IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 11 October 2023, accepted 9 November 2023, date of publication 13 November 2023, date of current version 21 November 2023.

*Digital Object Identifier 10.1109/ACCESS.2023.3332365*

RESEARCH ARTICLE

# Robust Adaptive Tracking Control for Nonlinearly Parameterized High-Order Systems With Unmatched Disturbances

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This work was supported in part by the Natural Science Foundation of Shandong Province under Grant ZR2022QF021, and in part by the National Natural Science Foundation of China under Grant 62173208 and Grant 62303264.

**ABSTRACT** The output tracking problem for a class of nonlinearly parameterized high-order uncertain systems with unmatched disturbances is studied. The developed robust adaptive control scheme is based on the combination of a decoupled backstepping control method for decoupling the coupling term in each step by elegantly using the Young's inequality and an adaptive technique with fixed $\sigma$-modification for ensuring the boundedness of parameter estimates. The proposed controller and adaptive laws guarantee the closed-loop signals global boundedness and output tracking error boundedness in a mean square sense. Finally, the desired control properties are verified by a numerical example and a mass-spring mechanical system, respectively.

**INDEX TERMS** Output tracking, function bounding technique, robust adaptive control, unmatched disturbances.

## I. INTRODUCTION

The stabilization or tracking problems of high-order systems have been studied extensively in recent years [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14]. In practice, various forms of uncertainties inevitably exist in the mathematical model of the controlled system due to the change of working condition, unmodeled error, system failure and so on. Among them, nonlinearly parameterized characteristics are also common in uncertain high-order nonlinear systems, such as diesel engine air path systems [15], air vehicle systems [16] and other physical systems [17], [18]. In addition, controlled systems are often affected by external disturbances [19], [20], [21], [22], for example, electromagnetic disturbances widely exist in the transmission process of electronic signals [20]. The existence of unknown parameters and external disturbances may lead to instability of the system and even cause great damage. Therefore, in order to reduce their impact on system performance, it is necessary to develop suitable control schemes for nonlinearly parameterized high-order systems with unmatched disturbances.

The nonlinear characteristics of high-order systems make it difficult to realize the desired control objectives. If the nonlinear system can be linearized, then a series of results, such as stabilization, regulation and tracking, can be obtained using linearized-based control methods [23], [24]. However, the linearized system at an equilibrium point is not always completely controllable, or linearized-based control methods can only achieve a local control performance. For this reason, a large number of studies were conducted and many effective methods were proposed without considering external disturbances as well as solving output tracking problems, such as adding an integrator method [3], [4], [5] and sliding mode control method [6]. Besides, parameter uncertainties in high-order systems can not be ignored in the control design process because they may damage the performance and even destroy the stability. For the purpose of solving the challenges caused by unknown parameters in the system, many control schemes were

The associate editor coordinating the review of this manuscript and approving it for publication was Qi Zhou.

proposed [8], [9], [10]. For example, in Reference [8], a continuous controller was proposed, which solves the problem of coexistence of unknown time-varying parameters and unknown time-varying control coefficients. In Reference [9], the problem of full errors constrained adaptive control for multi-input multi-output nonlinear systems is studied. When the unknown parameters appear in high-order systems in the form of nonlinear parameterization, a separation principle was introduced [11], which can estimate nonlinearly parameterized functions as linearly parameterized functions. This method was widely used in the study of nonlinearly parameterized systems [12], [13]. Meanwhile, some results have been achieved when external disturbances are considered in the uncertain nonlinear systems [26], [27], [28]. However, the above researches on uncertainly nonlinear systems with external disturbances have relatively strict requirements on external disturbances, for example, the external disturbances need to satisfy a $L^{1+\alpha}(\alpha \geq 1)$ condition in [26]. Therefore, the control problem of nonlinearly parameterized high-order systems with unmatched disturbances remains to be studied open.

When considering a tracking control problem of high-order uncertainly nonlinear systems with unmatched disturbances, some control schemes based on fuzzy control and neural network control were proposed [29], [30], [31], [32], [33], [34]. In Reference [29], a fuzzy adaptive controller for a class of multi-input multi-output nonlinear systems is designed to make the tracking error have a specified performance in a finite time by combining the adaptive backstepping method with nonlinear filters. In Reference [30], an adaptive fuzzy tracking control scheme based on high-order disturbance observer was proposed, which makes tracking errors converge to a compact set. In Reference [31], an adaptive fuzzy fault compensation mechanism is proposed to realize consensus tracking of unmanned aerial vehicles, so that the tracking error symbol remains unchanged during tracking process. In Reference [32], an adaptive prescribed performance neural control scheme was developed to track output signals. In Reference [35], the neural network control strategy is also applied, and a class of gain iterative disturbance observer is constructed to improve the control accuracy of the system under disturbances. Although neural network control and fuzzy control can solve tracking control problems for nonlinearly parameterized systems, they can only achieve semi-global control effects. Naturally, a global output tracking control scheme for nonlinearly parameterized high-order systems with externally unmatched disturbances is expected to be proposed. So, we propose the following problem:

*If a class of high-order nonlinearly parameterized systems are influenced by bounded unmatched disturbances, is it possible to design a robust adaptive output tracking controller to ensure the global boundedness of the closed-loop signals and mean convergence of the tracking error ?*

In order to solve this problem, the function bounding technique in Reference [16] is used to estimate nonlinearly parameterized terms which character as a known signal vector multiplied by an unknown parameter vector. Then, by means of the decoupled backstepping control method, a feasible robust adaptive output tracking controller is designed. And the fixed $\sigma$-modification technique is applied to correct adaptive laws to ensure the boundedness of parameter estimates, in the meantime, the robustness of the system is improved when there exists bounded unmatched disturbances. Under this control scheme, the results of global boundedness and mean convergence are summarized as an important conclusion. The main contributions of this paper lie in the following two aspects:

(i) It shows how to express the nonlinearly parametrized dynamics into upper-bound functions and how to estimate each upper-bound function as a product of an unknown parameter vector and a known signal vector using a function bounding technique.

(ii) It takes into account the influence of external disturbances on the system, and relaxes the requirements for external disturbances, so that the anti-interference ability of the system is enhanced.

(iii) It develops a robust adaptive tracking control strategy to ensure the global boundedness of the closed-loop signals and tracking error mean convergence, where the tracking error can be made arbitrarily small by adjusting the parameters designed.

The remainder of this paper is organized as follows. Section II presents the form of a class of nonlinearly parameterized systems studied in this paper, some lemmas and assumptions used in the controller design process. Section III addresses the robust adaptive tracking controller synthesis in detail and the proofs of the main results are given in Section IV. Section V verifies the rationality of the adaptive tracking controller through a numerical example and a practical example, respectively. Section VI presents the conclusions of this paper.

## II. PRELIMINARIES AND PROBLEM FORMULATION

### A. PROBLEM FORMULATION

This paper considers a class of high-order nonlinearly parameterized systems with unmatched disturbances as follows:

$$\begin{cases} \dot{x}_i(t) = \lambda_i(x, u, \theta)x_{i+1}^{p_i}(t) + \sum_{j=0}^{p_i-1} x_{i+1}^{j}(t)\phi_{ij}(\bar{x}_i, \theta) \\ \qquad + d_i(t), \ i = 1, \cdots, n-1, \\ \dot{x}_n(t) = \lambda_n(x, u, \theta)u^{p_n}(t) + \sum_{j=0}^{p_n-1} u^j(t)\phi_{nj}(x, \theta) + d_n(t), \\ y(t) = x_1(t), \end{cases} \tag{1}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is system state, $u(t) \triangleq x_{n+1}(t) \in \mathbb{R}$ is control input, and $y(t) \in \mathbb{R}$ is system output, respectively. $\theta \in \mathbb{R}^s$ is an unknown parameter vector. For each $i = 1, \ldots, n$, $\bar{x}_i(t) = [x_1(t), \ldots, x_i(t)]^T$,

$\bar{x}_n(t) = x(t)$; $p_i$ is an odd positive integer with $p_0 = 1$, $\lambda_i : \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^s \to \mathbb{R}$ and $\phi_{ij} : \mathbb{R}^i \times \mathbb{R}^s \to \mathbb{R}$ are continuous functions, and $\phi_{ij}(0, \ldots, 0, \theta) = 0$; $d_i : \mathbb{R} \to \mathbb{R}$ is known as an unmatched disturbance.

The control objective of this paper is to design a controller $u(t)$ through state feedback so that the output $y(t)$ can track any smooth reference signal $y_r(t)$. The following assumptions are necessary to achieve the above control objective.

*Assumption 1: For each $i = 1, \ldots, n$, there exist continuous functions $\eta_i(\bar{x}_i)$, $\mu_i(\bar{x}_i, \theta)$ and an unknown positive constant $a_i$ such that*

$$0 < a_i\eta_i(\bar{x}_i) \le |\lambda_i(x, u, \theta)| \le \mu_i(\bar{x}_i, \theta). \tag{2}$$

*Assumption 2: For each $i = 1, \ldots, n$, the unmatched disturbance $d_i(t)$ has upper bound $D_i$ that is*

$$|d_i(t)| \le D_i. \tag{3}$$

*Remark 1: Assumptions 1,2 are common in high-order uncertain nonlinear systems [11], [22], [37]. In addition, for many practical systems, the external disturbances are often bounded, such as the wind disturbances of the quadrotor UAV system [40], and the temperature changes generated by hypersonic vehicles during flight are bounded [41]. An upper bound of $\mu_i(\bar{x}_i, \theta)$ can be found by using the function bounding technique [16] and the external disturbances only need to be bounded, whose boundaries do not need to be known. Considering the assumption that $a_i\eta_i(x_1, \ldots, x_i) > 0$, $\lambda_i(x, u, \theta)$ is strictly positive or strictly negative. No loss of generality, the subsequent control design process of this paper is carried out on the premise that $\lambda_i(x, u, \theta)$ is a strictly positive function and $a_i$ is also strictly positive.*

### B. PRELIMINARIES

This part lists some lemmas and propositions used in control design and their proofs can refer to relevant references [3], [5], and [36] and the appendix.

