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RESEARCH ARTICLE

Notion of Complex Spherical Dombi Fuzzy Graph and Its Application in Decision-Making Problems

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ABSTRACT The complex spherical fuzzy graph (CSFG), which extends the concept of a spherical fuzzy graph (SFG), proves to be a more effective means of depicting relationships among diverse objects when these relationships are subject to uncertainty. In addition, the Dombi operators, featuring an adjustable operational parameter, offer valuable utility by accommodating distinct values. In this research paper, we present the concept of a complex spherical dombi fuzzy graph (CSDFG), an extension of a spherical dombi fuzzy graph (SDFG). Dombi operators are utilized as averaging operators, playing a crucial role in aggregating data into a single value for efficient decision-making. We implement Dombi operators on CSFGs. The complement of a CSDFG is defined, and self-complementry in CSDFGs is discussed. We explore homomorphism, isomorphism, weak isomorphism (W-isomorphism), and co-weak isomorphism (CW-isomorphism) to establish relationships between CSDFGs. We define regular, arc regular, and totally arc regular CSDFGs, explain their key properties, and demonstrate an application of CSDFG in decision-making problems.

INDEX TERMS CSDFG, complement, homomorphism, isomorphism, regular and total regular, application.

I. INTRODUCTION

Dombi operations are used in fuzzy graph theory to handle contradictory data, reduce uncertainty, visualise intricate connections, and provide a flexible method for merging and combining graphs. In situations where conventional clear-cut graphs are unable to fully capture the subtleties within the data, they support the adaptability and resilience of fuzzy graph representations, enabling more accurate decision-making. Dombi operations combine membership degrees from various nodes or edges to represent uncertainty. Dombi operations gracefully handle conflicting information, which is one of their key functions. Dombi operations can resolve conflicting information and provide representations of the fuzzy graph in circumstances where various sources provide incongruent information about the existence or strength of connections within a graph.

The notion of a fuzzy set (FU-S), which expands crisp set theory, was initially proposed by Zadeh [22] due to the existence of ambiguous data. A membership function within a FU-S assigns degrees of truth from the closed range [0, 1]. Fuzzy sets are a strong tool for handling uncertainty and ambiguity in a variety of applications because they combine linguistic variables with fuzzy logic.

Kaufman [11] was the first to propose the concept of fuzzy graphs (FG). The FG is established by expanding the notion of a standard graph to include FU-Ss and fuzzy relations. Instead of clear values for arcs, fuzzy graphs use membership degrees to describe the strength of links between nodes. The membership degrees can be described using fuzzy membership functions or fuzzy matrices, providing a more flexible representation of uncertain and imprecise

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connections in numerous applications. Fuzzy graphs are used to depict ambiguous and imprecise interactions between components, making it possible to represent complex systems and decision-making processes in a way that is more adaptable and realistic. Fuzzy graphs are very useful in the fields of pattern recognition, image processing, data analysis, and artificial intelligence. Later, Rosenfeld [15] investigated fuzzy relations on FU-Ss. Bhattacharya [4] made a few comments on FGs. Additionally, Shoaib et al. [17] introduced novel properties on picture fuzzy graphs. Recently, some researchers have been making contributions to the field of fuzzy theory recently [7], [8], [9], [18], [19], [20].

Triangle norms and conorms were first presented by Menger [13] in the context of probabilistic metric spaces, and after some time, they were studied by Schweizer and Sklar [14]. Other academics have offered different T-operators, [5], [10] as examples; among the most popular operators are products, [6]. During the selection of T-operators for specific uses, consider their characteristics, model suitability, ease of implementation on software and hardware, and so on.

The dombi FG was first introduced by Ashraf et al. [3]. The idea of CFS, where a range of degrees exists in the complex plane within the unit disc, was introduced by Ramot et al. [16]. The complex Pythagorean fuzzy graphs were studied by Akram and Khan [1] in the context of problems with decision-making. The complex picture fuzzy graphs were introduced by Shoaib et al. [21]. Karthick et al. [12] studied a material selection model based on SDFG.

We have introduced CSDFG, which is based on complex spherical fuzzy numbers. Complex spherical fuzzy numbers are used in graph theory to address complex relationships and uncertainties in network data. These numbers extend traditional spherical fuzzy numbers to include real and imaginary parts, allowing for the representation of both magnitude and phase information. This information is particularly valuable in directional networks, dynamic systems, and scenarios with oscillatory behaviors. Complex spherical fuzzy numbers can be used in complex relationships, decision-making, and uncertainty handling, enhancing their suitability for applications such as electrical circuit analysis, signal processing, and network dynamics.

The purpose of the paper can be summed up as follows:

- A CSDFG is useful in dealing with three-dimensional phenomena, including imprecise and intuitive knowledge, without losing information due to its phase term.
- Dombi operators, which cover a variety of widely used operator attributes, provide larger applications and excellent decision-making efficiency.

Here are some key points addressed in the paper:

- The idea of CSDFG is introduced.
- The degree of a node and its total degree, both in terms of amplitude and phase, are thoroughly explained with examples.
- The isomorphism, homomorphism, complement, W-Isomorphism, and CW-isomorphism are defined by their properties.
- The strong CSDFG and a complete CSDFG are introduced.
- Explanation and study of regular and completely regular graphs, as well as their key properties.

The paper's structure is as follows: In Section II, we provide fundamental definitions essential for understanding the paper. Section III covers the study of CSDFG, node degree, total degree, isomorphism, homomorphism, complement, W-Isomorphism, and CW-Isomorphism, strong CSDFG, complete CSDFG, and the detailed properties of regular and totally regular CSDFGs. Section IV explores the application of CSDFG. Lastly, in Section V, we present the conclusion and outline future plans.

II. PRELIMINARIES

Definition 1 ([1]): A FU-S on a universe Υ is an object $\mathcal{I} = \{< x, \omega_{\mathcal{I}}(x) > | x \in \Upsilon\}$, where $\omega_I : \Upsilon \to [0,1]$ represents the membership degree of $\mathcal{I}$.

Definition 2 ([1]): A FU-S on $\Upsilon \times \Upsilon$ is known as fuzzy relation on Υ, represented by $\mathcal{J} = \{< xb, \omega_{\mathcal{J}}(xb) > | xb \in \Upsilon \times \Upsilon\}$, where $\omega_J : \Upsilon \times \Upsilon \to [0,1]$ represents the degree of membership of $\mathcal{J}$.

Definition 3 ([1]): A FG on a non empty set Υ is a pair $\Psi = (\mathcal{I}, \mathcal{J})$ with $\mathcal{I}$ a FU-S on Υ and $\mathcal{J}$ a fuzzy relation on Υ such that $\omega_{\mathcal{J}}(xb) \leq \omega_{\mathcal{I}}(x) \wedge \omega_{\mathcal{I}}(b)$ for all $x, b \in \Upsilon$.

Definition 4 ([20]): On a universe Υ, a spherical fuzzy set (SFS) is defined as $\mathcal{I} = \{< x, \omega_{\mathcal{I}}(x), \tau_{\mathcal{I}}(x), \Gamma_{\mathcal{I}}(x) > | x \in \Upsilon\}$ which satisfies the axiom $0 \leq \omega_I^2(x) + \tau_I^2(x) + \Gamma_I^2(x) \leq 1$, for all $x \in \Upsilon$, where $\omega_{\mathcal{I}}, \tau_{\mathcal{I}}, \Gamma_{\mathcal{I}} : \Upsilon \to [0,1]$ represent the value of membership, value of non-membership and value of abstinence of $\mathcal{I}$ respectively.

Definition 5 ([20]): A SFS on $\Upsilon \times \Upsilon$ is said to be spherical fuzzy relation on Υ denoted by $\mathcal{J} = \{< xb, \omega_{\mathcal{J}}(xb), \tau_{\mathcal{J}}(xb), \Gamma_{\mathcal{J}}(xb) > | xb \in \Upsilon \times \Upsilon\}$ satisfies the axiam $0 \leq \omega_J^2(xb) + \tau_J^2(xb) + \Gamma_J^2(xb) \leq 1$, for all $x, b \in \Upsilon$, where $\omega_{\mathcal{J}}, \tau_{\mathcal{J}}, \Gamma_{\mathcal{J}} : \Upsilon \times \Upsilon \to [0,1]$ represent the value of membership, value of non-membership and value of abstinence of $\mathcal{J}$ respectively.

Definition 6 ([20]): On a non empty set Υ, a spherical fuzzy graph is a pair $\Psi = (\mathcal{I}, \mathcal{J})$ with $\mathcal{I}$ a SFS on Υ and $\mathcal{J}$ a spherical fuzzy relation on Υ such that

$$\omega_{\mathcal{J}}(xb) \leq \omega_{\mathcal{I}}(x) \wedge \omega_{\mathcal{I}}(x),$$
$$\tau_{\mathcal{J}}(xb) \leq \tau_{\mathcal{I}}(x) \vee \tau_{\mathcal{I}}(b),$$
$$\Gamma_{\mathcal{J}}(xb) \leq \Gamma_{\mathcal{I}}(x) \wedge \Gamma_{\mathcal{I}}(b),$$

and $0 \leq \omega_J^2(xb) + \tau_J^2(xb) + \Gamma_J^2(xb) \leq 1$ for all $x, b \in \Upsilon$, where $\omega_{\mathcal{J}}, \tau_{\mathcal{J}}, \Gamma_{\mathcal{J}} : \Upsilon \times \Upsilon \to [0,1]$ represent the value of membership, value of non-membership and value of abstinence of $\mathcal{J}$ respectively.

Definition 7 ([1]): A binary function $\mathcal{H} : [0,1] \times [0,1] \to [0,1]$ is known as triangular norm (t-norm) if for all x, b, e $\in$ [0,1], it follows the given conditions:

1. $\mathcal{H}(x, 1) = x$. (boundary axiam)
2. $\mathcal{H}(x, b) = \mathcal{H}(b, x)$. (commutativity)
3. $\mathcal{H}(x, \mathcal{H}(b, e)) = \mathcal{H}(\mathcal{H}(x, b), e)$. (associativity)
4. $\mathcal{H}(x, b) \leq \mathcal{H}(e, f)$ if $x \leq r$ and $b \leq f$. (monotonicity)

Replacing 1 by 0 in axiam (1), we obtain the concept of triangular conorm (t-conorm).

[1] The few popular t-norms are given as follows:

- $\mathcal{MN}(x, b) = minimum(x, b)$. (minimum operator $\mathcal{MN}$)
- $\mathcal{PR}(x, b) = xb$. (product operator $\mathcal{PR}$)
- $\mathcal{WL}(x, b) = max(x + b - 1, 0)$. (Lukasiewicz's t-norm $\mathcal{WL}$)
- $\mathcal{DMB}(x, b) = \frac{1}{1+[(\frac{1-x}{x})^{\mathcal{T}}+(\frac{1-b}{b})^{\mathcal{T}}]^{1/\mathcal{T}}}$: $\mathcal{T} > 0$. (Dombi's t-norm $\mathcal{DMB}$)

We derive another T-operator by using $\mathcal{T} = 1$ in dombi's t-norm that is $T(x, b) = \frac{xb}{x+b-xb}$.

The following are the related t-conorms::

- $\mathcal{MX}^*(x, b) = max(x, b)$. (maximum operator $\mathcal{MX}^*$)
- $\mathcal{PS}^*(x, b) = x + b - xb$. (probabilistic sum $\mathcal{PS}^*$)
- $\mathcal{WL}^*(x, b) = min(x + b, 1)$. (Lukasiewicz's t-conorm $\mathcal{WL}^*$)
- $\mathcal{DMB}^*(x, b) = \frac{1}{1+[(\frac{1-x}{x})^{-\mathcal{T}}+(\frac{1-b}{b})^{-\mathcal{T}}]^{1/-\mathcal{T}}}$: $\mathcal{T} > 0$. (Dombi's t-conorm $\mathcal{DMB}^*$)

We derive another T-operator by putting $\mathcal{T} = 1$ in Dombi's t-norm, which is $S(x, b) = \frac{x+b-2xb}{1-xb}$.

Definition 8 ([3]): A Dombi fuzzy graph over an underlying set $\mathcal{A}$ consists of an ordered pair $\Psi = (\mathcal{I}, \mathcal{J})$, where $\mathcal{I} : \mathcal{A} \to [0, 1]$ represents a fuzzy subset of $\mathcal{A}$ and $\mathcal{J} : \mathcal{A} \times \mathcal{A} \to [0, 1]$ is a symmetric fuzzy relation on $\mathcal{I}$ such that

$$\omega_{\mathcal{J}}(xb) \leq \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}$$

$\forall$ x,b$\in$ $\mathcal{A}$, where $\omega_{\mathcal{I}}$ and $\omega_{\mathcal{J}}$ represents the degree membership of $\mathcal{I}$ and $\mathcal{J}$ respectively.

Definition 9 ([12]): A SDFG on a non empty set $\mathcal{A}$ is a pair $\Psi = (\mathcal{I}, \mathcal{J})$ with $\mathcal{I} = (\omega_{\mathcal{I}}, \tau_{\mathcal{I}}, \Gamma_{\mathcal{I}}) : \mathcal{A} \to [0, 1]$ a SFS in $\mathcal{A}$ and $\mathcal{J} = (\omega_{\mathcal{J}}, \tau_{\mathcal{J}}, \Gamma_{\mathcal{J}}) : \mathcal{A} \times \mathcal{A} \to [0, 1]$ a spherical fuzzy relation on $\mathcal{I}$ such that

$$\omega_{\mathcal{J}}(xb) \leq \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}$$
$$\tau_{\mathcal{J}}(xb) \leq \frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}$$
$$\Gamma_{\mathcal{J}}(xb) \leq \frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}$$

and satisfies the axiam $0 \leq \omega_{\mathcal{J}}^2(xb) + \tau_{\mathcal{J}}^2(xb) + \Gamma_{\mathcal{J}}^2(xb) \leq 1$, for all $x, b \in \mathcal{A}$, where $\omega_{\mathcal{J}}, \tau_{\mathcal{J}}, \Gamma_{\mathcal{J}}$ represent the value of membership, value of non-membership and value of abstinence of $\mathcal{J}$ respectively.

Definition 10 ([20]): On a universe Υ, complex spherical fuzzy set (CSFS) is defined as $\mathcal{I} = \{< x, \omega_{\mathcal{I}}(x)e^{i\varkappa_{\mathcal{I}}(x)}, \tau_{\mathcal{I}}(x)e^{i\varsigma_{\mathcal{I}}(x)}, \Gamma_{\mathcal{I}}(x)e^{i\delta_{\mathcal{I}}(x)} > |x \in \Upsilon\}$, $i = \sqrt{-1}$ that satisfies the axiom $0 \leq \omega_I^2(x) + \tau_I^2(x) + \Gamma_I^2(x) \leq 1$ and $\varkappa_{\mathcal{I}}(x), \varsigma_{\mathcal{I}}(x), \delta_{\mathcal{I}}(x) \in [0, 2\pi]$, for all $x \in \Upsilon$, where $\omega_{\mathcal{I}}, \tau_{\mathcal{I}}, \Gamma_{\mathcal{I}} : \Upsilon \to [0, 1]$ represent the value of standard membership, value of standard non-membership and value of standard abstinence of $\mathcal{I}$ respectively. Note that $\omega_{\mathcal{I}}(x), \tau_{\mathcal{I}}(x), \Gamma_{\mathcal{I}}(x)$ are called amplitude terms and $\varkappa_{\mathcal{I}}(x), \varsigma_{\mathcal{I}}(x), \delta_{\mathcal{I}}(x)$ are called phase terms.

III. CSDFG

Definition 11: A CSFS on $\Upsilon \times \Upsilon$ is said to be complex spherical fuzzy relation (CSFR) denoted by $\mathcal{J} = \{< xb, \omega_{\mathcal{J}}(xb)e^{i\varkappa_{\mathcal{J}}(xb)}, \tau_{\mathcal{J}}(xb)e^{i\varsigma_{\mathcal{J}}(xb)}, \Gamma_{\mathcal{J}}(xb)e^{i\delta_{\mathcal{J}}(xb)} > |xb \in \Upsilon \times \Upsilon\}$, $i = \sqrt{-1}$ satisfies the axiam $0 \leq \omega_J^2(xb) + \tau_J^2(xb) + \Gamma_J^2(xb) \leq 1$ and $\varkappa_{\mathcal{J}}(xb), \varsigma_{\mathcal{J}}(xb), \delta_{\mathcal{J}}(xb) \in [0, 2\pi]$, for all $x, b \in \Upsilon$, where $\omega_{\mathcal{J}}, \tau_{\mathcal{J}}, \Gamma_{\mathcal{J}} : \Upsilon \times \Upsilon \to [0, 1]$ represent the value of standard membership, value of standard non-membership and value of standard abstinence of $\mathcal{J}$ respectively. Note that $\omega_{\mathcal{J}}(xb), \tau_{\mathcal{J}}(xb), \Gamma_{\mathcal{J}}(xb)$ are called amplitude terms and $\varkappa_{\mathcal{J}}(xb), \varsigma_{\mathcal{J}}(xb), \delta_{\mathcal{J}}(xb)$ are called phase terms.

Definition 12: A CSDFG on a universe Υ is an ordered pair $\Psi = (\mathcal{I}, \mathcal{J})$, where $\mathcal{I} = (\omega_{\mathcal{I}}e^{i\varkappa_{\mathcal{I}}}, \tau_{\mathcal{I}}e^{i\varsigma_{\mathcal{I}}}, \Gamma_{\mathcal{I}}e^{i\delta_{\mathcal{I}}}) : \Upsilon \to \{n : n \in C, |N| \leq 1\}$ is a CSF subset in Υ and $\mathcal{J} = (\omega_{\mathcal{J}}e^{i\varkappa_{\mathcal{J}}}, \tau_{\mathcal{J}}e^{i\varsigma_{\mathcal{J}}}, \Gamma_{\mathcal{J}}e^{i\delta_{\mathcal{J}}}) : \Upsilon \times \Upsilon \to \{n : n \in C, |N| \leq 1\}$ is a complex spherical fuzzy relation (CSFR) on $\mathcal{I}$ such that for amplitude terms

$$\omega_{\mathcal{J}}(xb) \leq \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)},$$
$$\tau_{\mathcal{J}}(xb) \leq \frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)},$$
$$\Gamma_{\mathcal{J}}(xb) \leq \frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)},$$

and for phase terms

$$\varkappa_{\mathcal{J}}(xb) \leq \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)},$$
$$\varsigma_{\mathcal{J}}(xb) \leq \frac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)},$$
$$\delta_{\mathcal{J}}(xb) \leq \frac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)},$$

for all x,b$\in$ Υ, where $i = \sqrt{-1}$, $0 \leq \omega_J^2(xb) + \tau_J^2(xb) + \Gamma_J^2(xb) \leq 1$ and $\varkappa_{\mathcal{J}}(xb), \varsigma_{\mathcal{J}}(xb), \delta_{\mathcal{J}}(xb) \in [0, 2\pi]$. We call $\mathcal{I}$ and $\mathcal{J}$ the CSF vertex set and CSF arc set, respectively.

Example 1: Let $\Psi = (\mathcal{I}, \mathcal{J})$ be a CSDFG on $\Psi^* = (\Upsilon, \mathcal{Z})$, where $\Upsilon = \{x, b, r, s\}$ and $\mathcal{Z} = \{xb, xr, xs, bs\}$ as shown in Figure 1. The set of nodes $\mathcal{I}$ and set of arcs $\mathcal{J}$ of Ψ are defined on Υ and $\mathcal{Z}$, respectively.

$$\mathcal{I} =< (\frac{x}{0.5e^{i2\pi(0.6)}}, \frac{b}{0.6e^{i2\pi(0.4)}}, \frac{r}{0.3e^{i2\pi(0.7)}}, \frac{s}{0.4e^{i2\pi(0.2)}}),$$
$$(\frac{x}{0.2e^{i2\pi(0.3)}}, \frac{b}{0.3e^{i2\pi(0.2)}}, \frac{r}{0.4e^{i2\pi(0.4)}}, \frac{s}{0.6e^{i2\pi(0.4)}}),$$
$$(\frac{x}{0.7e^{i2\pi(0.4)}}, \frac{b}{0.5e^{i2\pi(0.6)}}, \frac{r}{0.6e^{i2\pi(0.3)}}, \frac{s}{0.5e^{i2\pi(0.4)}}) >$$

and

$$\mathcal{J} =< (\frac{xb}{0.2e^{i2\pi(0.3)}}, \frac{xr}{0.21e^{i2\pi(0.4)}}, \frac{xs}{0.25e^{i2\pi(0.15)}}, \frac{bs}{0.27e^{i2\pi(0.13)}}),$$

$(\frac{xb}{0.37e^{i2\pi(0.37)}}, \frac{xr}{0.45e^{i2\pi(0.51)}}, \frac{xs}{0.61e^{i2\pi(0.40)}}, \frac{bs}{0.6e^{i2\pi(0.45)}}),$
$(\frac{xb}{0.51e^{i2\pi(0.55)}}, \frac{xr}{0.31e^{i2\pi(0.61)}}, \frac{xs}{0.13e^{i2\pi(0.25)}}, \frac{bs}{0.23e^{i2\pi(0.41)}})>$

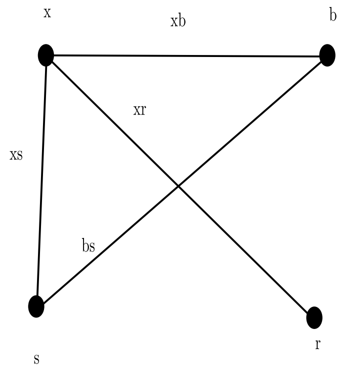

FIGURE 1. CSDFG.

$$\begin{aligned}
x &= (0.5e^{i2\pi(0.6)}, 0.2e^{i2\pi(0.3)}, 0.7e^{i2\pi(0.4)})\\
b &= (0.6e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.2)}, 0.5e^{i2\pi(0.6)})\\
r &= (0.3e^{i2\pi(0.7)}, 0.4e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.3)})\\
s &= (0.4e^{i2\pi(0.2)}, 0.6e^{i2\pi(0.4)}, 0.5e^{i3\pi(0.4)})\\
xb &= (0.2e^{i2\pi(0.3)}, 0.37e^{i2\pi(0.37)}, 0.51e^{i2\pi(0.55)})\\
xr &= (0.21e^{i2\pi(0.4)}, 0.45e^{i2\pi(0.51)}, 0.31e^{i2\pi(0.61)})\\
xs &= (0.25e^{i2\pi(0.15)}, 0.61e^{i2\pi(0.40)}, 0.13e^{i2\pi(0.25)})\\
bs &= (0.27e^{i2\pi(0.13)}, 0.6e^{i2\pi(0.45)}, 0.23e^{i3\pi(0.41)})
\end{aligned}$$

By calculations, one can see that $\Psi = (\mathcal{I}, \mathcal{J})$ is a CSDFG.

