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RESEARCH ARTICLE

Differential Game Analysis of Shared Manufacturing Platform Pricing Considering Cooperative Advertising Under Government Subsidies

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ABSTRACT Shared manufacturing is a new sustainable manufacturing model formed under the rapid development of the sharing economy. This paper studies the shared manufacturing model composed of one manufacturer and one shared manufacturing platform under the condition of government subsidies based on differential game. The pricing of the shared manufacturing platform, the effectiveness and profitability of collaboration advertising with the manufacturer are investigated. The best decision-making for the centralized, the decentralized, and the bilateral cost-sharing contract model are discussed and the solutions of differential game are analyzed. The bilateral cost-sharing contract is improved and analyzed that the improved profit level reaches the concentration level. The numerical examples by using Matlab are analyzed, and management implications are provided. According to the findings, the centralized decision-making leads to lower prices for shared manufacturing platform than the decentralized decision-making but higher advertising effort and profit levels. Although the amount of the decentralized decision-making, advertising effort, and profit have increased due to the introduction of the bilateral cost-sharing contract, the overall profits have yet to reach the level of centralized decision-making. This paper provides reference for government subsidy policies and shared manufacturing cooperation models.

INDEX TERMS Shared manufacturing, pricing, cooperative advertising, differential game, government subsidies.

I. INTRODUCTION

Manufacturing companies are embracing the Internet more quickly and using the shared manufacturing platform to implement shared manufacturing processes. Reasonable pricing of shared manufacturing platform is the key to achieving a win-win shared manufacturing process. Advertising for products is becoming increasingly important as businesses strive to grow their market share and revenue. Companies and shared manufacturing platforms employ the effect of advertising to draw customers and improve product sales. A realistic cooperative advertising strategy is essential to reaching a win-win situation between the shared manufacturing platform and other players. To fully mobilize the enthusiasm of members engaging in shared manufacturing and increase the total profit of shared manufacturing, the government has also adopted a subsidy measure to support the rapid development of this new shared manufacturing model.

Differential game can solve problems where multiple decision-makers make decisions in continuous time to pursue their respective goals, and these decisions will affect each other. It has the advantages of handling complex systems, making dynamic decisions, and considering changes in external factors.

This paper integrates government subsidies, shared manufacturing platform pricing, and advertising models into the analysis framework by using differential game. This paper

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studies the shared manufacturing model composed of one manufacturer and one shared manufacturing platform under the condition of government subsidies based on differential game. The pricing of the shared manufacturing platform, the effectiveness and profitability of collaboration advertising with the manufacturer are investigated.

The rest of this paper is organized as follows. Section II is literature review. Section III introduces the problem description and assumptions. Section IV proposes a cooperation strategy under government subsidies. A comparative analysis is given in Section V. A numerical example is given in Section VI. Section VII improves the bilateral cost-sharing contract model. Finally, Section VIII provides conclusions.

II. LITERATURE REVIEW

A new production model, shared manufacturing, is created as the ''sharing economy'' overgrew. Since 1990, when Ellen [1] initially put up the idea of ''shared manufacturing'' for the growth of small businesses, the academic field has made some progresses. Richard et al. [2] extended the concept of shared manufacturing to evaluate their exposure to large enterprises that are geographically distant. He et al. [3] analyzed the basic framework of shared manufacturing according to the current situation in China, pointed out the current development problems of shared manufacturing, and proposed more efficient methods. Rozman et al. [4] employed a cross-chain solution that scales the integration of blockchain technology into the concept of shared manufacturing. Yan et al. [5] supplemented the research progress on the operation management of the shared manufacturing platform, focusing on the analysis of ''order-capacity selection'' and ''order-capacity mutual selection'' problems. Zhang et al. [6] investigated the quality information disclosure strategies and incentive mechanism in the supply chain based on the third-party shared manufacturing platform. Chen and Tang [7] proposed the efficiency measurement of high-quality development of sharing manufacturing in China based on three-stage DEA-malmquist method. Li and Jiang [8] proposed enhanced self-organizing agents (ESAs) for shared factories to rematch idle resources in shared manufacturing. Wang et al. [9] proposed a digital dual-drive model to maintain the decision-making autonomy of resource providers in shared manufacturing. Zhang et al. [10] proposed a monitoring and maintenance method for idle resources in shared manufacturing using sensor perception and proved its effectiveness. Ji et al. [11] established a resource scheduling model that includes sustainable manufacturing in a shared manufacturing environment. Wei and Wu [12] studied the problem of two two-machine hybrid assembly workshops with fixed processing sequences to solve the scheduling problem of a series of selected processing sequences on a shared manufacturing platform. Liu and Liu [13] proposed a model based on the description of the dynamic allocation of tasks and resources in the shared manufacturing environment. Chen et al. [14] constructed a multi-belonging model with two shared manufacturing cloud platforms, conducted balanced analyses of four different situations, and compared and found the impact of different supply and demand structures on decision-making. The literature mentioned above investigates the shared manufacturing, however, they do not consider the pricing of the manufacturing platform.

In recent years, researches on pricing have begun to emerge in supply chains and capacity sharing. Zhu et al. [15] used the two-sided market theory to study and compare the pricing strategies of different charging models of cloud platforms under monopoly condition. Li et al. [16] proposed a dual-channel supply chain model based on cloud platforms and studied the optimal pricing strategies of two different cloud platforms. Zhao et al. [17] compared and analyzed the decision-making situations of profit maximization from the perspectives of society and the production capacity platform and found that there were decisions that were superior to both. Ye and Zhou [18] studied a dual-channel supply chain model that considered goodwill and emission reduction and introduced a cost-sharing contract to improve Pareto. Li et al. [19] studied the dual-channel supply chain model under the promotion method and coupon provision mode. Zhao and Chen [20] studied the pricing strategy of capacity-sharing platforms when the demand side of manufacturing capacity was sensitive to the delivery time of the supply side. When suppliers shared inventory in the supply chain, Martínez-de-Albéniz et al. [21] investigated the optimal decision-making and driving roles of platform pricing and the market driver. Zhao and Feng [22] explored the pricing strategies of the capacity-sharing platform under the two modes of registration fee and transaction fee in the monopoly manufacturing industry. Wang and Zhou [23] had different pricing strategies affected by platform discounts under the dual-channel supply chain.

The existing research ignores a critical issue in shared manufacturing platforms—advertising. Guo and Ma [24] used nonlinear dynamic system theory to establish a cooperative advertising model to find the optimal solution and analyze each parameter's influence on enterprises' cooperative advertising decision-making. Huo and Wu [25] examined the decisions of two competing retailers to invest in advertising under a dual-channel supply chain, considering goodwill. Wang et al. [26] studied the cooperative advertising model of the dual-channel supply chain under different models using the differential game model. Cozzolino et al. [27] investigated three types of advertising at various stages in the context of current digital platform challenges. Kennedy et al. [28] created a three-tier supply chain model with higher market recognition and found that it required less advertising than the first two tiers. Chen et al. [29] analyzed an advertising problem based on opportunity theory by using the dynamic programming method and given the optimal strategy. Cao et al. [30] studied the emergency handling of advertising effectiveness under different decisions in the environment of a closed-loop supply chain. Chen et al. [31] reviewed the most effective advertising decisions of the four cooperative advertising models in the supply chain environment consisting of one manufacturer and one retailer. Liu et al. [32] studied the impact of delay on advertising decision-making, considering marginal profit fluctuations and price competition. The above models that considered advertising did not involve shared manufacturing.

Although government subsidies are the primary means for the government to encourage enterprises to achieve high-quality development, government subsidies have different effects on different aspects of other enterprises. Wu et al. [33] studied the impact of equity concentration on the development of government green innovation subsidies. After analyzing the sample, Li et al. [34] concluded that government subsidies play a positive role in firm innovation and can offset some of the constraints caused by financial constraints. Yao et al. [35] established a model of six different subsidy methods, and the results showed that government subsidies have a relatively good incentive effect on green technology innovation and recycling services in the ecommerce supply chain when remanufacturers dominate them. Abhijit et al. [36] developed a duopoly-structured green supply model, which studied the optimal decision-making under different government subsidies and tax policies. He et al. [37] found that manufacturers can be encouraged to adopt an ideal channel structure by setting appropriate subsidy levels. Liu et al. [38] studied the decision-making results of different forms of government subsidies and the impact of government subsidy rates on decision-making results in the agricultural supply chain environment. Yu et al. [39] believed that government subsidies positively impact the problem of enterprise overcapacity, and this effect was more evident in large-scale enterprises. Wang and Chang [40] introduced the shareholding cooperation strategy into the supply chain model and found that it had a certain substitution effect on the government subsidy strategies.

In conclusion, scholars have become interested in the topic of shared manufacturing, and the concept of shared manufacturing has gained widespread acceptance. The concept's influence and significance for manufacturing have also been validated in theory and practice. The researchers have proposed various aspects and models and studied the pricing results of supply chain platforms. It is to improve platform efficiency and profitability by analyzing these results. These studies provide valuable references for shared manufacturing platform pricing and decision-making. Despite this progress, a critical research gap remains in shared manufacturing, where shared manufacturing platforms collaborate with manufacturers. Previous research has primarily focused on the analysis of platforms, overlooking the complexity of the multi-stakeholder environment and the regulatory role of the government involved. An in-depth exploration of shared manufacturing participants' collaborative governance is a necessary condition for formulating an overall solution to effectively develop shared manufacturing. The relationship between government subsidies, cooperative advertising behavior and shared manufacturing platform decision-making has received little attention.