*Lemma 1 [5]: Suppose $f : [a, b] \to \mathbb{R}(a < b)$ is a monotone continuous function that satisfies $f(a) = 0$, then we have $\int_a^b f(x)dx \le |f(b)| \cdot |b - a|$.*

*Lemma 2 [36]: For any $x \in \mathbb{R}$ and any $y \in \mathbb{R}$, if $p \ge 1$ is an odd integer, then:*

$$\begin{cases} |x+y|^{\frac{1}{p}} \le |x|^{\frac{1}{p}} + |y|^{\frac{1}{p}}, \\ |x-y| \le 2^{\frac{p-1}{p}} |x^p - y^p|^{\frac{1}{p}}. \end{cases}$$

*Lemma 3 [3]: For given constants $p > 0$, $q > 0$ and any $x \in \mathbb{R}$, $y \in \mathbb{R}$, $a \in \mathbb{R}$, one has*

$$|ax^p y^q| \le c|x|^{p+q} + \frac{q}{p+q}\left(\frac{p}{c(p+q)}\right)^{\frac{p}{q}}|a|^{\frac{p+q}{q}}|y|^{p+q}.$$

According to [37], there exist nonnegative smooth functions $\varphi_i(\bar{x}_i)$, $\overline{\mu}_i(\bar{x}_i)$ for $i = 1, \ldots, n$ such that

$$\sum_{j=0}^{p_i-1} x_{i+1}^j \phi_{ij}(\bar{x}_i, \theta) \le \frac{\lambda_i(x, u, \theta)|x_{i+1}|^{p_i}}{2} + \overline{\theta}_i\varphi_i(\bar{x}_i)\sum_{j=1}^{i}|x_j|, \tag{4}$$

$$\mu_i(\bar{x}_i, \theta) \le \overline{\theta}_i\overline{\mu}_i(\bar{x}_i), \tag{5}$$

where $\overline{\theta}_i \ge 1$ is a new unknown parameter. Then, some new unknown parameters are defined as

$$\Theta_i = \max\left\{\max_{1\le j\le i}\left\{\frac{|\overline{\theta}_j|}{a_i}, \left(\frac{\overline{\theta}_j}{a_i}\right)^2\right\}, \frac{1}{a_i^2}, \left(\frac{\overline{\theta}_{i-1}}{a_{i-1}}\right)^{2p_1\cdots p_{i-2}}\right\}, \quad i = 1, \ldots, n, \tag{6}$$

and coordinate transformations are introduced as follows:

$$\begin{cases} z_1(t) = x_1(t) - y_r(t), \\ z_i(t) = x_i^{p_1\cdots p_{i-1}}(t) - \alpha_{i-1}^{p_1\cdots p_{i-1}}(\bar{y}_r^{(i-1)}, \bar{x}_{i-1}, \hat{\bar{\Theta}}_i), \\ \qquad i = 2, \ldots, n, \end{cases} \tag{7}$$

where $z_1$ is the output tracking error, $\hat{\bar{\Theta}}_i(t)$ is the estimate of an unknown vector $\bar{\Theta}_i = [\Theta_1, \ldots, \Theta_i]^T$ defined in (6), and $\bar{y}_r^{(i)}(t) = [y_r(t), \dot{y}_r(t), \ddot{y}_r(t), \ldots, y_r^{(i)}(t)]$ with $y_r^{(i)}(t)$ being the $i$th derivative for $y_r(t)$ with respect to $t$. For the convenience of expression, we define $y_r(t) = \alpha_0(t)$. Remarkably, for each $i = 1, \ldots, n$, $\alpha_i^{p_1\cdots p_i} = -g_i(\bar{y}_r^{(i)}, \bar{x}_i, \hat{\bar{\Theta}}_i)z_i$, where $g_i(\bar{y}_r^{(i)}, \bar{x}_i, \hat{\bar{\Theta}}_i) > 0$ is smooth. Specifically, we set $u = \alpha_n$. By means of (4), we have the following proposition.

*Proposition 1: For $i = 1, \ldots, n$, there exist smooth nonnegative functions $\tilde{\varphi}_i$ and $\overline{\varphi}_i$ such that*

$$\sum_{j=0}^{p_i-1} x_{i+1}^j\phi_{ij} \le \frac{\lambda_i|x_{i+1}|^{p_i}}{2} + \tilde{\varphi}_i(\bar{y}_r^{(i-1)}, \bar{x}_i, \hat{\bar{\Theta}}_{i-1})\overline{\theta}_i + \overline{\varphi}_i(\bar{y}_r^{(i-1)}, \bar{x}_i, \hat{\bar{\Theta}}_{i-1})\sum_{j=1}^{i}|z_j|^{\frac{1}{p_1\cdots p_{i-1}}}\overline{\theta}_i. \tag{8}$$

*Proposition 2: For $i = 1, \ldots, k-1$, there exist nonnegative smooth functions $C_{ki}(\bar{y}_r^{(k)}, \bar{x}_{k+1}, \hat{\bar{\Theta}}_k)$, $D_{ki}(\bar{y}_r^{(k)}, \bar{x}_{k+1}, \hat{\bar{\Theta}}_k)$, $E_{ki}(\bar{y}_r^{(k)}, \bar{x}_{k+1}, \hat{\bar{\Theta}}_k)$ such that*

$$\frac{\partial\alpha_k^{p_1\cdots p_k}}{\partial x_i}\dot{x}_i \le (|z_1| + \cdots + |z_{k+1}|)C_{ki}(.)\overline{\theta}_i + D_{ki}(.)|d_i(t)| + E_{ki}(.)\overline{\theta}_i. \tag{9}$$

*Remark 2: Two technical Propositions are introduced above. Proposition 1 applies Lemma 2.1 in Reference [11] to separate the unknown parameter $\theta$ from the nonlinear term $\sum_{j=0}^{p_i-1} x_{i+1}^j\phi_{ij}(\bar{x}_i, \theta)$, and constructs a new unknown parameter $\overline{\theta}_i$. Proposition 2 gives an upper bound on an ideal form of $\frac{\partial\alpha_k^{p_1\cdots p_k}}{\partial x_i}\dot{x}_i$. These two propositions play important roles in the controller design process in the next section.*

## III. ROBUST ADAPTIVE CONTROL DESIGN

To begin with, we define functions $W_k(\bar{y}_r^{(k-1)}, \bar{x}_k, \bar{\hat{\Theta}}_{k-1}) : \mathbb{R}^+ \times \mathbb{R}^k \times \mathbb{R}^{k-1} \to \mathbb{R},\ k = 2, 3, \cdots, n$, as follows:

$$W_k = \int_{\alpha_{k-1}}^{x_k} (s^{p_1\cdots p_{k-1}} - \alpha_{k-1}^{p_1\cdots p_{k-1}})^{2-\frac{1}{p_1\cdots p_{k-1}}} ds, \tag{10}$$

where the definition of $\alpha_{k-1}$ is given in (7). It is easy to prove that $W_k$ is continuously differentiable, and there exists positive constants $N_1,\ N_2$ such that

$$N_1(x_k - \alpha_{k-1})^{2p_1\ldots p_{k-1}} \le W_k \le N_2 z_k^2. \tag{11}$$

Next, we design an applicable robust adaptive tracking controller for system (1) by the following recursive method.

**Step1.** Define $\tilde{\Theta}_1 = \Theta_1 - \hat{\Theta}_1$. Consider the function

$$V_1(y_r, x_1, \tilde{\Theta}_1) = \frac{1}{2a_1} z_1^2 + \frac{1}{2}\tilde{\Theta}_1^2. \tag{12}$$

Obviously, $V_1$ is continuously differentiable, positive definite and radically unbounded. The derivative of $V_1$ along (1) is

$$\dot{V}_1 = \frac{1}{a_1} z_1\Big(\lambda_1 x_2^{p_1} + \sum_{j=0}^{p_1-1} x_2^j \phi_{1j} + d_1(t) - \dot{y}_r(t)\Big) - \tilde{\Theta}_1\dot{\hat{\Theta}}_1. \tag{13}$$

From (7), Proposition 1 and Lemma 3, we obtain

$$\begin{aligned}\dot{V}_1 \le& \frac{1}{a_1}\lambda_1 z_1 z_2 + \frac{1}{a_1}\lambda_1 z_1 \alpha_1^{p_1} \\ &+ \frac{1}{a_1}|z_1|\big(\frac{\lambda_1}{2}(|z_2| + |\alpha_1^{p_1}|) + \bar{\theta}_1\overline{\varphi}_1|z_1|\big) \\ &+ \rho_1\Theta_1\tilde{\varphi}_1^2 z_1^2 + \frac{1}{4\rho_1} + \frac{1}{4r_1}d_1^2(t) + r_1\Theta_1 z_1^2 + \rho_1\Theta_1 z_1^2\dot{y}_r^2(t) \\ &+ \frac{1}{4\rho_1} - \tilde{\Theta}_1\dot{\hat{\Theta}}_1,\end{aligned} \tag{14}$$

where $r_1, \rho_1$ are positive constants to be designed. Taking $z_1\alpha_1^{p_1} < 0$ into account, we have

$$\begin{aligned}&\frac{1}{a_1} z_1\lambda_1\alpha_1^{p_1} + \frac{1}{a_1}\frac{\lambda_1}{2}|z_1 z_2| + \frac{1}{a_1}\frac{\lambda_1}{2}|z_1||\alpha_1|^{p_1} \\ &\quad\le \frac{1}{2a_1} z_1\lambda_1\alpha_1^{p_1} + \frac{\lambda_1}{2a_1}|z_1 z_2|.\end{aligned} \tag{15}$$

