IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 6 October 2023, accepted 22 October 2023, date of publication 6 November 2023, date of current version 9 November 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3330145

RESEARCH ARTICLE

Non-Linear Prediction-Based Trajectory Tracking for Non-Holonomic Mobile Robots

JULIO A. BÁEZ-HERNÁNDEZ [1], **MARTÍN VELASCO-VILLA** [1], **AND SABINE MONDIÉ** [2]

[1]Electrical Engineering Department, Center for Research and Advanced Studies of the National Polytechnic Institute, Mexico City 07360, Mexico

[2]Automatic Control Department, Center for Research and Advanced Studies of the National Polytechnic Institute, Mexico City 07360, Mexico

Corresponding author: Martín Velasco-Villa (velasco@cinvestav.mx)

ABSTRACT This work addresses the trajectory tracking problem for a non-holonomic differential drive mobile robot with a constant time delay h at the input signal. To compensate for the adverse effects of the input time delay on the vehicle, a non-linear prediction-observer scheme based on a sub-prediction strategy that asymptotically estimates the future values of the state, h units of time ahead was introduced, and, thanks to the characteristics of the system, a condition which depends only on the gains of the predictor-observer is obtained for the convergence of the predicted states. Non-linear feedback based on the estimated future state is proposed to tackle the trajectory tracking problem of a mobile robot. The closed-loop system describing the prediction strategy and trajectory tracking solution was formally analyzed, showing the asymptotic convergence of the prediction and tracking errors to the origin. Numerical and real-time experiments were performed to evaluate the prediction-based control scheme, which show adequate performance.

INDEX TERMS Non-holonomic mobile robot, non-linear prediction, time delays, trajectory tracking.

I. INTRODUCTION

Since the eighteenth century, delays have been essentials in studying dynamical systems because of the issues they raise. The study of input time delays for linear or non-linear systems is an important area of interest due to the influence that time delays have on the stability of the close-loop system when problems as regulation or trajectory tracking are faced, since the closed loop system results on a retarded differential equation. When the system, is not destabilized by the effects of the time delay, it suffers for a large degradation of the performance of the closed loop system [1], [2].

In particular, input time delays, which are often present in networked or teleoperated control systems, are well known to undermine the execution of feedback control, and significant problems are observed in data transmission through a network, such as variable sampling intervals or communication constraints of protocol scheduling, among others. These issues can be assessed using a time delay approach in networked control system [3].

The associate editor coordinating the review of this manuscript and approving it for publication was Mohammad AlShabi.

The complexity of the time delays has been addressed in several studies. For example, in a linear context, the celebrated Smith predictor [4] proposes a solution for open-loop linear stable systems that presents a strong restriction on the class of systems that can be applied. However, some modifications to Smith's work have overcome limitations such as large time delays and unstable systems [5], [6], [7].

Other approaches to control input delay-systems have also been explored. For example, [8] introduced the idea of finite spectrum assignment for input time delay systems, and further works improved this result by considering other types of delays and analyzing robustness [8], [9], [10], [11]. Meanwhile, [12] and [13] proposed a state-observer approach to estimate the future states of a system using the ideas of [14]. Finally, to address larger delays, an attractive solution inspired by the state observer approach is the introduction of a sub-predictor or chained-observer scheme that divides the time delay [15], [16].

Non-linear time delay systems present a new and generally difficult challenge. Some authors have proposed approaches based on non-continuous controllers or high-gain predictors with feedback controllers [17], [18]. In addition, a truncated predictor was presented in [19] to estimate a system's future state under the Lipschitz condition assumption. Also, the original linear sub-predictor approach was extended to a non-linear approach; see, for instance, [20], [21]. In [22], a prediction scheme that estimates the values of unknown delayed states from a cascade system was proposed.

Several studies on mobile robotics have been reported the literature. One of the most important approaches is related to prediction-based controls; an interesting survey was presented in [23]. For instance, considering delayed measurements, [24], [25] provides a solution for the kinematic model, and [26] uses a simplified dynamic model. In addition, using prediction-based solutions, [27] and [28] proposed a solution to the trajectory-tracking problem by incorporating fuzzy techniques with a parallel distributed strategy. The Smith predictor, together with sliding mode control, was considered in [29] as a strategy to compensate for a time delay on a differential-drive mobile robot, while in [30], an integral-based prediction was considered for the same purpose. A comparison of several control techniques for trajectory tracking was presented in [31] for a modified kinematic model of a mobile robot.

The dynamic model of a mobile robot with input or output time delays has also been studied. The tracking problem for small time-varying input delays was addressed in [32], whereas the consensus problem for time-varying delays was considered in [33] and [34]. A discrete-time approach has also been reviewed for the control of input-delay mobile robots, see for instance [35] for the tracking problem and [36] for a synchronization strategy under input time delays.

The time delay problem in teleoperated mobile robots has been a popular case study because of its various applications, such as exploration, rescue, and surveillance. Consequently, several authors have proposed GPI observers and state prediction schemes for mobile robots [24], [37], [38].

The present work is devoted to trajectory tracking problem using a non-holonomic mobile robot subject to an input time delay. To overcome the adverse delay effects on performance and stability of the closed-loop system, a generalization of the linear sequential predictor presented in [16] to the non-linear case was proposed, providing an alternative to the sub-predictor in [15]. The obtained future estimated state was used to design a non-linear feedback law that solves the aforementioned tracking problem. The main results of the work can be summarized as follows. a) It is presented a sub-predictor observer that asymptotically converge to the future value of the state, at time $t + h$ independent of the trajectory performed by the robot, subject to bounded linear and angular velocities; b) Based on a Lyapunov-Krasovskii approach, it is presented a sufficient condition for the convergence of the prediction state that depends on the design parameters of the observer stages and the involved original time delay h; c) The convergence of the trajectory tracking errors, by means of a feedback based on the estimated future states is formally proof for the closed loop system showing that theoretically, the proposed solution can handle any time delay by increasing the number of sub-predictors.

The present strategy is related to the problem considered in [27] or more recently in [28] where a parallel distributed compensation PDC is applied to a fuzzy model of a kinematic mobile robot. These two proposals have the disadvantage of local predictor convergence and the characteristic that stability results are based on linear matrix inequalities. The solution proposed in [25] considers as a disadvantage, a persistent excitation condition for the prediction strategy and an upper bound for the maximum value of the input time delay. In contrast to the restrictions founded in [25], [27], and [28], in this work it is provided a predictor-observer that globally asymptotically converges to the actual future state independently of the evolution of the robot where the limitation on the maximum time delay can be overcome by considering additional non-linear sub-predictors. Furthermore, it should be pointed out that prediction strategies based on the integral predictor of [39] or [40] produce approximate future values due to implementation problems [41], which motivates the use of approximate solutions of truncated predictors [42].

The contributions of this study are as follows: Section II develops the kinematic model of the delayed non-holonomic robot used in this study. Section III presents the non-linear sub-prediction scheme together with the corresponding prediction error stability analysis. Section IV introduces the prediction-based feedback control solution to the trajectory tracking problem and stability analysis of the closed-loop system. Finally, real-time experiments and numerical simulation results are presented in Section V, and concluding remarks are provided in Section VI.

II. KINEMATIC MODEL OF A (2,0) TYPE MOBILE ROBOT

Following [43] and [44], the kinematic model of a differential drive mobile robot shown in Figure 1 is represented as,

$$\begin{aligned}\dot{x}(t) &= v(t)\cos(\theta(t))\\ \dot{y}(t) &= v(t)\sin(\theta(t))\\ \dot{\theta}(t) &= \omega(t)\end{aligned} \tag{1}$$

where $\xi(t) = [x(t), y(t), \theta(t)]^T$ denotes the state of the robot. Point $P = (x, y)$ in Figure 1 represents the Cartesian position of the wheel axis center, $\theta(t)$ is the orientation of the vehicle measured from the X axis, and $u(t) = [v(t), \omega(t)]^T$ are the control inputs corresponding to the linear and angular velocities, respectively.

Kinematic model (1) is obtained based on the assumption that the robot has a rigid body that moves on a flat surface and that the vertical axis of the wheels is perpendicular to the ground. Under these conditions, the following non-holonomic constraint is satisfied [45], [46],

$$\dot{x}(t)\sin(\theta(t)) - \dot{y}(t)\cos(\theta(t)) = 0. \tag{2}$$

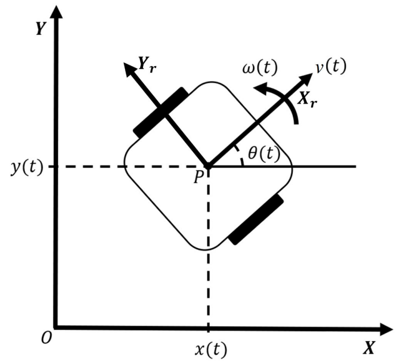


FIGURE 1. Differential drive mobile robot.

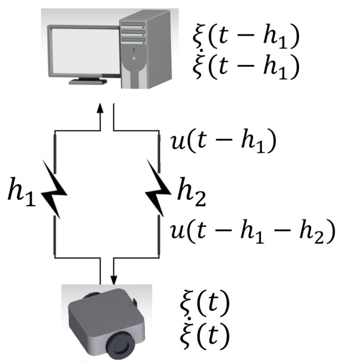


FIGURE 2. Mobile robot teleoperation scheme.

A. TIME DELAY IN THE CONTROL INPUTS

As mentioned previously, one of the problems associated with the teleoperation of robots is the time delay that can be produced by the environment or heavy data processing. A scheme for a remote-controlled mobile robot is presented in Figure 2. Here, $h_1 > 0$ is the time it takes the data to move from the robot sensors to the computer, whereas $h_2 > 0$ is the time the control input $u(t)$ is computed and injected into the robot. It should be noted that h_1 can be considered as a time delay at the output of the robot, whereas delay h_2 acts at the input of the vehicle; therefore, for the design purpose of a feedback law, a total time delay $h = h_1 + h_2$ in the input-output path is assumed.

