IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 17 October 2023, accepted 28 October 2023, date of publication 1 November 2023, date of current version 10 November 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3329242

RESEARCH ARTICLE

Circular Spherical Fuzzy Sugeno Weber Aggregation Operators: A Novel Uncertain Approach for Adaption a Programming Language for Social Media Platform

SHAHZAIB ASHRAF[1], WANIA IQBAL[1], SHAKOOR AHMAD[2], AND FAISAL KHAN[3]
[1]Institute of Mathematics, Khwaja Fareed University of Engineering and Information Technology, Rahim Yar Khan 64200, Pakistan
[2]Department of Mathematics, COMSATS University Islamabad, Islamabad Campus, Islamabad 45550, Pakistan
[3]College of Engineering and Informatics, National University of Ireland Galway, Galway, 91 CF50 Ireland

Corresponding authors: Shahzaib Ashraf (shahzaib.ashraf@kfueit.edu.pk) and Faisal Khan (f.khan4@nuigalway.ie)

This work was supported by the National University of Ireland Galway, Ireland.

ABSTRACT This study's major purpose is to highlight circular spherical fuzzy sets, that happens to be a prolongation of spherical fuzzy sets. The primary purpose of this research is to demonstrate the basic operations and theorems of circular spherical fuzzy structures (C-SFS), which give an effective way for dealing with data ambiguity. Aggregation operators (AOs) play a significant role in decision-making, particularly in situations where conflicting interests need to be taken into account. The Sugeno-Weber (SW) t-conorm and t-norm are employed in the C-SFS operating rules. The study describes in detail the fundamental operating criteria for C-SFS utilizing SW t-norms and t-conorms, as well as their crucial features. Furthermore, this research presents and fully investigates two novel operators, C-SFS Sugeno Weber weighted averaging (C-SFSWWA) and C-SFS Sugeno-Weber weighted geometric (C-SFSWWG), as well as their distinct applications and desired properties. A novel approach based on the C-SFSWWA and C-SFSWWG operators is suggested to address multiple attribute decision-making (MADM) problems utilizing C-SF information. A numerical example shows how this approach may be used to adapt a programming language for social media platform analytics, followed by a comparison study to highlight its advantages. The advised approach is successful, according to an investigation of authenticity and a comparison study. In reality, the recommended aggregation operators and decision-making approach are quite useful for decision analysis.

INDEX TERMS Fuzzy set, circular intuitionistic fuzzy set, sugeno weber, decision making.

I. INTRODUCTION

The 21st century has witnessed a dramatic transformation in communication and information dissemination, largely driven by the rise of social media. A bunch of online communities and web pages together referred to as ''social media'' enables people to produce, collaborate, and interact with content in a fake social environment. These platforms enable communication, networking, and information sharing among individuals, groups, enterprises, and communities all over the world. By providing businesses and brands with previously unheard-of opportunities to interact and engage with their target population, social media has fundamentally altered the marketing sector. Utilizing a number of social media platforms, social media marketing works to build client relationships while promoting products, services, and content. Businesses have the opportunity to interact with a huge global audience on social media, which has billions of active users across a number of platforms, and increase their visibility. It is now a vital part of modern marketing

The associate editor coordinating the review of this manuscript and approving it for publication was Yu-Da Lin.

© 2023 The Authors. This work is licensed under a Creative Commons Attribution 4.0 License.
For more information, see https://creativecommons.org/licenses/by/4.0/

strategies, giving businesses unparalleled opportunities to engage, communicate with, and influence their target market. As social media matures, society must navigate its challenges in order to fully realize its promise of connection and positive impact.

A social media analytics platform is required for a number of reasons, including the fact that it provides important insights and data-driven information that may tremendously benefit individuals, businesses, and organizations. It is a helpful tool that helps businesses and individuals to utilize the enormous volumes of data available on social media networks. It offers useful knowledge, facilitates decision-making, enhances marketing strategies, and strengthens customer relationships, all of which help firms to grow and succeed. Individuals and organizations interested in harnessing the potential of social media data may profit from a software company that offers a social media analytic platform application. The problem arises from the high volume of data and traffic on the application, which is causing issues. The firm wanted to transition from programming language of application i.e. Python to GoLang and considering some alternatives over some specific criterions. Compared to interpreted languages like Python, GoLang's execution speeds is faster since it immediately compiles to machine code. Thanks to the speed optimization of the compilation process, GoLang is a viable option for performance-critical applications. GoLang's sophisticated memory management and garbage collection system, which helps keep the memory footprint low, enable applications to handle a high number of concurrent connections and expand more successfully. Due to its low memory usage, it is particularly scalable.

The firm has taken into consideration some parameters over specific criterions, thus the problem can be tackle by decision making. Making choices with the parameters of fuzzy sets refers to the act of picking options or outcomes based on fuzzy logic principles. Fuzzy logic is highly useful in decision-making scenarios involving ambiguous, imprecise, or confusing facts. It provides more flexible and human-like decision-making by including language phrases and coping with ambiguity. Companies establish from DM opinions collects and ranks a range of view points ranging from great towards the most adverse possibilities on a regular basis. As a result, we may choose, categorize, develop, and conduct a thorough inquiry.

Multi-attribute decision-making (MADM) is the most effective technique for obtaining a significantly better result by considering all relevant aspects or criteria. The fundamental goal of overcoming the dilemmas in evaluation and taking decisions is to categorize and collect data for indices of judgment. But, because of the complexity of real-world systems, humans need to cope with a wide range of MADM challenges when the assessment information is ambiguous. The problem is coped by Zadeh [1], he introduced fuzzy sets in 1965 that deals with ambiguous information. Elements have degree of belonging-ness that range from [0,1]. Later on, Atanassov [2] gave the more extended idea on FS i.e. IFS. He added the concept of non-belonging-ness in FS. Xu [3] investigated aggregation operators, while Luo [4], [5] contributed to the concept of distance in his study. Other fuzzy concepts like q-rung fuzzy logic in optimizing urban parcel delivery strategies are explored in [6] and logarithmic bipolar fuzzy in [7].

Another genuine expansion of IFSs is suggested, whereby a circular region is allocated with a straightforward representation rather than a rectangle. It goes by the name Circular IFS (C-IFS). Atanassov [8] suggested the Circular IFS (C-IFS). A circle depicts the ambiguity of the belonging as well as non-belonging grades in a C-IFS. In other words, a circle with pair of non-negative real numbers in the center, given that their sum is less than one, represents the belonging and non-belonging grades of all parts to a C-IFS. C-IFSs provide more diplomatic management of modifications to both belonging and non-belonging degrees to indicate ambiguity. He also presented distances measures for C-IFSs [9]. Boltrk [10] proposed idea of interval valued in it and Cakir [11] proposed DM in it. As a result, circular intuitionistic fuzzy environments have been used with a variety of MCDM approach types [12], [13]. Oaty and Kahraman [14] extended AHP and VIKOR methods for C-IFSs and utilized them in the multi-expert supplier evaluation problem [15]. Chen [16], [17] explored distance in C-IFS. Khan and Kumam presented divergence measure for C-IFS and their application [18].

Despite the widespread interest in Atanassov's development of IFSs, decision-makers encounter constraints in assigning degrees of membership and non-membership. These restrictions complicated the decision-making process. Because, according to certain real-life choice theories, decision makers deal with scenarios involving distinct characteristics where the sum of their belonging degrees exceeds 1. In such a case, IFS will be unable to deliver an appropriate result. Then to tackle this hurdle, idea of PyFS is given by Yager [19] and some results [20], [21]. It meets the constraint that the value of the square sum of its grades is below or equal to one.i.e.$0 \leq \vartheta^2 + \kappa^2 \leq 1$. Olgun introduced PyF points [22], Peng presented PyFS results [23], Einstein operations [24] and norms were explored [25], and an extended TOPSIS method was proposed [26]. The study also covered TODIM [27], similarity measures [28], [29], and Biswas' work on TOPSIS with entropy [30]. Rahman investigated interval values in PyF [31] and Pythagorean vague normal operators are discussed in [32] by Palanikumar. These works have advanced decision-making approaches.

Just as PyFS are the elongation of IFS concept, similarly Murat and Bozyigit [33] proposed the notion of circular in PyFS. He represented the idea of belonging and non-belonging grades in term of cricle with condition $0 \leq \vartheta^2 + \kappa^2 \leq 1$. He proposed T-norms and T-conorms, which are utilized for describing numerous algebraic procedures for C-PFVs. A number of weighted averaging and geometric aggregation operations have been devised using these approaches. Khan and Kumam [34] expanded C-PyFS to disc PyFS in 2023.

Ashraf et al. [35] presented the more broader concept of SFS, taken into account of beloning degree, non- belonging degree and abstinence and the criteria $0 \leq \vartheta^2 + \iota^2 + \kappa^2 \leq 1$. This framework developed by Ashraf is significantly more in line with the traits of people than previous theories, resulting in one of a great deal of active fields in academia right now. Aggregation operators (AOs) play a big part in decision-making problems (DMPs). A number of academics have made rather significant contributions to the development of AOs for SF sets. Riaz investigated supply chain management in [36]. In his work, Ashraf explored t-conorms and t-norms, presented aggregation techniques [37] and its representation [38]. The WASPAS [39] and TOPSIS [40] techniques were introduced by Gundogdu. Mahmood's study [41] centered on making choices regarding diagnoses in medicine. While Ozceylan conducted a survey on SF information [42] and Peng concentrated on the subject of IIoT [43]. The concept of similarity and distance in SFS was developed by Khan [44].

This paper's main goal is to introduce a more generic representation of SFSs in order to enhance decision-making. Utilizing exact points, pairs of points, or triples of points from the closed interval [0, 1] is a common practice when working with fuzzy sets. While offering a rigid decision-making process, these techniques are in need that decision-makers (DMs) assign exact numerical values. Contrarily, Interval Valued (IV) SFSs provide for the flexible assignment of intervals for degrees of membership, non-membership, and abstention, but managing their representation can be difficult. A true extension of spherical fuzzy sets (SFSs) that uses circles and a more straightforward representation is proposed in this study to solve this.

We put forward the framework of circle in spherical fuzzy sets in this work. In this innovative fuzzy set concept, the degrees of membership, abstention, and non-membership for an element are represented by circles with centers $(\vartheta, \iota, \kappa)$ unlike integers, and with a more flexible state $0 \leq \vartheta^2 + \iota^2 + \kappa^2 \leq 1$. By employing this approach, we expand both the concept of C-IFS and C-PFS, in addition to enhancing the understanding of SFS concepts. Decision-making has grown more difficult as DMs can now acquire circles with certain characteristics rather than precise numbers. Instead of using exact numerical values, this feature enables DMs to design circles with certain attributes. In turn, this makes the decision-making process more sensitive and flexible, enabling DMs to handle uncertainties and difficult.

Sugeno introduced a class of nilpotent t-conorms in his PhD study [45], featuring the asymptotic components drastic, probabilistic sum. Weber [46], on the other hand, suggested a class of nilpotent t-norms with parametric elements product and drastic product. Both the Sugeno τ-CN with parameter λ and Weber τ-N with parameter ξ are identical to each other in the perspective of families τ-N and τ-CN. Both families are known as Sugeno-Weber t-norms and t-conorms (SW τ-N and τ-CN) in honor of Sugeno and Weber because of this duality. In his paper, Sarkar et al. [47] employed SW norms to define T-SFHySSs. When employing the variable parameter ξ in the SW τ-N and τ-CN, decision-makers (DM) have greater capacity since it enables them to accurately modify the parameter's value. As a consequence, the SW τ-N and τ-CN appear to be ideal for generating C-SFS operations and eliminating blunders and unnecessary information.

An AO is a methodical mathematical representation that is used in data analysis to integrate numerous pieces of evidence into a single data format, enabling rational decision-making in a variety of situations. Sugeno-Weber (SW)-based AOs are highly rated among the well-known classical AOs. The use of these AOs in decision-making processes poses a few challenges though, as they might not always yield the precise result that the decision-makers are looking for. In order to better meet the demands of certain decision-making processes, various AOs must be adapted and updated to solve these issues. In order to assess if SW AOs will motivate the continuing research and limits of C-SFSs mentioned previously, we shall offer SW AOs based on early data. The primary objectives of the article are as follows:

- Introduce the concept of circular spherical fuzzy sets (C-SFSs) to broaden the scope of the present framework for spherical fuzzy sets.
- To enhance the management of uncertainty, develop a technique for converting spherical fuzzy information into a circular spherical fuzzy structure.
- Establish basic operations of C-SF information using the Sugeno-Weber norms to lay the groundwork for further research.
- Sugeno-Weber norm is used to suggest novel C-SF aggregation techniques, and their properties are examined to see how useful they are in decision-making settings.
- Demonstrate a Multiple-Attribute Decision Making (MADM) method using the SW aggregation operators in the context of the C-SF information framework.
- To handle uncertainty and ambiguity in decision-making, compare the recommended SW aggregating operators to other existing aggregation operators.

By achieving these objectives, the project seeks to enhance decision making for addressing uncertainty in real-world.

The following serves as a basis for this paper's superfluity:

The research starts with an introductory section (Section I) that outlines context and creates the tone for the rest of the investigation. In Section II, basic definitions are given to help with the following topics. The third section gives important premise and supporting evidence while introducing Circular Spherical Fuzzy Sets (C-SFS), exploring different C-SFS operations. The scoring and accuracy measures that may be used to compare C-SFSs are also covered in this section. The computation of C-SFS radii is thoroughly discussed in section IV. Section V of the research examines Sugeno Weber (SW) norm operations and related identities and proofs. Section VI and VII discuss SW weighted geometric aggregation and weighted averaging methods.