Then by Assumption 1 and the fact that $z_1\alpha_1^{p_1} < 0$, we have

$$\begin{aligned}\dot{V}_1 \le& \frac{3\lambda_1}{2a_1}|z_1 z_2| + \frac{1}{2}\eta_1 z_1\alpha_1^{p_1} + \Theta_1 z_1^2\overline{\varphi}_1 + \rho_1\Theta_1 z_1^2\tilde{\varphi}_1^2 \\ &+ \frac{1}{4r_1}d_1^2(t) + r_1\Theta_1 z_1^2 + \rho_1\Theta_1 z_1^2\dot{y}_r^2(t) + \frac{1}{2\rho_1} - \tilde{\Theta}_1\dot{\hat{\Theta}}_1 \\ \le& z_1\bigg(\frac{\eta_1\alpha_1^{p_1}}{2} + z_1\sqrt{1+\hat{\Theta}_1^2}\big(\overline{\varphi}_1 + r_1 + \rho_1\tilde{\varphi}_1^2 + \dot{y}_r^2(t)\big)\bigg) \\ &+ \tilde{\Theta}_1 z_1^2\big(\overline{\varphi}_1 + r_1 + \rho_1\tilde{\varphi}_1^2 + \dot{y}_r^2(t) - \dot{\hat{\Theta}}_1\big) + \frac{1}{2\rho_1} \\ &+ \frac{1}{4r_1}d_1^2(t) + \frac{3\lambda_1}{2a_1}|z_1 z_2|.\end{aligned} \tag{16}$$

Therefore, we choose

$$\begin{cases}\alpha_1^{p_1} = -z_1\Bigg(\dfrac{2\big(l_1 + \sqrt{1+\hat{\Theta}_1^2}\big(\overline{\varphi}_1(x_1) + r_1 + \rho_1\tilde{\varphi}_1^2 + \dot{y}_r^2(t)\big)\big)}{\eta_1}\Bigg) \\ \quad\triangleq -z_1 g_1(\bar{y}_r^{(1)}, x_1, \hat{\Theta}_1), \\ \dot{\hat{\Theta}}_1 = z_1^2\big(\overline{\varphi}_1 + r_1 + \rho_1\tilde{\varphi}_1^2 + \dot{y}_r^2(t)\big) - \sigma_1\hat{\Theta}_1 \\ \quad\triangleq h_1(\bar{y}_r^{(1)}, x_1)z_1^2 - \sigma_1\hat{\Theta}_1,\end{cases} \tag{17}$$

where $l_1, \sigma_1$ are positive constants to be designed, and we have

$$\dot{V}_1 \le -l_1 z_1^2 + \sigma_1\hat{\Theta}_1\tilde{\Theta}_1 + \frac{3\lambda_1}{2a_1}|z_1 z_2| + \frac{1}{4r_1}d_1^2(t) + \frac{1}{2\rho_1}. \tag{18}$$

**Step k.** Suppose that $V_{k-1}(k = 3, 4, \ldots, n)$ satisfies

$$\begin{aligned}\dot{V}_{k-1} \le& -\sum_{i=1}^{k-1}\big(l_i - (k-i-1)\big)z_i^2 \\ &+ \sum_{i=1}^{k-1}\frac{k-i}{4r_i}d_i^2(t) + \sum_{i=1}^{k-1}\sigma_i\tilde{\Theta}_i\hat{\Theta}_i \\ &+ C_{k-1}\frac{\lambda_{k-1}}{a_{k-1}}|z_{k-1}|^{\frac{2p_1\cdots p_{k-2}-1}{p_1\cdots p_{k-2}}}|z_k|^{\frac{1}{p_1\cdots p_{k-2}}} + \sum_{i=1}^{k-1}\frac{1}{2\rho_i},\end{aligned} \tag{19}$$

where $C_{k-1} = 2^{\frac{p_1\cdots p_{k-2}-1}{p_1\cdots p_{k-2}}} + \frac{1}{2}$, and $l_i, r_i, \sigma_i, \rho_i, i = 1, \ldots, k-1$, are positive constants to be designed and $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i, i = 1, \ldots, k-1$. Then we choose

$$\begin{aligned}V_k &= V_{k-1} + \frac{1}{a_k}W_k(\bar{y}_r^{(k-1)}, \bar{x}_k, \bar{\hat{\Theta}}_{k-1}) + \frac{1}{2}\tilde{\Theta}_k^2 \\ &= \sum_{i=1}^{k}\frac{1}{a_i}W_i + \sum_{i=1}^{k}\tilde{\Theta}_i^2,\end{aligned} \tag{20}$$

where $\tilde{\Theta}_k = \Theta_k - \hat{\Theta}_k$. Obviously, $V_k$ is continuously differentiable, positive definite and radically unbounded. Differentiating (20), one has

$$\begin{aligned}\dot{V}_k \le& \dot{V}_{k-1} + \frac{1}{a_k}\Bigg(\frac{\partial W_k}{\partial x_k}\dot{x}_k + \sum_{i=1}^{k-1}\frac{\partial W_k}{\partial x_i}\dot{x}_i + \sum_{i=1}^{k-1}\frac{\partial W_k}{\partial\hat{\Theta}_i}\dot{\hat{\Theta}}_i \\ &+ \sum_{i=0}^{k-1}\frac{\partial W_k}{\partial y_r^{(i)}}y_r^{(i+1)}\Bigg) - \tilde{\Theta}_k\dot{\hat{\Theta}}_k.\end{aligned} \tag{21}$$

Combining (7), Lemmas 2, 3, Proposition B.1 in [5] with $z_k^{\frac{1}{p_1\cdots p_{k-1}}}\alpha_k^{p_k}<0$, we have

$$
\begin{aligned}
&\frac{1}{a_k}\frac{\partial W_k}{\partial x_k}\dot{x}_k \\
&\le \frac{\lambda_k}{a_k} z_k^{2-\frac{1}{p_1\cdots p_{k-1}}}(x_{k+1}^{p_k}-\alpha_k^{p_k})+\frac{\lambda_k}{2a_k}|z_k|^{2-\frac{1}{p_1\cdots p_{k-1}}} \\
&\quad (|z_{k+1}|^{\frac{1}{p_1\cdots p_{k-1}}}+|\alpha_k|^{p_k})+\frac{1}{4r_k}d_k^2(t)+\Theta_k z_k^2 h_{k1}(\cdot) \\
&\quad +\frac{\lambda_k}{a_k} z_k^{2-\frac{1}{p_1\cdots p_{k-1}}}\alpha_k^{p_k}+\sum_{i=1}^{k-1}\frac{1}{4}z_i^2+\frac{1}{8\rho_k} \\
&\le \frac{1}{2}\eta_k z_k^{2-\frac{1}{p_1\cdots p_{k-1}}}\alpha_k^{p_k}+\Theta_k h_{k1}(\cdot)z_k^2+\sum_{i=1}^{k-1}\frac{1}{4}z_i^2+\frac{1}{8\rho_k} \\
&\quad + C_k\frac{\lambda_k}{a_k}|z_k|^{2-\frac{1}{p_1\cdots p_{k-1}}}|z_{k+1}|^{\frac{1}{p_1\cdots p_{k-1}}}+\frac{1}{4r_k}d_k^2(t), \qquad (22)
\end{aligned}
$$

where $h_{k1}(\bar{y}_r^{(k-1)},\bar{x}_k,\bar{\hat{\Theta}}_{k-1})=(k-1)\frac{2p_1\cdots p_{k-1}-1}{2p_1\cdots p_{k-1}}\overline{\varphi}_k^{\frac{2p_1\cdots p_{k-1}}{2p_1\cdots p_{k-1}-1}}(\frac{2}{p_1\cdots p_{k-1}})^{\frac{1}{2p_1\cdots p_{k-1}-1}}+r_k(1+z_k^2)^{\frac{p_1\cdots p_{k-1}-1}{p_1\cdots p_{k-1}}}+2\rho_k\tilde{\varphi}_k^2(1+z_k^2)^{\frac{p_1\cdots p_{k-1}-1}{p_1\cdots p_{k-1}}}+\overline{\varphi}_k$ is a positive smooth function, $C_k=2^{\frac{p_1\cdots p_{k-1}-1}{p_1\cdots p_{k-1}}}+\frac{1}{2}$ and $r_k$, $\rho_k$ are positive constants to be designed. Using Proposition 2 and Lemma 3, we arrive at

$$
\frac{1}{a_k}\sum_{i=1}^{k-1}\frac{\partial W_k}{\partial x_i}\dot{x}_i \le \Theta_k h_{k2}(\cdot)z_k^2+\sum_{j=1}^{k-1}(\frac{1}{4r_j}d_j^2(t)+\frac{z_j^2}{4})+\frac{1}{8\rho_k}, \qquad (23)
$$

$$
\begin{aligned}
&\frac{1}{a_k}\sum_{i=1}^{k-1}\frac{\partial W_k}{\partial \hat{\Theta}_i}\dot{\hat{\Theta}}_i + C_{k-1}\frac{\lambda_{k-1}}{a_{k-1}}|z_{k-1}|^{\frac{2p_1\cdots p_{k-2}-1}{p_1\cdots p_{k-2}}}|z_k|^{\frac{1}{p_1\cdots p_{k-2}}} \\
&\le \frac{1}{2}\sum_{i=1}^{k-1}z_i^2+h_{k3}(\cdot)\Theta_k z_k^2, \qquad (24)
\end{aligned}
$$

$$
\frac{1}{a_k}\sum_{i=0}^{k-1}\frac{\partial W_k}{\partial y_r^{(i)}}y_r^{(i+1)} \le h_{k4}(\cdot)z_k^2\Theta_k+\frac{1}{4\rho_k}, \qquad (25)
$$