Thus, according to Figure 2, the system takes the input delay form,

$$\begin{aligned}\dot{x}(t) &= v(t-h)\cos(\theta(t))\\ \dot{y}(t) &= v(t-h)\sin(\theta(t))\\ \dot{\theta}(t) &= \omega(t-h).\end{aligned} \tag{3}$$

III. PREDICTION SCHEME

To design a prediction strategy to solve the trajectory tracking problem, instead of considering kinematic model (3), an alternative representation $\xi_r(t) = [x_r(t)\ y_r(t)\ \theta_r(t)]$ is obtained using the following globally invertible transformation,

$$\begin{bmatrix} x_r(t)\\ y_r(t)\\ \theta_r(t)\end{bmatrix} = \begin{bmatrix} \cos(\theta(t)) & \sin(\theta(t)) & 0\\ -\sin(\theta(t)) & \cos(\theta(t)) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix} x(t)\\ y(t)\\ \theta(t)\end{bmatrix} \tag{4}$$

that produces the body frame representation along the $X_r - Y_r$ frame,

$$\begin{aligned}\dot{x}_r(t) &= v(t-h) + \omega(t-h)y_r(t)\\ \dot{y}_r(t) &= -\omega(t-h)x_r(t)\\ \dot{\theta}_r(t) &= \omega(t-h).\end{aligned} \tag{5}$$

A. ADVANCED SYSTEM

The design of a feedback law that solves problems such as regulation or trajectory tracking requires the estimation of the future values of the states $\xi_r(t)$ of the mobile robot (5) to compensate for the time delay.

To obtain the future values, h units of time in the future, of the state $\xi_r(t)$ of system (5), a non-linear prediction strategy based on well-known linear sub-predictors [15], [16] is developed.

Assumption 1: For a fixed time delay h in system (5), there exists an integer $n \in N$ such that,

$$\bar{h} = \frac{h}{n}, \tag{6}$$

where $\bar{h}$ is a small enough time delay.

With condition (6), future values of $\xi_r(t)$ may be defined as,

$$\begin{aligned}w_{j1}(t) &= x_r(t+j\bar{h}) = w_{(j-1)1}(t+\bar{h})\\ w_{j2}(t) &= y_r(t+j\bar{h}) = w_{(j-1)2}(t+\bar{h})\\ w_{j3}(t) &= \theta_r(t+j\bar{h}) = w_{(j-1)3}(t+\bar{h})\end{aligned} \tag{7}$$

for $j = 1, 2, \ldots, n$.

From (7), the advanced systems are obtained for $j = 1, 2, \ldots, n$, as,

$$\begin{aligned}\dot{w}_{j1}(t) &= v(t-(n-j)\bar{h}) + \omega(t-(n-j)\bar{h})w_{j2}(t)\\ \dot{w}_{j2}(t) &= -\omega(t-(n-j)\bar{h})w_{j1}(t)\\ \dot{w}_{j3}(t) &= \omega(t-(n-j)\bar{h}).\end{aligned} \tag{8}$$

When $j = n$, a delay-free system that evolves h seconds ahead is obtained

$$\begin{aligned}\dot{w}_{n1}(t) &= v(t) + \omega(t)w_{n2}(t)\\ \dot{w}_{n2}(t) &= -\omega(t)w_{n1}(t)\\ \dot{w}_{n3}(t) &= \omega(t).\end{aligned} \tag{9}$$

B. SUB-PREDICTORS CHAIN

A Luenberger-type predictor-observer, based on advanced dynamics (8)-(9), is introduced below. The observer for system (9) corresponds to the predictor of system (5). For $j = 1, 2, \ldots, n$, it takes the form

$$\begin{aligned}\dot{\hat{w}}_{j1}(t) = {}& v(t-(n-j)\bar{h}) + \omega(t-(n-j)\bar{h})\hat{w}_{j2}(t)\\ &+ \lambda_{j1}e_{w_{j1}}(t-\bar{h})\end{aligned}$$

$$\dot{\hat{w}}_{j2}(t) = -\omega(t-(n-j)\bar{h})\hat{w}_{j1}(t) + \lambda_{j2}e_{w_{j2}}(t-\bar{h})$$
$$\dot{\hat{w}}_{j3}(t) = \omega(t-(n-j)\bar{h}) + \lambda_{j3}e_{w_{j3}}(t-\bar{h}) \quad (10)$$

where $\lambda_{j1}, \lambda_{j2}, \lambda_{j3} \in \mathbb{R}^{+}\setminus 0$ and the injection errors $e_{w_{ji}}$ are defined for $j=1$ in the form,

$$\begin{aligned} e_{w_{11}}(t) &= x_r(t+\bar{h}) - \hat{w}_{11}(t) = w_{11}(t) - \hat{w}_{11}(t) \\ e_{w_{12}}(t) &= y_r(t+\bar{h}) - \hat{w}_{12}(t) = w_{12}(t) - \hat{w}_{12}(t) \\ e_{w_{13}}(t) &= \theta_r(t+\bar{h}) - \hat{w}_{13}(t) = w_{13}(t) - \hat{w}_{13}(t) \end{aligned} \quad (11)$$

and for $j = 2, \ldots, n$ as,

$$\begin{aligned} e_{w_{j1}}(t) &= \hat{w}_{(j-1)1}(t+\bar{h}) - \hat{w}_{j1}(t) \\ e_{w_{j2}}(t) &= \hat{w}_{(j-1)2}(t+\bar{h}) - \hat{w}_{j2}(t) \\ e_{w_{j3}}(t) &= \hat{w}_{(j-1)3}(t+\bar{h}) - \hat{w}_{j3}(t). \end{aligned} \quad (12)$$

Remark 1: Note that the injection errors for $j=1$ defined in (11) have the property that $e_{w_{1i}}(t-h)$ is defined in time t, allowing the injection of these signals to the predictor-observer, referring to the actual position of the vehicle.

C. PREDICTION ERRORS

Unlike the injection errors, the prediction errors $\tilde{w}_j(t) = [\tilde{w}_{j1}(t)\ \tilde{w}_{j2}(t)\ \tilde{w}_{j3}(t)]^T$ for $j = 1, 2, \ldots, n$ are defined as,

$$\tilde{w}_j(t) = w_j(t) - \hat{w}_j(t). \quad (13)$$

Lemma 1: The prediction errors $\tilde{w}_j$ converge to zero if and only if the injection errors e_{w_k} converge to zero.

Proof:Given the definition of the injection errors (11)-(12),

$$\hat{w}_1(t) = \xi(t+\bar{h}) - e_{w_1}(t)$$
$$\hat{w}_2(t) = \hat{w}_1(t+\bar{h}) - e_{w_2}(t) = \xi(t+2\bar{h}) - e_{w_1}(t+\bar{h}) - e_{w_2}(t)$$

yielding,

$$\begin{aligned} \hat{w}_j(t) &= \xi(t+j\bar{h}) - \sum_{k=1}^{j} e_{w_k}(t+(j-k)\bar{h}) \\ &= w_j(t) - \sum_{k=1}^{j} e_{w_k}(t+(j-k)\bar{h}). \end{aligned} \quad (14)$$

Then, the prediction errors can be rewritten as,

$$\begin{aligned} \tilde{w}_j(t) &= w_j(t) + e_{w_j}(t) - \hat{w}_{j-1}(t+\bar{h}) \\ &= w_j(t) + e_{w_j}(t) + e_{w_{j-1}}(t+\bar{h}) - \hat{w}_{j-2}(t+2\bar{h}) \end{aligned}$$

this is,

$$\tilde{w}_j(t) = \sum_{k=1}^{j} e_{w_k}(t+(j-k)\bar{h}). \quad (15)$$

Because the prediction error $\tilde{w}_j(t)$ is the sum of the injection errors $e_{w_k}(t)$, the convergence of $e_{w_k}(t)$ implies convergence of the prediction error. ■

D. INJECTION ERRORS CONVERGENCE

As mentioned in Subsection III-C, the convergence of the injection errors $e_{w_{ji}}$, is directly related to the stability and efficiency of the prediction errors $\tilde{w}_n$. The time derivative of the injection errors is given by

$$\begin{aligned} \dot{e}_{w_{j1}}(t) &= \omega(t-(n-j)\bar{h})e_{w_{j2}}(t) + \lambda_{(j-1)1}e_{w_{(j-1)1}}(t) \\ &\quad - \lambda_{j1}e_{w_{j1}}(t-\bar{h}) \\ \dot{e}_{w_{j2}}(t) &= -\omega(t-(n-j)\bar{h})e_{w_{j1}}(t) + \lambda_{(j-1)2}e_{w_{(j-1)2}}(t) \\ &\quad - \lambda_{j2}e_{w_{j2}}(t-\bar{h}) \\ \dot{e}_{w_{j3}}(t) &= \lambda_{(j-1)3}e_{w_{(j-1)3}}(t) - \lambda_{j3}e_{w_{j3}}(t-\bar{h}) \end{aligned} \quad (16)$$

where, for $j=1$, it is considered $\lambda_{01} = \lambda_{02} = \lambda_{03} = 0$.

The definition $\bar{e}_{wj}(t)$,

$$\bar{e}_{wj}(t) = \begin{bmatrix} e_{wj1}(t) \\ e_{wj2}(t) \end{bmatrix}$$

allows rewriting system (16) as,

$$\dot{\bar{e}}_{wj}(t) = A_j(t)\bar{e}_{w_j}(t) + \Lambda_j\bar{e}_{w_j}(t-\bar{h}) - \Lambda_{j-1}\bar{e}_{w_{j-1}}(t) \quad (17a)$$
$$\dot{e}_{wj3}(t) = -\lambda_{j3}e_{w_{j3}}(t-\bar{h}) + \lambda_{(j-1)3}e_{w_{(j-1)3}}(t). \quad (17b)$$

where,

$$A_j(t) = \begin{bmatrix} 0 & \omega(t-(n-j)\bar{h} \\ -\omega(t-(n-j)\bar{h}) & 0 \end{bmatrix}$$
$$\Lambda_j = \begin{bmatrix} -\lambda_{j1} & 0 \\ 0 & -\lambda_{j2} \end{bmatrix}.$$

The convergence of the injection errors (17) is proven in two steps. First, the convergence of subsystem (17a) is addressed, followed by the convergence of subsystem (17b).

Before providing the stability conditions of subsystem (17a), we introduce the following auxiliary result.