A Multiple-Attribute Decision-Making (MADM) approach is also shown in section VIII, providing an algorithm that efficiently completes C-SFS aggregating operations. To help with comprehension, a real-world example is provided. A thorough comparison of all the methods is done in section IX. Finally, section X highlights the important conclusions and contributions of the study.

II. PRELIMINARIES

This section describes several important concepts related to C-IFS, C-PFS and S-IFS. These concepts lay a solid foundation for further exploration and analysis in the research and are essential for comprehending the complexity of the three fuzzy set frameworks. By examining these essential concepts, we establish a full framework, which lays the foundation for insightful discussions and informed interpretations.

A. CONCEPT OF C-IFS AND C-PFS

Definition 1 ([8]): The set χ be the subset of $\Im$. Then C-IFS is

$$\chi = \{(\ddot{z}, \vartheta_\chi(\ddot{z}), \kappa_\chi(\ddot{z}); r)|\ddot{z} \in \Im\} \tag{1}$$

where $\vartheta_\chi : \Im \to [0,1], \kappa_\chi : \Im \to [0,1]$ indicating MG, NMG and condition $0 \leq \vartheta_\chi(\ddot{z}) + \kappa_\chi(\ddot{z}) \leq 1$ and $r \in [0,1]$ is the radius of every element's circle.

Every element in demonstrated by a circle with a center $(\vartheta_\chi(\ddot{z}), \kappa_\chi(\ddot{z}))$ and radius r as opposed to the normal every point in the IFS indicates an element in the intuitionistic fuzzy interpretation triangle.

Definition 2 ([33]): Let $\Im$ be the universal set and $r \in [0,1]$. The C-PFS in $\Im$ is given by

$$\chi = \{(\ddot{z}, \vartheta_\chi(\ddot{z}), \kappa_\chi(\ddot{z}); r)|\ddot{z} \in \Im\}$$

often referred as a spherical fuzzy set, where $\vartheta_\chi : \Im \to [0,1], \kappa_\chi : \Im \to [0,1]$ are MG, NMG respectively, satisfying the condition below

$$0 \leq (\vartheta_\chi(\ddot{z}))^2 + (\kappa_\chi(\ddot{z}))^2 \leq 1$$

The point $(\vartheta_\chi(\ddot{z}), \kappa_\chi(\ddot{z}))$ on the plane is the radius of circle r.

B. OPERATIONS OF C-PFS

Consider two C-PFS $\alpha_{\check{r}_1}$ and $\beta_{\check{r}_2}$ in $\Im$:

$$\alpha_{\check{r}_1} = \left\{(\ddot{z}, \vartheta_\alpha(\ddot{z}), \kappa_\alpha(\ddot{z}); \check{r}_1|\ddot{z} \in \Im)\right\}$$
$$\beta_{\check{r}_2} = \left\{(\ddot{z}, \vartheta_\beta(\ddot{z}), \kappa_\beta(\ddot{z}); \check{r}_2|\ddot{z} \in \Im)\right\}$$

Given below are the definitions of different operations on C-PFSs:

1) $\alpha^c_{\check{r}_1} = \left\{(\ddot{z}, \kappa_\alpha(\ddot{z}), \vartheta_\alpha; \check{r}_1|\ddot{z} \in \Im)\right\}$.
2) $\alpha_{\check{r}_1} \subset \beta_{\check{r}_2}$ iff $\check{r}_1 \leq \check{r}_2$ and $\vartheta_\alpha \leq \vartheta_\beta$ and $\kappa_\alpha \geq \kappa_\beta$.
3) $\alpha_{\check{r}_1} = \beta_{\check{r}_2}$ iff $\check{r}_1 = \check{r}_2$ and $\vartheta_\alpha = \vartheta_\beta$, and $\kappa_\alpha = \kappa_\beta$.
4) $\alpha_{\check{r}_1} \cup_{min} \beta_{\check{r}_2} = \left\{(\ddot{z}, max(\vartheta_\alpha, \vartheta_\beta), min(\kappa_\alpha, \kappa_\beta); min(\check{r}_1, \check{r}_2)|\ddot{z} \in \Im)\right\}$.
5) $\alpha_{\check{r}_1} \cup_{max} \beta_{\check{r}_2} = \left\{(\ddot{z}, max(\vartheta_\alpha, \vartheta_\beta), min(\kappa_\alpha, \kappa_\beta); max(\check{r}_1, \check{r}_2)|\ddot{z} \in \Im)\right\}$.
6) $\alpha_{\check{r}_1} \cap_{min} \beta_{\check{r}_2} = \left\{(\ddot{z}, min(\vartheta_\alpha, \vartheta_\beta), max(\kappa_\alpha(\ddot{z}), \kappa_\beta); min(\check{r}_1, \check{r}_2)|\ddot{z} \in \Im)\right\}$.
7) $\alpha_{\check{r}_1} \cap_{max} \beta_{\check{r}_2} = \left\{(\ddot{z}, min(\vartheta_\alpha, \vartheta_\beta), max(\kappa_\alpha, \kappa_\beta); max(\check{r}_1, \check{r}_2)|\ddot{z} \in \Im)\right\}$.

Definition 3 ([35]): If $\Im$ is the universe of discourse, then

$$\chi = \left\{(\ddot{z}, \vartheta_\chi(\ddot{z}), \iota_\chi(\ddot{z}), \kappa_\chi(\ddot{z}))|\ddot{z} \in \Im\right\} \tag{2}$$

often referred as a spherical fuzzy set and $\vartheta_\chi, \iota_\chi, \kappa_\chi : \Im \to [0,1]$ are MG, Abstinenace, NMG respectively, fulfilling the criteria below

$$0 \leq (\vartheta_\chi(\ddot{z})^2 + \iota_\chi(\ddot{z})^2 + \kappa_\chi(\ddot{z}))^2 \leq 1 \tag{3}$$

For $\{(\ddot{z}, \vartheta_\chi(\ddot{z}), \iota_\chi(\ddot{z}), \kappa_\chi(\ddot{z}))|\ddot{z} \in \Im\}$, SFN are $(\vartheta_\chi(\ddot{z}), \iota_\chi(\ddot{z}), \kappa_\chi(\ddot{z}))$.

C. SUGENO-WEBER τ-N AND τ-CN [47]

During the early 1970s τ-CNs are developed by Michio Sugeno and in the early of 1980s, SW τ-Ns family are developed by Seigfried Weber.

Definition 4: The part T^ξ_{SW} of SW τ-Ns is given by

$$T^\xi_{SW}(a,b) = \begin{cases} T_{\bar{D}}(a,b), & \text{if } \xi = -1, \\ max\left(0, \dfrac{a+b-1+\xi ab}{1+\xi}\right), & \text{if } -1 < \xi < +\infty, \\ T_{\bar{P}}(a,b), & \text{if } \xi = +\infty, \end{cases}$$

where $T_{\bar{D}}(a,b)$ denote the drastic τ-N and $T_{\bar{P}}(a,b)$ denote the product τ-N (or, algebraic product).

$$S^\xi_{SW}(a,b) = \begin{cases} S_{\bar{D}}(a,b), & \text{if } \xi = -1, \\ min\left(1, a+b-\dfrac{\xi ab}{1+\xi}\right), & \text{if } -1 < \xi < +\infty, \\ S_{\bar{P}}(a,b), & \text{if } \xi = +\infty, \end{cases}$$

where $S_{\bar{D}}(a,b)$ denote the drastic τ-CNs and $S_{\bar{P}}(a,b)$ denote the probabilistic sum (or, algebraic sum).

III. FORMATION OF CIRCULAR SPHERICAL FUZZY SETS

In this section, we will elaborate on the extended concept of SFS, namely Circular Spherical Fuzzy Set (C-SFS). This enhancement enhances the representation and management of uncertainty in fuzzy collections by introducing circularity to the traditional SFS framework. The C-SFS concept offers a more complete and flexible way for modeling complex and uncertain information, providing insightful data for decision-making and problem-solving in a range of sectors.

Definition 5: Let χ denote a subset of $\Im$, a universe of discourse.

$$\chi = \left\{(\ddot{z}, \vartheta_\chi(\ddot{z}), \iota_\chi(\ddot{z}), \kappa_\chi(\ddot{z}); \check{r})|\ddot{z} \in \Im\right\} \tag{4}$$

will be described as CFS where $\vartheta_\chi : \Im \to [0,1], \iota_\chi : \Im \to [0,1]\kappa_\chi : \Im \to [0,1]$ are basically MG, Abstinence, NMG. And $\vartheta_\chi, \iota_\chi, \kappa_\chi$ satisfy the criteria

$$0 \leq (\vartheta_\chi(\ddot{z}))^2 + (\iota_\chi(\ddot{z}))^2 + (\kappa_\chi(\ddot{z}))^2 \leq 1. \tag{5}$$

The radius of circle is $\check{r}$ the point $(\vartheta_\chi(\ddot{z}), \iota_\chi(\ddot{z}), \kappa_\chi(\ddot{z}))$ on the plane. The circle demonstrate the membership grade, abstinence and non-membership grade of $\ddot{z} \in \Im$.
In the triangular fuzzy interpretation, each element in SFS is depicted as a point, but all element in C-SFS is expressed as a circle with a center $(\vartheta_\chi(\ddot{z}), \iota_\chi(\ddot{z}), \kappa_\chi(\ddot{z}))$ and a radius $\check{r}$.

Each typical SFS has a standard form, hence this new type of sets are a modification to the standard SFS

$$\chi = \{(\ddot{z}, \vartheta_\chi(\ddot{z}), \iota_\chi(\ddot{z}), \kappa_\chi(\ddot{z}); \breve{0})\}$$

but the C-SFS with $\check{r} > 0$ is not in accordance with a normal SFS.

Figure 1 shows some spherical fuzzy values (SFVs) where as Figure 2 represent some points in C-SF environment.

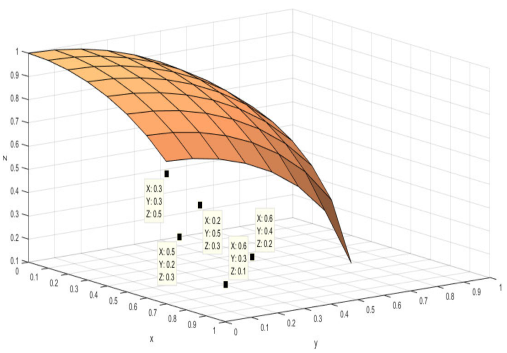

FIGURE 1. **Graphical representation of SFSVs.**

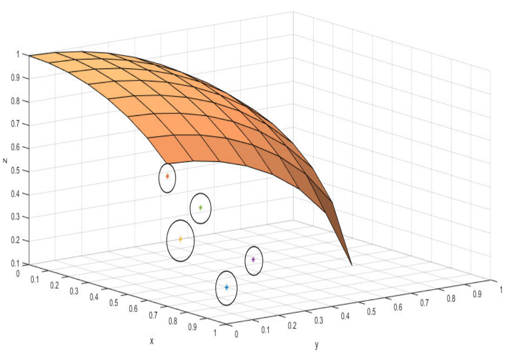

FIGURE 2. **Comparison chart.**

Example 1: Consider $\Im = \{\ddot{z}_1, \ddot{z}_2, \ddot{z}_3, \ddot{z}_4\}$. The CFS is be given as follows
$\chi = \big\{(\ddot{z}_1, 0.7, 0.2, 0.4; 0.6), (\ddot{z}_2, 0.6, 0.8, 0.1; 0.5),$
$(\ddot{z}_3, 0.2, 0.4, 0.5; 0.1), (\ddot{z}_4, 0.2, 0.9, 0.3; 0.5)\big\}$

A. OPERATIONS ON C-SFS

In this section, our focus will be on developing specific relations and operations on C-SFSs. In order to demonstrate the mathematical basis of these processes, we shall rigorously prove the required theorems. Additionally, we will explore methods for ranking C-SFS, enabling us to effectively compare and prioritize different C-SFS instances.

