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RESEARCH ARTICLE

# CCD-BSMG: Composite-Curve-Dilation-Based Brush Stroke Model Generator for Robotic Chinese Calligraphy

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**ABSTRACT** Brush stroke training models for Chinese brush play an important role in robotic Chinese calligraphy as the basis of stroke generation. How to combine the end-to-end method and physical model needs to be further clarified. As a large number of brush strokes collected by robotic arm for learning and training lacks operability. The simulated brush stroke model can be used as a source of training datasets for deep learning and training instead of real samples collected by robot. With different combinations of physical parameters of the robotic arm, a sufficient amount of sample data can be generated by the simulated brush stroke model and then to train a generator based on the datasets. In this way, we propose a composite-curve-dilation brush stroke model generator (CCD-BSMG) based on our composite-curve-dilation brush stroke model (CCD-BSM), which was formed by composite curve and morphological dilation directly according to the physical parameters of robotic arm and writing posture of the brush without a large number of samples for parameter estimation. The CCD-BSMG can generate the graphics with the dataset simulated by CCD-BSM for deep learning and training. Furthermore, with the output parameters of CCD-BSMG, the images reconstructed by robotic arm can have a better performance. Compared with other stroke generative models based on disordered pixels, our generator is based on parameterized brush strokes and provides a better foundation for robotic writing in deep learning or other fields. Compared with existing model and real stroke graphics written by robots, the results of several experiments prove that the proposed CCD-BSMG can generate stroke graphics well and show that it outperformed state-of-the-art stroke models. The results demonstrate the advantages of our proposed model in terms of high similarity and especially show the robustness and efficacy.

**INDEX TERMS** Stroke training model, Chinese brush, robotic Chinese calligraphy, brush stroke neural generator.

## I. INTRODUCTION

Chinese calligraphy is a charming and ancient form of artistic expression of Chinese characters. It is an important part of Chinese art and has a high artistic appreciation value with a long history [1]. As an indispensable calligraphy and painting tool for traditional Chinese culture, the soft hairy brush has occupied a lofty position in Chinese culture for thousands of years [2]. In recent years, the robotic arms have been employed to simulate human calligraphy behavior and calligraphy creation [3], [4]. As an important application of intelligent robot, robotic Chinese calligraphy can provide helpful ideas for solving other similar problems, which will promote the further research and development of intelligent robot. The application of calligraphy robots is not only employed in writing, but also can be extended into other

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operation fields, such as robot welding, robot painting, robot spraying and so on.

Unlike hard and elastic pens, which are not soft hairy brush, they are not required to consider the extrusion and dilation of brush pen when writing on paper [5]. In contrast, writing with a soft hairy brush is even more difficult especially for robotic arms. The brush is pressed on paper at some depth $H$ and angle of inclination $\alpha$ without moving to generate a particular graphics, namely the stroke, which is a basic unit of a trace of a character. $H$ and $\alpha$ are controlled by the robotic arm and will vary with the movement of the brush pen of robotic end effector when writing a character along trajectory points, thus robot parameters $H$ and $\alpha$ are important in the calligraphy task. Only with the stroke can we study robotic Chinese calligraphy. The construction of brush stroke model lays the foundation for Chinese calligraphy. Thus, how to train a brush stroke generator based on the images or computational model with physical parameters of robotic arm closer to real writing samples should be solved and addressed.

The current study of robotic Chinese calligraphy mainly focuses on trajectory generation of strokes. The trajectory models have been trained and optimized to generate the writing trajectory using reinforcement learning [6], [7] or generative adversarial networks (GANs) [8], [9], [10], [11]. The more important stroke model was not considered, especially the relationship between the parameters of the robotic arm and the generated stroke models. Even worse, the robotic arm was used to generate calligraphy images each time during learning and training, repeated writing may damage the hardware of the robotic arm seriously. Some brush stroke models have been synthesized based on graphic generation with physical parameters [12], [13], [14], [15]. Compared to other existing models, Bézier-curve-based stroke methods [12] have achieved better performance. Although the parameters of the robotic arm were integrated into the simulation model, they all used the linear regression method to predict the model parameters. Recently, neural renderers based on brushstrokes for painting have been proposed in deep reinforcement learning [16], [17], [18]. Since more attention has been paid to the generation of stroke trajectory or the control of robotic arm. Thus, the improvements or innovation of basic stroke model has somewhat been neglected, strokes have still been designed as traditional curves for training in neural renderers, such as Bézier curves, circles, or even straight strokes.

So far, the end-to-end method is essentially to build a generative model based on neural network, and the training models of some existing end-to-end methods were based on the classic brush simulation model, which is Bézier curve to generate a large number of training samples randomly. However, this classic Bézier curve has two main problems:

1) The definition of the parameters was mainly defined according to the characteristics of the model, there is no correlation between the brush specifications and posture parameters directly;

2) As the Bézier curve requires the samples for parameter estimation, it still needs the robotic arm to write hundreds of times or even more. Even worse, the parameters only for the fixed specifications of the brush pen, the replacement of the brush specifications will require re-sampling to update the model parameters.

3) The physical parameters of the proposed method are directly obtained from specifications of the brush and the posture of robotic arm. Moreover, the new and novelty of the designed composite curve can be directly solved, which not only addresses the above two problems, but also demonstrates the advantages in terms of high similarity in comparison with the real writing samples.

