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RESEARCH ARTICLE

Group Consensus of Hybrid Multi-Agent Systems With Event-Triggering Conditions

HUIQIN PEI, (Member, IEEE), YE ZHOU, AND JIANHENG TAN
School of Electrical and Automation Engineering, East China Jiaotong University, Nanchang 330013, China

Corresponding author: Huiqin Pei (peihuiqin@ecjtu.edu.cn)

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ABSTRACT This paper investigates the group consensus problem of hybrid multi-agent systems(HMASs) under event-triggering conditions. The communication between multi-agents can be effectively reduced by the event-triggering mechanism, and the energy consumption of system can be saved. First, a HMAS is constructed, which composes of individuals with continuous and discrete two states. A distributed hybrid multi-agent group consensus controller is proposed. Then considering the joint measurement method, appropriate event-triggering strategy is given, which effectively avoids the occurrence of Zeno phenomenon. Lyapunov's method is used to prove that system can finally achieve group consensus. Finally, the correctness of the research results is confirmed by simulation.

INDEX TERMS HMASs, event-triggering condition, group consensus.

I. INTRODUCTION

With research, it has become more common to combine multi-agents with other domains. These include neural networks [1], unmanned aircraft [2], electric transportation [3] and other fields. Nowadays, the importance of multi-agent research is self-evident. Among them, system consensus is a basic problem in related research. Consensus represents speed, position and other states of individuals in the system will eventually converge to similar values. With the continuous research of experts and scholars, numerous achievements have been achieved in consensus. These include aspects such as bipartite consensus [4], scale consensus [5], group consensus [6]. In the practical application process, in order to cope with different task requirements, the system will divide agents into multiple groups to cooperate. This is the reason for the emergence of group consensu.

In the application process, individuals in the system need to be controlled according to the actual situation. Therefore, individuals in the system will adopt different dynamic models, including continuous-time [7], discrete-time [8], linear [9], nonlinear [10] and other system models. When there are different dynamical models or continuous and discrete individuals components in a system, it is called a HMAS. At present, scholars have made a lot of researches. Liu et al. adopted the theorems of matrix theory, and the consensus criteria of individuals under delay constraint was obtained in [11]. Finally, the conditions for the stability were given. On this basis, a HMAS with continuous and discrete individuals is constructed in [12], and the system can achieved consensus with controller. In [13], two different HMAS models were explored, and the system consensus problem was solved. For hybrid systems, a new control protocol based on the containment control method was proposed in [14]. In summary, the control method of HMAS is more flexible. In the practical application process, the requirements of the system are increasing, and the HMAS can cope with the complex environment and task requirements [15]. At present, there are few researches on HMAS with mixed continuous and discrete individuals. We explore these systems and introduce the concept of group consistency. Such systems can achieve cluster collaboration, which is more advantageous in dealing with more complex tasks.

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In the application process, frequent communication between agents will increase the consumption of resources. In order to improve resource utilization and reduce controller updates, experts and scholars proposed an event-triggering mechanism [16]. In [17], event-triggering strategy was studied in nonlinear multi-agent systems(MASs), and symbolic functions were proposed for optimized control. Under the control of event-triggering strategy, the consensus problem of linear MASs affected by communication delay was investigated [18]. In [19], the system was controlled under dynamic event-triggering conditions. The event-triggering mechanism used a distributed dynamic framework. In the study of systems under the influence of disturbance factors, the fixed-time triggering scheme was proposed. So the formation tracking can be realized with less disturbance [20]. The distributed event-triggering strategy with state feedback was studied in [21], which is capable of feedback control even without complete state information. In [22], predictive control was explored and inter-agent state prediction was realized under the condition of event-triggering. In [23], an adaptive event-triggering strategy was proposed in which the triggering threshold can be dynamically adjusted. Compared with the traditional triggering method, it is more flexible. Then, an improved adaptive event-triggering mechanism was proposed in [24], and a fuzzy system model was designed for interference effects. According to Zeno phenomenon, an event-triggering strategy based on periodic sampling was proposed, which can avoid Zeno phenomenon fundamentally [25]. The disadvantage is that each agent controller not only updated at its own triggering time, but also was affected by the triggering time of the neighbor agent, which will increase the update times of the controller. Based on this, this paper introduces the method of joint measurement, so that the agent controller can be updated only at its own triggering time. Which avoided interference from neighbor agents and further reduced controller updates.

