIGURE 27. Comparison of the estimated mixing matrices in different environments.

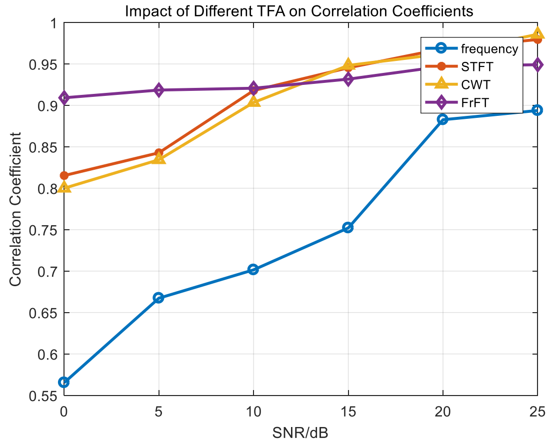

FIGURE 28. The correlation of the estimated source signals using different TFA.

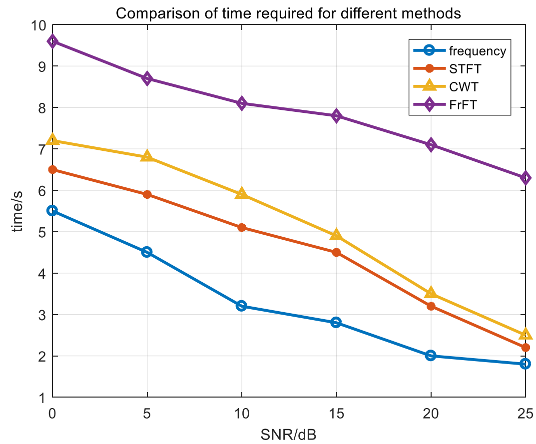

FIGURE 29. The time used to estimate the source signals using different TFA.

For the CWT to search for sparse characteristics, it is essential to select appropriate wavelet basis functions; an inappropriate choice can lead to inaccurate analysis results. Conversely, the WVD is susceptible to cross-term interference, potentially resulting in false spectral components. This poses a challenge to searching for sparse characteristics. In the case of the FrFT, due to the introduction of rotation factors, a more detailed search for global signal sparsity characteristics is feasible, but it comes at the cost of significantly increased computational burden. Therefore, when employing the "two-step" approach for UBSS, the choice of TFA method becomes of paramount importance.

To compare the performance of different TFA methods in source signal recovery, the observed channel number is set to M=2, and the number of source signals is N=3. The source signals are mixed, and various time-frequency analysis methods are used to estimate the source signals. In Figure 28, when the signal's time-domain sparsity is constant (sparsity = 0.7), the correlation of the estimated source signals using different time-frequency analysis methods changes with the variation in observed signal signal-to-noise ratio. Figure 29 illustrates the variation in the time used to estimate the source signals using different time-frequency analysis methods when the signal sparsity remains constant (sparsity = 0.7) and the observed signal signal-to-noise ratio changes.

Observing Figure 28, under constant sparsity, as SNR increases, the correlation coefficients between the recovered and actual source signals approach 1, indicating improved recovery. Among the methods, STFT and CWT exhibit the most significant improvement in correlation coefficients with increasing SNR. These methods perform exceptionally well in high SNR conditions. In contrast, source signal recovery based on frequency domain features performs poorly in noisy environments. FrFT shows relatively stable performance, although not as good as STFT and CWT in high SNR situations.

Observing the following Figure 29, under constant sparsity, as the SNR increases, the time required for recovering source signals decreases. This phenomenon occurs because algorithms consume some time due to noise interference. Among the methods, the FrFT algorithm consistently exhibits the highest time consumption, indicating it requires the most time and has relatively high complexity. On the other hand, recovering source signals using frequency domain features takes the least time, suggesting a relatively simple algorithm. STFT and CWT methods exhibit moderate time consumption in comparison to other methods [76].