Consider two C-SFS $\alpha_{\check{r}_1}$ and $\beta_{\check{r}_2}$ in $\Im$:

$$\alpha_{\check{r}_1} = \big\{(\ddot{z}, \vartheta_\alpha, \iota_\alpha, \kappa_\alpha; \check{r}_1 | \ddot{z} \in \Im)\big\}$$
$$\beta_{\check{r}_2} = \big\{(\ddot{z}, \vartheta_\beta, \iota_\beta, \kappa_\beta; \check{r}_2 | \ddot{z} \in \Im)\big\}$$

Given below are the definitions of different operations on C-SFSs:

1) $\alpha^c_{\check{r}_1} = \big\{(\ddot{z}, \kappa_\alpha, \iota_\alpha, \vartheta_\alpha; \check{r}_1 | \ddot{z} \in \Im)\big\}$.
2) $\alpha_{\check{r}_1} \subset \beta_{\check{r}_2}$ iff $\check{r}_1 \leq \check{r}_2$ and $\vartheta_\alpha \leq \vartheta_\beta$, $\iota_\alpha \leq \iota_\beta$ and $\kappa_\alpha \geq \kappa_\beta$.
3) $\alpha_{\check{r}_1} = \beta_{\check{r}_2}$ iff $\check{r}_1 = \check{r}_2$ and $\vartheta_\alpha = \vartheta_\beta$, $\iota_\alpha = \iota_\beta$ and $\kappa_\alpha = \kappa_\beta$.
4) $\alpha_{\check{r}_1} \cup_{min} \beta_{\check{r}_2} = \big\{(\ddot{z}, max(\vartheta_\alpha, \vartheta_\beta), min(\iota_\alpha, \iota_\beta), min(\kappa_\alpha, \kappa_\beta); min(\check{r}_1, \check{r}_2) | \ddot{z} \in \Im)\big\}$.
5) $\alpha_{\check{r}_1} \cup_{max} \beta_{\check{r}_2} = \big\{(\ddot{z}, max(\vartheta_\alpha, \vartheta_\beta), min(\iota_\alpha, \iota_\beta), min(\kappa_\alpha, \kappa_\beta); max(\check{r}_1, \check{r}_2) | \ddot{z} \in \Im)\big\}$.
6) $\alpha_{\check{r}_1} \cap_{min} \beta_{\check{r}_2} = \big\{(\ddot{z}, min(\vartheta_\alpha, \vartheta_\beta), min(\iota_\alpha, \iota_\beta), max(\kappa_\alpha, \kappa_\beta); min(\check{r}_1, \check{r}_2) | \ddot{z} \in \Im)\big\}$.
7) $\alpha_{\check{r}_1} \cap_{max} \beta_{\check{r}_2} = \big\{(\ddot{z}, min(\vartheta_\alpha, \vartheta_\beta), min(\iota_\alpha, \iota_\beta), max(\kappa_\alpha, \kappa_\beta); max(\check{r}_1, \check{r}_2) | \ddot{z} \in \Im)\big\}$.

Definition 6: The normalized Euclidean distance for two C-SFSs $\alpha_{\check{r}_1}$ and $\beta_{\check{r}_2}$ in a$\Im$ is given as:

$$d(\alpha_{\check{r}_1}, \beta_{\check{r}_2}) = \frac{|\check{r}_1 - \check{r}_2|}{\sqrt{2}} + \left(\sqrt{\frac{1}{h}\sum_{\ddot{z}=1}^{h}\left((\vartheta_\alpha - \vartheta_\beta)^2 + (\iota_\alpha - \iota_\beta)^2 + (\kappa_\alpha - \kappa_\beta)^2\right)}\right)$$

Theorem 1: Consider $\alpha = (\vartheta_\alpha, \iota_\alpha, \kappa_\alpha; \check{r}_\alpha)$ and $\beta = (\vartheta_\beta, \iota_\beta, \kappa_\beta; \check{r}_\beta)$ be two C-SFSs in $\Im$.
Then De-Morgan's law is given as:

1) $(\alpha \cap_{min} \beta)^c = \alpha^c \cup_{min} \beta^c$
2) $(\alpha \cap_{max} \beta)^c = \alpha^c \cup_{max} \beta^c$
3) $(\alpha \cup_{min} \beta)^c = \alpha^c \cap_{min} \beta^c$
4) $(\alpha \cup_{max} \beta)^c = \alpha^c \cap_{max} \beta^c$

Proof: For proof, see appendix A. □

B. RANKING OF C-SFSS

We have described the process for ranking C-SFSs in this section. The ranking process enables us to compare and prioritize different C-SFS instances based on their respective properties and characteristics.

Definition 7: Assume $\alpha = (\vartheta_\alpha, \iota_\alpha, \kappa_\alpha; \check{r}_\alpha)$ be any C-SFSs.

1) *Score Function:*
$\delta(\alpha) = \frac{1}{4}\left(\vartheta_\alpha - \iota_\alpha - \kappa_\alpha + \sqrt{2}\check{r}(2q - 1)\right)$
where $\delta(\alpha) \in [-1, 1]$ *and* $q \in [0, 1]$.
2) *Accuracy Function:* $\varsigma(\alpha) = \vartheta^2_\alpha + \iota^2_\alpha + \kappa^2_\alpha$ *and* $\varsigma(\alpha) \in [0, 1]$.

Now,assume α and β be two C-SFSs then,

- *If $\delta(\alpha) > \delta(\beta)$, then $\alpha > \beta$.*
- *If $\delta(\alpha) < \delta(\beta)$, then $\alpha < \beta$.*

If $\delta(\alpha) = \delta(\beta)$, then

- *If $\varsigma(\alpha) > \varsigma(\beta)$, then $\alpha > \beta$.*
- *If $\varsigma(\alpha) < \varsigma(\beta)$, then $\alpha < \beta$.*
- *If $\varsigma(\alpha) = \varsigma(\beta)$, then $\alpha \approx \beta$.*

Example 2: Consider collection of SFSs on $\Im = \{\bar{x_1}, \bar{x_2}, \bar{x_3}, \bar{x_4}\}$ is given as,

$\{\bar{x_1}, (0.3, 0.5, 0.2), (0.6, 0.2, 0.6), (0.4, 0.4, 0.7)\}$
$\{\bar{x_2}, (0.6, 0.3, 0.2), (0.4, 0.3, 0.2), (0.8, 0.2, 0.6)\}$
$\{\bar{x_3}, (0.8, 0.5, 0.3), (0.6, 0.6, 0.4), (0.9, 0.3, 0.5)\}$
$\{\bar{x_4}, (0.5, 0.2, 0.4), (0.3, 0.3, 0.4), (0.5, 0.3, 0.4)\}$

Now, we'll convert this collection into C-SFS.

$\big\{(\bar{x_1}, 0.5, 0.4, 0.5; 0.4), (\bar{x_2}, 0.6, 0.3, 0.4; 0.3), (\bar{x_3}, 0.8, 0.5, 0.4; 0.2), (\bar{x_4}, 0.4, 0.3, 0.4; 0.1)\big\}$

IV. DEVELOPMENT OF CIRCULAR SPHERICAL FUZZY SETS

We will talk about the computation process of the radius of C-SFS for the conversion of SFS to C-SFS in this section. To find the radius of SFS, we will use equations (6) and (7).

In an SFS $\digamma_m$, suppose spherical fuzzy pairs possess a shape $\{(\vartheta_{m,1}, \iota_{m,1}, \kappa_{m,1}), (\vartheta_{m,2}, \iota_{m,2}, \kappa_{m,2}), (\vartheta_{m,3}, \iota_{m,4}, \kappa_{m,5}), \ldots\}$, where m is a numeric value of SFS $\digamma_m$ each containing Π_m, which is the number of spherical fuzzy pairs $\digamma_m$.

The formula for calculating the spherical fuzzy pairs' arithematic average is as follows:

$$(\vartheta_{(\digamma_m)}, \iota_{(\digamma_m)}, \kappa_{(\digamma_m)}) = \left(\sqrt{\frac{\sum_{n=1}^{\Pi_m}\vartheta_{m,n}^2}{\Pi_m}}, \sqrt{\frac{\sum_{n=1}^{\Pi_m}\iota_{m,n}^2}{\Pi_m}}, \sqrt{\frac{\sum_{n=1}^{\Pi_m}\kappa_{m,n}^2}{\Pi_m}}\right) \tag{6}$$

The highest value of Euclidean distance is the radius of $(\vartheta_{(\digamma_m)}, \iota_{(\digamma_m)}, \kappa_{(\digamma_m)})$.

$$\breve{r} = \max_{1\leq m\leq \Pi_m} \sqrt{(\vartheta_{\digamma_m} - \vartheta_{m,n})^2 + (\iota_{\digamma_m} - \iota_{m,n})^2 + (\kappa_{\digamma_m} - \kappa_{m,n})^2} \tag{7}$$

thus, SFS is being changed into C-SFS.

V. SUGENO-WEBER OPERATIONS ON C-SFSS

In this part, we will talk about operations of Sugeno-Weber(SW) and its types in certain basic operations. Let us assume that τ-Ns T_{SW}^{ξ} denote the SW sum and τ-CNs S_{SW}^{ξ} denote the SW product. The generalization of union and intersection of SWCSFSs is SW sum $\alpha \bigoplus \beta$ and the SW product $\alpha \bigotimes \beta$, stated as:

1. $\alpha \bigoplus_{min} \beta = \begin{pmatrix} S_{SW}^{\xi}(\vartheta_\alpha, \vartheta_\beta), T_{SW}^{\xi}(\iota_\alpha, \iota_\beta), \\ T_{SW}^{\xi}(\kappa_\alpha, \kappa_\beta), T_{SW}^{\xi}(\breve{r}_\alpha, \breve{r}_\beta) \end{pmatrix}.$
2. $\alpha \bigoplus_{max} \beta = \begin{pmatrix} S_{SW}^{\xi}(\vartheta_\alpha, \vartheta_\beta), T_{SW}^{\xi}(\iota_\alpha, \iota_\beta), \\ T_{SW}^{\xi}(\kappa_\alpha, \kappa_\beta), S_{SW}^{\xi}(\breve{r}_\alpha, \breve{r}_\beta) \end{pmatrix}.$
3. $\alpha \bigotimes_{min} \beta = \begin{pmatrix} T_{SW}^{\xi}(\vartheta_\alpha, \vartheta_\beta), T_{SW}^{\xi}(\iota_\alpha, \iota_\beta), \\ S_{SW}^{\xi}(\kappa_\alpha, \kappa_\beta), T_{SW}^{\xi}(\kappa_\alpha, \kappa_\beta) \end{pmatrix}.$
4. $\alpha \bigotimes_{max} \beta = \begin{pmatrix} T_{SW}^{\xi}(\vartheta_\alpha, \vartheta_\beta), T_{SW}^{\xi}(\iota_\alpha, \iota_\beta), \\ S_{SW}^{\xi}(\kappa_\alpha, \kappa_\beta), S_{SW}^{\xi}(\kappa_\alpha, \kappa_\beta) \end{pmatrix}.$

Definition 8: Consider $\alpha = (\vartheta_\alpha, \iota_\alpha, \kappa_\alpha; \breve{r}_\alpha)$ and $\beta = (\vartheta_\beta, \iota_\beta, \kappa_\beta; \breve{r}_\beta)$ be any two C-SFSs and μ be a positive real number. Then, certain fundamental operations of C-SFSs on the basis of SW τ-N and τ-CNs are provided as:

1. $$\alpha \bigoplus_{min} \beta = \begin{pmatrix} \sqrt{\vartheta_\alpha^2 + \vartheta_\beta^2 - \frac{\xi}{1+\xi}\vartheta_\alpha^2\vartheta_\beta^2}, \\ \sqrt{\frac{\iota_\alpha^2+\iota_\beta^2-1+\xi\iota_\alpha^2\iota_\beta^2}{1+\xi}}, \\ \sqrt{\frac{\kappa_\alpha^2+\kappa_\beta^2-1+\xi\kappa_\alpha^2\kappa_\beta^2}{1+\xi}}, \\ \sqrt{\frac{\breve{r}_\alpha{}^2+\breve{r}_\beta{}^2-1+\xi\breve{r}_\alpha{}^2\breve{r}_\beta{}^2}{1+\xi}} \end{pmatrix}.$$
2. $$\alpha \bigoplus_{max} \beta = \begin{pmatrix} \sqrt{\vartheta_\alpha^2 + \vartheta_\beta^2 - \frac{\xi}{1+\xi}\vartheta_\alpha^2\vartheta_\beta^2}, \\ \sqrt{\frac{\iota_\alpha^2+\iota_\beta^2-1+\xi\iota_\alpha^2\iota_\beta^2}{1+\xi}}, \\ \sqrt{\frac{\kappa_\alpha^2+\kappa_\beta^2-1+\xi\kappa_\alpha^2\kappa_\beta^2}{1+\xi}}, \\ \sqrt{\breve{r}_\alpha{}^2 + \breve{r}_\beta{}^2 - \frac{\xi}{1+\xi}\breve{r}_\alpha{}^2\breve{r}_\beta{}^2} \end{pmatrix}.$$
3. $$\alpha \bigotimes_{min} \beta = \begin{pmatrix} \sqrt{\frac{\vartheta_\alpha^2+\vartheta_\beta^2-1+\xi\vartheta_\alpha^2\vartheta_\beta^2}{1+\xi}}, \\ \sqrt{\frac{\iota_\alpha^2+\iota_\beta^2-1+\xi\iota_\alpha^2\iota_\beta^2}{1+\xi}}, \\ \sqrt{\kappa_\alpha^2 + \kappa_\beta^2 - \frac{\xi}{1+\xi}\kappa_\alpha^2\kappa_\beta^2}, \\ \sqrt{\frac{\breve{r}_\alpha{}^2+\breve{r}_\beta{}^2-1+\xi\breve{r}_\alpha{}^2\breve{r}_\beta{}^2}{1+\xi}} \end{pmatrix}.$$
4. $$\alpha \bigotimes_{max} \beta = \begin{pmatrix} \sqrt{\frac{\vartheta_\alpha^2+\vartheta_\beta^2-1+\xi\vartheta_\alpha^2\vartheta_\beta^2}{1+\xi}}, \\ \sqrt{\frac{\iota_\alpha^2+\iota_\beta^2-1+\xi\iota_\alpha^2\iota_\beta^2}{1+\xi}}, \\ \sqrt{\kappa_\alpha^2 + \kappa_\beta^2 - \frac{\xi}{1+\xi}\kappa_\alpha^2\kappa_\beta^2}, \\ \sqrt{\breve{r}_\alpha{}^2 + \breve{r}_\beta{}^2 - \frac{\xi}{1+\xi}\breve{r}_\alpha{}^2\breve{r}_\beta{}^2} \end{pmatrix}.$$
5. $$\mu \bigodot_{min} \alpha = \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1 - \left(1 - \vartheta_\alpha^2(\frac{\xi}{1+\xi})\right)^\mu\right)}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\iota_\alpha^2+1}{1+\xi}\right)^\mu - 1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\kappa_\alpha^2+1}{1+\xi}\right)^\mu - 1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\breve{r}_\alpha{}^2+1}{1+\xi}\right)^\mu - 1\right)\frac{1}{\xi}} \end{pmatrix}.$$
6. $$\mu \bigodot_{max} \alpha = \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1 - \left(1 - \vartheta_\alpha^2(\frac{\xi}{1+\xi})\right)^\mu\right)}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\iota_\alpha^2+1}{1+\xi}\right)^\mu - 1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\kappa_\alpha^2+1}{1+\xi}\right)^\mu - 1\right)\frac{1}{\xi}}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1 - \left(1 - \breve{r}_\alpha{}^2(\frac{\xi}{1+\xi})\right)^\mu\right)} \end{pmatrix}.$$

