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RESEARCH ARTICLE

A Hybridized Optimal Algorithm for Multimodal Optimal Design of Inverse Problems

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ABSTRACT Particle swarm optimization (PSO) is an intelligent searching technique for solving complicated and multimodal design optimization's problems. The classical PSO algorithm is more flexible and efficient because of its ability to solve a diverse range of complex and real-world issues. Moreover, the primary deficiency of this method that it trapped and stuck to local minima during the optimization of multimodal, complex and inverse objective function. We introduce a crossover and mutation vectors in the conventional PSO to solve this deficiency. The differential evolution strategies inspired the novel vectors. The central idea of the proposal is that, the novel global best particle is updated through a mutation vector and crossover vector. The introduction of the global best particle maintains the swarm diversity at the final steps of the evolution process. Also, we designed a novel strategy for the control parameter, which will maintain a decent alignment of the candidates between the global and local searches. The performance evaluation table and trajectory curves illustrate that our proposed approach is the best compared to other methods.

INDEX TERMS Cross method, global best particle, global optimization, inverse problem, innovative process, mutation vector.

I. INTRODUCTION

In engineering, optimization is a very significant and crucial research topic. In various engineering fields, optimization techniques have been widely utilized. In recent era, we have seen a rise in the adoption of stochastic search algorithms for finding solutions of challenging problem in the optimization design problems. These days a number of optimization approaches, including simulated annealing, genetic algorithms, particle swarm optimization, evolutionary algorithms, and ant colony algorithms, are regarded as global search tools for high-dimensional, complicated and inverse problems.

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The somewhat recent particle swarm optimization, however, It is the easiest to utilize among these techniques of searching due to its restricted parameter set and simple implementation. In 1995, the PSO model was designed by two scientists: the electrical engineer Kennedy and the social scientist Eberhart. They were motivated from the natural habits and behaviors of animals, such as the schooling of fish and groups of birds during their pursuit of food [\[1\]. Th](#page-10-0)eir proposed technique was basically dynamic search based on population, stochastic search algorithm where every fish and bird position and velocity are initialized randomly. In PSOs, every particle shares information with others during the search process, as they move randomly toward their target region having different trajectories. Due to the random motion of the particles

in the evolution process, a large area of the search region is covered, facilitating the search for the feasible solution. Considering the optimal methods importance and popularity, various variants of PSO are successfully applied in many engineering fields due to its good charteristics, such as, robustness, stability, high convergence rate, and the ability to solve high-dimensional and complex problems. More concretely, PSOs were used in electrical robotics technology [\[2\],](#page-10-1) as well as other various fields of engineering. Currently, researchers are trying to overcome the problem arising from the ''No Free Lunch'' theorem. It stats that the optimal methods could not have the capability to address the issue of all kinds of mathematical standard function and inverse problems. Eberhart was the first to introduce an inertia weight term to the velocity update equation to establish a trade-off between the candidates' global and local exploring capabilities, respectively [\[3\]. T](#page-10-2)he learning updating factor attracts particles toward the current personal experience location and the current experience which the particle has globally location correspondingly and cooperate with other particles thereby speeding up the convergence while the varied control factor maintain a suitable alignment among the candidates in the PSO process [\[4\]. Th](#page-10-3)e literature shows that modifications in these three parameters play a primary role in establishing a proper weighing scale amongst the population candidates.

However, In the outdated PSO version, basic parameters values are not properly set to keep diversity among the candidates, where the candidates have no energy to find a feasible solution in the search region. Thus, it is essential to properly fix the main three parameter values in a novel way. Recently, the randomized values of the basic factors in PSO process are the need of the day. These days, scholars and researchers having tried to present appropriate model for the parameters in PSO algorithm, it is observed from the previous research work that novel formulations of inertia weight and for the learning factors make the algorithm more capable to solve multimodal and complicated and complex problems.

In this regard, our work demonstrates a novel formulation and mathematical for the inertia weight. Furthermore, an elite candidates participate in the update model where the new best particles or elite candidates make the search process to solve multimodal optimization problems.

Our novel and proposed PSO maintains proper alignment between the candidates during evolution. For performance comparison, our method is applied on different mathematical optimization problems including electromagnetic design problem. Observing the numerical analyses and calculations and the success rate of convergence shows that the quality rate of our PSO is the best comparable to other optimal methods. The major parts of the paper are organized as under. The paper's first part deals with the introduction's brief discussion. The basic PSO was reported in part two, and the third part of the paper provides the comprehensive reviews of PSO methods.