where $\overline{c}_k=2^{1-\frac{1}{p_1\cdots p_{k-1}}}(2-\frac{1}{p_1\cdots p_{k-1}})$ is a positive constant, and $h_{k2}(\bar{y}_r^{(k-1)},\bar{x}_k,\bar{\hat{\Theta}}_{k-1})=(k-1)\overline{c}_k^2(\sum_{i=1}^{k-1}C_{k-1,i}^2)+\overline{c}_k\sum_{i=1}^{k-1}C_{k-1,i}+2\rho_k\overline{c}_k^2(\sum_{i=1}^{k-1}E_{k-1,i}^2)+\overline{c}_k^2\sum_{i=1}^{k-1}r_iD_{k-1,i}^2$, $h_{k3}(\bar{y}_r^{(k-1)},\bar{x}_k,\bar{\hat{\Theta}}_{k-1})=\frac{1}{2}\overline{c}_k^2\sum_{i=1}^{k-2}(\frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial\hat{\Theta}_i}h_iz_i)^2+2\overline{c}_k^2(\frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial\hat{\Theta}_{k-1}}h_{k-1}z_{k-1})^2+2(k-1)\overline{c}_k^2\sum_{i=1}^{k-1}(\frac{\partial g_{k-1}}{\partial\hat{\Theta}_i}\sigma_i\hat{\Theta}_i)^2+\frac{1}{2p_1\cdots p_{k-2}}C_{k-1}^{2p_1\cdots p_{k-2}}(\frac{2(2p_1\cdots p_{k-2}-1)}{p_1\cdots p_{k-2}})^{2p_1\cdots p_{k-2}-1}\overline{\mu}_{k-1}^{2p_1\cdots p_{k-2}}$, $h_{k4}(\bar{y}_r^{(k)},\bar{x}_{k-1},\bar{\hat{\Theta}}_{k-1})=k\rho_k\overline{c}_k^2\sum_{i=0}^{k-1}(\frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial y_r^{(i)}}y_r^{(i+1)})^2$ are positive smooth functions. Substituting (22)-(25) into (21), we get

$$
\begin{aligned}
\dot{V}_k \le& -\sum_{i=1}^{k-1}\big(l_i-(k-i)\big)z_i^2+\sum_{i=1}^{k-1}\sigma_i\tilde{\Theta}_i\hat{\Theta}_i+\sum_{i=1}^{k}\frac{1}{2\rho_i} \\
&+\sum_{i=1}^{k}\frac{k-i+1}{4r_i}d_i^2(t)+h_k(\cdot)\tilde{\Theta}_k z_k^2-\tilde{\Theta}_k\dot{\hat{\Theta}}_k \\
&+z_k^{2-\frac{1}{p_1\cdots p_{k-1}}}\left(\frac{\eta_k\alpha_k^{p_k}}{2}+z_k^{\frac{1}{p_1\cdots p_{k-1}}}\sqrt{1+\hat{\Theta}_k^2}h_k(\cdot)\right) \\
&+C_k\frac{\lambda_k}{a_k}|z_k|^{2-\frac{1}{p_1\cdots p_{k-1}}}|z_{k+1}|^{\frac{1}{p_1\cdots p_{k-1}}}, \qquad (26)
\end{aligned}
$$

where $h_k(\bar{y}_r^{(k)},\bar{x}_k,\bar{\hat{\Theta}}_{k-1})=h_{k1}+h_{k2}+h_{k3}+h_{k4}$ is a positive smooth function. We choose

$$
\begin{cases}
\alpha_k^{p_1\cdots p_k} = -z_k\left(\dfrac{2\big(l_k+\sqrt{1+\hat{\Theta}_k^2}h_k(\bar{y}_r^{(k)},\bar{x}_k,\bar{\hat{\Theta}}_{k-1})\big)}{\eta_k}\right)^{p_1\cdots p_{k-1}} \\
\qquad = -z_k g_k(\bar{y}_r^{(k)},\bar{x}_k,\bar{\hat{\Theta}}_k), \\
\dot{\hat{\Theta}}_k = h_k(\bar{y}_r^{(k)},\bar{x}_k,\bar{\hat{\Theta}}_{k-1})z_k^2-\sigma_k\hat{\Theta}_k,
\end{cases} \qquad (27)
$$

where $l_k$, $\sigma_k$ are positive constants to be designed, according to Assumption 1 and the definition of $h_k(\cdot)$, we know that $\alpha_k^{p_1\ldots p_k}$ is always meaningful. Meanwhile, the order $p_1\ldots p_k$ is odd, so $\alpha_k$ is also meaningful, then we have

$$
\begin{aligned}
\dot{V}_k \le& -\sum_{i=1}^{k}\big(l_i-(k-i)\big)z_i^2 \\
&+\sum_{i=1}^{k}\frac{k-(i-1)}{4r_i}d_i^2(t)+\sum_{i=1}^{k}\sigma_i\tilde{\Theta}_i\hat{\Theta}_i \\
&+\sum_{i=1}^{k}\frac{1}{2\rho_i}+C_k\frac{\lambda_k}{a_k}|z_k|^{2-\frac{1}{p_1\cdots p_{k-1}}}|z_{k+1}|^{\frac{1}{p_1\cdots p_{k-1}}}. \qquad (28)
\end{aligned}
$$

When $k=n$, the Lyapunov function $V_n$ is chosen as

$$
V_n=\sum_{i=1}^{n}\frac{1}{a_i}W_i+\sum_{i=1}^{n}\tilde{\Theta}_i^2, \qquad (29)
$$

and the controller and adaptive laws are designed as

$$
\begin{cases}
u=\alpha_n(\bar{y}_r^{(n)},x,\bar{\hat{\Theta}}_n), \\
\dot{\hat{\Theta}}_i=h_i(\bar{y}_r^{(i)},\bar{x}_i,\bar{\hat{\Theta}}_{i-1})z_i^2-\sigma_i\hat{\Theta}_i,\ i=1,\cdots,n.
\end{cases} \qquad (30)
$$

Considering $z_{n+1}=0$, we have

$$
\dot{V}_n\le-\sum_{i=1}^{n}L_iz_i^2+\sum_{i=1}^{n}\frac{n-(i-1)}{4r_i}d_i^2(t)+\sum_{i=1}^{n}(\sigma_i\tilde{\Theta}_i\hat{\Theta}_i+\frac{1}{2\rho_i}), \qquad (31)
$$

where $L_i=l_i-(n-i)>0$ is a constant and $l_i,r_i,\sigma_i,\rho_i$, $i=1,\ldots,n$ are positive constants to be designed.

*Remark 3: We use the backstepping method to construct a robust adaptive tracking controller, the process of which is shown in Fig.1. In order to deal with the bounded disturbances in the system and improve the robustness of the*

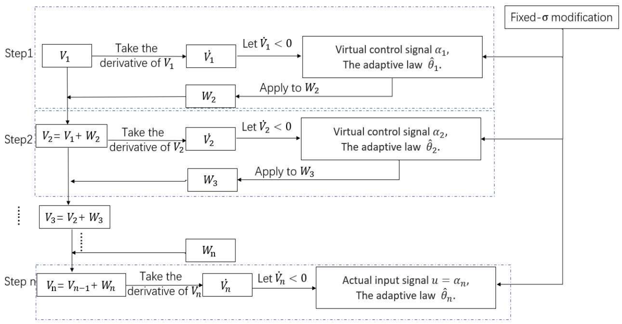


**FIGURE 1. The process of backstepping method.**

*system, we also apply the fixed $\sigma$-modification method to correct adaptive laws which also guarantees the boundedness of $\hat{\Theta}_i$.*

*Remark 4:* In (10), when the exponent of the integrand is $2 - \frac{1}{p_1 \cdots p_{k-1}}$, (11) can be obtained,which ensures that $W_k$ is positive definite and proper, on the other hand, we can deal with high-power terms with unknown coefficients in the system by means of it.

## IV. MAIN RESULTS

The first theorem gives the boundedness of the state variables of the closed-loop system.

*Theorem 1:* Consider the high-order nonlinearly parameterized system (1) with unmatched disturbances under Assumptions 1,2. If the robust adaptive tracking controller $u$ in (30) with the adaptive laws $\hat{\Theta}_i, i = 1, \cdots, n$ in (30) is applied, the global boundedness of the closed-loop state variables is guaranteed.

*Proof:* Consider the following Lyapunov function

$$V = \sum_{i=1}^{n} \frac{1}{a_i} W_i + \sum_{i=1}^{n} \tilde{\Theta}_i^2, \tag{32}$$

then according to the recursion procedure in section III, we arrive at

$$\begin{aligned} \dot{V} \leq & -\sum_{i=1}^{n} L_i z_i^2 + \sum_{i=1}^{n} \frac{n-(i-1)}{4r_i} d_i^2(t) \\ & + \sum_{i=1}^{n} \sigma_i \tilde{\Theta}_i \hat{\Theta}_i + \sum_{i=1}^{n} \frac{1}{2\rho_i}. \end{aligned} \tag{33}$$

Obviously, $\dot{V} < 0$ holds whenever

$$\sum_{i=1}^{n} \frac{n-(i-1)}{4r_i} d_i^2(t) + \sum_{i=1}^{n} \sigma_i \tilde{\Theta}_i \hat{\Theta}_i + \sum_{i=1}^{n} \frac{1}{2\rho_i} < 0. \tag{34}$$

Since $d_i(t)$ is bounded, that is $d_i(t) < D_i$, $\dot{V} < 0$ also holds when

$$\sum_{i=1}^{n} \frac{n-(i-1)}{4r_i} D_i^2 + \sum_{i=1}^{n} \sigma_i \tilde{\Theta}_i \hat{\Theta}_i + \sum_{i=1}^{n} \frac{1}{2\rho_i} < 0. \tag{35}$$

According to the definition of $\tilde{\Theta}_i$ and the method of completing the square we have if for each $i = 1, \ldots, n$,

$$|\hat{\Theta}_i(t)| \geq \frac{1}{2}|\Theta_i| + \frac{1}{2}\sqrt{\Theta_i^2 + \frac{n-(i-1)}{r_i \sigma_i} D_i^2 + \frac{2}{\rho_i \sigma_i}} \triangleq \theta_i^0, \tag{36}$$