The simulation stroke generator we proposed named CCD-BSMG, which is a composite-curve-dilation brush stroke generator considering both brush specification and posture for robotic Chinese calligraphy. The CCD-BSMG based on neural network requires a large number of samples to learn and train. Through a great deal of samples for learning and training, the strokes and even Chinese characters generated by the robotic arm are similar to the real calligraphy. The ideal training dataset is to directly utilize real samples collected by robotic arm for training, and samples are the corresponding images generated by the robotic arm with control parameters $(H, \alpha)$, so that the trained model can be established between physical parameters and brush stroke images. But this practice is not realistic at present, as plenty of repeated collecting will certainly lead to the hardware damage of the robotic arms, then cause rapid loss of the service life of the robotic arms. Due to the robotic arm, plenty of samples used for learning and training are difficult to obtain. What's worse, even under the same parameters, the samples generated by the robotic arm may be different from the reference images, which cannot be employed as dataset for training. In this case, we opted to use sample data generated by CCD-BSM instead of directly utilizing real samples for training. We need to solve directly or only need a small number of samples for parameter estimation model to approximate the real samples, and then to train the end-to-end generator by using deep learning method. From the perspective of geometry and graphics, geometric curves are used to simulate the real stroke contour and define the parameters of simulated model CCD-BSM, which is directly simulated from specifications of the brush and the posture of robotic arm without a large number of samples for parameter estimation. Moreover, the new and novelty of the designed composite curve can be directly solved. The simulated CCD-BSM images can be served as a source of training datasets to replace real samples collected by robot with different combinations of parameters $H$ and $\alpha$, Where $H$ and $\alpha$ denote the descent height and inclined angle of the brush pen affixed to the end-effector of the robot respectively. Then, the CCD-BSMG can be trained with parameters of CCD-BSM. By using simulated brush stroke images, rather than reference images, we can obtain a large number of images similar to the reference images effectively and rapidly. Experimental results demonstrate the advantages

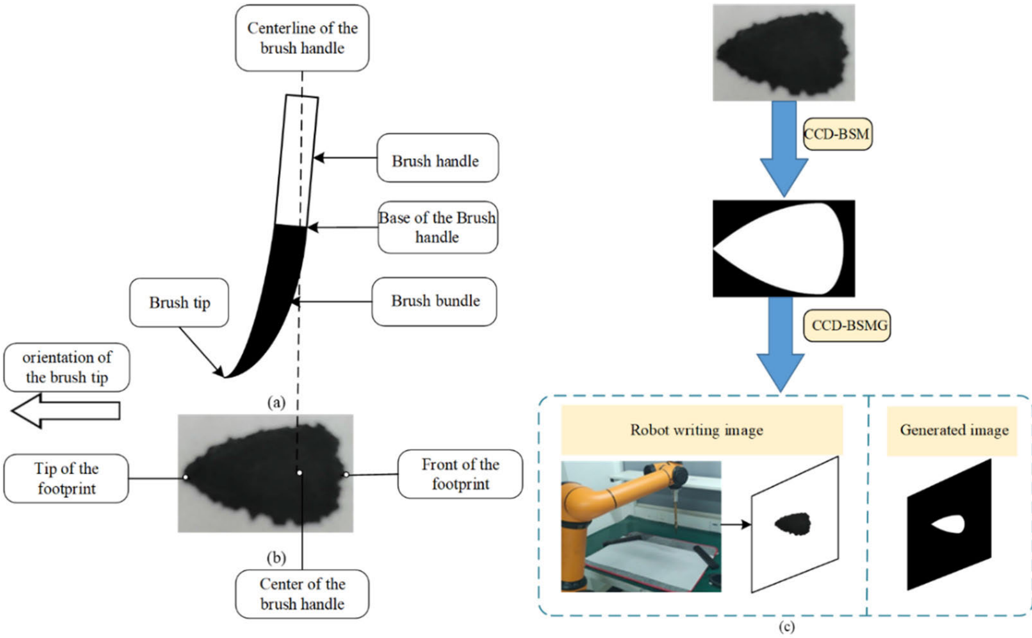



**FIGURE 1.** The image of a stroke written by a brush pen and the proposed CCD-BSMG. (a) shows the curve brush bundles produce the stroke by pressing the brush handle, (b) shows the stroke left by the brush on the paper, and (c) shows the proposed CCD-BSMG.

of CCD-BSM and CCD-BSMG in terms of high similarity in comparison with the real writing samples. FIGURE 1 shows the image of a stroke written by a brush pen and the proposed CCD-BSMG. In summary, the main contributions of this paper include the following:

1) A novel stroke-based generator based on measurable and controllable parameters with better generalization is proposed.

2) In the proposed generator, a differentiable neural generator is built for better performance and can support different stroke designs, which helps the generator to achieve better flexibility.

3) Several experiments show that the proposed generator based on our stroke model outperforms some existing neural renderers in stroke graphics generation for Chinese brush.

## II. RELATED WORKS

### A. EMIPIRICAL MODEL BASED ON VIRTUAL BRUSH

Wong and IP [13] focused on the design of the brush bundle. By the perpendicular intersection of the paper and the brush bundle, a cross-section was formed to generate a stroke. The statics model of Wong and IP is obviously inconsistent with the facts. In the actual writing process, it is impossible for the brush to penetrate the surface of the paper. Based on the model proposed by Wong and IP, the brush bundle of CCD-BSM [19] was approximated as a three-dimensional inverted cone. Zhang et al. [3] presented statistical methods to simulate the stroke with a closed area enclosed by two symmetrical 3rd Bézier curves and applied linear regression analysis to obtain the stroke model parameters. On this basis, Wang and Min [12] collected a large number of brush strokes written by the robotic arm to obtain a least-squares multiple linear regression model. In the model, the mathematical relationship was described between the length of the brush bundle tip, the length of the brush bundle base, the length of the brush bundle belly and the descending height and tilt angle of the brush. Inspired by Wang Yuzhuo, we collected the brush stroke samples based on this method, then constructed datasets for deep learning and training. Joshi [14] proposed the combination of trajectory and strokes, regarded the formation of brush strokes as a continuous superimposition of circular strokes along the trajectory.

### B. NEURAL GENERATOR FOR ROBOTIC CHINESE CALLIGRAPHY

Existing intelligent robot writing mainly includes stroke models based on GAN, RNN and LSTM [8], [9], [20], [21], [22], [23], Auto-Encoder [24] and deep reinforcement learning [6], [7], [16], [17], [18].