7. $\alpha^{\mu}_{min} = \begin{pmatrix} \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\vartheta_{\alpha}^{2}+1}{1+\xi}\right)^{\mu}-1\right)}, \\ \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\iota_{\alpha}^{2}+1}{1+\xi}\right)^{\mu}-1\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\kappa_{\alpha}^{2}(\frac{\xi}{1+\xi})\right)^{\mu}\right)}, \\ \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\breve{r_{\alpha}}^{2}+1}{1+\xi}\right)^{\mu}-1\right)} \end{pmatrix}.$

8. $\alpha^{\mu}_{max} = \begin{pmatrix} \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\vartheta_{\alpha}^{2}+1}{1+\xi}\right)^{\mu}-1\right)}, \\ \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\iota_{\alpha}^{2}+1}{1+\xi}\right)^{\mu}-1\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\kappa_{\alpha}^{2}(\frac{\xi}{1+\xi})\right)^{\mu}\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\breve{r_{\alpha}}^{2}(\frac{\xi}{1+\xi})\right)^{\mu}\right)} \end{pmatrix}.$

Theorem 2: Suppose three C-SFSs are $\alpha = (\vartheta_{\alpha}, \iota_{\alpha}, \kappa_{\alpha}; \breve{r_{\alpha}})$ and $\beta = (\vartheta_{\beta}, \iota_{\beta}, \kappa_{\beta}; \breve{r_{\beta}})$ $\gamma = (\vartheta_{\gamma}, \iota_{\gamma}, \kappa_{\gamma}; \breve{r_{\gamma}})$ and $\bar{x} \geq 1$. Then following properties are satisfied.

1) $\alpha \oplus \beta = \beta \oplus \alpha$
2) $\alpha \otimes \beta = \beta \otimes \alpha$
3) $(\alpha \oplus \beta) \oplus \gamma = \alpha \oplus (\beta \oplus \gamma)$
4) $(\alpha \otimes \beta) \otimes \gamma = \alpha \otimes (\beta \otimes \gamma)$
5) $\bar{x}\alpha \oplus \bar{x}\beta = \bar{x}(\alpha \oplus \beta), \quad \bar{x} \geq 0;$
6) $\bar{x}_{\alpha}\alpha \oplus \bar{x}_{\beta}\alpha = (\bar{x}_{\alpha} \oplus \bar{x}_{\beta})\alpha, \quad \bar{x}_{\alpha}$ and $\bar{x}_{\beta} \geq 0;$
7) $(\alpha \otimes \beta)^{\bar{x}} = \alpha^{\bar{x}} \otimes \beta^{\bar{x}}, \quad \bar{x} \geq 0$
8) $\alpha^{\bar{x}_{\alpha}} \otimes \alpha^{\bar{x}_{\beta}} = \alpha^{\bar{x}_{\alpha} \oplus \bar{x}_{\beta}}, \quad \bar{x}_{\alpha}$ and $\bar{x}_{\beta} \geq 0$

Proof: The proof is given in appendix B. □

VI. AVERAGING AGGREGATION OPERATORS FOR C-SFS

This section introduces the averaging aggregation operators for C-SFSs. We'll go through the operator's mathematical formulation and several key characteristics it satisfies. We will also provide thorough justifications for the accuracy and reliability of the operators in compiling C-SFS information.

A. C-SF SUGENO-WEBER WEIGHTED AVERAGING AGGREGATION OPERATORS

Definition 9: Assume $\alpha_i = (\vartheta_i, \iota_i, \kappa_j; \breve{r_i})$ be an assembely of C-SFNs, where $i = 1, 2, \ldots, n$.

If C-SFSWWA : $\Lambda^n \to \Lambda$, then C-SFSWWA is given as:

$$C - SFSWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{i=1}^{n} \varpi_i \alpha_i \tag{8}$$

where $\varpi_i > 0, \sum_{i=1}^{n} \varpi_i = 1$ denotes the weights of attributes.

Theorem 3: Assume $\alpha_i = (\vartheta_i, \iota_i, \kappa_i; \breve{r_i})$ be an assembely of C-SFNs, where $i = 1, 2, \ldots, n$. Then, aggregated values are also C-SFN and

$$C - SFSWA_{min}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{i=1}^{n} \varpi_i \alpha_i$$
$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\breve{r_i}^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}} \end{pmatrix}$$

$$C - SFSWA_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{i=1}^{n} \varpi_i \alpha_i$$
$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\breve{r_i}^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)} \end{pmatrix}$$

Proof: See appendix C. □

B. PROPERTIES

1) **IDEMPOTENCY:**
Consider $\alpha_i = \alpha = (\vartheta_i, \iota_i, \kappa_i, \breve{r_i})$ where $i = 1, 2 \ldots, n$ be a C-SFSNs, then

$$C - SFSWWA_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha$$

Proof: See appendix D. □

2) **HOMOGENITY:**
Consider τ be any positive real number, then

$$C - SFSWWA_{max}(\tau\alpha_1, \tau\alpha_2, \ldots, \tau\alpha_n)$$
$$= \tau C - SFSWWA_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$

Proof: See appendix E. □

3) **BOUNDEDNESS:** Assume $\alpha = (\vartheta_i, \iota_i, \kappa_i, \breve{r_i})$ be class of C-SFSNs where $i = 1, 2, \ldots, n$ and $\alpha_i^{+} = (\max_i(\vartheta_i), \min_i(\iota_i), \min_i(\kappa_i), \max_i(\breve{r_i}))$
$\alpha_i^{-} = (\min_i(\vartheta_i), \min_i(\iota_i), \max_i(\kappa_i), \max_i(\breve{r_i}))$. Then,

$$\alpha_i^{-} \leq C - SFSWWA_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \alpha_i^{+}$$

Proof: See appendix F. □

C. C-SF SUGENO-WEBER ORDERED WEIGHTED AVERAGING AGGREGATION OPERATORS

Definition 10: Assume $\alpha_i = (\vartheta_i, \iota_i, \kappa_j; \breve{r_i})$ be an assembely of C-SFNs, where $i = 1, 2, \ldots, n$. If C-SFSWOWA : $\Lambda^n \to$

Λ, then C-SFSWOWA is given as:

$$C - SFSSOWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{i=1}^{n} \varpi_i \alpha_{\delta(i)} \tag{9}$$

where $\varpi_i > 0, \sum_{i=1}^{n} \varpi_i = 1$ denotes the weights of attributes. Also $\delta(k)$ is the permutation as $\delta(1), \delta(2), \ldots, \delta(k)$.

Theorem 4: Assume $\alpha_i = (\vartheta_i, \iota_i, \kappa_i; \check{r}_i)$ be an assembely of C-SFNs, where $i = 1, 2, \ldots, n$. Then, aggregated values are also C-SFN and

$$C - SFSWOWA_{min}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{i=1}^{n} \varpi_i \alpha_{\delta(i)}$$

$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^{n}\left(1 - \vartheta^2_{\delta(i)}(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi \iota^2_{\delta(i)}+1}{1+\xi}\right)^{\varpi_i} - 1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi \kappa^2_{\delta(i)}+1}{1+\xi}\right)^{\varpi_i} - 1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi \check{r_{\delta(i)}}^2+1}{1+\xi}\right)^{\varpi_i} - 1\right)\frac{1}{\xi}} \end{pmatrix}$$

$$C - SFSWOWA_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{i=1}^{n} \varpi_i \alpha_{\delta(i)}$$

$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^{n}\left(1 - \vartheta^2_{\delta(i)}(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi \iota^2_{\delta(i)}+1}{1+\xi}\right)^{\varpi_i} - 1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi \kappa^2_{\delta(i)}+1}{1+\xi}\right)^{\varpi_i} - 1\right)\frac{1}{\xi}}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^{n}\left(1 - \check{r_{\delta(i)}}^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)} \end{pmatrix}$$

Proof: The proof is analogous to Theorem 3. □

D. C-SF SUGENO-WEBER HYBRID WEIGHTED AVERAGING AGGREGATION OPERATORS

Definition 11: Assume $\alpha_i = (\vartheta_i, \iota_i, \kappa_j; \check{r}_i)$ be an assembely of C-SFNs, where $i = 1, 2, \ldots, n$.

If C-SFSWHWA : $\Lambda^n \to \Lambda$, then C-SFSWHWA is given as:

$$C - SFSSHWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{i=1}^{n} \omega_i \alpha'_{\delta(i)} \tag{10}$$

where $\omega_i > 0, \sum_{i=1}^{n} \omega_i = 1$ is the associated weight and $\varphi_i > 0, \sum_{i=1}^{n} \varpi_i = 1$ denotes the weights of attributes. Also $\delta(k)$ is the permutation as $\delta(1), \delta(2), \ldots, \delta(k)$.

Theorem 5: Assume $\alpha_i = (\vartheta_i, \iota_i, \kappa_i; \check{r}_i)$ be an assembely of C-SFNs, where $i = 1, 2, \ldots, n$. Then, aggregated values are also C-SFN and

$$C - SFSWHWA_{min}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{i=1}^{n} \omega_i \alpha'_{\delta(i)}$$

$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^{n}\left(1 - \vartheta'^2_{\delta(i)}(\frac{\xi}{1+\xi})\right)^{\omega_i}\right)}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi \iota'^2_{\delta(i)}+1}{1+\xi}\right)^{\omega_i} - 1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi \kappa'^2_{\delta(i)}+1}{1+\xi}\right)^{\omega_i} - 1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi \check{r'_{\delta(i)}}^2+1}{1+\xi}\right)^{\omega_i} - 1\right)\frac{1}{\xi}} \end{pmatrix}$$

$$C - SFSWHWA_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{i=1}^{n} \omega_i \alpha'_{\delta(i)}$$

$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^{n}\left(1 - \vartheta'^2_{\delta(i)}(\frac{\xi}{1+\xi})\right)^{\omega_i}\right)}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi \iota'^2_{\delta(i)}+1}{1+\xi}\right)^{\omega_i} - 1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi \kappa'^2_{\delta(i)}+1}{1+\xi}\right)^{\omega_i} - 1\right)\frac{1}{\xi}}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^{n}\left(1 - \check{r'_{\delta(i)}}^2(\frac{\xi}{1+\xi})\right)^{\omega_i}\right)} \end{pmatrix}$$

Proof: The proof is analogous to Theorem 3. □

VII. WEIGHTED GEOMETRIC AGGREGATION OPERATORS

This section introduces the weighted geometric aggregation operator for C-SFSs. In this section, we'll go through the operator's mathematical definition and a thorough discussion of its key characteristics.

A. C-SF SUGENO-WEBER WEIGHTED GEOMETRIC AGGREGATION OPERATORS

Definition 12: Assume $\alpha_i = (\vartheta_i, \iota_i, \kappa_j; \check{r}_i)$ be an assembely of C-SFNs, where $i = 1, 2, \ldots, n$.

If C-SFSWWG : $\Lambda^n \to \Lambda$, then C-SFSWWG is given as:

$$C - SFSWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \prod_{i=1}^{n} \alpha_i^{\varpi_i} \tag{11}$$

where $\varpi_i > 0, \sum_{i=1}^{n} \varpi_i = 1$ denotes the weights of attributes.