(35) holds. In what follows we show that $\hat{\Theta}_i(t)$ is bounded by contradiction. In fact, if $\hat{\Theta}_i(t)$ is unbounded, then $|\sigma_i \tilde{\Theta}_i(t) \hat{\Theta}_i(t)| \to \infty$. On the other hand, we known from (33) and (36) that $\dot{V}_n(t) \leq 0$ if $\hat{\Theta}_i(t) \geq \theta_i^0$. Therefore, there exists a time $t_0 > 0$, such that

$$\begin{aligned} \sum_{i=1}^{n} \frac{1}{a_i} W_i(t) + \sum_{i=1}^{n} \tilde{\Theta}_i^2(t) &= V_n(t) \leq V_n(t_0) \\ &= \sum_{i=1}^{n} \frac{1}{a_i} W_i(t_0) + \sum_{i=1}^{n} \tilde{\Theta}_i^2(t_0) < \infty, \end{aligned} \tag{37}$$

so $\tilde{\Theta}_i(t)$ is bounded, which is contradict to $\hat{\Theta}_i(t)$ being unbounded. Consequently, $\hat{\Theta}_i(t)$ is bounded. By using the boundedness of $d_i(t)$ and $\hat{\Theta}_i(t)$, we can define a serial of finite constants

$$\tau_i = \sup_{t \geq 0} \left\{ \frac{n-(i-1)}{4r_i} d_i^2(t) + \sigma_i \tilde{\Theta}_i(t) \hat{\Theta}_i(t) + \frac{1}{2\rho_i} \right\}, \tag{38}$$

where $i = 1, \ldots, n$. This leads to

$$\dot{V}_n(t) \leq -\sum_{i=1}^{n} L_i z_i^2(t) + \sum_{i=1}^{n} \tau_i \triangleq -\sum_{i=1}^{n} L_i z_i^2(t) + \tau. \tag{39}$$

The inequality (39) shows that $z_i(t), i = 1, \ldots, n$ are bounded, together with the boundedness of $\hat{\Theta}_i(t)$ and the continuity of $\alpha_{i-1}(t)$, we can conclude that $x_i(t), i = 1, \ldots, n$ are bounded. □

The next theorem gives the robustness and convergence of the closed-loop system.

*Theorem 2:* Consider the high-order nonlinearly parameterized system (1) with unmatched disturbances under Assumptions 1,2. If the robust adaptive tracking controller $u$ in (30) with the adaptive laws $\hat{\Theta}_i, i = 1, \cdots, n$ in (30) is applied, for all $t_2 > t_1 \geq 0$, we have

$$\int_{t_1}^{t_2} z_i^2(t) dt \leq \frac{1}{L}\Omega + \frac{1}{L}(MD + \sigma^0 \Theta_s + P)(t_2 - t_1). \tag{40}$$

That is, $z_i(t), i = 1, \ldots, n$ are bounded by $\frac{1}{L}(MD + \sigma^0 \Theta_s + P)$ in the mean square sense, where $\Omega$ is a positive constant, $L$, $M$, $P$ and $\sigma^0$ are positive constants depending on design parameters $l_i, i = 1, \ldots, n$, $r_i, i = 1, \ldots, n$, $\rho_i, i = 1, \ldots, n$ and $\sigma_i, i = 1, \ldots, n$ respectively, $\Theta_s$ and $D$ are also positive constants depending on the system parameters and external disturbances, respectively.

*Proof:* In combination with Theorem 1, we introduce the following definitions:

$$M = \max_{1\leq i\leq n}\{\frac{n-(i-1)}{4r_i}\}, \sigma^0 = \max_{1\leq i\leq n}\{\sigma_i\}, L = \max_{1\leq i\leq n}\{L_i\},$$
$$D = \max_{1\leq i\leq n}\{nD_i\}, \Theta_s = \max_{1\leq i\leq n}\{n\tilde{\Theta}_{is}\hat{\Theta}_{is}\}, P = \max_{1\leq i\leq n}\{\frac{n}{2p_i}\}, \tag{41}$$

where $\tilde{\Theta}_{is}$, $\hat{\Theta}_{is}$ are the upper bounds of $|\tilde{\Theta}_i|$ and $|\hat{\Theta}_i|$ respectively. After that, from (31), we get

$$\begin{aligned} Lz_i^2 &\leq \sum_{i=1}^{n} L_i z_i^2 \leq -\dot{V}_n + \sum_{i=1}^{n}\frac{n-(i-1)}{4r_i}d_i^2 \\ &\quad + \sum_{i=1}^{n}(\sigma_i\tilde{\Theta}_i\hat{\Theta}_i + \frac{1}{2\rho_i}) \\ &\leq -\dot{V}_n + MD + \sigma^0\Theta_s + P. \end{aligned} \tag{42}$$

Integrating both sides of (42) on $[t_1, t_2]$, we have

$$\begin{aligned} \int_{t_1}^{t_2} z_i^2(t)dt &\leq \frac{1}{L}(MD + \sigma^0\Theta_s + P)(t_2 - t_1) \\ &\quad + \frac{1}{L}(V(t_1) - V(t_2)). \end{aligned} \tag{43}$$

Since $V(t)$ is bounded, we arrive at

$$\int_{t_1}^{t_2} z_i^2(t)dt \leq \frac{1}{L}\Omega + \frac{1}{L}(MD + \sigma^0\Theta_s + P)(t_2 - t_1), \tag{44}$$

where $\Omega = V(t_1) - V(t_2)$ is a positive constant. □

*Remark 5: According to (40), we can get $z_i, i = 1, 2, \ldots, n$ are bounded by $\frac{1}{L}(MD + \sigma^0\Theta_s + P)$ in the mean square sense, where $L$ depends on $l_i$, $M$ depends on $r_i$, $\sigma^0$ depends on $\sigma_i$, and $P$ depends on $\rho_i$, in the meantime, they are mutually independent. So we can reduce the bounds of $z_i, i = 1, 2, \ldots, n$ by adjusting the parameters $l_i, r_i, \sigma_i, \rho_i$ respectively, that is, we can reduce the bound of $z_i$ by increasing $l_i$,$r_i$,$\rho_i$ and decreasing $\sigma_i$, this makes the tracking error $z_1$ to be arbitrarily small.*

## V. SIMULATION EXAMPLE

To verify the rationality of the above robust adaptive controller, two simulation examples are given below.

*Example 5.1* The following dynamic system [5] is first considered, and take the displacement $x$ of the block of mass $m_1$ as the input of the system consisting of a ball of mass $m_2$ and a massless rod of length $l$, where $x$ is generated by the resultant force acting on $m_1$. And $k_s$ is the stiffness coefficient of the nonliear spring, $\alpha = \frac{\pi}{6}$ is the angle between the conical container wall and the ground.

For this, the following equation can be established for the angle $\theta$ of the bar where $f(t)$ is the amount of air resistance in the tangential direction of the bar during the motion of $m_2$ [42],

$$\ddot{\theta} = \frac{g}{l}\sin\theta + \frac{k_s}{m_2 l}(u - l\sin\theta)^3\cos\theta - \frac{f(t)}{m_2 l}, \tag{45}$$

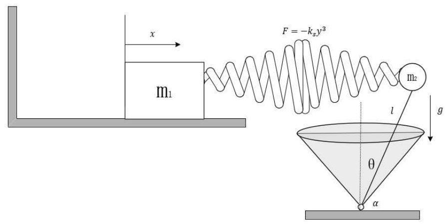


**FIGURE 2.** A class of dynamical systems.

where $\theta \in [-\frac{\pi}{3}, \frac{\pi}{3}]$. Let's take coordinate transformations $x_1 = \theta, x_2 = \dot{\theta}$, together with smooth state feedback $u = l\sin\theta + v$, transform (45) into

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \frac{k_s}{m_2 l}\cos x_1 v^3 + \frac{g}{l}\sin x_1 - \frac{f(t)}{m_2 l}. \end{cases} \tag{46}$$

For simplicity, let's take $\frac{g}{l} = 1$, $m_2 l = 1$ and $k_s$ is an unknown parameter, then the coefficients $\lambda_1 = 1$, $\lambda_2 = k_s\cos x_1$ satisfy Assumption 1 with $\mu_1 = \eta_1 = 1$, $\mu_2 = k_s$, $\eta_2 = \frac{1}{2}$. Meanwhile, Proposition 1 is satisfied with $\tilde{\varphi}_1 = \overline{\varphi}_1 = 0$, $\overline{\mu}_1 = 1$, $\tilde{\varphi}_2 = \sqrt{1 + y_r^2}$, $\overline{\varphi}_2 = 1$, $\overline{\mu}_2 = 1$, the desired angle of the rod trajectory is a periodic signal $y_r(t) = \sin 2t$. Here we take $f(t)$ as a random number in [0, 4]. Similarly, we define

$$z_1 = x_1 - y_r, z_2 = x_2 - \alpha_1.$$

According to the above robust adaptive tracking controller design process, we have

$$\dot{\hat{\Theta}}_1 = z_1^2(r_1 + \dot{y}_r^2) - \sigma_1\hat{\Theta}_1,$$
$$\alpha_1 = -z_1 \cdot 2(l_1 + \sqrt{1 + \hat{\Theta}_1^2}(r_1 + \dot{y}_r^2)) = -z_1 g_1.$$

Proposition 2 is also satisfied with

$$C_{11} = \sqrt{1 + \left(2(l_1 + \sqrt{1 + \hat{\Theta}_1^2}(r_1 + \dot{y}_r^2))\right)^2} \left[\frac{3}{2}\left(1 + 2(l_1 + \sqrt{1 + \hat{\Theta}_1^2}(r_1 + \dot{y}_r^2))\right)\right],$$

$$D_{11} = \sqrt{1 + \left(2(l_1 + \sqrt{1 + \hat{\Theta}_1^2}(r_1 + \dot{y}_r^2))\right)^2}, \; E_{11} = 0,$$

and then we get $h_{21} = 2 + r_2 + 2\rho_2(1 + y_r^2)$, $h_{22} = C_{11}^2 + r_1 D_{11}^2 + C_{11}$, $h_{23} = \frac{4\hat{\Theta}_1^2(r_1+\dot{y}_r^2)}{1+\hat{\Theta}_1^2}[2z_1^2(r_1 + \dot{y}_r^2)^2 + \sigma_1^2\hat{\Theta}_1^2] + \frac{9}{2}$, $h_{24} = 8\rho_2(l_1 + \sqrt{1 + \hat{\Theta}_1^2}(r_1 + \dot{y}_r^2))^2\dot{y}_r^2 + 32\rho_2(z_1\dot{y}_r\ddot{y}_r\sqrt{1 + \hat{\Theta}_1^2})^2$, $h_2 = h_{21} + h_{22} + h_{23} + h_{24}$. So we get the actual controller satisfying

$$v^3 = -4z_2 l_2 + 4z_2 h_2\sqrt{1 + \hat{\Theta}_2^2},$$

where $\hat{\Theta}_2$ is provided via adaptive law $\dot{\hat{\Theta}}_2 = z_2^2 h_2 - \sigma_2\hat{\Theta}_2$. We set the initial conditions $x_1(0) = 0.5$, $x_2(0) = 0.5$, $\hat{\Theta}_1(0) = 1.5$, $\hat{\Theta}_2(0) = 0.2$, then we assign different values to the parameters to show that we can reduce the tracking error by adjusting the parameters.