Definition 13: Let $\mathcal{J} = \{(xb, \omega_{\mathcal{J}}(xb)e^{i\varkappa_{\mathcal{J}}(xb)}, \tau_{\mathcal{J}}(xb)e^{i\varsigma_{\mathcal{J}}(xb)}, \Gamma_{\mathcal{J}}(xb)e^{i\delta_{\mathcal{J}}(xb)}) |xb \in \mathcal{Z}\}$ be a set of arcs in CSDFG Ψ, then

- *The degree of a node $x \in \Upsilon$ for amplitude term is expressed by $\mathcal{D}_\Psi(x) = (\mathcal{D}_\omega(x), \mathcal{D}_\tau(x), \mathcal{D}_\Gamma(x))$, where*

$$\mathcal{D}_\omega(x) = \sum_{x,b\neq x\in\Upsilon} \omega_{\mathcal{J}(xb)} = \sum_{x,b\neq x\in\Upsilon} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)},$$

$$\mathcal{D}_\tau(x) = \sum_{x,b\neq x\in\Upsilon} \tau_{\mathcal{J}(xb)} = \sum_{x,b\neq x\in\Upsilon} \frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)},$$

$$\mathcal{D}_\Gamma(x) = \sum_{x,b\neq x\in\Upsilon} \Gamma_{\mathcal{J}(xb)} = \sum_{x,b\neq x\in\Upsilon} \frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}.$$

The degree of a node $x \in \Upsilon$ for a phase term is represented by $\mathcal{D}_\Psi(x) = (\mathcal{D}_{e^{i\varkappa}}(x), \mathcal{D}_{e^{i\varsigma}}(x), \mathcal{D}_{e^{i\delta}}(x))$, where

$$\mathcal{D}_{e^{i\varkappa}}(x) = \sum_{x,b\neq x\in\Upsilon} \varkappa_{\mathcal{J}(xb)} = \sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)},$$

$$\mathcal{D}_{e^{i\varsigma}}(x) = \sum_{x,b\neq x\in\Upsilon} \varsigma_{\mathcal{J}(xb)} = \sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)},$$

$$\mathcal{D}_{e^{i\delta}}(x) = \sum_{x,b\neq x\in\Upsilon} \delta_{\mathcal{J}(xb)} = \sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}.$$

- *The total degree of vertex $x \in \Upsilon$ for amplitude term is represented by $\mathcal{TD}_\Psi(x) = (\mathcal{TD}_\omega(x), \mathcal{TD}_\tau(x), \mathcal{TD}_\Gamma(x))$, where*

$$\mathcal{TD}_\omega(x) = \sum_{x,b\neq x\in\Upsilon} \omega_{\mathcal{J}(xb)} + \omega_I(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} + \omega_I(x),$$

$$\mathcal{TD}_\tau(x) = \sum_{x,b\neq x\in\Upsilon} \tau_{\mathcal{J}(xb)} + \tau_I(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)} + \tau_I(x),$$

$$\mathcal{TD}_\Gamma(x) = \sum_{x,b\neq x\in\Upsilon} \Gamma_{\mathcal{J}(xb)} + \Gamma_I(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)} + \Gamma_I(x).$$

The total degree of a vertex $x \in \Upsilon$ for a phase term is represented by $\mathcal{TD}_\Psi(x) = (\mathcal{TD}_{e^{i\varkappa}}(x), \mathcal{TD}_{e^{i\varsigma}}(x), \mathcal{TD}_{e^{i\delta}}(x))$, where

$$\mathcal{TD}_{e^{i\varkappa}}(x) = \sum_{x,b\neq x\in\Upsilon} \varkappa_{\mathcal{J}(xb)} + \varkappa_I(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)} + \varkappa_I(x),$$

$$\mathcal{TD}_{e^{i\varsigma}}(x) = \sum_{x,b\neq x\in\Upsilon} \varsigma_{\mathcal{J}(xb)} + \varsigma_I(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)} + \varsigma_I(x),$$

$$\mathcal{TD}_{e^{i\delta}}(x) = \sum_{x,b\neq x\in\Upsilon} \delta_{\mathcal{J}(xb)} + \delta_I(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)} + \delta_I(x).$$

Example 2: From Example 1,

- *In Ψ, The degree of all nodes are as follows:*

$$\begin{aligned}
\mathcal{D}_\Psi(x) &= (0.66e^{i2\pi(0.85)}, 1.43e^{i2\pi(1.28)}, 0.95e^{i2\pi(1.41)}),\\
\mathcal{D}_\Psi(b) &= (0.47e^{i2\pi(0.43)}, 0.97e^{i2\pi(0.82)}, 0.74e^{i2\pi(0.96)}),\\
\mathcal{D}_\Psi(r) &= (0.21e^{i2\pi(0.4)}, 0.45e^{i2\pi(0.51)}, 0.31e^{i2\pi(0.61)}),\\
\mathcal{D}_\Psi(s) &= (0.52e^{i2\pi(0.28)}, 1.21e^{i2\pi(0.85)}, 0.36e^{i2\pi(0.66)}),
\end{aligned}$$

- *The total degree of all nodes in Ψ are as follows:*

$$\begin{aligned}
\mathcal{TD}_\Psi(x) &= (1.16e^{i2\pi(1.45)}, 1.63e^{i2\pi(1.58)}, 1.65e^{i2\pi(1.81)}),\\
\mathcal{TD}_\Psi(b) &= (1.07e^{i2\pi(0.83)}, 1.27e^{i2\pi(1.02)}, 1.24e^{i2\pi(1.56)}),
\end{aligned}$$

$$\mathcal{TD}_\Psi(r) = (0.51e^{i2\pi(1.1)}, 0.85e^{i2\pi(0.91)}, 0.91e^{i2\pi(0.91)}),$$
$$\mathcal{TD}_\Psi(s) = (0.92e^{i2\pi(0.48)}, 1.81e^{i2\pi(1.25)}, 0.86e^{i2\pi(1.06)}),$$

Definition 14: Let $\Psi = (\mathcal{I}, \mathcal{J})$ be a CSDFG. The complement of Ψ for amplitude term is defined by:

1. $\omega_{\bar{\mathcal{I}}}(x) = \omega_{\mathcal{I}}(x)$, $\tau_{\bar{\mathcal{I}}}(x) = \tau_{\mathcal{I}}(x)$ and $\Gamma_{\bar{\mathcal{I}}}(x) = \Gamma_{\mathcal{I}}(x)$.

2.

$$\omega_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}, \\ \quad \text{if } \omega_{\mathcal{J}}(xb) = 0. \\ \left(\dfrac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} - \omega_{\mathcal{J}}(xb)\right) \\ \quad \text{if } 0 < \omega_{\mathcal{J}}(xb) \leq 1. \end{cases}$$

$$\tau_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}, \\ \quad \text{if } \tau_{\mathcal{J}}(xb) = 0. \\ \left(\dfrac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)} - \tau_{\mathcal{J}}(xb)\right), \\ \quad \text{if } 0 < \tau_{\mathcal{J}}(xb) \leq 1. \end{cases}$$

$$\Gamma_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}, \\ \quad \text{if } \Gamma_{\mathcal{J}}(xb) = 0. \\ \left(\dfrac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)} - \Gamma_{\mathcal{J}}(xb)\right), \\ \quad \text{if } 0 < \Gamma_{\mathcal{J}}(xb) \leq 1. \end{cases}$$

In same way, the complement of Ψ for phase term is defined as:

1. $\varkappa_{\bar{\mathcal{I}}}(x) = \varkappa_{\mathcal{I}}(x)$, $\varsigma_{\bar{\mathcal{I}}}(x) = \varsigma_{\mathcal{I}}(x)$ and $\delta_{\bar{\mathcal{I}}}(x) = \delta_{\mathcal{I}}(x)$. 2.

$$\varkappa_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}, \\ \quad \text{if } \varkappa_{\mathcal{J}}(xb) = 0. \\ \left(\dfrac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)} - \varkappa_{\mathcal{J}}(xb)\right) \\ \quad \text{if } 0 < \varkappa_{\mathcal{J}}(xb) \leq 2\pi. \end{cases}$$

$$\varsigma_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}, \\ \quad \text{if } \varsigma_{\mathcal{J}}(xb) = 0. \\ \left(\dfrac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)} - \varsigma_{\mathcal{J}}(xb)\right), \\ \quad \text{if } 0 < \varsigma_{\mathcal{J}}(xb) \leq 2\pi. \end{cases}$$

$$\delta_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}, \\ \quad \text{if } \delta_{\mathcal{J}}(xb) = 0. \\ \left(\dfrac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)} - \delta_{\mathcal{J}}(xb)\right), \\ \quad \text{if } 0 < \delta_{\mathcal{J}}(xb) \leq 2\pi. \end{cases}$$

Again, the complement of a CSDFG Ψ is represented by $\bar{\Psi} = (\bar{\mathcal{I}}, \bar{\mathcal{J}})$.

Definition 15: A homomorphism $\vartheta : \Psi \to \Psi'$ of two CSDFGs $\Psi = (\mathcal{I}, \mathcal{J})$ and $\Psi' = (\mathcal{I}', \mathcal{J}')$ is a mapping $\vartheta : \Upsilon \to \Upsilon'$ follows:

1. $\omega_{\mathcal{I}}(x) \leq \omega_{\mathcal{I}'}(\vartheta(x))$, $\tau_{\mathcal{I}}(x) \leq \tau_{\mathcal{I}'}(\vartheta(x))$, $\Gamma_{\mathcal{I}}(x) \leq \Gamma_{\mathcal{I}'}(\Xi(x))$, $\varkappa_{\mathcal{I}}(x) \leq \varkappa_{\mathcal{I}'}(\vartheta(x))$, $\varsigma_{\mathcal{I}}(x) \leq \varsigma_{\mathcal{I}'}(\vartheta(x))$, $\delta_{\mathcal{I}}(x) \leq \delta_{\mathcal{I}'}(\vartheta(x))$ for all $x \in \Upsilon$.

2. $\omega_{\mathcal{J}}(xb) \leq \omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\tau_{\mathcal{J}}(xb) \leq \tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\Gamma_{\mathcal{J}}(xb) \leq \Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\varkappa_{\mathcal{J}}(xb) \leq \varkappa_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\varsigma_{\mathcal{J}}(xb) \leq \varsigma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\delta_{\mathcal{J}}(xb) \leq \delta_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$ for all $xb \in \mathcal{Z}$.

Definition 16: An isomorphism $\vartheta : \Psi \to \Psi'$ of two CSDFGs $\Psi = (\mathcal{I}, \mathcal{J})$ and $\Psi' = (\mathcal{I}', \mathcal{J}')$ is a bijective mapping $\vartheta : \Upsilon \to \Upsilon'$ follows:

1. $\omega_{\mathcal{I}}(x) = \omega_{\mathcal{I}'}(\vartheta(x))$, $\tau_{\mathcal{I}}(x) = \tau_{\mathcal{I}'}(\vartheta(x))$, $\Gamma_{\mathcal{I}}(x) = \Gamma_{\mathcal{I}'}(\vartheta(x))$, $\varkappa_{\mathcal{I}}(x) = \varkappa_{\mathcal{I}'}(\vartheta(x))$, $\varsigma_{\mathcal{I}}(x) = \varsigma_{\mathcal{I}'}(\vartheta(x))$, $\delta_{\mathcal{I}}(x) = \delta_{\mathcal{I}'}(\vartheta(x))$ for all $x \in \Upsilon$.

2. $\omega_{\mathcal{J}}(xb) = \omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\tau_{\mathcal{J}}(xb) = \tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\Gamma_{\mathcal{J}}(xb) = \Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\varkappa_{\mathcal{J}}(xb) = \varkappa_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\varsigma_{\mathcal{J}}(xb) = \varsigma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\delta_{\mathcal{J}}(xb) = \delta_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$ for all $xb \in \mathcal{Z}_1$.

Definition 17: A W-Isomorphism $\vartheta : \Psi \to \Psi'$ of two CSDFGs $\Psi = (\mathcal{I}, \mathcal{J})$ and $\Psi' = (\mathcal{I}', \mathcal{J}')$ is a bijective mapping $\vartheta : \Upsilon \to \Upsilon'$ follows:

1. ϑ is a homomorphism.

2. $\omega_{\mathcal{I}}(x) = \omega_{\mathcal{I}'}(\vartheta(x))$, $\tau_{\mathcal{I}}(x) = \tau_{\mathcal{I}'}(\vartheta(x))$, $\Gamma_{\mathcal{I}}(x) = \Gamma_{\mathcal{I}'}(\vartheta(x))$, $\varkappa_{\mathcal{I}}(x) = \varkappa_{\mathcal{I}'}(\vartheta(x))$, $\varsigma_{\mathcal{I}}(x) = \varsigma_{\mathcal{I}'}(\vartheta(x))$, $\delta_{\mathcal{I}}(x) = \delta_{\mathcal{I}'}(\vartheta(x))$ for all $x \in \Upsilon$.

Definition 18: An CW-Isomorphism $\vartheta : \Psi \to \Psi'$ of two CSDFGs $\Psi = (\mathcal{I}, \mathcal{J})$ and $\Psi' = (\mathcal{I}', \mathcal{J}')$ is a bijective mapping $\vartheta : \Upsilon \to \Upsilon'$ fulfilling

1. ϑ is a homomorphism.

2. $\omega_{\mathcal{J}}(xb) = \omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\tau_{\mathcal{J}}(xb) = \tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\Gamma_{\mathcal{J}}(xb) = \Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\varkappa_{\mathcal{J}}(xb) = \varkappa_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\varsigma_{\mathcal{J}}(xb) = \varsigma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\delta_{\mathcal{J}}(xb) = \delta_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$ for all $xb \in \mathcal{Z}$.

Definition 19: A CSDFG $\Psi = (\mathcal{I}, \mathcal{J})$ is said to be self complementary if $\bar{\Psi} \cong \Psi$.

Proposition 1: If $\Psi = (\mathcal{I}, \mathcal{J})$ is a self complementary CSDFG, then

$$\sum_{x \neq b} \omega_{\mathcal{J}}(xb) = \frac{1}{2}\Big(\sum_{x \neq b} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}\Big),$$
$$\sum_{x \neq b} \tau_{\mathcal{J}}(xb) = \frac{1}{2}\Big(\sum_{x \neq b} \frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}\Big),$$
$$\sum_{x \neq b} \Gamma_{\mathcal{J}}(xb) = \frac{1}{2}\Big(\sum_{x \neq b} \frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}\Big),$$
$$\sum_{x \neq b} \varkappa_{\mathcal{J}}(xb) = \frac{1}{2}\Big(\sum_{x \neq b} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}\Big),$$
$$\sum_{x \neq b} \varsigma_{\mathcal{J}}(xb) = \frac{1}{2}\Big(\sum_{x \neq b} \frac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}\Big),$$
$$\sum_{x \neq b} \delta_{\mathcal{J}}(xb) = \frac{1}{2}\Big(\sum_{x \neq b} \frac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}\Big).$$

Proof: Let Ψ is a self complementary CSDFG, then there exist an isomorphism $\vartheta : \Upsilon \to \Upsilon$ such that

$\overline{\omega_{\mathcal{I}}(\vartheta(x))} = \omega_{\mathcal{I}}(x)$, $\overline{\tau_{\mathcal{I}}(\vartheta(x))} = \tau_{\mathcal{I}}(x)$, $\overline{\Gamma_{\mathcal{I}}(\vartheta(x))} = \Gamma_{\mathcal{I}}(x)$, $\overline{\varkappa_{\mathcal{I}}(\vartheta(x))} = \varkappa_{\mathcal{I}}(x)$, $\overline{\varsigma_{\mathcal{I}}(\vartheta(x))} = \varsigma_{\mathcal{I}}(x)$, $\overline{\delta_{\mathcal{I}}(\vartheta(x))} = \delta_{\mathcal{I}}(x)$ for all $x \in \Upsilon$ $\overline{\omega_{\mathcal{J}}(\vartheta(x)\vartheta(b))} = \omega_{\mathcal{J}}(xb)$, $\overline{\tau_{\mathcal{J}}(\vartheta(x)\vartheta(b))} = \tau_{\mathcal{J}}(xb)$, $\overline{\Gamma_{\mathcal{J}}(\vartheta(x)\vartheta(b))} = \Gamma_{\mathcal{J}}(xb)$, $\overline{\varkappa_{\mathcal{J}}(\vartheta(x)\vartheta(b))} = \varkappa_{\mathcal{J}}(xb)$, $\overline{\varsigma_{\mathcal{J}}(\vartheta(x)\vartheta(b))} = \varsigma_{\mathcal{J}}(xb)$, $\overline{\delta_{\mathcal{J}}(\vartheta(x)\vartheta(b))} = \delta_{\mathcal{J}}(xb)$ for all $xb \in \mathcal{Z}$.

By utilizing the definition of complement, we have

$$\overline{\omega_{\mathcal{J}}(\vartheta(x)\vartheta(b))} = \frac{\overline{\omega_{\mathcal{I}}(\vartheta(x))\omega_{\mathcal{I}}(\vartheta(b))}}{\overline{\omega_{\mathcal{I}}(\vartheta(x))} + \overline{\omega_{\mathcal{I}}(\vartheta(b))} - \overline{\omega_{\mathcal{I}}(\vartheta(x))\omega_{\mathcal{I}}(\vartheta(b))}} - \omega_{\mathcal{J}}(\vartheta(x)\vartheta(b)).$$

$$\omega_{\mathcal{J}}(xb) = \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} - \omega_{\mathcal{J}}(\vartheta(x)\vartheta(b)),$$

$$\sum_{x\neq b}\omega_{\mathcal{J}}(xb) = \sum_{x\neq b}\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} - \sum_{x\neq b}\omega_{\mathcal{J}}(\vartheta(x)\vartheta(b)),$$

$$\sum_{x\neq b}\omega_{\mathcal{J}}(xb) + \sum_{x\neq b}\omega_{\mathcal{J}}(\vartheta(x)\vartheta(b)) = \sum_{x\neq b}\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)},$$

$$2\sum_{x\neq b}\omega_{\mathcal{J}}(xb) = \sum_{x\neq b}\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)},$$

$$\sum_{x\neq b}\omega_{\mathcal{J}}(xb) = \frac{1}{2}\sum_{x\neq b}\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)},$$

Similarly, the other parts can be proved. □

Proposition 2: If a CSDFG $\Psi = (I, J)$ on an crisp graph $\Psi^* = (\Upsilon, \mathcal{Z})$ satisfy the following:

$$\omega_{\mathcal{J}}(xb) = \frac{1}{2}(\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}),$$
$$\tau_{\mathcal{J}}(xb) = \frac{1}{2}(\frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}),$$
$$\Gamma_{\mathcal{J}}(xb) = \frac{1}{2}(\frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}),$$
$$\varkappa_{\mathcal{J}}(xb) = \frac{1}{2}(\frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}),$$
$$\varsigma_{\mathcal{J}}(xb) = \frac{1}{2}(\frac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}),$$
$$\delta_{\mathcal{J}}(xb) = \frac{1}{2}(\frac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}).$$

for all x,b∈ Υ, then Ψ is self complementary.

Proof: Take Ψ is CSDFG that obey

$$\omega_{\mathcal{J}}(xb) = \frac{1}{2}(\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}),$$
$$\tau_{\mathcal{J}}(xb) = \frac{1}{2}(\frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}),$$
$$\Gamma_{\mathcal{J}}(xb) = \frac{1}{2}(\frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}),$$

for all x,b∈ Υ, then the identity mapping $\mathcal{Y} : \Upsilon \to \Upsilon$ is an isomorphism from Ψ xb $\bar{\Psi}$ that obey the following axioms: $\overline{\omega_{\mathcal{I}}(\mathcal{Y}(x))} = \omega_{\mathcal{I}}(x)$, $\overline{\tau_{\mathcal{I}}(\mathcal{Y}(x))} = \tau_{\mathcal{I}}(x)$, $\overline{\Gamma_{\mathcal{I}}(\mathcal{Y}(x))} = \Gamma_{\mathcal{I}}(x)$, $\overline{\varkappa_{\mathcal{I}}(\mathcal{Y}(x))} = \varkappa_{\mathcal{I}}(x)$, $\overline{\varsigma_{\mathcal{I}}(\mathcal{Y}(x))} = \varsigma_{\mathcal{I}}(x)$, $\overline{\delta_{\mathcal{I}}(\mathcal{Y}(x))} = \delta_{\mathcal{I}}(x)$ for all $x \in \Upsilon$.

We know that the membership value of arc set to is specified as
$\omega_{\mathcal{J}}(xb) = \frac{1}{2}(\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)})$, for all x,b∈ Υ.
We have

$$\overline{\omega_{\mathcal{J}}(\mathcal{Y}(x)\mathcal{Y}(b))} = \overline{\omega_{\mathcal{J}}(xb)} = \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} - \omega_{\mathcal{J}}(xb)$$
$$= \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} - \frac{1}{2}(\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)})$$
$$= \frac{1}{2}(\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}) = \omega_{\mathcal{J}}(xb).$$

Similarly the other conditions of isomorphisms

$$\overline{\tau_{\mathcal{J}}(\mathcal{Y}(x)\mathcal{Y}(b))} = \tau_{\mathcal{J}}(xb),$$
$$\overline{\Gamma_{\mathcal{J}}(\mathcal{Y}(x)\mathcal{Y}(b))} = \Gamma_{\mathcal{J}}(xb),$$
$$\overline{\varkappa_{\mathcal{J}}(\mathcal{Y}(x)\mathcal{Y}(b))} = \varkappa_{\mathcal{J}}(xb),$$
$$\overline{\varsigma_{\mathcal{J}}(\mathcal{Y}(x)\mathcal{Y}(b))} = \varsigma_{\mathcal{J}}(xb),$$
$$\overline{\delta_{\mathcal{J}}(\mathcal{Y}(x)\mathcal{Y}(b))} = \delta_{\mathcal{J}}(xb),$$

are satisfied by $\mathcal{Y}$. Hence $\Psi = (I, J)$ is self complementary. □

Proposition 3: Let $\Psi = (\mathcal{I}, \mathcal{J})$ and $\Psi' = (\mathcal{I}', \mathcal{J}')$ be two CSDFGs, then $\Psi \cong \Psi'$ iff $\bar{\Psi} \cong \bar{\Psi}'$.

