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RESEARCH ARTICLE

Phase Retrieval-Based *Z* Parameter Estimation Method for Multiple-Input Multiple-Output Wireless Power Transfer Systems

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ABSTRACT Multiple-input multiple-output (MIMO) wireless power transfer (WPT) systems present various advantages to users, including improved power transfer efficiency, adjustable transmission power, and reduced magnetic field leakage. An essential aspect of controlling MIMO WPT systems is the estimation of Z parameters, which characterize the behavior of the linear electrical network in these systems. In this study, we propose a method to estimate all elements of the Z parameters without requiring synchronization between the transmitters and receivers. Instead, the elements are estimated based on the measured complex amplitudes of voltage and current at the transmitters, as well as the direct current (DC) at the output of the full-bridge rectifiers on the receivers. Importantly, all these measurements can be obtained at a minimal cost. Further, circuit simulations and experiments are also conducted to evaluate the performance of the proposed method. Specifically, a 2×2 MIMO WPT system is employed for the circuit simulations and experiments, and the Z parameters are estimated under various receiver position conditions. The evaluation of the simulation and experimental results is based on the power transmission efficiency of the system, considering the estimated Z parameters. The simulation and experimental results demonstrate that the difference between the power transfer efficiency based on the estimated Z parameters and the theoretical maximum efficiency based on the true Z parameters is found to fall within the range of 0.06 % and 0.4 %, respectively.

INDEX TERMS Inductive power transfer, multiple-input multiple-output (MIMO), phase retrieval, parameter estimation, wireless power transfer, Z parameters.

I. INTRODUCTION

Wireless power transfer (WPT) utilizing magnetic coupling has emerged as a highly efficient and safe method for charging batteries, leading to its increasing adoption in electronic devices and replacing conventional power cables. The proliferation of electronic devices employing WPT has resulted in a growing demand for simultaneous charging of multiple devices. To address this challenge [1], [2], [3], multiple-input multiple-output (MIMO) WPT systems have been extensively studied as a potential solution, offering improved power transfer efficiency [4], [5], adjustable power transmission to each receiver [6], [7], and reduced magnetic field leakage [8], [9] by leveraging control over both transmitters and receivers.

Effective control of MIMO WPT systems relies on accurate estimation of the Z parameters, which characterize the behavior of the linear electrical network in these systems. Various methods have been proposed for estimating the Z parameters in MIMO WPT systems. Several methods involve calculating the Z parameters based on Neumann's formula by using the information regarding the receivers'

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positions [2], [10]. However, obtaining precise receiver positions can be costly, and implementing a system to detect these positions adds to the overall expense. Moreover, these methods are only applicable in ideal environments where there are no objects around the coils, as mutual couplings are easily influenced by surrounding objects.

Alternatively, another method estimates the Z parameters by measuring the alternating current (AC) voltages and AC currents at the transmitters and receivers [11], [12]. However, this approach necessitates complex time synchronization between all transmitters and receivers, as well as the measurement of AC voltages and currents at the receivers. These requirements increase the implementation cost of WPT systems.

A method proposed by Zhou et.al. offers the estimation of certain Z parameters by utilizing multiple measurements of AC voltages and currents at the transmitters, as well as coil resistances at the receivers [13]. This method eliminates the need for complex time synchronization between transmitters and receivers, thereby reducing implementation costs. However, it disregards couplings among receivers, rendering it unsuitable for MIMO WPT systems where receivers are in close proximity or couplings among receivers are significant. Furthermore, this method is only applicable when the transmitters and receivers are completely compensated, owing to the derivation of the Z parameters. However, achieving complete compensation can be challenging as the coil inductance is subject to variations in the surrounding environment and degradation over time. These disadvantages limit the use of MIMO WPT systems and reduce the charging efficiency of WPT.

In this study, we propose a method to estimate all elements of the Z parameters without the need for time synchronization or complete compensation at the transmitters and receivers, overcoming the shortcomings of existing methods. These advantages allow users to place multiple receivers at any point on a transmitter array without any AC measurements or implementation of an automatic impedance matching network on the receiver side.

Our approach involves recovering the complex amplitudes of the currents at the receivers through a nonlinear simultaneous equation, using the AC voltages and currents measured at the transmitters, along with the DC currents at the rectifier output. By assuming that the transmitters are connected to a controller, the AC voltages and currents can be measured at a low cost without wireless synchronization. Additionally, the DC currents at the rectifier output can be obtained inexpensively by dividing the measured DC voltages by the load resistances.

We transform the recovering problem of the receiver currents to a phase retrieval problem [14] because efficient solutions have been developed in the field of signal processing [15], [16], [17], [18], and they can be leveraged to solve the problem for the recovery. The foundation of our method is based on the findings of our previous conference paper [19]. However, the paper only presents the results of numerical simulations without considering electric noise, assuming the use of linear components in the circuit. Furthermore, determining the signs of mutual inductances between transmitters and receivers necessitated the use of existing methods such as one-bit feedback [13], [20], thereby increasing system complexity and implementation costs of the receivers. In this study, we extend this method by incorporating circuit simulations and experiments that account for nonlinear components, such as class-D inverters and full-bridge rectifiers. Additionally, we demonstrate that the Z parameters estimated using our proposed method can control the power flow of MIMO WPT systems without the need to determine the signs of mutual inductances between the transmitters and receivers.

The remainder of this paper is organized as follows. Section II presents the principle of the proposed method. This principle is verified through the evaluation results of circuit simulations, which are described in Section III. Section IV describes an experimental evaluation of the proposed method. Lastly, Section V presents a summary of the study.