Theorem 6: Assume $\alpha_i = (\vartheta_i, \iota_i, \kappa_i; \check{r}_i)$ be an assembely of C-SFNs, where $i = 1, 2, \ldots, n$. Then, aggregated values are also C-SFN and

$$C-SFSWWG_{min}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\prod_{i=1}^{n}\alpha_i^{\varpi_i}$$
$$=\begin{pmatrix}\sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\vartheta_\alpha^2+1}{1+\xi}\right)^{\varpi_i}-1\right)},\\ \sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\iota_\alpha^2+1}{1+\xi}\right)^{\varpi_i}-1\right)},\\ \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\kappa_\alpha^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)},\\ \sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\breve{r_\alpha}^2+1}{1+\xi}\right)^{\varpi_i}-1\right)}\end{pmatrix}$$

$$C-SFSWWG_{max}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\prod_{i=1}^{n}\alpha_i^{\varpi_i}$$
$$=\begin{pmatrix}\sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\vartheta_\alpha^2+1}{1+\xi}\right)^{\varpi_i}-1\right)},\\ \sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\iota_\alpha^2+1}{1+\xi}\right)^{\varpi_i}-1\right)},\\ \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\kappa_\alpha^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)},\\ \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\breve{r_\alpha}^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)}\end{pmatrix}$$

Proof: The proof is analogous to Theorem 3. □

B. PROPERTIES

1) **IDEMPOTENCY:**
Consider $\alpha_i=\alpha=(\vartheta_i,\iota_i,\kappa_i,\breve{r}_i)$ where $i=1,2\ldots,n$ be a C-SFSNs, then

$$C-SFSWWG_{max}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\alpha$$

Proof: The proof is on the same manner as of appendix D. ■

2) **HOMOGENITY:**
Consider τ be any positive real number, then

$$C-SFSWWG_{max}(\tau\alpha_1,\tau\alpha_2,\ldots,\tau\alpha_n)$$
$$=\tau\ C-SFSWWA_{max}(\alpha_1,\alpha_2,\ldots,\alpha_n)$$

Proof: The proof is comparable to appendix E. ■

3) **BOUNDEDNESS:**

Assume $\alpha=(\vartheta_i,\iota_i,\kappa_i,\breve{r}_i)$ be class of C-SFSNs where $i=1,2,\ldots,n$ and
$\alpha_i^+=(\max_i(\vartheta_i),\min_i(\iota_i),\min_i(\kappa_i),\max_i(\breve{r}_i))$
$\alpha_i^-=(\min_i(\vartheta_i),\min_i(\iota_i),\max_i(\kappa_i),\max_i(\breve{r}_i))$. Then,

$$\alpha_i^-\leq C-SFSWWG_{max}(\alpha_1,\alpha_2,\ldots,\alpha_n)\leq\alpha_i^+$$

Proof: The proof resembles to appendix F. □

C. C-SF SUGENO-WEBER ORDERED WEIGHTED GEOMETRIC AGGREGATION OPERATORS

Now, we'll present the C-SFSWOWG aggregation operator in C-SFSs.

Definition 13: Assume $\alpha_i=(\vartheta_i,\iota_i,\kappa_j;\breve{r}_i)$ be an assembely of C-SFNs, where $i=1,2,\ldots,n$.
If C-SFSWOWG : $\Lambda^n\rightarrow\Lambda$, then C-SFSWOWG is given as:

$$C-SFSWOWG(\alpha_1,\alpha_2,\ldots,\alpha_n)=\prod_{i=1}^{n}\alpha_{\delta(i)}^{\varpi_i}\qquad(12)$$

where $\varpi_i>0,\sum_{i=1}^{n}\varpi_i=1$ denotes the weights of attributes. Also $\delta(k)$ is the permutation as $\delta(1),\delta(2),\ldots,\delta(k)$.

Theorem 7: Assume $\alpha_i=(\vartheta_i,\iota_i,\kappa_i;\breve{r}_i)$ be an assembely of C-SFNs, where $i=1,2,\ldots,n$. Then, aggregated values are also C-SFN and

$$C-SFSWOWG_{min}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\prod_{i=1}^{n}\alpha_{\delta(i)}^{\varpi_i}$$
$$=\begin{pmatrix}\sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\vartheta_{\delta(i)}^2+1}{1+\xi}\right)^{\varpi_i}-1\right)},\\ \sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\iota_{\delta(i)}^2+1}{1+\xi}\right)^{\varpi_i}-1\right)},\\ \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\kappa_{\delta(i)}^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)},\\ \sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\breve{r_{\delta(i)}}^2+1}{1+\xi}\right)^{\varpi_i}-1\right)}\end{pmatrix}$$

$$C-SFSWOWG_{max}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\prod_{i=1}^{n}\alpha_{\delta(i)}^{\varpi_i}$$
$$=\begin{pmatrix}\sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\vartheta_{\delta(i)}^2+1}{1+\xi}\right)^{\varpi_i}-1\right)},\\ \sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\iota_{\delta(i)}^2+1}{1+\xi}\right)^{\varpi_i}-1\right)},\\ \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\kappa_{\delta(i)}^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)},\\ \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\breve{r_{\delta(i)}}^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)}\end{pmatrix}$$

Proof: The proof resembles to Theorem 3. □

D. C-SF SUGENO-WEBER HYBRID WEIGHTED GEOMETRIC AGGREGATION OPERATORS

Now, we'll present the C-SFSWHWG aggregation operator in C-SFSs.

Definition 14: Assume $\alpha_i=(\vartheta_i,\iota_i,\kappa_j;\breve{r}_i)$ be an assembely of C-SFNs, where $i=1,2,\ldots,n$.
If C-SFSWHWG : $\Lambda^n\rightarrow\Lambda$, then C-SFSWHWG is given as:

$$C-SFSWHWG(\alpha_1,\alpha_2,\ldots,\alpha_n)=\prod_{i=1}^{n}\alpha_{\delta(i)}^{'\omega_i}\qquad(13)$$

where $\omega_i>0,\sum_{i=1}^{n}\omega_i=1$ is the associated weight and $\varpi_i>0,\sum_{i=1}^{n}\varpi_i=1$ denotes the weights of attributes. Also $\delta(k)$ is the permutation as $\delta(1),\delta(2),\ldots,\delta(k)$.

Theorem 8: Assume $\alpha_i=(\vartheta_i,\iota_i,\kappa_i;\breve{r}_i)$ be an assembely of C-SFNs, where $i=1,2,\ldots,n$. Then, aggregated values are *also C-SFN and*

$$C-SFSWHWG_{min}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\prod_{i=1}^{n}\alpha_{\delta(i)}^{'\omega_i}$$
$$=\left(\begin{array}{c}\sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\vartheta_{\delta(i)}^{'2}+1}{1+\xi}\right)^{\omega_i}-1\right)},\\ \sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\iota_{\delta(i)}^{'2}+1}{1+\xi}\right)^{\omega_i}-1\right)},\\ \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\kappa_{\delta(i)}^{'2}(\frac{\xi}{1+\xi})\right)^{\omega_i}\right)},\\ \sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\breve{r}_{\delta(i)}^{'2}+1}{1+\xi}\right)^{\omega_i}-1\right)}\end{array}\right)$$

$$C-SFSWHWG_{max}(\alpha_1,\alpha_2,\ldots,\alpha_n)=\prod_{i=1}^{n}\alpha_{\delta(i)}^{'\omega_i}$$
$$=\left(\begin{array}{c}\sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\vartheta_{\delta(i)}^{'2}+1}{1+\xi}\right)^{\omega_i}-1\right)},\\ \sqrt{\frac{1}{\xi}\left((1+\xi)\prod_{i=1}^{n}\left(\frac{\xi\iota_{\delta(i)}^{'2}+1}{1+\xi}\right)^{\omega_i}-1\right)},\\ \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\kappa_{\delta(i)}^{'2}(\frac{\xi}{1+\xi})\right)^{\omega_i}\right)},\\ \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{n}\left(1-\breve{r}_{\delta(i)}^{'2}(\frac{\xi}{1+\xi})\right)^{\omega_i}\right)}\end{array}\right)$$

Proof: The proof resembles to Theorem 3. □

VIII. CIRCULAR SF MULTI-ATTRIBUTE DECISON MAKING STRUCTURE

In this part, we'll present the MCDM method in Circular Spherical Fuzzy Information that accounts for the uncertainity. Finding the ideal answer to a problem is typically essential for getting the best outcomes. Algorithms are the means by which people make decisions. An algorithm is an anticipated, stated series of steps aimed at providing the ideal answer to an issue in particular. It is ideal to use an algorithm if precise detail is required since doing so enhances accuracy and lowers the risk of a mistake. With in this stage, we'll provide the algoritm using CSF Information, where we consider p alternatives $\Re = \{\nu_1, \nu_2, \ldots, \nu_p\}$ alongwith q criterias $\aleph = \{\zeta_1, \zeta_2, \ldots, \zeta_q\}$ for decision matrix $M_{p\times q}$.
The weight vectors for each criteria are given as $\varpi = \{\varpi_1, \varpi_2, \ldots, \varpi_q\}$, $\sum_{k=1}^{q}\varpi = 1$. Let the decision matrix is given as $M = (\xi_{ij})_{p\times q} = (\vartheta_{ij}, \iota_{ij}, \kappa_{ij}; \breve{r_{ij}})$ and their weight vectors are $\rho = \{\rho_1, \rho_2, \ldots, \rho_q\}$, $\sum_{k=1}^{q}\rho = 1$. Following are the steps of planned algorithm:

The flow chart of the algorithm is presented in figure 3 for better understanding.

Algorithm 1 Algorithm

Step 1: Collect Data of C-SF Information in the form of Decison matrices $M = (\xi_{ij})_{p\times q} = (\vartheta_{ij}, \iota_{ij}, \kappa_{ij}; \breve{r_{ij}})$ given by the decision makers.

Step 2: Normalize the C-SF data $M = (\xi_{ij})_{p\times q}$. If there is Cost Criteria, then normalize the decison matrices,otherwise there is no need of normalization.

$$M'_{ij} = \begin{cases}(\vartheta_{ij}, \iota_{ij}, \kappa_{ij}; \breve{r_{ij}}), & \text{if } C_I\\ (\kappa_{ij}, \iota_{ij}, \vartheta_{ij}; \breve{r_{ij}}), & \text{if } C_{II}\end{cases}$$

where C_I is benefit crterion and C_{II} being the cost criterion.

Step 3: Aggregate the data using $C-SFSWWA_{min}$, $C-SFSWWA_{max}$ or $C-SFSWWG_{min}$, $C-SFSWWG_{max}$ Operator.

Step 4(a): Evaluate C-SF Information by $C-SFSWWA_{min}$, $C-SFSWWA_{max}$, $C-SFSWWG_{min}$ and $C-SFSWWG_{max}$ Operator.

Step 4(b): Evaluate C-SF Information by $C-SFSWOWA_{min}$, $C-SFSWOWA_{max}$, $C-SFSWOWG_{min}$ and $C-SFSWOWG_{max}$ Operator.

Step 4(c): Evaluate C-SF Information by $C-SFSHWA_{min}$, $C-SFSWHWA_{max}$, $C-SFSWHWG_{min}$ and $C-SFSWHWG_{max}$ Operator.

Step 5: Find the score values of evaluated (aggregated) C-SF information and then give ranking by maximum score values using definition 5.4.
If scoring of two alternatives are same then use accuracy defined in definition 5.4.

Step 6: Choose the optimal alternative as per maximum value of scoring.

A. CASE STUDY WITH NUMERICAL EXAMPLE

In this case study, the decision-making procedure for changing a software project from Python to GoLang is examined. The focus is on a Python-based social media analytics application that has performance and scalability issues. The study examines the factors taken into account throughout the language switch, the chosen approach, and the impact on platform performance and the development team as a whole.
Software or hardware designed to collect, analyze, scrutinize, and interpret data from multiple social media channels are known as platforms for social media analytics. These tools help businesses, marketers, and individuals learn more about their target audiences' activity on social media, the success of their individual social media initiatives, and their overall social media strategy. Solutions for social media analytics are crucial in helping businesses and individuals

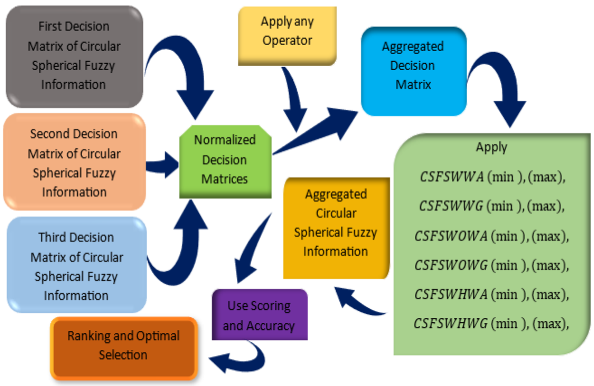

FIGURE 3. Flow chart of algorithm.

better understand their social media performance, boost engagement, and make informed decisions to enhance their online presence and marketing campaigns.

Phython is commonly used to build social networking networks for a number of compelling reasons. Its simplicity and user-friendliness make it possible for developers to write code rapidly in the quick-paced world of social media development, where constant updates and feature additions are necessary. Developers may quickly incorporate a variety of capabilities, such as image recognition, user authentication, and data processing, with the aid of this extensive set of tools. In addition, Python's syntax is simple and easy to comprehend, which is beneficial for complex projects like social networking networks since it makes code easier to understand and maintain. Because of its simplicity, versatility, and extensive ecosystem of libraries and frameworks, Python is generally regarded as one of the best options for creating social networking systems. Due to its rapid development time and strong community support, Python was first utilized to construct the social media analytic platform.

However, as the platform's user base developed, it struggled to adequately process enormous volumes of data and meet growing demand. Despite being a versatile and well-liked language, Python may struggle when compared to other languages with extremely large traffic and data amounts. Python is an interpreted language, thus when the program is running, each line of code is read and executed. This interpretation process may add a significant amount of overhead, making it slower than compiled languages when dealing with complicated computations and vast data processing. The problems caused by large traffic and data quantities can be resolved by using compiled languages like GoLang or Python in combination with technologies. Because of their increased speed, concurrent capabilities, and smaller memory footprint while handling extremely heavy traffic or processing vast quantities of data, additional languages like GoLang, Java, or C++ may be selected. After evaluating different options, the company made the decision to move the project from Python to the Go programming language (GoLang) in order to increase productivity and scalability.

Due to its several significant speed and scalability advantages, GoLang is a top choice for many high-performance and concurrent applications, especially those dealing with large volumes of data and high traffic. It can handle several connections and requests concurrently without significantly affecting performance. GoLang is a good option for managing big amounts of data and high traffic scenarios since it has grown in popularity for creating high-performance and scalable systems, such as web servers, microservices, cloud-based services, and networking applications.

While converting a program language from Python to GoLang, a number of decision-making options may be taken into consideration in order to assure a well-thought-out and effective strategy. The project's unique requirements and limitations will determine the best course of action.

