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RESEARCH ARTICLE

Observer-Based Control for High-Order Fully Actuated Systems

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ABSTRACT An observer-based control method for high-order fully actuated systems is proposed. First, a concept of exponentially stable observers is introduced, which, different from traditional state observers, requires not only asymptotic convergence of observation errors, but also exponential convergence. Further, inspired by existing results, two design methods for exponentially stable observers are developed, one of which is less conservative and the other is simpler and more straightforward to use. Secondly, a parametric control method based on the exponentially stable observer is proposed, which ensures the exponential stability of the closed-loop system. Moreover, the proposed method does not rely on the solution of nonlinear partial differential equations, and although the system is nonlinear and time-varying, the Separation Principle still holds under this control strategy, which is a significant advantage of the proposed method. Finally, the method is applied to the attitude control of flexible spacecraft with nonlinear inertia, and comparative simulation results verify the effect of the proposed approach.

INDEX TERMS Observer-based control, nonlinear control, high-order fully actuated systems, exponential stability, flexible spacecraft control.

I. INTRODUCTION

As we all know, strict linear systems do not exist in the practical physical world. In actual engineering, the theoretical basis for using linear system theory for control system design is often to linearly approximate the system near the operating point, which of course can only obtain results in a local sense. To obtain global results, nonlinear control methods can only be used. However, general nonlinear system control has been a difficult problem for decades, and there is currently no universal effective method. Most of the existing results are either for systems with special forms or for systems subject to some strict assumptions, and in many cases, the results of nonlinear system control can only be in a local sense (see, e.g., [1], [2], [3], [4], [5],).

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A. OBSERVER-BASED CONTROL UNDER THE STATE-SPACE APPROACH

Designing a state observer for a nonlinear system under the state space approach is as difficult as designing a stabilization control law for a nonlinear system, and results are usually not available for the general system case. Existing results are relatively few and most of them require the system to have some special structure or to satisfy certain assumptions. For example, some references require that the system has some "linear dominated" structure ([1], [2]), upper or lower triangular structure ([6], [7], [8]), or require that the nonlinearity of the system satisfies a strict global Lipschitz condition ([1], [2], [9]). In addition, although there are some results considering systems of general form (see, e.g., [10], [11], [12]), they usually transform the observer design problem into the solution of some complicated nonlinear partial differential equations.

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However, solving these equations is also quite difficult and there is no universal solution in general. They are only solvable for simple systems, while analytical solutions can not obtained generally for complicated systems. Such results are of great theoretical significance, but applying the methods to practical engineering systems is often difficult.

Compared to results of the state observer design for nonlinear systems, results of observer-based control are much fewer, and they are often more demanding in terms of system structure and assumptions. For example, some references require the system to be single-input ([13], [14]) or to have some kind of "triangular structure", typically a strict feedback structure ([13], [14], [15], [16], [17]), a semi-strict feedback structure ([18]), a pure feedback structure ([19], [20], [21]), a special nonstrict-feedback structure ([22], [23], [24], [26], [27]), a feedforward structure ([28], [29], [30]), and some other triangular structures ([31], [32]), etc. There are also lots of representative references that require systems to have some "linear dominant" structure (see e.g. [33], [34], [35]). Some other results are obtained for systems with certain special structures, see [36], [37] for instance.

Moreover, the existing results often hold only in a local sense and it is generally difficult to guarantee the global stability of closed-loop systems (see, e.g., [31], [33], [38]). Furthermore, many of the results only guarantee boundedness of the state of the augmented closed-loop system ([23], [34], [36], [38]), failing to obtain asymptotic convergence, not to say exponential convergence.

Even when discussing boundedness, it is also quite difficult to obtain results in a global sense, and many representative results have only yielded semi-global boundedness ([22], [37]), semi-global ultimately uniformly boundedness ([15], [18], [19], [24]) and global ultimately uniformly boundedness ([25]) for the state of the extended closed-loop system.

In summary, observer-based control under the state-space approach has the following difficulties:

1) There is currently no universal effective method for observer design and observer-based control of general nonlinear systems, and most of the existing results are either for systems with special forms or for systems subject to some strict assumptions, and in many cases the results can only be in a local sense.
2) In general, it is quite difficult to guarantee the asymptotic stability of the closed-loop system, many representative results can only guarantee the boundedness of augmented closed-loop systems in various senses.
3) Many existing results are obtained by transforming the observer design problem into the solution of some complicated nonlinear partial differential equations, which are also difficult to solve, and thus can usually not be applied to practical engineering systems.

B. OBSERVER-BASED CONTROL UNDER THE HOFA SYSTEM APPROACH

In recent years, a new methodology called the higher-order fully actuated (HOFA) system approach has been proposed in the two series of papers [39], [40], [41] and [42], [43], [44], [45], [46], [47], [48], [49], [50], [51], aiming to establish a unified architecture for the control of general nonlinear systems. It is argued in [39], [42], and [43] that the HOFA model, although subject to the full-actuation condition, is in fact a general model of a dynamic control system parallel to the state-space model, rather than representing a small class of nonlinear systems. The reason lies in two aspects: on one hand, the HOFA model of the dynamic system can be directly derived by modeling the controllable physical system through state transformation and variable elimination ([42], [48]); on the other hand, it has been proven that many nonlinear systems in the state-space form can be transformed into HOFA systems, such as strict feedback systems ([39], [51]) and generalized strict feedback systems ([43]), nonlinear systems in a kind of controllable canonical forms ([40]), feedback-linearizable systems ([39], [51]), and a more general class of nonlinear systems ([42]).

When the full state is measurable, the full-actuation characteristic of the HOFA system allows us to directly cancel the nonlinearity of the system, thereby transforming the nonlinear problem into a linear one, and finally obtaining a linear time invariant closed-loop system with arbitrarily assignable eigen-polynomials. This advantage provides great convenience for the control of the system, and this method has been proven to be quite simple and effective in dealing with the robust control ([44]), adaptive control ([45]), optimal control ([49]), tracking control ([50]), disturbance attenuation and decoupling ([47]), etc., of general nonlinear systems.

However, when only partial state is measurable, it becomes difficult to completely cancel the nonlinearity of the system, and in such a case using a state observer to observe the unmeasurable state may be the most direct idea. Then we naturally wonder whether the framework of the HOFA system method, compared with the framework of the state-space method, can also provide convenience for observer design and observer-based control.

In order to answer this question, first mention a fact that people usually do not pay attention to: the system state we care about is the same in the open-loop system and the closed-loop system. Therefore, we can design state observers for both open-loop systems and closed-loop systems. However, within the framework of the state-space approach, if the system is nonlinear, the closed-loop system is often also nonlinear. In this case, designing state observers for open-loop and closed-loop systems is equally difficult. This is the essential reason why this fact is not noticed. However, from the perspective of the HOFA system approach, the situation is quite different. Because (when all states are measurable) the closed-loop system is linear, we can now easily design state observers for linear closed-loop systems, which greatly reduces the complexity of the problem.

C. MOTIVATION AND CONTRIBUTIONS OF THIS PAPER

In our recent work [56], attitude control via state feedback of flexible spacecraft with nonlinear time-varying inertia is investigated in depth: we first establish a HOFA model for the original dynamic system, and then develop stabilization and tracking control laws based on the HOFA system theory, which guarantees the global asymptotic stability of the closed-loop system. However, as pointed out in the conclusion section in [56], flexible modes of spacecraft are often not measurable in practice, which motivates us to further consider attitude control of flexible spacecraft via output feedback. During the study of this specific control problem, we extracted a general control method from the control theory level, and finally reapplied this method to the control of flexible spacecraft in this current paper.

Specifically, this paper discusses the state observer design and observer-based control of HOFA systems in detail, with the following specific contributions:

1) The concept of an exponentially stable observer is introduced. This type of observers, different from traditional state observers, require the observation error of the state to converge exponentially rather than asymptotically. Furthermore, two design methods of exponentially stable observers based on linear matrix inequalities (LMIs) are developed respectively on the basis of previous works [9] and [52], the first one is less conservative, and the second one is more convenient and direct in application. Both of them only depend on solving LMIs, for which the LMI toolbox in MATLAB can be readily used.
2) A complete parametric control method based on the exponentially stable observer is proposed. Compared to many existing observer-based nonlinear control methods (see, e.g., [10], [11], [12]), the proposed approach does not rely on the solution of nonlinear partial differential equations, thus it is easier and more direct to use. In addition, the proposed method can ensure that the augmented closed-loop system is locally or globally exponentially stable (depending on whether the Lipschitz condition holds locally or globally), rather than bounded ([23], [34], [36], [38]), semi-globally bounded ([22], [37]), semi-globally uniformly ultimately bounded ([15], [18], [19], [24]), or global ultimately uniformly boundedness ([25]). It is noted that although the system is generalized to be nonlinear, the Separation Principle still holds under this control strategy, which is a significant advantage of the proposed method.
3) The proposed observer-based control method is applied to the attitude control of flexible spacecraft with nonlinear inertia for the output feedback case. Comparative simulation results show that the proposed method is able to achieve attitude stability control and flexible vibration suppression, while the traditional method fails.