Chao et al. [8] regarded the robot writing as an adversarial process, and the G network was used for generating the probability distribution of stroke trajectories. GANCC Robot [9] improved the traditional GAN framework, added the type of stroke and the latent code information representing the characteristics of different strokes. The RCCHP [20] framework proposed the random sample as input of G network to obtain the position of the stroke trajectory, and the output was the trajectory distribution of Chinese characters. The G network designed by Wu et al. [21] used U-Net structure. The D network was composed of strided convolutional layers, which can reduce the output dimension to 2D map. Each element in the map represented the possibility whether the input is true or false. Liang [22] proposed a robot writing method based on GAN to generate target fonts and realize style conversion. LSTM-GAN [23] generated the writing trajectory of Chinese character strokes by converting the pixel of stroke image into vector trajectory sequence controlled by the robotic arm.

Gao et al. [24] treated the random vector of writing trajectory as normal distribution. A convolutional auto-encoder was proposed to evaluate the writing quality. The differential evolution algorithm (DE) was applied to generate and optimize the writing trajectory model. The GAN-AC generated by Wu et al. [6] had a set of Gaussian noise as input and stroke images as output. GAN-AC [6] combined GAN with the actor-critic (AC) model of deep reinforcement learning, applied deep deterministic policy gradient (DDPG) algorithm to train the trajectory written by robotic arm and simulate human calligraphy. Wu et al. [7] adopted a stochastic policy gradient (SPG) to generate the probability distribution of the current writing result. All of the above mainly learn from stroke trajectory, without considering the brush stroke model. The writing trajectory are generated randomly by training the probability distribution of stroke images. Brushstrokes have also been incorporated an artistic style named stroke-based rendering by using deep reinforcement learning [16], [17], [18] recently. The input image can be converted to a series of brushstrokes based on Bézier curves [18] without physical parameters. The work of [25], [26], and [27] trained to generate brushstrokes and images that can be given by the parametric representation. The work of [25] considered the straight line brush stroke to train stroke-based neural network, which was used for painting and not suitable for the Chinese calligraphy. For better performance, our brush stroke neutral renderer was simulated with the brush specification and the physical parameters of the robotic end effector.

Except robotic calligraphy, more attentions have been paid to robotic painting with multicolor recently [28], [29], [30]. The work [28] proposed the custom-built robot which can accurately mix artistic dyes and make it possible to reproduce colors of brushstrokes. This work designed a novel mathematical model used for artistic paint mixing, developed the principles of mixing paints. The work [29] presented a robotic painting system to mix colors and achieved adaptive paint mixing.

## III. PROPOSED METHOD

When collecting brush stroke images, the real training data sets are very difficult to obtain due to various external factors, which are the state of hairy brush bundle of the brush pen, the thickness of the ink and the rice paper. As lack of the training samples, neural network cannot be performed. The CCD-BSM simulated brush stroke images can provide a source of training datasets. In this case, CCD-BSMG is divided into two parts. First, the simulated brush stroke model CCD-BSM is established. Then the CCD-BSM used as the ground truth is fed to neural network generator for learning and training brush stroke images. The overview of the proposed framework is shown in the FIGURE 2.

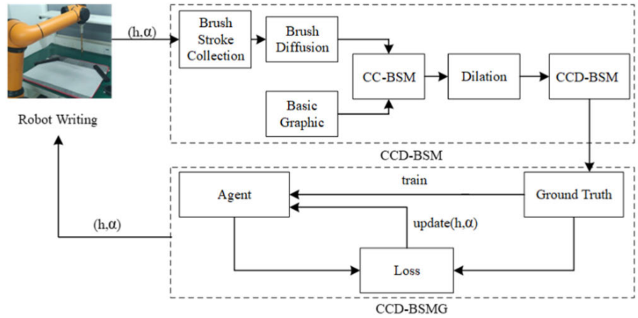



**FIGURE 2.** The overview of the proposed framework for robotic Chinese calligraphy.

### A. PROPOSED COMPOSITE-CURVE-DILATION BASED BRUSH STROKE MODEL

In order to reproduce the effect of real stroke precisely, we used two steps to build the proposed simulation stroke model CCD-BSM [19]. First, without considering dilation, CCD-BSM was established the mathematical expression between brush stroke and $L, R, H, \alpha$. The proposed stroke model CCD-BSM was defined as $M = \{L, R, H, \alpha\}$. It was consisted of four parameters, where $L$ and $R$ respectively denote the length and the maximum radius of the hairy brush bundle, which are parameters of the brush pen, and $H$ and $\alpha$ denote the descent height and inclined angle of the brush pen respectively, which are related to the control of robot end-effector. Based on these four parameters, an elliptical cross section of the approximate three-dimensional inverted cone of the brush bundle and paper can be determined. Then, two symmetrical parabolas were designed with two constraints: being tangent to the elliptical cross section, and intersecting at the brush tip. Thus, the two parabolas and half of the contour of the elliptical cross section formed a closed composite curve. It was the basic graphic of CCD-BSM.

The basic graphic was dilated in a single direction with a fixed coefficient, to simulate the extrusion diffusion of the brush hairs in writing. Two hypotheses were adopted and analyzed without parameter estimation, to obtain a theoretical and definite value of the coefficient $\lambda$. They were the constant perimeter assumption and area invariant assumption. A constant perimeter assumption assumed that the cross-section of the brush was extruded into two overlapping lines with the same length as the circumference of the section. Under the assumption of area invariance, the cross-section was composed of countless non-deformable circles. The extruded section was an isosceles triangle with the same area as the original circle section. As shown in FIGURE 3, by comparing the two hypothesis, the area invariance assumption dilation outperformed the others obviously.