II. PRINCIPLE

Phase retrieval involves estimating a vector, $\boldsymbol{x}$, using a model represented by the equation: $\boldsymbol{Ax} = \boldsymbol{b}$, where $\boldsymbol{A}$ denotes a known matrix representing the observation process and $\boldsymbol{b}$ represents a vector with elements that are known only by their absolute values. During phase retrieval, the phase information of the vector, $\boldsymbol{b}$, is missing. Thus, the solution for $\boldsymbol{x}$, which satisfies $|\boldsymbol{Ax}|^2 = |\boldsymbol{b}|^2$, where $|\ |^2$ denotes the square of the amplitude of each element, must be estimated.

A. CONDITIONS AND SCOPE OF APPLICATIONS

In this study, we estimate Z parameters by recovering the complex amplitudes of the voltages and currents at the receivers using a phase retrieval approach. This method utilizes multiple measurements of the DC voltages and currents at the rectifier output at the receivers, along with the complex amplitudes of the voltage and current at the transmitters. To transmit the measured DC voltage and currents from the receivers to the transmitters, load modulation is employed at the receiver without the need for complex synchronization between the transmitters and receivers. We assume that the off-diagonal components of the Z-parameter matrix are purely imaginary and that a part of the matrix at the transmitters, $\mathbf{Z_{TT}}$, is predetermined or measured beforehand. If there is a non-negligible real part in the off-diagonal part of the Z matrix, the estimation accuracy of the Z parameters by the proposed method deteriorates. However, when using high-Q coils, the real part of the off-diagonal part becomes very small. In practical WPT systems, the coils have a high Q factor to obtain high power transfer efficiency, so the presence of the real part of the off-diagonal part is practically not a problem. Additionally, these assumptions have been widely adopted in the existing

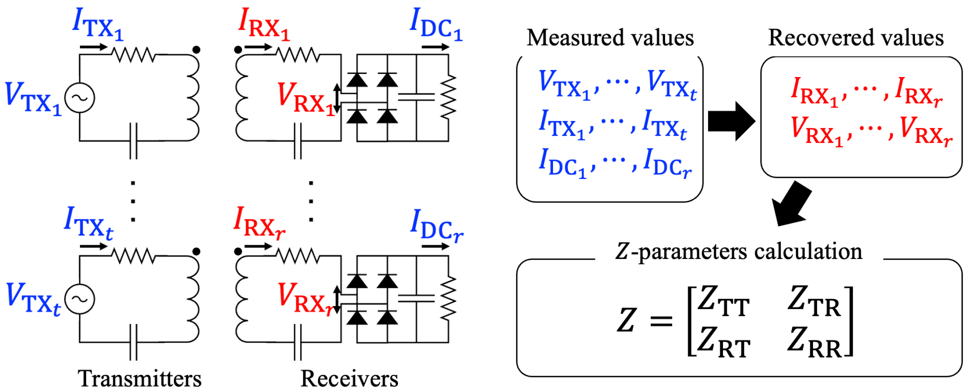


FIGURE 1. Circuit configuration of the MIMO WPT system used in this study (left) and flowchart of the Z-parameters estimation (right).

Z parameter estimation and control methods for MIMO WPT systems [20], [21], [22].

One limitation of our proposed estimation method is that the phase of the receiver current cannot be uniquely determined, and it can vary by 180°. This can be attributed to the use of the DC current from the rectifier output, which loses phase information. However, it is important to note that the input power to the rectifier remains unaffected regardless of the phase of the receiver current. Moreover, the difference in the 180° phase of the receiver current is equivalent to inverting the polarity of a port when calculating Z parameters. Therefore, despite this phase ambiguity, the proposed estimation method can still be applied for power flow control in MIMO WPT systems such as maximization of power transmission efficiency and adjustment of the power transmitted to each receiver.

Another limitation is that the number of transmitters must be greater than or equal to the number of receivers. This constraint is unique to the proposed method and is a mathematical constraint. Although the application is constrained when the number of transmitters is small, we believe that this is hardly a problem. This is because applications that use an array of transmitters generally use a large number of transmitters. In fact, in numerous studies using MIMO WPT systems, the number of transmitters is larger than the number of receivers in simulations and experiments. The main reason for this is that as the number of receivers increases, the power delivered to each receiver decreases due to the power limitations of each transmitter. In addition, when the operating frequency is near 100 kHz and the size of the transmitter and receiver are nearly equal, the receivers should be placed within the range of the transmitter array for high power transfer efficiency; however, it is impossible when the number of receivers is greater than the number of transmitters.

Considering the time required for multiple measurements of DC voltages and the calculation of Z parameters in the proposed method, the proposed method is suitable for charging static objects such as placed mobile devices or parked EVs.

B. Z-PARAMETERS ESTIMATION

Fig. 1 depicts the configuration of the MIMO WPT system used in this study along with a flowchart of the Z parameter estimation. Here, t denotes the number of transmitters and r represents the number of receivers. The circuit equation of the MIMO WPT system can be expressed as:

$$\begin{bmatrix} \boldsymbol{V_{TX}} \\ \boldsymbol{V_{RX}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z_{TT}} & \boldsymbol{Z_{TR}} \\ \boldsymbol{Z_{RT}} & \boldsymbol{Z_{RR}} \end{bmatrix} \begin{bmatrix} \boldsymbol{I_{TX}} \\ \boldsymbol{I_{RX}} \end{bmatrix}, \tag{1}$$

where $\boldsymbol{V_{TX}} \in C^t$ and $\boldsymbol{I_{TX}} \in C^t$ represent the complex voltage and current vectors of the transmitters, and $\boldsymbol{V_{RX}} \in C^r$ and $\boldsymbol{I_{RX}} \in C^r$ represent the complex voltage and current vectors of the receivers, respectively. In the proposed method, $\boldsymbol{V_{TX}}$ and $\boldsymbol{I_{TX}}$ are obtained from the measurements.