For convenience of description, following [42], and [43], we introduce the following notations for a vector $x \in \mathbb{R}^n$ and a set of matrices $A_i \in \mathbb{R}^{r\times r}$, $i = 0, 1, \cdots, n-1$:

$$x^{(0\sim k)} = \begin{bmatrix} x \\ \dot{x} \\ \vdots \\ x^{(k)} \end{bmatrix}, \; B_c = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I_r \end{bmatrix}, \tag{1}$$

$$A_{0\sim n-1} = \begin{bmatrix} A_0 \; A_1 \; \cdots \; A_{n-1} \end{bmatrix}, \tag{2}$$

$$\Phi(A_{0\sim n-1}) = \begin{bmatrix} 0 & I_r & & \\ & & \ddots & \\ & & & I_r \\ -A_0 & -A_1 & \cdots & -A_{n-1} \end{bmatrix}. \tag{3}$$

In addition, for a set of vectors $\eta_1, \eta_2, \ldots, \eta_m \in \mathbb{R}^n$, denote

$$\begin{aligned} &\mathrm{vec}\left(\begin{bmatrix} \eta_1 \; \eta_2 \; \cdots \; \eta_m \end{bmatrix}\right) \\ &\quad = \begin{bmatrix} \eta_1^{\mathrm{T}} \; \eta_2^{\mathrm{T}} \; \cdots \; \eta_m^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ &\mathrm{unvec}_{n,m}\left(\begin{bmatrix} \eta_1^{\mathrm{T}} \; \eta_2^{\mathrm{T}} \; \cdots \; \eta_m^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}\right) \\ &\quad = \begin{bmatrix} \eta_1 \; \eta_2 \; \cdots \; \eta_m \end{bmatrix}, \end{aligned}$$

and for a matrix $M \in \mathbb{R}^{r\times r}$, let

$$\mathrm{sym}\,(M) = M + M^{\mathrm{T}}. \tag{4}$$

$\det(M)$, $\mathrm{cond}\,(P)$, $\lambda_{\min}(M)$, $\lambda_{\max}(M)$, and $\lambda_i(M)$ represent the determinant, the condition number, the minimum eigenvalue, the maximum eigenvalue, and the i-th eigenvalue of the matrix M. Denote the real part of the i-th eigenvalue of the matrix M by $\mathrm{Re}\lambda_i(M)$.

II. PROBLEM FORMULATION

Consider the following system

$$\begin{cases} x^{(n)} = f(x^{(0\sim n-1)}, t) + B(Cx^{(0\sim n-1)}, t)u, \\ y = Cx^{(0\sim n-1)}, \end{cases} \tag{5}$$

where $x, u \in \mathbb{R}^r$ are the state and the control input, respectively, $y \in \mathbb{R}^m$ is the measured output, $f(x^{(0\sim n-1)}, t) \in \mathbb{R}^r$ and $B(Cx^{(0\sim n-1)}, t) \in \mathbb{R}^{r\times r}$ are a continuous vector function and a matrix function, respectively, satisfying $f(0, t) = 0$, and C is a constant matrix, satisfying the following assumptions:

Assumption A1:(see, e.g., [42], [43], [44]) $\det B(Cx^{(0\sim n-1)}, t) \neq 0$, $\forall x^{(i)} \in \mathbb{R}^r$, $i = 0, 1, \cdots, n-1$.

Assumption A2: (see, e.g., [9], [52]) The function $f(x^{(0\sim n-1)}, t)$ is local α-Lipschitz with respect to its first argument, that is, there exists a scalar $\alpha > 0$ and a convex subset $\Omega \subseteq \mathbb{R}^{nr}$ containing the origin such that:

$$\begin{aligned} &\left\| f(x_1^{(0\sim n-1)}, t) - f(x_2^{(0\sim n-1)}, t) \right\| \\ &\quad < \alpha \left\| x_1^{(0\sim n-1)} - x_2^{(0\sim n-1)} \right\|, \forall x_1^{(0\sim n-1)}, x_2^{(0\sim n-1)} \in \Omega. \end{aligned}$$

Design for system (5) the following observer-based control law:

$$\begin{cases} \dot{\hat{x}}^{(0\sim n-1)} = \Phi(A_{0\sim n-1})\hat{x}^{(0\sim n-1)} \\ \qquad +B_c v - L\left(y - C\hat{x}^{(0\sim n-1)}\right), \\ u = -B^{-1}(Cx^{(0\sim n-1)}, t) \\ \quad \times(f(\hat{x}^{(0\sim n-1)}, t) + A_{0\sim n-1}\hat{x}^{(0\sim n-1)} - v), \end{cases} \tag{6}$$

where $\hat{x}^{(0\sim n-1)} \in \mathbb{R}^{nr}$ is the state of the observer, which is the estimation of $x^{(0\sim n-1)}$, v is an external input, $A_{0\sim n-1} \in \mathbb{R}^{r\times nr}$ is a gain matrix such that $\Phi(A_{0\sim n-1})$ is Hurwitz, $L \in \mathbb{R}^{nr\times m}$ is another gain matrix to be designed. The control system structure is shown in Figure 1.

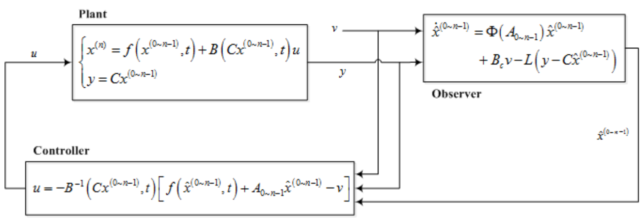

FIGURE 1. Observer-based control system structure.

As shown in Figure 1, $x^{(0\sim n-1)}$ and $\hat{x}^{(0\sim n-1)}$ perform essentially the state variables of the open-loop system (5) and the observer system (6). The estimation $\hat{x}^{(0\sim n-1)}$ of the state variable $x^{(0\sim n-1)}$ is obtained based on the measured output y and the external reference input v, and the control input u is finally computed by using signals y, v and the estimation $\hat{x}^{(0\sim n-1)}$. The control block diagram is presented in Figure 2.

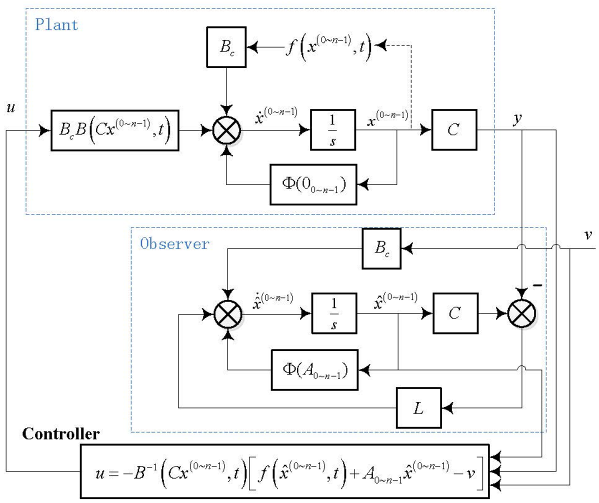

FIGURE 2. The control block diagram of the proposed control law.

Then, the corresponding closed-loop system can be given by

$$\begin{cases} x^{(n)} = f(x^{(0\sim n-1)}, t) - f(\hat{x}^{(0\sim n-1)}, t) \\ \qquad -A_{0\sim n-1}\hat{x}^{(0\sim n-1)} + v, \\ \dot{\hat{x}}^{(0\sim n-1)} = \Phi(A_{0\sim n-1})\hat{x}^{(0\sim n-1)} \\ \qquad +B_c v - L\left(y - C\hat{x}^{(0\sim n-1)}\right), \end{cases} \tag{7}$$

which can be further rewritten into the following state-space form:

$$\begin{bmatrix} \dot{x}^{(0\sim n-1)} \\ \dot{\hat{x}}^{(0\sim n-1)} \end{bmatrix} = \begin{bmatrix} \Phi(0_{0\sim n-1}) & -B_c A_{0\sim n-1} \\ -LC & \Phi(A_{0\sim n-1}) + LC \end{bmatrix} \begin{bmatrix} x^{(0\sim n-1)} \\ \hat{x}^{(0\sim n-1)} \end{bmatrix} + \begin{bmatrix} B_c \\ 0 \end{bmatrix}\left(f(x^{(0\sim n-1)}, t) - f(\hat{x}^{(0\sim n-1)}, t)\right) + \begin{bmatrix} B_c \\ B_c \end{bmatrix} v. \tag{8}$$

Based on the above preparations, the problem to be considered can be stated as follows.

Problem 1: For given system (5) satisfying Assumptions A1-A2, design the observer-based control law (6), such that the equilibrium $\left(x^{(0\sim n-1)}, \tilde{x}^{(0\sim n-1)}\right) = (0, 0)$ of the closed-loop system (8) is exponentially stable.

The above problem is solved in two steps, the first step is the design of the exponentially stable observer, and the second step is the design of the control system based on the exponentially stable observer. Before giving the results, let us present some preliminary results at first.

III. PRELIMINARIES

The following results plays important roles in the subsequent derivation, which are given by [44] and [47], respectively.

