

Received 23 October 2023, accepted 11 November 2023, date of publication 16 November 2023, date of current version 22 November 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3333867

## RESEARCH ARTICLE

# Credit Policy Strategies for Green Product With Expiry Date Dependent Deterioration via Grey Wolf Optimizer

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This work was supported by King Saud University, Riyadh, Saudi Arabia, through the Researchers Supporting Project, under Grant RSP2023R323.

**ABSTRACT** In a supply chain that consists of a supplier or manufacturer, a retailer, and a customer, the supplier regularly offers the retailer a pay in later facility in terms of  $S$  periods, while the retailer then gives their client a pay in later facility in terms of  $N$  periods to increase sales and decrease inventory. Offering trade credit benefits the seller's sales and profits, but it also increases default risk. As a result, understanding the credit period is becoming widely acknowledged as a key tactic for boosting seller profitability. This study suggests an EOQ model in the perspective of retailer point of view for which: (a) both the supplier and the retailer supply up-stream pay in later facility; (b) downstream trade credit provided from the retailer to the buyer increases opportunity cost and default risk in addition to sales and profitability; (c) items that are degrading not only continue to degrade over time but also have an expiration date. We employed the well-known metaheuristic algorithm Grey Wolf Optimizer (GWO) to solve the optimisation problem because the objective function is high nonlinear nature. In addition, we have compared the results with some other metaheuristic algorithm. In order to highlight the problem and provide managerial advice, we conclude by using some numerical examples.

**INDEX TERMS** Inventory model, perishable goods, expiration dates, selling price, green level dependent demand, grey wolf optimizer.

## I. INTRODUCTION

In order to boost sales and lower inventory, sellers frequently offer their buyers a legal payment delay. The buyer may accrue income during the credit period and receive interest on that amount. However, if the buyer is unable to pay the entire purchase price within the credit period, the seller will charge the buyer interest on the unpaid balance. Goyal's [1] work is among the earliest in this field of study. He identified the retailer's ideal economic order amount under circumstances where the supplier provides a fair payment delay (EOQ). Shah [2], on the other hand, later thought about a probabilistic inventory system for degrading goods when payment delays are acceptable. The EOQ model was then expanded by

The associate editor coordinating the review of this manuscript and approving it for publication was Md. Abdur Razzaque<sup>1</sup>.

Aggarwal and Jaggi [3] to include decaying items. In order to accommodate shortages, Using trade credit finance, Jamal et al. [4] generalised the Aggarwal and Jaggi [3] suggested model. Teng [5] then provided a straightforward closed-form analytical solution of similar type model. Huang [6] then broadened the scope of the credit policy issue to take into account the case where a supplier offers a store a pay in later facility, and the store in turn grants a second pay in later facility to its clients. Additionally, Liao [7] added an economic production quantity (EPQ) model for degrading goods to Huang's model. Teng [8] then offered the best ordering guidelines for a business to cope with both good and bad credit clients. On the other hand, Min et al. [9] proposed an EPQ model with two levels of pay in later facility and demand that is depending on stock. Subsequently, in an EPQ model with damaged items under trade credit policy, Kreng

and Tan [10] were able to determine the best replenishment choice. After that, Teng et al. [11] were able to determine the best ordering strategy for a demand that is stock-dependent using a progressive payment system. The demand pattern was further expanded by Teng et al. [12] changing throughout time from steady to growing. Ouyang and Chang [13] developed a production model with a flawless backlog and an imperfect production process. All of the aforementioned publications primarily studied the EOQ/EPQ models with pay in later facility in the perspective of buyer point of view. Only a few academics, like Teng and Lou [14] and Chern et al. [15] have focused on how to calculate the seller's ideal credit period. The trade credit literature has currently been organised by Seifert et al. [16], who also developed a thorough plan for future trade credit research. Wu et al. [17] studied a two-level pay in later framework, the appropriate credit duration and lot size for degrading commodities with expiration dates. Bhunia and Shaikh [18] solved a model by using PSO in a two-warehouse system with various inventory regulations and decaying items is permitted. Ouyang et al. [19] studied a capacity-constrained integrated system and a trade credit system that depends on order size. Yang et al. [20] studied an allocating dynamic trade credits and preservation technologies in the best possible way for a model of decaying inventory. Wu et al. [21] proposed a models of inventory for degrading goods having a maximum lifetime under partial trade credits to clients with high credit risk. Shaikh [22] introduced a two-storage inventory problem with a flexible trade credit alternative for decaying goods. Shaikh [23] studied a mixed-type inventory problem for degrading items with frequent advertising and demand that is based on selling price. Tiwari et al. [24] studied a price and inventory problem is utilised for goods that are degrading, have maximum lifetime, and partial backlogs. Two-level pay in later facility are recommended for the supply chain. Shaikh et al. [25] developed a deteriorating item with a three-parameter Weibull distribution that has changeable demand and is based on the item's price and how frequently it is advertised when it is being used as trade credit. Cárdenas-Barrón et al. [26] developed a nonlinear holding cost, nonlinear stock dependent demand under pay in later facility. Shaikh et al. [27] proposed an inventory model for degrading products with trade credit and a ramp-type preservation facility. Das et al. [28] examined a particle swarm optimised inventory model for non-instantaneous deteriorating goods that makes advantage of preservation technology and various credit arrangements for trade credit financing. Das et al. [29] introduced a model for manufacturing inventory that is trustworthy and has a partial trade credit policy. Rahman et al. [30] proposed a perishable commodities inventory model that combines a price-and-stock dependency with discounting options for in-advance payments. Taleizadeh et al. [31] introduced a carbon-emitting inventory model with price-related demand that operates with trade agreement credit and partial backordering. Das et al. [32] studied a green product

production system using an approach of parametric based on intervals and meta-heuristic algorithms with a single-level pay in later facility.

It is very obvious that many goods, including fruits, volatile liquids, vegetables, blood banks, clothing, and high-tech items, continuously degrade for a variety of reasons, including evaporation, spoiling, and obsolescence. Assuming an exponentially declining inventory, Ghare and Schrader [33] suggested an EOQ model in this course. By Covert and Philip [34], the continuous exponential deterioration rate was transformed into a two-parameter Weibull distribution. Subsequently, Dave and Patel [35] investigated a linearly growing demand EOQ model for decaying item without shortages. The EOQ model was then further expanded by Sachan [36] to incorporate shortages. On the other hand, a linear form of demand was generalised by Goswami and Chaudhuri [37] from a continuous demand pattern for decaying commodities. In parallel, Raafat [38] offered a review of the literature on the continually depreciating inventory model. For deteriorating goods with time-varying demand, Hariga [39] investigated the best EOQ models. Then, in 1999, Teng et al. [40] generalised the inventory models with shortages and varying demand. Later, Goyal and Giri [41] published an overview of current developments in the modelling of depreciating inventories. To incorporate partial backlogging, Teng et al. [42] further expanded the model. In 2009, Skouri et al. [43] developed inventory system that included ramp-type demand rates and Weibull deterioration rates. The ramp-type demand and permissible payment delay model for degraded goods was further generalised by Skouri et al. [44] in a subsequent publication. A production model for decaying item with retailer trade agreement credit policy was put forth by Mahata [45]. A research on the impact of technology investment on deteriorating goods was conducted by Dye [46]. A rework policy and probabilistic preventive maintenance production model for deteriorating products was created by Wee and Widyadana [47]. Bhunia et al. [48] proposed a two-warehouse system for decaying commodities with partial backlog and allowable payment delay. Sicilia et al. [49] introduced a model of inventory for degrading goods with shortages and variable demand over time. Ghiami and Williams [50] studied a two-echelon production system for numerous customers of decaying goods. Bhunia et al. [51] investigated a two-storage inventory system with fluctuating demand and partial backlog is used for items that are deteriorating. Wu et al. [21] looked into inventory models for products having a maximum lifespan under partial trade credits downstream to clients who might have credit issues. Banerjee and Agrawal [52] proposed the best ordering and discounting practises for an inventory model for decaying commodities with freshness- and price-dependent demand. Chan et al. [53] investigated an integrated production-inventory model for goods that deteriorate, taking the best production rate and delivery deterioration into account. Pal et al. [54] introduced