**Case 5.1.1.** To begin with, we choose parameters $l_1 = 2, l_2 = 1, r_1 = r_2 = 1, \rho_1 = \rho_2 = 1, \sigma_1 = \sigma_2 = 3$, at this time, the simulation results are shown in the figures below.

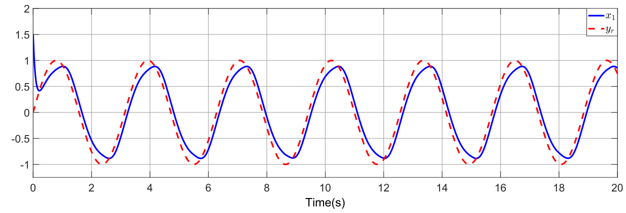


**FIGURE 3.** The trajectories of $x_1$,$y_r$ in Example 5.1 Case 5.1.1.

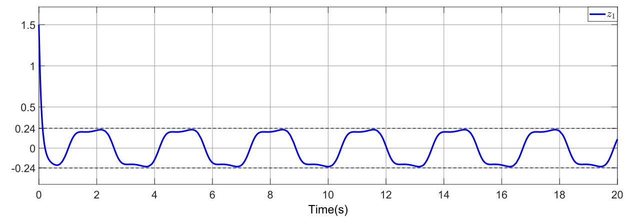


**FIGURE 4.** The trajectory of tracking error $z_1$ in Example 5.1 Case 5.1.1.

**Case 5.1.2.** When the parameters are adjusted to $l_1 = l_2 = 10, r_1 = r_2 = 5, \rho_1 = \rho_2 = 2, \sigma_1 = \sigma_2 = 0.1$, the simulation results at this time are obtained as follows.

It is clear that when the parameters are adjusted from Case 5.1.1 to Case 5.1.2, the absolute value of the tracking error $z_1$ decreases from 0.24 to 0.05, which indicates that increasing $l_1, l_2, r_1, r_2, \rho_1, \rho_2$ and decreasing $\sigma_1, \sigma_2$ can reduce the tracking error.

*Example 5.2* The nonlinear uncertain system with unmatched disturbances is considered

$$\begin{cases} \dot{x}_1 = \theta(2+\sin x_1)(2+\cos u)\cdot x_2 + \theta x_1 + d_1(t), \\ \dot{x}_2 = (1.5+\sin\theta x_1)\cdot u^3 + x_1 + u(1.5+\sin\theta x_1)x_2^{\frac{2}{3}} \\ \qquad + u^2(1.5+sin\theta x_1)^{\frac{2}{3}}x_1 + d_2(t), \\ y = x_1. \end{cases} \tag{47}$$

The tracking signal is $y_r(t) = \frac{1}{2}\sin t$, and the unmatched disturbances $d_1(t) = \sin t$, $d_2(t) = 4\sin t$. The system (47) can be easily verified to satisfy Assumption 1 with $\mu_1 = 3(2+\sin x_1)\theta, \eta_1 = 2+\sin x_1, \mu_2 = 2.5(1+\theta), \eta_2 = 0.5$, and $p_1 = 1, p_2 = 3$. At the same time, Proposition 1 is satisfied with $\tilde{\varphi}_1 = \sqrt{1+y_r^2}$, $\overline{\varphi}_1 = 1$, $\overline{\mu}_1 = 3(2+\sin x_1)$, $\tilde{\varphi}_2 = (1+\frac{4}{3}x_1^2)\sqrt{1+y_r^2}$, $\overline{\varphi}_2 = (1+\frac{4}{3}x_1^2)(1+g_1)$, $\overline{\mu}_2 = 2.5$, so we define

$$z_1 = x_1 - y_r, z_2 = x_2 - \alpha_1,$$

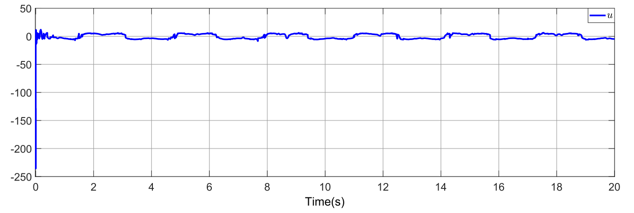


**FIGURE 5.** The trajectory of input *u* in Example 5.1 Case 5.1.1.

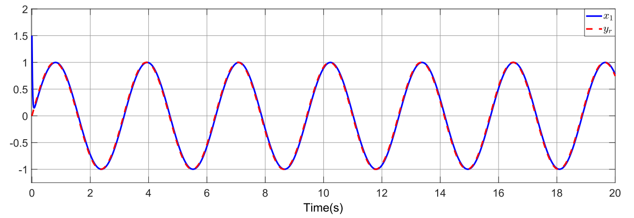


**FIGURE 6.** The trajectories of $x_1$,$y_r$ in Example 5.1 Case 5.1.2.

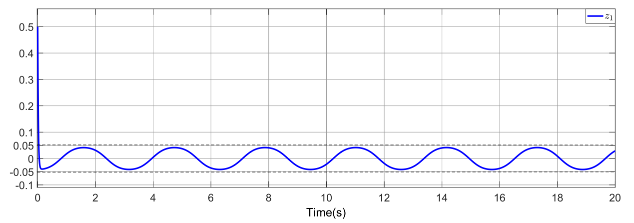


**FIGURE 7.** The trajectory of tracking error $z_1$ in Example 5.1 Case 5.1.2.

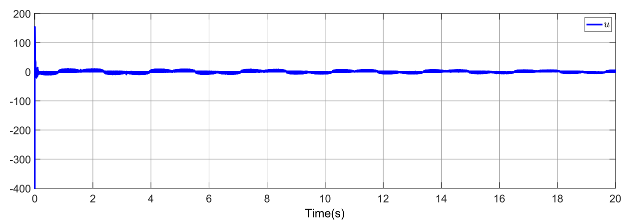


**FIGURE 8.** The response of input *u* in Example 5.1 Case 5.1.2.

then according to the above controller design process, we first get the virtual controller $\alpha_1$ as follows

$$\begin{aligned} \dot{\hat{\Theta}}_1 &= \left(1+\rho_1(1+y_r^2)+r_1+\dot{y}_r^2\right)z_1^2 - \sigma_1\hat{\Theta}_1 = h_1 z_1^2 - \sigma_1\hat{\Theta}_1, \\ \alpha_1 &= -z_1\frac{2\left(l_1+\sqrt{1+\hat{\Theta}_1^2}(r_1+1+\rho_1(1+y_r^2)+\dot{y}_r^2)\right)}{2+\sin x_1} \\ &= -z_1 g_1. \end{aligned}$$

Proposition 2 is satisfied with $\tau = \left[1+\left(-g_1+z_1\frac{2\cos x_1\left(l_1+\sqrt{1+\hat{\Theta}_1^2}(r_1+1+\rho_1(1+y_r^2)+\dot{y}_r^2)\right)}{(2+\sin x_1)^2}\right)^2\right]^{\frac{1}{2}}$, $C_{11} = \tau\left[1+\frac{3}{2}(2+\sin x_1)(1+g_1)\right]$, $D_{11} = E_{11} = \tau$, and then we get $h_{21} = (1+\frac{4}{3}x_1^2)^2(1+g_1)^2 + r_2 + 2\rho_2(1+y_r^2)(1+\frac{4}{3}x_1^2)^2 + (1+\frac{4}{3}x_1^2)(1+g_1), h_{22} = C_{11}^2 + r_1$

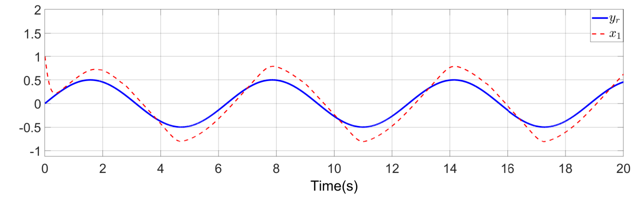


FIGURE 9. The trajectories of $x_1$,$y_r$ in Example 5.2 Case 5.2.1.

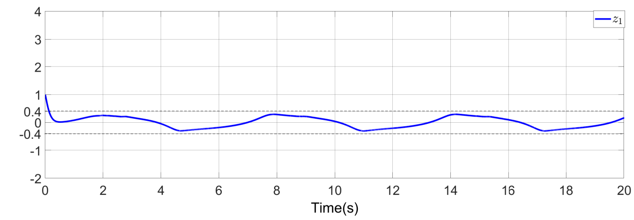


FIGURE 10. The trajectory of tracking error $z_1$ in Example 5.2 Case 5.2.1.