In summary, the dilation coefficient $\lambda = \pi$ was complemented in the basic graphic of CCD-BSM and the dilated

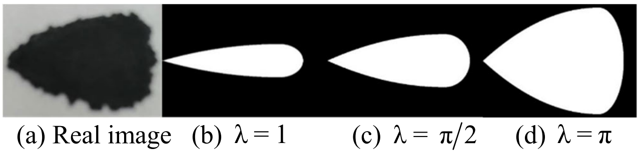



**FIGURE 3.** **The Comparison between real image and CCD-BSM with different dilation coefficient. (a) shows the real stroke image, (b) shows the simulated image with coefficient $\lambda = 1$, (c) shows the simulated image with coefficient $\lambda = \pi/2$, and (d) shows the simulated image with coefficient $\lambda = \pi$.**

model can be derived as follows:

$$y = \begin{cases} \pm\pi \cdot \sqrt{(O_3C)^2(1 - \frac{x^2}{(O_3A)^2})}, x \geq 0 \\ \pm\pi \cdot [\frac{O_3C}{(O_4O_3)^2}x^2 - O_3C],\ x < 0 \end{cases} \tag{1}$$

where $O_3A$, $O_3C$, $O_4O_3$ were expressed as follows:

$$O_3A = \frac{H}{\tan(90^\circ - \alpha)} - \frac{H}{\tan(90^\circ - \alpha + \theta)}, \tag{2}$$

$$O_3C = \frac{H \cdot R}{L \cdot \sin(90^\circ - \alpha)}, \tag{3}$$

$$O_4O_3 = \frac{H}{\sin(90^\circ - \alpha + \theta)} + \frac{H}{\tan(90^\circ - \alpha)} - \frac{H}{\tan(90^\circ - \alpha + \theta)}. \tag{4}$$

$$\theta = \arctan(\frac{R}{L}) \tag{5}$$

The parameters of the stroke model $M = \{L, R, H, \alpha\}$ were given by the specification parameters of hairy brush bundle and the posture parameters of the robotic arm, $\lambda$ was a constant. Thus, all the parameters of CCD-BSM were known, without parameter estimation and any other assumed variables to adjust the empirical parameter values.

### B. COMPOSITE-CURVE-DILATION BASED GENERATOR CCD-BSMG

The generator CCD-BSMG is based on composite-curve-dilation generator CCD-BSM. It is used to train the agent closed to the ground truth simulated by CCD-BSM. In addition, CCD-BSMG can be extended to other brush stroke models. All of the models can use the same network architectures structure. FIGURE 4 shows the CCD-BSMG based on CCD-BSM. From Eq.(1) to (5), CCD-BSM is represented by pressing depth $H$ and the tilt angle $\alpha$. These parameters together with the base point coordinate in the two-dimensional plane are fed to CCD-BSMG, which is used for ground truth to train and update the agent.

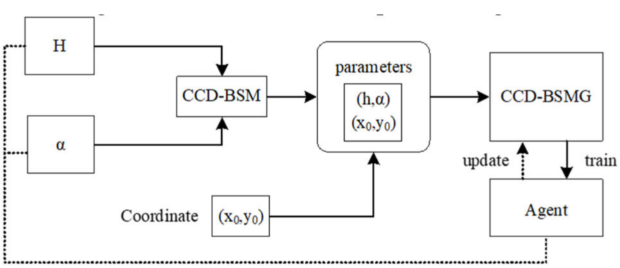



**FIGURE 4.** **CCD-BSMG based on simulated brush stroke CCD-BSM.**

#### 1) NEURAL GENERATOR CCD-BSMG

The input of CCD-BSMG is a set of parameter$M = \{L, R, H, \alpha\}$ and the starting coordinate $(x_0,y_0)$. The outputs are the brush stroke images. Computer graphics are used to generate training samples. The neural network generator can be learned and trained with supervised learning. The premise of learning and training is that the geometric curve of the simulated brush stroke model needs to have a closed-form gradient. The images of proposed CCD-BSM are all continuously differentiable. To train agent successfully, it is necessary to continuously approximate the discrete pixel position and pixel value of two-dimensional image when deriving the gradient, thus the error can be back-propagated.

There are two main advantages to using neural networks to generate brush stroke. First, the neural generator can flexibly generate the brush stroke of any given model. It is more efficient on GPU's powerful calculate capability. Second, the neural generator is differentiable, then end-to-end training can be performed to improve the performance of the agent.

#### 2) NETWORK ARCHITECTURES

The network architectures of CCD-BSMG is composed of several fully connected neural network (FCN/MLP) and convolutional neural network CNN. By taking the model parameters $(H, \alpha)$ of the robotic Chinese calligraphy as the input of FCN, two-dimensional matrix brush stroke images can be mapped into output by CNN. FIGURE 5 shows the network architectures of CCD-BSMG.

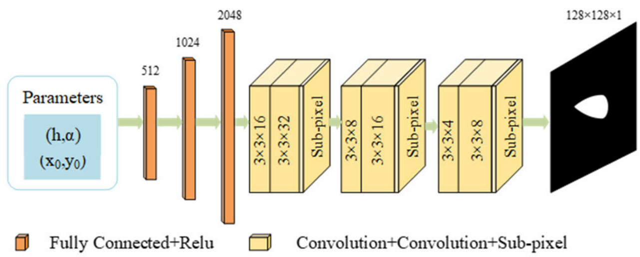



**FIGURE 5.** **Network architectures of CCD-BSMG.**

#### 3) LOSS FUNCTION

The used loss function of CCD-BSMG is the mean squared error(MSE) between the input data $y_i$ and the generated data $\hat{y}i$. All the parameters of CCD-BSMG can be easily optimized through a back-propagation algorithm. The loss function can be expressed as follow:

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{m} wi(yi - \hat{y}i)^2 \tag{6}$$

where $n$ denotes the size of the mini-batch training samples, $yi$ and $\hat{y}i$ denote the input sample and generated sample respectively.