The main innovations are as follows. In this paper, event-triggering conditions are added to the HMAS and the problem of group consensus is explored. At present, there are few researches on the mixed system of discrete and continuous individuals. Based on this HMAS research, the problem of group consensus in hybrid systems is discussed. Through the study, a new group consensus controller is designed. By adjusting the clustering coefficient of the controller, HMAS can reach the clustering consensus. Although the periodic sampling event-triggering method can avoid Zeno phenomenon, it will also lead to unnecessary updates of the controller. In this case, we introduce a joint measurement method, which can reduce the influence of neighbor agents and ultimately reduce the system resource loss. Compared with the existing research results, this paper makes the following innovations and contributions

1. The HMAS model with continuous and discrete states is constructed, and a hybrid group consensus theory is proposed.

2. The group consensus controller of HMAS is designed and given, through which the system can achieve mixed group consensus, and the group situation of the system can be flexibly changed by the group coefficient.

3. Based on the method of periodic sampling and joint measurement, the event-triggering strategy is designed, which can not only reduce the update of the controller, but also fundamentally avoid Zeno phenomenon.

This article has the following sections. In Section II, the basic knowledge is given first and then a dynamics model of HMAS is constructed. This system contains individuals with both continuous and discrete states. In the Section III of this paper, a distributed HMAS group consensus controller is given. Then an event-triggering strategy using the joint measurement method is devised. The condition for the system to achieve group consensus is given, and its feasibility is proved by Lyapunov method. In Section IV, simulation examples are given to prove the accuracy of the study.

Notation: R^n represents the n-dimensional Euclidean space. I_n represents the $n \times n$ identity matrix. $\|.\|$ represents the Euclidean norm. For any real symmetric matrix x, x^{-1} represents the inverse of the matrix, x^T represents the transpose of the matrix. $D = diag(d_1, d_2 \ldots\ldots, d_n)$ represents the diagonal matrix with the diagonal element of d_i.

II. PROBLEM DESCRIPTION

A. KNOWLEDGE OF GRAPH THEORY

In the study of multi-agent system, the knowledge of algebraic graph theory provides the relevant theoretical basis for study. A system communication topology is represented by $G = (V, \varepsilon, A)$. The $V = \{1, 2 \ldots\ldots, N\}$ denotes the set of nodes in the graph. The $\varepsilon \subseteq \{i, j \in V \times V\}$ denotes the edge set of connecting points, and $A = \left[a_{ij}\right] \in R^{N\times N}$ represents the adjacency matrix of the graph. If $(i, j) \in \varepsilon$, $a_{ij} = 1$, otherwise $a_{ij} = 0$. We define $D = diag\{d_{1,} d_2, \ldots, d_N\}$ as the degree matrix of the graph, and $d_i = \sum_{j=1}^{N} a_{ij}$. Then L stand for Laplacian matrix, and $L = (l_{ij})_{N\times N}$ satisfies the following definition:

$$l_{ij} = \begin{cases} \sum\limits_{j\in N, j\neq i} a_{ij}, & i = j \\ -a_{ij}, & i \neq j \end{cases} \qquad i, j = 1, 2, \ldots, N.$$

B. MODEL DESCRIPTION

Considering a HMAS composed of M agents with continuous dynamic and $(N - M)$ agents with discrete dynamic. The dynamic model of agents can be defined as:

$$\begin{cases} \dot{x}_i(t) = u_i(t), & i \in I_M, \\ x_i(t_{\kappa+1}) = x_i(t_\kappa) + u_i(t_\kappa),\ t_\kappa = c_\iota h_1, & i \in I_N/I_M, \end{cases} \tag{1}$$

where $x_i \in R^n, u_i \in R^n$ respectively stand for location and controller input of agent i. We use $c_\iota h_1$, $(c_\iota \in N^+)$ to describe the change of discrete-time agents state over time. And h_1 is the sampling interval of discrete-time multi-agents.