The proposed model and formulation of the novel PSO are presented in the fourth part of the paper. The part five

explained experimental tabulation and trajectory curves of the various optimization problems. Comparison of various optimization techniques results of inverse problem were demonstrated in the sixth part of the paper. At the end we conclude the paper.

II. PARTICLE SWARM OPTIMIZATION

PSO is an intelligent, population-based search algorithm originally developed by Kennedy and Eberhart [\[1\]. Co](#page-10-0)nsequently, the scientists further described and explained the social and psychological patterns of crowding in the PSO model. In the search process, each bird or fish has its location and speed.

In the PSO every birds and fishes behaves as particle and every particle is solution of the problems. During the iterative procedure, the particle's present location and mobility frequency establish its freshly modified position. Every particle in the PSO process stores and uses both his current best position indicated by (pbest) and the collective best position of all nearby particles represented by (gbest). Using the following equations, the velocity and the position are updated.

$$
velocity_i^{k+1} = w. velocity_i^k + c_1.r_1 \left(p_{besti}^k - x_i \right)
$$

$$
+ c_2 \cdot r_2 (g_{besti}^k - x_i) \tag{1}
$$

$$
x_i^{k+1} = x_i + velocity_i^{k+1}
$$
 (2)

where, speed is represented by V and xi represent position vectors of the particle respectively at the ith particles; Also, every particle has best experience in the search process which is indicated by *pbest* and the best the particle in the whole search space is represented by G best. For every dimension ranging from 0 to 1, the values of the parameters r1 and r2 were generated randomly. Learning variables c1 and c2 represent, respectively, the social and cognitive capabilities of the PSO. ''Inertia weight'' is the name given to the initial velocity-controlling parameter, w.

The updated history of p_{best} and gbest indicates that each particle in a search process tries to discover the best location compared to prior locations. Contrary to the velocity update equation, *p* best, gbest, and other particles deviate significantly from each other at the beginning of the evolution process. But when more newly generated particles are produced, some of them can become insignificant or equal to *p*best, gbest, or the difference between pbest and gbest. As an outcome, each particle's precise location and local maximum are the same. This phenomenon is referred to as stasis. It is impossible for any of the particles in this process to hop to some other location since they proceed at slow rates with a tendency to zero. To solve the aforementioned issues, we adjoined a new mechanism to the PSO process based on essential parameter adaptation, as well as novel and augmented parameters to the momentum update equation.

III. RELATED WORK

Wang et al. [\[5\]](#page-10-4) have suggested the concept of MLPSO, or ''Multi-Layer Particle Swarm Optimization,'' which considers more than just the classic PSO's two levels when considering a swarm. Mahmoodabadi et al. [\[6\], de](#page-10-5)veloped the

HEPSO method to improve the exploration ability with the PSO optimizer, in which two new operators have been incorporated in basic PSO. The first operator ''multi crossover mutation'' has been taken from the genetic algorithm, while the other operator is taken from the bee colony algorithm to update the position of the particles.

Yu et al. [\[7\], ha](#page-10-6)s further modified the idea of CLPSO "comprehensive learning PSO'' and gave it the name ECLPSO ''enhanced comprehensive learning PSO''. The enhancement has been done by adding two terms in CLPSO. In order to accomplish high performance exploitation, a perturbation term must initially be built into every particle's velocity update process. To faithfully trigger the perturbation-based exploitation, knowledge of the dimensional boundaries of one's optimal locations is essential. Furthermore, to foster convergence, the particles' learning probabilities are updated according to their exploitation progress and their position in the hierarchy of personal best fitness values.

Li et al. included a new smart candidate in the search process to improve the algorithm's evolutionary performance. The main idea of the proposal to introduce a weighted candidate into the search process, with the purpose to precise a more promising search direction for all particles during the optimization process. The novel candidate x_w can guide the population to achieve the more accurate direction in the search or feasible region. The advantage of new particle that it will improve the method performance and avoids premature.