III. PROBLEM DESCRIPTION AND MODEL ASSUMPTIONS

This paper examines the pricing strategy of the shared manufacturing platform under various scenarios, using the shared manufacturing platform as the research object. Since the shared manufacturing platform lacks the capacity to produce goods, it serves the interests of other shared manufacturing participants by controlling the demand for idle manufacturing resources and varying supply. The shared manufacturing platform earns the difference and aims for the maximum profit. With the enhancement of the sharing concept and cooperation awareness, it is particularly important for the shared manufacturing platform to complete the cooperative advertising process with other participants in the shared manufacturing process so as to gain greater market popularity and improve its competitiveness.

The shared manufacturing implementation process pulled by the shared manufacturing platform can be divided into three parts, as shown in Figure 1. The shared manufacturing platform converts the demand of the resource demander into a shared manufacturing order to provide to the resource supplier; The resource supplier collects the resource cost from the shared manufacturing platform, and the shared manufacturing platform charges the shared manufacturing platform pricing from the resource demander. In this way, the transformation of idle resources is completed through the shared manufacturing platform. At the same time, this paper considers the resource supply side, that is, the manufacturer, and the shared manufacturing platform to invest in advertising idle resources. Advertising effectiveness will stimulate resource demand.

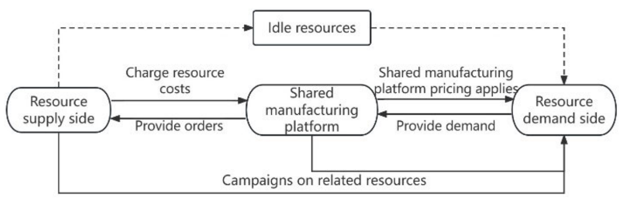


FIGURE 1. Shared manufacturing implementation process.

In the process of sharing manufacturing implementation, the advertising enthusiasm of shared manufacturing participants is affected by the natural decay rate, and the close connection between participants is low. This paper adopts the method of combining internal cost sharing among shared manufacturing members with shared external incentives for manufacturing. That is, the paper will carry out the bilateral cost-sharing contract at the same time as the government's advertising effort subsidy. By using a differential game, this paper studies the shared manufacturing model composed of one manufacturer and one shared manufacturing platform under the condition of government subsidies. Because both the manufacturer and the shared manufacturing platform participate in relevant resource advertising campaigns, both centralized and decentralized differential game models are developed. The parameters are shown in Table 1.

In the model of a shared manufacturing environment consisting of one manufacturer and one shared manufacturing

TABLE 1. Parameters and descriptions.

Parameters	Quantity
ρ	Discount factor.
α	Basic resource requirements.
β	The impact coefficient of shared manufacturing platform on demand.
η	The sensitivity coefficient of the demand function to market favorability.
δ	The natural decay rate of market favorability.
ω	The cost price of idle resources.
$P(t)$	Pricing of share manufacturing platform at time t.
$E_M(t)$、$E_C(t)$	The advertising effort levels of manufacturer and shared manufacturing platform at time t, respectively.
L_M、L_C	The advertising effort coefficients of manufacturer and shared manufacturing platform, respectively.
μ_M、μ_C	The sensitivity coefficients of market favorability to the advertising effort level of manufacturer and shared manufacturing platform, respectively.
$W(t)$	Market favorability of relevant resources at time t.
W_0	The initial value of the W(t) parameter
$\theta_M(t)$	The proportion of manufacturer advertising cost subsidies shared by the manufacturing platform at time t.
$\theta_C(t)$	The proportion of advertising cost subsidy for the manufacturing platform manufacturer sharing system at time t.
$\varepsilon_M(t), \varepsilon_C(t)$	The subsidy coefficients for advertising costs of manufacturer and shared manufacturing platform by the government at time t, respectively.
$\tau(t)$	The revenue sharing ratio under the improved bilateral cost-sharing contract model at time t.
J_T、J_M、J_C	The long-term profit functions of the shared manufacturing entity, the manufacturer, and the shared manufacturing platform, respectively.

platform, the shared manufacturing platform is paid to the manufacturer first. That is the acquisition cost of idle resource suppliers and then selling resources to resource demanders according to the pricing of the shared manufacturing platform. The specific decision-making order is as follows: First, the manufacturer invests in $E_M(t)$ for manufacturer advertising on resources, then the shared manufacturing platform sets the shared manufacturing platform pricing p and the shared manufacturing platform advertising investment $E_C(t)$, and finally, it determines the mutual cost subsidy ratios $\theta_M(t)$ and $\theta_C(t)$ between the manufacturer and the shared manufacturing platform.

Assumption 1: Based on the shared manufacturing model of one manufacturer and one shared manufacturing platform, to improve their competitiveness, the manufacturer and shared manufacturing platform invest in their brand image and self-awareness to enhance market favorability and increase market demand. According to Chen and Zhang [41], this paper sets up a manufacturer and a shared manufacturing platform to have advertising effort levels $E_M(t)$ and $E_C(t)$, respectively, and characterizes the impact of advertising effort levels on product market favorability as follows:

$$\dot{W}(t) = \mu_M E_M(t) + \mu_C E_C(t) - \delta W(t) \tag{1}$$

where δ denotes the natural decay rate, which decays over time; μ_M and μ_C are constants greater than 0, which are the sensitivity coefficients of market favorability to the advertising effort levels of manufacturers and shared manufacturing platforms, respectively. $W(t)$ indicates the market favorability of the resource at the time of t.

Assumption 2: In addition to the level of advertising effort considered in this article, market demand is also mainly affected by price [18]. So the market demand can be expressed as $D(t) = (W(t), h(t))$. $h(t)$ is the effect of price $p(t)$ on market demand, and let $h(t)$ be a linear function of relevant t. The market demand function is given as

$$D(W, p) = (\alpha - \beta p)\eta W(t) \tag{2}$$

where η is the sensitivity coefficient representing the demand function to market favorability, and the price $p(t) \in [0,\alpha/\beta]$ to ensure $h(p) > 0$.

Assumption 3: The cost of advertising input is related to the level of advertising effort, and there is a marginal increase between advertising effort and advertising cost. Therefore, this paper sets advertising cost as a convex function of advertising effort as follows:

$$\begin{cases} C_M(t) = \dfrac{L_M}{2} E_M^2(t) \\ C_C(t) = \dfrac{L_C}{2} E_C^2(t) \end{cases} \tag{3}$$

where $C_M(t)$, $C_C(t)$ is the cost of advertising efforts to share the manufacturing platform between manufacturers and retailers, respectively; L_M and L_C are the advertising effort coefficients of manufacturers and shared manufacturing platforms, respectively. Similarly, it assumes that advertising investment will not change the cost of managing the unit product. Therefore, the manufacturer's production cost and management cost per unit product are treated as constants, and for convenience, they are simplified to 0.

Assumption 4: The idle resource cost price ω is an exogenous variable, indicating the intensity of the manufacturer's competition, and the sluggish resource cost price is inversely proportional to the power of the manufacturer's competition. The manufacturer's advertising effort $E_M(t)$, the advertising effort of the shared manufacturing platform $E_C(t)$, and the shared manufacturing platform pricing $p(t)$ are the control variables.

$$J_M = \max \int_0^\infty e^{-\rho t}[\omega D(t) - (1 - \varepsilon_M)C_M(t)]dt \tag{4}$$

$$J_C = \max \int_0^\infty e^{-\rho t}[(p - \omega)D(t) - (1 - \varepsilon_C)C_C(t)]dt \tag{5}$$

$$J_T = \max \int_0^\infty e^{-\rho t}[pD(t) - (1 - \varepsilon_M)C_M(t) - (1 - \varepsilon_C)C_C(t)]dt \tag{6}$$

For the convenience of writing this article, the model part omits the t.

IV. COOPERATION STRATEGY UNDER GOVERNMENT SUBSIDIES

In the context of traditional cooperative advertising, considering the long-term collaborative profit problem of government subsidies, a centralized and decentralized differential game model and a bilateral cost-sharing contract model are established to solve the optimal pricing, optimal solution of market favorability, and optimal value of profit under different decisions.