Proof: Let Ψ and Ψ' be two isomorphic CSDFGs. Then utilizing isomorphism property, there exist a bijective mapping $\vartheta : \Upsilon \to \Upsilon'$ that fulfill
$\omega_{\mathcal{I}}(x) = \omega_{\mathcal{I}'}(\vartheta(x))$, $\tau_{\mathcal{I}}(x) = \tau_{\mathcal{I}'}(\vartheta(x))$, $\Gamma_{\mathcal{I}}(x) = \Gamma_{\mathcal{I}'}(\vartheta(x))$,
$\varkappa_{\mathcal{I}}(x) = \varkappa_{\mathcal{I}'}(\vartheta(x))$, $\varsigma_{\mathcal{I}}(x) = \varsigma_{\mathcal{I}'}(\vartheta(x))$, $\delta_{\mathcal{I}}(x) = \delta_{\mathcal{I}'}(\vartheta(x))$ for all $x \in \Upsilon_1$.
$\omega_{\mathcal{J}}(xb) = \omega_{\mathcal{J}'}(\vartheta(x)\Xi(b))$, $\tau_{\mathcal{J}}(xb) = \tau_{\mathcal{J}'}(\Xi(x)\Xi(b))$, $\Gamma_{\mathcal{J}}(xb) = \Gamma_{\mathcal{J}'}(\Xi(x)\Xi(b))$,
$\varkappa_{\mathcal{J}}(xb) = \varkappa_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\varsigma_{\mathcal{J}}(xb) = \varsigma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\delta_{\mathcal{J}}(xb) = \delta_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$ for all $xb \in \mathcal{Z}_1$. By applying the definition of complement, the membership value of an arc is

$$\overline{\omega_{\mathcal{J}}(xb)} = \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} - \omega_{\mathcal{J}}(xb),$$
$$\overline{\omega_{\mathcal{J}}(xb)} = \frac{\omega_{\mathcal{I}'}(\vartheta(x))\omega_{\mathcal{I}'}(\vartheta(b))}{\omega_{\mathcal{I}'}(\vartheta(x)) + \omega_{\mathcal{I}'}(\vartheta(b)) - \omega_{\mathcal{I}'}(\vartheta(x))\omega_{\mathcal{I}'}(\vartheta(b))} - \omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b)),$$
$$\overline{\omega_{\mathcal{J}}(xb)} = \overline{\omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b))}.$$

Also, the non membership value of an arc is

$$\overline{\tau_{\mathcal{J}}(xb)} = \frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)} - \tau_{\mathcal{J}}(xb),$$
$$\overline{\tau_{\mathcal{J}}(xb)} = \frac{\tau_{\mathcal{I}'}(\vartheta(x)) + \tau_{\mathcal{I}'}(\vartheta(b)) - 2\tau_{\mathcal{I}'}(\vartheta(x))\tau_{\mathcal{I}'}(\vartheta(b))}{1 - \tau_{\mathcal{I}'}(\vartheta(x))\tau_{\mathcal{I}'}(\vartheta(b))} - \tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b)),$$
$$\overline{\tau_{\mathcal{J}}(xb)} = \overline{\tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b))}.$$

And for the abstinence value of an arc xb is

$$\overline{\Gamma_{\mathcal{J}}(xb)} = \frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)} - \Gamma_{\mathcal{J}}(xb),$$
$$\overline{\Gamma_{\mathcal{J}}(xb)} = \frac{\Gamma_{\mathcal{I}'}(\vartheta(x)) + \Gamma_{\mathcal{I}'}(\vartheta(b)) - 2\Gamma_{\mathcal{I}'}(\vartheta(x))\Gamma_{\mathcal{I}'}(\vartheta(b))}{\Gamma_{\mathcal{I}'}(\vartheta(x)) + \Gamma_{\mathcal{I}'}(\vartheta(b)) - \Gamma_{\mathcal{I}'}(\vartheta(x))\Gamma_{\mathcal{I}'}(\vartheta(b))} - \Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b)),$$
$$\overline{\Gamma_{\mathcal{J}}(xb)} = \overline{\Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))}.$$

Likewise, other parts can be proved for the phase terms. Hence $\bar{\Psi} \cong \bar{\Psi}'$. Likely, the converse part can also be proved. □

Proposition 4: Suppose $\Psi = (\mathcal{I}, \mathcal{J})$ and $\Psi' = (\mathcal{I}', \mathcal{J}')$ are two W-Isomorphic CSDFGs, then $\bar{\Psi}$ and $\bar{\Psi}'$ are also W-Isomorphic to each other.

Proof: Let Ψ and Ψ' be two W-Isomorphic CSDFGs. Then by definition of W-Isomorphism, there occurs a bijective mapping $\vartheta : \Upsilon \to \Upsilon'$ that obey
$\omega_{\mathcal{I}}(x) = \omega_{\mathcal{I}'}(\vartheta(x))$, $\tau_{\mathcal{I}}(x) = \tau_{\mathcal{I}'}(\vartheta(x))$, $\Gamma_{\mathcal{I}}(x) = \Gamma_{\mathcal{I}'}(\vartheta(x))$, $\varkappa_{\mathcal{I}}(x) = \varkappa_{\mathcal{I}'}(\vartheta(x))$, $\varsigma_{\mathcal{I}}(x) = \varsigma_{\mathcal{I}'}(\vartheta(x))$, $\delta_{\mathcal{I}}(x) = \delta_{\mathcal{I}'}(\Xi(x))$ for all $x \in \Upsilon_1$.
and
$\omega_{\mathcal{J}}(xb) \leq \omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\tau_{\mathcal{J}}(xb) \leq \tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\Gamma_{\mathcal{J}}(xb) \leq \Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\varkappa_{\mathcal{J}}(xb) \leq \varkappa_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\varsigma_{\mathcal{J}}(xb) \leq \varsigma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$, $\delta_{\mathcal{J}}(xb) \leq \delta_{\mathcal{J}'}(\vartheta(x)\vartheta(b))$ for all $xb \in \mathcal{Z}_1$.

For the membership value of an arc, we have

$$\begin{aligned}
\omega_{\mathcal{J}}(xb) &\leq \omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b))\\
-\omega_{\mathcal{J}}(xb) &\geq -\omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b))\\
\mathcal{H}(\omega_{\mathcal{I}}(x), \omega_{\mathcal{I}}(b)) - \omega_{\mathcal{J}}(xb) &\geq \mathcal{H}(\omega_{\mathcal{I}}(x), \omega_{\mathcal{I}}(b)) - \omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b))\\
\mathcal{H}(\omega_{\mathcal{I}}(x), \omega_{\mathcal{I}}(b)) - \omega_{\mathcal{J}}(xb) &\geq \mathcal{H}(\omega_{\mathcal{I}'}(\vartheta(x)), \omega_{\mathcal{I}'}(\vartheta(b))) - \omega_{\mathcal{J}'}(\vartheta(x)\Xi(b))\\
\overline{\omega_{\mathcal{J}}(xb)} &\geq \overline{\omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b))}.
\end{aligned}$$

For the non membership value of an arc, we have

$$\begin{aligned}
\tau_{\mathcal{J}}(xb) &\leq \tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b))\\
-\tau_{\mathcal{J}}(xb) &\geq -\tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b))\\
\mathcal{S}(\tau_{\mathcal{I}}(x), \tau_{\mathcal{I}}(b)) - \tau_{\mathcal{J}}(xb) &\geq \mathcal{S}(\tau_{\mathcal{I}}(x), \tau_{\mathcal{I}}(b)) - \tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b))\\
\mathcal{S}(\tau_{\mathcal{I}}(x), \tau_{\mathcal{I}}(b)) - \tau_{\mathcal{J}}(xb) &\geq \mathcal{S}(\tau_{\mathcal{I}'}(\vartheta(x)), \tau_{\mathcal{I}'}(\vartheta(b))) - \tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b))\\
\overline{\tau_{\mathcal{J}}(xb)} &\geq \overline{\tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b))}.
\end{aligned}$$

And for the abstinence value of an arc, we have

$$\begin{aligned}
\Gamma_{\mathcal{J}}(xb) &\leq \Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))\\
-\Gamma_{\mathcal{J}}(xb) &\geq -\Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))\\
\mathcal{S}'(\Gamma_{\mathcal{I}}(x), \Gamma_{\mathcal{I}}(b)) - \Gamma_{\mathcal{J}}(xb) &\geq \mathcal{S}'(\Gamma_{\mathcal{I}}(x), \Gamma_{\mathcal{I}}(b)) - \Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))\\
\mathcal{S}'(\Gamma_{\mathcal{I}}(x), \Gamma_{\mathcal{I}}(b)) - \Gamma_{\mathcal{J}}(xb) &\geq \mathcal{S}'(\Gamma_{\mathcal{I}'}(\vartheta(x)), \Gamma_{\mathcal{I}'}(\vartheta(b))) - \Gamma_{\mathcal{J}'}(\vartheta(x)\Xi(b))\\
\overline{\Gamma_{\mathcal{J}}(xb)} &\geq \overline{\Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))}.
\end{aligned}$$

Similarly other parts of phase term can be proved. Therefore, we see that $\bar{\Psi}$ is W-Isomorphic to $\bar{\Psi}'$. □

Definition 20: A CSDFG is said to be complete if

$$\omega_{\mathcal{J}}(xb) = \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)},$$
$$\tau_{\mathcal{J}}(xb) = \frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)},$$
$$\Gamma_{\mathcal{J}}(xb) = \frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)},$$
$$\varkappa_{\mathcal{J}}(xb) = \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)},$$
$$\varsigma_{\mathcal{J}}(xb) = \frac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)},$$
$$\delta_{\mathcal{J}}(xb) = \frac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)},$$

for all x,b$\in \Upsilon$.

Definition 21: A CSDFG is said to be strong if

$$\omega_{\mathcal{J}}(xb) = \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)},$$
$$\tau_{\mathcal{J}}(xb) = \frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)},$$
$$\Gamma_{\mathcal{J}}(xb) = \frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)},$$
$$\varkappa_{\mathcal{J}}(xb) = \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)},$$
$$\varsigma_{\mathcal{J}}(xb) = \frac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)},$$
$$\delta_{\mathcal{J}}(xb) = \frac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)},$$

for all xb$\in \mathcal{Z}$.

Definition 22: Let $\Psi = (\mathcal{I}, \mathcal{J})$ be a strong CSDFG on a graph $\Psi^* = (\Upsilon, \mathcal{Z})$. The complement of Ψ for amplitude term is defined as:

1. $\omega_{\bar{\mathcal{I}}}(x) = \omega_{\mathcal{I}}(x)$, $\tau_{\bar{\mathcal{I}}}(x) = \tau_{\mathcal{I}}(x)$ and $\Gamma_{\bar{\mathcal{I}}}(x) = \Gamma_{\mathcal{I}}(x)$.

2.

$$\omega_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}, & \text{if } \omega_{\mathcal{J}}(xb) = 0.\\ 0, & \text{if } 0 < \omega_{\mathcal{J}}(xb) \leq 1. \end{cases}$$

$$\tau_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}, & \text{if } \tau_{\mathcal{J}}(xb) = 0. \\ 0, & \text{if } 0 < \tau_{\mathcal{J}}(xb) \leq 1. \end{cases}$$

$$\Gamma_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}, & \text{if } \Gamma_{\mathcal{J}}(xb) = 0. \\ 0, & \text{if } 0 < \Gamma_{\mathcal{J}}(xb) \leq 1. \end{cases}$$

In a same way, the complement of Ψ *for phase term is described as:*

1. $\varkappa_{\bar{\mathcal{I}}}(x) = \varkappa_{\mathcal{I}}(x)$, $\varsigma_{\bar{\mathcal{I}}}(x) = \varsigma_{\mathcal{I}}(x)$ *and* $\delta_{\bar{\mathcal{I}}}(x) = \delta_{\mathcal{I}}(x)$.
2.

$$\varkappa_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}, & \varkappa_{\mathcal{J}}(xb) = 0. \\ 0, & 0 < \varkappa_{\mathcal{J}}(xb) \leq 2\pi. \end{cases}$$

$$\varsigma_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}, & \varsigma_{\mathcal{J}}(xb) = 0. \\ 0, & 0 < \varsigma_{\mathcal{J}}(xb) \leq 2\pi. \end{cases}$$

$$\delta_{\bar{\mathcal{J}}}(xb) = \begin{cases} \dfrac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}, & \delta_{\mathcal{J}}(xb) = 0. \\ 0, & 0 < \delta_{\mathcal{J}}(xb) \leq 2\pi. \end{cases}$$

Moreover, the complement of a strong CSDFG Ψ *is represented by* $\bar{\Psi} = (\bar{\mathcal{I}}, \bar{\mathcal{J}})$.

Definition 23: A CSDFG $\Psi = (\mathcal{I}, \mathcal{J})$ *is called regular of degree* $(\mathcal{F}_1 e^{i\mathcal{F}_1^*}, \mathcal{F}_2 e^{i\mathcal{F}_2^*}, \mathcal{F}_3 e^{i\mathcal{F}_3^*})$ *or* $(\mathcal{F}_1 e^{i\mathcal{F}_1^*}, \mathcal{F}_2 e^{i\mathcal{F}_2^*}, \mathcal{F}_3 e^{i\mathcal{F}_3^*})$ *regular, if its each node has same degree. i.e,*

$$\mathcal{D}_{\omega}(x) = \sum_{x,b \neq x \in \Upsilon} \omega_{\mathcal{J}(xb)} = \mathcal{F}_1,$$

$$\mathcal{D}_{\tau}(x) = \sum_{x,b \neq x \in \Upsilon} \tau_{\mathcal{J}(xb)} = \mathcal{F}_2,$$

$$\mathcal{D}_{\Gamma}(x) = \sum_{x,b \neq x \in \Upsilon} \Gamma_{\mathcal{J}(xb)} = \mathcal{F}_3,$$

$$\mathcal{D}_{e^{i\varkappa}}(x) = \sum_{x,b \neq x \in \Upsilon} \varkappa_{\mathcal{J}(xb)} = \mathcal{F}_1^*,$$

$$\mathcal{D}_{e^{i\varsigma}}(x) = \sum_{x,b \neq x \in \Upsilon} \varsigma_{\mathcal{J}(xb)} = \mathcal{F}_2^*,$$

$$\mathcal{D}_{e^{i\delta}}(x) = \sum_{x,b \neq x \in \Upsilon} \delta_{\mathcal{J}(xb)} = \mathcal{F}_3^*.$$

for all $x \in \Upsilon$

Example 3: Let $\Psi = (\mathcal{I}, \mathcal{J})$ *be a CSDFG on* $\Psi^* = (\Upsilon, \mathcal{Z})$, *where* $\Upsilon = \{x, b, r, s, u, v\}$ *and*

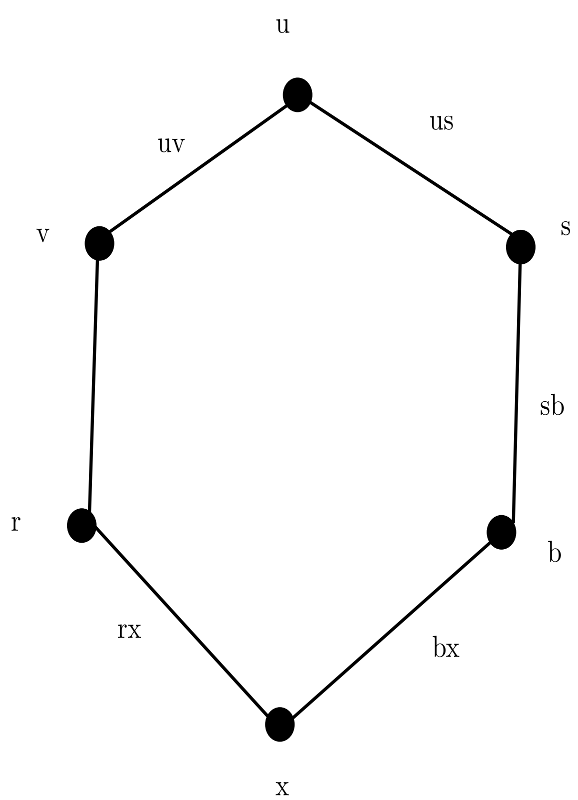

FIGURE 2. **Regular CSDFG.**

$\mathcal{Z} = \{xb, xr, bs, vr, vu, us\}$ *as shown in Figure 2. The node set* $\mathcal{I}$ *and the arc set* $\mathcal{J}$ *of* Ψ *are defined as.*

$$\mathcal{I} =< (\frac{x}{0.61e^{i2\pi(0.7)}}, \frac{b}{0.61e^{i2\pi(0.7)}}, \frac{r}{0.61e^{i2\pi(0.7)}}, \frac{s}{0.61e^{i2\pi(0.7)}}, \frac{u}{0.61e^{i2\pi(0.7)}}, \frac{v}{0.61e^{i2\pi(0.7)}}), (\frac{x}{0.15e^{i2\pi(0.2)}}, \frac{b}{0.15e^{i2\pi(0.2)}}, \frac{r}{0.15e^{i2\pi(0.2)}}, \frac{s}{0.15e^{i2\pi(0.2)}}, \frac{u}{0.15e^{i2\pi(0.2)}}, \frac{v}{0.15e^{i2\pi(0.2)}}), (\frac{x}{0.3e^{i2\pi(0.4)}}, \frac{b}{0.3e^{i2\pi(0.4)}}, \frac{r}{0.3e^{i2\pi(0.4)}}, \frac{s}{0.3e^{i2\pi(0.4)}}, \frac{u}{0.3e^{i2\pi(0.4)}}, \frac{v}{0.3e^{i2\pi(0.4)}}) >$$

and

$$\mathcal{J} =< (\frac{xb}{0.35e^{i2\pi(0.45)}}, \frac{xr}{0.35e^{i2\pi(0.45)}}, \frac{bs}{0.35e^{i2\pi(0.45)}}, \frac{vr}{0.35e^{i2\pi(0.45)}}, \frac{vu}{0.35e^{i2\pi(0.45)}}, \frac{us}{0.35e^{i2\pi(0.45)}}), (\frac{xb}{0.21e^{i2\pi(0.3)}}, \frac{xr}{0.21e^{i2\pi(0.3)}}, \frac{bs}{0.21e^{i2\pi(0.3)}}, \frac{vr}{0.21e^{i2\pi(0.3)}}, \frac{vu}{0.21e^{i2\pi(0.3)}}, \frac{us}{0.21e^{i2\pi(0.3)}}), (\frac{xb}{0.25e^{i2\pi(0.35)}}, \frac{xr}{0.25e^{i2\pi(0.35)}}, \frac{bs}{0.25e^{i2\pi(0.35)}}, \frac{vr}{0.25e^{i2\pi(0.35)}}, \frac{vu}{0.25e^{i2\pi(0.35)}}, \frac{us}{0.25e^{i2\pi(0.35)}}) >$$

$$x = (0.61e^{i2\pi(0.7)}, 0.15e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.4)})$$

$$\begin{aligned}
b &= (0.61e^{i2\pi(0.7)}, 0.15e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.4)})\\
r &= (0.61e^{i2\pi(0.7)}, 0.15e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.4)})\\
s &= (0.61e^{i2\pi(0.7)}, 0.15e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.4)})\\
u &= (0.61e^{i2\pi(0.7)}, 0.15e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.4)})\\
v &= (0.61e^{i2\pi(0.7)}, 0.15e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.4)})\\
xb &= (0.35e^{i2\pi(0.45)}, 0.21e^{i2\pi(0.3)}, 0.25e^{i2\pi(0.35)})\\
xr &= (0.35e^{i2\pi(0.45)}, 0.21e^{i2\pi(0.3)}, 0.25e^{i2\pi(0.35)})\\
bs &= (0.35e^{i2\pi(0.45)}, 0.21e^{i2\pi(0.3)}, 0.25e^{i2\pi(0.35)})\\
vr &= (0.35e^{i2\pi(0.45)}, 0.21e^{i2\pi(0.3)}, 0.25e^{i2\pi(0.35)})\\
vu &= (0.35e^{i2\pi(0.45)}, 0.21e^{i2\pi(0.3)}, 0.25e^{i2\pi(0.35)})\\
us &= (0.35e^{i2\pi(0.45)}, 0.21e^{i2\pi(0.3)}, 0.25e^{i2\pi(0.35)})
\end{aligned}$$

By calculations, one can see that $\Psi = (\mathcal{I}, \mathcal{J})$ is a $(0.7e^{i2\pi(0.9)}, 0.42e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.7)})$-regular CSDFG.

Definition 24: A CSDFG $\Psi = (\mathcal{I}, \mathcal{J})$ on a graph $\Psi^* = (\Upsilon, \mathcal{Z})$ is said to be totally regular of degree $(\mathcal{H}_1 e^{i\mathcal{H}_1^*}, \mathcal{H}_2 e^{i\mathcal{H}_2^*}, \mathcal{H}_3 e^{i\mathcal{H}_3^*})$ or $(\mathcal{H}_1 e^{i\mathcal{H}_1^*}, \mathcal{H}_2 e^{i\mathcal{H}_2^*}, \mathcal{H}_3 e^{i\mathcal{H}_3^*})$ totally regular, if its each node has same total degree, i.e,

$$\begin{aligned}
\mathcal{TD}_{\omega}(x) &= \sum_{x,b\neq x\in\Upsilon} \omega_{\mathcal{J}(xb)} + \omega_{\mathcal{I}}(x) = \mathcal{H}_1,\\
\mathcal{TD}_{\tau}(x) &= \sum_{x,b\neq x\in\Upsilon} \tau_{\mathcal{J}(xb)} + \tau_{\mathcal{I}}(x) = \mathcal{H}_2,\\
\mathcal{TD}_{\Gamma}(x) &= \sum_{x,b\neq x\in\Upsilon} \Gamma_{\mathcal{J}(xb)} + \Gamma_{\mathcal{I}}(x) = \mathcal{H}_3,\\
\mathcal{TD}_{e^{i\varkappa}}(x) &= \sum_{x,b\neq x\in\Upsilon} \varkappa_{\mathcal{J}(xb)} + \varkappa_{\mathcal{I}}(x) = \mathcal{H}_1^*,\\
\mathcal{TD}_{e^{i\varsigma}}(x) &= \sum_{x,b\neq x\in\Upsilon} \varsigma_{\mathcal{J}(xb)} + \varsigma_{\mathcal{I}}(x) = \mathcal{H}_2^*,\\
\mathcal{TD}_{e^{i\delta}}(x) &= \sum_{x,b\neq x\in\Upsilon} \delta_{\mathcal{J}(xb)} + \delta_{\mathcal{I}}(x) = \mathcal{H}_3^*.
\end{aligned}$$

for all $x \in \Upsilon$.