Lemma 1: For given matrix $F \in \mathbb{R}^{nr\times nr}$, all the matrices $A_{0\sim n-1}$ and $V \in \mathbb{R}^{nr\times nr}$ satisfying $\det V \neq 0$ and

$$\Phi\left(A_{0\sim n-1}\right) = VFV^{-1} \tag{9}$$

are given by

$$\begin{cases} A_{0\sim n-1} = -ZF^n V^{-1}(Z, F), \\ V = V(Z, F) = \begin{bmatrix} Z \\ ZF \\ \vdots \\ ZF^{n-1} \end{bmatrix}, \end{cases} \tag{10}$$

where $Z \in \mathbb{R}^{r\times nr}$ is a parameter matrix satisfying the following constraint:

$$\det V(Z, F) \neq 0. \tag{11}$$

Lemma 2: Let $A \in \mathbb{R}^{n\times n}$ satisfy

$$\mathrm{Re}\lambda_i(A) \leq -\frac{\mu}{2}, \ i = 1, 2, \cdots, n, \tag{12}$$

where $\mu > 0$. Then, there exists a positive definite matrix $P \in \mathbb{R}^{n\times n}$ satisfying

$$A^{\mathrm{T}}P + PA \leq -\mu P. \tag{13}$$

IV. OBSERVER DESIGN

This section treats the exponentially stable observer design. First, let us introduce the following definition of the exponentially stable observer.

Definition 1: A dynamic system

$$\dot{\hat{x}}^{(0\sim n-1)} = F\left(\hat{x}^{(0\sim n-1)}, y, u\right), \tag{14}$$

where $\hat{x} \in \mathbb{R}^r$, is called an (locally) exponentially stable observer of system (5) if there exist positive constant scalars c and β satisfying

$$\left\|\tilde{x}^{(0\sim n-1)}(t)\right\| \leq c\left\|\tilde{x}^{(0\sim n-1)}(0)\right\| e^{-\beta t},\ \tilde{x}^{(0\sim n-1)}(0) \in \Omega, \tag{15}$$

where $\Omega \subseteq \mathbb{R}^{nr}$ is a convex subset containing the origin, and

$$\tilde{x}^{(0\sim n-1)}(t) = x^{(0\sim n-1)}(t) - \hat{x}^{(0\sim n-1)}(t). \tag{16}$$

Particularly, if $\Omega = \mathbb{R}^{nr}$, then (14) is called a globally exponentially stable observer of system (5).

This section considers the design of state observers with the following specific form:

$$\begin{aligned}\dot{\hat{x}}^{(0\sim n-1)} &= \Phi(A_{0\sim n-1})\hat{x}^{(0\sim n-1)} \\ &\quad + B_c v - L\left(y - C\hat{x}^{(0\sim n-1)}\right),\end{aligned} \tag{17}$$

which is the first equation in control law (6), and the meaning and dimensions of the variables are given in the "Problem formulation" Section. Regarding how to design gain L to make (17) form an exponential stable observer of system (5), two design methods are given based on [9] and [52], respectively. The first method is more conservative, while the second method is more convenient and direct to use.

A. LINEAR PARAMETER-VARYING DESIGN METHOD

Firstly, let us introduce the first design method of the exponentially stable observer.

1) AN EQUIVALENT EXPRESSION OF LIPSCHITZ CONDITION

As a preparation, the following definition is introduced.

Definition 2: ([9]) For the following vectors

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n,\ Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \in \mathbb{R}^n, \tag{18}$$

define $X^{Y_i} \in \mathbb{R}^n, i = 0, 1, \ldots, n$ as

$$X^{Y_i} = \begin{cases} \begin{bmatrix} y_1 & \cdots & y_i & x_{i+1} & \cdots & x_n \end{bmatrix}^{\mathrm{T}}, \\ \qquad\qquad i = 1, 2, \ldots, n, \\ X,\ i = 0. \end{cases} \tag{19}$$

Denote

$$\mathcal{H}_{ij} = e_n(i)\, e_n^{\mathrm{T}}(j),\ i, j = 1, 2, \ldots, n,$$

where $e_n(i) \in \mathbb{R}^n$ represents the vector with its i-th element being 1 and the rest being 0. Then we have the following result, which is a minor generalization of Lemma 7 in [9].

Lemma 3: For given function $\Psi : \Omega \times [0, \infty) \to \mathbb{R}^n$, where $\Omega \subseteq \mathbb{R}^n$ is a convex subset containing the origin, the following two conditions are equivalent:

1) Ψ is local α_Ψ-Lipschitz with respect to its first variable, that is,

$$\begin{aligned}&\|\Psi(X, t) - \Psi(Y, t)\| \\ &\quad < \alpha_\Psi \|X - Y\|,\ \forall X, Y \in \Omega,\ t \geq 0;\end{aligned}$$

2) there exist

$$\psi_{ij} : \Omega \times \Omega \times [0, \infty) \to \mathbb{R},\ i, j = 1, 2, \ldots, n,$$

and constant scalars $\hat{\psi}_{ij}$ and $\check{\psi}_{ij}$ satisfying

$$\begin{aligned}&\Psi(X, t) - \Psi(Y, t) \\ &\quad = \sum_{i=1}^{n}\sum_{j=1}^{n} \psi_{ij}\left(X^{Y_{j-1}}, X^{Y_j}, t\right) \mathcal{H}_{ij}(X - Y),\end{aligned}$$

where the functions $\psi_{ij}(\cdot)$ satisfy

$$\check{\psi}_{ij} \leq \psi_{ij}\left(X^{Y_{j-1}}, X^{Y_j}, t\right) \leq \hat{\psi}_{ij},\ \forall X, Y \in \Omega,\ t \geq 0.$$

Proof: Although the independent variable t is added to the function $\Psi(\cdot)$, and the function $\Psi(\cdot)$ is only required to be local Lipschitz instead of global Lipschitz, the process of proof is the same as Lemma 7 in [9]. Thus the proof is omitted. ■

Remark 1: It is noted that the above Lemma 3 also covers the case that Ψ is global Lipschitz, and in such a case $\Omega = \mathbb{R}^n$.

Remark 2: Partition $\Psi(X)$ as

$$\Psi(X) = \begin{bmatrix} \Psi_1(X) & \Psi_2(X) & \cdots & \Psi_n(X) \end{bmatrix}^{\mathrm{T}}.$$

If $\Psi(X)$ is local α_Ψ-Lipschitz, then there exist a set of positive scalars $\alpha_{\Psi_i}, i = 1, 2, \ldots, n$ such that $\Psi_i(X), i = 1, 2, \ldots, n$ are local α_{Ψ_i} -Lipschitz. It is pointed out that the constant $\check{\psi}_{ij}$ and $\hat{\psi}_{ij}$ in Lemma 3 can be taken as $\check{\psi}_{ij} = -\alpha_{\Psi_i}$ and $\hat{\psi}_{ij} = \alpha_{\Psi_i}$, respectively (see the proofs of Lemmas 6 and 7 in [9]).

2) LMI-BASED DESIGN METHOD OF THE GAIN MATRIX L

Before giving the results, let us make some preparations at first. Recalling (16), and using Lemma 3, we have

$$f(x^{(0\sim n-1)}, t) - f(\hat{x}^{(0\sim n-1)}, t) = \mathcal{A}(\Theta)\, \tilde{x}^{(0\sim n-1)}, \tag{20}$$

where

$$\begin{cases} \Theta = \mathrm{vec}\left(\tilde{\Theta}\right) \in \mathbb{R}^{n^2} \\ \tilde{\Theta} = \left[\psi_{ij}(\cdot)\right]_{ij} \in \mathbb{R}^{n\times n}, \end{cases} \quad \mathcal{A}(\Theta) = \sum_{i=1}^{n}\sum_{j=1}^{n} \psi_{ij}(\cdot)\, \mathcal{H}_{ij}, \tag{21}$$

with

$$\psi_{ij}(\cdot) \triangleq \psi_{ij}\left(\left[x^{(0\sim n-1)}\right]^{\hat{x}_{j-1}^{(0\sim n-1)}}, \left[x^{(0\sim n-1)}\right]^{\hat{x}_{j}^{(0\sim n-1)}}, t\right) \tag{22}$$

satisfying

$$\check{\psi}_{ij} \leq \psi_{ij}(\cdot) \leq \hat{\psi}_{ij},\ \forall x^{(0\sim n-1)}, \hat{x}^{(0\sim n-1)} \in \Omega,\ t \geq 0. \tag{23}$$

In view of (21)-(23), we have $\Theta \in \mathcal{I}_n$, where

$$\mathcal{I}_n = \left\{ \Theta \in \mathbb{R}^{n^2} \middle| \check{\psi}_{ij} \leq \tilde{\Theta}_{ij} \leq \hat{\psi}_{ij},\ \tilde{\Theta} = \mathrm{unvec}_{n,n}(\Theta) \right\} \tag{24}$$

is obviously a convex set, with its extreme points set given by

$$v_{\mathcal{I}_n} = \left\{ \Theta \in \mathbb{R}^{n^2} \middle| \tilde{\Theta}_{ij} \in \{\check{\psi}_{ij}, \hat{\psi}_{ij}\},\ \tilde{\Theta} = \mathrm{unvec}_{n,n}(\Theta) \right\}. \tag{25}$$

Based on the above preparations, the following result can be obtained.