A. OPTIMAL PRICING UNDER CENTRALIZED DECISION-MAKING

The centralized decision-making (denoted by F in the upper-right corner) targets the overall profit of both the manufacturer and the shared manufacturing platform in the model. Thus, the objective function of centralized decision-making can be given by (7) as follows:

$$J_T^F(W^F) = \max_{E_M^F \geq 0, E_C^F \geq 0} \int_0^\infty e^{-\rho t}[p^F(\alpha - \beta p)\eta W^F(t) - (1-\varepsilon_M)\frac{L_M}{2}(E_M^F)^2 - (1-\varepsilon_C)\frac{L_C}{2}(E_C^F)^2]dt \tag{7}$$

Theorem 1: The solutions under centralized decision-making is:

(1) The optimal equilibrium strategies for manufacturer and shared manufacturing platform are

$$\begin{cases} p^{F*} = \dfrac{\alpha}{2\beta} \\ E_M^{F*} = \dfrac{\mu_M \eta \alpha^2}{4\beta(1-\varepsilon_M)L_M(\rho+\delta)} \\ E_C^{F*} = \dfrac{\mu_C \eta \alpha^2}{4\beta(1-\varepsilon_C)L_C(\rho+\delta)} \end{cases} \tag{8}$$

(2) The optimal trajectory of market favorability is

$$W^{F*}(t) = W_\infty^{F*} + (W_0 - W_\infty^{F*})e^{-\delta t} \tag{9}$$

where $W_\infty^{F*} = \frac{\mu_M^2 \eta \alpha^2}{4\delta\beta(1-\varepsilon_M)L_M(\rho+\delta)} + \frac{\mu_C^2 \eta \alpha^2}{4\delta\beta(1-\varepsilon_C)L_C(\rho+\delta)}$ (3) The optimal value function of both parties' overall profit is

$$J_T^{F*} = e^{-\rho t}[\frac{\eta\alpha^2}{4\beta(\rho+\delta)}W^{F*}(t) + \frac{\mu_M^2\alpha^4\eta^2}{32\rho\beta^2(\rho+\delta)^2(1-\varepsilon_M)L_M} + \frac{\mu_C^2\alpha^4\eta^2}{32\rho\beta^2(\rho+\delta)^2(1-\varepsilon_C)L_C}] \tag{10}$$

Proof of Theorem 1: To obtain the solutions in this case, the reverse induction method uses (7), and the optimal function of the system on t is:

$$J_T^{F*}(W^F) = e^{-\rho t}P_T(W^F) \tag{11}$$

As a result, optimal control theory states that $P_T(W^F)$ satisfies the Hamilton-Jacobi-Bellman (HJB) equation for all $W^F \geq 0$, as follows:

$$\rho P_T(W^F) = \max_{E_M^F \geq 0, E_C^F \geq 0} [p(\alpha - \beta p)\eta W^F(t) - (1-\varepsilon_M)\frac{L_M}{2}(E_M^F)^2 - (1-\varepsilon_C)\frac{L_C}{2}(E_C^F)^2 + P_T'(W^F)(\mu_M E_M^F + \mu_C E_C^F - \delta W^F(t))] \tag{12}$$

Finding the first-order partial derivative for p, E_M^F and E_C^F in (12), respectively, and making them 0, the result is as follows:

$$\begin{cases} p^{F*} = \dfrac{\alpha}{2\beta} \\ E_M^{F*} = \dfrac{\mu_M P_T'(W^F)}{(1-\varepsilon_M)L_M} \\ E_C^{F*} = \dfrac{\mu_C P_T'(W^F)}{(1-\varepsilon_C)L_C} \end{cases} \tag{13}$$

Substituting (13) into (12) gets the formula as follows:

$$\rho P_T(W^F) = [\frac{\eta\alpha^2}{4\beta} - \delta P_T'(W^F)]W^F(t) + \frac{\mu_M^2 P_T'^2}{2(1-\varepsilon_M)L_M} + \frac{\mu_C^2 P_T'^2}{2(1-\varepsilon_C)L_C} \tag{14}$$

Observing (14), $P_T(W^F)$ is a linear function of W^F. Let its analytic formula be $P_T(W^F)=t_1 W^F + t_2$, where t_1 and t_2 are constants, and the expression t_1 is

$$P_T'(W^F) = \frac{\eta\alpha^2}{4\beta(\rho+\delta)} \tag{15}$$

Substituting $P_T(W^F)$ and $P_T'(W^F)$ into (14), t_1 and t_2 are obtained by sorting out and comparing the coefficients of the same term on both sides of the equation. Substituting t_1 into (13) can solve the solutions E_M^{F*} and E_C^{F*} of the manufacturer and the shared manufacturing platform under centralized decision-making, that is, (8). At this time, substituting E_M^{F*} and E_C^{F*} into (1) can solve the market favorability as $W^{F*}(t)$. Then substituting $W^{F*}(t)$ into $P_T(W^F)$ can get $P_T^*(W^F)$. Finally, the optimal profit function of the system as a whole under centralized decision-making (10) can be obtained.

B. OPTIMAL PRICING UNDER DECENTRALIZED DECISION-MAKING

Under decentralized decision-making (indicated by D in the upper-right corner), manufacturers and shared manufacturing platforms make decisions with their profit optimization goals. So the objective functions of manufacturers and shared manufacturing platforms under decentralized decision-making are:

$$J_M^D(W^D) = \max_{E_M^F \geq 0} \int_0^\infty e^{-\rho t}[\omega(\alpha - \beta p)\eta W^D(t) - (1-\varepsilon_M)\frac{L_M}{2}(E_M^D)^2]dt \tag{16}$$

$$J_C^D(W^D) = \max_{E_C^F \geq 0} \int_0^\infty e^{-\rho t}[(p^D - \omega)(\alpha - \beta p)\eta W^D(t) - (1-\varepsilon_C)\frac{L_C}{2}(E_C^D)^2]dt \tag{17}$$

Theorem 2: The solutions under decentralized decision-making is:

(1) The optimal equilibrium strategies for manufacturers and shared manufacturing platforms are:

$$\begin{cases} p^{D*} = \dfrac{\alpha+\beta\omega}{2\beta} \\ E_M^{D*} = \dfrac{\mu_M\eta(\alpha-\beta\omega)\omega}{2(1-\varepsilon_M)L_M(\rho+\delta)} \\ E_C^{D*} = \dfrac{\mu_C\eta(\alpha-\beta\omega)^2}{4\beta(1-\varepsilon_C)L_C(\rho+\delta)} \end{cases} \tag{18}$$

(2) The optimal trajectory of market favorability is

$$W^{D*}(t) = \frac{\mu_M^2\eta\omega(\alpha-\beta\omega)}{2\delta(1-\varepsilon_M)L_M(\rho+\delta)} + \frac{\mu_C^2\eta(\alpha-\beta\omega)^2}{4\beta\delta(1-\varepsilon_C)L_C(\rho+\delta)} + (W_0 - W_\infty^{D*})\mathrm{e}^{-\delta t} \tag{19}$$

where $W_\infty^{D*} = \frac{\mu_M^2\eta\omega(\alpha-\beta\omega)}{2\delta(1-\varepsilon_M)L_M(\rho+\delta)} + \frac{\mu_C^2\eta(\alpha-\beta\omega)^2}{4\beta\delta(1-\varepsilon_C)L_C(\rho+\delta)}$ (3) The optimal value functions for the overall profit of both parties are

$$J_M^{D*} = e^{-\rho t}[\frac{(\alpha-\beta\omega)\omega\eta}{2(\rho+\delta)}W^{D*}(t) + \frac{\mu_M^2\eta^2(\alpha-\beta\omega)^2\omega^2}{8\rho(1-\varepsilon_M)L_M(\rho+\delta)^2} + \frac{\mu_C^2\eta^2(\alpha-\beta\omega)^3\omega}{8\rho\beta(1-\varepsilon_C)L_C(\rho+\delta)^2}] \tag{20}$$

$$J_C^{D*} = e^{-\rho t}[\frac{(\alpha-\beta\omega)^2\eta}{4\beta(\rho+\delta)}W^{D*}(t) + \frac{\mu_C^2\eta^2(\alpha-\beta\omega)^4}{32\rho\beta^2(1-\varepsilon_C)L_C(\rho+\delta)^2} + \frac{\mu_M^2\eta^2(\alpha-\beta\omega)^3\omega}{8\rho\beta(1-\varepsilon_M)L_M(\rho+\delta)^2}] \tag{21}$$

Proof of Theorem 2: Combined with (16) and (17), the optimal control theory shows that the maximum profit function of the manufacturer and the shared manufacturing platform satisfy the HJB equation:

$$\rho P_M^D(W^D) = \max_{E_M^F \geq 0}[\omega(\alpha-\beta p)\eta W^D(t) - (1-\varepsilon_M)\frac{L_M}{2}(E_M^D)^2 + P_M^{D'}(W^D)(\mu_M E_M^D + \mu_C E_C^D - \delta W^D(t))] \tag{22}$$

$$\rho P_C^D(W^D) = \max_{E_C^F \geq 0}[(p^D-\omega)(\alpha-\beta p)\eta W^D(t) - (1-\varepsilon_C)\frac{L_C}{2}(E_C^D)^2 + P_C^{D'}(W^D)(\mu_M E_M^D + \mu_C E_C^D - \delta W^D(t))] \tag{23}$$

From the first-order conditions, the advertising effort level of the manufacturer and the shared manufacturing platform are as follows:

$$\begin{cases} p^{D*} = \dfrac{\alpha+\beta\omega}{2\beta} \\ E_M^{D*} = \dfrac{\mu_M P_M^{D'}}{(1-\varepsilon_M)L_M} \\ E_C^{D*} = \dfrac{\mu_C P_C^{D'}}{(1-\varepsilon_C)L_C} \end{cases} \tag{24}$$

Substituting (24) into (22) and (23) yields the following formula:

$$\rho P_M^D(W^D) = [\frac{(\alpha-\beta\omega)\omega\eta}{2} - \delta P_M^{D'}]W^D(t) + \frac{\mu_M^2 P_M^{D'2}}{2(1-\varepsilon_M)L_M} + \frac{\mu_C^2 P_M^{D'} P_C^{D'}}{(1-\varepsilon_C)L_C} \tag{25}$$

$$\rho P_C^D(W^D) = [\frac{(\alpha-\beta\omega)^2\eta}{4\beta} - \delta P_C^{D'}]W^D(t) + \frac{\mu_C^2 P_C^{D'2}}{2(1-\varepsilon_C)L_C} + \frac{\mu_C^2 P_M^{D'} P_C^{D'}}{(1-\varepsilon_M)L_M} \tag{26}$$