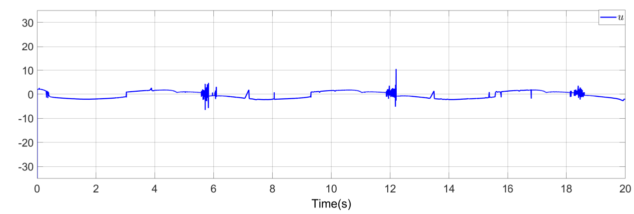


FIGURE 11. The response of input $u$ in Example 5.2 Case 5.2.1.

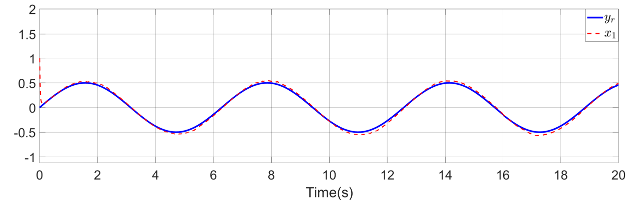


FIGURE 12. The trajectories of $x_1$,$y_r$ in Example 5.2 Case 5.2.2.

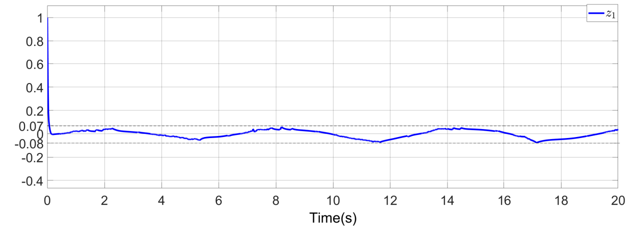


FIGURE 13. The response of tracking error $z_1$ in Example 5.2 Case 5.2.2.

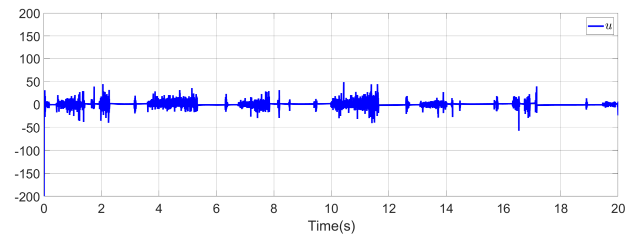


FIGURE 14. The response of input $u$ in Example 5.2 Case 5.2.2.

$D_{11}^2+2\rho_2E_{11}^2+C_{11}$, $h_{23} = \frac{4(1+\rho_1(1+y_r^2)+r_1+\dot{y}_r^2)^2\hat{\Theta}_1^2}{(2+\sin x_1)^2(1+\hat{\Theta}_1^2)}\left(2\rho_2 z_1^4(1+r_1+\rho_1(1+y_r^2)+\dot{y}_r^2)^2+\sigma_1^2\hat{\Theta}_1^2\right)+\frac{81}{2}(2+\sin x_1)^2$, $h_{24} = 2\rho_2\left[\left(\frac{2\left(l_1+\sqrt{1+\hat{\Theta}_1^2}(r_1+1+\rho_1(1+y_r^2)+\dot{y}_r^2)\right)}{2+\sin x_1}\right)^2\dot{y}_r^2 + \left(z_1\frac{4\dot{y}_r\sqrt{1+\hat{\Theta}_1^2}}{(2+\sin x_1)}\right)^2\ddot{y}_r^2\right]$, $h_2 = h_{21}+h_{22}+h_{23}+h_{24}$.

Finally we get the actual controller satisfying

$$u^3 = -2z_2l_2 - 2z_2h_2\sqrt{1+\hat{\Theta}_2^2},$$

where $\hat{\Theta}_2$ is provided via adaptive law $\dot{\hat{\Theta}}_2 = z_2^2h_2 - \sigma_2\hat{\Theta}_2$. We set the initial conditions $x_1(0) = x_2(0) = 1$, $\hat{\Theta}_1(0) = 1$, $\hat{\Theta}_2(0) = 0.3$, then we assign different values to the parameters to show that we can reduce the tracking error by adjusting the parameters.

**Case 5.2.1.** We choose the parameters $l_1 = l_2 = 3, r_1 = r_2 = 1, \rho_1 = \rho_2 = 1, \sigma_1 = \sigma_2 = 1$, at this point we get the following simulation results.

**Case 5.2.2.** When the parameters are adjusted to $l_1 = l_2 = 25, r_1 = r_2 = 5, \rho_1 = \rho_2 = 1.5, \sigma_1 = \sigma_2 = 0.1$, the simulation results at this time are obtained as follows.

It is obvious that when the parameters are adjusted from Case 5.2.1 to Case 5.2.2, the tracking error also decreases, which indicates that increasing parameters $l_1, l_2, r_1, r_2, \rho_1, \rho_2$ and decreasing $\sigma_1, \sigma_2$ can reduce the tracking error $z_1$.

## VI. CONCLUSION

The proposed robust adaptive tracking controller can effectively solve the tracking control problem of high-order nonlinearly parameterized systems with unmatched disturbances. Under reasonable assumptions, the controller guarantees that all the states and tracking error in the closed-loop system are globally bounded, and the tracking error can approach an arbitrarily small bound by selecting appropriate parameters. Simulation results illustrate the effectiveness of the proposed robust adaptive tracking controller. In the future work, We will consider whether the controller can be designed so that the tracking error converges to a specified range at a prescribed time when the system has an actuator fault.

## APPENDIX

### A. PROOF OF PROPOSITION 1

On the basis of the definitions of $\alpha_k^{p_1\cdots p_k}$ and $z_k,\ k = 1,\ldots,j$ in (7), one has

$$\alpha_j^{p_1\cdots p_j}(\bar{x}_j, \bar{y}_r^{(j)}, \bar{\hat{\Theta}}_j) = -\sum_{k=1}^{j}\Big(\prod_{l=k}^{j} g_l(\bar{y}_r^{(l)}, \bar{x}_l, \bar{\hat{\Theta}}_l)\Big)x_k^{p_1\cdots p_{k-1}} + \Big(\prod_{l=1}^{j} g_l(\bar{y}_r^{(l)}, \bar{x}_l, \bar{\hat{\Theta}}_l)\Big)\alpha_0, \tag{48}$$

further,

$$\begin{aligned}|x_j^{p_1\cdots p_{j-1}} - \alpha_{j-1}^{p_1\cdots p_{j-1}}| &\leq (1+x_j^2)^{\frac{p_1\cdots p_{j-1}}{2}} + (\prod_{l=1}^{j-1} g_l)(1+y_r^2)^{\frac{1}{2}} \\ &+ \sum_{k=1}^{j-1}(\prod_{l=k}^{j-1} g_l)(1+x_k^2)^{\frac{p_1\cdots p_{k-1}}{2}}. \quad (49)\end{aligned}$$

When $j = 2, \ldots, i$, according to Lemma 2, (49) and the definition of $\alpha_{j-1}^{p_1\cdots p_{j-1}}$, we get

$$\begin{aligned}|x_j| &= |x_j^{p_1\cdots p_{j-1}} - \alpha_{j-1}^{p_1\cdots p_{j-1}}(.) + \alpha_{j-1}^{p_1\cdots p_{j-1}}(.)|^{\frac{1}{p_1\cdots p_{j-1}}} \\ &\leq \varpi_j^{\frac{p_j\cdots p_{i-1}-1}{p_1\cdots p_{i-1}}} |z_j|^{\frac{1}{p_1\cdots p_{i-1}}} + g_{j-1}^{\frac{1}{p_1\cdots p_{j-1}}} \varpi_{j-1}^{\frac{p_j\cdots p_{i-1}-1}{p_1\cdots p_{i-1}}} |z_{j-1}|^{\frac{1}{p_1\cdots p_{i-1}}}, \quad (50)\end{aligned}$$

where $\varpi_{j-1}(\bar{y}_r^{(j-1)}, \bar{x}_j, \bar{\hat{\Theta}}_{j-1}) = (1 + x_j^2)^{\frac{p_1\cdots p_{j-1}}{2}} + \sum_{k=1}^{j-1}(\prod_{l=k}^{j-1} g_l)(1+x_k^2)^{\frac{p_1\cdots p_{k-1}}{2}} + (\prod_{l=1}^{j-1} g_l)(1+y_r^2)^{\frac{1}{2}} > 0$ is a smooth function, and when $j = 1$, we have

$$\begin{aligned}|x_1| &= |x_1 - y_r + y_r| \leq |x_1 - y_r| + |y_r| \\ &\leq \left(1 + (x_1-y_r)^2\right)^{\frac{p_1\cdots p_{i-1}-1}{2p_1\cdots p_{i-1}}} \cdot |z_1|^{\frac{1}{p_1\cdots p_{i-1}}} + \sqrt{1+y_r^2}. \quad (51)\end{aligned}$$

In combination with (4) and (5), we define nonnegative smooth functions

$$\begin{aligned}&\bar{\varphi}_i(\bar{y}_r^{(i-1)}, \bar{x}_i, \bar{\hat{\Theta}}_{i-1}) \\ &= \varphi_i\Bigg(\left(1+(x_1-y_r)^2\right)^{\frac{p_1\cdots p_{i-1}-1}{2p_1\cdots p_{i-1}}} \\ &\quad + \sum_{j=2}^{i} \varpi_j^{\frac{p_j\cdots p_{i-1}-1}{p_1\cdots p_{i-1}}} + \sum_{j=1}^{i-1} g_j^{\frac{1}{p_1\cdots p_j}} \varpi_j^{\frac{p_{j+1}\cdots p_{i-1}-1}{p_1\cdots p_{i-1}}}\Bigg), \\ &\tilde{\varphi}_i(\bar{y}_r^{(i-1)}, \bar{x}_i, \bar{\hat{\Theta}}_{i-1}) = \varphi_i\sqrt{1+y_r^2}, \quad (52)\end{aligned}$$

the following holds

$$\begin{aligned}\left|\sum_{j=0}^{p_i-1} x_{i+1}^j \phi_{i,j}\right| &\leq \frac{\lambda_i(x,u,\theta)|x_{i+1}|^{p_i}}{2} + \tilde{\varphi}_i\bar{\theta}_i \\ &+ \bar{\varphi}_i \sum_{j=1}^{i} |z_j|^{\frac{1}{p_1\cdots p_{i-1}}}\bar{\theta}_i. \quad (53)\end{aligned}$$