Example 4: Let $\Psi = (\mathcal{I}, \mathcal{J})$ be a CSDFG on $\Psi^* = (\Upsilon, \mathcal{Z})$, where $\Upsilon = \{x, b, r, s\}$ and $\mathcal{Z} = \{xb, br, xs, rs\}$ as shown in Figure 3. The nodes set $\mathcal{I}$ and arcs set $\mathcal{J}$ of Ψ are defined as.

$$\begin{aligned}
\mathcal{I} &=< (\frac{x}{0.6e^{i2\pi(0.45)}}, \frac{b}{0.6e^{i2\pi(0.45)}}, \frac{r}{0.6e^{i2\pi(0.45)}}, \frac{s}{0.6e^{i2\pi(0.45)}}),\\
&(\frac{x}{0.15e^{i2\pi(0.2)}}, \frac{b}{0.15e^{i2\pi(0.2)}}, \frac{r}{0.15e^{i2\pi(0.2)}}, \frac{s}{0.15e^{i2\pi(0.2)}}),\\
&(\frac{x}{0.3e^{i2\pi(0.4)}}, \frac{b}{0.3e^{i2\pi(0.4)}}, \frac{r}{0.3e^{i2\pi(0.4)}}, \frac{s}{0.3e^{i2\pi(0.4)}}) >
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{J} &=< (\frac{xb}{0.31e^{i2\pi(0.25)}}, \frac{br}{0.23e^{i2\pi(0.23)}}, \frac{xs}{0.23e^{i2\pi(0.23)}},\\
&\frac{rs}{0.31e^{i2\pi(0.25)}}), (\frac{xb}{0.21e^{i2\pi(0.3)}}, \frac{br}{0.2e^{i2\pi(0.3)}}, \frac{xs}{0.2e^{i2\pi(0.3)}},\\
&\frac{rs}{0.21e^{i2\pi(0.3)}}), (\frac{xb}{0.25e^{i2\pi(0.35)}}, \frac{br}{0.2e^{i2\pi(0.35)}},\\
&\frac{xs}{0.2e^{i2\pi(0.35)}}, \frac{rs}{0.25e^{i2\pi(0.35)}}) >
\end{aligned}$$

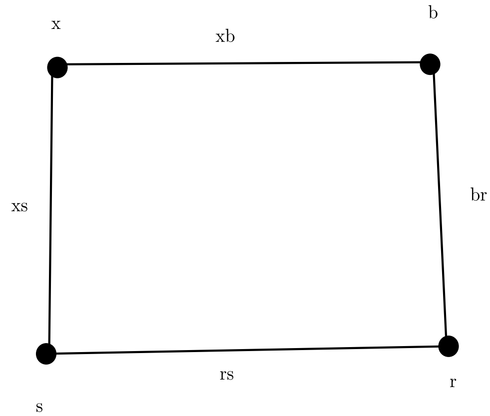

FIGURE 3. Totally regular CSDFG.

$$\begin{aligned}
x &= (0.6e^{i2\pi(0.45)}, 0.15e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.4)})\\
b &= (0.6e^{i2\pi(0.45)}, 0.15e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.4)})\\
r &= (0.6e^{i2\pi(0.45)}, 0.15e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.4)})\\
s &= (0.6e^{i2\pi(0.45)}, 0.15e^{i2\pi(0.2)}, 0.3e^{i3\pi(0.4)})\\
xb &= (0.31e^{i2\pi(0.25)}, 0.21e^{i2\pi(0.3)}, 0.25e^{i2\pi(0.35)})\\
br &= (0.23e^{i2\pi(0.23)}, 0.2e^{i2\pi(0.3)}, 0.2e^{i2\pi(0.35)})\\
xs &= (0.23e^{i2\pi(0.23)}, 0.2e^{i2\pi(0.3)}, 0.2e^{i2\pi(0.35)})\\
rs &= (0.31e^{i2\pi(0.25)}, 0.21e^{i2\pi(0.3)}, 0.25e^{i2\pi(0.35)})
\end{aligned}$$

By calculations, one can see that $\Psi = (\mathcal{I}, \mathcal{J})$ is a $(1.14e^{i2\pi(0.93)}, 0.56e^{i2\pi(0.8)}, 0.75e^{i2\pi(1.1)})$-totally regular CSDFG.

Theorem 1: Consider a CSDFG $\Psi = (\mathcal{I}, \mathcal{J})$ which is isomorphic to another CSDFG $\Psi' = (\mathcal{I}', \mathcal{J}')$;

1. If Ψ is regular, then Ψ' is also regular.

2. If Ψ is totally regular, then Ψ' is also totally regular.

Proof: 1. Let Ψ be isomorphic to Ψ' and Ψ be $(\mathcal{F}_1 e^{i\mathcal{F}_1^*}, \mathcal{F}_2 e^{i\mathcal{F}_2^*}, \mathcal{F}_3 e^{i\mathcal{F}_3^*})$-regular CSDFG, therefore degree of every node of Ψ is provided as:

$$\mathcal{D}_{\Psi}(x) = (\mathcal{D}_{\omega e^{i\varkappa}}(x), \mathcal{D}_{\tau e^{i\varsigma}}(x), \mathcal{D}_{\Gamma e^{i\delta}}(x))$$

$$\mathcal{D}_{\Psi}(x) = (\sum_{xb\in\mathcal{Z}} \omega_{\mathcal{J}}(xb) e^{i(\sum_{xb\in\mathcal{Z}} \varkappa_{\mathcal{J}}(xb))}, \sum_{xb\in\mathcal{Z}} \tau_{\mathcal{J}}(xb) e^{i(\sum_{xb\in\mathcal{Z}} \varsigma_{\mathcal{J}}(xb))}, \sum_{xb\in\mathcal{Z}} \Gamma_{\mathcal{J}}(xb) e^{i(\sum_{xb\in\mathcal{Z}} \delta_{\mathcal{J}}(xb))})$$

$$\mathcal{D}_{\Psi}(x) = (\sum_{x,b\neq x\in\Upsilon} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} e^{i(\sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)})},$$

$$\sum_{x,b\neq x\in\Upsilon}\frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)} e^{i(\sum_{x,b\neq x\in\Upsilon}\frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)})},$$
$$\sum_{x,b\neq x\in\Upsilon}\frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)} e^{i(\sum_{x,b\neq x\in\Upsilon}\frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)})})$$
$$=(\mathcal{F}_1e^{i\mathcal{F}_1^*},\mathcal{F}_2e^{i\mathcal{F}_2^*},\mathcal{F}_3e^{i\mathcal{F}_3^*})$$

Since $\Psi\cong\Psi'$, we must have

$$(\mathcal{F}_1e^{i\mathcal{F}_1^*},\mathcal{F}_2e^{i\mathcal{F}_2^*},\mathcal{F}_3e^{i\mathcal{F}_3^*})$$
$$=(\mathcal{D}_{\omega e^{i\varkappa}}(x),\mathcal{D}_{\tau e^{i\varsigma}}(x),\mathcal{D}_{\Gamma e^{i\delta}}(x))$$
$$=(\sum_{xb\in\mathcal{Z}}\omega_{\mathcal{J}}(xb)e^{i(\sum_{xb\in\mathcal{Z}}\varkappa_{\mathcal{J}}(xb))},\sum_{xb\in\mathcal{Z}}\tau_{\mathcal{J}}(xb)e^{i(\sum_{xb\in\mathcal{Z}}\varsigma_{\mathcal{J}}(xb))},\sum_{xb\in\mathcal{Z}}\Gamma_{\mathcal{J}}(xb)e^{i(\sum_{xb\in\mathcal{Z}}\delta_{\mathcal{J}}(xb))})$$
$$=(\sum_{x,b\neq x\in\Upsilon}\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} e^{i(\sum_{x,b\neq x\in\Upsilon}\frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)})},$$
$$\sum_{x,b\neq x\in\Upsilon}\frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)} e^{i(\sum_{x,b\neq x\in\Upsilon}\frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)})},$$
$$\sum_{x,b\neq x\in\Upsilon}\frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)} e^{i(\sum_{x,b\neq x\in\Upsilon}\frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)})})$$

$$(\mathcal{F}_1e^{i\mathcal{F}_1^*},\mathcal{F}_2e^{i\mathcal{F}_2^*},\mathcal{F}_3e^{i\mathcal{F}_3^*})$$
$$=(\sum_{x,b\neq x\in\Upsilon}\frac{\omega_{\mathcal{I}'}(\vartheta(x))\omega_{\mathcal{I}'}(\vartheta(b))}{\omega_{\mathcal{I}'}(\vartheta(x))+\omega_{\mathcal{I}'}(\vartheta(b))-\omega_{\mathcal{I}'}(\vartheta(x))\omega_{\mathcal{I}'}(\vartheta(b))} e^{i(\sum_{x,b\neq x\in\Upsilon}\frac{\varkappa_{\mathcal{I}'}(\vartheta(x))\varkappa_{\mathcal{I}'}(\vartheta(b))}{\varkappa_{\mathcal{I}'}(\vartheta(x))+\varkappa_{\mathcal{I}'}(\vartheta(b))-\varkappa_{\mathcal{I}'}(\vartheta(x))\varkappa_{\mathcal{I}'}(\vartheta(b))})},$$
$$\sum_{x,b\neq x\in\Upsilon}\frac{\tau_{\mathcal{I}'}(\vartheta(x))+\tau_{\mathcal{I}'}(\vartheta(b))-2\tau_{\mathcal{I}'}(\vartheta(x))\tau_{\mathcal{I}'}(\vartheta(b))}{1-\tau_{\mathcal{I}'}(\vartheta(x))\tau_{\mathcal{I}'}(\vartheta(b))} e^{i(\sum_{x,b\neq x\in\Upsilon}\frac{\varsigma_{\mathcal{I}'}(\vartheta(x))+\varsigma_{\mathcal{I}'}(\vartheta(b))-2\varsigma_{\mathcal{I}'}(\vartheta(x))\varsigma_{\mathcal{I}'}(\vartheta(b))}{1-\varsigma_{\mathcal{I}'}(\vartheta(x))\varsigma_{\mathcal{I}'}(\vartheta(b))})},$$
$$\sum_{x,b\neq x\in\Upsilon}\frac{\Gamma_{\mathcal{I}'}(\vartheta(x))+\Gamma_{\mathcal{I}'}(\vartheta(b))-2\Gamma_{\mathcal{I}'}(\vartheta(x))\Gamma_{\mathcal{I}'}(\vartheta(b))}{\Gamma_{\mathcal{I}'}(\vartheta(x))+\Gamma_{\mathcal{I}'}(\vartheta(b))-\Gamma_{\mathcal{I}'}(\vartheta(x))\Gamma_{\mathcal{I}'}(\vartheta(b))} e^{i(\sum_{x,b\neq x\in\Upsilon}\frac{\delta_{\mathcal{I}'}(\vartheta(x))+\delta_{\mathcal{I}'}(\vartheta(b))-2\delta_{\mathcal{I}'}(\vartheta(x))\delta_{\mathcal{I}'}(\vartheta(b))}{\delta_{\mathcal{I}'}(\vartheta(x))+\delta_{\mathcal{I}'}(\vartheta(b))-\delta_{\mathcal{I}'}(\vartheta(x))\delta_{\mathcal{I}'}(\vartheta(b))})})$$
$$=(\sum_{xb\in\mathcal{Z}}\omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b))e^{i(\sum_{xb\in\mathcal{Z}}\varkappa_{\mathcal{J}'}(\vartheta(x)\vartheta(b)))},$$
$$\sum_{xb\in\mathcal{Z}}\tau_{\mathcal{J}'}(\vartheta(x)(b))e^{i(\sum_{xb\in\mathcal{Z}}\varsigma_{\mathcal{J}'}(\vartheta(x)\vartheta(b)))},$$
$$\sum_{xb\in\mathcal{Z}}\Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b))e^{i(\sum_{xb\in\mathcal{Z}}\delta_{\mathcal{J}'}(\vartheta(x)\vartheta(b)))})$$
$$=(\mathcal{D}_{\omega'e^{i\varkappa'}}(x),\mathcal{D}_{\tau'e^{i\varsigma'}}(x),\mathcal{D}_{\Gamma'e^{i\delta'}}(x))$$
$$=\mathcal{D}_{\Psi'}(x)$$

Thus, Ψ' is a $(\mathcal{F}_1e^{i\mathcal{F}_1^*},\mathcal{F}_2e^{i\mathcal{F}_2^*},\mathcal{F}_3e^{i\mathcal{F}_3^*})$-regular CSDFG.

2. Let Ψ be isomorphic to Ψ' and Ψ is $(\mathcal{H}_1e^{i\mathcal{H}_1^*},\mathcal{H}_2e^{i\mathcal{H}_2^*},\mathcal{H}_3e^{i\mathcal{H}_3^*})$-totally regular CSDFG. So, the total degree of each node is given as:

$$\mathcal{TD}_{\Psi}(x)=(\mathcal{TD}_{\omega}(x),\mathcal{TD}_{\tau}(x),\mathcal{TD}_{\Gamma}(x))$$
$$=(\sum_{xb\in\mathcal{Z}}\omega_{\mathcal{J}}(xb)+\omega_{\mathcal{I}}(x),\sum_{xb\in\mathcal{Z}}\tau_{\mathcal{J}}(xb)+\tau_{\mathcal{I}}(x),\sum_{xb\in\mathcal{Z}}\Gamma_{\mathcal{J}}(xb)+\Gamma_{\mathcal{I}}(x))$$
$$=(\sum_{x,b\neq x\in\Upsilon}\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}+\omega_{\mathcal{I}}(x),$$
$$\sum_{x,b\neq x\in\Upsilon}\frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}+\tau_{\mathcal{I}}(x),$$
$$\sum_{x,b\neq x\in\Upsilon}\frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}+\Gamma_{\mathcal{I}}(x))$$
$$=(\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3).$$

Since $\Psi\cong\Psi'$, we must have

$$(\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3)$$
$$=(\mathcal{TD}_{\omega}(x),\mathcal{TD}_{\tau}(x),\mathcal{TD}_{\Gamma}(x))$$
$$=(\sum_{xb\in\mathcal{Z}}\omega_{\mathcal{J}}(xb)+\omega_{\mathcal{I}}(x),\sum_{xb\in\mathcal{Z}}\tau_{\mathcal{J}}(xb)+\tau_{\mathcal{I}}(x),\sum_{xb\in\mathcal{Z}}\Gamma_{\mathcal{J}}(xb)+\Gamma_{\mathcal{I}}(x))$$
$$=(\sum_{x,b\neq x\in\Upsilon}\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}+\omega_{\mathcal{I}}(x),$$
$$\sum_{x,b\neq x\in\Upsilon}\frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}+\tau_{\mathcal{I}}(x),$$
$$\sum_{x,b\neq x\in\Upsilon}\frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}+\Gamma_{\mathcal{I}}(x))$$
$$=(\sum_{x,b\neq x\in\Upsilon}\frac{\omega_{\mathcal{I}'}(\vartheta(x))\omega_{\mathcal{I}'}(\vartheta(b))}{\omega_{\mathcal{I}'}(\vartheta(x))+\omega_{\mathcal{I}'}(\vartheta(b))-\omega_{\mathcal{I}'}(\vartheta(x))\omega_{\mathcal{I}'}(\vartheta(b))}+\omega_{\mathcal{I}'}(\vartheta(x)),$$
$$\sum_{x,b\neq x\in\Upsilon}\frac{\tau_{\mathcal{I}'}(\vartheta(x))+\tau_{\mathcal{I}'}(\vartheta(b))-2\tau_{\mathcal{I}'}(\vartheta(x))\tau_{\mathcal{I}'}(\vartheta(b))}{1-\tau_{\mathcal{I}'}(\Xi(x))\tau_{\mathcal{I}'}(\Xi(b))}+\tau_{\mathcal{I}'}(\Xi(x)),$$
$$\sum_{x,b\neq x\in\Upsilon}\frac{\Gamma_{\mathcal{I}'}(\Xi(x))+\Gamma_{\mathcal{I}'}(\Xi(b))-2\Gamma_{\mathcal{I}'}(\Xi(x))\Gamma_{\mathcal{I}'}(\Xi(b))}{\Gamma_{\mathcal{I}'}(\Xi(x))+\Gamma_{\mathcal{I}'}(\Xi(b))-\Gamma_{\mathcal{I}'}(\Xi(x))\Gamma_{\mathcal{I}'}(\Xi(b))}+\Gamma_{\mathcal{I}'}(\Xi(x)))$$
$$=(\sum_{xb\in\mathcal{Z}}\omega_{\mathcal{J}'}(\vartheta(x)\vartheta(b))+\omega_{\mathcal{I}'}(\vartheta(x)),$$

$$\sum_{xb\in\mathcal{Z}} \tau_{\mathcal{J}'}(\vartheta(x)\vartheta(b)) + \tau_{\mathcal{I}}(\vartheta(x)),$$
$$\sum_{xb\in\mathcal{Z}} \Gamma_{\mathcal{J}'}(\vartheta(x)\vartheta(b)) + \Gamma_{\mathcal{I}}(\vartheta(x)))$$
$$= (\mathcal{TD}_{\omega'}(x), \mathcal{TD}_{\tau'}(x), \mathcal{TD}_{\Gamma'}(x))$$
$$= \mathcal{TD}_{\Psi'}(x).$$

Also for phase terms,

$$\begin{aligned}
&\mathcal{TD}_{\Psi}(x)\\
&= (\mathcal{TD}_{e^{i\varkappa}}(x), \mathcal{TD}_{e^{i\varsigma}}(x), \mathcal{TD}_{e^{i\delta}}(x))\\
&= (\sum_{xb\in\mathcal{Z}} \varkappa_{\mathcal{J}}(xb) + \varkappa_{\mathcal{I}}(x), \sum_{xb\in\mathcal{Z}} \varsigma_{\mathcal{J}}(xb) + \varsigma_{\mathcal{I}}(x),\\
&\quad \sum_{xb\in\mathcal{Z}} \delta_{\mathcal{J}}(xb) + \delta_{\mathcal{I}}(x))\\
&= (\sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)} + \varkappa_{\mathcal{I}}(x),\\
&\quad \sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)} + \varsigma_{\mathcal{I}}(x),\\
&\quad \sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)} + \delta_{\mathcal{I}}(x))\\
&= (\mathcal{H}_1^*, \mathcal{H}_2^*, \mathcal{H}_3^*).
\end{aligned}$$

Since $\Psi \cong \Psi'$, we must have

$$\begin{aligned}
&(\mathcal{H}_1^*, \mathcal{H}_2^*, \mathcal{H}_3^*)\\
&= (\mathcal{TD}_{e^{i\varkappa}}(x), \mathcal{TD}_{e^{i\varsigma}}(x), \mathcal{TD}_{e^{i\delta}}(x))\\
&= (\sum_{xb\in\mathcal{Z}} \varkappa_{\mathcal{J}}(xb) + \varkappa_{\mathcal{I}}(x),\\
&\quad \sum_{xb\in\mathcal{Z}} \varsigma_{\mathcal{J}}(xb) + \varsigma_{\mathcal{I}}(x), \sum_{xb\in\mathcal{Z}} \delta_{\mathcal{J}}(xb) + \delta_{\mathcal{I}}(x))\\
&= (\sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)} + \varkappa_{\mathcal{I}}(x),\\
&\quad \sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)} + \varsigma_{\mathcal{I}}(x),\\
&\quad \sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)} + \delta_{\mathcal{I}}(x))\\
&= (\sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}'}(\vartheta(x))\varkappa_{\mathcal{I}'}(\Xi(b))}{\varkappa_{\mathcal{I}'}(\Xi(x))+\varkappa_{\mathcal{I}'}(\Xi(b))-\varkappa_{\mathcal{I}'}(\Xi(x))\varkappa_{\mathcal{I}'}(\Xi(b))}\\
&\quad + \varkappa_{\mathcal{I}'}(\vartheta(x)),\\
&\quad \sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}'}(\vartheta(x))+\varsigma_{\mathcal{I}'}(\vartheta(b))-2\varsigma_{\mathcal{I}'}(\vartheta(x))\varsigma_{\mathcal{I}'}(\vartheta(b))}{1-\varsigma_{\mathcal{I}'}(\vartheta(x))\varsigma_{\mathcal{I}'}(\vartheta(b))}\\
&\quad + \varsigma_{\mathcal{I}'}(\vartheta(x)),\\
&\quad \sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}'}(\vartheta(x))+\delta_{\mathcal{I}'}(\vartheta(b))-2\delta_{\mathcal{I}'}(\vartheta(x))\delta_{\mathcal{I}'}(\vartheta(b))}{\delta_{\mathcal{I}'}(\vartheta(x))+\delta_{\mathcal{I}'}(\vartheta(b))-\delta_{\mathcal{I}'}(\vartheta(x))\delta_{\mathcal{I}'}(\vartheta(b))}\\
&\quad + \delta_{\mathcal{I}'}(\vartheta(x)))\\
&= (\sum_{xb\in\mathcal{Z}} \varkappa_{\mathcal{J}'}(\vartheta(x)\vartheta(b)) + \varkappa_{\mathcal{I}'}(\vartheta(x)),\\
&\quad \sum_{xb\in\mathcal{Z}} \varsigma_{\mathcal{J}'}(\vartheta(x)\vartheta(b)) + \varsigma_{\mathcal{I}}(\vartheta(x)),\\
&\quad \sum_{xb\in\mathcal{Z}} \delta_{\mathcal{J}'}(\vartheta(x)\vartheta(b)) + \delta_{\mathcal{I}}(\vartheta(x)))\\
&= (\mathcal{TD}_{\varkappa'}(x), \mathcal{TD}_{\varsigma'}(x), \mathcal{TD}_{\delta'}(x))\\
&= \mathcal{TD}_{\Psi'}(x).
\end{aligned}$$

□

Theorem 2: Suppose that $\Psi = (\mathcal{I}, \mathcal{J})$ is a CSDFG on a graph $\Psi^* = (\Upsilon, \mathcal{Z})$ with $\omega_{\mathcal{I}} e^{i\varkappa_{\mathcal{I}}}$, $\tau_{\mathcal{I}} e^{i\varsigma_{\mathcal{I}}}$ and $\Gamma_{\mathcal{I}} e^{i\delta_{\mathcal{I}}}$ as constant functions, then $\Psi = (\mathcal{I}, \mathcal{J})$ is a regular CSDFG iff Ψ is totally regular CSDFG.