Lemma 2: For subsystem (17a), given the positive real numbers α and η, there exists a symmetric matrix $P_j(t)$ such that,

$$\dot{P}_j(t) + A_j^T P_j(t) + P_j(t)A_j + 2\alpha_j P_j(t) + \eta_j I = 0. \quad (18)$$

Proof: Defining the symmetric matrix $P_j(t)$ as,

$$P_j(t) = \begin{bmatrix} p_{j1}(t) & p_{j2}(t) \\ p_{j2}(t) & p_{j3}(t) \end{bmatrix}$$

equation (18) rewrites as,

$$\dot{p}_{j1}(t) = A_j^*(t)p_{j1}(t) + \bar{\eta}_j \quad (19)$$

where $\omega_j^*(t) = \omega(t-(n-j)\bar{h})$ and,

$$p_{j1}(t) = \begin{bmatrix} p_{j1}(t) \\ p_{j2}(t) \\ p_{j3}(t) \end{bmatrix} \quad \bar{\eta}_j = -\begin{bmatrix} \eta_j \\ 0 \\ \eta_j \end{bmatrix}$$
$$A_j^*(t) = \begin{bmatrix} -2\alpha_j & 2\omega_j^*(t) & 0 \\ -\omega_j^*(t) & -2\alpha_j & \omega_j^*(t) \\ 0 & -2\omega_j^*(t) & -2\alpha_j \end{bmatrix}.$$

Using the transformation matrix

$$T_j = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \tag{20}$$

to get the real Jordan form $J(t) = T_j A_j^*(t) T_j^{-1}$ *of* $A^*(t)$, *the change of coordinates*

$$z_j(t) = T_j p_{j1}(t), \tag{21}$$

leads to the system,

$$\begin{bmatrix} \dot{z}_{j1}(t) \\ \dot{z}_{j2}(t) \\ \dot{z}_{j3}(t) \end{bmatrix} = \begin{bmatrix} -2\alpha_j & 0 & 0 \\ 0 & -2\alpha_j & 2\omega_j^*(t) \\ 0 & -2\omega_j^*(t) & -2\alpha_j \end{bmatrix} \begin{bmatrix} z_{j1}(t) \\ z_{j2}(t) \\ z_{j3}(t) \end{bmatrix} + \begin{bmatrix} -\eta_j \\ 0 \\ 0 \end{bmatrix}. \tag{22}$$

with solution,

$$\begin{aligned} z_{j1}(t) &= z_{j1}(0)e^{-2\alpha_j t} - \frac{\eta_j}{2\alpha_j}(1 - e^{-2\alpha_j t}) \\ z_{j2}(t) &= z_{j2}(0)e^{-2\alpha_j t} \cos(2\omega_j^*(t)) \\ &\quad + z_{j3}(0)e^{-2\alpha_j t} \sin(2\omega_j^*(t)) \\ z_{j3}(t) &= -z_{j2}(0)e^{-2\alpha_j t} \sin(2\omega_j^*(t)) \\ &\quad + z_{j3}(0)e^{-2\alpha_j t} \cos(2\omega_j^*(t)). \end{aligned}$$

The inverse transformation $p_{j1}(t) = T_j^{-1} z_j(t)$ *is*

$$P_j(t) = \begin{bmatrix} z_{j1} + z_{j2} & z_{j3} \\ z_{j3} & z_{j1} - z_{j2} \end{bmatrix}.$$

■

It is important to note that the error dynamics in (17) depend only on input $\omega_j^*(t)$. As a consequence, we present the following theorem inspired by [47]:

Theorem 1: A system of the form,

$$\dot{\bar{e}}_{w_j}(t) = A_j(t)\bar{e}_{w_j}(t) + \Lambda_j \bar{e}_{w_j}(t - \bar{h}) \tag{23a}$$

$$\dot{e}_{w_{j3}}(t) = -\lambda_{j3} e_{w_{j3}}(t - \bar{h}), \quad t \geq 0 \tag{23b}$$

where $\bar{h} > 0$, $\lambda_{j3} > 0$ *and* $\lambda_j > 0$, *is globally asymptotically stable if there exist positive real numbers* $\alpha, \beta, \epsilon, \eta_j$, *and a symmetric matrix* $P_j(t)$ *that satisfy the Lyapunov equation (18) and if the following inequalities hold*

$$\bar{h} < min\left\{ \frac{1}{\alpha} \ln\left(\frac{\sqrt{\epsilon(\eta_j - \epsilon)}}{\lambda_j \bar{p}} \right), \frac{\pi}{2\lambda_{j3}} \right\} \tag{24}$$

where, $\bar{p} = \sup_{t > t_0} ||P(t) + \beta I||$ *and* $2\alpha\beta > \eta_j > \epsilon$.

Proof: To prove the stability of subsystem (23a), the following Lyapunov-Krasovskii functional candidate is proposed,

$$V(t, \bar{e}_{w_j t}) = V_1(\bar{e}_{w_j}(t)) + V_2(t, \bar{e}_{w_j t}) \tag{25}$$

with,

$$V_1(\bar{e}_{w_j}(t)) = \bar{e}_{w_j}^T(t) P_j(t) \bar{e}_{w_j}(t) + \beta \bar{e}_{w_j}^T(t) \bar{e}_{w_j}(t) \tag{26}$$

$$V_2(t, \bar{e}_{w_j t}) = \epsilon \int_{t-\bar{h}}^{t} e^{2\alpha(s-t)} \bar{e}_{w_j}^T(s) \bar{e}_{w_j}(s) ds. \tag{27}$$

Differentiating the functional (26) gives,

$$\begin{aligned} \dot{V}_1(\bar{e}_{w_j}(t)) = & \ \bar{e}_{w_j}^T(t)[A_j^T(t)P_\beta(t) + P_\beta(t)A_j(t) + \dot{P}_j]\bar{e}_{w_j}(t) \\ & + 2\bar{e}_{w_j}^T(t - \bar{h})\Lambda_j^T P_\beta(t)\bar{e}_{w_j}(t) \end{aligned}$$

where

$$P_\beta = [P_j(t) + \beta I].$$

Note that if $\eta_j < 2\alpha\beta$ *then* $P_\beta > 0$.

Defining $\bar{p}$ *and* λ_j *as* $\bar{p} = \sup_{t > t_0} ||P_\beta(t)||$ *and* $\lambda_j = \lambda_{max}\{\lambda_{j1}, \lambda_{j2}\} = sup||\Lambda_j||$, *the next inequality is obtained,*

$$\begin{aligned} \dot{V}_1(\bar{e}_{w_j}(t)) \leq & \ \bar{e}_{w_j}^T(t)[A_j^T(t)P_\beta(t) + P_\beta(t)A_j(t) + \dot{P}_j]\bar{e}_{w_j}(t) \\ & + 2\bar{p}\lambda_j||\bar{e}_{w_j}(t - \bar{h})|| \, ||\bar{e}_{w_j}(t)||. \end{aligned}$$

The time derivative of (27), yields,

$$\dot{V}_2(t, \bar{e}_{w_j t}) \leq -2\alpha V_2 + \epsilon||\bar{e}_{w_j}(t)||^2 - \epsilon e^{-2\alpha\bar{h}}||\bar{e}_{w_j}(t - \bar{h})||^2.$$

Given that $A_j^T(t) + A_j(t) = 0_{2\times 2}$, *the next inequality follows*

$$\dot{V}(t, \bar{e}_{w_j t}) \leq -[\eta_j - \epsilon - \frac{\bar{p}^2 \lambda_j^2 e^{2\bar{h}\alpha}}{\epsilon}]||\bar{e}_{w_j}(t)||^2 \tag{28}$$

Hence, if λ_j *is such that,*

$$\lambda_j < \frac{\sqrt{\epsilon(\eta_j - \epsilon)}}{\bar{p}e^{\alpha\bar{h}}}. \tag{29}$$

If $\eta_j > \epsilon$ *then subsystem (23a) is asymptotically stable. Similarly, subsystem (23b) is asymptotically stable if the following delay condition holds [2],*

$$\bar{h} < \frac{\pi}{2\lambda_{j3}}. \tag{30}$$

Equations (29) and (30) indicate that system (23) is asymptotically stable for every fixed time delay which satisfies the following condition,

$$\bar{h} < min\left\{ \frac{1}{\alpha} \ln\left(\frac{\sqrt{\epsilon(\eta_j - \epsilon)}}{\lambda_j \bar{p}} \right), \frac{\pi}{2\lambda_{j3}} \right\}.$$

Furthermore, because origin $e_{w_j} = [0\ 0\ 0]^T$ *is the only equilibrium point, system (23) is globally asymptotically stable.* ■

From Theorem 1, it is possible to provide a more straightforward existence condition for the prediction scheme which depends only on the predictor parameters.

Corollary 1: There always exists positive gains λ_i, $i = 1, 2, 3$, *such that system (23) is globally asymptotically stable if,*

$$\bar{h} < min\left\{ \frac{1}{\lambda_j e}, \frac{\pi}{2\lambda_{j3}} \right\}. \tag{31}$$

Proof: Directly from equation (29) and condition (30).

Lemma 3: Assume that system (16)-(17) for $j = 1, 2, \ldots, n$ satisfies inequality (24). Subsequently, the injection errors $e_{w_j}(t)$ asymptotically converge to the origin.

Proof: To carry out the proof, note that the injection error dynamic (17) for $j = 1$ corresponds to that considered in Theorem 1 for the case $\Lambda_0 = 0$ and $\lambda_{03} = 0$. Therefore, system (17) for $j = 1$, satisfies condition (23) in Theorem 1. Under these conditions, injection error $e_{w_1}(t)$ converges asymptotically to the origin.

To show that $e_{w_2}(t)$ also converges to the origin, note first that system (17) for $j = 2$ takes the form,

$$\begin{aligned}\dot{\bar{e}}_{w_2}(t) &= A_2(t)\bar{e}_{w_2}(t) + \Lambda_2\bar{e}_{w_2}(t-\bar{h}) - \Lambda_1\bar{e}_{w_1}(t)\\ \dot{e}_{w_{23}}(t) &= -\lambda_{23}e_{w_{23}}(t-\bar{h}) + \lambda_{13}e_{w_{13}}(t).\end{aligned} \tag{32}$$

Because $e_{w_1}(t)$ is a vanishing term for system (32), the convergence of $e_{w_2}(t)$ is obtained by considering $e_{w_1}(t) = 0$, obtaining

$$\begin{aligned}\dot{\bar{e}}_{w2}(t) &= \begin{bmatrix} 0 & \omega(t-(n-2)\bar{h} \\ -\omega(t-(n-2)\bar{h}) & 0 \end{bmatrix}\begin{bmatrix} e_{w21}(t) \\ e_{w22}(t)\end{bmatrix}\\ &\quad + \begin{bmatrix} -\lambda_{21} & 0 \\ 0 & -\lambda_{22}\end{bmatrix}\begin{bmatrix} e_{w21}(t-\bar{h}) \\ e_{w22}(t-\bar{h})\end{bmatrix}\\ \dot{e}_{w23}(t) &= -\lambda_{23}e_{w_{23}}(t-\bar{h}).\end{aligned} \tag{33}$$

Finally, Theorem 1 implies the convergence of $e_{w_2}(t)$. These arguments can be repeated until $j = n$. ∎

Remark 2: Note that the prediction-observer scheme is based on the body-frame representation (5) of the kinematic model (1) obtained by employing global transformation (4). This transformation allows global convergence to the future values of the states, independent of the displacements of the mobile robot, in contrast to the results obtained in [25], [27], and [28] where the convergence to the future value depends on the linear velocity of the vehicle.