The amplitudes of $\boldsymbol{I_{RX}}$ can be derived from the DC current measured at the receiver rectifier output. At the n-th receiver, the correlation between the amplitude of the RF input current, I_{RX_n}, and the DC output current, I_{DC_n}, is expressed by $I_{RX_n} = \frac{\pi}{2} I_{DC_n}$, as shown in Fig. 2. The equations for I_{RX_n} and I_{DC_n} are derived as (2) [23].

$$I_{DC_n} = \frac{1}{T} \int_0^T |i_{RX_n}| dt = \frac{2}{\pi} I_{RX_n}, \tag{2}$$

where $i_{RX_n} = I_{RX_n} \sin(\omega t)$,

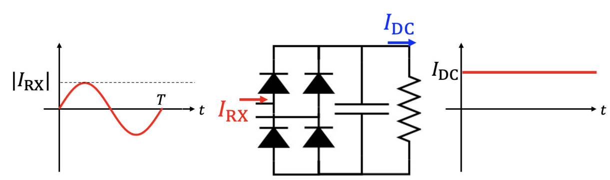


FIGURE 2. I_{RX} and I_{DC} at a full-bridge rectifier.

When the matrix of the equivalent resistance at the full-bridge rectifier on the receiver ($\boldsymbol{R_{RX}} \in C^{r \times r}$) is known, $\boldsymbol{V_{RX}}$ can be derived using $\boldsymbol{V_{RX}} = -\boldsymbol{R_{RX}} \boldsymbol{I_{RX}}$. When using full-bridge rectifiers, $\boldsymbol{R_{RX}}$ is approximately calculated as $\boldsymbol{R_{RX}} = \frac{8}{\pi^2} \boldsymbol{R_{Load}}$, where $\boldsymbol{R_{Load}}$ represents the matrix of the load resistance at the rectifier output. This approximation ignores the voltage drops in the diodes.

After $\boldsymbol{I_{RX}}$ is obtained, $\boldsymbol{V_{RX}}$ can also be obtained: Therefore, $\boldsymbol{I_{RX}}$ is first obtained using the proposed method. To apply the existing phase retrieval method to the estimation of $\boldsymbol{I_{RX}}$, the measured voltage and current information are expressed as $|\boldsymbol{Ax}|^2 = |\boldsymbol{b}|^2$ by using the circuit equation.

The correlations among $\boldsymbol{V_{TX}}$, $\boldsymbol{I_{TX}}$, and $\boldsymbol{I_{RX}}$ are expressed as follows:

$$\boldsymbol{V_{TX}} - \boldsymbol{Z_{TT}}\boldsymbol{I_{TX}} = \boldsymbol{Z_{TR}}\boldsymbol{I_{RX}}. \tag{3}$$

By multiplying both sides of (3) by $(\boldsymbol{Z_{RT}}\boldsymbol{Z_{TR}})^{-1}\boldsymbol{Z_{RT}}$, taking the transpose of both sides, and squaring the absolute value of each element on both sides, we obtain

$$\left| \left[\boldsymbol{V_{TX}}^T - \boldsymbol{I_{TX}}^T \boldsymbol{Z_{TT}}^T \right] \underbrace{\left[((\boldsymbol{Z_{RT}}\boldsymbol{Z_{TR}})^{-1}\boldsymbol{Z_{RT}})^T \right]}_{\boldsymbol{W}} \right|^2 = |\boldsymbol{I_{RX}}^T|^2, \tag{4}$$

where $\boldsymbol{W} \in C^{t \times r}$. This equation holds true when the number of transmitters (t) is greater than or equal to the number of receivers (r).

By substituting the data for $\boldsymbol{V_{TX}}$, $\boldsymbol{I_{TX}}$, and $|\boldsymbol{I_{RX}}|$ (which are measured multiple times), and $\boldsymbol{Z_{TT}}$ (which can be measured in advance), (4) can be expressed as

$$\left| \begin{bmatrix} (\boldsymbol{V_{TX}^1})^T - (\boldsymbol{I_{TX}^1})^T \boldsymbol{Z_{TT}}^T \\ (\boldsymbol{V_{TX}^2})^T - (\boldsymbol{I_{TX}^2})^T \boldsymbol{Z_{TT}}^T \\ (\boldsymbol{V_{TX}^3})^T - (\boldsymbol{I_{TX}^3})^T \boldsymbol{Z_{TT}}^T \\ \vdots \end{bmatrix} \boldsymbol{W} \right|^2 = \begin{bmatrix} |(\boldsymbol{I_{RX}^1})^T|^2 \\ |(\boldsymbol{I_{RX}^2})^T|^2 \\ |(\boldsymbol{I_{RX}^3})^T|^2 \\ \vdots \end{bmatrix}, \tag{5}$$

where $\boldsymbol{V_{TX}^i}$, $\boldsymbol{I_{TX}^i}$, and $|\boldsymbol{I_{RX}^i}|$ represent the values of the ith measurement.

Assuming that the total number of measurements is m, (5) can be expressed as follows:

$$|\boldsymbol{AW}|^2 = \left| \underbrace{\begin{bmatrix} b_{1,1} & \cdots & b_{1,r} \\ \vdots & \ddots & \vdots \\ b_{m,1} & \cdots & b_{m,r} \end{bmatrix}}_{\boldsymbol{B}} \right|^2, \tag{6}$$

where $\boldsymbol{A} \in C^{m \times t}$ and $\boldsymbol{B} \in C^{m \times r}$. Eq. (6) is a nonlinear simultaneous equation for deriving $\boldsymbol{W}$ and $\boldsymbol{B}$. We transform the equation into phase-retrieval problems to apply the existing efficient solutions.