Proof: Let $\omega_{\mathcal{I}} e^{i\varkappa_{\mathcal{I}}}$, $\tau_{\mathcal{I}} e^{i\varsigma_{\mathcal{I}}}$ and $\Gamma_{\mathcal{I}} e^{i\delta_{\mathcal{I}}}$ be constant functions, i.e, $\omega_{\mathcal{I}}(x) e^{i\varkappa_{\mathcal{I}}(x)} = c_1 e^{ic_1^*}$, $\tau_{\mathcal{I}}(x) e^{i\varsigma_{\mathcal{I}}(x)} = c_2 e^{ic_2^*}$ and $\Gamma_{\mathcal{I}}(x) e^{i\delta_{\mathcal{I}}(x)} = c_3 e^{ic_3^*}$ are constant functions $\forall\, x \in \Upsilon$, where $c_1 e^{ic_1^*}$, $c_2 e^{ic_2^*}$ and $c_3 e^{ic_3^*}$ are constants.
Let $\Psi = (\mathcal{I}, \mathcal{J})$ be $(\mathcal{F}_1 e^{i\mathcal{F}_1^*}, \mathcal{F}_2 e^{i\mathcal{F}_2^*}, \mathcal{F}_3 e^{i\mathcal{F}_3^*})$-regular CSDFG, then

$$\mathcal{D}_{\omega}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} = \mathcal{F}_1,$$
$$\mathcal{D}_{\tau}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)} = \mathcal{F}_2,$$
$$\mathcal{D}_{\Gamma}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)} = \mathcal{F}_3,$$
$$\mathcal{D}_{e^{i\varkappa}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)} = \mathcal{F}_1^*,$$
$$\mathcal{D}_{e^{i\varsigma}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)} = \mathcal{F}_2^*,$$
$$\mathcal{D}_{e^{i\delta}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)} = \mathcal{F}_3^*.$$

The total degree of a vertex is given by

$$\begin{aligned}
\mathcal{TD}_{\omega}(x) &= \sum_{x,b\neq x\in\Upsilon} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}\\
&\quad + \omega_{\mathcal{I}}(x) = \mathcal{F}_1 + c_1,\\
\mathcal{TD}_{\tau}(x) &= \sum_{x,b\neq x\in\Upsilon} \frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}\\
&\quad + \tau_{\mathcal{I}}(x) = \mathcal{F}_2 + c_2,\\
\mathcal{TD}_{\Gamma}(x) &= \sum_{x,b\neq x\in\Upsilon} \frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}\\
&\quad + \Gamma_{\mathcal{I}}(x) = \mathcal{F}_3 + c_3,\\
\mathcal{TD}_{e^{i\varkappa}}(x) &= \sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}\\
&\quad + \varkappa_{\mathcal{I}}(x) = \mathcal{F}_1^* + c_1^*,\\
\mathcal{TD}_{e^{i\varsigma}}(x) &= \sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}\\
&\quad + \varsigma_{\mathcal{I}}(x) = \mathcal{F}_2^* + c_2^*,
\end{aligned}$$

$$\mathcal{TD}_{e^{i\delta}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)} + \delta_{\mathcal{I}}(x) = \mathcal{F}_3^* + c_3^*.$$

Hence, Ψ is a $((\mathcal{F}_1+c_1)e^{i(\mathcal{F}_1^*+c_1^*)}, (\mathcal{F}_2+c_2)e^{i(\mathcal{F}_2^*+c_2^*)}, (\mathcal{F}_3+c_3)e^{i(\mathcal{F}_3^*+c_3^*)})$-totally regular CSDFG.

Conversely, suppose that $\Psi = (\mathcal{I}, \mathcal{J})$ is $(\mathcal{H}_1e^{i\mathcal{H}_1^*}, \mathcal{H}_2e^{i\mathcal{H}_2^*}, \mathcal{H}_3e^{i\mathcal{H}_3^*})$-totally regular CSDFG, then

$$\mathcal{TD}_{\omega}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} + \omega_{\mathcal{I}}(x) = \mathcal{H}_1,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} + c_1 = \mathcal{H}_1,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} = \mathcal{H}_1 - c_1,$$
$$\mathcal{D}_{\omega}(x) = \mathcal{H}_1 - c_1 = \mathcal{F}_1.$$
$$\mathcal{TD}_{e^{i\varkappa}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)} + \varkappa_{\mathcal{I}}(x) = \mathcal{H}_1^*,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)} + c_1^* = \mathcal{H}_1^*,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)} = \mathcal{H}_1^* - c_1^*,$$
$$\mathcal{D}_{e^{i\varkappa}}(x) = \mathcal{H}_1^* - c_1^* = \mathcal{F}_1^*.$$

Similarly for non membership value,

$$\mathcal{TD}_{\tau}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)} + \tau_{\mathcal{I}}(x) = \mathcal{H}_2,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)} + c_2 = \mathcal{H}_2,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)} = \mathcal{H}_2 - c_2,$$
$$\mathcal{D}_{\tau}(x) = \mathcal{H}_2 - c_2 = \mathcal{F}_2.$$
$$\mathcal{TD}_{e^{i\varsigma}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)} + \varsigma_{\mathcal{I}}(x) = \mathcal{H}_2^*,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)} + c_2^* = \mathcal{H}_2^*,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)} = \mathcal{H}_2^* - c_2^*,$$
$$\mathcal{D}_{e^{i\varsigma}}(x) = \mathcal{H}_2^* - c_2^* = \mathcal{F}_2^*.$$

Also for abstinence value,

$$\mathcal{TD}_{\Gamma}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)} + \Gamma_{\mathcal{I}}(x) = \mathcal{H}_3,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)} + + c_3 = \mathcal{H}_3,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)} = \mathcal{H}_3 - c_3,$$
$$\mathcal{D}_{\Gamma}(x) = \mathcal{H}_3 - c_3 = \mathcal{F}_3$$
$$\mathcal{TD}_{e^{i\delta}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)} + + \delta_{\mathcal{I}}(x) = \mathcal{H}_3^*,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)} + + c_3^* = \mathcal{H}_3^*,$$
$$\sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)} = \mathcal{H}_3^* - c_3^*,$$
$$\mathcal{D}_{e^{i\delta}}(x) = \mathcal{H}_3^* - c_3^* = \mathcal{F}_3^*.$$

So Ψ is a $(\mathcal{F}_1e^{i\mathcal{F}_1^*}, \mathcal{F}_2e^{i\mathcal{F}_2^*}, \mathcal{F}_3e^{i\mathcal{F}_3^*})$-regular CSDFG.

Theorem 3: Suppose that $\Psi = (\mathcal{I}, \mathcal{J})$ is a CSDFG on a graph $\Psi^* = (\Upsilon, \mathcal{Z})$. If Ψ is both $(\mathcal{F}_1e^{i\mathcal{F}_1^*}, \mathcal{F}_2e^{i\mathcal{F}_2^*}, \mathcal{F}_3e^{i\mathcal{F}_3^*})$-regular and $(\mathcal{H}_1e^{i\mathcal{H}_1^*}, \mathcal{H}_2e^{i\mathcal{H}_2^*}, \mathcal{H}_3e^{i\mathcal{H}_3^*})$-totally regular CSDFG, then $\omega_{\mathcal{I}}e^{i\varkappa_{\mathcal{I}}}$, $\tau_{\mathcal{I}}e^{i\varsigma_{\mathcal{I}}}$ and $\Gamma_{\mathcal{I}}e^{i\delta_{\mathcal{I}}}$ as constant functions.

Proof: LetΨ be $(\mathcal{F}_1e^{i\mathcal{F}_1^*}, \mathcal{F}_2e^{i\mathcal{F}_2^*}, \mathcal{F}_3e^{i\mathcal{F}_3^*})$-regular and $(\mathcal{H}_1e^{i\mathcal{H}_1^*}, \mathcal{H}_2e^{i\mathcal{H}_2^*}, \mathcal{H}_3e^{i\mathcal{H}_3^*})$ totally regular CSDFG. Then, the degree of a node is defined as

$$\mathcal{D}_{\omega}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} = \mathcal{F}_1,$$
$$\mathcal{D}_{\tau}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)} = \mathcal{F}_2,$$
$$\mathcal{D}_{\Gamma}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)} = \mathcal{F}_3,$$
$$\mathcal{D}_{e^{i\varkappa}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)} = \mathcal{F}_1^*,$$

$$\mathcal{D}_{e^{i\varsigma}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)} = \mathcal{F}_2^*,$$
$$\mathcal{D}_{e^{i\delta}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)} = \mathcal{F}_3^*.$$

The total degree is given by

$$\mathcal{TD}_{\omega}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x)+\omega_{\mathcal{I}}(b)-\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)} + +\omega_{\mathcal{I}}(x) = \mathcal{H}_1,$$
$$\mathcal{TD}_{\tau}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)} + +\tau_{\mathcal{I}}(x) = \mathcal{H}_2,$$
$$\mathcal{TD}_{\Gamma}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)} + +\Gamma_{\mathcal{I}}(x) = \mathcal{H}_3,$$
$$\mathcal{TD}_{e^{i\varkappa}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)} + +\varkappa_{\mathcal{I}}(x) = \mathcal{H}_1^*,$$
$$\mathcal{TD}_{e^{i\varsigma}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)} + +\varsigma_{\mathcal{I}}(x) = \mathcal{H}_2^*,$$
$$\mathcal{TD}_{e^{i\delta}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)} + +\delta_{\mathcal{I}}(x) = \mathcal{H}_3^*.$$

It follows that

$$\mathcal{TD}_{\omega}(g) = \mathcal{F}_1 + \omega_{\mathcal{I}}(x) = \mathcal{H}_1,$$
$$\omega_{\mathcal{I}}(x) = \mathcal{H}_1 - \mathcal{F}_1.$$
$$\mathcal{TD}_{\tau}(g) = \mathcal{F}_2 + \tau_{\mathcal{I}}(x) = \mathcal{H}_2,$$
$$\tau_{\mathcal{I}}(x) = \mathcal{H}_2 - \mathcal{F}_2.$$
$$\mathcal{TD}_{\Gamma}(g) = \mathcal{F}_3 + \Gamma_{\mathcal{I}}(x) = \mathcal{H}_3,$$
$$\Gamma_{\mathcal{I}}(x) = \mathcal{H}_3 - \mathcal{F}_3.$$
$$\mathcal{TD}_{e^{i\varkappa}}(g) = \mathcal{F}_1^* + \varkappa_{\mathcal{I}}(x) = \mathcal{H}_1^*,$$
$$\varkappa_{\mathcal{I}}(x) = \mathcal{H}_1^* - \mathcal{F}_1^*.$$
$$\mathcal{TD}_{e^{i\varsigma}}(g) = \mathcal{F}_2^* + \varsigma_{\mathcal{I}}(x) = \mathcal{H}_2^*,$$
$$\varsigma_{\mathcal{I}}(x) = \mathcal{H}_2^* - \mathcal{F}_2^*.$$
$$\mathcal{TD}_{e^{i\delta}}(g) = \mathcal{F}_3^* + \delta_{\mathcal{I}}(x) = \mathcal{H}_3^*,$$
$$\delta_{\mathcal{I}}(x) = \mathcal{H}_3^* - \mathcal{F}_3^*.$$

Hence, $\omega_{\mathcal{I}}e^{i\varkappa_{\mathcal{I}}} = (\mathcal{H}_1 - \mathcal{F}_1)e^{i(\mathcal{H}_1^*-\mathcal{F}_1^*)}$, $\tau_{\mathcal{I}}e^{i\varsigma_{\mathcal{I}}} = (\mathcal{H}_2 - \mathcal{F}_2)e^{i(\mathcal{H}_2^*-\mathcal{F}_2^*)}$ and
$\Gamma_{\mathcal{I}}e^{i\delta_{\mathcal{I}}} = (\mathcal{H}_3 - \mathcal{F}_3)e^{i(\mathcal{H}_3^*-\mathcal{F}_3^*)}$ are constant functions.

Converse of theorem 3 need not to be true in general as given in the following example 5. □

Example 5: Let $\Psi = (\mathcal{I}, \mathcal{J})$ be a CSDFG on $\Psi^* = (\Upsilon, \mathcal{Z})$, where $\Upsilon = \{x, b, r, s, u, v\}$ and $\mathcal{Z} = \{xb, xr, bs, vr, vu, us\}$ as shown in Figure 4. The node set $\mathcal{I}$

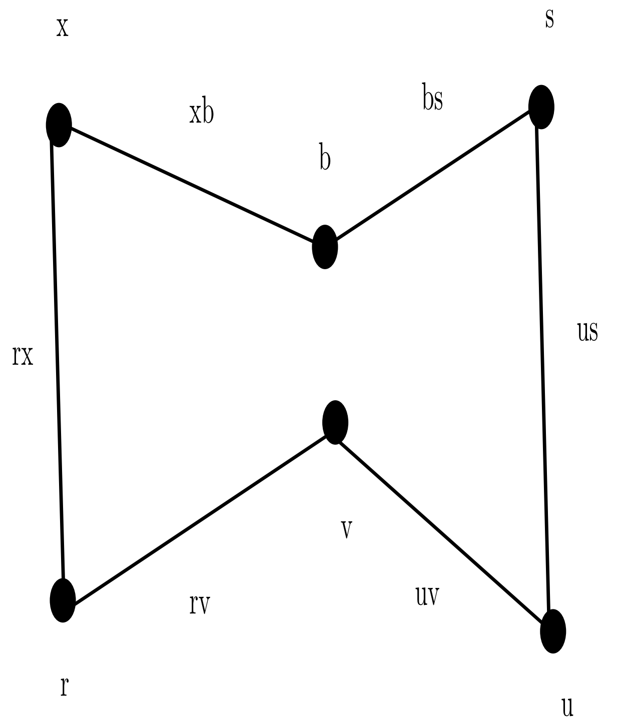

FIGURE 4. **Neither regular nor totally regular CSDFG.**

and arc set $\mathcal{J}$ of Ψ are defined as.

$$\mathcal{I} =< (\frac{x}{0.62e^{i2\pi(0.45)}}, \frac{b}{0.62e^{i2\pi(0.45)}}, \frac{r}{0.62e^{i2\pi(0.45)}}, \frac{s}{0.62e^{i2\pi(0.45)}}, \frac{u}{0.62e^{i2\pi(0.45)}}, \frac{v}{0.62e^{i2\pi(0.45)}}), (\frac{x}{0.34e^{i2\pi(0.15)}}, \frac{b}{0.34e^{i2\pi(0.15)}}, \frac{r}{0.34e^{i2\pi(0.15)}}, \frac{s}{0.34e^{i2\pi(0.15)}}, \frac{u}{0.34e^{i2\pi(0.15)}}, \frac{v}{0.34e^{i2\pi(0.15)}}), (\frac{x}{0.43e^{i2\pi(0.42)}}, \frac{b}{0.43e^{i2\pi(0.42)}}, \frac{r}{0.43e^{i2\pi(0.42)}}, \frac{s}{0.43e^{i2\pi(0.42)}}, \frac{u}{0.43e^{i2\pi(0.42)}}, \frac{v}{0.43e^{i2\pi(0.42)}}) >$$

and

$$\mathcal{B} =< (\frac{xb}{0.31e^{i2\pi(0.21)}}, \frac{xr}{0.25e^{i2\pi(0.13)}}, \frac{bs}{0.23e^{i2\pi(0.25)}}, \frac{vr}{0.15e^{i2\pi(0.19)}}, \frac{vu}{0.13e^{i2\pi(0.17)}}, \frac{us}{0.14e^{i2\pi(0.18)}}), (\frac{xb}{0.43e^{i2\pi(0.3)}}, \frac{xr}{0.36e^{i2\pi(0.4)}}, \frac{bs}{0.27e^{i2\pi(0.35)}}, \frac{vr}{0.25e^{i2\pi(0.45)}}, \frac{vu}{0.22e^{i2\pi(0.21)}}, \frac{us}{0.21e^{i2\pi(0.23)}}), (\frac{xb}{0.34e^{i2\pi(0.2)}}, \frac{xr}{0.55e^{i2\pi(0.35)}}, \frac{bs}{0.31e^{i2\pi(0.4)}}, \frac{vr}{0.21e^{i2\pi(0.3)}}, \frac{vu}{0.32e^{i2\pi(0.22)}}, \frac{us}{0.24e^{i2\pi(0.31)}}) >$$

Here $\omega_{\mathcal{A}}e^{i\sigma_{\mathcal{A}}}$, $\tau_{\mathcal{A}}e^{i\varsigma_{\mathcal{A}}}$ and $\kappa_{\mathcal{A}}e^{i\rho_{\mathcal{A}}}$ for x,b,r,s,u,v are constant functions. But $\mathcal{D}_{\mathcal{G}}(x) = (0.56e^{i2\pi(0.38)}, 0.79e^{i2\pi(0.7)}, 0.89e^{i2\pi(0.55)}) \neq (0.4e^{i2\pi(0.32)}, 0.61e^{i2\pi(0.85)}, 0.76e^{i2\pi(0.65)}) = \mathcal{D}_{\mathcal{G}}(r)$.

$$\begin{aligned}
x &= (0.62e^{i2\pi(0.45)}, 0.34e^{i2\pi(0.15)}, 0.43e^{i2\pi(0.42)})\\
b &= (0.62e^{i2\pi(0.45)}, 0.34e^{i2\pi(0.15)}, 0.43e^{i2\pi(0.42)})\\
r &= (0.62e^{i2\pi(0.45)}, 0.34e^{i2\pi(0.15)}, 0.43e^{i2\pi(0.42)})\\
s &= (0.62e^{i2\pi(0.45)}, 0.34e^{i2\pi(0.15)}, 0.43e^{i2\pi(0.42)})\\
u &= (0.62e^{i2\pi(0.45)}, 0.34e^{i2\pi(0.15)}, 0.43e^{i2\pi(0.42)})\\
v &= (0.62e^{i2\pi(0.45)}, 0.34e^{i2\pi(0.15)}, 0.43e^{i2\pi(0.42)})\\
xb &= (0.31e^{i2\pi(0.21)}, 0.43e^{i2\pi(0.3)}, 0.34e^{i2\pi(0.2)})\\
xr &= (0.25e^{i2\pi(0.13)}, 0.36e^{i2\pi(0.4)}, 0.55e^{i2\pi(0.35)})\\
bs &= (0.23e^{i2\pi(0.25)}, 0.27e^{i2\pi(0.35)}, 0.31e^{i2\pi(0.4)})\\
vr &= (0.15e^{i2\pi(0.19)}, 0.25e^{i2\pi(0.45)}, 0.21e^{i2\pi(0.3)})\\
vu &= (0.13e^{i2\pi(0.17)}, 0.22e^{i2\pi(0.21)}, 0.32e^{i2\pi(0.22)})\\
us &= (0.14e^{i2\pi(0.18)}, 0.21e^{i2\pi(0.23)}, 0.24e^{i2\pi(0.31)}).
\end{aligned}$$

Here $\omega_{\mathcal{I}}e^{i\varkappa_{\mathcal{I}}}$, $\tau_{\mathcal{I}}e^{i\varsigma_{\mathcal{I}}}$ and $\Gamma_{\mathcal{I}}e^{i\delta_{\mathcal{I}}}$ for x,b,r,s are constant functions. But

$\mathcal{D}_{\Psi}(x) = (0.56e^{i2\pi(0.34)}, 0.79e^{i2\pi(0.7)}, 0.89e^{i2\pi(0.6)}) \neq (0.38e^{i2\pi(0.44)}, 0.52e^{i2\pi(0.8)}, 0.52e^{i2\pi(0.7)}) = \mathcal{D}_{\Psi}(r)$ *and*

$\mathcal{TD}_{\mathcal{G}}(x) = (1.18e^{i2\pi(0.83)}, 1.13e^{i2\pi(0.85)}, 1.32e^{i2\pi(0.97)}) \neq (1.02e^{i2\pi(0.77)}, 0.95e^{i2\pi(1.00)}, 1.19e^{i2\pi(1.07)}) = \mathcal{TD}_{\mathcal{G}}(r)$

Hence, $\Psi = (\mathcal{I}, \mathcal{J})$ is neither regular nor totally regular CSDFG.

Definition 25: Let $\mathcal{J} = \{(xb, \omega_{\mathcal{J}}(xb)e^{i\varkappa_{\mathcal{J}}(xb)}, \tau_{\mathcal{J}}(xb)e^{i\varsigma_{\mathcal{J}}(xb)}, \Gamma_{\mathcal{J}}(xb)e^{i\delta_{\mathcal{J}}(xb)})|xb \in \mathcal{Z}\}$ be the set of arcs in CSDFG Ψ, then

- *The degree of an arc $xb \in \mathcal{Z}$ is represented by $\mathcal{D}_{\Psi}(xb) = (\mathcal{D}_{\omega e^{i\varkappa}}(xb), \mathcal{D}_{\tau e^{i\varsigma}}(xb), \mathcal{D}_{\Gamma e^{i\delta}}(xb))$, where*

$$\begin{aligned}
\mathcal{D}_{\omega}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \omega_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \omega_{\mathcal{J}}(br)\\
&= \mathcal{D}_{\omega_{\mathcal{I}}}(x) + \mathcal{D}_{\omega_{\mathcal{I}}}(b) - 2\omega_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\omega_{\mathcal{I}}}(x) + \mathcal{D}_{\omega_{\mathcal{I}}}(b)\\
&\quad - 2(\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}).\\
\mathcal{D}_{\tau}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \tau_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \tau_{\mathcal{J}}(br)\\
&= \mathcal{D}_{\tau_{\mathcal{I}}}(x) + \mathcal{D}_{\tau_{\mathcal{I}}}(b) - 2\tau_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\tau_{\mathcal{I}}}(x) + \mathcal{D}_{\tau_{\mathcal{I}}}(b)\\
&\quad - 2(\frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}).\\
\mathcal{D}_{\Gamma}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \Gamma_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \Gamma_{\mathcal{J}}(br)\\
&= \mathcal{D}_{\Gamma_{\mathcal{I}}}(x) + \mathcal{D}_{\Gamma_{\mathcal{I}}}(b) - 2\Gamma_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\Gamma_{\mathcal{I}}}(x) + \mathcal{D}_{\Gamma_{\mathcal{I}}}(b)\\
&\quad - 2(\frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}).\\
\mathcal{D}_{e^{i\varkappa}}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \varkappa_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \varkappa_{\mathcal{J}}(br)\\
&= \mathcal{D}_{\varkappa_{\mathcal{I}}}(x) + \mathcal{D}_{\varkappa_{\mathcal{I}}}(b) - 2\varkappa_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\varkappa_{\mathcal{I}}}(x) + \mathcal{D}_{\varkappa_{\mathcal{I}}}(b)\\
&\quad - 2(\frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}).\\
\mathcal{D}_{e^{i\varsigma}}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \varsigma_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \varsigma_{\mathcal{J}}(br)\\
&= \mathcal{D}_{\varsigma_{\mathcal{I}}}(x) + \mathcal{D}_{\varsigma_{\mathcal{I}}}(b) - 2\varsigma_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\varsigma_{\mathcal{I}}}(x) + \mathcal{D}_{\varsigma_{\mathcal{I}}}(b)\\
&\quad - 2(\frac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}).\\
\mathcal{D}_{e^{i\delta}}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \delta_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \delta_{\mathcal{J}}(br)\\
&= \mathcal{D}_{\delta_{\mathcal{I}}}(x) + \mathcal{D}_{\delta_{\mathcal{I}}}(b) - 2\delta_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\delta_{\mathcal{I}}}(x) + \mathcal{D}_{\delta_{\mathcal{I}}}(b)\\
&\quad - 2(\frac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}).
\end{aligned}$$