Remark 3: Note also, that the estimate of the future value is obtained for $\bar{h}$ under condition (31) of Corollary 1, which only involves the design gains of sub-predictors λ_j. Therefore, increasing the number of sub-predictors allows the handling of a larger input time delay for the original system (1). It should be noticed that due to the chain configuration of the observer-predictor (10), or the injection errors (23), the convergence to the future values at time $t+h$ is obtained step by step converging first to the future value at time $t+\bar{h}$ and then at time $t+2\bar{h}$ and so on until time $t+n\bar{h} = t+h$. The convergence conditions (31) are obtained for each step of the prediction chain on Corollary 1 which only involves the design gains of each sub-predictor. Therefore, condition (31) is relaxed by increasing the number n of sub-predictors, this fact is directly related to the size of the original time delay h that can be handled by the prediction strategy to produce the desired future value.

Remark 4: Even when theoretically the predictor-observer can handle any time delay, this fact is not relevant for a practical implementation since small time delays can destabilize the response of a possible prediction-based feedback, and therefore, for small delays a short chain of sub-predictors will be necessary. The number of sub-predictors required to get the future values depends on the conditions given by Corollary 1 that state the minimal numbers of sub-predictors that must be used. Notice also that, when the input time delay h increases, it also increases the transient response of the prediction error affecting the time to get the future value of the state necessary to implement a feedback law for tracking or regulation.

IV. TRAJECTORY TRACKING PROBLEM FOR THE DIFFERENTIAL DRIVE MOBILE ROBOT

For the original delay-free system (1)) the trajectory tracking problem can be solved by first defining the tracking error,

$$e_s(t) = \begin{bmatrix} e_{s1}(t) \\ e_{s2}(t) \\ e_{s3}(t)\end{bmatrix} = \begin{bmatrix} x_{rd}(t) - x_r(t) \\ y_{rd}(t) - y_r(t) \\ \theta_{rd}(t) - \theta_r(t)\end{bmatrix}. \tag{34}$$

The desired trajectory $\xi_{rd}(t) = [x_{rd}(t)\ y_{rd}(t)\ \theta_{rd}(t)]^T$ that the robot must follow is generated by a virtual mobile robot,

$$\begin{aligned}\dot{x}_{rd}(t) &= v_d(t) + \omega_d(t)y_{rd}(t)\\ \dot{y}_{rd}(t) &= -\omega_d(t)x_{rd}(t)\\ \dot{\theta}_{rd}(t) &= \omega_d(t)\end{aligned} \tag{35}$$

that satisfies the equivalent non-holonomic restriction (2).

For the delay-free mobile robot (1), the feedback

$$\begin{aligned}v(t) &= v_d(t)\cos(e_{s3}(t)) + k_1e_{s1}(t)\\ \omega(t) &= \omega_d(t) + k_2v_d(t)\frac{\sin(e_{s3}(t))}{e_{s3}(t)}e_{s2}(t) + k_3e_{s3}(t)\end{aligned} \tag{36}$$

introduced in [48] solves the trajectory tracking problem employing the correct real positive gains k_1, k_2, and k_3.

To solve the trajectory tracking problem for the advanced system (9), feedback (36) is expressed in terms of the estimated future values of the state, obtained from the state of the n-th sub-predictor, namely,

$$\begin{aligned}v(t) &= v_d(t)\cos(\hat{e}_{n3}(t)) + k_1\hat{e}_{n1}(t)\\ \omega(t) &= \omega_d(t) + k_2v_d(t)\frac{\sin(\hat{e}_{n3}(t))}{\hat{e}_{n3}(t)}\hat{e}_{n2}(t) + k_3\hat{e}_{n3}(t)\end{aligned} \tag{37}$$

where the desired trajectory originates from the virtual robot (35). The trajectory tracking error is described as

$$\begin{aligned}\begin{bmatrix} e_{n1}(t) \\ e_{n2}(t) \\ e_{n3}(t)\end{bmatrix} &= \begin{bmatrix} e_{s1}(t+h) \\ e_{s2}(t+h) \\ e_{s3}(t+h)\end{bmatrix}\\ &= \begin{bmatrix} x_{rd}(t+h) - x_r(t+h) \\ y_{rd}(t+h) - y_r(t+h) \\ \theta_{rd}(t+h) - \theta_r(t+h)\end{bmatrix}\\ &= \begin{bmatrix} w_{1d}(t) - w_{n1}(t) \\ w_{2d}(t) - w_{n2}(t) \\ w_{3d}(t) - w_{n3}(t).\end{bmatrix}\end{aligned}$$

Thus the estimated tracking error is described by

$$\begin{bmatrix} \hat{e}_{n1}(t) \\ \hat{e}_{n2}(t) \\ \hat{e}_{n3}(t) \end{bmatrix} = \begin{bmatrix} w_{1d}(t) - \hat{w}_{n1}(t) \\ w_{2d}(t) - \hat{w}_{n2}(t) \\ w_{3d}(t) - \hat{w}_{n3}(t) \end{bmatrix} = \begin{bmatrix} \tilde{w}_{n1}(t) + e_{n1}(t) \\ \tilde{w}_{n2}(t) + e_{n2}(t) \\ \tilde{w}_{n3}(t) + e_{n3}(t) \end{bmatrix}.$$

A. TRAJECTORY TRACKING ERROR DYNAMICS

After some computations, the closed-loop system (9)-(37) takes the form

$$\dot{e}_{wj}(t) = A_1(t)e_{wj}(t) + \Lambda_{(j-1)}e_{w(j-1)}(t) - \Lambda_j e_{wj}(t - \bar{h}) \quad (38a)$$

$$\dot{e}_n(t) = f(e_n(t)) + \Phi(e_n(t), e_{wj}(t)) \quad (38b)$$

where,

$$f(e_n) = \begin{bmatrix} -k_1 & \omega_d(t) & 0 \\ -\omega_d(t) & 0 & 0 \\ 0 & 0 & -k_3 \end{bmatrix} \begin{bmatrix} e_{n1}(t) \\ e_{n2}(t) \\ e_{n3}(t) \end{bmatrix} + \begin{bmatrix} k_2 v_d(t)\gamma_s e_{n2}^2(t) + k_3 e_{n3}(t)e_{n2}(t) \\ -k_2 v_d(t)\gamma_s e_{n2}(t)e_{n1}(t) - k_3 e_{n3}(t)e_{n1}(t) \\ 0 \end{bmatrix}$$

$$\Phi(e_n, e_{wj}) = \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -k_3 \end{bmatrix} \begin{bmatrix} \tilde{w}_{n1}(t) \\ \tilde{w}_{n2}(t) \\ \tilde{w}_{n3}(t) \end{bmatrix} + \begin{bmatrix} \gamma_1(t) \\ \gamma_2(t) \\ \gamma_3(t) \end{bmatrix}$$

with

$$\begin{aligned} \gamma_s(t) &= \frac{\sin(\tilde{w}_{n3}(t) + e_{n3}(t))}{\tilde{w}_{n3}(t) + e_{n3}(t)} \\ \gamma_1(t) &= v_d(t)[1 - \cos(\tilde{w}_{n3}(t) + e_{n3}(t))] \\ &\quad - k_2 v_d(t)\gamma_s(t) \\ &\quad \times \left[\tilde{w}_{n2}(t)(w_{2d}(t) - e_{n2}(t)) + e_{n2}(t)w_{2d}(t)\right] \\ &\quad - k_3 \left[\tilde{w}_{n3}(t)(w_{2d}(t) - e_{n2}(t)) + e_{n3}(t)w_{2d}(t)\right] \\ \gamma_2(t) &= k_2 v_d(t)\gamma_s(t)\left[\tilde{w}_{n2}(t)(w_{1d}(t) - e_{n1}(t)) + e_{n2}(t)w_{1d}(t)\right] \\ &\quad + k_3 \left[\tilde{w}_{n3}(t)(w_{1d}(t) - e_{n1}(t)) + e_{n3}(t)w_{1d}(t)\right] \\ \gamma_3(t) &= -k_2 v_d(t)\gamma_s(t)[\tilde{w}_{n2}(t) + e_{n2}(t)]. \end{aligned}$$

For details on the computation of system (38), refer to Appendix A.

B. STABILITY ANALYSIS OF THE CLOSED-LOOP SYSTEM

To formally prove the convergence to the origin of the tracking error (38b), recall that the prediction error $e_{wj}(t)$ converges to zero independently of the dynamics of $e_n(t)$. For the tracking error, consider the case where $\Phi(e_n, e_{wj}) = 0$ in (38b)

$$\begin{bmatrix} \dot{e}_{n1}(t) \\ \dot{e}_{n2}(t) \\ \dot{e}_{n3}(t) \end{bmatrix} = \begin{bmatrix} -k_1 & \omega_d(t) & 0 \\ -\omega_d(t) & 0 & 0 \\ 0 & 0 & -k_3 \end{bmatrix} \begin{bmatrix} e_{n1}(t) \\ e_{n2}(t) \\ e_{n3}(t) \end{bmatrix} + \begin{bmatrix} k_2 v_d(t)\gamma_s e_{n2}^2 + k_3 e_{n3}e_{n2} \\ -k_2 v_d(t)\gamma_s e_{n2}e_{n1} - k_3 e_{n3}e_{n1} \\ 0 \end{bmatrix} \quad (39)$$

where it is clear that $(e_{n1}, e_{n2}, e_{n3}) = (0, 0, 0)$ is an equilibrium point of (39). Time derivative of the Lyapunov candidate function,

$$V(t) = \frac{1}{2}e_{n1}^2 + \frac{1}{2}e_{n2}^2 + \frac{1}{2}e_{n3}^2 \quad (40)$$

yields,

$$\begin{aligned} \dot{V}(t) &= e_{n1}\dot{e}_{n1} + e_{n2}\dot{e}_{n2} + e_{n3}\dot{e}_{n3} \\ &= e_{n1}(-k_1 e_{n1} + \omega_d e_{n2} + k_2 v_d \gamma_s e_{n2}^2 + k_3 e_{n3}e_{n2}) \\ &\quad + e_{n2}(-\omega_d e_{n1} - k_2 v_d(t)\gamma_s e_{n2}e_{n1} - k_3 e_{n3}e_{n1}) - k_3 e_{n3}^2 \\ &= -k_1 e_{n1}^2 - k_3 e_{n3}^2. \end{aligned}$$

Since $\dot{V}(t)$ is negative semidefinite, the Barbalat lemma allows proving the asymptotic stability of subsystem (39). Term $\ddot{V}(t)$ is computed as follows,

$$\begin{aligned} \ddot{V}(t) &= -2k_1 e_{n1}\dot{e}_{n1} - 2k_3 e_{n3}\dot{e}_{n3} \\ &= -2k_1 e_{n1}(-k_1 e_{n1} + \omega_d e_{n2} + k_2 v_d \gamma_s e_{n2}^2 + k_3 e_{n3}e_{n2}) \\ &\quad + 2k_3^2 e_{n3}^2. \end{aligned}$$

Thus, $\dot{V}(t)$ is uniformly continuous and $\dot{V}(t) \to 0$ as $t \to \infty$, which in turn implies that $e(t) \to 0$ as $t \to \infty$.