Theorem 1: Suppose that system (5) satisfies Assumptions A1-A2. If the gain matrix L is taken as

$$L = P^{-1}Y, \tag{26}$$

where $P > 0$ and Y satisfy the following LMIs:

$$\begin{aligned} &\mathrm{sym}\left(P\Phi(0_{0\sim n-1}) + YC + PB_c\mathcal{A}(\Theta)\right) \\ &\quad < -\varepsilon I,\ \forall\Theta \in v_{\mathcal{I}_n}, \end{aligned} \tag{27}$$

with ε being a certain positive scalar. Then (17) forms a locally exponentially stable observer of system (5).

Particularly, if Assumption A2, namely, the Lipschitz condition, holds globally, then (17) forms a globally exponentially stable observer of system (5).

Proof: It follows from the closed-loop system (8) that the observation error given by (16) satisfies

$$\begin{aligned} \dot{\tilde{x}}^{(0\sim n-1)} &= \left(\Phi(0_{0\sim n-1}) + LC\right)\tilde{x}^{(0\sim n-1)} \\ &\quad + B_c\left[f(x^{(0\sim n-1)}, t) - f(\hat{x}^{(0\sim n-1)}, t)\right]. \end{aligned} \tag{28}$$

In view of (20), the above equation can be further rewritten into

$$\dot{\tilde{x}}^{(0\sim n-1)} = \left(\Phi(0_{0\sim n-1}) + B_c\mathcal{A}(\Theta) + LC\right)\tilde{x}^{(0\sim n-1)}. \tag{29}$$

Choose $V\left(\tilde{x}^{(0\sim n-1)}\right) = \left[\tilde{x}^{(0\sim n-1)}\right]^{\mathrm{T}} P\tilde{x}^{(0\sim n-1)}$, then the derivative of $V(\cdot)$ can be calculated along the trajectory of system (29) as

$$\dot{V}\left(\tilde{x}^{(0\sim n-1)}\right) = \left[\tilde{x}^{(0\sim n-1)}\right]^{\mathrm{T}} \Gamma(\Theta)\,\tilde{x}^{(0\sim n-1)}, \tag{30}$$

where

$$\Gamma(\Theta) = \mathrm{sym}\left[P\left(\Phi(0_{0\sim n-1}) + LC + B_c\mathcal{A}(\Theta)\right)\right]. \tag{31}$$

If the LMIs in (27) hold, then it follows from the convex optimization theory (see, e.g., [53]) and (26) that $\Gamma(\Theta) < -\varepsilon I,\ \forall\Theta \in \mathcal{I}_n$. Thus we have

$$\begin{aligned} \dot{V}\left(\tilde{x}^{(0\sim n-1)}\right) &< -\varepsilon \left\|\tilde{x}^{(0\sim n-1)}\right\|^2 \\ &\leq -cV\left(\tilde{x}^{(0\sim n-1)}\right),\ \forall\tilde{x}^{(0\sim n-1)} \in \Omega, \end{aligned} \tag{32}$$

where $c = \varepsilon\lambda_{\max}^{-1}(P)$. Therefore, according to the well-known Comparison Theorem, we have from (32) that

$$V(\tilde{x}^{(0\sim n-1)}(t)) \leq V\left(\tilde{x}^{(0\sim n-1)}(0)\right)e^{-ct}. \tag{33}$$

In view of

$$\begin{aligned} \lambda_{\min}(P)\left\|\tilde{x}^{(0\sim n-1)}\right\|^2 &\leq V\left(\tilde{x}^{(0\sim n-1)}, t\right) \\ &\leq \lambda_{\max}(P)\left\|\tilde{x}^{(0\sim n-1)}\right\|^2 \end{aligned}$$

and (33), we have

$$\left\|\tilde{x}^{(0\sim n-1)}(t)\right\| \leq \sqrt{\mathrm{cond}(P)}\left\|\tilde{x}^{(0\sim n-1)}(0)\right\| e^{-\frac{c}{2}t}. \tag{34}$$

Thus the conclusion can be immediately obtained according to Definition 1.

As for the global case, just note that $\Omega = \mathbb{R}^{nr}$, except that the proof process is exactly the same as in the local case, so it is omitted. Then the proof is completed. ■

B. ANOTHER DESIGN METHOD BASED ON LMIs

In this subsection, inspired by the method in [52], another design approach of exponentially stable observer is proposed as follows.

Theorem 2: Suppose that system (5) satisfies Assumptions A1-A2. If the gain matrix L is taken as

$$L = P^{-1}Y, \tag{35}$$

where Y and $P > 0$ satisfy the following LMI:

$$\begin{bmatrix} \mathrm{sym}\left[P\Phi(0_{0\sim n-1}) + YC\right] + \varepsilon_1\left(\alpha^2 + \varepsilon_2\right)I & P \\ P & -\varepsilon_1 I \end{bmatrix} < 0, \tag{36}$$

with ε_1 and ε_2 being positive scalars. Then (17) forms a locally exponentially stable observer of system (5).

Particularly, if Assumption A2, namely, the Lipschitz condition holds globally, then (17) forms a globally exponentially stable observer of system (5).

Proof: It follows from the closed-loop system (8) that the observation error defined by (16) satisfies (28). If the Lyapunov function candidate is chosen as $V\left(\tilde{x}^{(0\sim n-1)}\right) = \left[\tilde{x}^{(0\sim n-1)}\right]^{\mathrm{T}} P\tilde{x}^{(0\sim n-1)}$, where $P \in \mathbb{R}^{nr\times nr}$ is a positive definite matrix. Then, taking the derivative of $V(\cdot)$ along the trajectory of the system (28), yields

$$\dot{V}\left(\tilde{x}^{(0\sim n-1)}\right) = \xi^{\mathrm{T}}\Pi\xi, \tag{37}$$

where

$$\xi = \begin{bmatrix} \tilde{x}^{(0\sim n-1)} \\ f(x^{(0\sim n-1)}, t) - f(\hat{x}^{(0\sim n-1)}, t) \end{bmatrix}, \tag{38}$$

$$\Pi = \begin{bmatrix} \mathrm{sym}\left[P\left(\Phi(0_{0\sim n-1}) + LC\right)\right] & P \\ P & 0 \end{bmatrix}. \tag{39}$$

It follows from (35)-(36) that

$$\Pi - \varepsilon_1 M < 0, \tag{40}$$

where

$$M = \begin{bmatrix} -\left(\alpha^2 + \varepsilon_2\right)I & 0 \\ 0 & I \end{bmatrix}. \tag{41}$$

Substituting the above equation (40) into (37), and in view of Assumption A2, we have

$$\begin{aligned}&\dot{V}\left(\tilde{x}^{(0\sim n-1)}\right)\\&\quad<\varepsilon_1\left(-\left(\alpha^2+\varepsilon_2\right)\left\|\tilde{x}^{(0\sim n-1)}\right\|^2+\|\Delta f(\cdot)\|^2\right)\\&\quad\leq-\varepsilon_1\varepsilon_2\left\|\tilde{x}^{(0\sim n-1)}\right\|^2\\&\quad\leq-cV\left(\tilde{x}^{(0\sim n-1)}\right),\end{aligned}\tag{42}$$

where $c = \lambda_{\max}^{-1}(P)\,\varepsilon_1\varepsilon_2$. Repeating the proof process in (32)-(34), we can immediately obtain the conclusion according to Definition 1.

The proof for the global case is exactly the same as the proof for the local case except $\Omega = \mathbb{R}^{nr}$ and is therefore omitted. Then the proof is completed. ■

Remark 3: In Theorems 1 and 2, the observer design problem has been transformed into the solutions of the LMIs (26)-(27) and (35)-(36), respectively. The feasibility of the LMIs can be directly verified by using the LMI toolbox in MATLAB.

V. OBSERVER-BASED CONTROL

It is pointed out at the end of the "Problem formulation" Section that Problem 1 is to be solved in two steps. The first step is the design of an exponentially stable observer, which has been discussed in depth in the previous section. In this section, we investigate the second step, namely, the control law design based on the exponentially stable observer. The core of this problem is to discuss the stability of the closed-loop system (8).

A. STABILITY OF THE CLOSED-LOOP SYSTEM

Recalling (16), the closed-loop system (8) can be rewritten into

$$\begin{cases}\dot{x}^{(0\sim n-1)}=\Phi(A_{0\sim n-1})x^{(0\sim n-1)}+B_cA_{0\sim n-1}\tilde{x}^{(0\sim n-1)}\\\qquad\qquad+B_c\Delta f\left(x^{(0\sim n-1)},\tilde{x}^{(0\sim n-1)},t\right),\\\dot{\tilde{x}}^{(0\sim n-1)}=\left(\Phi(0_{0\sim n-1})+LC\right)\tilde{x}^{(0\sim n-1)}\\\qquad\qquad+B_c\Delta f\left(x^{(0\sim n-1)},\tilde{x}^{(0\sim n-1)},t\right),\end{cases}\tag{43}$$

where

$$\begin{aligned}&\Delta f\left(x^{(0\sim n-1)},\tilde{x}^{(0\sim n-1)},t\right)\\&\quad=f(x^{(0\sim n-1)},t)-f(x^{(0\sim n-1)}-\tilde{x}^{(0\sim n-1)},t),\end{aligned}\tag{44}$$

satisfies

$$\begin{aligned}&\left\|\Delta f\left(x^{(0\sim n-1)},\tilde{x}^{(0\sim n-1)},t\right)\right\|\\&\quad\leq\alpha\left\|\tilde{x}^{(0\sim n-1)}\right\|,\ \forall x^{(0\sim n-1)},\tilde{x}^{(0\sim n-1)}\in\Omega,\end{aligned}\tag{45}$$

in view of Assumption A2. Regarding the stability of the above system (43), we have the following result.