a stochastic production system with a finite life cycle for products that are deteriorating. Sharma et al. [55] proposed a time-varying holding cost and expiration date inventory model for goods that degrade over time. Khan et al. [56] studied an inventory model for degrading goods with two warehouses, some backlog, and advance payment plan. Panda et al. [57] introduced a credit policy technique for degrading items in two-storage system with demand that is price- and stock-dependent under partial backlog. Khakzad and Gholamian [58] presented an inventory model to examine the effects of inspection on the rate of deterioration of items with advanced payment. Rahman et al. [59] introduced a parametric technique for interval differential equations in an inventory model for depreciating goods with demand dependent on selling price. Shaikh et al. [27] investigated an inventory model for degrading products with trade credit and a ramp-type preservation facility. Mahata [60], [61] proposed different inventory models based on the payment policy for deteriorating item. Duary et al. [62] studied advance and deferred payments for degrading goods under capacity constraints and partially backlogged shortages are possible with the price-discount inventory model. De and Mahata [63] introduced an inventory model with backlogging situations under disruption. Choudhury et al. [64] proposed an inventory model for degrading goods that takes expiration date into account using the Stackelberg game technique. Mahato and Mahata [65] suggested two level trade credit policy in an inventory system for deteriorating item. Mahato et al. [66] investigated a models of inventory for depreciating goods with fixed lifetimes, partial backordering, and carbon emission regulations.

A collection of computational approaches known as “soft computing” is created to handle issues that are challenging to tackle with traditional methodologies. Soft computing algorithms are frequently employed when the data is complicated and challenging to represent, has a high level of uncertainty or ambiguity, or both. Many different applications, such as control systems, robotics, image processing, data mining, and decision support systems, make extensive use of soft computing algorithms. They are especially helpful when more flexible and adaptive approaches are required or when traditional methods are ineffective.

There are three different types of meta-heuristics: evolutionary algorithms, physics-based algorithms, and swarm intelligence based algorithms. Swarm intelligence algorithms are based on the collective behaviour of social insects and are used for optimisation and decision-making tasks, whereas evolutionary algorithms are based on the principles of natural selection and are used for optimisation and search issues. GA is the most popular algorithm in this field. This approach was put forth by Holland [67] and simulates the concepts of Darwinian evolution. A thorough research into the engineering applications of GA was done by Goldberg [68], [69]. EAs frequently carry out the optimisation by incrementally improving a beginning random solution.

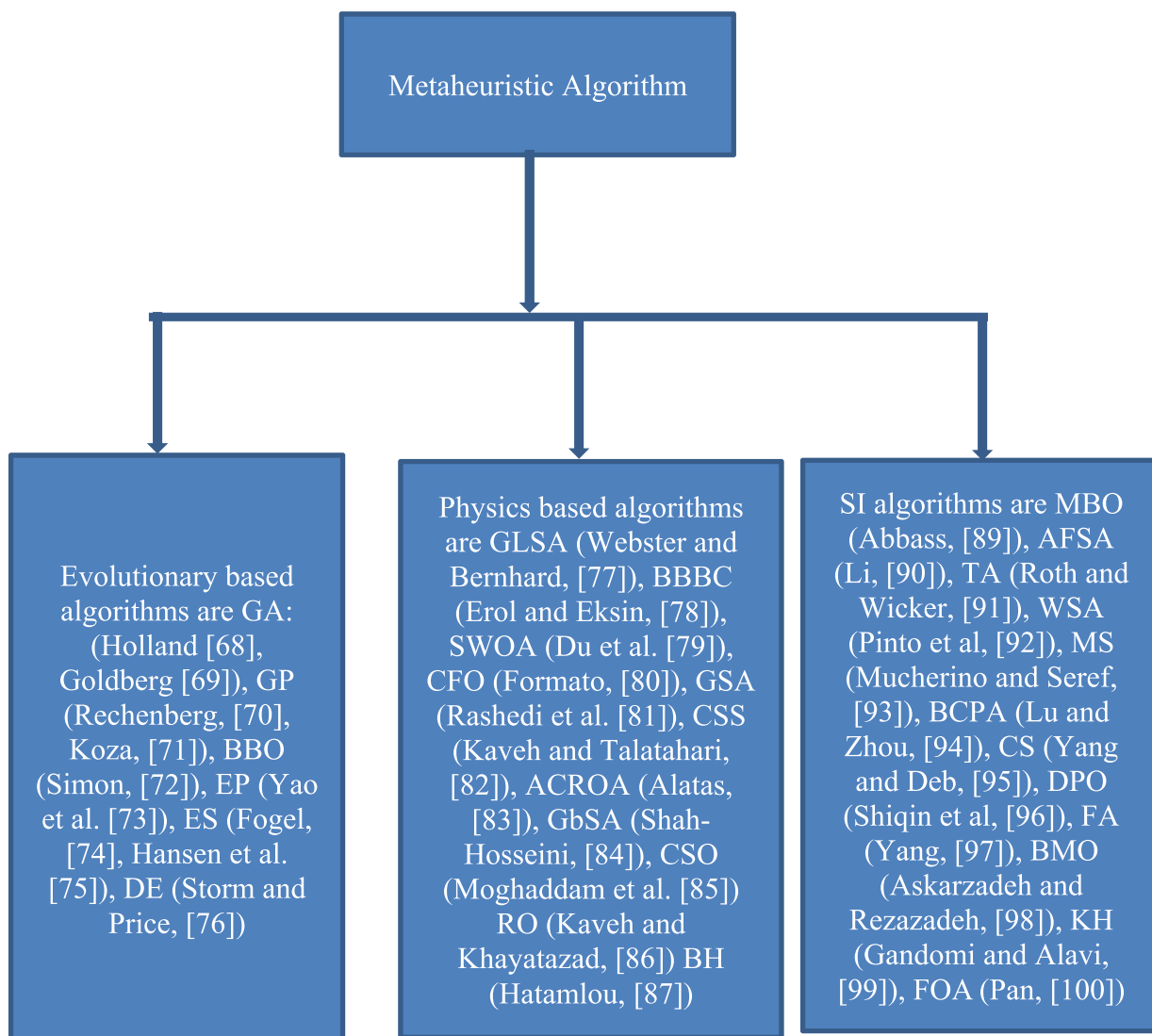
Every new populace is made by combining and altering the members of the preceding generation. The following generation is likely to be better than the one before it since the best people are likely to contribute to its construction (s). By doing so, it will be ensured that the starting random population is increased over many generations. A few EAs include genetic programming (GP) [70], [71], biogeography-based optimizer (BBO) [72], evolutionary programming (EP) [73], evolution strategy (ES) [74], [75], and differential evolution (DE) [76].

Physicistically based methods make up the second major subfield of meta-heuristics. Usually, these optimisation techniques imitate physical laws. Some of the popular algorithms that are reported by several researchers viz. Gravitational Local Search (GLSA) [77], Big-Bang Big-Crunch (BBBC) [78], Small-World Optimization Algorithm (SWOA) [79], Central Force Optimization (CFO) [80], Gravitational Search Algorithm (GSA) [81], Charged System Search (CSS) [82], Artificial Chemical Reaction Optimization Algorithm (ACROA) [83], Galaxy-based Search Algorithm (GbSA) [84], Curved Space Optimization (CSO) [85] Ray Optimization (RO) [86] algorithm and Black Hole (BH) [87] algorithm. In contrast to EAs, these algorithms employ by using the randomly search agents that interact and fly about the search space in line with physical laws. For instance, this movement is carried out by the forces of gravity, ray casting, electromagnetic force, inertia force, weights, and other forces.