C. LINEAR GROWTH OF THE INTERCONNECTION TERM

To complete the convergence proof of error (38b), it is necessary to verify the linear growth of the interconnection term $\Phi(e_n(t), e_{wj}(t))$. For that matter, the following definition [49] is recalled.

Definition 1: Function $\Psi(z, \zeta)$ has linear growth in z if there exist two class-κ functions $\delta_1(\cdot)$ and $\delta_2(\cdot)$, differentiable at $\zeta = 0$, such that,

$$||\Psi(z, \zeta)|| \leq \delta_1(||\zeta||)\,||z|| + \delta_2(||\zeta||). \quad (41)$$

Straightforward computations show that the function $\Phi(e_n(t), e_{wj})$ is bounded as in (42), shown at the bottom of the next page, where γ_1, γ_2 and γ_3 are bounded as,

$$\begin{aligned} ||\gamma_1(t)|| &\leq \delta_{21}(||e_{wj}(t)||) + \delta_{11}(||e_{wj}(t)||)||e(t)|| \\ ||\gamma_2(t)|| &\leq \delta_{22}(||e_{wj}(t)||) + \delta_{12}(||e_{wj}(t)||)||e(t)|| \\ ||\gamma_3(t)|| &\leq \delta_{23}(||e_{wj}(t)||) + \delta_{13}(||e_{wj}(t)||)||e(t)|| \end{aligned} \quad (43)$$

with

$$\begin{aligned} \delta_{11}(||e_{wj}(t)||) &= ||v_d(t)|| + ||k_2 v_d(t)e_{wj_2}(t)|| + ||k_3 e_{wj_3}(t)|| \\ \delta_{21}(||e_{wj}(t)||) &= ||v_d(t)e_{wj_3}(t)|| + ||k_2 v_d(t)w_{2d}(t)e_{wj_2}(t)|| \\ &\quad + ||k_3 w_{2d}(t)e_{wj_3}(t)|| \\ \delta_{12}(||e_{wj}(t)||) &= ||k_2 v_d(t)e_{wj_2}(t)|| + ||k_3 e_{wj_3}(t)|| \\ \delta_{22}(||e_{wj}(t)||) &= ||k_2 v_d(t)w_{1d}(t)e_{wj_2}(t)|| \\ &\quad + ||k_3 w_{1d}(t)e_{wj_3}(t)|| \\ \delta_{13}(||e_{wj}(t)||) &= ||k_2 v_d(t)|| \\ \delta_{23}(||e_{wj}(t)||) &= ||k_2 v_d(t)\tilde{w}_{n2}(t)||. \end{aligned}$$

It is easy to see that there exist $\delta_1(||e_{wj}(t)||)$ and $\delta_2(||e_{wj}(t)||)$ that satisfy the inequality (41) for the function

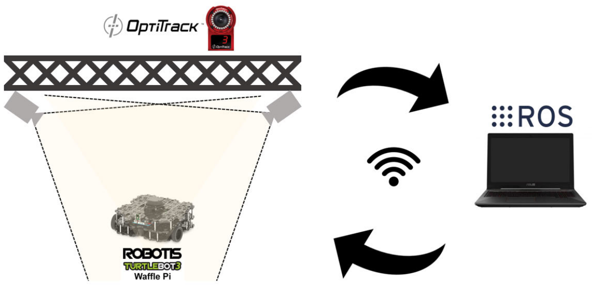


FIGURE 3. Experimental platform diagram.

FIGURE 4. Passive markers in the differential drive robot and elements of the experimental platform.

$\Phi(e_n(t), e_{wj}(t))$. Appendix (B) shows how to obtain $||\gamma_i(t)||$ for $i = 1, 2, 3$..

The above developments allow us to conclude on the stability of the closed-loop system (38).

Theorem 2: Consider system (9) and the prediction-based feedback (37) and assume that Lemma 3 is satisfied. Then, positive gains k_1, k_2, k_3 exist such that the overall closed-loop system (38) is asymptotically stable.

Proof: To demonstrate the theorem, notice first that the convergence of the prediction errors is insured by Lemma 3. The cascade structure of system (38), and the linear growth of function $\Phi(e_n(t), e_{wj}(t))$ given in (42) allows the application of Proposition 4.1 in [49], which establishes the convergence of the tracking errors $e_n(t)$. ■

V. NUMERICAL AND EXPERIMENTAL EVALUATION

A ©MATLAB simulation and an experimental implementation are carried out to evaluate the proposed prediction scheme and the trajectory tracking solution. The experimental platform, depicted in Figure 3, is composed of an ©OptiTrack motion capture camera system and a ©ROBOTIS differential drive robot "Turtlebot3 Waffle Pi" with passive markers as indicators for 3D position and orientation, as shown in Figure 4, which are interconnected by the Robotic Operating System (ROS).

We consider a lemniscate-type trajectory described as follows,

$$x = A\cos(pt)$$
$$y = B\sin(2pt)$$

with $A = 0.7$, $B = 0.4$, $p = \frac{\pi}{20}$.

For the differential drive model (5), the input delay is assumed to be $h = 0.12\ s$. This delay time was artificially introduced through programming in the system for the numerical and experimental cases. For the prediction strategy (10), a set of $n = 3$ sub-predictors provide, according to (31), an appropriate delay $\bar{h}_{max} = 0.092$. The gain parameters proposed for the predictor (10) and the feedback (37) are given in Table 1.

TABLE 1. Predictor and control parameters.

Parameter	Simulation	Experiment
$\lambda_{11}, \lambda_{12}, \lambda_{13}$	4	4
$\lambda_{21}, \lambda_{22}, \lambda_{23}$	3	4
$\lambda_{31}, \lambda_{32}, \lambda_{33}$	2	1
$k1, k3$	0.4	0.2
$k2$	1.5	3.5

The initial condition are set as $x = 0.7\,m$, $y = -0.5\,m$ and $\theta = \frac{\pi}{2}\ rad$.

For the simulation results, the prediction and trajectory tracking errors are shown in Figure 5 and 6, respectively, where it is noticeable that both errors converge correctly and smoothly to the origin. For illustrative purposes, the injection errors are presented in Figure 7, showing that the injection error $e_{w1}(t)$ tends to zero before $e_{w2}(t)$, which in turn converges before $e_{w3}(t)$. This evolution is expected from the sub-predictor scheme because the future values are estimated step by step, a delay $\bar{h}$ ahead. Observe that the trajectory tracking errors $e_s(t)$ converge after the prediction errors because the control law (37) requires the future state estimates. The evolution of control signals $v(t)$, $\omega(t)$, depicted in Figure 8, is smooth.

For the real-time experiments, the time evolution of the prediction error $\tilde{w}_3(t)$ and the trajectory tracking error evolution $e_s(t)$ are shown in Figures 9 and 10, respectively. The errors converged to the origin, following the convergence pattern of the numerical results. Observe that the orientation errors $\tilde{w}_{33}(t)$ and $e_{s3}(t)$ were more sensitive to inaccurate measurements. Figure 11 shows that the experimental control signals $v(t)$, $\omega(t)$ exhibited adequate behavior.

Finally, Figure 12 shows the desired trajectory generated by a virtual robot on the $X - Y$ plane, the path of the real-time experimental trajectories of the differential drive

$$||\Phi(e_n(t), e_w(t))|| \leq \sqrt{|| - k_1\tilde{w}_{n1}(t) + \gamma_1(t)||^2 + ||\gamma_2(t)||^2 + || - k_3\tilde{w}_{n3}(t) + \gamma_3(t)||^2}. \quad (42)$$

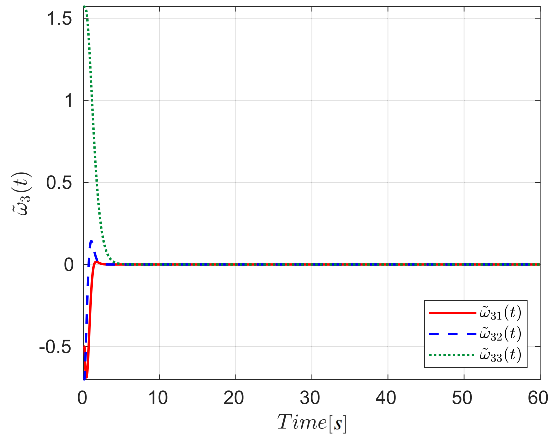


FIGURE 5. Numerical prediction errors.

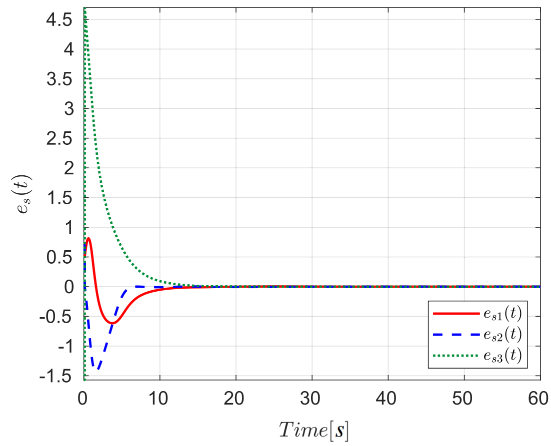


FIGURE 6. Numerical trajectory tracking errors.

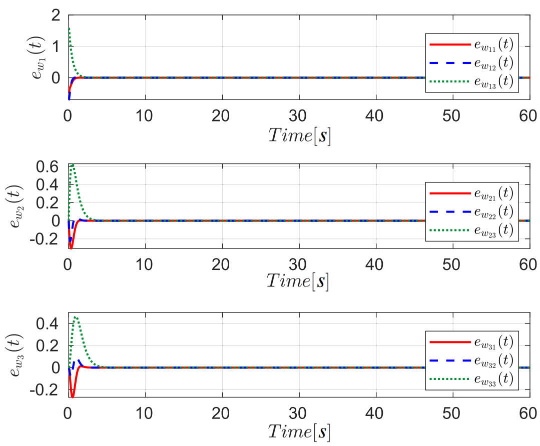


FIGURE 7. Numerical injection errors evolution.

robot, and the numerical results obtained from simulations of the closed-loop scheme. As expected, the numerical experiment converged better to the desired trajectory than the real-time experiments due to measurement errors in the latter case.