According to the loss function, the training processes of CCD-BSMG were performed to update the parameters of network. The neural generator can write brush strokes based on the $H$ and $\alpha$. In each round of training, the parameters $H$ and $\alpha$ of CCD-BSM used as ground truth data were fed to the generator of CCD-BSMG. By using the Adam algorithm, which is an advanced gradient optimization algorithm, the iterations continue until the objective is optimized or the maximum number of iterations reaches. The pseudo-code of the entire training procedures is summarized in Algorithm 1.

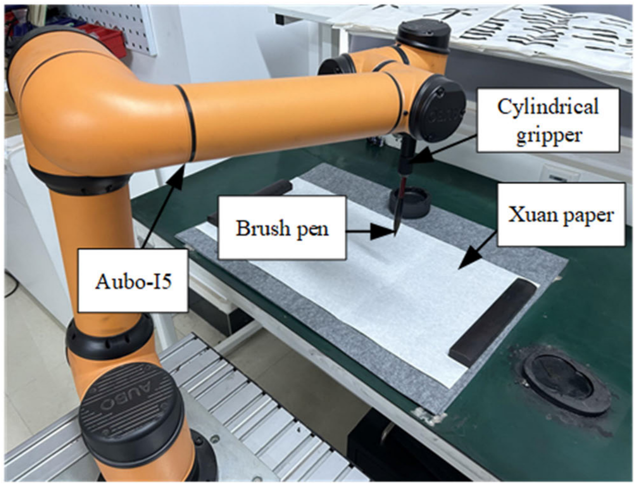
**Algorithm 1** Training procedure of CCD-BSMG

```
Require: R: brush radius; L: brush-bundle length; H:the descending height of robotic arm; α: the tilt angle of robotic arm; pix2len: the conversion coefficient of pixel unit into length (mm) unit; m: the iteration number of output image; G: the maximum iteration number.
Ensure: brush stroke generated by CCD-BSMG, posture parameter H, α;
Initialize R, L, H, α, pix2len;
repeat
   for CCD-BSMG-step do
      produce a set of random number H, α;
      Input H, α into CCD-BSM(H, α);
      Computer O3A, O3C, O4O3 by Eq.(2)(3)(4);
      simulation of CCD-BSM by Eq.(1);
      Input CCD-BSM(H, α) into CCD-BSMG;
      calculate loss by Eq.(6);
       update CCD-BSMG;
       Output loss;
       Output H, α;
   end for
    if step mod m==0
       Output simulated brush stroke image CCD-BSM;
       Output generated brush stroke image CCD-BSMG;
    end if
    CCD-BSMG-step= CCD-BSMG-step+1
until model converges or CCD-BSMG-step>G
```

## IV. EXPERIMENTS AND ANALYSIS

### A. EXPERIMENTAL HARDWARE SYSTEM

All the comparative experiments were based on "Aubo-I5". The configuration of our experimental hardware system is shown in FIGURE 6, the specifications of the system are as follow: (1) collaborative robot: "Aubo-I5", which is a high-precision robotic writing system with repeatability of ±0.05 mm. (2) Hairy brush pen: Langhao brush pen with brush radius R=6mm and brush-bundle length L=48mm (3) Xuan paper: specially used for hairy brush pen. The cylindrical gripper was affixed to the end-effector of the robot, and the brush pen was affixed to the center of the cylindrical gripper, which can be shown in details in FIGURE 6.




(a)Hardware architecture of the Robotic Calligraphy System

(b)The brush pen

**FIGURE 6. Hardware architecture of the Robotic Calligraphy System. (a) shows the Hardware architecture of the Robotic Calligraphy System, and (b) shows the brush pen used in the experiment.**

### B. TRAINING AND TESTING DATASET FOR CCD-BSMG

#### 1) DATA COLLECTION AND PREPROCESSING

In the experiment, the robot first descended vertically to the initial state. The initial state is defined as the brush pen being vertical and perpendicular to the paper, with the tip of the brush just touching the paper without pressure. Then the robot descended to the position $H$ and posture $\alpha$ along the Z-axis. A brush collection with the parameter $(H, \alpha)$ of the robot was completed. We first collected samples with $H = \{11\text{mm},13\text{mm},15\text{mm},17\text{mm},20\text{mm}\}$ and $\alpha = \{0°,5°,10°\}$ respectively, since the parameters $H$ and $\alpha$ are controllable with the high-precision robot writing system. For each combination, 10 stroke images were collected at least using the robotic writing system. After each stroke was collected, the robot's end effector performed the ink dip operation in order to avoid the situation of the poor brush strokes collected caused by split brush bristles, curved brush bristles and the lacking of ink. The collected images needed to be preprocessed for size normalization in order to comparison between the real images and the simulated graphics within the same group of parameters. Then the images were cropped and aligned. Finally, all images were binarized for comparison and similarity calculation. FIGURE 7 shows some of samples of the collected stroke images after preprocessing.

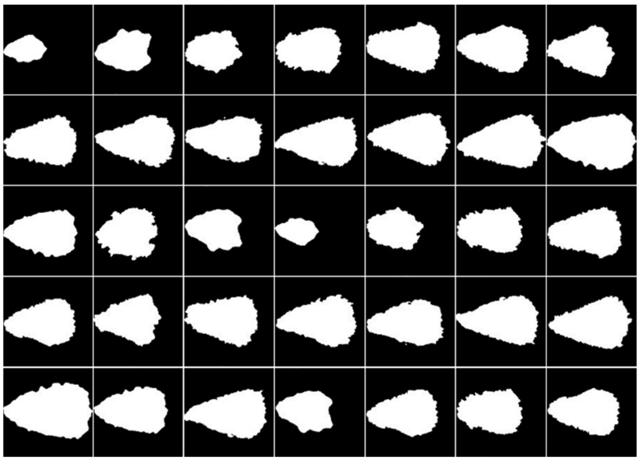


**FIGURE 7. Samples of stroke images.**

### 2) TRAINING AND TESTING DATASET

After data collection and preprocessing, we can generate plenty of simulated CCD-BSM images in the range of $H$=(11mm-20mm) and $\alpha$= {0°-10°} respectively. $H$ and $\alpha$ were corresponded to the range of samples collected by the robotic arm. Especially, the length value of L, R, and H needed to be converted to the pixel value both in the real collected images and the generated images. For normalization, we defined a transformation coefficient $\lambda_{pix2len}$= 0.05 which meant that one pixel in the image was equivalent to the length of 20mm in the fact. Thus, all images can be resized to a consistent resolution of length according to the $\lambda_{\mathrm{pix2len}}$. FIGURE 8 shows some of stroke images in training and testing dataset.