Sun et al. proposed PSO (TSCPSO) [\[9\]. Th](#page-10-7)e main proposal, one is the secondary swarm and the other is the main swarm. As the particles of the secondary swarm are updated without the information from the current velocities. The particles achieve pieces of information from the dimension of the neighboring particle. The salve swarm candidates converge to solution space due to presence of mentioned property. A novel method was included in the usual PSO for the purpose to enhance the searching process of the solution space in outdated PSO [\[10\]. A](#page-10-8)skarzadeh et al. [\[11\], p](#page-10-9)roposed two different modifications in the conventional PSO process.

Furthermore, the author validated his approach to the daily optimal chiller loading problem, to reduce the overall system's power consumption. In [\[12\], t](#page-10-10)he inertia weight model was modified and a novel model was presented for the learning factors. The proposed mathematical formulation for the basic parameters has a key role in the PSO process. Consequently, the author applied the improved version of PSO on the Photovoltaic model. In [\[13\], a](#page-10-11) new approach was proposed to employ the genetic evolution with the basic PSO to guide the search process. According to his idea, the two respective layers of the candidates were participating in the optimization process, the layer which is near to more feasible solution acts as best exemplar and crowd which is remained it will guide the particles. Wang et al. [\[14\], d](#page-10-12)eveloped a new variant of PSO, which proposes three new strategies in the PSO method. According to his idea, every best neighbor shares information with the global best particle in the swarm. Due to the information sharing among the candidates, the algorithm

finds the best optimal solution and the candidates accelerated during the evolution process as result the particles converge to the global optima [\[14\].](#page-10-12)

In [\[15\], t](#page-10-13)he random swarm concept in the conventional PSO process was introduced. In the proposed idea a new formula designed for adaptive inertia weights and a novel competition operator are introduced to encourage the rate of convergence while control premature convergence process. Azab et al. [\[16\], m](#page-10-14)odified the traditional PSO using the approach data clustering. The new method overcomes the drawbacks of the average value of mentioned alphabetic (K) algorithm currently used for data clustering. According to them, it uses the neighbor particles to get each cluster centroid's optimal position. The entire swarm represents a solution to the clustering problem during the optimization process.

A. MODIFICATION OF PSO PARAMETERS

Spichakova et al. [\[17\]](#page-10-15) suggested a novel PSO in which the user will not define the learning coefficients but will ultimately be calculated by the stochastic search methods based on gravitational law. According to the proposed idea, the gravitational search algorithm is quite similar to the basic PSO algorithm, but the only difference is that the gravitational algorithm having bodies with masses, while the remaining ideas of position and velocity vectors are the same. The uniqueness of this idea is updating the PSO process by gravitational force to provide more guides to the individuals of the swarm. Juneja et al. [\[18\]](#page-10-16) developed a review article about the PSO variants, facilitating the readers of PSO fellow researchers. This study describes the basic idea of evolutionary algorithms and comparing PSO with other methods in detail. The critical parameters for the PSO update rule are discussed in detail, giving each parameter's connection [\[18\].](#page-10-16) Huang et al. using the usual PSO method where candidates share with one another main proposal of the mechanism, that they described two locations in his approach, one is the personal-best position and second are the improved cognition term, with three positions, in order to enhance search capability where the individuals easily moving and speedup for the purpose to explore more search region and escape from local optima [\[19\]. I](#page-10-17)n [\[20\], th](#page-10-18)e author introduced in PSO process to multiple groups of candidates, where every group have different tasks for the purpose to control the premature optimization process. From their work, we knew that one group or swarm is to adopt uniform explore the procedure while the dependent swarm is used to enhance exploration for quickness of the convergence. In [\[21\], m](#page-10-19)odified method was introduced where the basic idea that evolution operators are combined in usual PSO. The central idea of his work is the emotional status of every candidate in the feasible space. Furthermore, the best objective function values could be chosen from the entire search region, where the author categorize the swarm into 3 sub population. The best global particle keeps decent alignment of the candidates and escape the method from premature convergence.

In an effort to minimize the cost of localization as well as improve the appropriate algorithm in services based on location (LBS), Wang et al. [\[22\]](#page-10-20) suggested combining an enhanced PSO with RFID technology. The intent of the aforementioned method is to enhance indoor localization and avert system issues.