Observing (25) and (26), $P_M(W^D)$ and $P_C(W^D)$ are linear functions of W^D. Let their analytic formulas be $P_M(W^D){=}m_1^{D*}W^D{+}m_2^{D*}$ and $P_C(W^D){=}c_1^{D*}W^D{+}c_2^{D*}$, where m_1, m_2, c_1 and c_2 are constants, namely $P_M^{D'}(W^D) = m_1^{D*}$ and $P_C^{D'}(W^D) = c_1^{D*}$. $P_M^{D'}(W^D)$ and $P_C^{D'}(W^D)$ can be expressed as

$$\begin{cases} P_M^{D'}(W^D) = \dfrac{\eta(\alpha-\beta\omega)\omega}{2(\rho+\delta)} \\ P_C^{D'}(W^D) = \dfrac{\eta(\alpha-\beta\omega)^2}{4\beta(\rho+\delta)} \end{cases} \tag{27}$$

Substituting $P_M^D(W^D), P_M^{D'}(W^D), P_C^D(W^D)$, and $P_C^{D'}(W^D)$ into (25) and (26), m_1^{D*}, m_2^{D*}, c_1^{D*} and c_2^{D*} are obtained by sorting out and comparing the coefficients of the homogeneous terms on both sides of the equation. Substituting them into (24), the solutions E_M^{D*} and E_C^{D*} of the manufacturer and the shared manufacturing platform under decentralized decision-making, that is, (18) can be obtained.

At this time, substituting E_M^{D*} and E_C^{D*} into (1) can solve the market favorability as $W^{D*}(t)$. Then substituting $W^{D*}(t)$ into $P_M^D(W^D), P_C^D(W^D)$ can get $P_M^{D*}(W^D)$ and $P_C^{D*}(W^D)$. Finally, the optimal profit functions (20) and (21) of the system as a whole under decentralized decision-making can be obtained.

C. OPTIMAL PRICING UNDER BILATERAL COST-SHAREING CONTRACT

This section designs an improved cooperative advertising cost-sharing contract. There are two ways to improve the collaborative advertising enthusiasm of the manufacturer and the shared manufacturing platform. In addition to adopting government subsidy incentives, it also adopts mutual incentives between members within shared manufacturing. That is, the manufacturer accepts the shared manufacturing platform to share the ratio θ_M of the advertising effort cost, and the shared manufacturing platform carries the manufacturer to share the ratio θ_C of the advertising effort cost, where $0\leq \theta_M \leq 1$ and $0\leq \theta_C \leq 1$.

After the introduction of the contract, the decision-making goals of the manufacturer and the shared manufacturing platform are as follows:

$$J_M^B(W^B) = \max_{E_M^B \geq 0, E_C^B \geq 0} \int_0^\infty e^{-\rho t}[\omega(\alpha - \beta p)\eta W^B(t) - (1 - \varepsilon_M - \theta_M^B)\frac{L_M}{2}(E_M^B)^2 - \theta_C^B \frac{L_C}{2}(E_C^B)^2]dt \tag{28}$$

$$J_C^B(W^B) = \max_{E_M^B \geq 0, E_C^B \geq 0} \int_0^\infty e^{-\rho t}[(p^B - \omega)(\alpha - \beta p)\eta W^B(t) - (1 - \varepsilon_C - \theta_C^B)\frac{L_C}{2}(E_C^B)^2 - \theta_M^B \frac{L_M}{2}(E_M^B)^2]dt \tag{29}$$

Theorem 3: The solutions under the decision-making of bilateral cost-sharing contracts is:

(1) The pricing of the shared manufacturing platform, the optimal effort level by the manufacturer and the shared manufacturing platform, the proportion of the optimal advertising effort cost shared by the shared manufacturing platform for the manufacturer and the proportion of the optimal advertising effort cost shared by the manufacturer for the shared manufacturing platform are as follows:

$$\begin{cases} p^{B*} = \dfrac{\alpha + \beta\omega}{2\beta} \\ E_M^{B*} = \dfrac{\mu_M \eta(\alpha - \beta\omega)\omega}{2(1 - \varepsilon_M - \theta_M^B)L_M(\rho + \delta)} \\ E_C^{B*} = \dfrac{\mu_C \eta(\alpha - \beta\omega)^2}{4\beta(1 - \varepsilon_C - \theta_C^B)L_C(\rho + \delta)} \\ \theta_M^{B*} = 1 - \varepsilon_M^B - \dfrac{2\beta\omega(1 - \varepsilon_M)(\alpha - \beta\omega)}{\alpha^2} \\ \theta_C^{B*} = 1 - \varepsilon_C^B - \dfrac{(1 - \varepsilon_C)(\alpha - \beta\omega)^2}{\alpha^2} \end{cases} \tag{30}$$

(2) The optimal trajectory of market favorability is as follows:

$$W^{B*}(t) = \frac{\mu_M^2 \eta\omega(\alpha - \beta\omega)}{2\delta(1 - \varepsilon_M - \theta_M^B)L_M(\rho + \delta)} + \frac{\mu_C^2 \eta(\alpha - \beta\omega)^2}{4\beta\delta(1 - \varepsilon_C - \theta_C^B)L_C(\rho + \delta)} + (W_0 - W_\infty^{B*})\mathrm{e}^{-\delta \mathrm{t}} \tag{31}$$

where $W_\infty^{B*} = \frac{\mu_M^2 \eta\omega(\alpha-\beta\omega)}{2\delta(1-\varepsilon_M-\theta_M^B)L_M(\rho+\delta)} + \frac{\mu_C^2 \eta(\alpha-\beta\omega)^2}{4\beta\delta(1-\varepsilon_C-\theta_C^B)L_C(\rho+\delta)}$

(3) After the introduction of bilateral cost-sharing contracts, the optimal profit functions of manufacturers and shared manufacturing platforms are as follows:

$$J_M^{B*} = e^{-\rho t}[\frac{(\alpha - \beta\omega)\omega\eta}{2(\rho + \delta)}W^{B*}(t) + \frac{\mu_M^2 \eta^2(\alpha - \beta\omega)^2\omega^2}{8\rho(1 - \theta_M^B - \varepsilon_M)L_M(\rho + \delta)^2} - \frac{\theta_C^B \mu_C^2 \eta^2(\alpha - \beta\omega)^4}{32\beta^2\rho(1 - \theta_C^B - \varepsilon_C)^2 L_C(\rho + \delta)^2} + \frac{\mu_C^2 \eta^2(\alpha - \beta\omega)^3\omega}{8\rho\beta(1 - \theta_C^B - \varepsilon_C)L_C(\rho + \delta)^2}] \tag{32}$$

$$J_C^{B*} = e^{-\rho t}[\frac{(\alpha - \beta\omega)^2\eta}{4\beta(\rho + \delta)}W^{B*}(t) + \frac{\mu_C^2 \eta^2(\alpha - \beta\omega)^4}{32\rho\beta^2(1 - \theta_C^B - \varepsilon_C)L_C(\rho + \delta)^2} - \frac{\theta_M^B \mu_M^2 \eta^2(\alpha - \beta\omega)^2\omega^2}{8\rho\beta(1 - \theta_M^B - \varepsilon_M)L_M(\rho + \delta)^2} + \frac{\mu_M^2 \eta^2(\alpha - \beta\omega)^3\omega}{8\rho\beta(1 - \theta_M^B - \varepsilon_M)L_M(\rho + \delta)^2}] \tag{33}$$

Proof of Theorem 3. The HJB equation for the maximum profit function of the manufacturer and the shared manufacturing platform is as follows:

$$\rho P_M^B = \max[\omega(\alpha - \beta\omega)\eta W^B(t) - (1 - \varepsilon_M - \theta_M^B)\frac{L_M}{2}(E_M^B)^2 - \theta_C^B \frac{L_C}{2}(E_M^B)^2 + P_M^{B'}(\mu_M E_M^B + \mu_C E_C^B - \delta W^B(t))] \tag{34}$$

$$\rho P_C^B = \max[(p - \omega)(\alpha - \beta\omega)\eta W^B(t) - (1 - \varepsilon_C - \theta_C^B)\frac{L_C}{2}(E_C^B)^2 - \theta_M^B \frac{L_M}{2}(E_M^B)^2 + P_C^{B'}(\mu_M E_M^B + \mu_C E_C^B - \delta W^B(t))] \tag{35}$$

Finding the first-order partial derivation for E_S and E_M in (34) and (35), respectively, and making them 0, p^B, E_M^B, and E_C^B can be expressed as

$$\begin{cases} p^B = \dfrac{\alpha + \beta\omega}{2\beta} \\ E_M^B = \dfrac{\mu_M P_M^{B'}}{(1 - \varepsilon_M - \theta_M^B)L_M} \\ E_C^B = \dfrac{\mu_C P_C^{B'}}{(1 - \varepsilon_C - \theta_C^B)L_C} \end{cases} \tag{36}$$