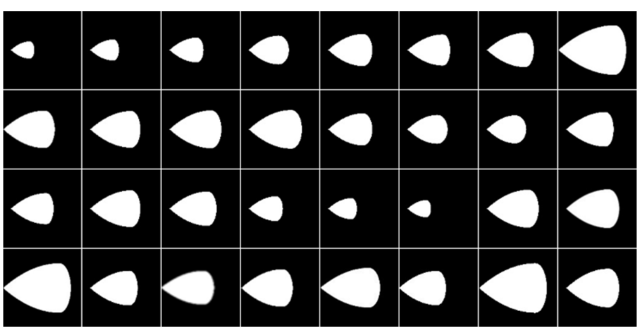
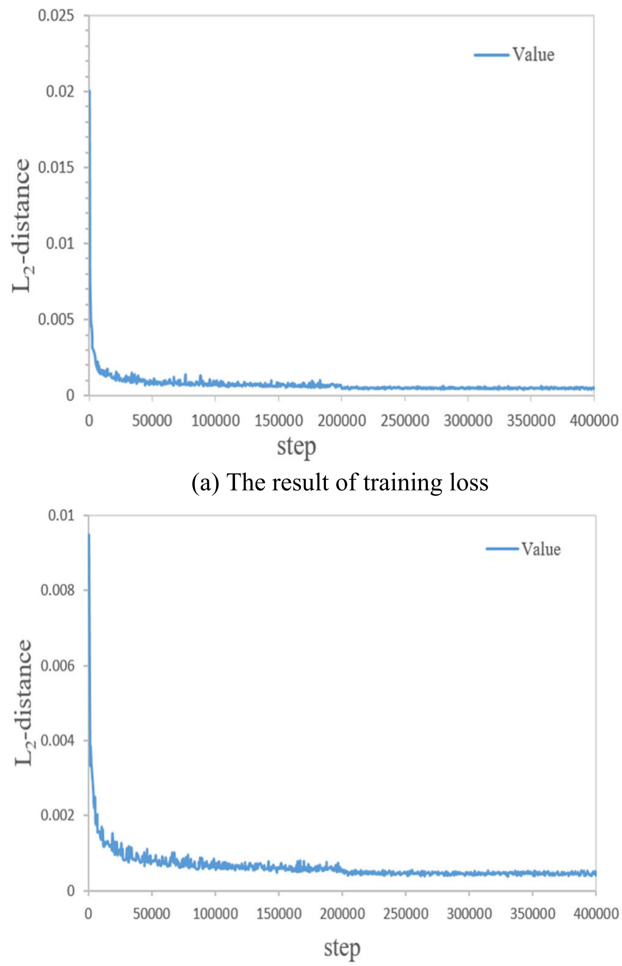


**FIGURE 8. Sample of stroke images in training and testing dataset.**

## C. TRAINING OF CCD-BSMG

All images were resized to the resolution of 128 × 128 pixels before feeding the agent. The whole training procedure was performed on an NVIDIA GeForce RTX 2080 Ti GPU. We used Adam as optimization algorithm and MSE as loss function.

Different batch sizes have an effect on training time, and from a large number of training, we found the relationship between batch size and training time. Considering both the model training time and the performance of the generated images comprehensively, the batch size was set to 64. The results of training loss and validating loss are shown in FIGURE 9.




(a) The result of training loss



(b) The result of validating loss

**FIGURE 9. The result of training loss and validating loss. (a) shows the result of training loss, and (b) shows result of validating loss.**

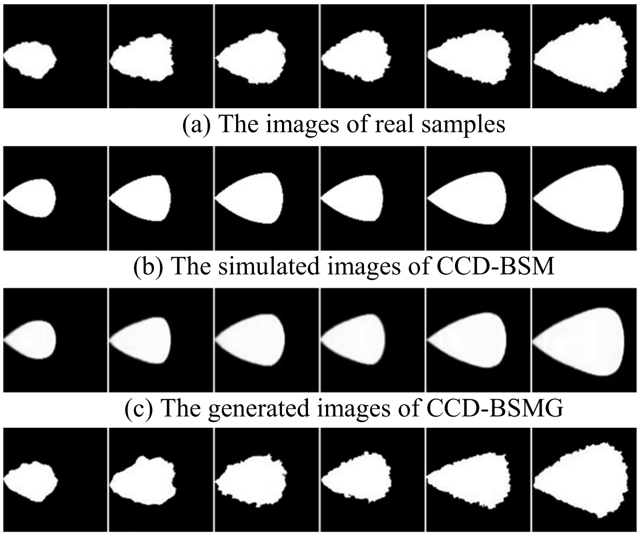
In our experiment, we randomly generated 20000 images by CCD-BSM for our training set with different combinations of parameters $H$ and $\alpha$. Then, we randomly sampled 2000 images as the validation set to validate the generalization ability of the trained model.

## D. RESULTS AND DISCUSSIONS

### 1) COMPARISON WITH CCD-BSM AND REAL SAMPLES

The brush strokes generated by the proposed CCD-BSMG were compared with CCD-BSM and real collected samples.