Theorem 3: Suppose that system (5) satisfies Assumptions A1-A2. If

1) the first equation in the control law (6) forms a locally exponentially stable observer of system (5), and
2) for arbitrarily given positive constant scalar μ, there hold

$$\mathrm{Re}\lambda_i\left(\Phi(A_{0\sim n-1})\right)\leq-\frac{\mu}{2},\ i=1,2,\ldots,nr.\tag{46}$$

Then, the observer-based control law (6), with $v = 0$, guarantees that the equilibrium $\left(x^{(0\sim n-1)},\tilde{x}^{(0\sim n-1)}\right)=(0,0)$ of the closed-loop system (43) is locally exponentially stable.

Particularly, if Assumption A2, namely, the Lipschitz condition, holds globally, and the first equation in the control law (6) forms a globally exponentially stable observer of system (5), then the equilibrium $\left(x^{(0\sim n-1)},\tilde{x}^{(0\sim n-1)}\right)=(0,0)$ of the closed-loop system (43) is globally exponentially stable.

Proof: It follows from the assumptions in this theorem that there exist positive constant scalars c and β such that

$$\begin{aligned}&\left\|\tilde{x}^{(0\sim n-1)}(t)\right\|\\&\quad\leq c\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|e^{-\beta t},\ \tilde{x}^{(0\sim n-1)}(0)\in\Omega.\end{aligned}\tag{47}$$

According to Lemma 2, when (46) holds, there exists $P > 0$ satisfying

$$P\Phi(A_{0\sim n-1})+\Phi^{\mathrm{T}}(A_{0\sim n-1})P=-\mu P.\tag{48}$$

Let $V_x(x^{(0\sim n-1)})=\left[x^{(0\sim n-1)}\right]^{\mathrm{T}}Px^{(0\sim n-1)}$, and denote

$$\varphi(\cdot)\triangleq B_cA_{0\sim n-1}\tilde{x}^{(0\sim n-1)}+B_c\Delta f\left(x^{(0\sim n-1)},\tilde{x}^{(0\sim n-1)},t\right),$$

then, using the Young Inequality and Assumption A2, we have

$$\begin{aligned}\dot{V}_x&=-\mu V_x+2\left[x^{(0\sim n-1)}\right]^{\mathrm{T}}P\varphi(\cdot)\\&\leq-\mu V_x+2\|P\|\left\|x^{(0\sim n-1)}\right\|\|\varphi(\cdot)\|\\&\leq-\mu V_x+\|P\|\left(\varepsilon\left\|x^{(0\sim n-1)}\right\|^2+\frac{1}{\varepsilon}\|\varphi(\cdot)\|^2\right)\\&\leq-\beta_1V_x+\beta_2\left\|\tilde{x}^{(0\sim n-1)}\right\|^2,\end{aligned}\tag{49}$$

with

$$\begin{aligned}\beta_1&=\mu-\varepsilon\|P\|\lambda_{\min}^{-1}(P)>0,\\\beta_2&=\frac{\|P\|}{\varepsilon}\left(\|B_cA_{0\sim n-1}\|+\alpha\|B_c\|\right)^2,\end{aligned}$$

where $\varepsilon > 0$ is a sufficiently small constant scalar satisfying

$$\beta_1\neq2\beta.\tag{50}$$

According to the Comparison Theorem, it follows from (49) that

$$\begin{aligned}&V_x(x^{(0\sim n-1)}(t))\\&\quad\leq e^{-\beta_1t}V_x(x^{(0\sim n-1)}(0))\\&\qquad+\beta_2\int_0^te^{-\beta_1(t-\tau)}\left\|\tilde{x}^{(0\sim n-1)}(\tau)\right\|^2\mathrm{d}\tau.\end{aligned}$$

Substituting (47) into the above equation, and in view of (50), we have

$$\begin{aligned}
&V_x(x^{(0\sim n-1)}(t))\\
&\quad\leq e^{-\beta_1 t}V_x(x^{(0\sim n-1)}(0))\\
&\qquad+\beta_2\int_0^t e^{-\beta_1(t-\tau)}\left(c\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|e^{-\beta\tau}\right)^2\mathrm{d}\tau\\
&\quad=e^{-\beta_1 t}V_x(x^{(0\sim n-1)}(0))\\
&\qquad+\beta_2c^2\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|^2e^{-\beta_1 t}\int_0^t e^{(\beta_1-2\beta)\tau}\mathrm{d}\tau\\
&\quad=e^{-\beta_1 t}V_x(x^{(0\sim n-1)}(0))\\
&\qquad+\beta_2c^2\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|^2e^{-\beta_1 t}\\
&\qquad\times\frac{1}{\beta_1-2\beta}\left(e^{(\beta_1-2\beta)t}-1\right)\\
&\quad=e^{-\beta_1 t}V_x(x^{(0\sim n-1)}(0))\\
&\qquad+\frac{\beta_2c^2\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|^2}{\beta_1-2\beta}\\
&\qquad\times\left(e^{-2\beta t}-e^{-\beta_1 t}\right). \qquad (51)
\end{aligned}$$

If $\beta_1>2\beta$, then it follows from (51) that

$$\begin{aligned}
&V_x(x^{(0\sim n-1)}(t))\\
&\quad\leq e^{-\beta_1 t}V_x(x^{(0\sim n-1)}(0))\\
&\qquad+\frac{\beta_2c^2\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|^2}{\beta_1-2\beta}e^{-2\beta t}\\
&\quad\leq (V_x(x^{(0\sim n-1)}(0))\\
&\qquad+\frac{\beta_2c^2\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|^2}{\beta_1-2\beta})e^{-2\beta t}. \qquad (52)
\end{aligned}$$

If $\beta_1<2\beta$, then we know from (51) that

$$\begin{aligned}
&V_x(x^{(0\sim n-1)}(t))\\
&\quad\leq e^{-\beta_1 t}V_x(x^{(0\sim n-1)}(0))\\
&\qquad-\frac{\beta_2c^2\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|^2}{\beta_1-2\beta}e^{-\beta_1 t}\\
&\quad\leq (V_x(x^{(0\sim n-1)}(0))\\
&\qquad+\frac{\beta_2c^2\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|^2}{2\beta-\beta_1})e^{-\beta_1 t}. \qquad (53)
\end{aligned}$$

In view of (50), the following conclusion can be obtained by combining the above two cases (52) and (53):

$$V_x(x^{(0\sim n-1)}(t))\leq (V_x(x^{(0\sim n-1)}(0))+\sigma)e^{-\tilde{\beta}t}, \qquad (54)$$

where

$$\sigma=\frac{\beta_2c^2}{|2\beta-\beta_1|}\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|^2,\ \tilde{\beta}=\min\{\beta_1,2\beta\}. \qquad (55)$$

It is obvious that

$$\begin{aligned}
c_1'\left\|x^{(0\sim n-1)}(t)\right\|^2&\leq V_x(x^{(0\sim n-1)}(t))\\
&\leq c_2'\left\|x^{(0\sim n-1)}(t)\right\|^2,
\end{aligned}$$

where $c_1'=\lambda_{\min}(P)$ and $c_2'=\lambda_{\max}(P)$, thus it follows from (54) that

$$\begin{aligned}
&\left\|x^{(0\sim n-1)}(t)\right\|^2\\
&\quad\leq\frac{1}{c_1'}V_x(x^{(0\sim n-1)}(t))\\
&\quad\leq\frac{1}{c_1'}(V_x(x^{(0\sim n-1)}(0))+\sigma)e^{-\tilde{\beta}t}\\
&\quad\leq\frac{1}{c_1'}(c_2'\left\|x^{(0\sim n-1)}(0)\right\|^2\\
&\qquad+\frac{\beta_2c^2}{|2\beta-\beta_1|}\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|^2)e^{-\tilde{\beta}t}\\
&\quad=(\frac{c_2'}{c_1'}\left\|x^{(0\sim n-1)}(0)\right\|^2\\
&\qquad+\frac{\beta_2c^2}{c_1'|2\beta-\beta_1|}\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|^2)e^{-\tilde{\beta}t}\\
&\quad\leq(\frac{c_2'}{c_1'}\left\|X^{(0\sim n-1)}(0)\right\|^2\\
&\qquad+\frac{\beta_2c^2}{c_1'|2\beta-\beta_1|}\left\|X^{(0\sim n-1)}(0)\right\|^2)e^{-\tilde{\beta}t}\\
&\quad\leq\mu\left\|X^{(0\sim n-1)}(0)\right\|^2e^{-\tilde{\beta}t}, \qquad (56)
\end{aligned}$$

where

$$X^{(0\sim n-1)}(t)=\begin{bmatrix}x^{(0\sim n-1)}(t)\\ \tilde{x}^{(0\sim n-1)}(t)\end{bmatrix},\ \mu=\frac{c_2'}{c_1'}+\frac{\beta_2c^2}{c_1'|2\beta-\beta_1|}.$$