Eq. (6) can be considered as r-times the phase retrieval problem, one of which is expressed as:

$$\left| \boldsymbol{A} \underbrace{\begin{bmatrix} x_{1,i} \\ \vdots \\ x_{t,i} \end{bmatrix}}_{\boldsymbol{x_i}} \right|^2 = \left| \underbrace{\begin{bmatrix} b_{1,i} \\ \vdots \\ b_{m,i} \end{bmatrix}}_{\boldsymbol{b_i}} \right|^2, \tag{7}$$

where $i = 1, \cdots, r$. These r-times phase retrieval problems can be regarded as a one-time phase retrieval problem by stacking all column vectors of $\boldsymbol{W}$ and $\boldsymbol{B}$ as

$$\left| \begin{bmatrix} \boldsymbol{A} & & 0 \\ & \ddots & \\ 0 & & \boldsymbol{A} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_1} \\ \vdots \\ \boldsymbol{x_r} \end{bmatrix} \right|^2 = \left| \begin{bmatrix} \boldsymbol{b_1} \\ \vdots \\ \boldsymbol{b_r} \end{bmatrix} \right|^2.$$

During phase retrieval, the relative phases between the elements of $\boldsymbol{x_i}$ can be obtained, whereas the absolute phases cannot be determined owing to the uncertainty in the phase of $\boldsymbol{b}$. In this study, the phase of $\boldsymbol{x_i}$ is narrowed based on the assumption that the off-diagonal components of the Z parameter matrix, $\boldsymbol{Z_{TR}}$ and $\boldsymbol{Z_{RT}}$, are purely imaginary. When $\boldsymbol{Z_{RT}}$ and $\boldsymbol{Z_{TR}}$ are purely imaginary, $\boldsymbol{W}$ is also purely imaginary. Subsequently, $x_{1,i}$ can be considered as a purely imaginary value, $x_{1,i} = \pm j|x'_{1,i}|$. The other elements of $\boldsymbol{x}$ are derived as $x_{k,i} = x_{1,i}x'_{k,i}/x'_{1,i}$, where $k = 2, 3, \cdots, t$. The sign of $x_{1,i}$ determines the sign of the mutual inductances between all the transmitters and the i-th receiver since it determines the phase of I_{RX_i}. As previously explained, the phase of the receiver current does not affect the power flow of the MIMO WPT systems. Therefore, all the signs of $\boldsymbol{x_i}$ can be set instantaneously, and $\boldsymbol{W}$ can be determined using $[\boldsymbol{x_1}, \cdots, \boldsymbol{x_r}]$. The calculation of $\boldsymbol{Z_{RT}}$ is performed as $\boldsymbol{Z_{RT}} = (\boldsymbol{W}^T\boldsymbol{W})^{-1}\boldsymbol{W}^T$ using the estimated $\boldsymbol{W}$.

Once $\boldsymbol{W}$ is estimated, $\boldsymbol{I_{RX}}$ can be derived using the equation, $\boldsymbol{AW} = \boldsymbol{B}$: The other unknown elements of the Z matrix, $\boldsymbol{Z_{TR}}$ and $\boldsymbol{Z_{RR}}$, are obtained as follows: First, $\boldsymbol{Z_{TR}}$ is obtained by transposing the derived $\boldsymbol{Z_{RT}}$. Subsequently, $\boldsymbol{Z_{RR}}$ is estimated as (8), which is based on (1) and the equation: $\boldsymbol{V_{RX}} = -\boldsymbol{R_{RX}}\boldsymbol{I_{RX}}$.

$$\boldsymbol{Z_{RR}} = -\boldsymbol{Z_{RT}} \left[\boldsymbol{I_{TX}^1} \cdots \boldsymbol{I_{TX}^r} \right] \left[\boldsymbol{I_{RX}^1} \cdots \boldsymbol{I_{RX}^r} \right]^{-1} - \boldsymbol{R_{RX}}. \tag{8}$$

To calculate $\boldsymbol{Z_{RR}}$, the number of measurements must be greater than or equal to the number of receivers, $m \geq r$, as (8) uses r times the measured transmitter and receiver currents. Additionally, during phase retrieval, the greater the number of measurements, the better is the estimation accuracy [15]. When the number of measurements is greater than the number of receivers, $m > r$, r time measurements must be selected to calculate $\boldsymbol{Z_{RR}}$. Therefore, $\boldsymbol{Z_{RR}}$ is calculated for all combinations of the r time measurements, and then, averaged $\boldsymbol{Z_{RR}}$ is calculated.

To account for the voltage drop at the diodes in the rectifier, the equivalent resistance of the rectifier, R_{Rec}, is calculated as (9), where I_s denotes the saturation current, N represents the emission coefficient, and V_{D} refers to the voltage drop at the diode.

$$V_{\mathrm{D}} = (\log_{10}\left(\frac{I_{\mathrm{DC}}}{I_s}\right) + 1)(N \times V_t),$$
$$R_{\mathrm{Rec}} = \frac{V_{\mathrm{D}}}{I_{\mathrm{DC}}} \times 2. \tag{9}$$

R_{Rec} varies in each simulation owing to the change in the output DC for each simulation condition. Therefore, the average DC current is used to derive R_{Rec}, and this value is subtracted from the calculated $\boldsymbol{Z_{\mathrm{RR}}}$.