Swarm intelligence methods make up the third category of meta-heuristics. These algorithms closely resemble the social behaviour of natural swarms, herds, flocks, or schools of living organisms. Although adopting a method that is mostly physics-based, search agents navigate by mimicking the collective and social intelligence of living things. PSO is the SI approach that is most commonly utilised. Kennedy and Eberhart [88] used the flocking of birds as the inspiration for their PSO algorithm. The PSO method uses a number of particles, each of which moves in accordance with the best particle and its own most optimal places to date. In other words, while moving a particle, it takes into account both its own best solution and the best solution found by the swarm. The graphic representation of several SI algorithms, physics-based algorithms, and evolutionary algorithms is shown in Fig. 1.

This list demonstrates how many SI strategies have been proposed thus far, many of which were influenced by behaviours related to searching and hunting.

In order to determine the retailer's ideal credit duration and cycle time, this study suggests an EOQ model. (a) a down-stream trade credit of  $S$  years is provided to the buyer by the retailer, whereas an up-stream trade credit of  $S$  years is granted to the retailer by the supplier; (b) downstream trade credit provided from the retailer to the purchaser raises opportunity cost and default risk in addition to sales and revenue; and (c) A retailer's goal functions are



**FIGURE 1.** Literature of different metaheuristic algorithms.

then established for a variety of probable circumstances. An item that is decaying not only deteriorates continually but also raises opportunity cost. Demand of the product is considered here non-linear function of credit period, selling price and green level of the product. In addition, purchase cost also considered as the green level dependent. In Due to high complexity of the objective function, we have used soft computing technique GWO and compared with other metaheuristic algorithms for numerical illustration.

#### A. NOVELTY AND CONTRIBUTION

In this model, we have incorporated trade credit and the green level of the product together. According to the literature, in most papers, downstream trade credit time is taken as an input parameter. They also don't take this impact as part of the demand. The main contributions of this paper are given below:

- i) Downstream trade credit is taken as a decision variable in part of the demand. It is also taken as a nonlinear form of the downstream trade credit period.
- ii) The green level of the product is taken as a part of the demand for the product. It is also taken as a nonlinear form of demand.
- iii) To solve this high nonlinear optimisation problem, we have used a metaheuristic algorithm (Grey Wolf Optimizer Algorithm (GWOA)).

To the best of our knowledge, no paper is published by taking all the above mentioned key points together.

#### II. NOTATION AND ASSUMPTIONS

For the purpose of building an EOQ interval inventory model for perishable goods with cash and carbon tax, the following notation and presumptions are introduced:

**A. NOTATION**

The problem is developed using the ensuing variables and parameters.

Notation	Definition
$K$	Setup cost /order
$D(N, s_p, \rho)$	Selling price and nonlinear green level dependent demand (units)
$\varepsilon, a, b, c, d$ and $\kappa$	Demand related parameters
$c_p$	Purchase cost (units)
$c_1, c_2$	Purchase cost parameters
$\phi_c$	Rate of interest charged
$\phi_e$	Rate of interest earn
$c_h$	Holding cost/unit
$\xi$	Maximum lifetime of a product
$\tau$	Cycle length
$S$	Upstream credit period
$V$	Total order quantity (unit)
$q(t)$	Inventory level at any time $t$
$TP$	Total profit of the system (\$)
$ATP$	Average profit of the system (\$/year)
<b>Decision variables:</b>	
$N$	Downstream credit period
$s_p$	Selling price of the item (\$/unit)
$\tau$	Cycle length of the buyer's replenishment in units of time (year)

**B. ASSUMPTIONS**

i. Anything that ages has an expiration date. Hence, when the expiration date  $m$  draws closer, the rate of deterioration must be reduced to 1. It's reasonable to suppose that the rate of decline is  $\varpi(\xi) = \frac{\mu}{(\mu + \xi - t)}$  or  $\varpi(\xi) = e^{\mu(t - \xi)}$  where  $\mu$  be constant. To make the issue manageable, however, we make the following assumptions about the pace of deterioration:

$$\phi(\xi) = \frac{1}{(1 + \xi - t)}, 0 \leq t \leq \tau \leq \xi. \quad (1)$$

ii. Demand for a product is influenced by the product's green level, selling price, and credit period. Mathematically it can be presented as:

$$D(N, s_p, \rho) = aN^\varepsilon + \alpha - bs_p + c\rho^d \quad \text{where } a, \varepsilon, \alpha, b, c, d > 0. \quad (2)$$

iii. The retailer faces a greater default risk the longer its down-stream credit duration is. We can estimate the default risk rate given the retailer's downstream credit duration  $N$  is considered to be as follows for the sake of simplicity:

$$U(N) = 1 - e^{-\kappa N} \quad \text{where } \kappa > 0. \quad (3)$$

iv. Dollars obtained at time  $t$  are comparable to dollars received at time  $t$  today if the annual compound interest rate is  $e^{-\theta t}$ . The merchant grants the customer  $N$  days of credit. After subtracting default risk and opportunity cost, the retailer's net income is as follows:

$$s_p D(N, s_p, \rho) [1 - U(N)] e^{-\theta t} = s_p (aN^\varepsilon + \alpha - bs_p + c\rho^d) e^{-(\kappa + \theta)t} \quad (4)$$

v. Infinite time horizon is considered with negligible lead-time.  
 vi. Once the permitted delay has ended, the supplier bills the retailer at prime rate  $\phi_c$ . The retailer, on the other hand, is free to use the sales money to make investments in the stock market or to produce new goods and generate a profit of  $\phi_e$  during the permitted wait time.  
 vii. Instant replenishments are available for a single product. Purchase cost is also taken as the green level of the product. It can be represented mathematically in the following way

$$c_p = c_1 + c_2 \rho^\gamma \quad \text{where } c_1, c_2, \gamma > 0 \quad (5)$$

viii. Deterioration and shortages are not allowed.

**III. MATHEMATICAL DERIVATION OF THE MODEL**

According to the merchants' perspective, this model is designed based on the aforementioned hypotheses. Initially, a retailer place an order of  $S$  units and kept in his store room. Demand and degradation over the replenishment cycle cause

the inventory level to decrease over  $[0, \tau]$ , and is therefore controlled by the differential equation shown below:

$$\frac{dq(t)}{dt} + \varpi(\xi) = -D(N, s_p, \rho), \quad 0 < t \leq \tau \quad (6)$$

$$\text{With the condition } q(0) = S \text{ and } q(\tau) = 0. \quad (7)$$

From the equations (6) and (7), we get

$$q(t) = D(N, s_p, \rho) (1 + \xi - t) \log\left(\frac{1 + \xi - t}{1 + \xi - \tau}\right), \quad 0 < t \leq \tau. \quad (8)$$

Hence for finding the ordering quantity, we have used

$$S = q(0) = D(N, s_p, \rho) (1 + \xi) \log\left(\frac{1 + \xi}{1 + \xi - \tau}\right), \quad 0 < t \leq \tau. \quad (9)$$

Sales revenue throughout the cycle is

$$(SR) = s_p D(N, s_p, \rho). \quad (10)$$

Holding cost to the entire cycle

$$(HC) = \frac{c_h D(N, s_p, \rho)}{\tau} \left[ \frac{(1 + \xi)^2}{2} \log\left(\frac{1 + \xi}{1 + \xi - \tau}\right) + \frac{\tau^2}{4} - \frac{(1 + \xi)\tau}{2} \right]. \quad (11)$$

$$\text{Ordering cost (OC)} = \frac{K}{\tau} \quad (12)$$

According to the vales of  $N$  and  $S$ , following cases may arise.