Definition 1: When the intelligence in HMAS can satisfy the following equation in any initial-states $x_i(0)$

$$\begin{cases} \lim\limits_{t\to\infty} \left\|x_j(t)-x_i(t)\right\| = 0, & x_{\sigma j} = x_{\sigma i},\, i \in I_N, \\ \lim\limits_{t_k\to\infty} \left\|x_j(t_\kappa)-x_i(t_\kappa)\right\| = 0, & x_{\sigma j} = x_{\sigma i},\, i \in I_N, \\ \lim\limits_{t\to\infty, t_k\to\infty} \left\|x_j(t)-x_i(t_\kappa)\right\| = 0, & x_{\sigma j} = x_{\sigma i},\, i \in I_N. \end{cases} \tag{2}$$

This indicates that the HMAS reaches group consensus.

Lemma 1: For an connected undirected graph, the corresponding Laplace matrix is L. Assuming that the eigenvalues of L are $\lambda_1, \lambda_2, .., \lambda_n$, and $\lambda_1 \leq \lambda_2, \ldots, \leq \lambda_n$. Let $x = [x_1, x_2, \ldots, x_n]^T$, then equation $\lambda_2(L)x^TLx \leq x^TL^2x \leq \lambda_n(L)x^TLx$ holds.

III. MAIN RESULTS

Combined with event-triggering mechanism, a distributed hybrid multi-agent group consensus controller as shown below

$$\begin{cases} u_i(t) = \sum\limits_{j\in N_i} a_{ij}(x_j(t_\varsigma^i) - x_i(t_\varsigma^i)) + \alpha \sum\limits_{j\in N_i} l_{ij}x_{\sigma j}, \\ t_\varsigma^i = c_\iota h_2\ , c_\iota \in N^+,\ i \in I_M, \\ u_i(t_\kappa) = h_1(\sum\limits_{j\in N_i} a_{ij}(x_j(t_{\kappa}{}_\xi^i) - x_i(t_{\kappa}{}_\xi^i)) + \alpha \sum\limits_{j\in N_i} l_{ij}x_{\sigma j}), \\ t_\kappa \in [\, t_{\kappa}{}_\xi^i, t_{\kappa}{}_{\xi+1}^i)\ ,\ i \in I_N/I_M\ , \end{cases} \tag{3}$$

where h_2 represents the sampling period of continuous-time multi-agents under the event-triggering mechanism. The $x_{\sigma j}$ represent grouping coefficient. When agent states with the same $x_{\sigma j}$ are tend to converge into a subgroup. $t_\varsigma^i, t_{\kappa}{}_\xi^i$ respectively represents the triggering moments of corresponding agent i. When the agent is between two adjacent triggering moments, control input u_i will keep the status value of previous triggering moment.

The definition of joint measurement is given

$$\begin{cases} f_i(t) = \sum\limits_{j\in N_i} a_{ij}(x_j(t) - x_i(t)), & i \in I_M, \\ f_i(t_\kappa) = \sum\limits_{j\in N_i} a_{ij}(x_j(t_\kappa) - x_i(t_\kappa)), & i \in I_N/I_M, \end{cases} \tag{4}$$

when agent $i \in I_M$ is in continuous-time state, its triggering time are assumed to be $t_o^i, t_1^i, t_2^i \ldots\ldots t_\varsigma^i$ in sequence. When agent $i \in I_N/I_M$ is in discrete-time state, its triggering time are assumed to be $t_{\kappa}{}_o^i, t_{\kappa}{}_1^i, t_{\kappa}{}_2^i \ldots\ldots t_{\kappa}{}_\xi^i$ in sequence. Then under the joint measurement condition, the mixed combination measurement error of agents can be obtained by the equation

$$\begin{cases} e_i(t) = f_i(t_\varsigma^i) - f_i(t), & t \in [\, t_\varsigma^i, t_{\varsigma+1}^i), i \in I_M\ , \\ e_i(t_\kappa) = f_i(t_{\kappa}{}_\xi^i) - f_i(t_\kappa), & t_\kappa \in [t_{\kappa}{}_\xi^i, t_{\kappa}{}_{\xi+1}^i), i \in I_N/I_M. \end{cases} \tag{5}$$

An event-triggering strategy is designed based on the joint measurement method. When the error norm reaches the preset threshold, the controller is updated. Otherwise, the controller maintained the state of the last triggering moment. Then the event-triggering conditions are:

$$\begin{cases} \|e_i\,(t_s)\| \leq \gamma_i \,\|f_i(t_s)\|\,, & t_s = c_\iota h_{2,}\ i \in I_M, \\ \|e_i\,(t_\kappa)\| \leq \gamma_i \,\|f_i(t_\kappa)\|\,, & t_\kappa = ch_{1,}\ i \in I_N/I_M, \end{cases} \tag{6}$$

where t_s represents the time change of the continuous-time multi-agents under the sampling event-triggering mechanism. In engineering applications, the system needs to maintain the corresponding formation to complete the task. Individuals in the system change controller updates by sensing the position status of neighboring agents, ensuring that the system formation is maintained. The event-triggering condition designed in this paper can not only ensure the formation of the system, but also reduce the updating frequency of the controller and reduce the energy loss.