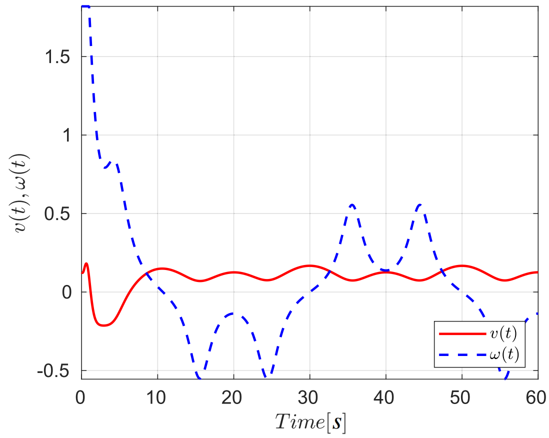


FIGURE 8. Numerical control signed evolution.

TABLE 2. Predictor parameters for comparison.

Parameter	Simulation
$\lambda_{11},\ \lambda_{12},\ \lambda_{13}$	0.4
$\lambda_{21},\ \lambda_{22},\ \lambda_{23}$	0.4
$\lambda_{31},\ \lambda_{32},\ \lambda_{33}$	0.75

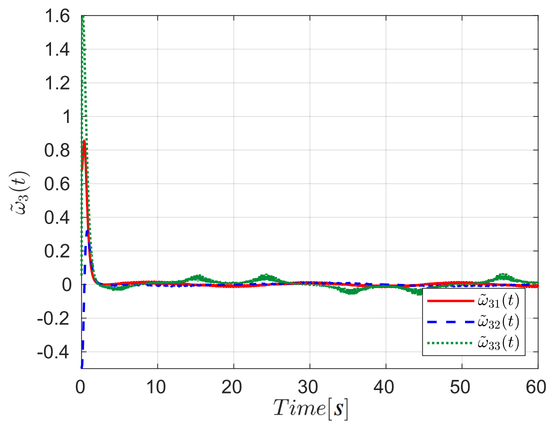


FIGURE 9. Real-time prediction errors.

A. PREDICTION SCHEME COMPARISON

To illustrate the main difference between the prediction-based solution presented in this work and the one proposed in [25], Figure 13 shows the robot evolution on the $X - Y$ plane with an input time delay of $h = 1.7\ s$ For the solution in [25] and the one in equation (37), with $n = 1$ and $n = 3$, there were considered the gains parameters given in Table 2.

It is clear from Figure 13, that the performance of the solution in [25] (purple and dotted line) is comparable to the case $n = 1$ (green and discontinuous line). Nevertheless, in the present case, it is possible to improve the obtained performance simply by increasing the value of n, as seen in Figure 13 for $n = 3$ (blue and discontinuous line).

Remark 5: The proposed strategy is based on the kinematic model of the non-holonomic mobile robot,

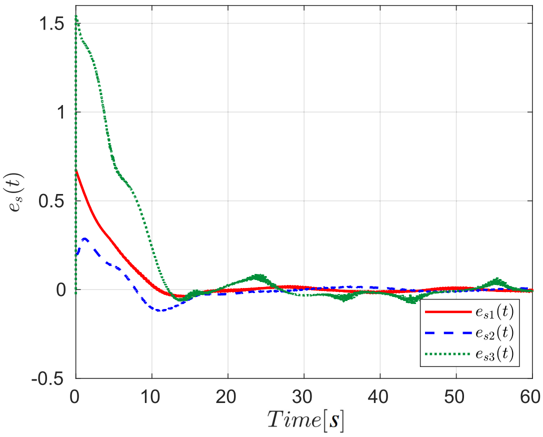


FIGURE 10. Real-time trajectory tracking errors.

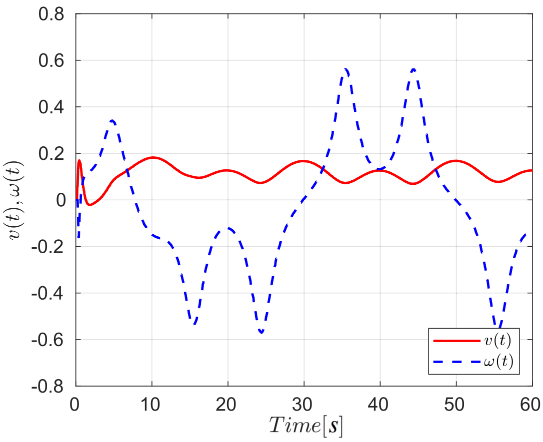


FIGURE 11. Experimental control inputs.

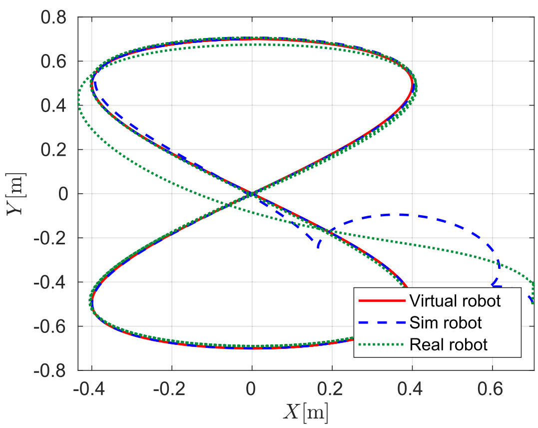


FIGURE 12. Desired trajectory (red and continuous), traveled path in the simulation (blue and discontinuous), and traveled path in the experiment (green and doted) with time delay $h = 0.12$ s.

therefore, there exists forces and inertia that are not considered in the design that may interfere with the overall performance.

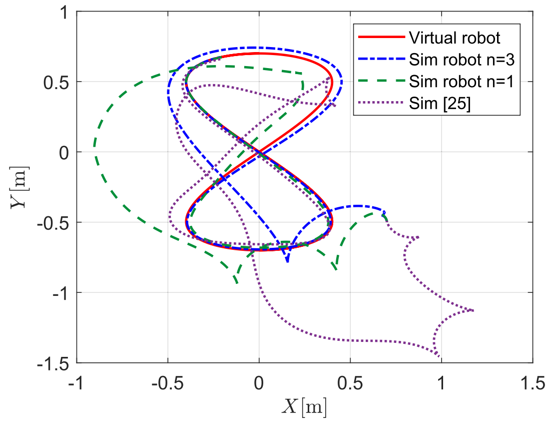


FIGURE 13. Comparison of solutions proposed in [25] with the present solution for $n = 1$ and $n = 3$.

Remark 6: Note that for implementation purposes, a possibly large amount of data processing due to complex environments or a formation composed of many non-holonomic mobile robots, it is necessary to increase the number of sub-predictors to ensure the fulfillment of the obtained stability condition. Also, as shown in the experimental results, unmodelled time delay fluctuations can deteriorate the overall performance.

VI. CONCLUSION

This contribution addresses the trajectory tracking problem of a non-holonomic differentially driven mobile robot subject to an input time delay h. A generalization of the non-linear case of a linear sub-predictor strategy is introduced to estimate the state of the system h units of time ahead. It was shown that it is possible to predict the state of the system independently of the trajectory that the mobile robot must track. Furthermore, a Lyapunov-Krasovskii functional-based analysis proved that the sub-predictor scheme can manage large time delays by increasing the number of sub-predictors in the observer chain. For appropriate gains in the prediction scheme, the introduction of the estimated future states into the control law was shown to achieve closed-loop system stability. Real-time experiments and numerical simulations were conducted to evaluate the effectiveness of the proposed prediction-based strategy. It should be pointed out that the presented prediction-based strategy is based on the knowledge of the constant time delay affecting the input signal, and even when theoretically it can be considered as large as desired, large time delay induces a non-adequate transient state for the prediction and tracking errors. The asymptotic rate of the obtained convergence of the prediction and tracking errors will depend on the stability margin obtained in Corollary 1. Future work includes taking into account time-varying delays arising in remote control due to wireless communication protocols, testing feedback laws with better convergence rates, and extending our work to the formation problem for a set of mobile robots with time delays.

APPENDIX A
TRAJECTORY TRACKING ERROR DYNAMICS

The evolution of the trajectory tracking errors can be obtained as follows.

$$\begin{aligned}
&\dot{e}_1(t)\\
&= \dot{w}_{1d}(t) - \dot{w}_{n1}(t)\\
&= \dot{w}_{1d}(t) - v(t) - \omega(t)w_{n2}(t)\\
&= \dot{w}_{1d}(t) - v_d(t)\cos(\hat{e}_3(t)) - k_1\hat{e}_1(t)\\
&\quad - \left[\omega_d(t) + k_2v_d(t)\frac{\sin(\hat{e}_3(t))}{\hat{e}_3(t)}\hat{e}_2(t) + k_3\hat{e}_3(t)\right]w_{n2}(t)\\
&= \dot{w}_{1d}(t) - v_d(t)\cos(\tilde{w}_{n3}(t)+e_3(t)) - k_1[\tilde{w}_{n1}(t)+e_1(t)]\\
&\quad - \left[\omega_d(t) + k_2v_d(t)\frac{\sin(\tilde{w}_{n3}(t)+e_3(t))}{\tilde{w}_{n3}(t)+e_3(t)}[\tilde{w}_{n2}(t)+e_2(t)]\right.\\
&\quad \left. + k_3[\tilde{w}_{n3}(t)+e_3(t)]\right][w_{2d}(t)-e_2(t)].
\end{aligned}$$