Combining (47) and (56), we have

$$\begin{aligned}
&\left\|X^{(0\sim n-1)}(t)\right\|\\
&\quad\leq\left\|x^{(0\sim n-1)}(t)\right\|+\left\|\tilde{x}^{(0\sim n-1)}(t)\right\|\\
&\quad\leq\sqrt{\mu}\left\|X^{(0\sim n-1)}(0)\right\|e^{-\frac{\tilde{\beta}}{2}t}+c\left\|\tilde{x}^{(0\sim n-1)}(0)\right\|e^{-\beta t}\\
&\quad\leq\sqrt{\mu}\left\|X^{(0\sim n-1)}(0)\right\|e^{-\frac{\tilde{\beta}}{2}t}+c\left\|X^{(0\sim n-1)}(0)\right\|e^{-\beta t}\\
&\quad\leq\left(\sqrt{\mu}+c\right)\left\|X^{(0\sim n-1)}(0)\right\|e^{-\varpi t},
\end{aligned}$$

where $\varpi=\min\left\{\frac{\tilde{\beta}}{2},\beta\right\}$, revealing that the equilibrium $\left(x^{(0\sim n-1)},\tilde{x}^{(0\sim n-1)}\right)=(0,0)$ is locally exponentially stable. From the definition of the exponential stability, the assumptions in the global sense certainly lead to the stability results in the global sense. Then the proof is completed. ■

Remark 4: It should be noted that the second condition in Theorem 3 does not bring any conservatism to the design of the control gain $A_{0\sim n-1}$, because the constant μ can be selected arbitrarily small. As long as $\Phi(A_{0\sim n-1})$ is Hurwitz, there always exists a constant μ satisfying (46), thus this condition essentially requires $\Phi(A_{0\sim n-1})$ to be Hurwitz. We only need to select proper $A_{0\sim n-1}$ such that all the eigenvalues of $\Phi(A_{0\sim n-1})$ have negative real part.

The above Theorem 3 also reveals an important fact: the controller part and the observer part of the observe-based control law (6) can be independently designed, which means that although the system is generalized to be nonlinear, the Separation Principle is still valid under the proposed control strategy. The observer design part has been discussed in depth in the "Observer design" Section. Next we further discuss the design of the control gain $A_{0\sim n-1}$.

B. ALGORITHM OF THE OBSERVER-BASED CONTROL METHOD

Based on Lemma 1, we can use the following algorithm to directly parameterize the gain matrix $A_{0\sim n-1}$.

Algorithm 1: [Parameterization of the control gain $A_{0\sim n-1}$]

1) Choose a positive constant scalar μ, and select appropriate $F \in \mathbb{R}^{nr\times nr}$ according to practical requirements, such that

$$\mathrm{Re}\lambda_i(F) \leq -\frac{\mu}{2},\ i=1,2,\ldots,nr. \tag{57}$$

2) Establish the following complete parametric expression for the gain matrix $A_{0\sim n-1}$ using Lemma 1:

$$\begin{cases} A_{0\sim n-1} = -ZF^nV^{-1}(Z,F), \\ V(Z,F) = \begin{bmatrix} Z \\ ZF \\ \vdots \\ ZF^{n-1} \end{bmatrix}, \end{cases} \tag{58}$$

where $Z \in \mathbb{R}^{r\times nr}$ is an arbitrary parameter matrix satisfying the following constraint:

$$\det V(Z,F) \neq 0. \tag{59}$$

3) Optimize the parameter matrix Z to meet additional design requirements or achieve better performance. For example, Z can be appropriately selected to make the system have better anti-disturbance performance, lower sensitivity to parameter perturbations, and smaller control gain (see, e.g., [54], [55], [56] for details).
4) Substitute the optimal parameter Z^* into the parametric expression (58) to obtain the final design of $A^*_{0\sim n-1}$.

Remark 5: The above parametric design of the gain $A_{0\sim n-1}$ preserves all design degrees of freedom, which come not only from Z, but also from F. In fact, F can be optimized together with Z to improve system performance, while F only needs to satisfy the constraint (57).

Combining the above Algorithm with Theorems 1 or 2, gives the following algorithm, which summarizes the specific design process of the proposed observer-based control law.

Algorithm 2: [Procedure of the observer-based controller design]

1) Establish the HOFA model (5) for the considered system, and calculate the Lipschitz constant α of the function $f(x^{(0\sim n-1)}, t)$.
2) Obtain the observation gain L by using Theorems 1 or 2.
3) Design the control gain $A_{0\sim n-1}$ using Algorithm 1.
4) Substitute the obtained observation gain L and control gain $A_{0\sim n-1}$ into (6) to give the observer-based control law.

C. AN ILLUSTRATED EXAMPLE

Consider the following numerical example

$$\begin{cases} \ddot{x} = f\left(x^{(0\sim 1)}\right) + B(x)u \\ y = Cx, \end{cases} \tag{60}$$

where

$$f\left(x^{(0\sim 1)}\right) = 0.3\sqrt{x^2+3} + \frac{\dot{x}^2}{2\left(\dot{x}^2+1\right)} + 0.2\arctan\left(x+\frac{1}{2}\dot{x}\right),$$
$$B(x) = x^2+5,\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

First, it is easy to see

$$\frac{\partial f}{\partial x} = \frac{0.3x}{\sqrt{x^2+3}} + \frac{0.2}{1+\left(x+\frac{1}{2}\dot{x}\right)^2} \leq 0.5,$$
$$\frac{\partial f}{\partial \dot{x}} = \frac{0.5}{\left(\dot{x}^2+1\right)^2} + \frac{0.1}{1+\left(x+\frac{1}{2}\dot{x}\right)^2} \leq 0.6,$$

thus $f\left(x^{(0\sim 1)}\right)$ is globally Lipschitz with the following Lipschitz constant

$$\alpha = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial \dot{x}}\right)^2} = 0.78.$$

Then, the observation gain L can be immediately derived by using Theorem 2 as follows

$$L = \begin{bmatrix} -8.8759 \\ -16.5894 \end{bmatrix}. \tag{61}$$

Secondly, choose

$$F = \begin{bmatrix} -1 & 0.5 \\ -0.5 & -1 \end{bmatrix}, \tag{62}$$

with $\mu = 1$. Then, the controller gain $A_{0\sim 1}$ can be obtained using Algorithm 1 as

$$A_{0\sim 1} = \begin{bmatrix} 1.25 & 2 \end{bmatrix}. \tag{63}$$

Finally, substituting (61) and (63) into (6), gives the control law.

To verify the effect of the proposed method, numerical simulation is carried out with the following initial value

$$x^{(0\sim 1)} = \begin{bmatrix} 2 \\ -1 \end{bmatrix},\ \hat{x}^{(0\sim 1)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

and the simulation result is shown in Figure 3.

Is is seen from Figure 3 that the state variables $\hat{x}^{(0\sim 1)}$ of the observer can realize the estimation of the unmeasurable state of the system, and all state variables converge to zero smoothly, which fully verifies the effect of the proposed method.

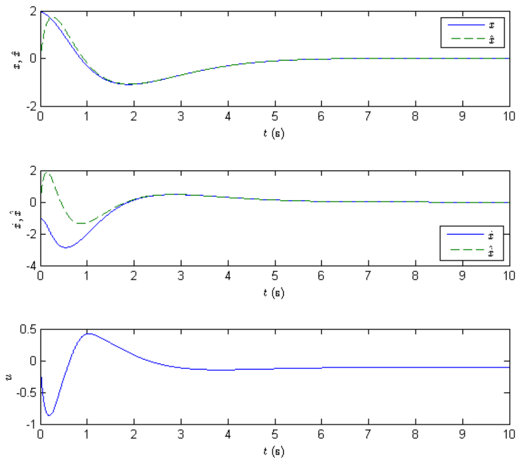

FIGURE 3. Simulation results of the illustrated example (60).

VI. ATTITUDE CONTROL OF FLEXIBLE SPACECRAFT WITH NONLINEAR INERTIA

In this section, the attitude control of flexible spacecraft with nonlinear inertia via output feedback is considered. In our recent work [56], the HOFA system model is derived from the original dynamic equation of the system, and then the control system via state feedback is designed based on the HOFA system theory. This paper further treats the case that only partial state can be measured, for which the observer-based control method proposed in this paper is applied, guaranteeing the exponential stability of the closed-loop system. Simulation is carried out based on practical engineering parameters to verify the effect of the proposed method.

A. SYSTEM MODEL

The attitude system of a flexible spacecraft with nonlinear time-varying inertia is given by ([56]):

$$\begin{cases} I\left(\theta,\dot{\theta},t\right)\ddot{\theta}+b\ddot{q}=u, \\ \ddot{q}+2\xi\Lambda\dot{q}+\Lambda^2 q+b\ddot{\theta}=0. \end{cases} \tag{64}$$

where θ is the attitude angle, q is the flexible mode, u is the control torque, b, I and are coefficients, $I\left(\theta,\dot{\theta},t\right)$ is the moment of inertia associated with θ, $\dot{\theta}$ and t, satisfying the following assumptions:

Assumption A3: Λ, ξ and b are non-zero constants.