For better analysis, we make the $h_1 = h_2 = h$ through assumptions. This indicates that all multi-agents have the same sampling period. Then define $t_\kappa = c_\iota h, (c_\iota \in N^+)$ to denote the scale of all intelligences over time, so the above evevt-triggering conditions can be redefined as:

$$\|e_i\,(t_\kappa)\| \leq \gamma_i \,\|f_i(t_\kappa)\|\,, \quad t_\kappa = c_\iota h,\ i \in I_N. \tag{7}$$

Theorem 1: The HMAS (1) composed of continuous and discrete agents meets the event-triggering condition (6), and under the action of controller (3). When $h < \frac{-2-2\gamma_M}{2\gamma_M\lambda_n+\lambda_n+\gamma^2{}_M\lambda_n}$ and $\alpha < 0$, the HMAS reaches group consensus. Where $\gamma_M = \max\{\gamma_i | i = 1, 2, 3 \ldots\ldots, N\}$ and λ_n are the largest eigenvalue of Laplace matrix L.

Proof: Combining the hybrid system model (1) and controller (3), we can get

$$\begin{cases} x_i(t) = x_i(t_\kappa) + (t - t_\kappa)(\sum\limits_{j\in N_i} a_{ij}(x_j(t_{\kappa}{}_\varsigma^i) - x_i(t_{\kappa}{}_\varsigma^i)) \\ +\alpha \sum\limits_{j\in N_i} l_{ij}x_{\sigma j}), \qquad t \in (t_\kappa, t_{\kappa+1}],\ i \in I_M, \\ x_i(t_{\kappa+1}) = x_i(t_\kappa) + h(\sum\limits_{j\in N_i} a_{ij}(x_j(t_{\kappa}{}_\xi^i) - x_i(t_{\kappa}{}_\xi^i)) \\ +\alpha \sum\limits_{j\in N_i} l_{ij}x_{\sigma j}), \qquad i \in I_N/I_M\ . \end{cases} \tag{8}$$

According to the condition of event-triggering, no individual will triggered when the agent is at $(k, k+1)$. So the system can be uniformly represented as:

$$x_i(t_{\kappa+1}) = x_i(t_\kappa) + h(\sum_{j\in N_i} a_{ij}(x_j(t_{\kappa}{}_\xi^i) - x_i(t_{\kappa}{}_\xi^i)) + \alpha \sum_{j\in N_i} l_{ij}x_{\sigma j}), \quad i \in I_N. \tag{9}$$

Set

$$x(t_\kappa) = [x_1(t_\kappa), x_2(t_\kappa), \ldots, x_N(t_\kappa)]^T,$$
$$e(t_\kappa) = [e_1(t_\kappa), e_2(t_\kappa), \ldots, e_N(t_\kappa)]^T,$$

$$f(t_\kappa) = [f_1(t_\kappa), f_2(t_\kappa), \ldots, f_N(t_\kappa)]^T,$$
$$x_\sigma(t_\kappa) = [x_{\sigma 1}(t_\kappa), x_{\sigma 2}(t_\kappa), \ldots, x_{\sigma N}(t_\kappa)]^T.$$

Because of $f(t_{\kappa_\xi^i}) = f(t_\kappa) + e(t_\kappa)$, so we can get

$$x(t_{\kappa+1}) = x(t_\kappa) + h(Lx(t_\kappa) + e(t_\kappa) + \alpha L x_\sigma(t_\kappa)), \qquad i \in I_N. \tag{10}$$