- (i) Case 1:  $N \leq S$  and (ii) Case 2:  $N > S$

**Case 1:  $N \leq S$**

There are two alternative sub-cases that are according to the time  $S$  (i.e., the date by which the retailer shall have make payment to the supplier the entire cost of the purchase in order to avoid paying interest) and  $\tau + N$  (i.e., the cut off date for when the store will receive payment from the final client). The retailer pays off all units sold by  $S - N$  at time  $S$ , keeps the profits, and begins paying for the interest charges on the products sold after  $S - N$ , that are available [17], if  $\tau + N > S$  (i.e., interest charge is applicable). The store will receive the entire revenue at time  $\tau + N$  and will repay the entire purchase price at time  $S$  if  $\tau + N \leq S$  (i.e., there is no interest charge). Again two situations may arise. Now, all the situations are discussed in details.

**Situation 1.1:  $S \leq \tau + N$**

The supplier's offers up-stream pay in later facility period  $S \leq \tau + N$  in this sub-case which is less than or equal to the customer's most recent payment time  $\tau + N$ . This means that the shop will have to finance all things sold after time  $S - N$  at an interest rate of  $\phi_c$  per dollar per year as they cannot pay off the purchase price at time  $S$ . As a results retailers must pay an interest on that amount and which is given by

$$\frac{c_p \phi_c (aN^\epsilon + \alpha - bs_p + c\rho^d) (\tau + N - S)}{2\tau}. \quad (13)$$

This calculation is done as per [17].

In the meantime, retailers start to sells decaying goods at time zero, but is paid at time  $N$ . As a result, the store builds up earnings in an account that generates with the rate  $\phi_e$  from  $N$  to  $S$  annually. Hence, retailers earn interest during the time period is given by

$$\frac{s_p \phi_e (aN^\epsilon + \alpha - bs_p + c\rho^d) (S - N)^2}{2\tau}. \quad (14)$$

Hence the objective function can be written as

$$ATP_{1.1}(N, s_p, \tau) = \left[ \frac{s_p (aN^\epsilon + \alpha - bs_p + c\rho^d)}{c_h (aN^\epsilon + \alpha - bs_p + c\rho^d)} \left[ \frac{(1 + \xi)^2}{2} \log\left(\frac{1 + \xi}{1 + \xi - \tau}\right) + \frac{\tau^2}{4} - \frac{(1 + \xi)\tau}{2} \right] - \frac{K}{\tau} - \frac{c_p \phi_c (aN^\epsilon + \alpha - bs_p + c\rho^d) (\tau + N - S)}{2\tau} + \frac{s_p \phi_e (aN^\epsilon + \alpha - bs_p + c\rho^d) (S - N)^2}{2\tau} \right] \quad (15)$$

The corresponding optimization problem can represent mathematically in the following way

$$\begin{aligned} & \text{Maximize } ATP_{1.1}(N, s_p, \tau) \\ & \text{subject to } N > 0, s_p > 0, \tau > 0 \end{aligned} \quad (16)$$

**Situation 1.2:  $S > \tau + N$**

In this particular situation, the retailer is able to make payment the entire purchase price at time  $S$  after receiving the total revenue at  $\tau + N$ . Hence, no interest will be imposed to the retailer, and instead, as calculated from [17], with the rate  $\phi_e$  over the time period  $[N, S]$ . Hence, the annual interest collected by the retailer is  $s_p \phi_e (aN^\epsilon + \alpha - bs_p + c\rho^d) (S - N - \frac{\tau}{2})$

Hence the objective function can be written as

$$ATP_{1.2}(N, s_p, \tau) = \left[ \frac{s_p (aN^\epsilon + \alpha - bs_p + c\rho^d)}{c_h (aN^\epsilon + \alpha - bs_p + c\rho^d)} \left[ \frac{(1 + \xi)^2}{2} \log\left(\frac{1 + \xi}{1 + \xi - \tau}\right) + \frac{\tau^2}{4} - \frac{(1 + \xi)\tau}{2} \right] - \frac{K}{\tau} + s_p \phi_e (aN^\epsilon + \alpha - bs_p + c\rho^d) \left( S - N - \frac{\tau}{2} \right) \right] \quad (17)$$

The corresponding optimization problem can represent mathematically in the following way

$$\begin{aligned} & \text{Maximize } ATP_{1.2}(N, s_p, \tau) \\ & \text{subject to } N > 0, s_p > 0, \tau > 0 \end{aligned} \quad (18)$$

**Case 1:**  $N > S$

$N > S$  means that the store does not earn interest. The retailer must also finance the total amount of the purchase at time  $S$  and repay the loan from time  $N$  to time  $\tau + N$ . As a result, the interest charged with the rate  $\phi_c$  every cycle is on the interval  $[S, \tau + N]$ . As a result, the annual interest rate is provided by

$$\frac{c_p \phi_c (aN^\epsilon + \alpha - bs_p + c\rho^d) [2(N - S) + \tau]}{2} \tag{19}$$

Hence the objective function can be written as

$$ATP_{1.3}(N, s_p, \tau) = \left[ \begin{array}{l} s_p (aN^\epsilon + \alpha - bs_p + c\rho^d) \\ c_h (aN^\epsilon + \alpha - bs_p + c\rho^d) \\ \frac{(1 + \xi)^2}{2} \log \left( \frac{1 + \xi}{1 + \xi - \tau} \right) \\ + \frac{\tau^2}{4} - \frac{(1 + \xi)\tau}{2} \\ \frac{K}{\tau} \\ c_p \phi_c \left( \frac{aN^\epsilon + \alpha}{-bs_p + c\rho^d} \right) [2(N - S) + \tau] \end{array} \right] \tag{20}$$

The corresponding optimization problem can represent mathematically in the following way

$$\begin{aligned} & \text{Maximize } ATP_{1.3}(N, s_p, \tau) \\ & \text{subject to } N > 0, s_p > 0, \tau > 0 \end{aligned} \tag{21}$$

Now, we have employed a meta-heuristic method called Grey Wolf Optimizer to resolve the aforementioned three issues (GWO). It is a recently created, widely used, and well-known optimisation approach. The metaheuristic algorithm known as Grey Wolf Optimization (GWO) was influenced by the social structure and hunting methods of grey wolves. This algorithm is frequently used to resolve different optimisation issues, including issues with inventory management. The following justifies why GWO might be a wise decision for handling inventory issues:

- (i) It has been demonstrated that the effective optimisation algorithm GWO performs effectively on a range of optimisation tasks, including challenging inventory management issues.
- (ii) GWO is a population-based method, allowing for a more complete exploration of the solution space than single-point algorithms. This is particularly crucial in inventory management, where it is frequently necessary to identify numerous optimal solutions.
- (iii) GWO is a strong algorithm that can deal with erratic and noisy data. This is especially important for inventory management because demand, lead times, and supply chain disruptions can all cause the system to become unpredictable.
- (iv) GWO is simple to use and may be modified to meet the unique requirements of the current inventory issue.

Examples of constraints that can be easily added into the algorithm include inventory capacity, lead time specifications, and safety stock levels.

In summary, we can say that GWO is a strong optimisation technique and can be an excellent option for issues relating to inventory management. It is a useful tool for inventory optimisation because of its capacity to efficiently explore the solution space, manage uncertainty, and take restrictions into account. The solution process section includes a detailed overview of GWO.

**IV. SOLUTION PROCEDURE**

The GWO technique has been covered in this section.