Assumption A4: For any $\theta,\dot{\theta}\in\mathbb{R}$ and $t\in[0,+\infty)$, there holds

$$\Delta\left(\theta,\dot{\theta},t\right)\triangleq I\left(\theta,\dot{\theta},t\right)-b^2\neq 0. \tag{65}$$

It is proved in our recent work [56] that system (64) satisfying the above Assumptions A3-A4 can be transformed into the following HOFA system form:

$$x_1^{(4)}=f\left(\ddot{x}_1,\dddot{x}_1,\theta,\dot{\theta},t\right)+B\left(\theta,\dot{\theta},t\right)u, \tag{66}$$

where

$$f\left(\ddot{x}_1,\dddot{x}_1,\theta,\dot{\theta},t\right)=-\frac{2\Lambda\xi I\left(\theta,\dot{\theta},t\right)}{\Delta\left(\theta,\dot{\theta},t\right)}\dddot{x}_1-\frac{\Lambda^2 I\left(\theta,\dot{\theta},t\right)}{\Delta\left(\theta,\dot{\theta},t\right)}\ddot{x}_1, \tag{67}$$

and

$$B\left(\theta,\dot{\theta},t\right)=-\frac{b\Lambda^3}{2\xi\Delta\left(\theta,\dot{\theta},t\right)}\neq 0. \tag{68}$$

Moreover, the relation between the variables $x_1,\dot{x}_1,\ddot{x}_1,\dddot{x}_1$ of the HOFA system (66) and the variables $\theta,q,\dot{\theta},\dot{q}$ of the original system (64) can be described by the following invertible state transformation:

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \\ \ddot{x}_1 \\ \dddot{x}_1 \end{bmatrix}=\frac{1}{2\xi}\begin{bmatrix} -b\Lambda & \Lambda\left(4\xi^2-1\right) & 2\xi b & 2\xi \\ 0 & -2\xi\Lambda^2 & -b\Lambda & -\Lambda \\ 0 & \Lambda^3 & 0 & 0 \\ 0 & 0 & 0 & \Lambda^3 \end{bmatrix}\times\begin{bmatrix} \theta \\ q \\ \dot{\theta} \\ \dot{q} \end{bmatrix}, \tag{69}$$

or, equivalently,

$$\begin{bmatrix} \theta \\ q \\ \dot{\theta} \\ \dot{q} \end{bmatrix}=-\frac{2\xi}{b\Lambda^3}\begin{bmatrix} \Lambda^2 & 2\xi\Lambda & 1 & 0 \\ 0 & 0 & -b & 0 \\ 0 & \Lambda^2 & 2\xi\Lambda & 1 \\ 0 & 0 & 0 & -b \end{bmatrix}\times\begin{bmatrix} x_1 \\ \dot{x}_1 \\ \ddot{x}_1 \\ \dddot{x}_1 \end{bmatrix}. \tag{70}$$

On one hand, in practical situation, only the attitude angle θ and the attitude angular velocity $\dot{\theta}$ can be measured, thus it follows from (70) that the measured output equation is given by

$$y=\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}=Cx_1^{(0\sim 3)}, \tag{71}$$

where

$$C=-\frac{2\xi}{b\Lambda^3}\begin{bmatrix} \Lambda^2 & 2\xi\Lambda & 1 & 0 \\ 0 & \Lambda^2 & 2\xi\Lambda & 1 \end{bmatrix}. \tag{72}$$

On the other hand, substituting the relation (71) into (66)-(68), yields

$$x_1^{(4)}=f\left(x_1^{(0\sim 3)},t\right)+B\left(Cx_1^{(0\sim 3)},t\right)u, \tag{73}$$

where

$$f\left(x_1^{(0\sim 3)},t\right)=-\frac{2\Lambda\xi I\left(Cx_1^{(0\sim 3)},t\right)}{\Delta\left(Cx_1^{(0\sim 3)},t\right)}\dddot{x}_1-\frac{\Lambda^2 I\left(Cx_1^{(0\sim 3)},t\right)}{\Delta\left(Cx_1^{(0\sim 3)},t\right)}\ddot{x}_1,$$

and

$$B\left(Cx_1^{(0\sim3)},t\right)=-\frac{b\Lambda^3}{2\xi\Delta\left(Cx_1^{(0\sim3)},t\right)}. \tag{74}$$

Combining (71) with (73), gives the following HOFA system model with a measured output equation for system (64):

$$\begin{cases} x_1^{(4)}=f\left(x_1^{(0\sim3)},t\right)+B\left(Cx_1^{(0\sim3)},t\right)u,\\ y=Cx_1^{(0\sim3)}, \end{cases} \tag{75}$$

which is in the same form as (5).

B. CONTROL SYSTEM DESIGN

It follows from Assumptions A3-A4 that the above system (75) obviously satisfies Assumptions A1-A2. Therefore, according to the control law (6) given in the "Problem formulation" Section, we can design for the system (73)-(74) the following observer-based control law:

$$\begin{cases} \dot{\hat{x}}_1^{(0\sim3)}=\Phi(a_{0\sim3})\hat{x}_1^{(0\sim3)}+B_c v-L\left(y-C\hat{x}_1^{(0\sim3)}\right),\\ u=-B^{-1}(Cx_1^{(0\sim3)},t)f(\hat{x}_1^{(0\sim3)},t)+a_{0\sim3}\hat{x}_1^{(0\sim3)}-v, \end{cases} \tag{76}$$

where $B_c=\left[\,0\;0\;0\;1\,\right]^{\mathrm{T}}$.

First, design the control gain $a_{0\sim3}$. Select the desired eigenvalues of $\Phi(a_{0\sim3})$ as

$$(-1.1915\pm1.3995)\times10^{-2}\,\mathrm{i},-0.6049,-7.1630\times10^{-3}.$$

Then, according to Algorithm 1, the gain $a_{0\sim3}$ in the control law (76) can be calculated as

$$\begin{aligned} &a_0=1.4637\times10^{-6},\ a_1=3.1001\times10^{-4},\\ &a_2=1.9255\times10^{-2},\ a_3=0.6359. \end{aligned} \tag{77}$$

Secondly, design the observation gain L. In a certain convex subset $\Omega\subseteq\mathbb{R}^4$ containing the origin, the Lipschitz constant of $f\left(x_1^{(0\sim3)},t\right)$ is $\alpha=0.3404$, and thus the gain L can be designed using Theorem 2 as

$$L=\begin{bmatrix} 124.5428 & -1.0016\times10^4\\ -5.3065\times10^3 & -5.2440\times10^3\\ -180.8737 & -233.1855\\ -3.3240 & -6.1768 \end{bmatrix}. \tag{78}$$

Finally, substitute (77) and (78) into (76), gives the designed control law.

For comparison, we also design the observer-based control law by using the method in [54] for the nominal system case that $I=20667.25\ \left(\mathrm{kg}\cdot\mathrm{m}^2\right)$. To make an equal comparison, the desired closed-loop poles set is selected as

$$\operatorname{eig}\left(\Phi(a_{0\sim3})\right)\cup\operatorname{eig}\left(\Phi(0_{0\sim n-1})+LC\right),$$

in such a case the control law is given by

$$\begin{cases} \dot{\hat{X}}=A\hat{X}+Bu-L\left(y-C_m\hat{X}\right),\\ u=K\hat{X}, \end{cases} \tag{79}$$

where

$$y=\begin{bmatrix}\theta\\ \dot\theta\end{bmatrix},\ A=\begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{b\Lambda^2}{I-b^2} & 0 & \frac{2b\Lambda\xi}{I-b^2}\\ 0 & \frac{-\Lambda^2 I}{I-b^2} & 0 & \frac{-2\Lambda\xi I}{I-b^2} \end{bmatrix},$$

$$B=\begin{bmatrix}0\\0\\ \frac{1}{I-b^2}\\ \frac{-b}{I-b^2}\end{bmatrix},\ C_m=\begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix},$$

$$K^{\mathrm{T}}=\begin{bmatrix}-1.4329\times10^{-2}\\ 169.3030\\ -3.0347\\ -49.6332\end{bmatrix},$$

$$L=\begin{bmatrix} -1.2834\times10^{-2} & -1.0000\\ -12.8410 & -187.7868\\ 3.1068 & -0.4863\\ 589.3338 & -92.2486 \end{bmatrix}.$$

C. SIMULATION RESULTS

In this subsection, numerical simulations are carried out based on the following practical parameters of a flexible spacecraft with a large antenna ([54]):

$$\begin{aligned} &b=-108.88\sqrt{\mathrm{kg}}\cdot\mathrm{m},\ \xi=0.005,\ \Lambda=2\pi\times0.151,\\ &I=527.78\sin\left[3.424\theta\,(t)+1.072\dot\theta\,(t)\right]\\ &\quad+20667.25\ \left(\mathrm{kg}\cdot\mathrm{m}^2\right). \end{aligned}$$