Construct the following Lyapunov function

$$V(t_\kappa) = \frac{1}{2} x^T(t_\kappa) Lx(t_\kappa). \tag{11}$$

So we get

$$\begin{aligned} V(t_{\kappa+1}) &= \frac{1}{2} x(t_{\kappa+1})^T Lx(t_{\kappa+1}) \\ &= \frac{1}{2}(x(t_\kappa) + hLx(t_\kappa) + he(t_\kappa) + \alpha hLx_\sigma(t_\kappa))^T \\ &\quad \times L(x(t_\kappa) + hLx(t_\kappa) + he(t_\kappa) + \alpha hLx_\sigma(t_\kappa)) \\ &= \frac{1}{2}(x(t_\kappa)^T + hx(t_\kappa)^T L^T + he(t_\kappa)^T + \alpha hx_\sigma(t_\kappa)^T \\ &\quad \times L^T)L(x(t_\kappa) + hLx(t_\kappa) + he(t_\kappa) + \alpha hLx_\sigma(t_\kappa)) \\ &= \frac{1}{2}(x(t_\kappa)^T Lx(t_\kappa) + 2hx(t_\kappa)^T L^2 x(t_\kappa) + h^2 x(t_\kappa)^T \\ &\quad \times L^T LLx(t_\kappa) + h^2 e(t_\kappa)^T LLx(t_\kappa) + h^2 x_\sigma(t_\kappa)^T \alpha L^T L \\ &\quad \times Lx(t_\kappa) + \alpha hx(t_\kappa)^T L^2 x_\sigma(t_\kappa) + \alpha h^2 x(t_\kappa)^T L^T L^2 x_\sigma \\ &\quad \times (t_\kappa) + \alpha h^2 e(t_\kappa)^T LLx_\sigma(t_\kappa) + hx(t_\kappa)^T Le(t_\kappa) + h^2 x \\ &\quad \times (t_\kappa)^T L^T L\, e(t_\kappa) + h^2 e(t_\kappa)^T Le(t_\kappa) + \alpha h^2 x_\sigma(t_\kappa)^T L^T \\ &\quad \times Le(t_\kappa) + he(t_\kappa)^T Lx(t_\kappa) + \alpha hx_\sigma(t_\kappa)^T L^T Lx(t_\kappa)). \end{aligned} \tag{12}$$

Further, there is

$$\begin{aligned} \Delta V(t_\kappa) &= V(t_{\kappa+1}) - V(t_\kappa) \\ &= \frac{1}{2}(2hx(t_\kappa)^T L^2 x(t_\kappa) + h^2 x(t_\kappa)^T L^T LLx(t_\kappa) + h^2 e \\ &\quad \times (t_\kappa)^T LLx(t_\kappa) + h^2 x_\sigma(t_\kappa)^T \alpha L^T LLx(t_\kappa) + \alpha hx(t_\kappa)^T \\ &\quad \times L^2 x_\sigma(t_\kappa) + \alpha h^2 x(t_\kappa)^T L^T L^2 x_\sigma(t_\kappa) + \alpha h^2 e(t_\kappa)^T LL \\ &\quad \times x_\sigma(t_\kappa) + hx(t_\kappa)^T Le(t_\kappa) + h^2 x(t_\kappa)^T L^T Le(t_\kappa) + h^2 e \\ &\quad \times (t_\kappa)^T Le(t_\kappa) + \alpha h^2 x_\sigma(t_\kappa)^T L^T Le(t_\kappa) + he(t_\kappa)^T Lx \\ &\quad \times (t_\kappa) + \alpha hx_\sigma(t_\kappa)^T L^T Lx(t_\kappa)). \end{aligned} \tag{13}$$

Set $\vartheta(t_\kappa) = Lx(t_\kappa)$, then can obtain

$$\begin{aligned} &\Delta V(t_\kappa) \\ &= \frac{1}{2}(2h\vartheta(t_\kappa)^T \vartheta(t_\kappa) + he(t_\kappa)^T \vartheta(t_\kappa) + h \\ &\quad \times x_\sigma(t_\kappa)^T \alpha L\vartheta(t_\kappa) + h^2 \vartheta(t_\kappa)^T L\vartheta(t_\kappa) + h^2 e(t_\kappa)^T L \\ &\quad \times \vartheta(t_\kappa) + \alpha h^2 x_\sigma(t_\kappa)^T L^2 \vartheta(t_\kappa) + h\vartheta(t_\kappa)^T e(t_\kappa) + h^2 \\ &\quad \times \vartheta(t_\kappa)^T Le(t_\kappa) + h^2 e(t_\kappa)^T Le(t_\kappa) + \alpha h^2 x_\sigma(t_\kappa)^T L^T \\ &\quad \times Le(t_\kappa) + \alpha h\vartheta(t_\kappa)^T Lx_\sigma(t_\kappa) + \alpha h^2 \vartheta(t_\kappa)^T L^2 x_\sigma(t_\kappa) \\ &\quad + \alpha h^2 e(t_\kappa)^T L^2 x_\sigma(t_\kappa)). \end{aligned} \tag{14}$$