**A. MOTIVATION**

The grey wolf is a member of the canid family (*Canis lupus*). Grey wolves are thought of being apex predators since they are at the top of the food chain. Grey wolves typically like to live in packs. There are typically 5 to 12 people in each group. They have a rather tight social dominance structure, which is quite interestingly and that are available in Mirjalili et al. [101].

The leaders, or alphas, might be either male or female. The alpha is mainly in charge of choosing where to sleep, when to wake up, how to hunt, and other factors. The alpha's instructions must be followed by the pack. Nonetheless, an alpha has also been observed to act democratically by sticking with the additional wolves in the group. At gatherings, the entire pack bows to the alpha by keeping their tails down. The alpha wolf is also known as the dominating wolf because the group is expected to follow his orders. The only wolves allowed to mate are the alphas. It's fascinating to notice that the alpha is frequently the pack's best leader rather than its strongest physical member. This demonstrates that a pack's organisation and discipline are far more crucial than its physical power.

The position after alpha in the grey wolf hierarchy is beta. The wolves below the alpha that help him make decisions or take part in other pack activities are known as betas. In the case that one of the other wolves dies or becomes too old to function as alpha, the beta wolf, which can be either male or female, is most likely the best candidate. The beta wolf should respect the alpha as well as lead the other subordinate wolves. It acts as both the alpha's advisor and the pack's disciplinary authority. The beta reinforces the alpha's directives throughout the pack while also giving input to the alpha.

The grey wolf with the lowest ranking is named Omega. The perpetrator is the omega. Omega wolves usually get the upper hand over all other dominating wolves. These are the last wolves that are allowed to eat. Although the omega may appear to be a relatively unimportant pack member, it has been observed that when the omega is lost, the complete pack experiences internal conflict and problems. The aggressive and enraged wolf outbursts of the Omega are to blame for this (s). This helps to maintain the hierarchy of authority and

please the entire pack. The omega are occasionally the pack's nannies.

One who is not an alpha, beta, or omega wolf is referred to as a "subordinate" wolf (or delta in some references). Delta wolves, who are subjugated by alpha and beta wolves, rule over omega wolves. Scouts, sentinels, seniors, hunters, and carers are among those who belong to this category. Scouts are in charge of keeping an eye on the limits of the area and warning the pack if something is amiss. Sentinels keep an eye on and ensure the group's security. Older wolves that have previously been alpha or beta are known as elders. The betas and alphas are assisted by hunters in capturing game and acquiring nourishment for the group. The weak, ill, and injured wolves are also taken care of by the pack's carers.

Group hunting is another distinctive aspect of grey wolves' social conduct, in addition to their social hierarchy. The following are the key stages of grey wolf hunting.

- Locating, pursuing, and approaching the prey.
- After the target stops moving, it is pursued, cornered, and threatened.
- Direct your attack at the prey.

These actions are available in Mirjalili et al. [101].

## B. ALGORITHM REPRESENT MATHEMATICALLY

In this section of the article, mathematical formulas are given for the social structure, prey detection, prey surround, and prey hunting. The GWO algorithm is then explained.

### 1) SOCIAL STRUCTURE

Using the fittest solution, or the alpha, we mathematically simulate the social structure of the wolf's alpha when developing GWO (a). Hence, the second- and third-best responses, beta (b) and delta (d), are given those names. The last candidate is assumed to be omega (x). The GWO algorithm uses alpha (a), beta (b), and delta (d) as a guide for its hunting (optimisation). The omega (x) wolves come after this pack of three wolves.

### 2) ENCIRCLING OF PREY

Grey wolves, as was already said, circle their victim while hunting. The following equations are presented to predict accurately encircling behaviour:

$$\vec{E}_c = \left| \vec{L} \vec{U}_p(t) - \vec{U}(t) \right| \quad (22)$$

$$\vec{U}(t+1) = \vec{U}_p(t) - \vec{M} \cdot \vec{E}_c \quad (23)$$

where  $t$  denote the current iteration,  $\vec{L}$  and  $\vec{M}$  be the coefficient vectors. The position vector of prey  $\vec{U}_p(t)$  and the position vector of the Grey  $\vec{U}(t)$ .

The calculation of the vector  $\vec{L}$  and  $\vec{M}$  are in the following way:

$$\vec{M} = 2\vec{l} \cdot \vec{r}_1 - \vec{l} \quad (24)$$

$$\vec{L} = 2\vec{r}_2 \quad (25)$$

where  $\vec{l}$  be decreased linearly from 2 to 0 throughout iterations and  $\vec{r}_1, \vec{r}_2$  be random number [0, 1].

### 3) HUNTING OF PREY

Grey wolves have the ability to detect prey and encircle them. The alpha usually takes the lead in the search. The beta and delta may occasionally engage in hunting as well. Yet, we are unaware of the location of the ideal site within a hypothetical search space (prey). We assume that the alpha, beta, and delta have superior knowledge of the probable locations of prey in order to mathematically recreate the hunting behaviour of grey wolves (the best candidate answer). To encourage the other search agents, including the omegas, to update their locations in line with the position of the best search agents, we save the first three best solutions that we have so far identified. The following formula is used to calculate the position's update:

$$\begin{aligned} \vec{E}_{ca} &= \left| \vec{L}_a \vec{U}_a(t) - \vec{U}(t) \right|, \vec{E}_{cb} = \left| \vec{L}_b \vec{U}_b(t) - \vec{U}(t) \right|, \\ \vec{E}_{cd} &= \left| \vec{L}_d \vec{U}_d(t) - \vec{U}(t) \right| \end{aligned} \quad (26)$$

$$\vec{U}_1(t+1) = \vec{U}_a(t) - \vec{M}_a \cdot \vec{E}_{ca},$$

$$\vec{U}_2(t+1) = \vec{U}_b(t) - \vec{M}_b \cdot \vec{E}_{cb},$$

$$\vec{U}_3(t+1) = \vec{U}_d(t) - \vec{M}_d \cdot \vec{E}_{cd} \quad (27)$$

$$\vec{U}(t+1) = \frac{\vec{U}_1(t+1) + \vec{U}_2(t+1) + \vec{U}_3(t+1)}{3} \quad (28)$$

### 4) ATTACKING OF PREY

As already mentioned, the prey is attacked by the grey wolves when it stops moving to conclude the chase. By reducing the value of  $\vec{l}$ , we could simulate going after the prey mathematically. Remember that an additionally narrows the range of fluctuation of  $\vec{M}$ . Alternatively, let's say,  $\vec{M}$  is a number chosen at random from the available options  $[-2\vec{l}, 2\vec{l}]$  where  $\vec{l}$  decreases from 2 to 0 during the iterations. Any position between the current location of a search agent and the location of the prey is feasible if the random values of  $\vec{M}$  are in the range  $[1, -1]$ . If the value of  $|\vec{M}| < 1$  then the wolves attack on prey.

### 5) SEARCHING OF PREY

Grey wolves primarily conduct their search according to the order of alpha, beta, and delta positions. They disperse from one another to hunt prey; they assemble to attack prey. To get the search agent to leave the target and go elsewhere, we use  $\vec{M}$  to represent divergence mathematically, we use random numbers larger than 1 or smaller than -1. This promotes exploration and makes it possible to find the GWO method globally. When  $\vec{M} > 1$ , grey wolves must break away from their victim in order to find a better prey. GWO's component  $\vec{L}$  also encourages investigation. Eq. (25) shows that the  $\vec{L}$  vector has random values in the range  $[0, 2]$ . This part offers random weights for the prey, which can be used to stochastically accentuate ( $\vec{L} > 1$ ) or deemphasize ( $\vec{L} < 1$ ) the prey's contribution to determining the distance in Eq. (22). Due to a more irregular behaviour during optimisation that emphasises exploration and steers clear of local optima, GWO benefits from this. It should be noted that, unlike



$\vec{M}$ , the decline in  $\vec{L}$  is not linear. We purposely force  $\vec{L}$  to supply random values at all times to place an emphasis on exploration both during initial rounds and final iterations. This component is quite helpful when local optima stagnate, especially in the last iterations.