- **Incremental Migration** (ν_1)**:** A big-bang migration may be replaced by transferring modules or features one at a time. This approach enables the team to focus on specific areas, thoroughly test each component, and gradually develop the GoLang code base while preserving the Python functionality. The possibility of abruptly disrupting the entire application is decreased by doing this. Basically, incremental migration allows for a planned and deliberate transition while reducing the risks associated with a complete redesign and gaining access to GoLang's concurrency and performance benefits.
- **Hybrid Microservices** (ν_2)**:** Microservices is an architectural framework that builds an application as a collection of small, independent services that communicate with one another using well-defined APIs. Every single microservice is in charge of a distinct piece of a company's functionality and may be built, implemented, and expanded separately. In a microservices framework, the application is divided into loosely linked services, and each service may be constructed using multiple technologies or programming languages. The organization may consider breaking the program up into smaller components as microservices, each of which would be developed in the language that best suited its requirements. While Python may still be utilized for some components, GoLang may be used to build microservices that are performance-critical.
- **Using a Polyglot Architecture** (ν_3)**:** A polyglot architecture is a software design approach that involves using multiple programming languages and technologies within a single application or system. In the context of transitioning from Python to GoLang, a polyglot architecture can be employed to facilitate a gradual and controlled migration. While the rest elements of the program would still be written in Python, the most important performance-intensive sections may be redone in GoLang. This approach allows the team to

utilize GoLang's parallelism and performance benefits where they are most needed without having to substantially rework the code.

- **Improving Existing Python Code** (ν_4)**:** Before migrating from Python to GoLang, it's critical to optimize the existing Python codebase for efficiency, maintainability, and readability. Improving the Python code will make the transfer easier and provide a solid basis for the future migration to GoLang. Performance bottlenecks may be located using profiling tools, and performance-critical code parts can be strengthened by rewriting or optimization. Even while the scalability and concurrency problems may not be entirely resolved by this method, it can nevertheless significantly improve performance immediately.

Making an informed and sensible decision on whether to migrate from Python to GoLang as a software language requires consideration of a number of significant factors. Assessing the feasibility, benefits, and potential downside of the shift will be made simpler by these requirements. Below is a list of the primary decision elements for the case study:

- **Performance Requirements** (ζ_1)**:** Examine the performance requirements of the application. Determine whether Python's interpreted nature is causing any performance concerns, and then determine whether GoLang's compiled nature could significantly speed up and improve the application's efficiency. Through comprehensive performance testing, the team compares the application's performance in Python and GoLang. They keep an eye on response times, resource consumption, the system's ability to handle a lot of simultaneous requests, and user traffic.
- **Scalability Demands** (ζ_2)**:** Analyze the application's scalability requirements. Scalability refers to a language's ability to manage increasing workloads and resource demands as an application grows. Scalable applications may perform several tasks at once while maximizing the use of available resources. Think about if GoLang's concurrency architecture and capacity to fully utilize multiple CPU cores will better fit the application's expansion and accommodate rising user demand.
- **Development Team's Expertise and Productivity** (ζ_3)**:** Evaluate the development team's level of expertise and knowledge of Python and GoLang. Consider the GoLang phase of learning and if the team has the knowledge and resources necessary to complete the conversion successfully. If the team has previous experience working with GoLang or other statically typed languages, they may be able to shift more quickly to GoLang development. Check to see if GoLang can boost productivity and optimize development processes. Because to GoLang's usability and simplicity, the development team can be more productive, which gives them more time to focus on developing new features and enhancing performance.
- **Concurrency and Parallelism** (ζ_4)**:** Analyze the parallelism and concurrency requirements of the application. Determine if GoLang's goroutines and channels are a more efficient and user-friendly alternative to Python for managing several tasks at once. When an application does multiple tasks at once, only one of them is done in its forefront while the others are completed in the background. If the computer only has one CPU, the program might not be able to perform several tasks at once, but it can manage multiple tasks on its own. It doesn't finish one activity before beginning the next. This is known as Concurrency. While a program that uses parallelism divides its tasks into smaller ones so that they may be handled simultaneously by several CPUs in parallel.

As an illustration, consider a group of technicians assembling a monitor. Multiple technicians can construct various display components simultaneously, yet they all use the same workstation to put everything together. At the workstation, only one technician may assemble at once while the others work on their pieces in the background. With parallelism, technicians may concurrently construct components on many workbenches. As a result, it is clear that a language supports concurrency. It is ideal for large-scale programs.

The transition from Python to GoLang as the programming language was advantageous for the social media analytic platform application. With the help of GoLang's improved concurrency, scalability, and performance characteristics, the application was able to effectively handle an increasing user base and rising data processing requirements. The end users and the development team recognized long-term benefits that made the shift desirable even though it required careful planning and adaption.

Step 1: Consider three decision matrices provided in Table 1,2,3, each containing $\Re = \{\nu_1, \nu_2, \nu_3, \nu_4\}$ as alternatives assessed in light of four criterions $\aleph = \{\zeta_1, \zeta_2, \zeta_3, \zeta_4\}$.

TABLE 1. C-SF decision matrix 1.

	ζ_1	ζ_2
ν_1	$(0.2, 0.5, 0.3; 0.8)$	$(0.5, 0.4, 0.3; 0.2)$
ν_2	$(0.2, 0.5, 0.4; 0.7)$	$(0.5, 0.4, 0.4; 0.4)$
ν_3	$(0.2, 0.5, 0.5; 0.6)$	$(0.5, 0.4, 0.5; 0.6)$
ν_4	$(0.2, 0.5, 0.6; 0.5)$	$(0.5, 0.4, 0.6; 0.8)$
	ζ_3	ζ_4
ν_1	$(0.4, 0.6, 0.2; 0.1)$	$(0.1, 0.7, 0.3; 0.5)$
ν_2	$(0.4, 0.6, 0.5; 0.3)$	$(0.1, 0.7, 0.5; 0.5)$
ν_3	$(0.4, 0.6, 0.3; 0.5)$	$(0.1, 0.7, 0.7; 0.1)$
ν_4	$(0.4, 0.6, 0.4; 0.7)$	$(0.1, 0.7, 0.2; 0.3)$

Step 2: There is no cost criteria so no need of normalization.

Step 3: In Table 4, the C-SF matrices have been aggregated using the C-SFSWW$A_{\min}$ operator

TABLE 2. C-SF decision matrix 2.

	ζ_1	ζ_2
ν_1	$(0.3, 0.5, 0.3; 0.2)$	$(0.1, 0.4, 0.3; 0.3)$
ν_2	$(0.3, 0.5, 0.4; 0.1)$	$(0.1, 0.4, 0.4; 0.2)$
ν_3	$(0.3, 0.5, 0.5; 0.4)$	$(0.1, 0.4, 0.5; 0.5)$
ν_4	$(0.3, 0.5, 0.6; 0.5)$	$(0.1, 0.4, 0.6; 0.1)$
	ζ_3	ζ_4
ν_1	$(0.6, 0.6, 0.2; 0.3)$	$(0.2, 0.7, 0.3; 0.2)$
ν_2	$(0.6, 0.5, 0.5; 0.4)$	$(0.3, 0.7, 0.5; 0.3)$
ν_3	$(0.6, 0.5, 0.3; 0.7)$	$(0.4, 0.7, 0.5; 0.3)$
ν_4	$(0.6, 0.4, 0.4; 0.6)$	$(0.1, 0.7, 0.2; 0.7)$

TABLE 3. C-SF decision matrix 3.

	ζ_1	ζ_2
ν_1	$(0.2, 0.4, 0.3; 0.8)$	$(0.5, 0.3, 0.3; 0.1)$
ν_2	$(0.2, 0.5, 0.4; 0.4)$	$(0.3, 0.4, 0.4; 0.4)$
ν_3	$(0.2, 0.4, 0.5; 0.4)$	$(0.2, 0.4, 0.5; 0.1)$
ν_4	$(0.2, 0.5, 0.6; 0.5)$	$(0.5, 0.4, 0.3; 0.8)$
	ζ_3	ζ_4
ν_1	$(0.5, 0.6, 0.2; 0.2)$	$(0.2, 0.7, 0.3; 0.5)$
ν_2	$(0.4, 0.2, 0.5; 0.4)$	$(0.1, 0.4, 0.5; 0.4)$
ν_3	$(0.5, 0.6, 0.3; 0.4)$	$(0.2, 0.7, 0.5; 0.1)$
ν_4	$(0.4, 0.3, 0.4; 0.1)$	$(0.3, 0.7, 0.2; 0.2)$

with the provided decision weight vectors $\rho = \{0.25, 0.40, 0.35\}$ and $\xi = 2$.

TABLE 4. Aggregated C-SF decision matrix.

	ζ_1	ζ_2
ν_1	$(0.245, 0.466, 0.3; 0.588;)$	$(0.4, 0.367, 0.3; 0.22)$
ν_2	$(0.245, 0.5, 0.4; 0.399)$	$(0.318, 0.4, 0.4; 0.33)$
ν_3	$(0.245, 0.466, 0.5; 0.453)$	$(0.29, 0.4, 0.5; 0.422)$
ν_4	$(0.245, 0.5, 0.6; 0.5)$	$(0.399, 0.4, 0.504; 0.571)$
	ζ_3	ζ_4
ν_1	$(0.52, 0.6, 0.2; 0.227)$	$(0.18, 0.7, 0.3; 0.397)$
ν_2	$(0.494, 0.438, 0.5; 0.377)$	$(0.206, 0.599, 0.5; 0.389)$
ν_3	$(0.524, 0.561, 0.3; 0.549)$	$(0.286, 0.7, 0.551; 0.202)$
ν_4	$(0.494, 0.422, 0.4; 0.492)$	$(0.196, 0.7, 0.2; 0.451)$

Step 4(a): Apply the C-SFSWWA_{min}, C-SFSWWA_{max}, C-SFSWWG_{min}, and C-SFSWWG_{max} operators with the weight vector of criteria $\varphi = \{0.25, 0.27, 0.22, 0.26\}$ to the information provided in Table 4, see Table 5.

TABLE 5. Aggregated operations.

	ζ_1	ζ_2
ν_1	$(0.359, 0.580, 0.432; 0.488)$	$(0.359, 0.580, 0.432; 0.397)$
ν_2	$(0.330, 0.552, 0.533; 0.487)$	$(0.330, 0.552, 0.533; 0.374)$
ν_3	$(0.349, 0.579, 0.547; 0.535)$	$(0.349, 0.579, 0.547; 0.424)$
ν_4	$(0.352, 0.559, 0.557; 0.587)$	$(0.352, 0.559, 0.557; 0.507)$
	ζ_3	ζ_4
ν_1	$(0.488, 0.580, 0.281; 0.488)$	$(0.488, 0.580, 0.281; 0.397)$
ν_2	$(0.468, 0.552, 0.452; 0.487)$	$(0.468, 0.552, 0.452; 0.374)$
ν_3	$(0.472, 0.579, 0.481; 0.535)$	$(0.472, 0.579, 0.481; 0.424)$
ν_4	$(0.483, 0.559, 0.458; 0.587)$	$(0.483, 0.559, 0.458; 0.507)$

Step 4(b): Table 6 provide the results of applying the C-SFSWOWA_{min}, C-SFSWOWA_{max}, C-SFSWOWG_{min} and C-SFSWOWG_{max} operator by takng $q = 0.1$.

TABLE 6. Ordered aggregated operations.

	ζ_1	ζ_2
ν_1	$(0.372, 0.588, 0.429, 0.479)$	$(0.372, 0.588, 0.429, 0.385)$
ν_2	$(0.343, 0.551, 0.537, 0.487)$	$(0.343, 0.551, 0.537, 0.374)$
ν_3	$(0.357, 0.604, 0.547, 0.518)$	$(0.357, 0.604, 0.547, 0.434)$
ν_4	$(0.361, 0.577, 0.518, 0.583)$	$(0.361, 0.577, 0.518, 0.509)$
	ζ_3	ζ_4
ν_1	$(0.496, 0.588, 0.277; 0.479)$	$(0.496, 0.588, 0.277; 0.385)$
ν_2	$(0.476, 0.551, 0.457; 0.487)$	$(0.476, 0.551, 0.457; 0.374)$
ν_3	$(0.482, 0.604, 0.473; 0.518)$	$(0.482, 0.604, 0.473; 0.434)$
ν_4	$(0.486, 0.577, 0.465; 0.583)$	$(0.486, 0.577, 0.465; 0.509)$

Step 4(c): Use the information in Table 4 and apply the C-SFSWHWA_{min}, C-SFSWHWA_{max}, C-SFSWHWG_{min} and C-SFSWOWG_{max} operator. Finding hybrid values is the initial step in this, after which scoring is used to sort the aggregated data, see Table 7.

TABLE 7. Hybrid aggregated operations.

	ζ_1	ζ_2
ν_1	$(0.359, 0.565, 0.424; 0.485)$	$(0.359, 0.565, 0.424; 0.378)$
ν_2	$(0.324, 0.541, 0.523; 0.488)$	$(0.324, 0.541, 0.523; 0.368)$
ν_3	$(0.340, 0.591, 0.548; 0.501)$	$(0.340, 0.591, 0.548; 0.426)$
ν_4	$(0.350, .594, 0.509; 0.578)$	$(0.350, .594, 0.509; 0.509)$
	ζ_3	ζ_4
ν_1	$(0.483, 0.565, 0.284; 0.485)$	$(0.483, 0.565, 0.284; 0.380)$
ν_2	$(0.459, 0.541, 0.450; 0.491)$	$(0.459, 0.541, 0.450; 0.369)$
ν_3	$(0.485, 0.608, 0.468; 0.513)$	$(0.485, 0.608, 0.468; 0.451)$
ν_4	$(0.486, 0.579, 0.455; 0.583)$	$(0.486, 0.579, 0.455; 0.512)$

Step 5: Next, we calculate the scoring of each alternative as presented in Table 8. The ranking is then determined in descending order based on the obtained scores. The results are presented in Table 9.