Substituting the evolution of the desired trajectory $\dot{w}_{1d}(t)$,

$$\begin{aligned}
\dot{e}_1(t) &= v_d(t) + \omega_d(t)w_{2d}(t) - v_d(t)\cos(\tilde{w}_{n3}(t)+e_3(t))\\
&\quad - k_1[\tilde{w}_{n1}(t)+e_1(t)]\\
&\quad - \left[\omega_d(t) + k_2v_d(t)\frac{\sin(\tilde{w}_{n3}(t)+e_3(t))}{\tilde{w}_{n3}(t)+e_3(t)}[\tilde{w}_{n2}(t)+e_2(t)]\right.\\
&\quad \left. + k_3[\tilde{w}_{n3}(t)+e_3(t)]\right][w_{2d}(t)-e_2(t)]\\
&= v_d(t)[1-\cos(\tilde{w}_{n3}(t)+e_3(t))]\\
&\quad - k_1[\tilde{w}_{n1}(t)+e_1(t)] + \omega_d(t)e_2(t)\\
&\quad - \left[k_2v_d(t)\frac{\sin(\tilde{w}_{n3}(t)+e_3(t))}{\tilde{w}_{n3}(t)+e_3(t)}[\tilde{w}_{n2}(t)+e_2(t)]\right.\\
&\quad \left. + k_3[\tilde{w}_{n3}(t)+e_3(t)]\right][w_{2d}(t)-e_2(t)]\\
&= -k_1[\tilde{w}_{n1}(t)+e_1(t)] + \omega_d(t)e_2(t) + k_2v_d\gamma_s e_2^2(t)\\
&\quad + k_3e_3(t)e_2(t) + \gamma_1(t).
\end{aligned}$$

Repeating the procedure with $e_2(t)$ and $e_3(t)$,

$$\begin{aligned}
\dot{e}_2(t) &= \dot{w}_{2d}(t) - \dot{w}_{n2}(t)\\
&= \dot{w}_{2d}(t) + \omega(t)w_{n1}(t)\\
&= \dot{w}_{2d}(t)\\
&\quad + \left[\omega_d(t) + k_2v_d(t)\frac{\sin(\hat{e}_3(t))}{\hat{e}_3(t)}\hat{e}_2(t) + k_3\hat{e}_3(t)\right]w_{n1}(t)\\
\dot{e}_2(t) &= -\omega_d(t)w_{1d}(t) + \left[\omega_d(t) + k_2v_d(t)\frac{\sin(\hat{e}_3(t))}{\hat{e}_3(t)}\hat{e}_2(t)\right.\\
&\quad \left. + k_3\hat{e}_3(t)\right]w_{n1}(t)\\
&= -\omega_d(t)w_{1d}(t) + \left[\omega_d(t) + k_2v_d(t)\frac{\sin(\hat{e}_3(t))}{\hat{e}_3(t)}\hat{e}_2(t)\right.\\
&\quad \left. + k_3\hat{e}_3(t)\right][w_{1d}(t)-e_1(t)]\\
&= \left[k_2v_d(t)\frac{\sin(\hat{e}_3(t))}{\hat{e}_3(t)}\hat{e}_2(t) + k_3\hat{e}_3(t)\right][w_{1d}(t)-e_1(t)]\\
&\quad - \omega_d(t)e_1(t)\\
&= \left[k_2v_d(t)\frac{\sin(\tilde{w}_{n3}(t)+e_3(t))}{\tilde{w}_{n3}(t)+e_3(t)}[\tilde{w}_{n2}(t)+e_2(t)]\right.\\
&\quad \left. + k_3[\tilde{w}_{n3}(t)+e_3(t)]\right][w_{1d}(t)-e_1(t)] - \omega_d(t)e_1(t)\\
&= -\omega_d(t)e_1(t) - k_2v_d\gamma_s e_2(t)e_1(t) - k_3e_3(t)e_1(t)\\
&\quad + \gamma_2(t)\\
\dot{e}_3(t) &= \dot{w}_{3d}(t) - \dot{w}_{n3}(t)\\
&= \dot{w}_{3d}(t) - \omega(t)\\
&= \omega_d(t) - \omega_d(t) - k_2v_d(t)\frac{\sin(\hat{e}_3(t))}{\hat{e}_3(t)}\hat{e}_2(t) - k_3\hat{e}_3(t)\\
&= -k_2v_d(t)\frac{\sin(\tilde{w}_{n3}(t)+e_3(t))}{\tilde{w}_{n3}(t)+e_3(t)}[\tilde{w}_{n2}(t)+e_2(t)]\\
&\quad - k_3[\tilde{w}_{n3}(t)+e_3(t)]\\
&= -k_3[\tilde{w}_{n3}(t)+e_3(t)] + \gamma_3(t)
\end{aligned}$$

APPENDIX B
BOUNDS OF γ_1, γ_2 AND γ_3

The bound of the γ_i terms for $i = 1, 2, 3.$ can be done as follows.

$$\begin{aligned}
\gamma_1(t) &\le ||v_d(t)[1-\cos(\tilde{w}_{n3}(t)+e_3(t))]||\\
&\quad + \left|\left|k_2v_d(t)\gamma_s(t)\left[\tilde{w}_{n2}(t)(w_{2d}(t)\right.\right.\right.\\
&\quad \left.\left.\left.-e_2(t)) + e_2(t)w_{2d}(t)\right]\right|\right|\\
&\quad + \left|\left|k_3\left[\tilde{w}_{n3}(t)(w_{2d}(t)-e_2(t)) + e_3(t)w_{2d}(t)\right]\right|\right|\\
&\le \left|\left|2v_d(t)\sin^2\left(\frac{\sin(e_{w_3}(t)+e_3(t))}{2}\right)\right|\right|\\
&\quad + ||k_2v_d(t)e_{w_2}(t) + k_3e_{w_3}(t)||\;||w_{2d}(t)-e_2(t)||\\
&\le ||v_d(t)(e_{w_3}(t)+e_3(t))||\\
&\quad + ||k_2v_d(t)e_{w_2}(t) + k_3e_{w_3}(t)||\;||w_{2d}(t)-e_2(t)||\\
&\le ||v_d(t)e_{w_3}(t)||\\
&\quad + ||v_d(t)e_3(t)|| + ||k_2v_d(t)w_{2d}(t)e_{w_2}(t)||\\
&\quad + ||k_3w_{2d}(t)e_{w_3}(t)|| + ||k_2v_d(t)e_2(t)e_{w_2}(t)||\\
&\quad + ||k_3e_2(t)e_{w_3}(t)||\\
&\le \alpha_{21}(||e_w||) + \alpha_{11}(||e_w||)||e||\\
\gamma_2(t) &\le \left|\left|k_2v_d(t)\frac{\sin(e_{w_3}(t)+e_3(t))}{e_{w_3}(t)+e_3(t)}e_{w_2}(t)\right.\right.\\
&\quad \left.\left.+k_3e_{w_3}(t)\right|\right|\;||w_{1d}(t)-e_1(t)||\\
&\le ||k_2v_d(t)e_{w_2}(t) + k_3e_{w_3}(t)||\;||w_{1d}(t)-e_1(t)||\\
&\le ||k_2v_d(t)w_{1d}(t)e_{w_2}(t)|| + ||k_3w_{1d}(t)e_{w_3}(t)||\\
&\quad + ||k_2v_d(t)e_1(t)e_{w_2}(t)|| + ||k_3e_1(t)e_{w_3}(t)||\\
&\le \alpha_{22}(||e_w||) + \alpha_{12}(||e_w||)||e||\\
\gamma_3(t) &\le \left|\left|k_2v_d(t)\frac{\sin(e_{w_3}(t)+e_3(t))}{e_{w_3}(t)+e_3(t)}[\tilde{w}_{n2}(t)+e_2(t)]\right|\right|\\
&\le ||k_2v_d(t)[\tilde{w}_{n2}(t)+e_2(t)]||\\
&\le ||k_2v_d(t)\tilde{w}_{n2}(t)|| + ||k_2v_d(t)e_2(t)||\\
&\le \alpha_{23}(||e_w||) + \alpha_{13}(||e_w||)||e||
\end{aligned}$$

REFERENCES

[1] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*. Boston, MA, USA: Birkhäuser, 2003.

[2] S.-I. Niculescu, *Delay Effects on Stability: A Robust Control Approach*. London, U.K.: Springer, 2001.

[3] E. Fridman, *Introduction to Time-Delay Systems*, 1st ed. Cham, Switzerland: Birkhäuser, 2014.

[4] O. J. M. Smith, "Closer control of loops with deadtime," *Chem. Eng. Prog.*, vol. 53, no. 5, pp. 217–219, 1957.

[5] Z. J. Palmor and D. V. Powers, "Improved dead-time compensator controllers," *AIChE J.*, vol. 31, no. 2, pp. 215–221, Feb. 1985.

[6] Z. J. Palmor, "Time-delay compensation Smith predictor and its modifications," in *The Control Handbook*, vol. 1. Boca Raton, FL, USA: CRC Press, 1996, pp. 224–229.

[7] K. J. Astrom, C. C. Hang, and B. C. Lim, "A new Smith predictor for controlling a process with an integrator and long dead-time," *IEEE Trans. Autom. Control*, vol. 39, no. 2, pp. 343–345, Feb. 1994.

[8] A. Manitius and A. Olbrot, "Finite spectrum assignment problem for systems with delays," *IEEE Trans. Autom. Control*, vol. AC-24, no. 4, pp. 541–552, Aug. 1979.

[9] V. L. Kharitonov, "An extension of the prediction scheme to the case of systems with both input and state delay," *Automatica*, vol. 50, no. 1, pp. 211–217, Jan. 2014.

[10] V. L. Kharitonov, "Predictor-based controls: The implementation problem," *Differ. Equ.*, vol. 51, no. 13, pp. 1675–1682, Dec. 2015.

[11] L. Juárez, S. Mondié, and V. L. Kharitonov, "Dynamic predictor for systems with state and input delay: A time-domain robust stability analysis," *Int. J. Robust Nonlinear Control*, vol. 30, no. 6, pp. 2204–2218, Apr. 2020.

[12] M. Dalla Mora, A. Germani, and C. Manes, "Design of state observers from a drift-observability property," *IEEE Trans. Autom. Control*, vol. 45, no. 8, pp. 1536–1540, Aug. 2000.

[13] M. Hou, P. Zitek, and R. J. Patton, "An observer design for linear time-delay systems," *IEEE Trans. Autom. Control*, vol. 47, no. 1, pp. 121–125, Jan. 2002.

[14] F. E. Thau, "Observing the state of non-linear dynamic systems," *Int. J. Control*, vol. 17, no. 3, pp. 471–479, Mar. 1973.

[15] M. Najafi, S. Hosseinnia, F. Sheikholeslam, and M. Karimadini, "Closed-loop control of dead time systems via sequential sub-predictors," *Int. J. Control*, vol. 86, no. 4, pp. 599–609, Apr. 2013.

[16] V. Fragoso-Rubio, M. Velasco-Villa, M. A. Hernández-Pérez, B. del Muro-Cuéllar, and J. F. Márquez-Rubio, "Prediction-observer scheme for linear systems with input-output time-delay," *Int. J. Control, Autom. Syst.*, vol. 17, no. 8, pp. 2012–2025, Aug. 2019.