Alternately, the effect of natural obstacles on moving prey might be considered  $\vec{L}$  vector. Wolves generally run into problems with nature along their hunting routes, which makes it difficult for them to approach prey quickly and conveniently. The vector  $\vec{L}$  functions in just this way. Depending on where a wolf is positioned, it may be randomly assign the prey a weight that makes it harder and more difficult for wolves to reach, or it can do the opposite.

In conclusion, as the first phase in the search process, the GWO algorithm generates a random population of grey wolves (potential solutions). The potential position of the prey is calculated by the alpha, beta, and delta wolves across a number of iterations. Each possible response alters the prey's distance from it. To highlight the importance of exploitation and exploration, the value  $\vec{l}$  is dropped from 2 to 0. With  $|\vec{M}| > 1$  and  $|\vec{M}| < 1$ , candidate solutions usually depart from the prey and then return to it. The GWO algorithm is eventually completed when an end criterion is met. There is information about the pseudocode and specifics in [101].

## 6) COMPLEXITY OF TIME OF GWO

The GWO's time difficulties can be summed up as follows:

- i) The GWO require  $O(N \times m)$  time during the initialization phase, where  $N$  stands for the population size and  $m$  for the problem's dimension.
- ii) The GWO's control parameters must be calculated in  $O(N \times m)$  time.
- iii) It takes  $O(N \times m)$  time to update the agents' positions in the GWO equations.
- iv) It takes  $O(N \times m)$  time to evaluate each agent's fitness value. Based on the aforementioned analysis, the overall time complexity for each generation is  $O(N \times m)$ , and the total time complexity of the GWO, given a maximum number of iterations, is  $O(N \times m \times Maxit)$ , where  $Maxit$  is the maximum number of iterations.

## 7) ADVANTAGE AND DISADVANTAGE OF GWO

The Grey Wolf Optimizer (GWO) is an optimization algorithm inspired by nature and based on the social structure and hunting habits of grey wolves. Like other optimization algorithms, GWO has benefits and drawbacks that might affect how well it works to solve optimization issues. The following are some of the main benefits and drawbacks of the Grey Wolf Optimizer:

Advantages of GWO:

- Compared to certain other optimisation methods, GWO is comparatively easy to use and comprehend. It is

founded on instinctive ideas drawn from grey wolves' behaviour.

- GWO is simpler to configure than algorithms with many hyper parameters because it has a small number of control parameters.
- GWO frequently finds solutions rapidly and in a manageable length of time, which might be helpful when trying to solve issues with constrained computational resources.
- GWO strikes a balance between exploration and exploitation, allowing it to both explore the search space for global optima and improve viable solutions close to them.
- Numerous optimisation issues, including continuous, discrete, and mixed-integer issues, can be solved with GWO.

The algorithms GWO have some limitations which are given below:

- Although GWO includes fewer parameters than some other algorithms, the selection of these factors, such as the number of wolves and the initial values, can affect how well the system performs.
- The performance of the algorithm can be greatly influenced by the quality of the starting grey wolf population. Slow or inefficient convergence may result from poor initializations.
- In certain circumstances, especially in multimodal or deceptive fitness landscapes, GWO may converge prematurely to local optima and struggle to escape from them.

## 8) PSEUDO CODE OF GWO

The pseudo code of GWO algorithm are given below:

```

Initialize the population of Grey wolf  $U_i(i = 1, 2, 3, 4, \dots, n)$ 
Initialize the values of  $l$ ,  $M$  and  $L$ 
Calculate the fitness of each search agent
 $U_\alpha$  be the best search agent
 $U_\beta$  be the second best search agent
 $U_\delta$  be the third best search agent
while ( $t < Max\_it$ )
    for each search agent
        update the current search agent with the help of
        equation (25)
    end for
    Update the values of  $l$ ,  $M$  and  $L$ 
    Calculate the fitness of all search agent
    Update  $U_\alpha$ ,  $U_\beta$  and  $U_\delta$ .
     $t = t + 1$ 
end while
return  $U_\alpha$ 

```

## V. NUMERICAL EXAMPLE

Three examples are taken into consideration for each of the three difficulties to show and solve them, and each problem

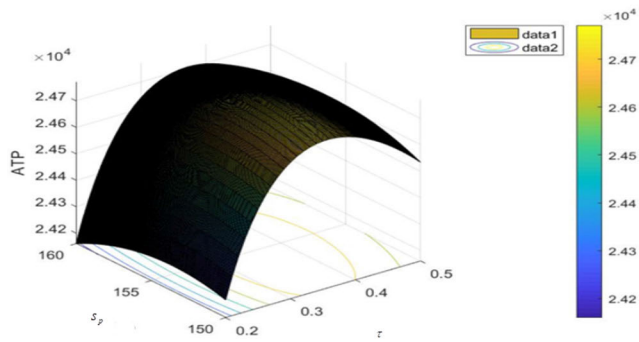


FIGURE 2. Concavity of the objective function of problem 1 with respect to Example-1.

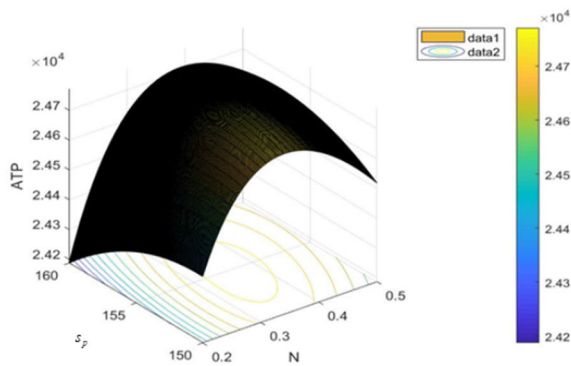


FIGURE 3. Concavity of the objective function of problem 1 with respect to Example-1.

is then resolved using GWO in the MATLAB programming interface.

*Example 1:* The values of the various inventory parameters are based on assumptions. These numbers appear to be plausible, though. We have used an example for each issue to help us tackle the corresponding optimisation issue. Also, for each objective function’s corresponding examples, we plotted the 3D figures. Below are the values for the various parameters:

$$\alpha = 180; \varepsilon = 0.06; c_h = 2; a = 500; b = 2.5; \kappa = 0.15; c_1 = 40; c_2 = 0.5; \gamma = 0.2; \phi_e = 0.04; \phi_c = 0.06; K = 500; S = 0.15; \xi = 1.2; \rho = 0.89; \theta = 0.05; c = 1; d = 0.5;$$

The optimal solutions of Example 1 are presented in the following Table 1.

The concavity of the objective function is shown in Fig.2. This figure is drawn with respect to the decision variable  $s_p, \tau$ .

The concavity of the objective function is shown in Fig.3. This figure is drawn with respect to the decision variable  $s_p, N$ .

The concavity of the objective function is shown in Fig.4. This figure is drawn with respect to the decision variable  $\tau, N$ .

TABLE 1. Best found results of Example-1 by using GWO.