- *The total degree of an arc $xb \in \mathcal{Z}$ is represented by $\mathcal{TD}_{\Psi}(xb) = (\mathcal{TD}_{\omega e^{i\varkappa}}(xb), \mathcal{TD}_{\tau e^{i\varsigma}}(xb), \mathcal{TD}_{\Gamma e^{i\delta}}(xb))$, where*

$$\begin{aligned}
\mathcal{TD}_{\omega}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \omega_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \omega_{\mathcal{J}}(br) + \omega_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\omega_{\mathcal{I}}}(x) + \mathcal{D}_{\omega_{\mathcal{I}}}(b) - \omega_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\omega_{\mathcal{I}}}(x) + \mathcal{D}_{\omega_{\mathcal{I}}}(b)\\
&\quad - (\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}).\\
\mathcal{TD}_{\tau}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \tau_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \tau_{\mathcal{J}}(br) + \tau_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\tau_{\mathcal{I}}}(x) + \mathcal{D}_{\tau_{\mathcal{I}}}(b) - \tau_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\tau_{\mathcal{I}}}(x) + \mathcal{D}_{\tau_{\mathcal{I}}}(b)\\
&\quad - (\frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}).\\
\mathcal{TD}_{\Gamma}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \Gamma_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \Gamma_{\mathcal{J}}(br) + \Gamma_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\Gamma_{\mathcal{I}}}(x) + \mathcal{D}_{\Gamma_{\mathcal{I}}}(b) - \Gamma_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\Gamma_{\mathcal{I}}}(x) + \mathcal{D}_{\Gamma_{\mathcal{I}}}(b)\\
&\quad - (\frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}).\\
\mathcal{TD}_{e^{i\varkappa}}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \varkappa_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \varkappa_{\mathcal{J}}(br) + \varkappa_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\varkappa_{\mathcal{I}}}(x) + \mathcal{D}_{\varkappa_{\mathcal{I}}}(b) - \varkappa_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\varkappa_{\mathcal{I}}}(x) + \mathcal{D}_{\varkappa_{\mathcal{I}}}(b)\\
&\quad - (\frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}).\\
\mathcal{TD}_{e^{i\varsigma}}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \varsigma_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \varsigma_{\mathcal{J}}(br) + \varsigma_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\varsigma_{\mathcal{I}}}(x) + \mathcal{D}_{\varsigma_{\mathcal{I}}}(b) - \varsigma_{\mathcal{J}}(xb)\\
&= \mathcal{D}_{\varsigma_{\mathcal{I}}}(x) + \mathcal{D}_{\varsigma_{\mathcal{I}}}(b)\\
&\quad - (\frac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}).\\
\mathcal{TD}_{e^{i\delta}}(xb) &= \sum_{xr\in\mathcal{Z}, r\neq b} \delta_{\mathcal{J}}(xr) + \sum_{br\in\mathcal{Z}, x\neq r} \delta_{\mathcal{J}}(br) + \delta_{\mathcal{J}}(xb)
\end{aligned}$$

$$= \mathcal{D}_{\delta_\mathcal{I}}(x) + \mathcal{D}_{\delta_\mathcal{I}}(b) - \delta_\mathcal{J}(xb)$$
$$= \mathcal{D}_{\delta_\mathcal{I}}(x) + \mathcal{D}_{\delta_\mathcal{I}}(b) - (\frac{\delta_\mathcal{I}(x) + \delta_\mathcal{I}(b) - 2\delta_\mathcal{I}(x)\delta_\mathcal{I}(b)}{\delta_\mathcal{I}(x) + \delta_\mathcal{I}(b) - \delta_\mathcal{I}(x)\delta_\mathcal{I}(b)}).$$

Definition 26: A CSDFG $\Psi = (\mathcal{I}, \mathcal{J})$ is said to be arc regular, if its every arc has same degree. i.e,

$$\mathcal{D}_\omega(xb) = \mathcal{D}_{\omega_\mathcal{I}}(x) + \mathcal{D}_{\omega_\mathcal{I}}(b) - 2\omega_\mathcal{J}(xb) = \mathcal{D}_{\omega_\mathcal{I}}(x) + \mathcal{D}_{\omega_\mathcal{I}}(b) - 2(\frac{\omega_\mathcal{I}(x)\omega_\mathcal{I}(b)}{\omega_\mathcal{I}(x) + \omega_\mathcal{I}(b) - \omega_\mathcal{I}(x)\omega_\mathcal{I}(b)}) = \mathcal{L}_1.$$

$$\mathcal{D}_\tau(xb) = \mathcal{D}_{\tau_\mathcal{I}}(x) + \mathcal{D}_{\tau_\mathcal{I}}(b) - 2\tau_\mathcal{J}(xb) = \mathcal{D}_{\tau_\mathcal{I}}(x) + \mathcal{D}_{\tau_\mathcal{I}}(b) - 2(\frac{\tau_\mathcal{I}(x) + \tau_\mathcal{I}(b) - 2\tau_\mathcal{I}(x)\tau_\mathcal{I}(b)}{1 - \tau_\mathcal{I}(x)\tau_\mathcal{I}(b)}) = \mathcal{L}_2.$$

$$\mathcal{D}_\Gamma(xb) = \mathcal{D}_{\Gamma_\mathcal{I}}(x) + \mathcal{D}_{\Gamma_\mathcal{I}}(b) - 2\Gamma_\mathcal{J}(xb) = \mathcal{D}_{\Gamma_\mathcal{I}}(x) + \mathcal{D}_{\Gamma_\mathcal{I}}(b) - 2(\frac{\Gamma_\mathcal{I}(x) + \Gamma_\mathcal{I}(b) - 2\Gamma_\mathcal{I}(x)\Gamma_\mathcal{I}(b)}{\Gamma_\mathcal{I}(x) + \Gamma_\mathcal{I}(b) - \Gamma_\mathcal{I}(x)\Gamma_\mathcal{I}(b)}) = \mathcal{L}_3.$$

$$\mathcal{D}_{e^{i\varkappa}}(xb) = \mathcal{D}_{\varkappa_\mathcal{I}}(x) + \mathcal{D}_{\varkappa_\mathcal{I}}(b) - 2\varkappa_\mathcal{J}(xb) = \mathcal{D}_{\varkappa_\mathcal{I}}(x) + \mathcal{D}_{\varkappa_\mathcal{I}}(b) - 2(\frac{\varkappa_\mathcal{I}(x)\varkappa_\mathcal{I}(b)}{\varkappa_\mathcal{I}(x) + \varkappa_\mathcal{I}(b) - \varkappa_\mathcal{I}(x)\varkappa_\mathcal{I}(b)}) = \mathcal{L}_1^*.$$

$$\mathcal{D}_{e^{i\varsigma}}(xb) = \mathcal{D}_{\varsigma_\mathcal{I}}(x) + \mathcal{D}_{\varsigma_\mathcal{I}}(b) - 2\varsigma_\mathcal{J}(xb) = \mathcal{D}_{\varsigma_\mathcal{I}}(x) + \mathcal{D}_{\varsigma_\mathcal{I}}(b) - 2(\frac{\varsigma_\mathcal{I}(x) + \varsigma_\mathcal{I}(b) - 2\varsigma_\mathcal{I}(x)\varsigma_\mathcal{I}(b)}{1 - \varsigma_\mathcal{I}(x)\varsigma_\mathcal{I}(b)}) = \mathcal{L}_2^*.$$

$$\mathcal{D}_{e^{i\delta}}(xb) = \mathcal{D}_{\delta_\mathcal{I}}(x) + \mathcal{D}_{\delta_\mathcal{I}}(b) - 2\delta_\mathcal{J}(xb) = \mathcal{D}_{\delta_\mathcal{I}}(x) + \mathcal{D}_{\delta_\mathcal{I}}(b) - 2(\frac{\delta_\mathcal{I}(x) + \delta_\mathcal{I}(b) - 2\delta_\mathcal{I}(x)\delta_\mathcal{I}(b)}{\delta_\mathcal{I}(x) + \delta_\mathcal{I}(b) - \delta_\mathcal{I}(x)\delta_\mathcal{I}(b)}) = \mathcal{L}_3^*.$$

for all $xb \in \mathcal{Z}$. Ψ is called $(\mathcal{L}_1 e^{i\mathcal{L}_1^*}, \mathcal{L}_2 e^{i\mathcal{L}_2^*}, \mathcal{L}_3 e^{i\mathcal{L}_3^*})$-arc regular CSDFG.

Definition 27: A CSDFG $\Psi = (\mathcal{I}, \mathcal{J})$ is said to be totally arc regular, if its every arc has same degree. i.e,

$$\mathcal{TD}_\omega(xb) = \mathcal{D}_{\omega_\mathcal{I}}(x) + \mathcal{D}_{\omega_\mathcal{I}}(b) - \omega_\mathcal{J}(xb) = \mathcal{D}_{\omega_\mathcal{I}}(x) + \mathcal{D}_{\omega_\mathcal{I}}(b) - (\frac{\omega_\mathcal{I}(x)\omega_\mathcal{I}(b)}{\omega_\mathcal{I}(x) + \omega_\mathcal{I}(b) - \omega_\mathcal{I}(x)\omega_\mathcal{I}(b)}) = \mathcal{K}_1.$$

$$\mathcal{TD}_\tau(xb) = \mathcal{D}_{\tau_\mathcal{I}}(x) + \mathcal{D}_{\tau_\mathcal{I}}(b) - \tau_\mathcal{J}(xb) = \mathcal{D}_{\tau_\mathcal{I}}(x) + \mathcal{D}_{\tau_\mathcal{I}}(b) - (\frac{\tau_\mathcal{I}(x) + \tau_\mathcal{I}(b) - 2\tau_\mathcal{I}(x)\tau_\mathcal{I}(b)}{1 - \tau_\mathcal{I}(x)\tau_\mathcal{I}(b)}) = \mathcal{K}_2.$$

$$\mathcal{TD}_\Gamma(xb) = \mathcal{D}_{\Gamma_\mathcal{I}}(x) + \mathcal{D}_{\Gamma_\mathcal{I}}(b) - \Gamma_\mathcal{J}(xb) = \mathcal{D}_{\Gamma_\mathcal{I}}(x) + \mathcal{D}_{\Gamma_\mathcal{I}}(b) - (\frac{\Gamma_\mathcal{I}(x) + \Gamma_\mathcal{I}(b) - 2\Gamma_\mathcal{I}(x)\Gamma_\mathcal{I}(b)}{\Gamma_\mathcal{I}(x) + \Gamma_\mathcal{I}(b) - \Gamma_\mathcal{I}(x)\Gamma_\mathcal{I}(b)}) = \mathcal{K}_3.$$

$$\mathcal{TD}_{e^{i\varkappa}}(xb) = \mathcal{D}_{\varkappa_\mathcal{I}}(x) + \mathcal{D}_{\varkappa_\mathcal{I}}(b) - \varkappa_\mathcal{J}(xb) = \mathcal{D}_{\varkappa_\mathcal{I}}(x) + \mathcal{D}_{\varkappa_\mathcal{I}}(b) - (\frac{\varkappa_\mathcal{I}(x)\varkappa_\mathcal{I}(b)}{\varkappa_\mathcal{I}(x) + \varkappa_\mathcal{I}(b) - \varkappa_\mathcal{I}(x)\varkappa_\mathcal{I}(b)}) = \mathcal{K}_1^*.$$

$$\mathcal{TD}_{e^{i\varsigma}}(xb) = \mathcal{D}_{\varsigma_\mathcal{I}}(x) + \mathcal{D}_{\varsigma_\mathcal{I}}(b) - \varsigma_\mathcal{J}(xb) = \mathcal{D}_{\varsigma_\mathcal{I}}(x) + \mathcal{D}_{\varsigma_\mathcal{I}}(b) - (\frac{\varsigma_\mathcal{I}(x) + \varsigma_\mathcal{I}(b) - 2\varsigma_\mathcal{I}(x)\varsigma_\mathcal{I}(b)}{1 - \varsigma_\mathcal{I}(x)\varsigma_\mathcal{I}(b)}) = \mathcal{K}_2^*.$$

$$\mathcal{TD}_{e^{i\delta}}(xb) = \mathcal{D}_{\delta_\mathcal{I}}(x) + \mathcal{D}_{\delta_\mathcal{I}}(b) - \delta_\mathcal{J}(xb) = \mathcal{D}_{\delta_\mathcal{I}}(x) + \mathcal{D}_{\delta_\mathcal{I}}(b) - (\frac{\delta_\mathcal{I}(x) + \delta_\mathcal{I}(b) - 2\delta_\mathcal{I}(x)\delta_\mathcal{I}(b)}{\delta_\mathcal{I}(x) + \delta_\mathcal{I}(b) - \delta_\mathcal{I}(x)\delta_\mathcal{I}(b)}) = \mathcal{K}_3^*.$$

for all $xb \in \mathcal{Z}$. Ψ is called $(\mathcal{K}_1 e^{i\mathcal{K}_1^*}, \mathcal{K}_2 e^{i\mathcal{K}_2^*}, \mathcal{K}_3 e^{i\mathcal{K}_3^*})$-totally arc regular CSDFG.

Example 6: Let $\Psi = (\mathcal{I}, \mathcal{J})$ be a CSDFG on $\Psi^* = (\Upsilon, \mathcal{Z})$, where $\Upsilon = \{x, b, r, s\}$ and $\mathcal{Z} = \{xb, xs, bs, xr, br, rs\}$ as shown in Figure 5. The node set $\mathcal{I}$ and the arc set $\mathcal{J}$ of Ψ are defined as.

$$\mathcal{I} =< (\frac{x}{0.42e^{i2\pi(0.36)}}, \frac{b}{0.34e^{i2\pi(0.43)}}, \frac{r}{0.51e^{i2\pi(0.37)}}, \frac{s}{0.36e^{i2\pi(0.56)}}), (\frac{x}{0.27e^{i2\pi(0.19)}}, \frac{b}{0.35e^{i2\pi(0.26)}}, \frac{r}{0.28e^{i2\pi(0.26)}}, \frac{s}{0.18e^{i2\pi(0.39)}}), (\frac{x}{0.33e^{i2\pi(0.46)}}, \frac{b}{0.47e^{i2\pi(0.52)}}, \frac{r}{0.35e^{i2\pi(0.45)}}, \frac{s}{0.47e^{i2\pi(0.39)}}) >$$

and

$$\mathcal{J} =< (\frac{xb}{0.12e^{i2\pi(0.15)}}, \frac{xs}{0.12e^{i2\pi(0.15)}}, \frac{bs}{0.12e^{i2\pi(0.15)}}, \frac{xr}{0.12e^{i2\pi(0.15)}}, \frac{br}{0.12e^{i2\pi(0.15)}}, \frac{rs}{0.12e^{i2\pi(0.15)}}), (\frac{xb}{0.25e^{i2\pi(0.28)}}, \frac{xs}{0.25e^{i2\pi(0.28)}}, \frac{bs}{0.25e^{i2\pi(0.28)}}, \frac{xr}{0.25e^{i2\pi(0.28)}}, \frac{br}{0.25e^{i2\pi(0.28)}}, \frac{rs}{0.25e^{i2\pi(0.28)}}), (\frac{xb}{0.11e^{i2\pi(0.13)}}, \frac{xs}{0.11e^{i2\pi(0.13)}}, \frac{bs}{0.11e^{i2\pi(0.13)}}, \frac{xr}{0.11e^{i2\pi(0.13)}}, \frac{br}{0.11e^{i2\pi(0.13)}}, \frac{rs}{0.11e^{i2\pi(0.13)}})>.$$

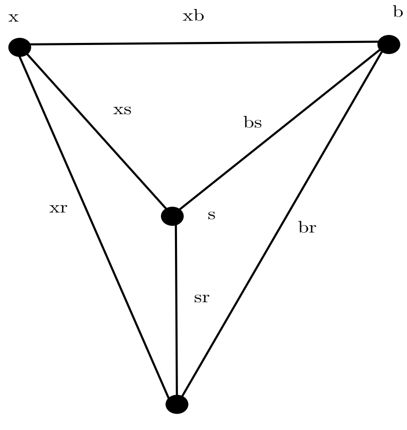

FIGURE 5. **Edge regular and totally edge regular CSDFG.**

Since degree of each arc is
$(0.48e^{i2\pi(0.6)}, 1.0e^{i2\pi(1.12)}, 0.44e^{i2\pi(0.52)})$
and total degree of each arc is
$(0.6e^{i2\pi(0.75)}, 1.25e^{i2\pi(1.4)}, 0.55e^{i2\pi(0.65)})$.

$$\begin{aligned}
x &= (0.42e^{i2\pi(0.36)}, 0.27e^{i2\pi(0.19)}, 0.33e^{i2\pi(0.46)})\\
b &= (0.34e^{i2\pi(0.43)}, 0.35e^{i2\pi(0.26)}, 0.47e^{i2\pi(0.52)})\\
r &= (0.51e^{i2\pi(0.37)}, 0.28e^{i2\pi(0.26)}, 0.35e^{i2\pi(0.45)})\\
s &= (0.36e^{i2\pi(0.56)}, 0.18e^{i2\pi(0.39)}, 0.47e^{i2\pi(0.39)})\\
xb &= (0.12e^{i2\pi(0.15)}, 0.25e^{i2\pi(0.28)}, 0.11e^{i2\pi(0.13)})\\
xs &= (0.12e^{i2\pi(0.15)}, 0.25e^{i2\pi(0.28)}, 0.11e^{i2\pi(0.13)})\\
bs &= (0.12e^{i2\pi(0.15)}, 0.25e^{i2\pi(0.28)}, 0.11e^{i2\pi(0.13)})\\
xr &= (0.12e^{i2\pi(0.15)}, 0.25e^{i2\pi(0.28)}, 0.11e^{i2\pi(0.13)})\\
br &= (0.12e^{i2\pi(0.15)}, 0.25e^{i2\pi(0.28)}, 0.11e^{i2\pi(0.13)})\\
rs &= (0.12e^{i2\pi(0.15)}, 0.25e^{i2\pi(0.28)}, 0.11e^{i2\pi(0.13)})
\end{aligned}$$

So, $\Psi = (\mathcal{I}, \mathcal{J})$ *is* $(0.48e^{i2\pi(0.6)}, 1.0e^{i2\pi(1.12)}, 0.44e^{i2\pi(0.52)})$*-arc regular and* $(0.6e^{i2\pi(0.75)}, 1.25e^{i2\pi(1.4)}, 0.55e^{i2\pi(0.65)})$*-totally arc regular CSDFG.*

Theorem 4: Suppose $\Psi = (\mathcal{I}, \mathcal{J})$ *is* $(\mathcal{F}_1 e^{i\mathcal{F}_1^*}, \mathcal{F}_2 e^{i\mathcal{F}_2^*}, \mathcal{F}_3 e^{i\mathcal{F}_3^*})$*-regular CSDFG. If* $\omega_\mathcal{J} e^{i\varkappa_\mathcal{J}}$*,* $\tau_\mathcal{J} e^{i\varsigma_\mathcal{J}}$ *and* $\Gamma_\mathcal{J} e^{i\delta_\mathcal{J}}$ *are constant functions, then* Ψ *is* $(\mathcal{L}_1 e^{i\mathcal{L}_1^*}, \mathcal{L}_2 e^{i\mathcal{L}_2^*}, \mathcal{L}_3 e^{i\mathcal{L}_3^*})$*-arc regular CSDFG.*

Proof: Suppose that $\Psi = (\mathcal{I}, \mathcal{J})$ is a $(\mathcal{F}_1 e^{i\mathcal{F}_1^*}, \mathcal{F}_2 e^{i\mathcal{F}_2^*}, \mathcal{F}_3 e^{i\mathcal{F}_3^*})$-regular CSDFG, then

$$\mathcal{D}_{\omega e^{i\varkappa}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\omega_\mathcal{I}(x)\omega_\mathcal{I}(b)}{\omega_\mathcal{I}(x)+\omega_\mathcal{I}(b)-\omega_\mathcal{I}(x)\omega_\mathcal{I}(b)} e^{i(\sum_{x,b\neq x\in\Upsilon} \frac{\varkappa_\mathcal{I}(x)\varkappa_\mathcal{I}(b)}{\varkappa_\mathcal{I}(x)+\varkappa_\mathcal{I}(b)-\varkappa_\mathcal{I}(x)\varkappa_\mathcal{I}(b)})} = \mathcal{F}_1 e^{i\mathcal{F}_1^*}.$$

$$\mathcal{D}_{\tau e^{i\varsigma}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\tau_\mathcal{I}(x)+\tau_\mathcal{I}(b)-2\tau_\mathcal{I}(x)\tau_\mathcal{I}(b)}{1-\tau_\mathcal{I}(x)\tau_\mathcal{I}(b)} e^{i(\sum_{x,b\neq x\in\Upsilon} \frac{\varsigma_\mathcal{I}(x)+\varsigma_\mathcal{I}(b)-2\varsigma_\mathcal{I}(x)\varsigma_\mathcal{I}(b)}{1-\varsigma_\mathcal{I}(x)\varsigma_\mathcal{I}(b)})} = \mathcal{F}_2 e^{i\mathcal{F}_2^*}.$$

$$\mathcal{D}_{\Gamma e^{i\delta}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\Gamma_\mathcal{I}(x)+\Gamma_\mathcal{I}(b)-2\Gamma_\mathcal{I}(x)\Gamma_\mathcal{I}(b)}{\Gamma_\mathcal{I}(x)+\Gamma_\mathcal{I}(b)-\Gamma_\mathcal{I}(x)\Gamma_\mathcal{I}(b)} e^{i(\sum_{x,b\neq x\in\Upsilon} \frac{\delta_\mathcal{I}(x)+\delta_\mathcal{I}(b)-2\delta_\mathcal{I}(x)\delta_\mathcal{I}(b)}{\delta_\mathcal{I}(x)+\delta_\mathcal{I}(b)-\delta_\mathcal{I}(x)\delta_\mathcal{I}(b)})} = \mathcal{F}_3 e^{i\mathcal{F}_3^*}.$$

Now $\omega_\mathcal{J} e^{i\varkappa_\mathcal{J}}$, $\tau_\mathcal{J} e^{i\varsigma_\mathcal{J}}$ and $\Gamma_\mathcal{J} e^{i\delta_\mathcal{J}}$ are constant functions, therefore, $\omega_\mathcal{J}(xb)e^{i\varkappa_\mathcal{J}(xb)} = c_1 e^{ic_1^*}$, $\tau_\mathcal{J}(xb)e^{i\varsigma_\mathcal{J}(xb)} = c_2 e^{ic_2^*}$ and $\Gamma_\mathcal{J}(xb)e^{i\delta_\mathcal{J}(xb)} = c_3 e^{ic_3^*}$ for all $xb \in \mathcal{Z}$.