Yapici et al. [\[23\]](#page-10-21) added a novel strategy to the typical PSO procedure. According to his strategy, the main tenet is that it manages the optimization process in two processes, first of all the global search ability of the candidates were random while in the later iteration individuals were an intense local search. The particles become motivated to locate the global ideal the running using this two-stage methodology.

Ramiro et al. have presented a new approach to the outdated PSO process version to find the excellent solution (particle) discovering the entire solutions set population. According to their novel approach, the local optimal feasible solution evolves to choose the best results in the form linear and rotation motion. Furthermore, they use the MRI and single photon emission computed tomography image [\[21\].](#page-10-19)

Khan et al. [\[24\], p](#page-10-22)roposed a novel model for the traditional PSO method in which the learning and control parameters were updated in a new way for the purpose to empower the search method while solving the complex and complicated optimization design problem.

B. HYBRIDIZATION

In order to encourage example particles to seek larger areas and the universe, the author inserts the evolution of genes into the core PSO system. The primary principle behind the methodology is that the swarm is composed of an upper layer that creates the best example and two lower levels that imitate it. The remaining layer acts as a particle guide. As exemplars are produced using genetic operators, from which particles can learn, historical search data of the particles offers impetus for the maturation of the exemplars. The created exemplars are well-diversified and highly qualified by executing crossover, mutation, and selection on the historical understanding of particles. These procedures improve the PSO's capacity for broad searches and its effectiveness [\[13\]. T](#page-10-11)he PSO-GA algorithm was devised by Gong et al. They contend that introducing genetic operators in the PSO algorithm improves the basic technique of discovering and using the particles' search capabilities. The proposal's significance lies in its ability to improve the PSO algorithm's potential for social thought by combining it with the genetic algorithm's local search functionality. Given that the hybrid PSO-GA technique suggests a more appropriate and best optimum algorithm for locating the most effective global solution to the optimization issues $[25]$. The rivalry index is known as a psychological condition to each component of the hive, according to the novel method for counting individual differences. In the revolutionary process, the entire swarm has been separated into three subgroups, and each particle is given a unique evolutionary route depending on its emotional state and fitness value. The learning factor value is dynamically

updated during the evolution process each particular particle has evolved. Utilizing the restarting technique to replenish relevant particles and promote population variety is another imaginative feature of this concept was introduced [\[26\].](#page-10-24) A hybrid optimal strategy was reported and recorded in [\[27\].](#page-10-25)

Their method's most significant suggestion is that to boost the convergence potential of the usual PSO process, the author literally put the mechanism of the usual process with the Grey Wolf Optimizer. Although hybrid algorithmic operators have been incorporated, the combination of them possesses the properties of co-evolution and low level. The algorithm versions are implemented sequentially to find the best optimum solution in the search space throughout the optimization process $[8]$. Using a crossover operator with a fundamental PSO, Chen et al. [\[28\]](#page-11-0) established a technique. (PSOCO) with the objective of disseminating the best expertise throughout the particles and minimizing the process of early convergence. The recommended proposal updates the PSO process and maintains a healthy balance between the global and local searches utilizing two crossover operatives, the differential evolution crossover operative and new design update formation of speed vector, respectively. To improve the ability of the individuals to be exploited, a dynamic correcting conduct is additionally included [\[6\].](#page-10-5)

In summary, a lot of effort is devoted to the various directions of classical PSO's technique with the purpose to make the PSO algorithms more robust and stable but all the present variants of PSO failed to solve the high dimensional and multimodal problems and converge to local optima region. We developed a novel method to solve this issue that will handle complex and high-dimensional problems.

IV. PROPOSED APPROACH

In the past decades, many novel PSO variants were presented in the literature; whilst most of them have good searching capabilities, but the main cause that they are often stuck in local minima during the first run due to the stationary positions of the personal best and global best particles and the constant basic parameters, which causes the algorithm to converge to the local optimum region. In addition, the outdated PSO method encounters premature convergence due to perturbations in the particle trade-off when considering into account challenging, complex, and multimodal issues.

A. METHODOLOGY OF HYBRIDIZED PSO METHOD

The previous research reveals that the PSO and DE methods are regularly utilized brilliantly to solve various optimization issues in many engineering areas. The current version of PSO offers a wide range of aspects, such as ease in coding, short computing times, resiliency and rapid processing. The literature describes that premature convergence is main cause of the usual PSO and DE technique. The outdated PSO and DE algorithms converge quickly premature during the starting of the optimization process, when it solves inverse and multimodal problems. To handle the aforementioned issues, we interleaved DE operators into the PSO algorithm.