FIGURE 10 shows the results of these generated models and the corresponding real images collected in our proposed dataset. It can be shown from FIGURE 10 that images of CCD-BSMG were closed to our CCD-BSM with the same model parameters. This shows that our generator based on the CCD-BSM was effective and available. By multiple of training and learning, the parameters of CCD-BSMG can be optimized to reach a lower loss value. What's more, with the output $H$ and $\alpha$, the images of real strokes written by robotic arm can be further generated besides the CCD-BSMG. Evidently, from FIGURE 10 (a) and (d), the real samples and robot writing images are similar to each other with the shape of stroke graphic.



(a) The images of real samples

(b) The simulated images of CCD-BSM

(c) The generated images of CCD-BSMG

(d) The generated images of the robotic arm

**FIGURE 10. Comparison of proposed generated model and the corresponding real images. (a) shows the images of real samples, (b) shows the simulated images of CCD-BSM, (c) shows the generated images of CCD-BSMG, and (d) shows the generated images of the robotic arm.**

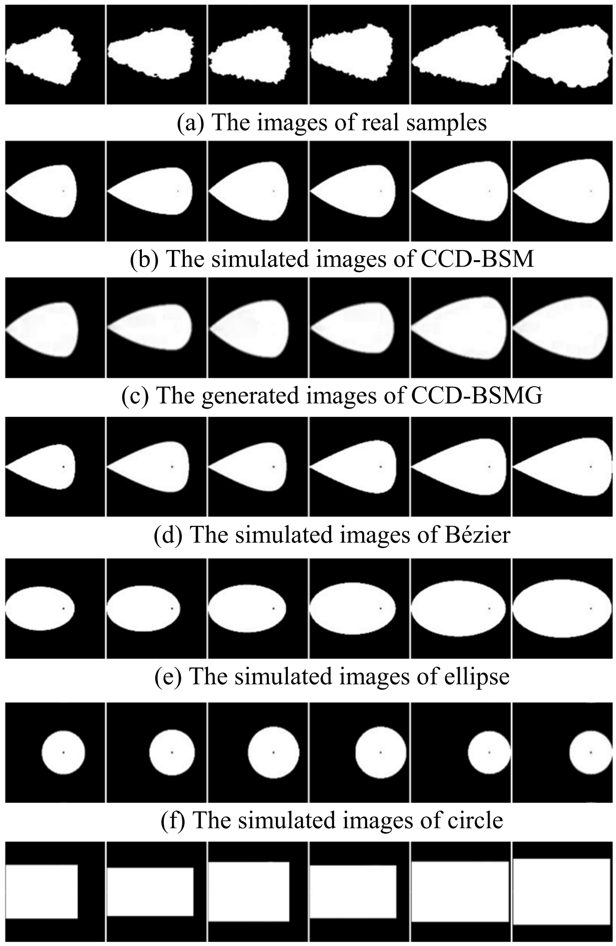
The cosine similarity(CSIM) [7], [24] and Structural Similarity Index Measure (SSIM) [31] were all employed as quantitatively evaluation metrics to indicate the similarity between generated stroke graphics and real stroke images. They were used to evaluate the quality of simulated strokes. TABLE 1 shows the results of different models. From TABLE 1, we can infer that compared to CCD-BSM, the CSIM and SSIM of our CCD-BSMG and the generated images of real strokes written by the robotic arm were higher than that of CCD-BSM, especially the results of robot writing with a maximum similarity of 98.21%. This fully demonstrates that we can reconstruct stroke images by the robotic arm and maintain a good performance with the corresponding output parameters.

### 2) COMPARISON WITH OTHER MODELS

The proposed brush strokes generator CCD-BSMG based on our CCD-BSM was compared with some other typical models.

**TABLE 1. The performance (%) of generated models and the corresponding real images.**

| Model | CSIM | SSIM |
|---|---|---|
| CCD-BSM | 94.25(max) | 90.13(max) |
| CCD-BSM | 88.33(min) | 85.16(min) |
| CCD-BSM | 91.84(avg) | 87.71(avg) |
| CCD-BSMG | 97.29(max) | 90.58(max) |
| CCD-BSMG | 89.22(min) | 85.49(min) |
| CCD-BSMG | 92.65(avg) | 88.16(avg) |
| robot writing | 98.21(max) | 95.62(max) |
| robot writing | 89.42(min) | 85.85(min) |
| robot writing | 93.31(avg) | 90.12(avg) |



(a) The images of real samples

(b) The simulated images of CCD-BSM

(c) The generated images of CCD-BSMG

(d) The simulated images of Bézier

(e) The simulated images of ellipse

(f) The simulated images of circle

(g) The simulated images of straight line

**FIGURE 11. Comparison of real images and the corresponding simulated models. (a) shows the images of real samples, (b) shows the simulated images of CCD-BSM, (c) shows the generated images of CCD-BSMG, (d) shows the simulated images of Bézier, (e) shows the simulated images of ellipse, (f) shows the simulated images of circle, and (g) shows the simulated images of straight line.**

The strokes were constructed as the two symmetrical cube Bézier curves in [3] and [12]. A circle was designed in Joshi [14] and Wang et al. [15]. Wong and IP [13] proposed an ellipse to simulate the stroke graphic. The straight line was considered to represent shape parameters of strokes [25]. FIGURE 11 shows the results of real images and the corresponding simulated models.

In this subsection, the comparison of different methods was based on the same parameters for a fair comparison. From the aspect of morphology, the Bézier-curve-based model, our proposed CCD-BSMG and CCD-BSM can simulate the real stroke graphic better than the other models.

We also calculated the two evaluations CSIM and SSIM for each method based on our proposed dataset, to quantitatively evaluate and compare different methods. the results were recorded in best case, worst case, and average of all images for each method. The comparison values of CSIM and SSIM are shown in TABLE 2. From TABLE 2, compared with other simulated stroke models, the CCD-BSMG exhibited the higher similarity with the real stroke images. According to the average of CSIM and SSIM, CCD-BSMG>CCD-BSM>Bézier>Ellipse>Straight Line>Circle. This fully shows that our proposed CCD-BSMG based on CCD-BSM outperformed the others significantly.