Thus, Z matrix can be estimated using $\boldsymbol{Z_{\mathrm{TT}}}$, $\boldsymbol{Z_{\mathrm{RT}}}$, $\boldsymbol{Z_{\mathrm{TR}}}$ and $\boldsymbol{Z_{\mathrm{RR}}}$:

$$Z = \begin{bmatrix} \boldsymbol{Z_{\mathrm{TT}}} & \boldsymbol{Z_{\mathrm{TR}}} \\ \boldsymbol{Z_{\mathrm{RT}}} & \boldsymbol{Z_{\mathrm{RR}}} \end{bmatrix}. \tag{10}$$

C. PROGRAM FOR SOLVING PHASE RETRIEVAL PROBLEMS

In this paper, we employ the "PhaseLift" method [15], which is a widely used and efficient approach for phase retrieval. Although the phase retrieval problem is not convex, PhaseLift approximates the problem as a convex problem by relaxing the constraints. To implement the proposed Z-parameter estimation method, we developed a program in Python using the CVXPY library [24]. In PhaseLift, when a signal is noiseless, the phase-retrieval problem is reduced to:

$$\begin{aligned} &\underset{\boldsymbol{X}}{\text{minimize}} \quad \mathrm{Tr}(\boldsymbol{X}) \\ &\text{subject to} \quad \mathcal{A}_i\boldsymbol{X} = |b_i|^2 \;\; \forall i, \;\; \boldsymbol{X} \succeq 0, \end{aligned} \tag{11}$$

where $\boldsymbol{X} = xx^H$, $\mathcal{A}_i = a_i a_i^H$, and a_i denotes the ith row of $\boldsymbol{A}$. When the measured signal contains noise, the problem is reduced to:

$$\begin{aligned} &\underset{\boldsymbol{X}}{\text{minimize}} \quad \lambda\mathrm{Tr}(\boldsymbol{X}) + \frac{1}{2m}\sum_{i=1}^{m} ||\mathcal{A}_i\boldsymbol{X} - |b_i|^2||^2 \\ &\text{subject to} \quad \boldsymbol{X} \succeq 0, \end{aligned} \tag{12}$$

where $\lambda > 0$ denotes the regularization parameter. We used (12) in the circuit simulations and experiments because the simulations and experiments contain electrical noise generated from nonlinear components such as class-D inverters and full-bridge rectifiers.

III. CIRCUIT SIMULATIONS

To evaluate the proposed method in a circuit with non-linear elements, the circuit simulations were performed using the LTspice (Analog Devices) circuit simulator. The estimated parameters were evaluated based on the power transfer efficiency when inputting the optimal currents at the transmitters. The optimal input currents were derived to maximize the power transfer efficiency under constant load impedances at the receivers, based on the assumption of a system with estimated Z parameters. The optimal input currents were derived based on the method proposed by [25].

A. SIMULATION SETTINGS

In circuit simulations, the 2×2 MIMO WPT system depicted in Fig. 3 is evaluated by changing the positions of the receivers. The operating frequency is set to 100 kHz and the positions of the receivers are selected from the two points marked in red in Fig. 5. The combinations of all the positions are: {(0, 0"), (2, 0"), (2, 1"), (2, 2"), (3, 0"), (3, 1"), (3, 2"), (3, 3"), (4, 2")}. The center of the receiver is set at the point of the marker. The proposed estimation method is evaluated by using power transfer efficiency (PTE). The PTE is defined as the RF output power divided by the RF input power.

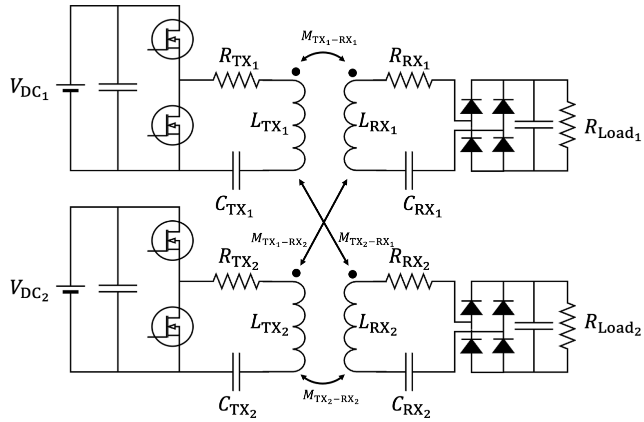


FIGURE 3. Circuit diagram of the 2×2 MIMO WPT system.

TABLE 1. Parameters in circuit simulations.

Parameters	
$C_{\mathrm{TX}_1}, C_{\mathrm{TX}_2}, C_{\mathrm{RX}_1}, C_{\mathrm{RX}_2}$	47.0 [nF]
$L_{\mathrm{TX}_1}, L_{\mathrm{TX}_2}, L_{\mathrm{RX}_1}, L_{\mathrm{RX}_2}$	53.3 [μH]
$R_{\mathrm{TX}_1}, R_{\mathrm{TX}_2}, R_{\mathrm{RX}_1}, R_{\mathrm{RX}_2}$	0.70 [Ω]
$R_{\mathrm{Load}_1}, R_{\mathrm{Load}_2}$	10 [Ω]
$V_{\mathrm{DC}_1}, V_{\mathrm{DC}_2}$	18 [V]
MOSFET	EPC2007C, EPC
Diode	RSX205L-30, Rohm

Table 1 lists the circuit parameters of the transmitters and receivers. GaN MOSFET models (EPC2007C, EPC) are used for the class-D inverters in the transmitters. Schottky diode models (RSX205L-30, Rohm) are used for the full-bridge rectifiers at the receivers. These circuit parameters and models are determined based on the measured values and elements used in the experiments, which are described in Section IV. Additionally, the coupling coefficients between the coils are determined based on the values measured in the experiments. Fig. 4 presents a schematic of the circuit simulation in LTspice.