The PSO algorithm plays a crucial role in the evolution process by improving the particle exploration while the differential evolution mechanism improves the particle local searches. The new cross over limit and the best selection mechanism were combined to the outdated PSO in accordance with the suggested methodology. As the fundamental differential evolution algorithm served as the inspiration for the two tactics.

A novel strategy of exploration and exploitation should be used to address unbalance issue of the search process. The first PSO parameters are chosen to boost exploration while decreasing exploitation phenomena. Furthermore, the exploitation process could be controlled by inertia weight values. They were extracted literally from DE and incorporated into PSO design to prevent electromagnetic optimizing problems in the early stages. The paragraph that follows is an exhaustive overview of the parameter selection and hybridized global best particle selection processes used for tackling premature convergence issues in electromagnetic optimization problems.

B. ADAPTIVE PARAMETER CONTROL

Many models and approaches for global optimization have recently been parameterized, enabling them to be adjusted for a specific context. Researchers were also attempting to determine the best settings to employ for different kinds of algorithms. The previous research describes that suitable tuning of the parameter values gives and provides the best feasible solution during the iterative process. From earlier work we conclude that the constant value of inertia weight converges prematurely, once PSO deals high-dimensional and multimodal optimization issues.

As the constant value of inertia weight is unable to push the candidates to the optimal region especially when the algorithm deals with dynamic problems due to the outdated memory of the particles. During the evolution process, the balance of the candidates is disordered due to diversity loss of the algorithm and the PSO converge prematurely [\[29\].](#page-11-1) Also, in the literature, various scientists developed different variants of control parameter formulations but none of this is performing well for inverse and multimodal optimization problems.

In this regard, the adaptive mechanism was proposed for the control parameters to incorporate it with the new hybrid strategy. The system automatically identifies the best optimum solution all over the search process uses search results as input in the adaptable characteristics. In this study, the w_ i parameter was given a unique formulation, and the values of the adaptive parameter have been determined by dividing the absolute difference between ith objective function fitness value and neighborhood fitness to the square of the global maximum fitness in the current iteration index of population.

$$
W_i = \frac{rand()}{2} + \frac{|f(X_i) - f(X_j)|}{(f(g_{best} - value))^2}
$$
 (3)

 X_i neighbor particle of X_i

C. HYBRIDIZED GLOBAL BEST PARTICLE SELECTION USING THRESHOLD PARAMETER

We introduced a threshold selection value to divide the current swarm into two subpopulations to explore more search space and get dynamic information about the particles. The candidates whose objective function values are less than the threshold value it will be considered a bad subpopulation, and the reminders of a good subpopulation. Based on this threshold value, we pick the elite particles for the next generation during the search process.

$$
U = h \times \sum_{i=1}^{m} pop_value_i / m
$$
 (4)

where pop_valuei is the objective function value of each candidate i, m is the size of the population, the value of h is set to 0.05 by error and trail method. The given threshold value gathers the dynamic information of the candidates which plays a primary role in the evaluation process.

In the literature one can easily found that the values of personal best and global best candidates directly affect the search process, the more suitable value improves the evolution process, and the candidates easily find the global optimal space in the search region. The existing PSO velocity update formulation reveals that at the initial stages of the evolution process the gap between personal best individual candidates, global best candidates and other individuals increased during the optimization procedure. In the PSO process number of particles was increasing and as consequence, the particles may be equal to personal best experience, global best experience of the candidates, the same situation could have explained in such way that during the iterative process the gap is minimum between the personal and global candidates or any individual which are evolving in the evolution process. It means diversity of the swarm is nearly zero. We incorporated the two new operator's mutation and cross over to address the above issue. The main idea of the mechanism that trial vector is generated in a novel way on the bases of mutation strategy. The trial particles X∗ is the sum of the global best particle in the whole feasible region, namely, ''gbest'' and the difference of best candidate and worst candidates chosen from the current population based on threshold value.

$$
X* = \text{gbesti} + \text{rand}() * (\text{Xbest} - \text{Xworst}) \tag{5}
$$