Substituting (36) into (34) and (35), the HJB equation can be expressed as

$$\rho P_M^{B*} = [\frac{(\alpha - \beta\omega)\omega\eta}{2} - \delta P_M^{B'}]W^B(t) + \frac{\mu_M^2 (P_M^{B'})^2}{2(1 - \varepsilon_M - \theta_M^B)L_M} - \frac{\theta_C^B \mu_C^2 P_C^{B\prime 2}}{2(1 - \varepsilon_C - \theta_C^B)^2 L_C} + \frac{\mu_C^2 P_M^{B\prime} P_C^{B\prime 2}}{(1 - \varepsilon_C - \theta_C^B)L_C} \tag{37}$$

$$\rho P_C^{B*} = [\frac{(\alpha - \beta\omega)^2\eta}{4\beta} - \delta P_C^{B'}]W^B(t) + \frac{\mu_C^2 (P_C^{B'})^2}{2(1 - \varepsilon_C - \theta_C^B)L_C} - \frac{\theta_M^B \mu_M^2 P_M^{\prime 2}}{2(1 - \varepsilon_M - \theta_M^B)^2 L_M} + \frac{\mu_M^2 P_M^{B\prime} P_C^{\prime 2}}{(1 - \varepsilon_M - \theta_M^B)L_M} \tag{38}$$

Observing (37) and (38), $P_M(W^B)$ and $P_C(W^B)$ are linear functions of W^B. Let their analytic formulas be $P_M(W^B)$=$m_1 W^B$+m_2 and $P_C(W^B)$=$c_1 W^B$+c_2, where they are constants, namely $P_M^{B'}(W^B) = \mathrm{m}_1$, $P_C^{B'}(W^B) = \mathrm{c}_1$.

$$\begin{cases} P_M^{B'}(W^D) = \dfrac{\eta(\alpha-\beta\omega)\omega}{2(\rho+\delta)} \\ P_C^{B'}(W^D) = \dfrac{\eta(\alpha-\beta\omega)^2}{4\beta(\rho+\delta)} \end{cases} \tag{39}$$

When the decision-making model after the introduction of bilateral cost-sharing contracts is equal to the optimal decision-making of shared manufacturing in the centralized situation, the overall coordination of shared manufacturing can be realized. Only then would the introduction of bilateral cost-sharing models be relevant. In the model, that is, a and b are established at the same time, from which the following equation can be obtained.

$$\begin{cases} \theta_M^{B*} = 1-\varepsilon_M - \dfrac{(1-\varepsilon_M)P_M^{B'}}{P_T'} \\ \theta_C^{B*} = 1-\varepsilon_C - \dfrac{(1-\varepsilon_C)P_C^{B'}}{P_T'} \end{cases} \tag{40}$$

Substituting $P_M(W^B), P_M'(W^B), P_C(W^B)$ and $P_C'(W^B)$ into (37) and (38), $\mathrm{m}_1^{B*}, \mathrm{m}_2^{B*}, c_1^{B*}$ and c_2^{B*} are obtained by sorting out and comparing the coefficients of the same term on both sides of equation. Substituting them into (36) and (40), the solutions E_M^{B*} and E_C^{B*} of the manufacturer and the shared manufacturing platform under bilateral cost-sharing contract can be solved, that is, (30).

At this time, substituting E_M^{B*} and E_C^{B*} into the (1) can solve the market favorability as $W^{B*}(t)$. Then substituting $W^{B*}(t)$ into $P_M^B(W^B)$ and $P_C^B(W^B)$ can get $P_M^{B*}(W^B)$ and $P_C^{B*}(W^B)$. Finally, the optimal profit function (32) and (33) of the system as a whole under the bilateral cost-sharing contract can be obtained.

V. COMPARATIVE ANALYSIS

The three cases in section III are compared, and their relevance is explored.

Corollary 1: (1) The pricing comparison, market favorability, and profit comparison of manufacturers and shared manufacturing platforms under different decisions are as follows:

$$p^{F*} < p^{D*} = p^{B*}, W^{F*}(t) > W^{B*}(t) > W^{D*}(t), J_M^{F*} > J_M^{B*} > J_M^{D*}, J_C^{F*} > J_C^{B*} > J_C^{D*}$$

(2) The influences of the range of advertising cost subsidy coefficients between manufacturers and shared manufacturing platforms on the level of advertising effort are as follows:

When $0 < \theta_M^B < 1-\varepsilon_M - \frac{2\beta\omega(1-\varepsilon_M)(\alpha-\beta\omega)}{\alpha^2}$, $E_M^{F*} > E_M^{B*} > E_M^{D*}$;

When $\theta_M^B = 1-\varepsilon_M - \frac{2\beta\omega(1-\varepsilon_M)(\alpha-\beta\omega)}{\alpha^2}$, $E_M^{F*} = E_M^{B*} > E_M^{D*}$;

When $1-\varepsilon_M - \frac{2\beta\omega(1-\varepsilon_M)(\alpha-\beta\omega)}{\alpha^2} < \theta_M^B < 1-\varepsilon_M$, $E_M^{B*} > E_M^{F*} > E_M^{D*}$.

Similarly, when $0 < \theta_C^B < 1-\varepsilon_C - \frac{(1-\varepsilon_C)(\alpha-\beta\omega)^2}{\alpha^2}$, $E_C^{F*} > E_C^{B*} > E_C^{D*}$;

When $E_C^{F*} > E_C^{B*} > E_C^{D*}$, $0 < \theta_C^B < 1-\varepsilon_C - \frac{(1-\varepsilon_C)(\alpha-\beta\omega)^2}{\alpha^2}$;

When $E_C^{B*} > E_C^{F*} > E_C^{D*}$, $1-\varepsilon_C - \frac{(1-\varepsilon_C)(\alpha-\beta\omega)^2}{\alpha^2} < \theta_C^B < 1-\varepsilon_C$.

Proof of Corollary 1.

$$E_M^{F*} - E_M^{B*} = \frac{\mu_M\eta}{2\beta L_M(\rho+\delta)(1-\varepsilon_M)(1-\varepsilon_M-\theta_M^B)} \cdot [\alpha^2(1-\varepsilon_M-\theta_M^B) - 2\beta\omega(\alpha-\beta\omega)(1-\varepsilon_M)],$$

when $[\alpha^2(1-\varepsilon_M-\theta_M^B) - 2\beta\omega(\alpha-\beta\omega)(1-\varepsilon_M)] > 0$, $E_M^{F*} - E_M^{B*} > 0$;

when $[\alpha^2(1-\varepsilon_M-\theta_M^B) - 2\beta\omega(\alpha-\beta\omega)(1-\varepsilon_M)] < 0$, $E_M^{F*} - E_M^{B*} < 0$.

Similarly, the impact of the value range on the comparison of advertising efforts on shared manufacturing platforms can be found.

The comparison of *Corollary* 1 (1) and *Corollary* 1 (2) shows that the feedback of the optimal strategy under the three decisions is a time-independent parameter. That is the optimal feedback of different decisions changing at other times, which is of practical significance in line with the actual situation. At the same time, using differential game to verify the close relationship between three different decisions from a dynamic perspective. Under centralized decision-making, in addition to the fact that the pricing of the shared manufacturing platform is lower than the pricing under the decentralized decision, the advertising effort, market favorability, and profit solutions of the manufacturer and the shared manufacturing platform under the centralized decision are higher than the solutions under the decentralized decision. The introduction of bilateral cost-sharing contracts improves the advertising effort, market favorability, and profit solutions of the manufacturer and the shared manufacturing platform under the decentralized decision. However, the profits of the two have yet to reach the level of centralized decision-making.

Corollary 2: Under the three decisions, the advertising effort level and profit of manufacturers and shared manufacturing platforms positively correlate with the respective government subsidy rate. The optimal trajectory of market favorability for different decisions is also positively correlated with the government subsidy rate.

Proof of Corollary 2. Taking centralized decision-making as an example, find the first-order partial derivative as follows:

$$\frac{dE_M^{F*}}{d\varepsilon_M} = \frac{\mu_M\eta\alpha^2}{4\beta(1-\varepsilon_M)^2 L_M(\rho+\delta)} > 0$$
$$\frac{dJ^{F*}}{d\varepsilon_M} = \frac{e^{-\rho t}\mu_M^2\alpha^4\eta^2}{32\rho\beta^2(\rho+\delta)^2(1-\varepsilon_M)^2 L_M} > 0$$

Similarly, the first-order bias of manufacturers on the advertising effort level and profit of the shared manufacturing platform under the three decisions is greater than 0. Therefore, their separate subsidy rates have a positive correlation with the government's subsidy rate.

Corollary 2 shows that the advertising effort willingness of manufacturers and shared manufacturing platforms is not only affected by the advertising effort coefficient, the sensitivity coefficient of market favorability to effort, and the sensitivity coefficient of the demand function to market favorability, but also affected by the government subsidy coefficient. Increasing the government subsidy rate can enhance their advertising efforts, thereby increasing market favorability, when the natural decline rate is large. The government's subsidy policy can effectively encourage the willingness of shared manufacturing participants to cooperate and promote the overall development of shared manufacturing by increasing the government subsidy rate while also improving the profits of manufacturers and shared manufacturing platforms.

Corollary 3: Given the decision to introduce bilateral cost-sharing contracts, the advertising cost-sharing ratio of manufacturers to shared manufacturing platforms is negatively correlated with the government's subsidy rate for shared manufacturing platforms. The proportion of advertising cost sharing between shared manufacturing platforms and manufacturers is negatively correlated with the subsidy rates of governments and manufacturers.