From inequality $\|e_i(t_\kappa)\| \le \gamma_i \|f_i(t_\kappa)\|$, $f_i(t_\kappa) = -\vartheta_i(t_\kappa)$, then $\|e_i(t_\kappa)\| \le \gamma_i \|\vartheta_i(t_\kappa)\|$. Combined with Lemma 1 gives the following formula

$$\begin{aligned} e(t_\kappa)^T e(t_\kappa) &\le \gamma^2{}_M \vartheta(t_\kappa)^T \vartheta(t_\kappa), \\ e(t_\kappa)^T Le(t_\kappa) &\le \gamma^2{}_M \lambda_n \vartheta(t_\kappa)^T \vartheta(t_\kappa), \\ \vartheta(t_\kappa)^T e(t_\kappa) &\le \gamma_M \vartheta(t_\kappa)^T \vartheta(t_\kappa), \\ \vartheta(t_\kappa)^T Le(t_\kappa) &\le \gamma_M \lambda_n \vartheta(t_\kappa)^T \vartheta(t_\kappa), \\ e(t_\kappa)^T \vartheta(t_\kappa) &\le \gamma_M \vartheta(t_\kappa)^T \vartheta(t_\kappa), \\ e(t_\kappa)^T L\vartheta(t_\kappa) &\le \gamma_M \lambda_n \vartheta(t_\kappa)^T \vartheta(t_\kappa), \\ \vartheta(t_\kappa)^T L\vartheta(t_\kappa) &\le \lambda_n \vartheta(t_\kappa)^T \vartheta(t_\kappa). \end{aligned}$$

Among them $\gamma_M = \max\{\gamma_i | i = 1, 2, \ldots\ldots, N\}$, λ_n indicates the largest eigenvalue of the Laplacian matrix L. By substituting all the above inequalities into (14), the above formula becomes

$$\begin{aligned} \Delta V(t_\kappa) &\le \frac{1}{2}((2h + 2h\gamma_M + h^2\lambda_n + 2h^2\gamma_M\lambda_n + h^2 \\ &\quad \times \gamma^2{}_M \lambda_n)\vartheta(t_\kappa)^T \vartheta(t_\kappa) + hx_\sigma(t_\kappa)^T \alpha L\vartheta(t_\kappa) + \alpha h^2 \\ &\quad \times x_\sigma(t_\kappa)^T L^2 \vartheta(t_\kappa) + \alpha h^2 x_\sigma(t_\kappa)^T L^T Le(t_\kappa) + \alpha h\vartheta \\ &\quad \times (t_\kappa)^T Lx_\sigma(t_\kappa) + \alpha h^2 \vartheta(t_\kappa)^T L^2 x_\sigma(t_\kappa) + \alpha h^2 e(t_\kappa)^T \\ &\quad \times L^2 x_\sigma(t_\kappa)). \end{aligned} \tag{15}$$

Then according to the property of norm, we can get

$$\begin{aligned} &\Delta V(t_\kappa) \\ &\le \frac{1}{2}((2h + 2h\gamma_M + h^2\lambda_n + 2h^2\gamma_M\lambda_n \\ &\quad + h^2\gamma^2{}_M\lambda_n) \left\|\vartheta(t_\kappa)^T \vartheta(t_\kappa)\right\| + \alpha h \left\|x_\sigma(t_\kappa)^T\right\| \\ &\quad \times \|L\| \|\vartheta(t_\kappa)\| + \alpha h^2 \left\|x_\sigma(t_\kappa)^T\right\| \left\|L^2\right\| \|\vartheta(t_\kappa)\| \\ &\quad + \alpha h^2 \left\|x_\sigma(t_\kappa)^T\right\| \left\|L^2\right\| \|e(t_\kappa)\| + \alpha h \left\|\vartheta(t_\kappa)^T\right\| \|L\| \\ &\quad \times \|x_\sigma(t_\kappa)\| + \alpha h^2 \left\|\vartheta(t_\kappa)^T\right\| \left\|L^2\right\| \|x_\sigma(t_\kappa)\| + \alpha h^2 \\ &\quad \times \left\|e(t_\kappa)^T\right\| \left\|L^2\right\| \|x_\sigma(t_\kappa)\|). \end{aligned} \tag{16}$$