**TABLE 2. The performance (%) of generated models and other models.**

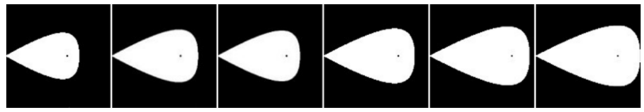
| Model | CSIM (max) | CSIM (min) | CSIM (avg) | SSIM (max) | SSIM (min) | SSIM (avg) |
|---|---|---|---|---|---|---|
| CCD-BSMG | 97.29 | 89.22 | 92.65 | 90.58 | 85.49 | 88.16 |
| CCD-BSM | 94.25 | 88.33 | 91.84 | 90.13 | 85.16 | 87.71 |
| Bézier | 90.36 | 85.48 | 88.02 | 83.14 | 78.21 | 81.03 |
| Ellipse | 86.43 | 81.62 | 84.15 | 76.81 | 71.05 | 73.46 |
| Circle | 71.40 | 64.36 | 67.03 | 63.85 | 57.21 | 60.23 |
| Line | 85.99 | 81.26 | 83.50 | 77.38 | 67.22 | 72.77 |

### 3) ABLATION STUDY

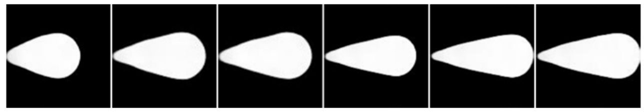
To investigate the effectiveness of introduced network structure in our proposed CCD-BSMG method, we performed an ablation study. We used neural generator with the same network architectures to implement the rendering of different stroke designs, as different stroke representations only needed to change the input of the network. Besides the CCD-BSMG, we trained the generator for QBC and the Bézier-based renderer (BBR) was also generated. The strokes were designed as the QBC with 8 parameters to fit the shape of brush stroke, which are the coordinates of three control points, the thickness of two endpoints of the QBC, respectively. Thus, the shape of the QBC has been specified by these parameters. The comparison of QBC and BBR are shown in FIGURE 12. This shows that our network can generalize other stroke designs.

### 4) COMPARISON WITH CHINESE CHARCTER

In order to verify the robustness and efficacy of the proposed model, complete Chinese characters were formed based on the superimposition of a series of generated stroke graphics CCD-BSMG along the trajectory. Thus, for any Chinese character, we can control the robot to write with brush pen. A simulation image can be generated by a series of stroke graphics in the trajectory points and a real image can be obtained through the robotic writing. Take Chinese character "zi" for example, FIGURE 13 shows the reference "yanti" image of "zi", the simulated image of CCD-BSMG and real character written by robotic arm. For Chinese character "zi", more than 10 writing tests were used, and the quantitative evaluating results were recorded in TABLE 3.



(a) The simulated images of QBC



(b) The simulated images of BBR

**FIGURE 12. Comparison of QBC and BBR. (a) shows the simulated images of QBC, and (b) shows the simulated images of BBR.**

(a) Chinese character "zi" (b) The simulated image (c) The image written by robot

**FIGURE 13. Comparison of Chinese character "zi". (a) shows the reference simulated images, (b) shows the simulated images, and (c) shows the image written by the robot.**

**TABLE 3. The performance (%) of generated image and the corresponding real image of chinese character "zi".**

| Model | CSIM (max) | CSIM (min) | CSIM (avg) | SSIM (max) | SSIM (min) | SSIM (avg) |
|---|---|---|---|---|---|---|
| CCD-BSMG | 97.71 | 91.45 | 94.58 | 79.26 | 72.56 | 75.91 |
| robot writing | 95.82 | 91.36 | 93.59 | 78.78 | 70.42 | 74.60 |

As shown in TABLE 3, both the generated images of CCD-BSMG and the robot writing images can obtain a good performance with the Chinese character "zi". Our proposed brush stroke model generator based on CCD-BSM exhibited the highest similarity with the corresponding real image, with a maximum similarity of 97.71%. The average value of CSIM written by the robot was more than 90%, Which proved that the parameters $H_i$ and $\alpha_i$ obtained from CCD-BSMG are consistent with expectations. This especially shows that the robustness and generalization of CCD-BSMG with physical parameters of Chinese calligraphy robot.

## V. CONCLUSION

This paper defined the mathematical description of multivariate parameters between the real brush stroke model and the simulated stroke model. On this basis, a differentiable neural network generator named CCD-BSMG was proposed to build a simulation brush stroke model. The CCD-BSMG can train the agent to generate corresponding brush stroke images according to the curves or geometries of the brush stroke model. The control parameters $(H, \alpha)$ of robotic arm were used as the input parameters of the neural network CCD-BSMG. The loss function of CCD-BSMG was designed to update the parameters for rewritten by robotic arm. After multiple iterations and optimizations, the generated images were close to the stroke data set.

The main purpose of this paper was to generate brush stroke based on CCD-BSM and obtain the optimal control parameters by model-based learning and training directly, which can be used for robotic arm writing only by fine-tuning these control parameters. It lays a foundation for the generation and reconstruct of simulation stroke model. The robotic arm can rewrite the strokes to generator images using the updated control parameters.

The stroke images are composed of many brush strokes, thus the stroke images can also map to the position and posture of a six-degree-of-freedom robotic arm. On this basis, we will build the mapping connection between the neural network and the robotic arm, which make it possible to generate interpretable neural network to compensate the black box defects of neural network. We will further focus on the transparency and the deformation of the brush stroke model generator by the rotation and motion. In the future, the stroke trajectory generation based on the generator will be paid more attention to simulate the movement and change of the stroke graphic during writing by robotic arm. Then, a generative system which is closed to the actual situation can be developed in order to train and teach robots. In this way, the robots can perform writing task and complete Chinese calligraphy well.

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