The estimation of one Z matrix involved repeating the simulations 10 times by changing the phase of the input voltages at the transmitters for each condition of the receiver positions. The amplitude of the square-wave voltage output from the inverters is set to 18 V for each simulation. The phases of the square-wave voltage are randomly set from 0 to 2π [rad]. The load resistances at the output of the rectifiers are set to 10 Ω. The equivalent resistance of the rectifier, R_{Rec}, is calculated using (9) based on a datasheet of RSX205L-30. In this environment, $I_s = 18.434 \times 10^{-6}$ [A], $N = 1.0515$ and $V_t = 0.026$ [V].

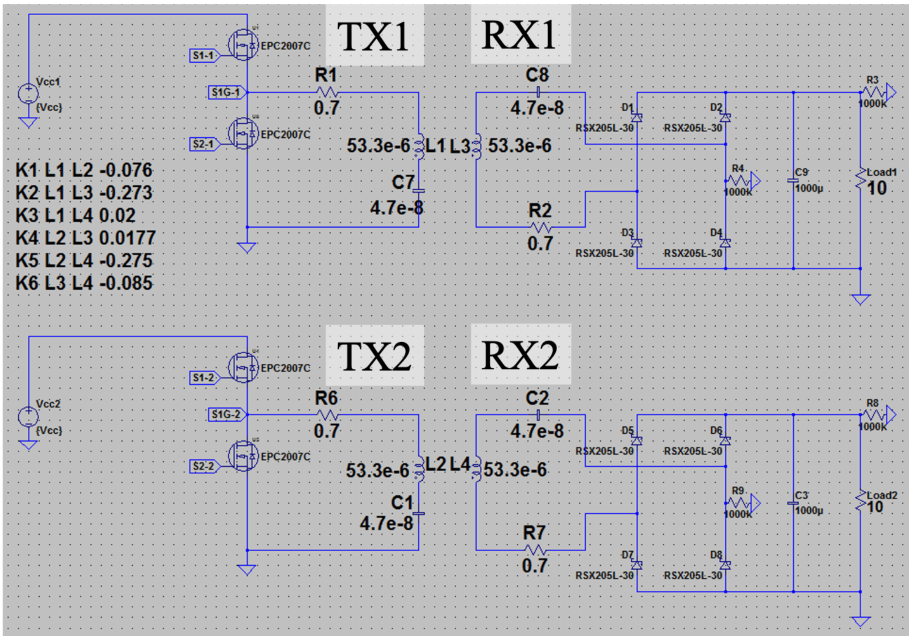


FIGURE 4. Schematic of circuit simulations using LTspice.

TABLE 2. Original *i*, *j* elements of *Z* matrix for two transmitters and two receivers when receivers are placed at (0,0'') in circuit simulations. The unit is [Ω].

$i\backslash j$	1	2	3	4
1	$0.70-0.37j$	$-2.54j$	$-9.14j$	$0.67j$
2	$-2.54j$	$0.70-0.37j$	$0.59j$	$-9.21j$
3	$-9.14j$	$0.59j$	$0.70-0.37j$	$-2.85j$
4	$0.67j$	$-9.21j$	$-2.85j$	$0.70-0.37j$

TABLE 3. Estimated *i*, *j* elements of *Z* matrix for two transmitters and two receivers when receivers are placed at (0,0'') in circuit simulations. The unit is [Ω].

$i\backslash j$	1	2	3	4
1	$0.70-0.37j$	$-2.54j$	$-9.34j$	$0.50j$
2	$-2.54j$	$0.70-0.37j$	$0.37j$	$-9.46j$
3	$-9.34j$	$0.37j$	$0.85-0.41j$	$-3.05j$
4	$0.50j$	$-9.46j$	$-3.05j$	$0.91-0.38j$

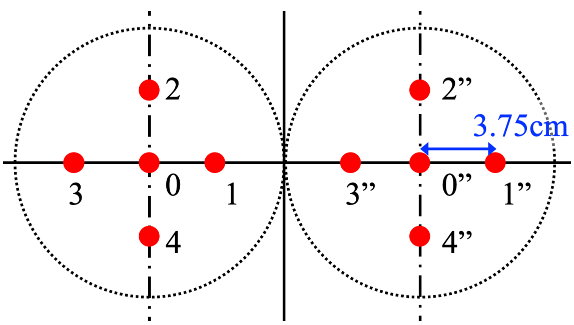


FIGURE 5. Positions of receivers.

B. SIMULATION RESULTS

The Z parameters are estimated based on the simulated results. As an example of the proposed estimation result, Tables 2 and 3 present the true and estimated Z parameters, respectively, when the receivers are placed at (0, 0''). Additionally, the maximum PTE values are calculated based on both the true and estimated Z parameters. To provide a comparison, we also calculate the PTE without transmitter control, where the input current is set to the same phase and amplitude for all transmitters. Fig. 6 presents the calculated PTEs, where "Estimated" indicates the PTE based on the estimated Z parameters, "Measured" represents the PTE based on the true Z parameters, and "Uniform" denotes the PTE obtained using the currents with the same amplitudes and phases.

These results clearly demonstrate that the differences between the PTE based on the estimated Z parameters and the PTE based on the true Z parameters are less than 0.06 %. Furthermore, the PTE based on the estimated Z parameters is higher than the PTE of "Uniform". As for the control of "Uniform", the amplitudes and phases of input currents at the transmitters are always constant, and the control is not based on the location of the receivers. Therefore, a certain constant current flows to the transmitter even when the receiver is not placed above a transmitter, and this constant current causes increased losses at the transmitter. This is the reason for the relatively low PTE for "Uniform".