When $h < \frac{-2-2\gamma_M}{2\gamma_M\lambda_n + \lambda_n + \gamma^2{}_M\lambda_n}$ and $\alpha < 0$, we can calculate that $\Delta V \le 0$. The analysis shows that when $\|n_i(t_\kappa)\| = 0$, there is $\Delta V = 0$. If $\|n_i(t_\kappa)\| = 0$, because of $f_i(t_\kappa) = -n_i(t_\kappa)$, then $f_i(t_\kappa) = 0$ is obtained, so $\|e_i(t_\kappa)\| = \gamma_i \|f_i(t_\kappa)\| = 0$. According to formula (5), all $f_i(t), f_i(t_\kappa)$ are equal to 0. Combined with the definition of joint measurement given by formula (4), it can be concluded that the system meets the requirement of group consensus (2). Therefore, the HMAS achieves group consensus.

When the event-triggering mechanism is used, the system may produce an infinite number of triggers in a limited time. The event-triggering condition designed in this paper is sampled at intervals of $h (h > 0)$. This ensures that all adjacent triggering intervals are either equal to or greater than the sampling interval. It effectively avoids the occurrence of countless triggers within a limited time. So the Zeno phenomenon was fundamentally avoided.

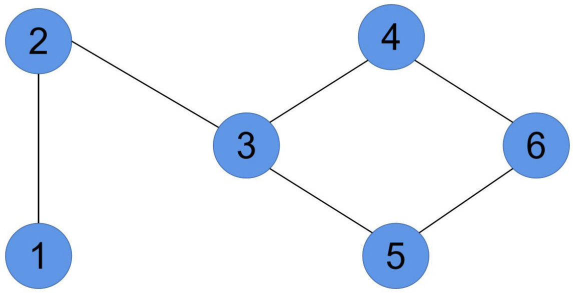

FIGURE 1. Structure topology of hybrid multi-agent system.

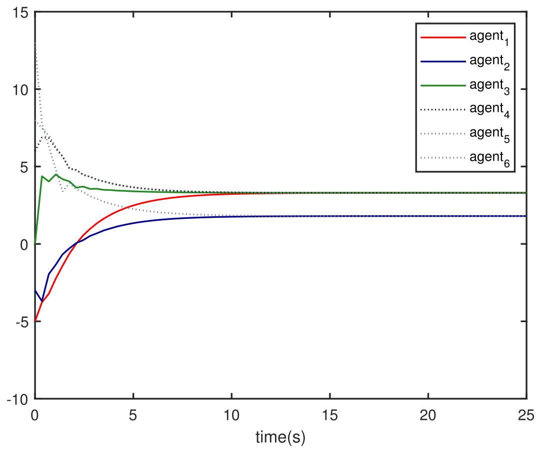

FIGURE 2. Agents position status change trend.

IV. CASE SIMULATION

Considering the HMAS is composed of 6 multi-agents, among which $1, 2, 3$ represent the continuous-time multi-agents, and $4, 5, 6$ represent the discrete-time multi-agents. The communication structure of whole system is shown in the Fig. 1 below

Through the communication topology of the system, the Laplacian matrix L expressed as:

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & -1 & 0 \\ 0 & 0 & -1 & 2 & 0 & -1 \\ 0 & 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}.$$

Its maximum eigenvalue can be obtained by the above Laplace matrix as $\lambda_n = 3$. Combined with the above system stabilization conditions, sampling cycle time $h = 0.2s$ and $\alpha = -1$. Assuming the initial positions are $x(0) = [-5\ \text{-}3\ 0\ 6\ 8\ 13\ ]^T$. Under the action of the controller, the position states of multi-agents in the system vary with time as shown in Fig.2. Then, Fig.3 represents the input of controller and Fig.4 represents the triggering time of each agent.