The new global best particle is achieved through a novel crossover mechanism that is inspired by the DE algorithm. It randomly swaps vector components between a previous global best particle vector and X∗ vector to produce a new global best particle. The novel best particle participates in the velocity update formulation on the bases of the following condition.

$$
Gbest^* = X^*if \ U \le C \tag{6}
$$

Previous global best otherwise (7)

From the above mathematical, we conclude that the new global best particle is chosen based on the crossover method inspired by the traditional DE method. The old global best particle will be replaced at every generation by the new one,

while the new global best particle is chosen according to the crossover strategy, where the crossover rate value is fixed by 0.7 and the threshold value is randomly varied according to its formulation. According to the said approach, the novel global best particle behaves like the best leader which could make the search process more effective. Consider an intuition or any organization an example, where the intelligent and smart Head of a department performs well and guides all the staff members politely as compared to the layman. The above methodology uses a crossover mechanism having the main feature to design a smart leader at every iteration during the searching process.

To strengthen the PSO performance in relationships of solution quality and convergence speed different methods are used to produce and improve the position update equation of PSO. Moreover, the modified global best particle provides and shares information with the best particles due to the randomized candidates which are selected through crossover rate and threshold rate. The crossover value is fixed 0.5 by error and trial method while the threshold value is randomly varied at every generation. In the interim, it also reminiscences the current individual positions and specifically swarm particles that are trapped into local minima. By using the proposed notion, the global particle quickly attracts a swarm toward the global minima which results in the particle could reach the global minima instead of trapping.

The global best particles have a range of abilities that could really enable other candidates during the evolution process. The suggested approach will also help the process be able to successfully educate the particles in the next generations. The particle that adopts the proposed methodology will reach the global optimum region during the optimization process, as contrary to other techniques and algorithms. The proposed PSO takes significantly less time to implement and compute when compared to other well-developed techniques and other ideal algorithms.

V. MATHNUMERICAL RESULT ANALYSIS

The proposed modified PSO was compared with well-known strategies and methods for the purpose to understand the performance, stability and robustness of the proposed algorithm. All the parameters (the basic control parameter, cognitive factor, social factor, and iteration number swarm size and dimension problem) were set accordingly same. In this work we use very famous optimization problems as shown in the table [1.](#page-5-0) The optimal algorithms were validated on above table [1,](#page-5-0) typically all these problems are considered as standard problems in the field of computer science and engineering. For this research we choose ten benchmark model to judge the effectiveness of proposed method. For performance comparison of the various strategies along with our approach we consider unimodal and multimodal problems, and our hybrid model was compared to the different PSO's variants such as, GPSO [\[32\], A](#page-11-2)MPSO [\[30\], M](#page-11-3)PSOED [\[24\],](#page-10-22) GCMPSO [\[31\]](#page-11-4) and MPSOEG [\[34\].](#page-11-5)

TABLE 1. Bench mark optimization problems.

Table [2](#page-7-0) demonstrates the best objective function, mean, variance, and worst values of the different strategies and algorithms. Also, the comparison graph of the different algorithms was presented and indicated from 1∼10.In summary, we can say that our novel approach performs well as compared to ones.

A. COMPARISON OF DIFFERENT PSO ALGORITHM

Viewing the comparison analyses of the optimal algorithms, we categorized the optimal algorithm into three different categories on the bases of their performances.

Category One: If an algorithm's final outcome enormously surpasses that of its counterpart, it is commonly referred to as a category ''One Algorithm'';

Category 2: If an algorithm's final answer does not significantly improve upon that of its competitors, it is often referred to as a ''Category 2 Algorithm'';

Category three: If an algorithm's outcome is weaker than its competitors, it will be considered a category three algorithm.