Proof of Corollary 3. Find θ_C^B for the first-order partial derivative of ε_M as follows:

$$\frac{d\theta_M^B}{d\varepsilon_M} = -1 + \frac{2\beta\omega(\alpha - \beta\omega)}{\alpha^2} = \frac{-(\alpha - \beta\omega)^2 - \beta^2\omega^2}{\alpha^2} < 0$$

Similarly, $\frac{d\theta_C^B}{d\varepsilon_C} < 0$.

Corollary 3 shows that the proportion of internal mutual subsidies among shared manufacturing members is not only affected by the parameters that determine pricing but also by external subsidies, that is, the subsidy rate of the government to the shared manufacturing participants.

VI. NUMERICAL ANALYSIS

The impact of various parameters on the pricing and market acceptability of shared manufacturing platforms, the comparison of multiple decisions and the effects of government subsidies on the solutions are also covered in this article, which are simulated by using the MATLAB program. Our simulation settings are extracted from Mustard Network, a shared manufacturing platform, which includes 1081 orders and 23 providers. Referring to Ye and Zhou [18], the following parameter values of the simulation are set: $\alpha= 8;\beta= 2;\rho= 0.3;\ E_M= 0.4;\ E_C= 0.4;\eta= 0.75;\delta= 0.2;\ L_M=1;\ L_C= 1;W_0= 10;\ t=1;\omega= 2;\tau=0.4;\mu_M= 0.5;\ \mu_C= 0.5;\theta= 0.2$.

A. FACTORS INFLUENCING THE PRICING OF THE SHARED MANUFACTURING PLATFORM

The effects of the parameters α, β and ω on the pricing of the shared manufacturing platform are shown in Figures 2(a)-(c), respectively. As can be seen in the previous section, the introduction of bilateral cost-sharing contracts did not change the pricing of shared manufacturing platforms under decentralized decision-making. Therefore, this section considers only the change in the pricing of the shared manufacturing platform under centralized and decentralized decision-making.

The figures show that for centralized and decentralized decision-making, the price p is positively correlated with the fundamental demand for the parameter α resources and negatively correlated with the influence coefficient of the parameter β shared manufacturing platform on the market. In contrast, the parameter ω resource cost price is positively correlated with the pricing under decentralized decision-making and has no bearing on centralized decision-making. This is due to the fact that, under decentralized decision-making, both manufacturers and shared manufacturing platforms seek to maximize their profits. The higher the cost of shared resources, the more the price of the shared manufacturing platform must increase to maximize profits. In addition, under some circumstances, the pricing of shared manufacturing platforms under decentralized decision-making is higher than that of centralized decision-making.

B. COMPARISION OF DIFFERENT DECISION

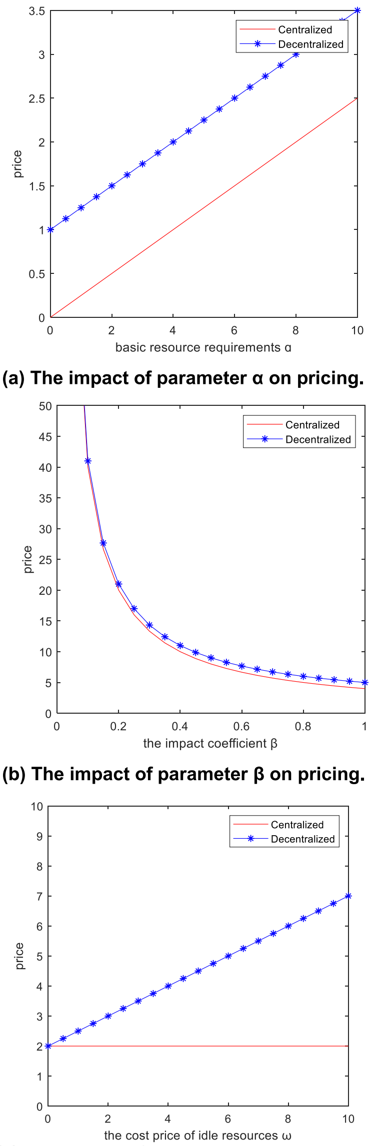
1) COMPARISON OF MARKET FAVORABILITY UNDER DIFFERENT DECISION

Figure 3 illustrates the optimal trajectory of market favorability under different decisions. It shows that the optimal trajectory of centralized decision-making at any time is higher than that of decentralized decision-making. The growth rate is faster in the early stages. The optimal course of market favorability under decentralized decision-making has improved with the introduction of bilateral cost subsidy contracts in terms of growth rate and stability. However, it still falls short of the level of market favorability under centralized decision-making. This demonstrates that while bilateral cost-sharing contract can enhance the combined effect of decentralized decision-making, they still need to be improved to achieve the desired result under centralized decision-making. Centralized decision-making can promote the collaborative advertising process between manufacturers and shared manufacturing platforms.

2) COMPARISON OF ADVERTISING EFFORTS UNDER DIFFERENT DECISIONS

Figures 4 and 5 depict the advertising efforts of manufacturers and shared manufacturing platforms under different decisions and their intersections.

It can be seen from Figure 4 that before the intersection (0.6,15), the advertising efforts of manufacturers under centralized decision-making are higher than those under decentralized decision-making and bilateral cost-sharing contracts, and after the intersection, the advertising efforts under bilateral cost-sharing contract decision-making exceed those under centralized decision-making. It can be seen from Figure 5 that the same situation occurs after the intersection point (0.72, 21.4286). It shows that centralized decision-making can stimulate manufacturers and shared manufacturing platforms to pay more attention to



(a) The impact of parameter α on pricing.

(b) The impact of parameter β on pricing.

(c) The impact of parameter ω on pricing.

FIGURE 2. **The impact of parameters α, β and ω on the pricing.**

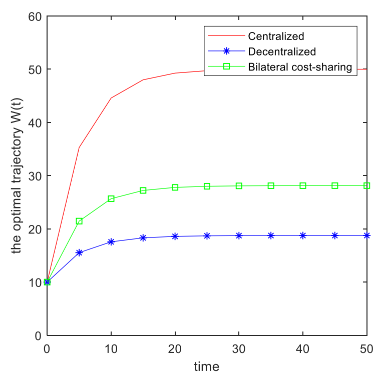


FIGURE 3. **The optimal trajectory of market favorability under different decisions.**

advertising efforts, while decentralized decision-making can improve the enthusiasm of shared manufacturing advertising

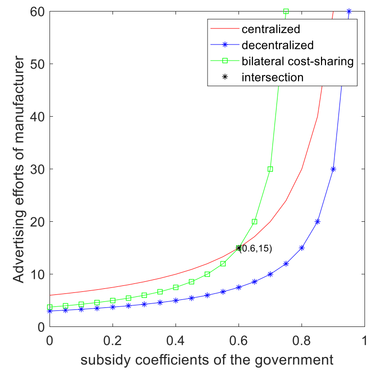


FIGURE 4. **Advertising efforts of manufacturer under different decisions.**

efforts after introducing bilateral cost sharing and even control the correlation coefficient to make advertising investment reach or be expected to reach a higher level than the level of advertising efforts under centralized decision-making.

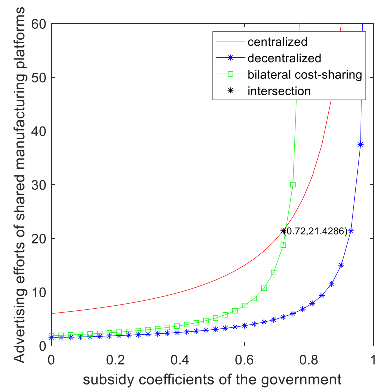


FIGURE 5. **Advertising efforts of the shared manufacturing platform under different decisions.**

3) COMPARISON OF PROFIT UNDER DIFFERENT DECISIONS

The overall profit of shared manufacturing under centralized and decentralized decision-making, as well as the profit of manufacturers and shared manufacturing platforms under decentralized decision-making and the inclusion of bilateral cost-sharing contract, are compared in Figures 6–8.

Figure 6 shows that the profit of centralized decision-making is always higher than that of decentralized decision-making, demonstrating that the efficiency of shared manufacturing is higher under centralized decision-making and that decentralized decision-making is not conducive to the overall advantages of shared manufacturing. Figures 7 and 8 show that the implementation of bilateral cost-sharing contract based on decentralized decision-making can enhance not only the respective profits of manufacturers and shared manufacturing platforms, but also the overall profits of shared manufacturing. However, the degree of improvement has not yet reached the general profit level of shared manufacturing under centralized decision-making.

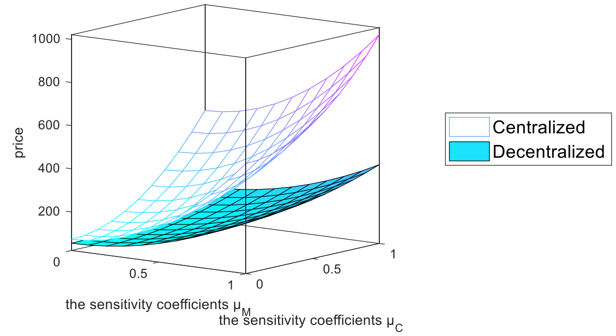


FIGURE 6. Overall profit of shared manufacturing under centralized and decentralized.