TABLE 8. Scoring.

	$\delta(\nu_1)$	$\delta(\nu_2)$	$\delta(\nu_3)$	$\delta(\nu_4)$
C-SFSWWA_{min}	-0.3606	-0.3860	-0.4011	-0.4078
C-SFSWWA_{max}	-0.3412	-0.3617	-0.3783	-0.3926
C-SFSWWG_{min}	-0.2910	-0.3313	-0.3537	-0.3502
C-SFSWWG_{max}	-0.2716	-0.3070	-0.3308	-0.3350
C-SFSWOWA_{min}	-0.3571	-0.3837	-0.4020	-0.3995
C-SFSWOWA_{max}	-0.3368	-0.3593	-0.3848	-0.3854
C-SFSWOWG_{min}	-0.2881	-0.3303	-0.3524	-0.3551
C-SFSWOWG_{max}	-0.2678	-0.3060	-0.3352	-0.3410
C-SFSWHWA_{min}	-0.3546	-0.3827	-0.3999	-0.4033
C-SFSWHWA_{max}	-0.3319	-0.3566	-0.3844	-0.3901
C-SFSWHWG_{min}	-0.2887	-0.3313	-0.3505	-0.3532
C-SFSWHWG_{max}	-0.2659	-0.3049	-0.3379	-0.3396

Step 6: The optimal choice is ρ_1.

Thus, implementing an incremental migration to Golang is the best solution to the limitations of Python in handling high traffic and the complexity of a language transfer due to scalability for the social media analytics platform. This approach obviously exceeds other choices and effectively

TABLE 9. Ranking.

	Ranking
C-SFSWWA_{min}	$\nu_1 > \nu_2 > \nu_3 > \nu_4$
C-SFSWWA_{max}	$\nu_1 > \nu_2 > \nu_3 > \nu_4$
C-SFSWWG_{min}	$\nu_1 > \nu_2 > \nu_3 > \nu_4$
C-SFSWWG_{max}	$\nu_1 > \nu_2 > \nu_3 > \nu_4$
C-SFSWOWA_{min}	$\nu_1 > \nu_2 > \nu_4 > \nu_3$
C-SFSWOWA_{max}	$\nu_1 > \nu_2 > \nu_3 > \nu_4$
C-SFSWOWG_{min}	$\nu_1 > \nu_2 > \nu_3 > \nu_4$
C-SFSWOWG_{max}	$\nu_1 > \nu_2 > \nu_3 > \nu_4$
C-SFSWHWA_{min}	$\nu_1 > \nu_2 > \nu_3 > \nu_4$
C-SFSWHWA_{max}	$\nu_1 > \nu_2 > \nu_3 > \nu_4$
C-SFSWHWG_{min}	$\nu_1 > \nu_2 > \nu_3 > \nu_4$
C-SFSWHWG_{max}	$\nu_1 > \nu_2 > \nu_3 > \nu_4$

solves each criterion for a seamless language transfer inside the parameters of the social media analytics platform.

Incremental migration reflects a deliberate, smooth shift that has a number of significant advantages over more abrupt ones. Transitioning of language becomes less disruptive, by using gradual migration, allowing the incorporation of Golang while maintaining essential Python features. The powerful capabilities of Golang to manage massive amounts of data and concurrent activities may be fully utilized through incremental migration. In summary, incremental migration ensures a smooth transfer while maximizing performance, scalability, and overall usefulness. It is not just an option; it is a deliberate investment in the platform's present and future effectiveness.

IX. COMPARATIVE ANALYSIS

In this significant section of our research, we conduct a detailed and in-depth comparison of the properties of the proposed Sugeno-Weber aggregation operators (SW AOs) and the MADM approach presented in this work. We highlight the unique advantages of our reliable approach that set it apart from others. To give a comprehensive evaluation, we compare the unique properties of our multiple aggregation operators to logarithmic-based aggregation approaches [48] and the Einstein aggregation method presented by Abdullah and Ashraf [49]. This extensive comparison demonstrates how our distictive technique succeeds in coping with confusing real-world decision-making problems (DMPs), demonstrating its exceptional efficacy and durability.

We adopted a rigorous technique to compare our proposed aggregating procedures thoroughly. Table 10 shows an aggregated normalized SF information dataset from Abdullah and Ashraf's study [49].

TABLE 10. SF matrix for comparison.

	ζ_1	ζ_2
ν_1	$(0.78, 0.22, 0.31)$	$(0.78, 0.31, 0.20)$
ν_2	$(0.80, 0.07, 0.27)$	$(0.67, 0.24, 0.34)$
ν_3	$(0.81, 0.12, 0.22)$	$(0.89, 0.16, 0.09)$
ν_4	$(0.70, 0.26, 0.27)$	$(0.53, 0.32, 0.39)$
ν_5	$(0.63, 0.33, 0.27)$	$(0.64, 0.29, 0.36)$
	ζ_3	ζ_4
ν_1	$(0.69, 0.40, 0.29)$	$(0.76, 0.23, 0.37)$
ν_2	$(0.81, 0.16, 0.22)$	$(0.91, 0.18, 0.09)$
ν_3	$(0.74, 0.28, 0.22)$	$(0.67, 0.15, 0.39)$
ν_4	$(0.61, 0.42, 0.43)$	$(0.57, 0.25, 0.42)$
ν_5	$(0.72, 0.36, 0.22)$	$(0.68, 0.41, 0.29)$

This data was then converted into our proposed C-SF structure see Table 11, which allows us to execute different aggregate operations on the dataset.

TABLE 11. Circular intuitionistic fuzzy decision matrix.

	ζ_1	ζ_2
ν_1	$(0.78, 0.22, 0.31; 0.08)$	$(0.78, 0.31, 0.20; 0.10)$
ν_2	$(0.80, 0.07, 0.27; 0.10)$	$(0.67, 0.24, 0.34; 0.17)$
ν_3	$(0.81, 0.12, 0.22; 0.07)$	$(0.89, 0.16, 0.09; 0.19)$
ν_4	$(0.70, 0.26, 0.27; 0.15)$	$(0.53, 0.32, 0.39; 0.07)$
ν_5	$(0.63, 0.33, 0.27; 0.04)$	$(0.64, 0.29, 0.36; 0.09)$
	ζ_3	ζ_4
ν_1	$(0.69, 0.40, 0.29; 0.11)$	$(0.76, 0.23, 0.37; 0.09)$
ν_2	$(0.81, 0.16, 0.22; 0.03)$	$(0.91, 0.18, 0.09; 0.19)$
ν_3	$(0.74, 0.28, 0.22; 0.10)$	$(0.67, 0.15, 0.393; 0.17)$
ν_4	$(0.61, 0.42, 0.43; 0.10)$	$(0.57, 0.25, 0.42; 0.08)$
ν_5	$(0.72, 0.36, 0.22; 0.08)$	$(0.68, 0.41, 0.29; 0.05)$

Using these techniques, we generated a variety of alternative rankings. We ranked the alternatives using score function to make assessment and comparison easier, see Table 12.

TABLE 12. Score value of circular intuitionistic fuzzy set.

	$\delta(\nu_1)$	$\delta(\nu_2)$	$\delta(\nu_3)$	$\delta(\nu_4)$	$\delta(\nu_5)$
L-SFWA [48]	0.982	0.998	0.984	0.737	0.934
L-SFOWA [48]	0.980	0.993	0.987	0.613	0.903
L-SFHWA [48]	0.9995	0.9999	0.9997	0.646	0.984
L-SFWG [48]	0.979	0.995	0.972	0.622	0.926
L-SFOWG [48]	0.976	0.979	0.973	0.330	0.892
L-SFHWG [48]	0.9998	0.9999	0.9991	0.822	0.998
	$\delta(\nu_1)$	$\delta(\nu_2)$	$\delta(\nu_3)$	$\delta(\nu_4)$	$\delta(\nu_5)$
SFEWA [49]	0.472	0.529	0.523	0.426	0.458
SFEWG [49]	0.553	0.609	0.593	0.477	0.510
GSFEWA [49]	0.512	0.561	0.555	0.470	0.495
GSFEWG [49]	0.516	0.566	0.549	0.433	0.473
GSFEOWA [49]	0.512	0.562	0.556	0.470	0.496
GSFEOWG [49]	0.516	0.567	0.549	0.433	0.474
	$\delta(\nu_1)$	$\delta(\nu_2)$	$\delta(\nu_3)$	$\delta(\nu_4)$	$\delta(\nu_5)$
CSFSWWA_{min}	-0.1995	-0.168	-0.170	-0.254	-0.225
CSFSWWA_{max}	-0.119	-0.104	-0.108	-0.178	-0.133
CSFSWWG_{min}	-0.161	-0.126	-0.130	-0.216	-0.180
CSFSWWG_{max}	-0.081	-0.062	-0.067	-0.141	-0.089
CSFSWOWA_{min}	-0.1995	-0.162	-0.171	-0.248	-0.225
CSFSWOWA_{max}	-0.119	-0.097	-0.108	-0.173	-0.135
CSFSWOWG_{min}	-0.161	-0.1196	-0.131	-0.212	-0.182
CSFSWOWG_{max}	-0.081	-0.054	-0.068	-0.137	-0.091

The ranks of our suggested aggregation operations, aggregation operations based on logarithms, and Einstein aggregation operators were then summarized in Table 13. By reviewing the results of this wide comparison, we give convincing evidence of the superiority and practicability of our technique. Its ability to deliver reliable responses in complex decision-making settings demonstrates its potential to alter decision sciences and related disciplines.

Our findings are based on the coherence discover in the C-SF information ranking lists generated by both of the suggested SW aggregation operators and comparison techniques. As a consequence, we are certain that the SW Aggregation Operators presented in this research, which operate inside

TABLE 13. Ranking.

	Ranking
L-SFWA [48]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
L-SFOWA [48]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
L-SFHWA [48]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
L-SFWG [48]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
L-SFOWG [48]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
L-SFHWG [48]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
	Ranking
SFEWA [49]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
SFEWG [49]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
GSFEWA [49]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
GSFEWG [49]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
GSFEOWA [49]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
GSFEOWG [49]	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
	Ranking
CSFSWWA_{min}	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
CSFSWWA_{max}	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
CSFSWWG_{min}	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
CSFSWWG_{max}	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
CSFSWOWA_{min}	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
CSFSWOWA_{max}	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
CSFSWOWG_{min}	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$
CSFSWOWG_{max}	$\nu_2 > \nu_3 > \nu_1 > \nu_5 > \nu_4$

a circular spherical fuzzy set environment, offer a unique and adaptive technique for dealing with ambiguity in DM scenarios. The proven coherence and efficacy of our proposed strategy pave the way for more informed and confident decision-making in difficult conditions. Our method fully uses the potential of SFS in dealing with complexity and uncertainty by using the C-SF framework, allowing for more informed and robust decisions. These SW aggregation operators' adaptability makes them useful tools across a wide range of fields, resulting in improvements in decision sciences and crucial insights for practitioners faced with tough and dynamic decision-making situations.

To facilitate a clearer understanding of the comparison, we have analyzed the data as a bar chart. The figure 4 graphically displays the order of our recommended C-SF aggregation operators, logarithmic-based aggregation operators, and Einstein aggregation operators. When the data is presented in this graphical manner, it is easier to comprehend the relative effectiveness and utility of each strategy in addressing real-world decision-making challenges.

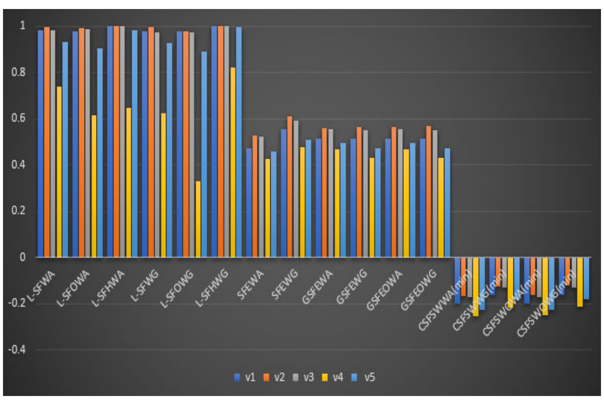

FIGURE 4. Comparison chart.

X. CONCLUSION

This paper developed the circular spherical fuzzy structure along with key operations and theorems of circular spherical fuzzy structures. This study examined the basic operations of SW (Sugeno-Weber) and then generated a number of fundamental identities. We demonstrated the validity and significance of these operations and identities in the context of decision-making processes through our research. The study's findings and insights establish the groundwork for future advancements in the use of SW approaches for a variety of applications and problem-solving scenarios. This study contributes to the greater subject of decision sciences by offering a detailed explanation of SW operations and identities, as well as practical tools for practitioners looking for effective solutions to real-world situations.