The efficacy analyses of different approaches for 100-dimensional problems are shown in Table [2.](#page-7-0) These categories' definitions and findings from Table [2](#page-7-0) lead to the subsequent outcomes:

- The proposed MPSO is a type one algorithm for test functions f_1 , among all algorithms;
- The MPSO-ED is a type two algorithm for test functions *f*¹ among all algorithms.
- For the test function f_1 , the performance of AMPSO and the other optimal algorithm are not well. So, it is considered type category two algorithm.
- The MPSO for second mathematical model performs well and GCMPSO, MPSOED shows good results while the AMPSO, GPSO and MPSOEG shows weak performance when compared to the proposed PSO.
- For the test function f_3 the Modified PSO and AMPSO, MPSO-ED shows best performance compared to GCMPSO and GPSO. From the result we conclude that MPSO, AMPSO and MPSO-ED are type one algorithm means category one.
- Also, viewing the calculation of the mathematical test function f_4 , we observe that modified PSO finds the optimal solution while other methods converge prematurely. Obviously, our novel approach shows well on the test function *f*4.
- The mathematical model of f_5 illustrates that our hybrid PSO has the best performance compared to AMPSO, GPSO, and GCMPSO. While the MPSO_ED shows also little bit of good performance.
- The statistical analysis of the benchmark problem f_6 indicates that our novel approach and MPSO_ED come in the first category while other algorithms perform badly so some of them, come in the second category and some in the third.
- Considering the mean value of optimization problem *f*⁷ then we knew from the table results that our hybrid optimal algorithm has the ability for solving complex and complicated problems, MPSO_ED and modified MPSO are considered type one optimal algorithms. While the remaining algorithms are considered type two and type three based on their statistical results.
- The proposed approach illustrates an interesting result on the happy cat function. The GCMPSO, and

MPSO_ED are considered in the second cate-gory for the said benchmark problem, while the remaining optimal methods come in the third category.

• The mean value of the two MPSO and MPSO-ED for the Griewank optimization problem are approximately the same. It means that the suggested method comes in the first category.

While MPSO-EG are considered in the second category on the bases of their results. The above observation demonstrates that our hybrid model PSO is appropriate to find the feasible solution of the complex design problems compared to the others. Similarly, the result of the six-test model demonstrates that our novel approach has good results compared to the AMPSO and other well-known strategies. Considering the statistical calculation of the mathematical test function *f*⁷ our proposed approach comes in the first category while the MPSO_ED and GCMPSO also shows and illustrates best performance as compared to global particle swarm optimization and adaptive model of PSO. In other words, the GPSO indicates worst performance among all the algorithms.

If we observe the best objective function, mean, variance and worst values of the mathematical test function f_8 then we conclude that our hybrid method comes in the first class as compared to other optimal techniques which illustrates the stability and robust ness of our novel method. Also, the results of GCMPSO, MPSO_ED and MPSO EG good and due to this it comes in second category.

Also, the novel method illustrates best results on the mathematical test function f_9 and f_{10} .

B. CONVERGENCE PERFORMANCE OF DIFFERENT OPTIMAL ALGORITHM

We investigated the convergence graphs of the optimum methods to verify the proposed PSO's performance. The various figures depict the convergence traits of several optimum algorithms. The following findings are typical ones. Considering the convergence plot of the test function f_1 , we observe that our proposed modified PSO converges to the optimal solution at the end of generations while the AMPSO, MPSOED, GCMPSO, MPSO EG could not finds the optimal solution throughout the search process and GPSO also shows better performance as compared to the other optimal algorithms. From the convergence plot of the test function f_2 it is clear that the MPSO-ED, GCMPSO, GPSO and MPSOEG cannot reach the optimal point during the whole search process. Form the plot we observe that only MPSO finds the feasible region and also the GPSO shows little bit good performance. Viewing the curve of the third model we observe that our new method obtains the feasible solution of the search space after 1000 generations. On other hand the methods namely ''GCMPSO'' and ''MPSOED'' illustrated significant performance for the third model of the function. At the same time, the remaining optimal algorithms perform worst during the whole search process. Observing the fourth mathematical model trajectory curve or plot shows, that only our hybrid method converges to the more feasible solution

TABLE 2. Comparison results of mathematical test function using different search algorithms.

space while the compared optimization never escapes from the local optima. The plot comparison curve of the mathematical test functions of five, six and eight demonstrated the performance of the compared algorithms, where we concluded that all the optimal algorithms balance are disordered during the evolution and the compared algorithms are failed to finds the optimal solution of the given or mentioned optimization problems. Only our novel hybrid method has the ability to optimize the mentioned problems. From the foregoing information, it is readily apparent that the novel PSO technique executes well for high dimensional optimization problems when compared to previous optimum algorithms.

It also exhibits exceptional performance when compared to the well-established, proven inertia weight schemes.

FIGURE 1. Convergence Curve of various algorithms of test f1.