Since the degree of an arc $xb \in \mathcal{Z}$ is given by $\mathcal{D}_\Psi(xb) = (\mathcal{D}_{\omega e^{i\varkappa}}(xb), \mathcal{D}_{\tau e^{i\varsigma}}(xb), \mathcal{D}_{\Gamma e^{i\delta}}(xb))$, where

$$\begin{aligned}
\mathcal{D}_\omega(xb) &= \mathcal{D}_{\omega_\mathcal{I}}(x) + \mathcal{D}_{\omega_\mathcal{I}}(b)\\
&\quad - 2(\frac{\omega_\mathcal{I}(x)\omega_\mathcal{I}(b)}{\omega_\mathcal{I}(x)+\omega_\mathcal{I}(b)-\omega_\mathcal{I}(x)\omega_\mathcal{I}(b)})\\
&= \mathcal{F}_1 + \mathcal{F}_1 - 2c_1 = 2(\mathcal{F}_1 - c_1) = \mathcal{L}_1.\\
\mathcal{D}_\tau(xb) &= \mathcal{D}_{\tau_\mathcal{I}}(x) + \mathcal{D}_{\tau_\mathcal{I}}(b)\\
&\quad - 2(\frac{\tau_\mathcal{I}(x)+\tau_\mathcal{I}(b)-2\tau_\mathcal{I}(x)\tau_\mathcal{I}(b)}{1-\tau_\mathcal{I}(x)\tau_\mathcal{I}(b)})\\
&= \mathcal{F}_2 + \mathcal{F}_2 - 2c_2 = 2(\mathcal{F}_2 - c_2) = \mathcal{L}_2.\\
\mathcal{D}_\Gamma(xb) &= \mathcal{D}_{\Gamma_\mathcal{I}}(x) + \mathcal{D}_{\Gamma_\mathcal{I}}(b)\\
&\quad - 2(\frac{\Gamma_\mathcal{I}(x)+\Gamma_\mathcal{I}(b)-2\Gamma_\mathcal{I}(x)\Gamma_\mathcal{I}(b)}{\Gamma_\mathcal{I}(x)+\Gamma_\mathcal{I}(b)-\Gamma_\mathcal{I}(x)\Gamma_\mathcal{I}(b)})\\
&= \mathcal{F}_3 + \mathcal{F}_3 - 2c_3 = 2(\mathcal{F}_3 - c_3) = \mathcal{L}_3.\\
\mathcal{D}_{e^{i\varkappa}}(xb) &= \mathcal{D}_{\varkappa_\mathcal{I}}(x) + \mathcal{D}_{\varkappa_\mathcal{I}}(b)\\
&\quad - 2(\frac{\varkappa_\mathcal{I}(x)\varkappa_\mathcal{I}(b)}{\varkappa_\mathcal{I}(x)+\varkappa_\mathcal{I}(b)-\varkappa_\mathcal{I}(x)\varkappa_\mathcal{I}(b)}) = 2\mathcal{F}_1^* - 2c_1^*\\
&= 2(\mathcal{F}_1^* - c_1^*) = \mathcal{L}_1^*.\\
\mathcal{D}_{e^{i\varsigma}}(xb) &= \mathcal{D}_{\varsigma_\mathcal{I}}(x) + \mathcal{D}_{\varsigma_\mathcal{I}}(b)\\
&\quad -2(\frac{\varsigma_\mathcal{I}(x)+\varsigma_\mathcal{I}(b)-2\varsigma_\mathcal{I}(x)\varsigma_\mathcal{I}(b)}{1-\varsigma_\mathcal{I}(x)\varsigma_\mathcal{I}(b)}) = 2\mathcal{F}_2^* - 2c_2^*\\
&= 2(\mathcal{F}_2^* - c_2^*) = \mathcal{L}_2^*.\\
\mathcal{D}_{e^{i\delta}}(xb) &= \mathcal{D}_{\delta_\mathcal{I}}(x) + \mathcal{D}_{\delta_\mathcal{I}}(b)\\
&\quad - 2(\frac{\delta_\mathcal{I}(x)+\delta_\mathcal{I}(b)-2\delta_\mathcal{I}(x)\delta_\mathcal{I}(b)}{\delta_\mathcal{I}(x)+\delta_\mathcal{I}(b)-\delta_\mathcal{I}(x)\delta_\mathcal{I}(b)}) = 2\mathcal{F}_3^* - 2c_3^*\\
&= 2(\mathcal{F}_3^* - c_3^*) = \mathcal{L}_3^*.
\end{aligned}$$

□

Hence Ψ is $(\mathcal{L}_1 e^{i\mathcal{L}_1^*}, \mathcal{L}_2 e^{i\mathcal{L}_2^*}, \mathcal{L}_3 e^{i\mathcal{L}_3^*})$-arc regular CSDFG.

Theorem 5: Suppose a CSDFG Ψ *is* $(\mathcal{L}_1 e^{i\mathcal{L}_1^*}, \mathcal{L}_2 e^{i\mathcal{L}_2^*}, \mathcal{L}_3 e^{i\mathcal{L}_3^*})$*-arc regular and* $(\mathcal{K}_1 e^{i\mathcal{K}_1^*}, \mathcal{K}_2 e^{i\mathcal{K}_2^*}, \mathcal{K}_3 e^{i\mathcal{K}_3^*})$*-totally arc regular, then* $\omega_\mathcal{J} e^{i\varkappa_\mathcal{J}}$*,* $\tau_\mathcal{J} e^{i\varsigma_\mathcal{J}}$ *and* $\Gamma_\mathcal{J} e^{i\delta_\mathcal{J}}$ *are constant functions.*

Proof: Suppose that Ψ is $(\mathcal{L}_1 e^{i\mathcal{L}_1^*}, \mathcal{L}_2 e^{i\mathcal{L}_2^*}, \mathcal{L}_3 e^{i\mathcal{L}_3^*})$-arc regular CSDFG, then the degree of its each arc is

$$\begin{aligned}
\mathcal{D}_\omega(xb) &= \mathcal{D}_{\omega_\mathcal{I}}(x) + \mathcal{D}_{\omega_\mathcal{I}}(b)\\
&\quad - 2(\frac{\omega_\mathcal{I}(x)\omega_\mathcal{I}(b)}{\omega_\mathcal{I}(x)+\omega_\mathcal{I}(b)-\omega_\mathcal{I}(x)\omega_\mathcal{I}(b)}) = \mathcal{L}_1.\\
\mathcal{D}_\tau(xb) &= \mathcal{D}_{\tau_\mathcal{I}}(x) + \mathcal{D}_{\tau_\mathcal{I}}(b)\\
&\quad - 2(\frac{\tau_\mathcal{I}(x)+\tau_\mathcal{I}(b)-2\tau_\mathcal{I}(x)\tau_\mathcal{I}(b)}{1-\tau_\mathcal{I}(x)\tau_\mathcal{I}(b)}) = \mathcal{L}_2.
\end{aligned}$$

$$\mathcal{D}_{\Gamma}(xb) = \mathcal{D}_{\Gamma_{\mathcal{I}}}(x) + \mathcal{D}_{\Gamma_{\mathcal{I}}}(b) - 2(\frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}) = \mathcal{L}_3.$$

$$\mathcal{D}_{e^{i\varkappa}}(xb) = \mathcal{D}_{\varkappa_{\mathcal{I}}}(x) + \mathcal{D}_{\varkappa_{\mathcal{I}}}(b) - 2(\frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}) = \mathcal{L}_1^*.$$

$$\mathcal{D}_{e^{i\varsigma}}(xb) = \mathcal{D}_{\varsigma_{\mathcal{I}}}(x) + \mathcal{D}_{\varsigma_{\mathcal{I}}}(b) - 2(\frac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}) = \mathcal{L}_2^*.$$

$$\mathcal{D}_{e^{i\delta}}(xb) = \mathcal{D}_{\delta_{\mathcal{I}}}(x) + \mathcal{D}_{\delta_{\mathcal{I}}}(b) - 2(\frac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}) = \mathcal{L}_3^*.$$

Also Ψ is $(\mathcal{K}_1 e^{i\mathcal{K}_1^*}, \mathcal{K}_2 e^{i\mathcal{K}_2^*}, \mathcal{K}_3 e^{i\mathcal{K}_3^*})$ totally arc regular CSDFG, then the degree of each arc is

$$\mathcal{TD}_{\omega}(xb) = \mathcal{D}_{\omega_{\mathcal{I}}}(x) + \mathcal{D}_{\omega_{\mathcal{I}}}(b) - (\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}) = \mathcal{K}_1.$$

$$\mathcal{TD}_{\tau}(xb) = \mathcal{D}_{\tau_{\mathcal{I}}}(x) + \mathcal{D}_{\tau_{\mathcal{I}}}(b) - (\frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}) = \mathcal{K}_2.$$

$$\mathcal{TD}_{\Gamma}(xb) = \mathcal{D}_{\Gamma_{\mathcal{I}}}(x) + \mathcal{D}_{\Gamma_{\mathcal{I}}}(b) - (\frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}) = \mathcal{K}_3.$$

$$\mathcal{TD}_{e^{i\varkappa}}(xb) = \mathcal{D}_{\varkappa_{\mathcal{I}}}(x) + \mathcal{D}_{\varkappa_{\mathcal{I}}}(b) - (\frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}) = \mathcal{K}_1^*.$$

$$\mathcal{TD}_{e^{i\varsigma}}(xb) = \mathcal{D}_{\varsigma_{\mathcal{I}}}(x) + \mathcal{D}_{\varsigma_{\mathcal{I}}}(b) - (\frac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}) = \mathcal{K}_2^*.$$

$$\mathcal{TD}_{e^{i\delta}}(xb) = \mathcal{D}_{\delta_{\mathcal{I}}}(x) + \mathcal{D}_{\delta_{\mathcal{I}}}(b) - (\frac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}) = \mathcal{K}_3^*.$$

Further, it follows that

$$\mathcal{TD}_{\omega}(xb) = \mathcal{K}_1$$
$$\mathcal{D}_{\omega_{\mathcal{I}}}(x) + \mathcal{D}_{\omega_{\mathcal{I}}}(b) - (\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}) = \mathcal{K}_1$$
$$\mathcal{D}_{\omega_{\mathcal{I}}}(x) + \mathcal{D}_{\omega_{\mathcal{I}}}(b) - 2(\frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}) + \omega_{\mathcal{J}}(xb) = \mathcal{K}_1$$
$$\omega_{\mathcal{J}}(xb) = \mathcal{K}_1 - \mathcal{F}_1.$$

$$\mathcal{TD}_{\varkappa}(xb) = \mathcal{K}_1^*$$
$$\mathcal{D}_{\varkappa_{\mathcal{I}}}(x) + \mathcal{D}_{\varkappa_{\mathcal{I}}}(b) - (\frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}) = \mathcal{K}_1^*$$
$$\mathcal{D}_{\varkappa_{\mathcal{I}}}(x) + \mathcal{D}_{\varkappa_{\mathcal{I}}}(b) - 2(\frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x) + \varkappa_{\mathcal{I}}(b) - \varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}) + \varkappa_{\mathcal{J}}(xb) = \mathcal{K}_1^*$$
$$\varkappa_{\mathcal{J}}(xb) = \mathcal{K}_1^* - \mathcal{F}_1^*.$$

Similarly, for non membership value

$$\mathcal{TD}_{\tau}(xb) = \mathcal{K}_2$$
$$\mathcal{D}_{\tau_{\mathcal{I}}}(x) + \mathcal{D}_{\tau_{\mathcal{I}}}(b) - (\frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}) = \mathcal{K}_2$$
$$\mathcal{D}_{\tau_{\mathcal{I}}}(x) + \mathcal{D}_{\tau_{\mathcal{I}}}(b) - 2(\frac{\tau_{\mathcal{I}}(x) + \tau_{\mathcal{I}}(b) - 2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1 - \tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}) + \tau_{\mathcal{J}}(xb) = \mathcal{K}_2$$
$$\tau_{\mathcal{J}}(xb) = \mathcal{K}_2 - \mathcal{F}_2.$$

$$\mathcal{TD}_{\varsigma}(xb) = \mathcal{K}_2^*$$
$$\mathcal{D}_{\varsigma_{\mathcal{I}}}(x) + \mathcal{D}_{\varsigma_{\mathcal{I}}}(b) - (\frac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}) = \mathcal{K}_2^*$$
$$\mathcal{D}_{\varsigma_{\mathcal{I}}}(x) + \mathcal{D}_{\varsigma_{\mathcal{I}}}(b) - 2(\frac{\varsigma_{\mathcal{I}}(x) + \varsigma_{\mathcal{I}}(b) - 2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1 - \varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}) + \varsigma_{\mathcal{J}}(xb) = \mathcal{K}_2^*$$
$$\varsigma_{\mathcal{J}}(xb) = \mathcal{K}_2^* - \mathcal{F}_2^*.$$

Also, similarly for the abstinence value

$$\mathcal{TD}_{\Gamma}(xb) = \mathcal{K}_3$$
$$\mathcal{D}_{\Gamma_{\mathcal{I}}}(x) + \mathcal{D}_{\Gamma_{\mathcal{I}}}(b) - (\frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}) = \mathcal{K}_3$$
$$\mathcal{D}_{\Gamma_{\mathcal{I}}}(x) + \mathcal{D}_{\Gamma_{\mathcal{I}}}(b) - 2(\frac{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - 2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x) + \Gamma_{\mathcal{I}}(b) - \Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}) + \varsigma_{\mathcal{J}}(xb) = \mathcal{K}_3$$
$$\varsigma_{\mathcal{J}}(xb) = \mathcal{K}_3 - \mathcal{F}_3.$$

$$\mathcal{TD}_{\delta}(xb) = \mathcal{K}_3^*$$
$$\mathcal{D}_{\delta_{\mathcal{I}}}(x) + \mathcal{D}_{\delta_{\mathcal{I}}}(b) - (\frac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}) = \mathcal{K}_3^*$$
$$\mathcal{D}_{\delta_{\mathcal{I}}}(x) + \mathcal{D}_{\delta_{\mathcal{I}}}(b) - 2(\frac{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - 2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x) + \delta_{\mathcal{I}}(b) - \delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}) + \delta_{\mathcal{J}}(xb) = \mathcal{K}_3^*$$
$$\delta_{\mathcal{J}}(xb) = \mathcal{K}_3^* - \mathcal{F}_3^*.$$

□

Hence, $\omega_{\mathcal{J}} e^{i\varkappa_{\mathcal{J}}}$, $\tau_{\mathcal{J}} e^{i\varsigma_{\mathcal{J}}}$ and $\Gamma_{\mathcal{J}} e^{i\delta_{\mathcal{J}}}$ are constant functions. □

Theorem 6: Suppose $\Psi = (\mathcal{I}, \mathcal{J})$ is a CSDFG. Then the functions $\omega_{\mathcal{J}} e^{i\varkappa_{\mathcal{J}}}$, $\tau_{\mathcal{J}} e^{i\varsigma_{\mathcal{J}}}$ and $\Gamma_{\mathcal{J}} e^{i\delta_{\mathcal{J}}}$ are constant iff Ψ is regular as well as totally arc regular CSDFG.

Proof: Suppose that Ψ is a CSDFG. Assume that $\omega_{\mathcal{J}} e^{i\varkappa_{\mathcal{J}}}$, $\tau_{\mathcal{J}} e^{i\varsigma_{\mathcal{J}}}$ and $\Gamma_{\mathcal{J}} e^{i\delta_{\mathcal{J}}}$ are constant functions, therefore, $\omega_{\mathcal{J}}(xb)e^{i\varkappa_{\mathcal{J}}(xb)} = c_1 e^{ic_1^*}$, $\tau_{\mathcal{J}}(xb)e^{i\varsigma_{\mathcal{J}}(xb)} = c_2 e^{ic_2^*}$ and $\Gamma_{\mathcal{J}}(xb)e^{i\delta_{\mathcal{J}}(xb)} = c_3 e^{ic_3^*}$ for all $xb \in \mathcal{Z}$, where $c_1 e^{ic_1^*}$, $c_2 e^{ic_2^*}$ and $c_3 e^{ic_3^*}$ are constants.

Since the degree of a node $x \in \Upsilon$ is given by $\mathcal{D}_{\Psi}(x) = (\mathcal{D}_{\omega e^{i\varkappa}}(x), \mathcal{D}_{\tau e^{i\varsigma}}(x), \mathcal{D}_{\Gamma e^{i\delta}}(x))$, where

$$\mathcal{D}_{\omega e^{i\varkappa}}(x) = \sum_{xb\in\mathcal{Z}} \omega_{\mathcal{J}}(xb) e^{i(\sum_{xb\in\mathcal{Z}} \varkappa_{\mathcal{J}}(xb))}$$

$$\mathcal{D}_{\omega e^{i\varkappa}}(x) = \sum_{x,b\neq x\in\Upsilon} \frac{\omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}{\omega_{\mathcal{I}}(x) + \omega_{\mathcal{I}}(b) - \omega_{\mathcal{I}}(x)\omega_{\mathcal{I}}(b)}$$

$$e^{i(\sum_{x,b\neq x\in\Upsilon}\frac{\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)}{\varkappa_{\mathcal{I}}(x)+\varkappa_{\mathcal{I}}(b)-\varkappa_{\mathcal{I}}(x)\varkappa_{\mathcal{I}}(b)})}$$

$$\mathcal{D}_{\omega e^{i\varkappa}}(x)=\sum_{xb\in\mathcal{Z}}c_1e^{i(\sum_{xb\in\mathcal{Z}}c_1)}$$

$$\mathcal{D}_{\omega e^{i\varkappa}}(x)=\mathcal{F}c_1e^{i\mathcal{F}c_1^*}.$$

$$\mathcal{D}_{\tau e^{i\varsigma}}(x)=\sum_{xb\in\mathcal{Z}}\tau_{\mathcal{J}}(xb)e^{i(\sum_{xb\in\mathcal{Z}}\varsigma_{\mathcal{J}}(xb))}$$

$$\mathcal{D}_{\tau e^{i\varsigma}}(x)=\sum_{x,b\neq x\in\Upsilon}\frac{\tau_{\mathcal{I}}(x)+\tau_{\mathcal{I}}(b)-2\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}{1-\tau_{\mathcal{I}}(x)\tau_{\mathcal{I}}(b)}$$

$$e^{i(\sum_{x,b\neq x\in\Upsilon}\frac{\varsigma_{\mathcal{I}}(x)+\varsigma_{\mathcal{I}}(b)-2\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)}{1-\varsigma_{\mathcal{I}}(x)\varsigma_{\mathcal{I}}(b)})}$$

$$\mathcal{D}_{\tau e^{i\varsigma}}(x)=\sum_{xb\in\mathcal{Z}}c_2e^{i(\sum_{xb\in\mathcal{Z}}c_2)}$$

$$\mathcal{D}_{\tau e^{i\varsigma}}(x)=\mathcal{F}c_2e^{i\mathcal{F}c_2^*}.$$

$$\mathcal{D}_{\Gamma e^{i\delta}}(x)=\sum_{xb\in\mathcal{Z}}\Gamma_{\mathcal{J}}(xb)e^{i(\sum_{xb\in\mathcal{Z}}\delta_{\mathcal{J}}(xb))}$$

$$\mathcal{D}_{\Gamma e^{i\delta}}(x)=\sum_{x,b\neq x\in\Upsilon}\frac{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-2\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}{\Gamma_{\mathcal{I}}(x)+\Gamma_{\mathcal{I}}(b)-\Gamma_{\mathcal{I}}(x)\Gamma_{\mathcal{I}}(b)}$$

$$e^{i(\sum_{x,b\neq x\in\Upsilon}\frac{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-2\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)}{\delta_{\mathcal{I}}(x)+\delta_{\mathcal{I}}(b)-\delta_{\mathcal{I}}(x)\delta_{\mathcal{I}}(b)})}$$

$$\mathcal{D}_{\Gamma e^{i\delta}}(x)=\sum_{xb\in\mathcal{Z}}c_3e^{i(\sum_{xb\in\mathcal{Z}}c_3)}$$

$$\mathcal{D}_{\Gamma e^{i\delta}}(x)=\mathcal{F}c_3e^{i\mathcal{F}c_3^*}.$$

Thus, Ψ is $(\mathcal{F}c_1e^{i\mathcal{F}c_1^*},\mathcal{F}c_2e^{i\mathcal{F}c_2^*},\mathcal{F}c_3e^{i\mathcal{F}c_3^*})$-regular CSDFG.

As for an arc $xb\in\mathcal{Z}$, its total degree is given as $\mathcal{TD}_{\Psi}(xb)=(\mathcal{TD}_{\omega e^{i\varkappa}}(xb),\mathcal{TD}_{\tau e^{i\varsigma}}(xb),\mathcal{TD}_{\Gamma e^{i\delta}}(xb))$, where

$$\begin{aligned}\mathcal{TD}_{\omega}(xb)&=\sum_{xr\in\mathcal{Z},r\neq b}\omega_{\mathcal{J}}(xr)+\sum_{br\in\mathcal{Z},x\neq r}\omega_{\mathcal{J}}(br)+\omega_{\mathcal{J}}(xb)\\&=\sum_{xr\in\mathcal{Z},r\neq b}c_1+\sum_{br\in\mathcal{Z},x\neq r}c_1+c_1\\&=c_1(\mathcal{F}-1)+c_1(\mathcal{F}-1)+c_1\\&=c_1(2\mathcal{F}-1).\end{aligned}$$

$$\begin{aligned}\mathcal{TD}_{\varkappa}(xb)&=\sum_{xr\in\mathcal{Z},r\neq b}\varkappa_{\mathcal{J}}(xr)+\sum_{br\in\mathcal{Z},x\neq r}\varkappa_{\mathcal{J}}(br)+\varkappa_{\mathcal{J}}(xb)\\&=\sum_{xr\in\mathcal{Z},r\neq b}c_1^*+\sum_{br\in\mathcal{Z},x\neq r}c_1^*+c_1^*\\&=c_1^*(\mathcal{F}-1)+c_1^*(\mathcal{F}-1)+c_1^*\\&=c_1^*(2\mathcal{F}-1).\end{aligned}$$

$$\begin{aligned}\mathcal{TD}_{\tau}(xb)&=\sum_{xr\in\mathcal{Z},r\neq b}\tau_{\mathcal{J}}(xr)+\sum_{br\in\mathcal{Z},x\neq r}\tau_{\mathcal{J}}(br)+\tau_{\mathcal{J}}(xb)\\&=\sum_{xr\in\mathcal{Z},r\neq b}c_2+\sum_{br\in\mathcal{Z},x\neq r}c_2+c_2\\&=c_2(\mathcal{F}-1)+c_2(\mathcal{F}-1)+c_2\\&=c_2(2\mathcal{F}-1).\end{aligned}$$

$$\begin{aligned}\mathcal{TD}_{\varsigma}(xb)&=\sum_{xr\in\mathcal{Z},r\neq b}\varsigma_{\mathcal{J}}(xr)+\sum_{br\in\mathcal{Z},x\neq r}\varsigma_{\mathcal{J}}(br)+\varsigma_{\mathcal{J}}(xb)\\&=\sum_{xr\in\mathcal{Z},r\neq b}c_2^*+\sum_{br\in\mathcal{Z},x\neq r}c_2^*+c_2^*\\&=c_2^*(\mathcal{F}-1)+c_2^*(\mathcal{F}-1)+c_2^*\\&=c_2^*(2\mathcal{F}-1).\end{aligned}$$

$$\begin{aligned}\mathcal{TD}_{\Gamma}(xb)&=\sum_{xr\in\mathcal{Z},r\neq b}\Gamma_{\mathcal{J}}(xr)+\sum_{br\in\mathcal{Z},x\neq r}\Gamma_{\mathcal{J}}(br)+\Gamma_{\mathcal{J}}(xb)\\&=\sum_{xr\in\mathcal{Z},r\neq b}c_3+\sum_{br\in\mathcal{Z},x\neq r}c_3+c_3\\&=c_3(\mathcal{F}-1)+c_3(\mathcal{F}-1)+c_3\\&=c_3(2\mathcal{F}-1).\end{aligned}$$

$$\begin{aligned}\mathcal{TD}_{\delta}(xb)&=\sum_{xr\in\mathcal{Z},r\neq b}\delta_{\mathcal{J}}(xr)+\sum_{br\in\mathcal{Z},x\neq r}\delta_{\mathcal{J}}(br)+\delta_{\mathcal{J}}(xb)\\&=\sum_{xr\in\mathcal{Z},r\neq b}c_3^*+\sum_{br\in\mathcal{Z},x\neq r}c_3^*+c_3^*\\&=c_3^*(\mathcal{F}-1)+c_3^*(\mathcal{F}-1)+c_3^*\\&=c_3^*(2\mathcal{F}-1).\end{aligned}$$

Hence Ψ is $(c_1(2\mathcal{F}-1)e^{i(c_1^*(2\mathcal{F}-1))}, c_2(2\mathcal{F}-1)e^{i(c_2^*(2\mathcal{F}-1))}, c_3(2\mathcal{F}-1)e^{i(c_3^*(2\mathcal{F}-1))})$-totally arc regular CSDFG. So Ψ is a regular graph and it is also totally arc regular.