Choose the initial values as

$$\begin{bmatrix}\theta\,(0)\\ q\,(0)\\ \dot\theta\,(0)\\ \dot q\,(0)\end{bmatrix}=\begin{bmatrix}-45^\circ\\ 2^\circ\\ 0.5\,(^\circ/s)\\ -1\,(^\circ/s)\end{bmatrix},$$

$$\hat{x}_1^{(0\sim3)}(0)=\begin{bmatrix}-8.1149\times10^3\\ 90.6377\\ 0.5962\\ -0.4472\end{bmatrix}.$$

The simulation results based on the proposed control law are shown in Figures 4 and 5, where $\hat q$ and $\hat{\dot q}$ are estimations of q and $\dot q$, respectively, which, in view of (70), are given by

$$\begin{bmatrix}\hat q\\ \hat{\dot q}\end{bmatrix}=-\frac{2\xi}{b\Lambda^3}\begin{bmatrix}0&0&-b&0\\0&0&0&-b\end{bmatrix}\hat{x}_1^{(0\sim3)},$$

$\tilde q$ and $\tilde{\dot q}$ are observation errors of q and $\dot q$, respectively, defined by

$$\tilde q=q-\hat q,\ \tilde{\dot q}=\dot q-\hat{\dot q}.$$

As shown in Figure 4, the attitude angle and angular velocity converge to zero smoothly in about 700 seconds without overshoot. Besides, it is seen form Figure 5 that the estimates $\hat q$ and $\hat{\dot q}$ of the states asymptotically track the flexible modes q and $\dot q$, and the flexible modes gradually decay to 0, indicating that the vibrations are effectively suppressed, thus revealing the effect of the designed observer.

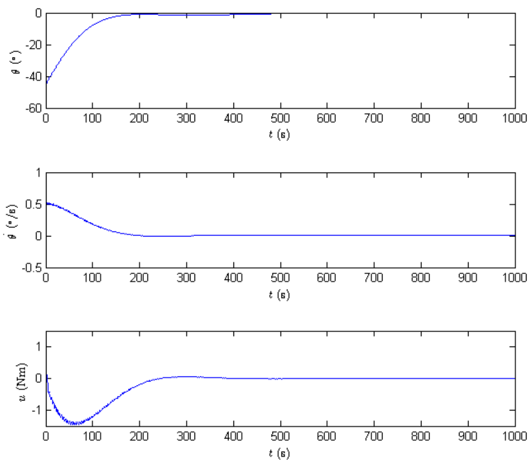

FIGURE 4. Simulation results of system (64) based on the proposed control law.

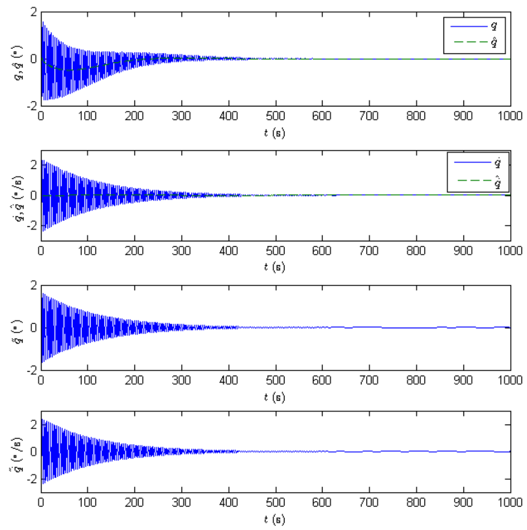

FIGURE 5. Response of flexible modes and observation errors of system (64) based on the proposed control law.

Furthermore, although the control torque bears the burden of canceling the nonlinearity of the system, its amplitude is still within 1.5 Nm, which is acceptable in practical engineering. Thus the effect of the proposed observer-based control method is fully verified.

For comparison, we select another set of initial values closer to the origin as follows

$$\begin{bmatrix} \theta(0) \\ q(0) \\ \dot{\theta}(0) \\ \dot{q}(0) \end{bmatrix} = \begin{bmatrix} 2^\circ \\ 0.5^\circ \\ -0.5\,(^\circ/s) \\ -0.2\,(^\circ/s) \end{bmatrix},$$

$$\hat{X}(0) = \begin{bmatrix} 2^\circ \\ 0^\circ \\ -0.5\,(^\circ/s) \\ 0\,(^\circ/s) \end{bmatrix},$$

and carry out the simulation based on the control method in [54], the simulation results are shown in Figure 6.

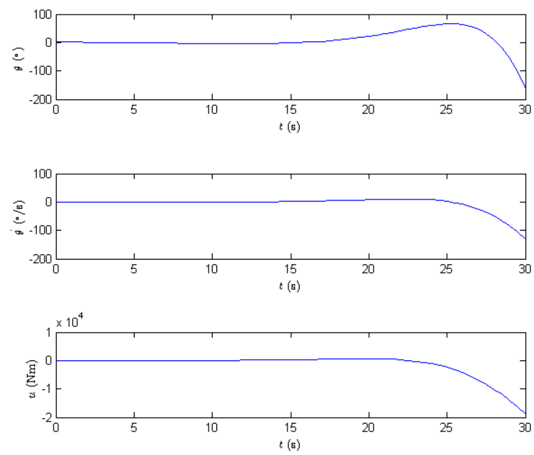

FIGURE 6. Simulation results of system (64) based on the control law in [54].

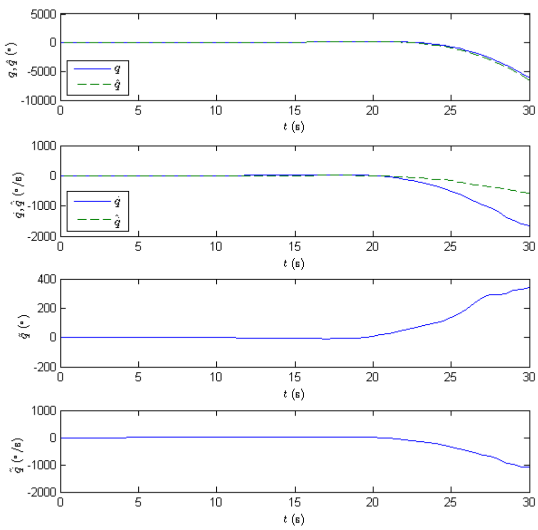

FIGURE 7. Response of flexible modes and observation errors of system (64) based on the control law in [54].

As shown in Figures 6 and 7, although the initial values of the state are chosen to be very small, when the control method in [54] is applied, the state of the system diverges rapidly, and the amplitude of the control torque quickly exceeded 10^4 Nm in about 30 seconds. This shows that the observer-based linear control law designed for the nominal system fails when the inertia has nonlinear characteristics.

VII. CONCLUSION

In [54], [55], and [57], the observer-based parametric control of linear systems are discussed from different perspectives.

In this paper, we extended this parametric design method to nonlinear time-varying systems based on the HOFA system approach. The HOFA system studied in this paper is first proposed in two recent series of papers [39], [40], [41] and [42], [43], [44], [45], [46], [47], [48], [49], [50], [51]. These two series of papers have fully demonstrated that the HOFA model, although subject to the full-actuation condition, is actually a general model of dynamic control systems parallel to the state-space model, rather than representing a small class of nonlinear systems. This paper develops an observer-based parametric control method for HOFA systems, and makes the following contributions:

1) An exponentially stable observer is proposed, which, different from traditional nonlinear observers, requires not only asymptotic convergence of observation errors, but also exponential convergence. Afterwards, two design methods for exponentially stable observers are developed inspired by the theories in [9] and [52], respectively. The former is less conservative, while the latter is more convenient and direct to use.
2) A parametric control method based on the exponentially stable observer is proposed. Unlike many existing observer-based nonlinear control approaches (see, e.g., [10], [11], [12]), the method developed in this paper does not rely on the solution of nonlinear partial differential equations, so it is easier and more direct to apply. Furthermore, the proposed method can ensure that the augmented closed-loop system is locally or globally exponentially stable (depending on whether the Lipschitz condition holds locally or globally), rather than bounded ([23], [34], [36], [38]), semi-globally bounded ([22], [37]), semi-globally uniformly ultimately bounded ([15], [18], [19], [24]), or global ultimately uniformly boundedness ([25]). It is noted that although the system is generalized to be nonlinear, the Separation Principle still holds under this control strategy, which is a significant advantage of the proposed method.
3) The proposed method is successfully applied to the attitude control of flexible spacecraft with nonlinear inertia, and comparative simulations are carried out based on practical engineering parameters, verifying the effect and superiority of the proposed method.

Finally, it should be pointed out that, for the sake of simplicity, this paper considers a single-order HOFA system, but in fact the proposed method can be extended to multi-order HOFA systems in parallel. Considering that the design ideas and processes are identical and the length of the paper is limited, we do not discuss the multi-order HOFA systems.

It should be noted that, as shown in (5), the control matrix $B(Cx^{(0\sim n-1)}, t)$ is required to be associated with the measured output $y = Cx^{(0\sim n-1)}$ instead of the state variable $x^{(0\sim n-1)}$. In fact, control problems of systems for the general case of $B(x^{(0\sim n-1)}, t)$ is one of our future research directions. In addition, by combining the proposed method with the design approaches in [44], [45] and [47], it may also be possible to further extend the control method to disturbed systems case, uncertain systems case, and the case of systems with unknown parameters, etc. These may also become future research directions.

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