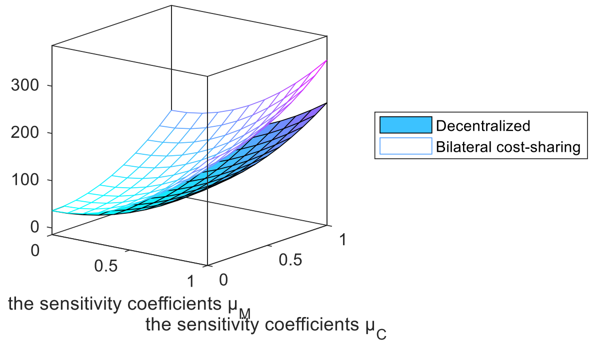


FIGURE 7. Manufacturers' profits under decentralized and bilateral cost-sharing contract.

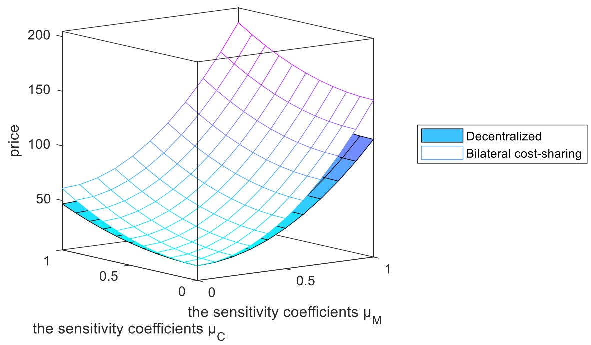


FIGURE 8. Shared manufacturing platform profits under decentralized and bilateral cost-sharing.

C. SENSITIVITY ANALYSIS OF MARKET FAVORABILITY

This section only chooses pertinent variables for sensitivity analysis. The pictures below show how these variables may affect market favorability.

The effects of advertising effort coefficient L_m and effort sensitivity coefficient μ_m on market favorability under various decisions are depicted in Figures 9 and 10. Figure 9 shows that, for the three decisions in this paper, market favorability at any time decreases as the cost coefficient of advertising effort increases, and the rate gets smaller and smaller. This is because the more extensive the parameter L_m, the higher the cost of advertising effort, which in turn reduces the enthusiasm for advertising investment and results in less market favorability. It demonstrates how shared manufacturing participants might enhance market favorability by advancing associated technology to lower advertising costs.

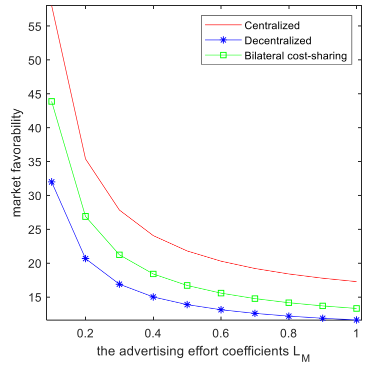


FIGURE 9. The influence of parameter L_M on market favorability.

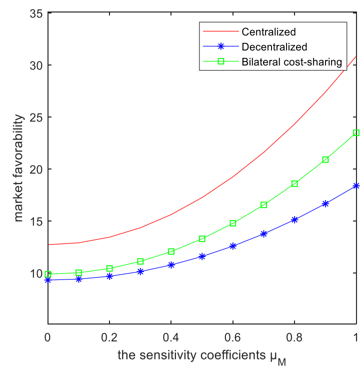


FIGURE 10. The influence of parameter μ_M on market favorability.

Similarly, Figure 10 shows that the market favorability for the three decisions in the figure increases and the rate increases as the coefficient m increases. It is also clear that the bilateral cost-sharing contract causes market favorability to grow at the rate of growth due to the influence of parameters. This is because the more sensitive the market is to advertising efforts, the more effective advertising is at converting market favorability. It demonstrates that the greater the impact of advertising efforts on market favorability, the greater the conversion rate between them. In addition to the fact that the coefficient is the same and other conditions remain the same, the introduction of bilateral cost-sharing contracts can further improve market favorability.

Setting various values under centralized decision-making, Figures 11 and 12 are compared to examine the impact of parameters η and δ on market favorability.

As seen from Figure 11, when the sensitivity coefficient of demand to market favorability is specific, the most favorable trajectory of market favorability increases over time. The rate becomes smaller and smaller and finally tends to level off. In other words, the cooperative advertising process of this shared manufacturing is controllable. In addition, at any exact moment, the slope corresponding to the optimal trajectory of market favorability increases with the increase of the coefficient η. This change shows that the greater the demand is affected by market favorability, the better the cooperative

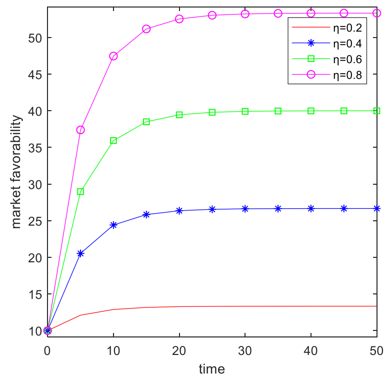


FIGURE 11. The influence of parameter η on market favorability.

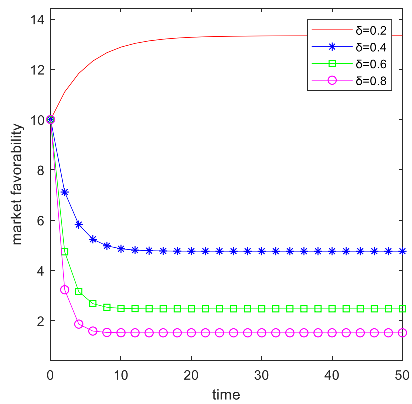


FIGURE 12. The influence of parameter δ on market favorability.

advertising effect between manufacturers participating in shared manufacturing and shared manufacturing platforms. Figure 12 shows that advertising efforts' impact on market favorability is shorter. The greater the natural decay rate, the larger the δ, and the smaller the product occupancy all at once. The less effective the cooperative advertising between the manufacturers participating in shared manufacturing and the shared manufacturing platform.

D. ANALYSIS OF THE IMPACT OF THE GOVERNMENT SUBSIDY RATE COEFFICIENT

Figures 13–15 are depicted to explore the effects of government subsidy rates on shared manufacturing. Figure 13 illustrates how market favorability is affected by the government subsidy rate ε_M in both centralized and decentralized decision-making scenarios. The effect of the government subsidy rate ε_M on manufacturers' earnings under centralized decision-making is depicted in Figure 14. Following bilateral cost-sharing contracts, Figure 15 illustrates the effect of government subsidy rates ($\varepsilon_M, \varepsilon_C$) on the sharing coefficients (θ_M, θ_C) of the manufacturer and shared manufacturing platform.

Figure 13 shows that for centralized and decentralized decision-making, the market favorability rises over time, and the growth rate decreases and tends to stabilize when the government subsidy coefficient is fixed. Market favorability rises concurrently with rising government subsidy rates, and

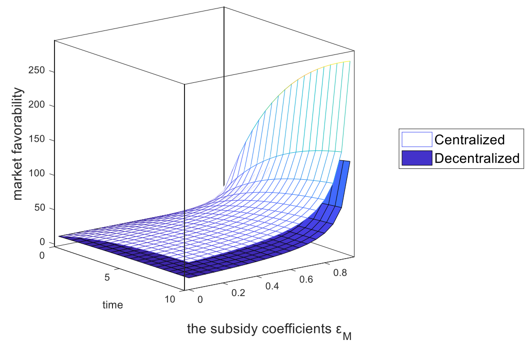


FIGURE 13. The effect of parameters t and ε_M on market favorability.

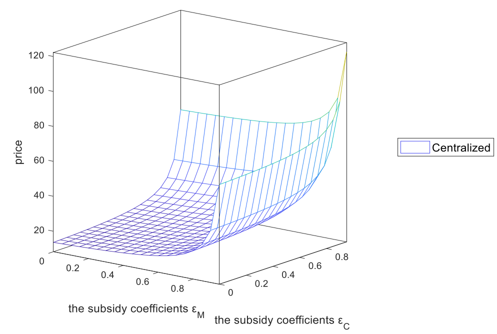


FIGURE 14. The effect of the parameters ε_M and ε_C on profit.

growth rates are rising. It demonstrates how boosting government subsidies might encourage spending on joint advertising and enhance collaborative outcomes. Figure 14 illustrates that, in centralized decision-making, the manufacturer's and the shared manufacturing platform's earnings rise in tandem with both an increase in government subsidies for each party and an increase in government subsidies for each party individually. It demonstrates how the government can either establish a win-win collaboration by boosting the subsidies of the manufacturer or by directly increasing subsidies to the shared manufacturing platform to increase its revenues.

According to Figure 15, the government subsidy coefficient is inversely linked with the cost-sharing coefficient under the bilateral cost-sharing contract, demonstrating that the government subsidy coefficient has some complementary effects on the cost-sharing coefficient.

VII. IMPROVED BILATERAL COST-SHARING CONTRACT MODEL

As the preceding section shows, adding bilateral subsidies does not cause joint manufacturing to reach its full potential. Although the overall profit of shared manufacturing is higher than that of decentralized decision-making, the bilateral cost-sharing contract model still needs to be improved if the overall profit is to match that of centralized decision-making. Additionally, the manufacturer and the shared manufacturing platform are not permitted to select the bilateral cost-sharing contract model if their profits under the decentralized decision under the upgraded bilateral cost-sharing contract are within those profits. To ensure that the improved model can be implemented and that manufacturer and shared

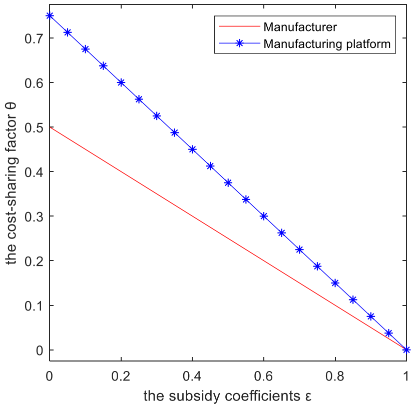


FIGURE 15. The effect of the parameter ε on the cost-sharing factor θ.

manufacturing platform profits are not lower than those under decentralized decision-making, the improved model must make sure that centralized decision-making is achieved on a general level and that shared manufacturing platforms and manufacturers enjoy higher profitability than under decentralized decision-making.