Simulation analysis: Fig.2 indicates the HMAS (1) under the event-triggering condition makes agents 2, 6 and 1, 3, 4, 5 realize clustering. And the location trajectory of each subgroup eventually tends to be consistent with the passage of time. Fig.3 reflects the change trend of control input of each agents over time. From the figure, we can see the control input of agents in the system presents discrete distribution, and the control input eventually tends to 0. Fig.4 shows the triggering time of each agents under the event-triggering condition. It can be seen that the controller of the agent is updated less frequently under the event-triggering condition. And the Zeno phenomenon is not occur during operation.

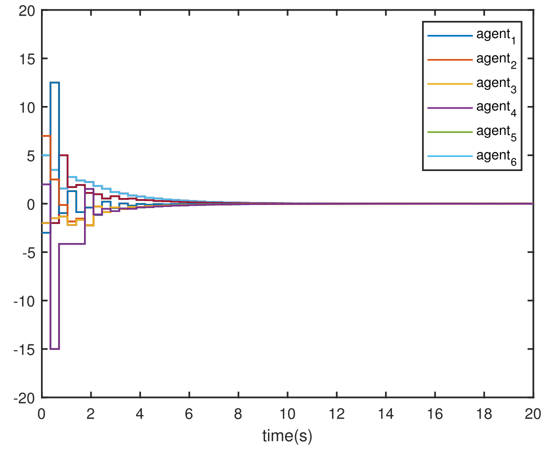

FIGURE 3. Control input of agents under event-triggering condition.

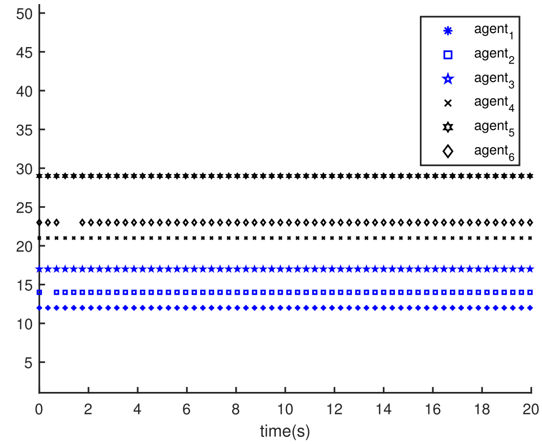

FIGURE 4. Triggering moment of agents under event-triggering condition.

V. CONCLUSION

After research, we explored the group consensus problem of HMASs under the event-triggering conditions. First, a HMAS has been constructed in which the intelligences have both continuous and discrete states. Then a distributed group consensus controller has been proposed, by which the effect of subgroup consensus can be achieved. In order to reduce controller updates, we proposed an event-triggering condition based on joint measurement method. The event-triggering strategy has been sampled with time interval h. So the occurrence of Zeno phenomenon has been avoided fundamentally.

Then Lyapunov's method has been used to obtain the condition of system stability. Finally, the research results of this paper have been tested by numerical simulation. In the future, we will study more complex system models and apply the research results to directed topology.

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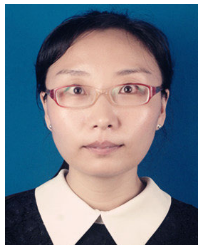

HUIQIN PEI (Member, IEEE) received the B.Sc. degree in automation from East China Jiaotong University, Nanchang, China, in 2007, the M.S. degree in control theory and control engineering, in 2010, and the Ph.D. degree in control science and engineering from East China Jiaotong University, in 2017.

She is currently an Associate Professor with the School of Electrical and Automation Engineering, East China Jiaotong University. Her current research interests include swarm dynamics and cooperative control and multi-agent systems.

YE ZHOU received the B.Sc. degree in electrical engineering and automation from the School of Electrical and Automation Engineering, Yancheng Normal College, Yancheng, China, in June 2020. He is currently pursuing the M.S. degree in control engineering with the School of Electrical and Automation Engineering, East China Jiaotong University, Nanchang, China.

JIANHENG TAN received the B.Sc. degree in automation from the School of Information Science and Engineering, Dalian Polytechnic University, Dalian, China, in June 2021. He is currently pursuing the M.S. degree in control engineering with the School of Electrical and Automation Engineering, East China Jiaotong University, Nanchang, China.