[17] J. Lei and H. K. Khalil, "High-gain-predictor-based output feedback control for time-delay nonlinear systems," *Automatica*, vol. 71, pp. 324–333, Sep. 2016.

[18] P. Pepe and E. Fridman, "On global exponential stability preservation under sampling for globally Lipschitz time-delay systems," *Automatica*, vol. 82, pp. 295–300, Aug. 2017.

[19] Z. Zuo, Z. Lin, and Z. Ding, "Truncated predictor control of Lipschitz nonlinear systems with time-varying input delay," *IEEE Trans. Autom. Control*, vol. 62, no. 10, pp. 5324–5330, Oct. 2017.

[20] A. Germani, C. Manes, and P. Pepe, "A new approach to state observation of nonlinear systems with delayed output," *IEEE Trans. Autom. Control*, vol. 47, no. 1, pp. 96–101, Jan. 2002.

[21] F. Mazenc and M. Malisoff, "Stabilization of nonlinear time-varying systems through a new prediction based approach," *IEEE Trans. Autom. Control*, vol. 62, no. 6, pp. 2908–2915, Jun. 2017.

[22] C. Guo and X.-J. Xie, "Global output feedback control of nonlinear time-delay systems with input matching uncertainty and unknown output function," *Int. J. Syst. Sci.*, vol. 50, no. 4, pp. 713–725, Mar. 2019.

[23] Y. Deng, V. Léchappé, E. Moulay, Z. Chen, B. Liang, F. Plestan, and Q.-L. Han, "Predictor-based control of time-delay systems: A survey," *Int. J. Syst. Sci.*, vol. 53, no. 12, pp. 2496–2534, Sep. 2022.

[24] K. Kojima, T. Oguchi, A. Alvarez-Aguirre, and H. Nijmeijer, "Predictor-based tracking control of a mobile robot with time-delays," in *Proc. 8th IFAC Symp. Nonlinear Control Syst.*, 2010, pp. 167–170.

[25] A. Alvarez-Aguirre, N. van de Wouw, T. Oguchi, and H. Nijmeijer, "Predictor-based remote tracking control of a mobile robot," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 6, pp. 2087–2102, Nov. 2014.

[26] E.-H. Guechi, J. Lauber, M. Dambrine, and M. Defoort, "Output feedback controller design of a unicycle-type mobile robot with delayed measurements," *IET Control Theory Appl.*, vol. 6, no. 5, p. 726, 2012.

[27] E.-H. Guechi, J. Lauber, M. Dambrine, G. Klančar, and S. Blažič, "PDC control design for non-holonomic wheeled mobile robots with delayed outputs," *J. Intell. Robot. Syst.*, vol. 60, nos. 3–4, pp. 395–414, Dec. 2010.

[28] E.-H. Guechi, K. Belharet, and S. Blažič, "Tracking control for wheeled mobile robot based on delayed sensor measurements," *Sensors*, vol. 19, no. 23, p. 5177, Nov. 2019.

[29] D.-H. Lee, J.-H. Jung, H.-N. Yoon, Y.-S. Park, and J.-M. Lee, "Simulation of time delay compensation controller for a mobile robot using the SMC and Smith predictor," in *Intelligent Autonomous Systems 14*, W. Chen, K. Hosoda, E. Menegatti, M. Shimizu, and H. Wang, Eds. Cham, Switzerland: Springer, 2017, pp. 687–694.

[30] Q. Xue and X. Yu, "State feedback finite-time tracking controller of nonholonomic mobile robot with actuator delay," in *Proc. Chin. Control Decis. Conf. (CCDC)*, Jun. 2018, pp. 2832–2836.

[31] B. Sarabia, D. Chávez, O. Camacho, and K. Patiño, "A control techniques comparison for trajectory tracking for a robotics platform with time delay," in *Proc. IEEE 5th Ecuador Tech. Chapters Meeting (ETCM)*, Cuenca, Ecuador, Oct. 2021, pp. 1–6.

[32] B. S. Park and S. J. Yoo, "A low-complexity tracker design for uncertain nonholonomic wheeled mobile robots with time-varying input delay at nonlinear dynamic level," *Nonlinear Dyn.*, vol. 89, no. 3, pp. 1705–1717, Aug. 2017.

[33] E. Nuño, A. Loría, T. Hernández, M. Maghenem, and E. Panteley, "Distributed consensus-formation of force-controlled nonholonomic robots with time-varying delays," *Automatica*, vol. 120, Oct. 2020, Art. no. 109114.

[34] W. Lu, C. Zong, J. Li, and D. Liu, "Bipartite consensus-based formation control of high-order multi-robot systems with time-varying delays," *Trans. Inst. Meas. Control*, vol. 44, no. 6, pp. 1297–1308, 2022.

[35] M. Velasco-Villa, A. Alvarez-Aguirre, and G. Rivera-Zago, "Discrete-time control of an omnidireccional mobile robot subject to transport delay," in *Proc. Amer. Control Conf.*, Jul. 2007, pp. 2171–2176.

[36] M. Velasco-Villa, R. Castro-Linares, F. Rosales-Hernández, B. del Muro-Cuéllar, and M. A. Hernández-Pérez, "Discrete-time synchronization strategy for input time-delay mobile robots," *J. Franklin Inst.*, vol. 350, no. 10, pp. 2911–2935, Dec. 2013.

[37] H. Sira-Ramírez, C. López-Uribe, and M. Velasco-Villa, "Trajectory-tracking control of an input delayed omnidirectional mobile robot," in *Proc. 7th Int. Conf. Electr. Eng. Comput. Sci. Autom. Control (CCE)*, Sep. 2010, pp. 470–475.

[38] I. Estrada-Sánchez, M. Velasco-Villa, and H. Rodríguez-Cortés, "Prediction-based control for nonlinear systems with input delay," *Math. Problems Eng.*, vol. 2017, pp. 1–11, Oct. 2017, doi: 10.1155/2017/7415418.

[39] A. Manitius and A. Olbrot, "Finite spectrum assignment problem for systems with delays," *IEEE Trans. Autom. Control*, vol. AC-24, no. 4, pp. 541–552, Aug. 1979.

[40] Z. Artstein, "Linear systems with delayed controls: A reduction," *IEEE Trans. Autom. Control*, vol. AC-27, no. 4, pp. 869–879, Aug. 1982.

[41] S. Mondie and W. Michiels, "Finite spectrum assignment of unstable time-delay systems with a safe implementation," *IEEE Trans. Autom. Control*, vol. 48, no. 12, pp. 2207–2212, Dec. 2003.

[42] B. Zhou, Z. Lin, and G.-R. Duan, "Truncated predictor feedback for linear systems with long time-varying input delays," *Automatica*, vol. 48, no. 10, pp. 2387–2399, Oct. 2012.

[43] G. Campion, G. Bastin, and B. Dandrea-Novel, "Structural properties and classification of kinematic and dynamic models of wheeled mobile robots," *IEEE Trans. Robot. Autom.*, vol. 12, no. 1, pp. 47–62, Feb. 1996.

[44] C. C. de Wit, G. Bastin, and B. Siciliano, *Theory of Robot Control*, 1st ed. Berlin, Germany: Springer-Verlag, 1996.

[45] R. M. Murray and S. S. Sastry, "Nonholonomic motion planning: Steering using sinusoids," *IEEE Trans. Autom. Control*, vol. 38, no. 5, pp. 700–716, May 1993.

[46] R. W. Brockett, "Asymptotic stability and feedback stabilization," in *Differential Geometric Control Theory*. Boston, MA, USA: Birkhäuser, 1983, pp. 181–191.

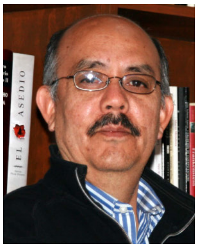
[47] P. Niamsup, K. Mukdasai, and V. N. Phat, "Improved exponential stability for time-varying systems with nonlinear delayed perturbations," *Appl. Math. Comput.*, vol. 204, no. 1, pp. 490–495, Oct. 2008.

[48] C. Samson and K. Ait-Abderrahim, "Mobile robot control. Part 1: Feedback control of a non-holonomic wheeled cart in Cartesian space," Nat. Inst. Res. Digit. Sci. Technol., Chesnay-Rocquencourt, France, Tech. Rep. 1288, 1990.

[49] R. Sepulchre, M. Jankovic, and P. V. Kokotovic, *Constructive Nonlinear Control*, 1st ed. London, U.K.: Springer, 1997.

JULIO A. BÁEZ-HERNÁNDEZ received the B.S. degree in mechatronics engineering from Veracruz University, Veracruz, in 2017, and the M.S. degree in electrical engineering from the Center for Research and Advanced Studies of the National Polytechnic Institute (CINVESTAV-IPN), Mexico City, Mexico, in 2019, where he is currently pursuing the Ph.D. degree in electrical engineering.

His research interests include mobile robotics and non-linear systems with time delays.



MARTÍN VELASCO-VILLA received the B.S. degree in electronics and communication engineering from the National Polytechnic Institute (IPN), Mexico City, Mexico, in 1987, and the M.S. and Ph.D. degrees in electrical engineering from the Center for Research and Advanced Studies of the National Polytechnic Institute (CINVESTAV-IPN), in 1994.

He made a postdoctoral research stay with Ecole Centrale de Nantes, Nantes, France. He is currently a full-time Professor with the Mechatronics Section, Electrical Engineering Department, CINVESTAV-IPN. His research interests include the analysis and control of linear and non-linear systems with time delay, in particular, the prediction problem associated with this type of system is of great interest. In the field of non-linear systems, the area of mobile robotics associated with the formation and synchronization problems and the analysis and control of electromechanical systems are some other topics of interest.

SABINE MONDIÉ received the B.S. degree in industrial engineering from ITESM, Mexico City, the M.S. degree in electrical engineering from the Center for Research and Advanced Studies of the National Polytechnic Institute (CINVESTAV), Mexico City, Mexico, in 1883, and the Ph.D. degree in electrical engineering from IRCyN, Nantes, France, in 1996. She has been a Professor with the Department of Automatic Control, CINVESTAV, since 1996. She has been the Chair for Education and she is currently the Vice-Chair of the IFAC Technical Committee 2.2. on "Linear Control Systems." She has served as an Associate Editor for several journals in control, including *Systems and Control Letters* and the *European Journal of Control*. She is also an Associate Editor of IEEE TRANSACTIONS ON AUTOMATIC CONTROL. Her research interests include time-delay systems, their stability and robustness properties, and delay applications in engineering and biology. She has directed/co-directed over 15 Ph.D. and 25 master theses and authored/coauthored 75 journals and 125 conference papers.