The estimation accuracy is statistically evaluated by calculating the root mean square deviation (RMSD) of the

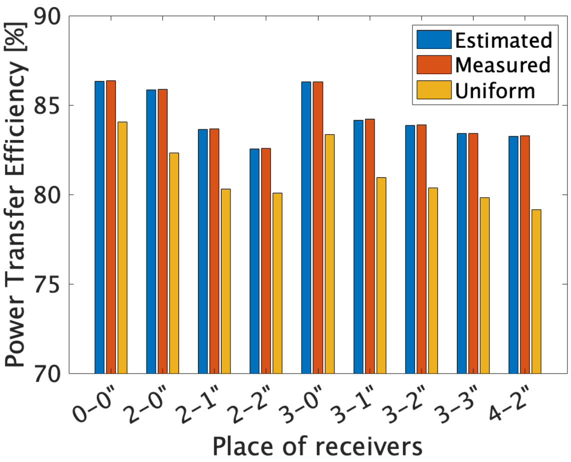


FIGURE 6. Power transfer efficiency at each receiver's position when using data obtained in the circuit simulations.

TABLE 4. RMSD [Ω] and RMS [Ω] of the estimated Z parameters and the original Z parameters in circuit simulations.

RXs	0-0"	2-0"	2-1"	2-2"	3-0"	3-1"	3-2"	3-3"	4-2"
RMSD	0.18	0.15	0.14	0.10	0.18	0.16	0.13	0.11	0.12
RMS	4.81	4.37	3.86	3.99	4.42	3.96	3.88	4.00	3.76

estimated Z parameters based on the measured data and the root mean square (RMS) of the measured Z parameters when all the phases of the receiver currents are set correctly. Table 4 presents the RMSD and the RMS. It is observed that the RMSD of the estimated values is approximately 3.4 % of the RMS of the true values. The primary source of estimation error is the voltage drop at the full-bridge rectifier. Despite correcting the resistances of the receivers using (9), it is challenging to fully consider the effect of the voltage drop in the estimation process. The error decreases when a high DC voltage is output from the rectifier, as the voltage drop becomes negligible in comparison to the output voltage.

IV. EXPERIMENTS

A. EXPERIMENTAL SETTINGS

Fig. 7 depicts the experimental setup. The transmitter and receiver coils are constructed using Litz wire, consisting of comprised 235 thin strands with a diameter of 0.08 mm. The operating frequency is set to 100 kHz and the circuit parameters are listed in Table 5. For the experimental setup, the parameters of the transmitter and receiver coils are provided in Table 6. These parameters are measured using an LCR meter, specifically the IM3536 model by Hioki. The vertical distance between the transmitters and receivers is set to 2 or 3 cm. The AC voltages and currents at the transmitters are measured using an oscilloscope (MDO4054-3, Tektronix). From the measured values obtained by the oscilloscope, the amplitudes and phases at the fundamental frequency are calculated using Fourier transform techniques in MATLAB.

The class-D inverters in the transmitters utilize GaN MOSFETs (EPC2007Cm EPC). The Schottky diodes (RSX205L-30, Rohm) are used as the full-bridge rectifiers in the receivers. The DC voltages and currents at the receivers are measured using the electric loads of LN-300A, KEISOKUGIKEN. A field-programmable gate array (FPGA; DE0 Nano, Terasic) is used to control the inverters. This measurement is repeated 10 times for each receiver position. The amplitude of the square-wave voltage output from the inverters is consistently set to 18 V for each measurement, while the voltage phase is randomly varied.

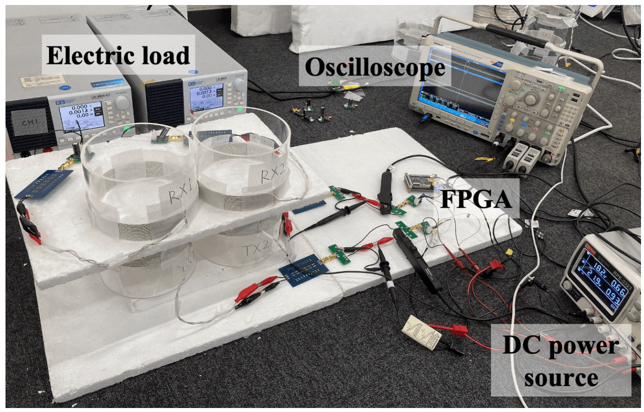


FIGURE 7. Experimental environment.

TABLE 5. Coil parameters.

Parameters	Values
Diameter of litz wire	1.6 mm
Diameter of coil	150 mm
Number of turns	15

TABLE 6. Circuit parameters in experiments.

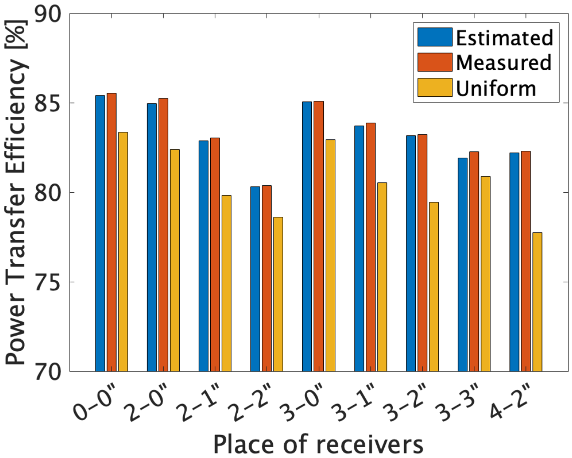
Parameters	
$C_{\mathrm{TX}_1}, C_{\mathrm{TX}_2}, C_{\mathrm{RX}_1}, C_{\mathrm{RX}_2}$	46.6, 46.7, 47.0, 46.9 [nF]
$L_{\mathrm{TX}_1}, L_{\mathrm{TX}_2}, L_{\mathrm{RX}_1}, L_{\mathrm{RX2}}$	53.3, 53.1, 53.1, 53.5 [μH]
$R_{\mathrm{TX}_1}, R_{\mathrm{TX}_2}, R_{\mathrm{RX}_1}, R_{\mathrm{RX}_2}$	0.72, 0.77, 0.68, 0.66 [Ω]
$R_{\mathrm{Load}_1}, R_{\mathrm{Load}_2}$	10 [Ω]
$V_{\mathrm{DC}_1}, V_{\mathrm{DC}_2}$	18 [V]
MOSFET	EPC2007C, EPC
Diode	RSX205L-30, Rohm