D. NONLINEAR TFA METHODS FOR BSS IMPLEMENTATION

Nonlinear time-frequency analysis has developed rapidly in recent years [77], [78]. In nonlinear TFA, researchers initiate the process by identifying local maxima or the energy centroid on the time-frequency plane. These points directly influence the extraction of crucial parameters such as Instantaneous Frequency (IF) and Group Delay (GD) from non-stationary signals. The approach involves reallocating the TFA representation to the energy distribution centroid associated with each time-frequency point, a method known as Time-Frequency Reassignment Method (RM) [79].

The RM enhances the interpretability of the TF representation by redistributing energy from each point on the TF spectrogram to its corresponding local spectral centroid position. This results in a concentrated energy distribution for the isolated signal components, making them more distinguishable. The spectral centroid's frequency coordinate signifies the IF, while the time coordinate signifies the GD. The use of the Instantaneous Frequency Operator (IFO) and the Group Delay Operator (GDO) aids in estimating the spectral centroid positions, facilitating the process of TF reassignment. Subsequent research has built upon this method [80].

While the RM holds theoretical promise for achieving optimal results, its simultaneous compression of signal energy along the frequency and time axes, coupled with its disregard for phase information, poses challenges for inverse transforms and reconstructing time-domain signals. This limitation becomes particularly problematic in signal separation tasks.

To address this limitation, Huang et al. proposed the widely recognized Adaptive Empirical Mode Decomposition (EMD) [81]. The aim was to develop a method capable of both time-domain signal reconstruction and providing a clearer TFA representation. An iterative approach was introduced to identify a finite number of Intrinsic Mode Functions (IMFs) that effectively represent the signal. These IMFs enable the estimation of critical data, including instantaneous frequency and amplitude, thereby facilitating efficient signal separation. Due to EMD's adaptive nature, it offers high TF resolution, rendering it a popular TFA choice in recent years. Researchers have continually enhanced EMD and integrated it with traditional or parameterized TFA methods [82], [83], [84], [85], [86], [87]. However, EMD still faces challenges like mode mixing. Although Huang et al. introduced Ensemble Empirical Mode Decomposition (EEMD) [88] to mitigate such issues, the addition of zero-mean Gaussian white noise to the signal affects the algorithm's robustness.

Drawing inspiration from Empirical Mode Decomposition (EMD) and phase information estimation techniques in wavelet transforms, researchers introduced the Synchrosqueezing Transform (SST). This technique enables simultaneous energy reallocation for each component in multi-component signals [89]. SST achieves this by reorganizing the energy distribution of each TF point. This dual benefit enhances the energy of relevant signals, reducing noise impact and improving noise resilience. Simultaneously, it compresses the transformation coefficients obtained through the time-frequency transformation, thereby enhancing the interpretability of the time-frequency representation. Consequently, SST has found widespread application and emerged as a research focal point in recent years, owing to its advantages such as high TF concentration, reversibility, and signal reconstruction capability. Numerous research breakthroughs have emerged in this domain [90], [91], [92], [93], [94].

In addition to the traditional TFA-based and two-step approach for BSS, non-linear TFA methods are also rapidly evolving in the field of BSS. From EMD to the currently popular VMD, these methods are increasingly playing a significant role in BSS systems [95], [96], [97], [98], [99], [100], [101], [102]. The main concept involves employing non-linear algorithms to decompose the observed signals, thereby simplifying the underdetermined problem into an overdetermined one.

In [95], G. X. Zhong introduces an innovative approach for processing EEG signals, combining Independent Component Analysis (ICA) and Empirical Mode Decomposition (EMD). The proposed method effectively eliminates noisy artifacts from EEG signals with remarkable accuracy. This fusion technique enhances the overall accuracy of artifact removal, ensuring the reliability of processed EEG data. Aiming at the uncertainty of single-channel blind source separation amplitude, [96] proposed an adaptive filtering amplitude correction method based on the minimum distortion criterion. In [97], Hao proposed a single-channel blind source separation method combining EMD and constrained independent component analysis (CICA). Through EMD decomposition of the collected fault mixed signal to achieve noise reduction and single-channel expansion, the effective IMF component is selected based on the combination of white noise statistical characteristics and kurtosis value, which is used as the input signal of BSS, and the target vibration signal is extracted by CICA method to identify fault characteristics. In [98], It is very important for UBSS to use EMD to realize the number identification of source signals. The improved EMD is used to realize blind source separation. The simulation results further verify the effectiveness of EMD.