Cases/Situations	Variables/unknown parameters	Optimal values
Situations 1.1	$N$	0.3625 years
	$s_p$	\$ 154.6384
	$\tau$	0.3630 years
	$V$	105.0744 unit
	$ATP_{1.1}(N, s_p)$	24771.0826
Cases/Situations	Variables/unknown parameters	Optimal values
Situations 1.2	$N$	0.2970 years
	$s_p$	\$ 152.9555
	$\tau$	0.3515 years
	$V$	100.5923 unit
	$ATP_{1.2}(N, s_p)$	24378.9874
Cases/Situations	Variables/unknown parameters	Optimal values
Case 2.1	$N$	0.3443 years
	$s_p$	\$ 154.2606
	$\tau$	0.3751 years
	$V$	108.7198 unit
	$ATP_{2.1}(N, s_p)$	24798.4593

## VI. SENSITIVITY ANALYSIS

To investigate the impact of the different inventory parameters on the centre of average profit (APC), selling price

TABLE 2. Best found results of Example-1 by using AEFA.

Cases/Situations	Variables/unknown parameters	Optimal values
Situations 1.1	$N$	0.3615 years
	$s_p$	\$ 154.6184
	$\tau$	0.3520 years
	$V$	105.0644 unit
	$ATP_{1.1}(N, s_p, \tau)$	24771.0412
Cases/Situations	Variables/unknown parameters	Optimal values
Situations 1.2	$N$	0.2868 years
	$s_p$	\$ 152.8955
	$\tau$	0.3482 years
	$V$	100.5713 unit
	$ATP_{1.2}(N, s_p, \tau)$	24378.8765
Cases/Situations	Variables/unknown parameters	Optimal values
Case 2.1	$N$	0.3337 years
	$s_p$	\$ 154.1908
	$\tau$	0.3689 years
	$V$	108.6997 unit
	$ATP_{2.1}(N, s_p, \tau)$	24798.4481

( $p$ ) and total length of replenishment cycle ( $\tau$ ), the sensitivity analyses are executed and supplied in Table 4.

The following implications are observed:

- (i) From Table 4, it follows that average profit ( $ATP$ ) is highly sensitive with respect to  $a, \xi, \alpha$  whereas it is

TABLE 3. Best found results of Example-1 by using WOA.

Cases/Situations	Variables/unknown parameters	Optimal values
Situations 1.1	$N$	0.3604 years
	$s_p$	\$ 154.6179
	$\tau$	0.3525 years
	$V$	105.0634 unit
	$ATP_{1.1}(N, s_p, \tau)$	24771.0426
Cases/Situations	Variables/unknown parameters	Optimal values
Situations 1.2	$N$	0.2867 years
	$s_p$	\$ 152.8856
	$\tau$	0.3484 years
	$V$	100.5923 unit
	$ATP_{1.2}(N, s_p, \tau)$	24378.9664
Cases/Situations	Variables/unknown parameters	Optimal values
Case 2.1	$N$	0.3347 years
	$s_p$	\$ 154.1808
	$\tau$	0.3670 years
	$V$	108.6887 unit
	$ATP_{2.1}(N, s_p, \tau)$	24798.4572

less sensible with respect to the rest of the inventory parameters.

- (ii) It is observed from Table 4 that the selling price ( $s_p$ ) is equally sensitive with respect to  $a, \xi, \alpha$  whereas it is less sensible with respect to the rest of the inventory parameters.

TABLE 4. Post optimality experiment of Example 1.

Inventory parameters	Decision variables/average profit	Change in percentages				
		-20%	-10%	0%	20%	10%
$\epsilon$	$ATP$	<u>2.4</u>	<u>1.2</u>	<u>0</u>	<u>-1.18</u>	<u>-2.37</u>
	$N$	<u>-1.24</u>	<u>-0.29</u>	<u>0</u>	<u>0.2</u>	<u>1.14</u>
	$\tau$	<u>1.09</u>	<u>0.03</u>	<u>0</u>	<u>-0.14</u>	<u>-1.08</u>
	$s_p$	<u>10.02</u>	<u>5.01</u>	<u>0</u>	<u>-5.02</u>	<u>-10.01</u>
	$V$	<u>2.12</u>	<u>1.13</u>	<u>0</u>	<u>-1.28</u>	<u>-2.33</u>
$\kappa$	$ATP$	<u>1.6</u>	<u>0.8</u>	<u>0</u>	<u>-0.79</u>	<u>-1.58</u>
	$N$	<u>1.3</u>	<u>0.2</u>	<u>0</u>	<u>-0.2</u>	<u>-1.1</u>
	$\tau$	<u>1.17</u>	<u>0.25</u>	<u>0</u>	<u>-0.36</u>	<u>-1.01</u>
	$s_p$	<u>1.05</u>	<u>0.05</u>	<u>0</u>	<u>-0.03</u>	<u>-0.9</u>
	$V$	<u>1.15</u>	<u>0.27</u>	<u>0</u>	<u>-0.34</u>	<u>-1.04</u>
$a$	$ATP$	<u>-37.38</u>	<u>-18.68</u>	<u>0</u>	<u>18.69</u>	<u>37.37</u>
	$N$	<u>1.06</u>	<u>0.18</u>	<u>0</u>	<u>-0.08</u>	<u>-1.04</u>
	$\tau$	<u>0.09</u>	<u>0.09</u>	<u>0</u>	<u>0.07</u>	<u>0.17</u>
	$s_p$	<u>5.03</u>	<u>1.02</u>	<u>0</u>	<u>-1.01</u>	<u>-4.02</u>
	$V$	<u>-35.43</u>	<u>-17.63</u>	<u>0</u>	<u>17.77</u>	<u>35.67</u>
$c_1$	$ATP$	<u>9.49</u>	<u>4.75</u>	<u>0</u>	<u>-4.74</u>	<u>-9.49</u>
	$N$	<u>-1.04</u>	<u>-0.51</u>	<u>0</u>	<u>0.12</u>	<u>2.09</u>
	$\tau$	<u>2.07</u>	<u>1.05</u>	<u>0</u>	<u>-0.95</u>	<u>-1.28</u>
	$s_p$	<u>5.20</u>	<u>2.03</u>	<u>0</u>	<u>-2.13</u>	<u>-4.20</u>
	$V$	<u>1.07</u>	<u>0.51</u>	<u>0</u>	<u>-0.32</u>	<u>-1.10</u>
$K$	$ATP$	<u>1.07</u>	<u>0.54</u>	<u>0</u>	<u>-0.54</u>	<u>-1.08</u>
	$N$	<u>-1.08</u>	<u>-0.20</u>	<u>0</u>	<u>0.50</u>	<u>1.15</u>
	$\tau$	<u>2.07</u>	<u>1.09</u>	<u>0</u>	<u>-1.05</u>	<u>-2.29</u>
	$s_p$	<u>0.92</u>	<u>0.22</u>	<u>0</u>	<u>-0.12</u>	<u>-1.02</u>
	$V$	<u>1.03</u>	<u>0.38</u>	<u>0</u>	<u>-0.41</u>	<u>-1.27</u>

TABLE 4. (Continued.) Post optimality experiment of Example 1.