FIGURE 2. Convergence Curve of various algorithms of test f2.

FIGURE 3. Convergence curve of various algorithms of test f3.

VI. APPLICATIONS

In electrical engineering, the researchers and scientists use problem 22, which is one of most complicated and multimodal problems and generally serves as standard problem for the validation of various optimal techniques, especially in

FIGURE 4. Convergence curve of various algorithms of test f4.

FIGURE 5. Convergence curve of various algorithms of test f5.

FIGURE 6. Convergence curve of various algorithms of test f6.

the field electromagnetic assessment. The optimization structure of a superconducting magnetic energy storage (SMES) illustrated in the figure *[11](#page-10-27)*. The problem definition and optimization structure are like that's a three-parameter optimization problem. To fully comprehend the main conditions of the envisioned application, the generated magnetic field must also comply with another criterion known as the quench

FIGURE 7. Convergence curve of various algorithms of test f7.

FIGURE 8. Convergence curve of various algorithms of test f8.

FIGURE 9. Convergence curve of various algorithms of test f9.

condition, it demonstrates that the appliance will be equipped with a 180-kilowatt-hour power reserve capacity. Control the variability of the coupling of the field lines to the magnetic field. The following cost objective function and constraint's

FIGURE 10. Convergence curve of various algorithms of test f10.

TABLE 3. Comparison results of inverse problem using different search algorithms.

ALGORITH	AVERAGE	$R2 \in [2.6]$	$H2/2 \in [0.204,$	$D2 \in [0.1, 0.4]$
M	OBJECTIV	, 3.4]	1.11	
	E			
	FUNCTIO			
	N			
AMPSO	0.1207	3.0052	0.8265	0.2786
GCMPSO	0.851	2.7619	0.2037	0.3192
MPSO ED	0.283	2.6927	0.5729	0.3382
MPSO	0.0712	2.0513	0.2028	0.2739

structure can be used as model.

$$
\min f = B_{\text{stary}}^2 / \left| B_{\text{norm}}^2 \right| + \left| E - E_{\text{ref}} \right| / E_{\text{ref}}
$$
\n
$$
\text{subject to } Ji \le (-6.4 \left| (B_{\text{max}})_i \right| + 56(A / \text{mm}^2)(i = 1, 2) \tag{8}
$$

According to the previously discussed mathematical model, and corresponds to the linkage, and these can be established at 22 points that are equidistant using the line a and line b, shown in Fig. [11](#page-10-27) while laid out as and is the amount of magnetic energy held in the winding as opposed to reference energy, which is represented by *E*ref and is the optimum magnetic flux level of exertion and current density that exists in the ith coil, accordingly

$$
B_{\text{stray}}^2 = \sum_{i=1}^{22} |B_{\text{stray},i}|^2 / 22 \tag{9}
$$

In the present analysis, we're applying the methodology of finite elements to determine the values for the function that defines the objective model and B stray model.

For the experimental analyses, the total size of the searching process is fixed to 15, the maximum iteration number is fixed at 50, and the search space of the optimization problem runs in the decision number is 3. All the optimal methods run equally in order to fair compared their mean values. The optimized values of inverse problems, In Table [3,](#page-9-0) measurements of height $(h₂/2)$, diameter $(d₂)$, and radius $(r₂)$ have been depicted and tabulated. As an outcome, the average data provided information on how successfully every method performed. The average values of the algorithms are shown in Table *[3](#page-9-0)* after all search techniques were simulated in

a soft math lab environment. Our research clearly illustrates and demonstrates that the functionality and performance of the proposed modified PSO is preferable to the other ones.

FIGURE 11. The schematic diagram of a SMES.

VII. CONCLUSION

In the proposed work, we designed a novel mechanism for the inertia weight and introduced elite candidates to the velocity update model to improve the search process. The novel elite or best candidates are inspired by the differential evolution strategies, which uses crossover mechanism and mutation vector, where the innovative best particle directs the whole search and aids the particles in eluding local optima. In addition, the adaptive parameter (inertia weight) ensures an excellent balance between the particles' potential for exploration and exploitation searches. The Table [3](#page-9-0) demonstrates that the modified PSO that has been developed is more suited for high-dimensional, intricate, and challenging optimization and inverse tasks.

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Authors declare no conflict for this research.

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