A. ANALYSIS OF THE IMPROVED MODEL

This paper designs an improved bilateral cost-sharing contract model where the manufacturer and the shared manufacturing platform share the idle resource transaction income *(p*Q)* in proportion to τ and $(1-\tau)$, improving the overall profit, which necessitates lowering the pricing of the shared manufacturing platform.

$$J_M^E = \max \int_0^\infty e^{-\rho t}[\tau p^E(\alpha - \beta p^E)\eta W^E(t) - (1-\varepsilon_M - \theta_M^E)\frac{L_M}{2}(E_M^E)^2 - \theta_C^E \frac{L_C}{2}(E_C^E)^2]dt \tag{41}$$

$$J_C^E = \max \int_0^\infty e^{-\rho t}[(1-\tau)p^E(\alpha - \beta p^E)\eta W^E(t) - (1-\varepsilon_C - \theta_C^E)\frac{L_C}{2}(E_C^E)^2 - \theta_M^E \frac{L_M}{2}(E_M^E)^2]dt \tag{42}$$

According to optimal control theory, the following can be obtained according to the HJB equation:

$$\begin{cases} p^{E*} = \dfrac{\alpha}{2\beta} \\ E_M^{E*} = \dfrac{\tau \mu_M \alpha^2 \eta}{4\beta(1-\varepsilon_M - \theta_M^E)L_M(\rho+\delta)} \\ E_C^{E*} = \dfrac{(1-\tau)\mu_C \alpha^2 \eta}{4\beta(1-\varepsilon_C - \theta_C^E)L_C(\rho+\delta)} \end{cases} \tag{43}$$

To ensure that the improved bilateral cost-sharing contract achieves a centralized decision-making level, the model needs to guarantee that $E_M^{F*} = E_M^{E*}, E_C^{F*} = E_C^{E*}$. The model needs to ensure that the expressions for calculating θ_M^E and θ_C^E are

$$\begin{cases} \theta_M^E = (1-\tau)(1-\varepsilon_M) \\ \theta_C^E = \tau(1-\varepsilon_C) \end{cases} \tag{44}$$

After substituting the formula into the objective function, the accurate function expression is as follows:

$$\begin{cases} J_M^{E*} = \tau J^{F*} \\ J_C^{E*} = (1-\tau)J^{F*} \\ J_M^{E*} + J_C^{E*} = J^{F*} \end{cases} \tag{45}$$

As a result, the overall profit of the improved bilateral cost-sharing model has reached the level of centralized decision-making. It can be seen from (45) that the essence of profit sharing is that the manufacturer negotiates with the shared manufacturing platform to determine a revenue mechanism to reasonably divide the total profit, and the segmentation coefficient is τ.

According to the above analysis, it can be seen that in order to ensure the smooth implementation of the improved model, it needs to be satisfied.

$$\begin{cases} J_M^{E*} - J_M^{D*} \geq 0 \\ J_C^{E*} - J_C^{D*} \geq 0 \end{cases} \tag{46}$$

Substituting the equation into the range of values that can be found τ, $\tau \in [\tau_{\min}, \tau_{\max}]$,where

$$\tau_{\min} = T_1/T_3, \tau_{\max} = T_2/T_3$$

$$T_1 = \frac{(\alpha-\beta\omega)\omega}{2} \cdot A \cdot W^{D*} + \frac{(\alpha-\beta\omega)^2\omega^2}{8} \cdot B + \frac{(\alpha-\beta\omega)^3\omega}{8} \cdot C$$

$$T_2 = \frac{\alpha^2 \cdot W^{E*} - (\alpha-\beta\omega)^2 \cdot W^{D*}}{4\beta} \cdot A + (\frac{\alpha^4}{32\beta^2} - \frac{(\alpha-\beta\omega)^3\omega}{8\beta}) \cdot B + \frac{\alpha^4 - (\alpha-\beta\omega)^4}{32\beta} \cdot C$$

$$T_3 = \frac{\alpha^2}{4\beta} \cdot A \cdot W^{E*} + \frac{\alpha^4}{32\beta^2} \cdot B + \frac{\alpha^4}{32\beta} \cdot C$$

$$A = \frac{\eta}{\rho+\delta}, B = \frac{\mu_M^2\eta^2}{\rho(1-\varepsilon_M)L_M(\rho+\delta)^2},$$

$$C = \frac{\mu_C^2\eta^2}{\rho\beta(1-\varepsilon_C)L_C(\rho+\delta)^2}$$

B. ANALYSIS OF THE IMPROVED EFFECT

The effect of the improved bilateral cost-sharing contract model is further analyzed using MATLAB software, and $\tau = 0.5$ (that is, the manufacturer and the shared manufacturing platform share the benefits equally) and the other assignments remain unchanged. The data is obtained from Mustard Network, a shared manufacturing platform, which includes 1081 orders and 23 providers. The solutions of the differential game for the three cases of concentration, dispersion, and the improved contract in this chapter are compared, as shown in Table 2.

As seen from Table 2, the level of advertising effort, market favorability, and overall profit of shared manufacturing platforms of the improved bilateral cost-sharing contract manufacturers and shared manufacturing platforms have reached the level of centralized decision-making. It demonstrated that the improvement of the expected incentive advertising effort and profit level has been achieved; At the same time, the respective profits of manufacturers and shared manufacturing platforms are higher than the profit level under decentralized decision-making, indicating that manufacturers and shared manufacturing models will choose the improved contract model out of their own interests, which ensures the smooth realization of the improvement contract.

TABLE 2. Comparison of solutions of differential games under different decisions.

Symbol	Centralized decision-making	Decentralized decision-making	Bilateral cost-sharing	Improved bilateral cost-sharing
Manufacturer's advertising efforts	10.0000	5.0000	10.0000	10.0000
Share advertising efforts on manufacturing platforms	10.0000	2.5000	10.0000	10.0000
Market favorability	17.2508	11.5861	17.2508	17.2508
Manufacturer profits	-	88.5401	137.4403	150.7599
Shared manufacturing platform profits	-	48.9002	137.4403	150.7599
Shared overall manufacturing profits	301.5199	137.4403	274.8806	301.5199

VIII. CONCLUSION AND MANAGERIAL IMPLICATIONS

In practice, the cooperative advertising game relationship between the shared manufacturing platform and other participants in the shared manufacturing platform is long-standing. And the performance of advertising investment decreases over time. Therefore, it is more realistic to study decision-making behaviors such as shared manufacturing platform pricing under the dynamic game model. In this work, government subsidies are also considered, the introduction of bilateral cost subsidy contracts is explored under the scenario of a continuous time dynamic game, and bilateral cost subsidy decisions are improved. The results show that: (1) The pricing p of the shared manufacturing platform is positively correlated with the primary demand for resources, negatively correlated with the influence coefficient of the shared manufacturing platform on demand, and is also related to the cost price of resources under decentralized decision-making; (2) the shared manufacturing platform has the lowest pricing under centralized decision-making, and the level and effect of cooperative advertising efforts and the overall efficiency of shared manufacturing are the highest; (3) after the introduction of bilateral cost sharing, the pricing of the shared manufacturing platform remains unchanged, and the efficiency and profit of cooperative advertising are improved, but the overall profit is still lower than the solutions under centralized decision-making; (4) the improved bilateral cost subsidy contract makes the pricing of the shared manufacturing platform equal to the centralized one; the level and effect of cooperative advertising efforts and the overall profit have reached the centralized level; and the improved contract is essentially a reasonable division of the overall profit of shared manufacturing on the basis of the bilateral cost subsidy before the improvement.

Through the analysis of the model results, we got some important managerial implications as follows:(1) Under the background of shared economy, managers should transform their ''single-handed'' strategy into cooperation with the upstream and downstream firms so as to improve shared manufacturing efficiency and maximize profits. (2) Centralized decision-making or the introduction of bilateral cost-sharing contracts can improve the profitability of manufacturers and shared manufacturing platforms. Shared manufacturing members can adopt similar contracts to improve their performance. (3) In the process of shared manufacturing cooperation, while pursuing to improve the overall profit, the reasonable distribution of profits among the members of the shared manufacturing should also be ensured. (4) The government can take measures to promote the development of shared manufacturing. For example, the government can increase the incentive of manufacturers and shared manufacturing platforms by increasing the government subsidy rate.

This paper's limitation is that it ignores the scenario in which multiple shared manufacturing platforms serve multiple manufacturers to carry out shared manufacturing simultaneously, as well as the effect of idle resources traded by other channels on shared manufacturing platform transactions. The research of government subsidy on shared manufacturing considering taxation or other government policy is the next stage of future work. The research of some potential external disturbances during model evaluation and deduction, such as the entry and exit of shared manufacturing resources, is also the next stage of our work.

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