This completes the proof. □

### B. PROOF OF PROPOSITION 2

It can be proved by mathematical induction method. According to Assumption 1, Proposition 1, the definition of $\alpha_1^{p_1}$ and (6), we have

$$\begin{aligned}\left|\frac{\partial \alpha_1^{p_1}}{\partial x_1}\dot{x}_1\right| &\leq \left|\frac{\partial \alpha_1^{p_1}}{\partial x_1}\right| \cdot \left|\frac{3}{2}\bar{\mu}_1\bar{\theta}_1\left(|z_2| + g_1|z_1|\right) + \bar{\varphi}_1\bar{\theta}_1|z_1|\right| \\ &+ \left|\frac{\partial \alpha_1^{p_1}}{\partial x_1}\right|\tilde{\varphi}_1\bar{\theta}_1 + \left|\frac{\partial \alpha_1^{p_1}}{\partial x_1}\right||d_1(t)| \\ &\leq (|z_1| + |z_2|)C_{11}\bar{\theta}_1 + D_{11}|d_1(t)| + E_{11}\bar{\theta}_1, \quad (54)\end{aligned}$$

where $C_{11}(\bar{y}_r^{(1)}, \bar{x}_2, \bar{\hat{\Theta}}_1) = \sqrt{1 + (\frac{\partial \alpha_1^{p_1}}{\partial x_1})^2}(\frac{3}{2}\bar{\mu}_1(1+g_1) + \bar{\varphi}_1) > 0$, $D_{11}(\bar{y}_r^{(1)}, \bar{x}_2, \bar{\hat{\Theta}}_1) = \sqrt{1 + (\frac{\partial \alpha_1^{p_1}}{\partial x_1})^2} > 0$, $E_{11}(\bar{y}_r^{(1)}, \bar{x}_2, \bar{\hat{\Theta}}_1) = \tilde{\varphi}_1\sqrt{1 + (\frac{\partial \alpha_1^{p_1}}{\partial x_1})^2} > 0$ are smooth functions. Then we assume that, there exist smooth functions $C_{k-1,i} \geq 0, D_{k-1,i} \geq 0, E_{k-1,i} \geq 0$ for $i = 1, \ldots, k-1$ such that

$$\begin{aligned}\left|\frac{\partial \alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial x_i}\dot{x}_i\right| &\leq (|z_1| + \cdots + |z_k|)C_{k-1,i}\bar{\theta}_i \\ &+ D_{k-1,i}|d_i(t)| + E_{k-1,i}\bar{\theta}_i. \quad (55)\end{aligned}$$

In order to prove that there exist nonnegative smooth functions $C_{ki}, D_{ki}, E_{ki}, i = 1, \ldots, k$ such that (9) is satisfied, we first consider a case that $i = 1, \ldots, k-1$,

$$\begin{aligned}\left|\frac{\partial \alpha_k^{p_1\cdots p_k}}{\partial x_i}\dot{x}_i\right| &\leq |z_k|\left|\frac{\partial g_k}{\partial x_i}\right|\Bigg|\frac{3}{2}\bar{\mu}_i(\bar{x}_i)\bar{\theta}_i|x_{i+1}|^{p_i} \\ &+ \varphi_i(\bar{x}_i)\bar{\theta}_i\sum_{l=1}^{i}|x_l|\Bigg| + |z_k|\left|\frac{\partial g_k}{\partial x_i}\right||d_i(t)| \\ &+ |g_k| \cdot \big|(|z_1| + \cdots + |z_k|)C_{k-1,i}\bar{\theta}_i \\ &+ D_{k-1,i}|d_i(t)| + E_{k-1,i}\bar{\theta}_i\big| \\ &\leq C_{ki}(.)\bar{\theta}_i\left(\sum_{l=1}^{k}|z_l|\right) + D_{ki}(.)|d_i(t)| \\ &+ E_{ki}(.)\bar{\theta}_i, \quad (56)\end{aligned}$$

where $C_{ki}(\bar{y}_r^{(k)}, \bar{x}_k, \bar{\hat{\Theta}}_k) = \left(1 + (\frac{\partial g_k}{\partial x_i})^2\right)^{\frac{1}{2}}\left(\frac{3}{2}\bar{\mu}_i(1+x_{i+1}^2)^{\frac{p_i}{2}} + \varphi_i\sum_{l=1}^{i}(1+x_l^2)^{\frac{1}{2}}\right) + C_{k-1,i}(1+g_k^2)^{\frac{1}{2}} > 0$, $D_{ki}(\bar{y}_r^{(k)}, \bar{x}_k, \bar{\hat{\Theta}}_k) = \left(1 + (z_k\frac{\partial g_k}{\partial x_i})^2\right)^{\frac{1}{2}} + D_{k-1,i}(1+g_k^2)^{\frac{1}{2}} > 0$, $E_{ki}(\bar{y}_r^{(k)}, \bar{x}_k, \bar{\hat{\Theta}}_k) = \left(1+g_k^2\right)^{\frac{1}{2}}E_{k-1,i} > 0$ are smooth functions. For another case that $i = k$, we have

$$\begin{aligned}&\left|\frac{\partial \alpha_k^{p_1\cdots p_k}}{\partial x_k}\dot{x}_k\right| \\ &= \left|\frac{\partial g_k z_k}{\partial x_k}\dot{x}_k\right| \\ &\leq \left|z_k\frac{\partial g_k}{\partial x_k}\dot{x}_k\right| + (p_1\cdots p_{k-1})|g_k x_k^{p_1\cdots p_{k-1}-1}\dot{x}_k| \\ &\leq \left|z_k\frac{\partial g_k}{\partial x_k}\right| \cdot \Bigg|\frac{3}{2}\bar{\mu}_k(\bar{x}_k)\bar{\theta}_k|x_{k+1}|^{p_k} + \varphi_k(\bar{x}_k)\bar{\theta}_k(|x_1| \\ &\quad + \cdots + |x_k|)\Bigg| + \left|z_k\frac{\partial g_k}{\partial x_k}\right||d_k(t)| + (p_1\cdots p_{k-1}) \\ &\quad |g_k x_k^{p_1\cdots p_{k-1}-1}\dot{x}_k|. \quad (57)\end{aligned}$$

For the term $|x_k^{p_1\cdots p_{k-1}-1}\dot{x}_k|$, using Lemma 3 and (7), there exists smooth functions $\gamma_k, \hat{\gamma}_k, \tilde{\gamma}_k \geq 0$ such that

$$\begin{aligned}|x_k^{p_1\cdots p_{k-1}-1}\dot{x}_k| &\leq |x_k^{p_1\cdots p_{k-1}-1}| \cdot \Big|\frac{3}{2}\overline{\mu}_k\overline{\theta}_k|x_{k+1}|^{p_k} + (|x_1| \\ &\quad + \cdots + |x_k|)\varphi_k(\bar{x}_k)\overline{\theta}_k + d_k(t)\Big| \\ &\leq \gamma_k(\bar{x}_k)\overline{\theta}_k(|x_1|^{p_1\cdots p_{k-1}} + \cdots + |x_k|^{p_1\cdots p_{k-1}} \\ &\quad + |x_{k+1}|^{p_1\cdots p_k}) + |x_k|^{p_1\cdots p_{k-1}-1}|d_k(t)| \\ &\leq \hat{\gamma}_k(\bar{y}_r^k, \bar{x}_k, \hat{\bar{\Theta}}_k)\overline{\theta}_k\left(\sum_{l=1}^{k+1}|z_l|\right) + \tilde{\gamma}_k(\bar{x}_k, y_r)\overline{\theta}_k \\ &\quad + |x_k|^{p_1\cdots p_{k-1}-1}|d_k(t)|. \end{aligned} \tag{58}$$

Therefore, there are positive smooth functions $C_{kk}(\bar{y}_r^{(k)}, \bar{x}_{k+1}, \hat{\bar{\Theta}}_k) = \big(1 + (\frac{g_k}{x_k})^2\big)^{\frac{1}{2}}\big(\frac{3}{2}\overline{\mu}_k(1 + x_{k+1}^2)^{\frac{p_k}{2}} + \varphi_k\sum_{l=1}^{k}(1 + x_l^2)^{\frac{1}{2}}\big) + p_1\cdots p_{k-1}\big(1 + g_k^2\big)^{\frac{1}{2}}\hat{r}_k$, $D_{kk}(\bar{y}_r^{(k)}, \bar{x}_{k+1}, \hat{\bar{\Theta}}_k) = \big(1 + (z_k\frac{\partial g_k}{\partial x_k})^2\big)^{\frac{1}{2}} + p_1\cdots p_{k-1}\big(1 + g_k^2\big)^{\frac{1}{2}}\big(1 + x_k^2\big)^{\frac{p_1\cdots p_{k-1}-1}{2}}$ and $E_{kk}(\bar{y}_r^{(k)}, \bar{x}_{k+1}, \hat{\bar{\Theta}}_k) = p_1\cdots p_{k-1}\big(1 + g_k^2\big)^{\frac{1}{2}}\tilde{r}_k$ such that

$$\begin{aligned}\left|\frac{\partial\alpha_k^{p_1\cdots p_k}}{\partial x_k}\dot{x}_k\right| &\leq (|z_1| + \cdots + |z_{k+1}|)C_{kk}\overline{\theta}_k \\ &\quad + D_{kk}|d_k(t)| + E_{kk}\overline{\theta}_k. \end{aligned} \tag{59}$$

So far, the above proposition is proved. □

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