In the application of VMD in BSS system, [99] proposed a method of SCBSS based on variational mode decomposition (VMD) and principal component analysis (PCA). The observed signal is decomposed into a number of modes simultaneously using VMD. Then, PCA is used to select the corresponding source components from the decomposed modes. In [100], An UBSS method based on VMD is proposed. The paper transforms the problem of single-channel underdetermined blind source separation into a non-underdetermined problem by creating virtual multi-channel signals based on VMD. The separation of signals is achieved through the Joint Approximate Diagonalization of Eigen-matrices (JADE) of fourth-order cumulant matrices. This method establishes virtual multi-channel signals from the observation signals using VMD, followed by signal separation through JADE of fourth-order cumulant matrices, effectively addressing the underdetermined blind source separation problem and providing an efficient approach to determine the optimal number of decomposition layers for VMD.

As a crucial component of the nonlinear TFA method, SST is frequently employed as a post-processing technique. SST enhances the energy aggregation of the signal in the TF plane by reorganizing the energy of each TF point based on its frequency. This process enhances the sparsity of the TF domain and mitigates the influence of noise [101], [102]. It serves as an effective strategy to enhance performance in BSS system.

E. CONTRIBUTION OF TIME-FREQUENCY ANALYSIS IN BLIND SOURCE SEPARATION SYSTEMS

BSS is a complex signal processing problem that may encounter various challenges and difficulties in practical applications. The application of time-frequency analysis offers highly favorable conditions for addressing these challenges in blind source separation systems, making it a crucial tool in dealing with such scenarios. The following are some challenges that blind source separation algorithms may face, along with the contributions of time-frequency analysis in addressing these challenges:

F. UNDERDETERMINED PROBLEM

Underdetermined scenarios arise when the number of mixed signals is fewer than the number of source signals, resulting in an infinite number of solutions and complicating the separation process. This makes it difficult to solve the problem using simple linear algebra methods. Overcoming underdetermined scenarios may require additional information or constraints.

TF Contribution: Time-frequency analysis aids in identifying frequency components and time-varying characteristics within mixed signals, thereby narrowing down the solution space for source signals to a certain extent. By analyzing frequency and temporal information, the estimation of source signal quantity and characteristics can be achieved more accurately.

G. NON-GAUSSIANITY AND NON-INDEPENDENCE

Many blind source separation methods rely on the non-Gaussianity and mutual independence of signals. However, in some cases, signals may exhibit similar non-Gaussian characteristics or fail to meet the assumption of mutual independence, affecting separation outcomes.

TF Contribution: Time-frequency analysis methods may have lower requirements for non-Gaussianity and non-independence during the separation process. Time-frequency analysis focuses more on the signal's characteristics in the time-frequency domain, beyond mere statistical properties.

H. NONLINEAR MIXING

If the mixing process is nonlinear, such as oscillations or modulation, conventional linear separation methods may not be applicable. Nonlinear mixing leads to the intertwining of signal components in the time-frequency domain, increasing the complexity of separation.

TF Contribution: Time-frequency analysis can be employed to capture the time-frequency characteristics of nonlinear mixing processes. Some nonlinear time-frequency analysis methods can model nonlinear features like modulation and oscillation, thereby facilitating better separation.

I. NOISE AND AMBIGUITY

Noise in the data can introduce uncertainty during the separation process, disrupting the integrity of separated signals. Additionally, due to the ambiguity inherent in the mixing process, the recovery of source signals might be inherently uncertain and challenging to accurately separate.