Conversely, let Ψ be a $(\mathcal{L}_1e^{i\mathcal{L}_1^*},\mathcal{L}_2e^{i\mathcal{L}_2^*},\mathcal{L}_3e^{i\mathcal{L}_3^*})$ regular and $(\mathcal{K}_1e^{i\mathcal{K}_1^*},\mathcal{K}_2e^{i\mathcal{K}_2^*},\mathcal{K}_3e^{i\mathcal{K}_3^*})$ totally arc regular CSDFG. The total degree of an arc xb is given by

$\mathcal{TD}_{\Psi}(xb)=(\mathcal{TD}_{\omega e^{i\varkappa}}(xb),\mathcal{TD}_{\tau e^{i\varsigma}}(xb),\mathcal{TD}_{\Gamma e^{i\delta}}(xb))$, where

$$\begin{aligned}\mathcal{TD}_{\omega}(xb)&=\mathcal{D}_{\omega_{\mathcal{I}}}(x)+\mathcal{D}_{\omega_{\mathcal{I}}}(b)-\omega_{\mathcal{J}}(xb)\\\mathcal{K}_1&=\mathcal{F}_1+\mathcal{F}_1-\omega_{\mathcal{J}}(xb)\\\omega_{\mathcal{J}}(xb)&=2\mathcal{F}_1-\mathcal{K}_1.\\\mathcal{TD}_{e^{i\varkappa}}(xb)&=\mathcal{D}_{\varkappa_{\mathcal{I}}}(x)+\mathcal{D}_{\varkappa_{\mathcal{I}}}(b)-\varkappa_{\mathcal{J}}(xb)\\\mathcal{K}_1^*&=\mathcal{F}_1^*+\mathcal{F}_1^*-\varkappa_{\mathcal{J}}(xb)\\\varkappa_{\mathcal{J}}(xb)&=2\mathcal{F}_1^*-\mathcal{K}_1^*.\text{ for all }xb\in\mathcal{Z}.\\\mathcal{TD}_{\tau}(xb)&=\mathcal{D}_{\tau_{\mathcal{I}}}(x)+\mathcal{D}_{\tau_{\mathcal{I}}}(b)-\tau_{\mathcal{J}}(xb)\\\mathcal{K}_2&=\mathcal{F}_2+\mathcal{F}_2-\tau_{\mathcal{J}}(xb)\\\tau_{\mathcal{J}}(xb)&=2\mathcal{F}_2-\mathcal{K}_2.\\\mathcal{TD}_{e^{i\varsigma}}(xb)&=\mathcal{D}_{\varsigma_{\mathcal{I}}}(x)+\mathcal{D}_{\varsigma_{\mathcal{I}}}(b)-\varsigma_{\mathcal{J}}(xb)\\\mathcal{K}_2^*&=\mathcal{F}_2^*+\mathcal{F}_2^*-\varsigma_{\mathcal{J}}(xb)\\\varsigma_{\mathcal{J}}(xb)&=2\mathcal{F}_2^*-\mathcal{K}_2^*.\text{ for all }xb\in\mathcal{Z}.\\\mathcal{TD}_{\Gamma}(xb)&=\mathcal{D}_{\Gamma_{\mathcal{I}}}(x)+\mathcal{D}_{\Gamma_{\mathcal{I}}}(b)-\Gamma_{\mathcal{J}}(xb)\\\mathcal{K}_3&=\mathcal{F}_3+\mathcal{F}_3-\Gamma_{\mathcal{J}}(xb)\\\Gamma_{\mathcal{J}}(xb)&=2\mathcal{F}_3-\mathcal{K}_3.\\\mathcal{TD}_{e^{i\delta}}(xb)&=\mathcal{D}_{\delta_{\mathcal{I}}}(x)+\mathcal{D}_{\delta_{\mathcal{I}}}(b)-\delta_{\mathcal{J}}(xb)\\\mathcal{K}_3^*&=\mathcal{F}_3^*+\mathcal{F}_3^*-\delta_{\mathcal{J}}(xb)\\\delta_{\mathcal{J}}(xb)&=2\mathcal{F}_3^*-\mathcal{K}_3^*.\text{ for all }xb\in\mathcal{Z}.\end{aligned}$$

Hence, $\omega_{\mathcal{J}}e^{i\varkappa_{\mathcal{J}}}$, $\tau_{\mathcal{J}}e^{i\varsigma_{\mathcal{J}}}$ and $\Gamma_{\mathcal{J}}e^{i\delta_{\mathcal{J}}}$ are constant functions.

IV. APPLICATION

The Financial Action Task Force (FATF) is critical to worldwide efforts to prevent money laundering and terrorist funding. FATF, founded in 1989, is a non-profit group that establishes worldwide standards and promotes actions to protect the integrity of the global financial system. This is accomplished through the use of many important functions. First and foremost, FATF creates and disseminates a collection of guidelines known as the "FATF Recommendations," which serve as a complete foundation for anti-money laundering (AML) and counter-terrorist financing (CTF) efforts. These suggestions, among other things, include guidance on customer due diligence, reporting suspicious transactions, and seizure of illegal assets. Furthermore, FATF conducts reciprocal reviews of member and non-member countries to examine their AML/CTF systems and verify compliance with the guidelines. In addition, the organisation monitors and supports nations' attempts to adopt these measures, urging them to align their AML/CTF regimes with international norms. FATF encourages worldwide collaboration and coordination in combating financial crimes through its function as a global standard-setter and assessor. Furthermore, it involves both the public and private sectors, with a focus on coordination among multiple stakeholders such as financial institutions, law enforcement agencies, and regulators. FATF's work has been crucial in improving the efficacy of AML/CTF procedures and protecting the international financial system from misuse by criminals and terrorists, thanks to research and guidance. In situations of noncompliance, FATF can place nations on its "grey list" or "blocklist," emphasising its role in ensuring global responsibility. Overall, the FATF's multidimensional strategy is critical to worldwide efforts to combat financial crime and protect the financial system's integrity. In this section, we propose a strategy and address the difficult task of deciding which country should be taken off the FATF's grey list. Understanding the issue will help you comprehend the chosen approach.

A. ALGORITHM

The algorithm is as follows:

INPUT: A discrete set of proper alternatives $B = \{B_1, B_2, \ldots, B_n\}$ under some parameters in order to achieve the target and creation of complex fuzzy preference relation (CFPR) $Q = (d_{kq})_{n\times n}$.

OUTPUT: The selection of a suitable alternative.

1. Take $d_{kq} = (\omega_{kq}e^{i\varkappa_{kq}}, \tau_{kq}e^{i\varsigma_{kq}}, \Gamma_{kq}e^{i\delta_{kq}})$ (k,b= 1,2,3,4…,n) and set of alternatives $B = \{B_1, B_2, \ldots, B_n\}$.

2. Aggregate all $d_{kq} = (\omega_{kq}e^{i\varkappa_{kq}}, \tau_{kq}e^{i\varsigma_{kq}}, \Gamma_{kq}e^{i\delta_{kq}})$ (k,b= 1,2,3,4,…,n) relating to the alternative B_k and get the complex fuzzy element (CFE) d_k of the alternative B_k over all other alternatives by using complex dombi fuzzy operator, as shown in the equation at the bottom of the page.

3. Take the following equation to find the score functions:

$$s(d_k) = (\omega^2 - \tau^2 - \Gamma^2) + \frac{1}{4\pi^2}(\varkappa^2 - \varsigma^2 - \delta^2)$$

4. By using the equation of the score function, calculate the value of the score function $s(d_k)$ of the combined overall preference value d_k (k = 1, 2,…,n).

5. On the basis of the score function $s(d_k)$ (k=1,2,3,4…,n), sort through the alternatives B_k(k=1,2,3,4…,n).

6. Using the scoring functions that were acquired in step 4 of the process, output the suitable choice.

B. SELECTION OF SUITABLE COUNTRY FOR RELEASING FUNDS AND EXISTED FROM THE GREY LIST OF FATF

In this application, we create a few assumptions that play a beneficial role in decision-making problems.

FATF is an intergovernmental organisation that implements effective regulatory and operational measures to tackle

$$d_k = CDFoperator(d_{k1}, d_{k2}, \ldots, d_{kn})$$

$$d_k = \left(\sqrt{\left(1 - \frac{1}{1 + [\sum_{b=1}^{n} \frac{1}{n}(\frac{\omega_{kq}^2}{1-\omega_{kq}^2})^{\xi}]^{1/\xi}}\right)} e^{i2\pi\sqrt{\left(1-\frac{1}{1+[\sum_{b=1}^{n}\frac{1}{n}(\frac{(\frac{\varkappa_{kq}}{2\pi})^2}{1-(\frac{\varkappa_{kq}}{2\pi})^2})^{\xi}]^{1/\xi}}\right)}},\right.$$

$$\left(1 - \frac{1}{1+[\sum_{b=1}^{n}\frac{1}{n}(\frac{1-\tau_{kq}}{\tau_{kq}})^{\xi}]^{1/\xi}}\right)e^{i2\pi\left(1-\frac{1}{1+[\sum_{b=1}^{n}\frac{1}{n}(\frac{1-\frac{\varkappa_{kq}}{2\pi}}{\frac{\varkappa_{kq}}{2\pi}})^{\xi}]^{1/\xi}}\right)},$$

$$\left.\sqrt{\left(1 - \frac{1}{1 + [\sum_{b=1}^{n} \frac{1}{n}(\frac{\Gamma_{kq}^2}{1-\Gamma_{kq}^2})^{\xi}]^{1/\xi}}\right)} e^{i2\pi\sqrt{\left(1-\frac{1}{1+[\sum_{b=1}^{n}\frac{1}{n}(\frac{(\frac{\delta_{kq}}{2\pi})^2}{1-(\frac{\delta_{kq}}{2\pi})^2})^{\xi}]^{1/\xi}}\right)}}\right)$$

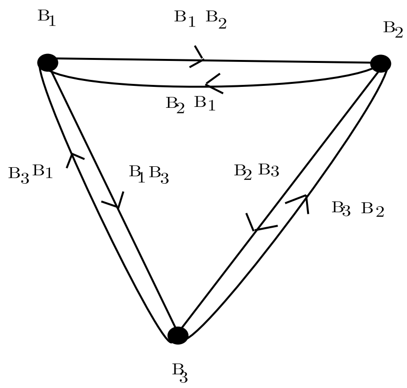

FIGURE 6. CSDFG directed network.

money laundering, terrorist financing, and similar threats to the global financial system. Over time, its mandate has grown to include a wider range of duties. All nations should adhere to the instructions made by the FATF in order to play their respective roles in effectively addressing the threat to financial institutions. These suggestions seek to increase the financial system's transparency and give the nation a framework for looking into and taking action against criminal behaviours. The FATF, along with other international partners, keeps track of how well each nation is doing in fending off its own dangers to the financial system and the black money that might be used to finance terrorism or other related crimes. The FATF is a decision-making body, and its Session meets three times a year to assess the development of the various states' judicial, administrative, and operational measures to combat money laundering and terrorism funding. To determine whether members are adhering to FATF regulations or not, it relies on self-evaluations and recurring mutual evaluation reports from experts.

The Financial Action Task Force (FATF) provided some funds to improve and maintain the situation of country to remove them from the grey list. In this way a country can take advantage to stabilize economically and defensively. The following are some of the criteria that were taken into consideration for this purpose.

- Control of terror financing activities.
- To reduce corruption of various departments.
- Available resources.
- Money laundering.
- Improving financial sector operations.
- Building up social safety nets.
- Strengthening public financial management.
- Minimum level of international reserves.
- Budget consistent with fiscal framework.
- Restriction on government previous policies and enhancing new policies.

$$B_1 = (0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)})$$
$$B_2 = (0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)})$$
$$B_3 = (0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)})$$

$$B_1B_2 = (0.6e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.2)}, 0.1e^{i2\pi(0.1)})$$
$$B_2B_1 = (0.3e^{i2\pi(0.2)}, 0.6e^{i2\pi(0.4)}, 0.1e^{i2\pi(0.1)})$$
$$B_1B_3 = (0.7e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.3)}, 0.2e^{i2\pi(0.1)})$$
$$B_3B_1 = (0.4e^{i2\pi(0.3)}, 0.7e^{i2\pi(0.5)}, 0.2e^{i2\pi(0.1)})$$
$$B_2B_3 = (0.4e^{i2\pi(0.4)}, 0.5e^{i2\pi(0.3)}, 0.1e^{i2\pi(0.2)})$$
$$B_3B_2 = (0.5e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.4)}, 0.1e^{i2\pi(0.2)})$$

The team specialists present their preference data in the form of CSFPR $Q = (d_{kq})_{3\times 3}$, where $d_{kq} = (\omega_{kq}e^{i\varkappa_{kq}}, \tau_{kq}e^{i\varsigma_{kq}}, \Gamma_{kq}e^{i\delta_{kq}})$ is a complex spherical fuzzy element (CSFE) assigned by the expert. Members of the team should choose three countries in which they are planning to release fund and removing from the grey list. The team make pairwise comparison in the three countries to select the suitable one.

Consider $(0.6e^{i2\pi(0.55)}, 0.3e^{i2\pi(0.4)}, 0.1e^{i2\pi(0.05)})$, the amplitude term for the grade 0.6 shows that sixty percent of the expert says B_1 is preferable country over B_2 to be put out from the grey list, 0.3 shows thirty percents are not in favor of the country B_1 and 0.1 shows 10 percents are neutral. Now the phase term 0.55 shows that fifty five percent of the experts says that the country B_1 will stand-by all instructions in future over the country B_2, 0.4 shows that forty percents are against B_1 and 0.05 shows 5 percents are neutral.

The directed network of CSFPR Q given in 2 and is displayed in figure 6.

In the first scenario, we proceed as follows. Let $B = \{J_1 = Afghanistan, B_2 = Syria, B_3 = Africa\}$ be the set of countries where the team wishes to conclude which country will be most suitable to exist from grey list and to deliver funds. Let 65 percent of the team's specialists feel that Afghanistan should be chosen to be out of grey list, 20 percent of the specialists are against Afghanistan and 5 percent are neutral after carefully analyzing the parameters. Therefore, we can determine the terms of all membership, non membership and neutral functions. It is necessary to compute the phase term, which defines which country will comply all instructions in future and improve their situation. Let 40 percent of the specialists are in favour of Afghanistan, 15 percent are opposite and 10 percent are neutral. The CSFPR $Q = (d_{kq})_{3\times 3}$ is shown in Table 2 and displed in Figure 6.

By using complex dombi fuzzy operator, we calculate $d_{kq} = (\omega_{kq}e^{i\varkappa_{kq}}, \tau_{kq}e^{i\varsigma_{kq}}, \Gamma_{kq}e^{i\delta_{kq}})$ (k,b = 1,2,3) of the countries B_k over others. We have taken $\xi = 1$. The combined overall preference value d_k (k=1,2,3) which is given below:

$$d_1 = (0.6183e^{i2\pi(0.4714)}, 0.6170e^{i2\pi(0.7097)}, 0.3373e^{i2\pi(0.3247)})$$
$$d_2 = (0.4146e^{i2\pi(0.3982)}, 0.4706e^{i2\pi(0.6170)}, 0.3247e^{i2\pi(0.3373)})$$
$$d_3 = (0.4714e^{i2\pi(0.3878)}, 0.4940e^{i2\pi(0.5385)}, 0.3373e^{i2\pi(0.3373)})$$

TABLE 1. CSFPR.

Q	B_1	B_2	B_3
B_1	$(0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)})$	$(0.6e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.2)}, 0.1e^{i2\pi(0.1)})$	$(0.7e^{i2\pi(0.5)}, 0.4e^{i2\pi(0.3)}), 0.2e^{i2\pi(0.1)})$
B_2	$(0.3e^{i2\pi(0.2)}, 0.6e^{i2\pi(0.4)}, 0.1e^{i2\pi(0.1)})$	$(0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)})$	$(0.4e^{i2\pi(0.4)}, 0.5e^{i2\pi(0.3)}, 0.1e^{i2\pi(0.2)})$
B_3	$(0.4e^{i2\pi(0.3)}, 0.7e^{i2\pi(0.5)}, 0.2e^{i2\pi(0.1)})$	$(0.5e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.4)}, 0.1e^{i2\pi(0.2)})$	$(0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)}, 0.5e^{i2\pi(0.5)})$

TABLE 2. SFPR of the expert.

Q	A_1	A_2	A_3
A_1	(0.5, 0.5, 0.5)	(0.6, 0.3, 0.1)	(0.7, 0.4, 0.2)
A_2	(0.3, 0.6, 0.1)	(0.5, 0.5, 0.5)	(0.4, 0.5, 0.1)
A_3	(0.4, 0.7, 0.2)	(0.5, 0.4, 0.1)	(0.5, 0.5, 0.5)

TABLE 3. Comparison among proposed and existing operator.

Operators	$Ranking of alternatives$
$CSDF - operator$	$A_1 \succ A_3 \succ A_2$
$SDF - operator$	$A_3 \succ A_1 \succ A_2$

The score function $s(d_k)$ (k=1,2,3) is calculated by using

$$s(d_k) = (\omega^2 - \tau^2 - \Gamma^2) + \frac{1}{4\pi^2}(\varkappa^2 - \varsigma^2 - \delta^2)$$

which is given below:

$$s(d_1) = -0.4991$$
$$s(d_2) = -0.4909$$
$$s(d_3) = -0.3889$$

We determine the three countries rankings using the scoring functions B_k as:

$$B_3 \succ B_2 \succ B_1.$$

The results of the ranking indicate that B_3, Africa is the most suitable country to release funds and put out from the grey list of FATF.

C. COMPARATIVE ANALYSIS

In this section, we write a numerical example to compare withb the outcomes of our newly introduced operators with those of existing operator, specifically the spherical dombi fuzzy operator. The context involves a scenario where a firm is in need of acquiring a machine for their office. To make an informed decision, they consider machines offered by three different companies denoted as A_k $(k = 1, 2, 3)$. The selection process revolves around four essential parameters:

1. Rate of Injection:

This parameter quantifies how fast the machine can perform the injection process.

2. Back Pressure:

This refers to the level of resistance that the machine exerts when executing its functions.

3. Mould and Material Temperature:

These parameters indicate the temperature conditions at which the machine operates, which can significantly affect performance.

4. Brand Tag:

This parameter evaluates the reputation and branding of the machine manufacturer, which is often a crucial factor in decision-making.

An expert of the office provides his preference information in the form of complex spherical fuzzy preference relation (CSFPR) $Q = (d_{kq})_{3\times3}$, where $d_{kq} = (\omega_{kq}e^{i\sigma_{kq}}, \tau_{kq}e^{i\varsigma_{kq}}, \Gamma_{kq}e^{i\delta_{kq}})$ is the complex spherical fuzzy element (CSFE) assigned by the expert. The amplitude term for membership and non-membership grades represents the preference and non-preference values, respectively, for the speed of the machine.

Now the phase term for membership and non-membership grades represents the preference and non-preference values, respectively, for the accuracy of PCs. CSFPR $Q = (d_{kq})_{3\times3}$ is represented in Table 1. If we consider only the amplitude terms, then the data in the form of PFPR are given in Table 2. From the ranking results given in Table 3, it is easy to see that the best alternatives using the proposed operators are A_1 and A_4; whereas by using the existing operators, the best alternative is A_4. Thus, the best alternatives are A_1 and A_4. While computing the results using the complex spherical dombi fuzzy graph operator, we consider both the amplitude and phase terms. The results using the presented operators provide complete information and avoid any loss of information. On the other hand, in calculating the results using the spherical dombi fuzzy operator, we consider only the amplitude term. Thus, to compute the results of information having two terms, namely amplitude and phase term, complex spherical dombi fuzzy operators are able to draw conclusions. The operators that we have applied can handle two-dimensional phenomena. Therefore, to tackle imprecise or unclear information, our proposed method will provide a platform to handle such information.

V. CONCLUSION

Graphs are useful for graphically displaying information and modelling relationships between various items. They are extensively employed in a range of biological, social, and physical systems, as well as in industrial and communication network diagnostics. Their importance is derived from their capacity to digest data from many sources and deliver critical insights for decision-making. This study proposes the notion of CSDFG, which is an extension of SDFG that provides greater flexibility and comparability. It establishes CSDFG's complement and investigates notions like as homomorphism, isomorphism, W-Isomorphism, and CW-Isomorphism, yielding a variety of findings. CSDFGs that are both regular and completely regular are also examined. In addition, the application of CSDFG is illustrated. Future work might include specifying operations on CSDFGs and investigating CSDFG energy.

USE OF AI TOOLS DECLARATION

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

CONFLICT OF INTEREST

The authors state that there is no conflict of interest.

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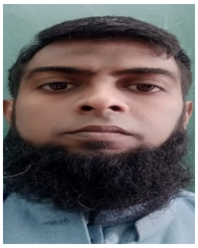

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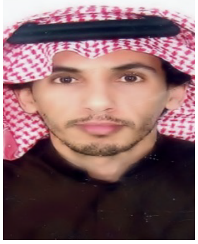

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