B. EXPERIMENTAL RESULTS

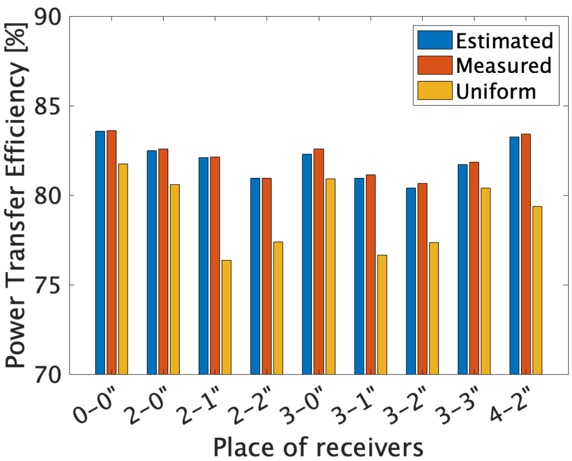
Tables 7 and 8 present the measured and estimated Z parameters when the receivers are placed at (0, 0'') and the distance between transmitters and the receivers is 2 cm. Fig. 8 presents the PTEs calculated using the estimated Z parameters and the measured Z parameters, along with the PTE obtained by inputting currents with identical amplitudes and phases at all transmitters. These results clearly demonstrate that the differences between the PTE based on the estimated Z parameters and the PTE based on the measured Z parameters are less than 0.4 %. Furthermore, the PTE based on the estimated Z parameters is higher than the PTE obtained by using the same amplitudes and phases.

TABLE 7. Measured i, j elements of Z matrix for two transmitters and two receivers when the receivers are placed at (0,0"). The unit is [Ω].

$i\backslash j$	1	2	3	4
1	0.72 −0.66j	−2.53j	−9.13j	0.67j
2	−2.53j	0.77 −0.72j	0.59j	−9.21j
3	−9.13j	0.59j	0.68 −0.50j	−2.84j
4	0.67j	−9.21j	−2.84j	0.66 −0.32j



(a) TXs-RXs distance: 2 cm



(b) TXs-RXs distance: 3 cm

FIGURE 8. Power transfer efficiency at each receiver's position when using data obtained in the experiments. The distance between the transmitters and receivers is set to 2 cm and 3 cm.

The RMSD of the estimated Z parameters based on the RMS of the measured Z parameters are calculated when all the phases of the receiver currents are set accurately. Tables 9 and 10 present the RMSD and the RMS values. It can be observed that the RMSD of the estimated values is approximately 12 % of the RMS of the measured values.

TABLE 8. Estimated i, j elements of Z matrix for two transmitters and two receivers when receivers are placed at (0,0") in experiments. The unit is [Ω].

$i\backslash j$	1	2	3	4
1	0.72 −0.66j	−2.53j	−9.03j	0.17j
2	−2.53j	0.77 −0.72j	0.86j	−9.62j
3	−9.03j	0.86j	0.94 +0.11j	−2.75j
4	0.17j	−9.62j	−2.75j	1.18 −0.26j

This estimation error is attributed to the voltage drop at the full-bridge rectifier. Additionally, the measurement error of $\mathbf{Z_{TT}}$ is not negligible in the experiments. Therefore, improving the measurement accuracy of $\mathbf{Z_{TT}}$ is crucial for obtaining more precise estimates of the Z parameters.

Regarding computation time for the proposed Z-parameter estimation, it is 273.5 ms when the PhaeLift program is run on a PC with an Intel Core i7-7500U.

TABLE 9. RMSD [Ω] and RMS [Ω] of the estimated Z parameters and the measured Z parameters in experiments when the distance between transmitters and receivers is 2 cm.

RXs	0-0"	2-0"	2-1"	2-2"	3-0"	3-1"	3-2"	3-3"	4-2"
RMSD	0.48	0.47	0.43	0.47	0.51	0.50	0.43	0.44	0.30
RMS	4.83	4.38	3.88	3.99	4.41	3.97	3.90	4.02	3.79

TABLE 10. RMSD [Ω] and RMS [Ω] of the estimated Z parameters and the measured Z parameters in experiments when the distance between transmitters and receivers is 3 cm.

RXs	0-0"	2-0"	2-1"	2-2"	3-0"	3-1"	3-2"	3-3"	4-2"
RMSD	0.33	0.48	0.45	0.40	0.44	0.53	0.49	0.38	0.54
RMS	3.93	3.66	3.25	3.44	3.55	3.28	3.29	3.45	3.26

V. CONCLUSION

In this study, we propose a method for estimating the Z parameters in MIMO wireless power transfer systems by employing phase retrieval. Our method utilizes the complex amplitudes of the voltage and current at the transmitters and the DC and the load resistance at the output of the full-bridge rectifier on the receivers. The proposed method eliminates the need for complex time synchronization between the transmitters and receivers to estimate all elements of the Z parameters. Additionally, the Z parameters estimated using our proposed method can control the power flow of MIMO WPT systems without the need to determine the signs of mutual inductances between transmitters and receivers.

The simulation and experimental results demonstrate that the difference between the power transfer efficiency based on the estimated Z parameters and that based on the true Z parameters lies within the range of 0.06 % and 0.4 %, respectively. These findings highlight the effectiveness of our proposed Z parameter estimation method and its capability to achieve accurate control of MIMO WPT systems.

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