$c_h$	$ATP$	<u>0.03</u>	<u>0.02</u>	<u>0</u>	<u>-0.02</u>	<u>-0.03</u>
	$N$	<u>0.24</u>	<u>0.06</u>	<u>0</u>	<u>-0.02</u>	<u>-0.28</u>
	$\tau$	<u>0.13</u>	<u>0.03</u>	<u>0</u>	<u>-0.19</u>	<u>-0.25</u>
	$s_p$	<u>0.13</u>	<u>0.04</u>	<u>0</u>	<u>-0.06</u>	<u>-0.13</u>
	$V$	<u>0.11</u>	<u>0.02</u>	<u>0</u>	<u>-0.01</u>	<u>-0.15</u>
$S$	$ATP$	<u>-0.05</u>	<u>-0.03</u>	<u>0</u>	<u>0.03</u>	<u>0.05</u>
	$N$	<u>1.02</u>	<u>0.16</u>	<u>0</u>	<u>-0.21</u>	<u>-1.12</u>
	$\tau$	<u>1.04</u>	<u>0.13</u>	<u>0</u>	<u>-0.11</u>	<u>1.18</u>
	$s_p$	<u>2.02</u>	<u>0.5</u>	<u>0</u>	<u>-0.3</u>	<u>-1.03</u>
	$V$	<u>1.01</u>	<u>0.12</u>	<u>0</u>	<u>-0.14</u>	<u>-0.94</u>
$\xi$	$ATP$	<u>-14.34</u>	<u>-7.17</u>	<u>0</u>	<u>7.17</u>	<u>14.34</u>
	$N$	<u>-0.28</u>	<u>-0.1</u>	<u>0</u>	<u>0.01</u>	<u>-0.27</u>
	$\tau$	<u>-0.12</u>	<u>-0.08</u>	<u>0</u>	<u>0.13</u>	<u>0.2</u>
	$s_p$	<u>-18.01</u>	<u>-5.02</u>	<u>0</u>	<u>5.01</u>	<u>17.01</u>
	$V$	<u>-13.73</u>	<u>-6.76</u>	<u>0</u>	<u>6.94</u>	<u>13.82</u>
$\alpha$	$ATP$	<u>30.74</u>	<u>15.36</u>	<u>0</u>	<u>-15.35</u>	<u>-30.71</u>
	$N$	<u>-2.57</u>	<u>-0.31</u>	<u>0</u>	<u>0.21</u>	<u>3.34</u>
	$\tau$	<u>0.11</u>	<u>0.1</u>	<u>0</u>	<u>-0.03</u>	<u>-0.12</u>
	$s_p$	<u>20.05</u>	<u>10.21</u>	<u>0</u>	<u>-10.03</u>	<u>20.95</u>
	$V$	<u>29.24</u>	<u>14.67</u>	<u>0</u>	<u>-14.58</u>	<u>-29.14</u>

(iii) From Table 4, it follows that the credit period ( $N$ ) is less sensible with respect to the rest of the inventory parameters.

(iv) From Table 4, it follows that the cycle length ( $\tau$ ) is less sensible with respect to the rest of the inventory parameters.

#### A. MANAGERIAL INSIGHT

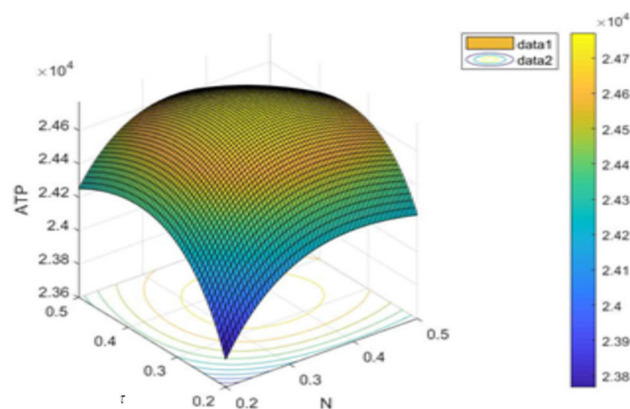
Trade credit dependent demand refers to a situation where a firm's customers rely heavily on trade credit to finance their purchases. This type of demand can have both positive and negative effects on a firm's financial performance.

On the one hand, offering trade credit can increase sales and help a firm gain market share by making it easier for

customers to purchase its products. However, trade credit also carries risks, such as the possibility of bad debt and the potential for cash flow problems if customers do not pay their bills on time.

As a manager, it is important to carefully consider the trade-offs involved in offering trade credit to customers. To minimize the risks of bad debt and cash flow problems, it is important to establish clear credit policies and procedures, including credit limits and payment terms.

Additionally, managers should closely monitor customer payment behavior and take action when necessary to ensure timely payment. This may involve implementing collection policies, such as offering discounts for early payment or charging late fees for overdue balances.



**FIGURE 4.** Concavity of the objective function of problem 1 with respect to Example-1.

Green level dependent demand refers to a situation where a firm's customers demand products that are environmentally sustainable or "green". This type of demand can have both opportunities and challenges for a firm's financial performance.

On the one hand, catering to green demand can help a firm gain a competitive advantage by differentiating its products and appealing to environmentally conscious consumers. This can result in increased sales and market share. Moreover, investing in environmentally sustainable practices can also result in cost savings over time, for instance through reduced energy usage or waste reduction.

However, meeting green demand can also require significant investments in research and development, product redesign, and operational changes. These investments may be difficult to justify if the expected increase in sales does not materialize. Additionally, environmental regulations and standards may vary by region, making it challenging for firms to navigate the regulatory landscape and ensure compliance.

As a manager, it is important to carefully evaluate the potential benefits and costs of meeting green demand. This may involve conducting market research to assess the level of green demand in target markets, as well as assessing the feasibility and cost-effectiveness of environmentally sustainable practices.

Managers should also consider partnering with suppliers, industry associations, and regulatory bodies to stay up-to-date on best practices and compliance requirements. Additionally, investing in employee training and engagement can help to build a culture of sustainability within the organization and ensure that sustainability practices are embedded in all aspects of the firm's operations.

Selling price dependent demand refers to a situation where a firm's customers are sensitive to changes in the price of its products. This type of demand can have both opportunities and challenges for a firm's financial performance.

On the one hand, lowering prices can help a firm gain market share by making its products more affordable and

appealing to price-sensitive consumers. Additionally, raising prices can increase profit margins and allow a firm to invest in research and development, marketing, and other areas that can enhance its competitive position.

However, changes in price can also result in changes in demand that can be difficult to predict and manage. For instance, if a firm raises its prices too much, it may lose customers to competitors with lower prices. Conversely, if a firm lowers its prices too much, it may struggle to maintain profitability.

As a manager, it is important to carefully evaluate the relationship between price and demand and develop pricing strategies that balance the need for profitability with the need to remain competitive in the marketplace. This may involve conducting market research to assess customer sensitivity to price changes, as well as analyzing data on historical sales and pricing trends.

Managers should also consider implementing dynamic pricing strategies that adjust prices based on factors such as seasonality, demand fluctuations, and competitor pricing. Additionally, implementing effective marketing and promotional strategies can help to increase demand and mitigate the potential negative effects of price changes.

Overall, while trade credit can be an effective tool for increasing sales, it is important for managers to carefully manage the associated risks and take steps to ensure timely payment from customers. While meeting green demand can offer significant opportunities for firms, it is important for managers to carefully evaluate the potential costs and benefits and take a strategic approach to managing sustainability practices within the organization. Selling price dependent demand can present challenges for firms, it also presents opportunities for managers to develop effective pricing and marketing strategies that can enhance the firm's competitive position and financial performance.

## VII. CONCLUSIONS AND FUTURE SCOPE

Credit financing is an interesting strategy for inventory management. In this study, we develop a price- and green-level-dependent inventory model for demand. The product's greenness has an impact on production costs as well. There is extensive coverage of a variety of conceivable situations and subsituations concerning credit facilities. Due to the nonlinearity of the objective function, these optimisation problems are resolved using the Grey Wolf Optimizer (GWO) technique and compared with some other algorithms. It is observed that GWO performs better for solving the optimization problem. The concavity of the objective function is graphically represented using the MATLAB 2018a tool. The numerical results suggest that, from an economic standpoint, circumstance 2.1 is more profitable. This business strategy can be used to produce commodities for bakeries, pharmacies, cosmetics, cement, chemicals, food products (such as sugar and powdered milk), alcoholic beverages, and other industries.

By incorporating nonlinear stock dependent demand, preservation technologies, carbon cap and trade legislation,

trade credit (both single and two level), etc., anyone can broaden a study. When tackling this challenging problem using the soft computing technique, anyone can add an interval objective.

## CONFLICT OF INTEREST

The author is declared that there is no conflict of interest.

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