TF Contribution: Time-frequency analysis can be employed to mitigate the impact of noise on signal separation. Through appropriate filtering and signal processing techniques, signal separability can be enhanced.

J. INSUFFICIENT PRIOR INFORMATION

Lack of prior information about the source signals can limit the accuracy of separation algorithms. Prior information can help better constrain the solution space during separation.

TF Contribution: Time-frequency analysis methods can perform analysis based on inherent data characteristics, often requiring less reliance on extensive prior information. Time-frequency analysis approaches extract information from the data, utilizing the signal's time-frequency properties.

IV. CONCLUSION

This paper addresses the prevalent issue of non-stationary signals in practical applications and provides an overview of the application of various time-frequency analysis (TFA) methods in blind source separation (BSS) systems over the past decade. By analyzing several major factors influencing the performance of time-frequency analysis, improved algorithms based on these factors are explored and their advantages and limitations in blind source separation systems are summarized. However, as research advances and signals become more complex and variable, along with environmental factors, the application of time-frequency analysis in blind source separation systems faces heightened demands. The following are some directions and trends for the future development of time-frequency analysis:

i. High-Resolution Time-Frequency Representations: Researchers are striving to develop higher-resolution time-frequency analysis methods to accurately capture the temporal variations of signals. This involves enhancing window function design, wavelet basis function selection, and more.
ii. Nonlinear Time-Frequency Analysis: The study of nonlinear time-frequency analysis methods is gaining significance when dealing with nonlinear signals or nonlinear mixing scenarios. These methods can better capture the nonlinear features of signals.
iii. Sparse Representation and Compressed Sensing: Applying sparse representation and compressed sensing theories to time-frequency analysis can reduce the number of measurements and samples, thereby decreasing computational complexity and enhancing analysis efficiency.
iv. Integration of Deep Learning and Time-Frequency Analysis: Deep learning techniques have achieved remarkable success in signal processing, including time-frequency analysis. Researchers are exploring ways to combine deep learning methods with time-frequency analysis to enhance analysis performance.

v. Multi-Source Signal Separation: Time-frequency analysis finds extensive application in multi-source signal separation problems, and researchers are developing more efficient methods to address multi-source signal mixing and separation challenges.
vi. Adaptive Time-Frequency Analysis: Adaptive time-frequency analysis methods can automatically select appropriate analysis parameters based on signal characteristics, thereby improving analysis accuracy and adaptability.

In summary, this paper provides an overview of the research development in blind source separation systems through a decade of time-frequency analysis methods. The field of time-frequency analysis is continuously progressing towards higher resolution, greater accuracy, and adaptability to diverse signal types. These efforts contribute to a better understanding and handling of time-varying signals, playing a larger role in various application domains. We have reason to believe that as applications such as medical, radar, sonar, speech processing, etc., continue to expand, and as demands for signal quality rise, blind source separation systems will undergo accelerated development.

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YANGYANG LI received the B.Sc. degree from Xuchang University, China, and the M.Sc. degree from Lanzhou Jiaotong University, China. She is currently pursuing the Ph.D. degree with the School of Electrical and Electronic Engineering, Universiti Sains Malaysia, Malaysia, with a focus on in-depth research in the field of signal processing. Her primary research interests include recognition and detection of signals, source separation, and related key areas. Her research not only centers around theoretical exploration but also emphasizes the application of outcomes in practical scenarios, contributing to daily life and technological advancements.

DZATI ATHIAR RAMLI (Senior Member, IEEE) received the B.Sc. and M.Sc. degrees in mathematics from Universiti Sains Malaysia, Malaysia, and the Ph.D. degree in electrical, electronic and system engineering from Universiti Kebangsaan Malaysia.

She is currently a Professor and a Ph.D. Supervisor with Universiti Sains Malaysia. Her research interests include the application of biometric sensing systems, image processing, computer vision, data fusion, and detection recognition algorithms.