The research then focused on providing many SW circular spherical fuzzy aggregation operators, such as weighted average, weighted geometric e.t.c.. By investigating the formal definitions and features of these aggregation operators, derived from the operational laws of C-SFSs using the SW τ-N and τ-CN, the paper addressed the Multiple-Attribute Decision Making (MADM) problem with innovative techniques. A comprehensive study of the language transition for the social media analytics platform was examined within the context of CSF frameworks and comparison with existing methodologies demonstrated the novel method's validity and use. The findings highlighted the method's dependability as well as its potential to improve decision-making processes in uncertain contexts.

We will concentrate on investigating and developing techniques like EDAS, TOPSIS, and TODIM e.t.c in the framework of Circular Spherical Fuzzy (C-SF) environments in the future.

COMPETING INTERESTS

All authors are here with confirm that there are no competing interests between them.

APPENDIX A

(1). To prove, $(\alpha \cap_{min} \beta)^c = \alpha^c \cup_{min} \beta^c$.

First we'll consider,

$$L.H.S = (\alpha \cap_{min} \beta)^c$$
$$= ((\vartheta_\alpha, \iota_\alpha, \kappa_\alpha; \check{r_\alpha}) \cap_{min} (\vartheta_\beta, \iota_\beta, \kappa_\beta; \check{r_\beta}))^c$$
$$= \left(min(\vartheta_\alpha, \vartheta_\beta), min(\iota_\alpha, \iota_\beta), max(\kappa_\alpha, \kappa_\beta); min(\check{r}_1, \check{r}_2)\right)^c$$
$$= \left(max(\kappa_\alpha, \kappa_\beta), min(\iota_\alpha, \iota_\beta), min(\vartheta_\alpha, \vartheta_\beta); min(\check{r}_1, \check{r}_2)\right)$$

Now, consider

$$R.H.S = \alpha^c \cup_{min} \beta^c$$
$$= (\vartheta_\alpha, \iota_\alpha, \kappa_\alpha; \check{r_\alpha})^c \cup_{min} (\vartheta_\beta, \iota_\beta, \kappa_\beta; \check{r_\beta}^c)$$
$$= (\kappa_\alpha, \iota_\alpha, \vartheta_\alpha; \check{r_\alpha}) \cup_{min} (\kappa_\beta, \iota_\beta, \vartheta_\beta; \check{r_\beta})$$
$$= \left(max(\kappa_\alpha, \kappa_\beta), min(\iota_\alpha, \iota_\beta), min(\vartheta_\alpha, \vartheta_\beta); min(\check{r}_1, \check{r}_2)\right)$$

Hence proved.
The remaining proofs are on similar way.

APPENDIX B

(1). To prove, $\alpha \oplus_{max} \beta = \beta \oplus_{max} \alpha$.
First we'll consider,

$$L.H.S = \alpha \oplus_{max} \beta$$
$$= (\vartheta_\alpha, \iota_\alpha, \kappa_\alpha; \breve{r_\alpha}) \oplus (\vartheta_\beta, \iota_\beta, \kappa_\beta; \breve{r_\beta})$$
$$= \begin{pmatrix} \sqrt{\vartheta_\alpha^2 + \vartheta_\beta^2 - \frac{\xi}{1+\xi}\vartheta_\alpha^2\vartheta_\beta^2}, \\ \sqrt{\frac{\iota_\alpha^2+\iota_\beta^2-1+\xi\iota_\alpha^2\iota_\beta^2}{1+\xi}}, \\ \sqrt{\frac{\kappa_\alpha^2+\kappa_\beta^2-1+\xi\kappa_\alpha^2\kappa_\beta^2}{1+\xi}}, \\ \sqrt{\breve{r_\alpha}^2 + \breve{r_\beta}^2 - \frac{\xi}{1+\xi}\breve{r_\alpha}^2\breve{r_\beta}^2} \end{pmatrix}$$
$$= \begin{pmatrix} \sqrt{\vartheta_\beta^2 + \vartheta_\alpha^2 - \frac{\xi}{1+\xi}\vartheta_\beta^2\vartheta_\alpha^2}, \\ \sqrt{\frac{\iota_\beta^2+\iota_\alpha^2-1+\xi\iota_\beta^2\iota_\alpha^2}{1+\xi}}, \\ \sqrt{\frac{\kappa_\beta^2+\kappa_\alpha^2-1+\xi\kappa_\beta^2\kappa_\alpha^2}{1+\xi}}, \\ \sqrt{\breve{r_\beta}^2 + \breve{r_\alpha}^2 - \frac{\xi}{1+\xi}\breve{r_\beta}^2\breve{r_\alpha}^2} \end{pmatrix}$$
$$R.H.S = \beta \oplus_{max} \alpha$$

Hence proved.

(2). To prove, $\alpha \otimes_{max} \beta = \beta \otimes_{max} \alpha$.
First we'll consider,

$$L.H.S = \alpha \otimes_{max} \beta$$
$$= (\vartheta_\alpha, \iota_\alpha, \kappa_\alpha; \breve{r_\alpha}) \otimes (\vartheta_\beta, \iota_\beta, \kappa_\beta; \breve{r_\beta})$$
$$= \begin{pmatrix} \sqrt{\frac{\vartheta_\alpha^2+\vartheta_\beta^2-1+\xi\vartheta_\alpha^2\vartheta_\beta^2}{1+\xi}}, \\ \sqrt{\frac{\iota_\alpha^2+\iota_\beta^2-1+\xi\iota_\alpha^2\iota_\beta^2}{1+\xi}}, \\ \sqrt{\kappa_\alpha^2 + \kappa_\beta^2 - \frac{\xi}{1+\xi}\kappa_\alpha^2\kappa_\beta^2}, \\ \sqrt{\breve{r_\alpha}^2 + \breve{r_\beta}^2 - \frac{\xi}{1+\xi}\breve{r_\alpha}^2\breve{r_\beta}^2} \end{pmatrix}$$
$$= \begin{pmatrix} \sqrt{\frac{\vartheta_\beta^2+\vartheta_\alpha^2-1+\xi\vartheta_\beta^2\vartheta_\alpha^2}{1+\xi}}, \\ \sqrt{\frac{\iota_\beta^2+\iota_\alpha^2-1+\xi\iota_\beta^2\iota_\alpha^2}{1+\xi}}, \\ \sqrt{\kappa_\beta^2 + \kappa_\alpha^2 - \frac{\xi}{1+\xi}\kappa_\beta^2\kappa_\alpha^2}, \\ \sqrt{\breve{r_\beta}^2 + \breve{r_\alpha}^2 - \frac{\xi}{1+\xi}\breve{r_\beta}^2\breve{r_\alpha}^2} \end{pmatrix}$$
$$R.H.S = \beta \otimes_{max} \alpha$$

Hence proved.

(7). To prove, $(\alpha \otimes_{max} \beta)^{\bar{x}} = \alpha^{\bar{x}} \otimes_{max} \beta^{\bar{x}}$. First we'll consider,

$$L.H.S = (\alpha \otimes_{max} \beta)^{\bar{x}}$$
$$= ((\vartheta_\alpha, \iota_\alpha, \kappa_\alpha; \breve{r_\alpha}) \otimes_{max} (\vartheta_\beta, \iota_\beta, \kappa_\beta; \breve{r_\beta}))^{\bar{x}}$$
$$= \begin{pmatrix} \sqrt{\frac{\vartheta_\alpha^2+\vartheta_\beta^2-1+\xi\vartheta_\alpha^2\vartheta_\beta^2}{1+\xi}}, \\ \sqrt{\frac{\iota_\alpha^2+\iota_\beta^2-1+\xi\iota_\alpha^2\iota_\beta^2}{1+\xi}}, \\ \sqrt{\kappa_\alpha^2 + \kappa_\beta^2 - \frac{\xi}{1+\xi}\kappa_\alpha^2\kappa_\beta^2}, \\ \sqrt{\breve{r_\alpha}^2 + \breve{r_\beta}^2 - \frac{\xi}{1+\xi}\breve{r_\alpha}^2\breve{r_\beta}^2} \end{pmatrix}_{max}^{\bar{x}}$$
$$= \begin{pmatrix} \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\vartheta_\alpha^2+\xi\vartheta_\beta^2+\xi^2\vartheta_\alpha^2\vartheta_\beta^2+1}{1+\xi}\right)^{\bar{x}}-1\right)}, \\ \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\iota_\alpha^2+\xi\iota_\beta^2+\xi^2\iota_\alpha^2\iota_\beta^2+1}{1+\xi}\right)^{\bar{x}}-1\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\frac{\xi\kappa_\alpha^2+\xi\kappa_\beta^2}{1+\xi}+\frac{\xi^2\kappa_\alpha^2\kappa_\beta^2}{(1+\xi)^2}\right)^{\bar{x}}\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\frac{\xi\breve{r_\alpha}^2+\xi\breve{r_\beta}^2}{1+\xi}+\frac{\xi^2\breve{r_\alpha}^2\breve{r_\beta}^2}{(1+\xi)^2}\right)^{\bar{x}}\right)}, \end{pmatrix}$$

Now, we'll consider

$$R.H.S = \alpha^{\bar{x}} \otimes_{max} \beta^{\bar{x}}$$
$$= (\vartheta_\alpha, \iota_\alpha, \kappa_\alpha; \breve{r_\alpha})_{max}^{\bar{x}} \otimes_{max} (\vartheta_\beta, \iota_\beta, \kappa_\beta; \breve{r_\beta})_{max}^{\bar{x}}$$
$$= \begin{pmatrix} \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\vartheta_\alpha^2+1}{1+\xi}\right)^{\bar{x}}-1\right)}, \\ \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\iota_\alpha^2+1}{1+\xi}\right)^{\bar{x}}-1\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\kappa_\alpha^2(\frac{\xi}{1+\xi})\right)^{\bar{x}}\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\breve{r_\alpha}^2(\frac{\xi}{1+\xi})\right)^{\bar{x}}\right)} \end{pmatrix} \otimes_{max} \begin{pmatrix} \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\vartheta_\beta^2+1}{1+\xi}\right)^{\bar{x}}-1\right)}, \\ \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\iota_\beta^2+1}{1+\xi}\right)^{\bar{x}}-1\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\kappa_\beta^2(\frac{\xi}{1+\xi})\right)^{\bar{x}}\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\breve{r_\beta}^2(\frac{\xi}{1+\xi})\right)^{\bar{x}}\right)} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{(\xi\vartheta_\alpha^2+1)(\xi\vartheta_\beta^2+1)}{(1+\xi)^2}\right)^{\bar{x}}-1\right)}, \\ \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{(\xi\iota_\alpha^2+1)(\xi\iota_\beta^2+1)}{(1+\xi)^2}\right)^{\bar{x}}-1\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\left(\frac{1-\xi\kappa_\alpha^2}{1+\xi}\right)\left(\frac{1-\xi\kappa_\beta^2}{1+\xi}\right)\right)^{\bar{x}}\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\left(\frac{1-\xi\breve{r}_\alpha{}^2}{1+\xi}\right)\left(\frac{1-\xi\breve{r}_\beta{}^2}{1+\xi}\right)\right)^{\bar{x}}\right)} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\vartheta_\alpha^2+\xi\vartheta_\beta^2+\xi^2\vartheta_\alpha^2\vartheta_\beta^2+1}{1+\xi}\right)^{\bar{x}}-1\right)}, \\ \sqrt{\frac{1}{\xi}\left((1+\xi)\left(\frac{\xi\iota_\alpha^2+\xi\iota_\beta^2+\xi^2\iota_\alpha^2\iota_\beta^2+1}{1+\xi}\right)^{\bar{x}}-1\right)}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\frac{\xi\kappa_\alpha^2+\xi\kappa_\beta^2}{1+\xi}+\frac{\xi^2\kappa_\alpha^2\kappa_\beta^2}{(1+\xi)^2}\right)^{\bar{x}}\right)} \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\frac{\xi\breve{r}_\alpha{}^2+\xi\breve{r}_\beta{}^2}{1+\xi}+\frac{\xi^2\breve{r}_\alpha{}^2\breve{r}_\beta{}^2}{(1+\xi)^2}\right)^{\bar{x}}\right)} \end{pmatrix}$$

Hence proved.

On similar way, we can prove the remaining identities.

APPENDIX C

Mathematical induction could be used for proving the C-SFSWWA operator.

We will prove for $n = 2$. Then we have

$$C-SFSWA_{min}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \sum_{i=1}^{2} \varpi_i\alpha_i\varpi_1 \odot \alpha_1$$

$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\vartheta_1^2(\frac{\xi}{1+\xi})\right)^{\varpi_1}\right)}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\iota_1^2+1}{1+\xi}\right)^{\varpi_1}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\kappa_1^2+1}{1+\xi}\right)^{\varpi_1}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\breve{r}_1{}^2+1}{1+\xi}\right)^{\varpi_1}-1\right)\frac{1}{\xi}} \end{pmatrix}$$

$$\varpi_2 \odot \alpha_2 = \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\vartheta_2^2(\frac{\xi}{1+\xi})\right)^{\varpi_2}\right)}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\iota_2^2+1}{1+\xi}\right)^{\varpi_2}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\kappa_2^2+1}{1+\xi}\right)^{\varpi_2}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\breve{r}_2{}^2+1}{1+\xi}\right)^{\varpi_2}-1\right)\frac{1}{\xi}} \end{pmatrix}$$

$$C-SFSWA_{min}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \varpi_1\alpha_1 \oplus \varpi_2\alpha_2$$

$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\vartheta_1^2(\frac{\xi}{1+\xi})\right)^{\varpi_1}\right)}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\iota_1^2+1}{1+\xi}\right)^{\varpi_1}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\kappa_1^2+1}{1+\xi}\right)^{\varpi_1}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\breve{r}_1{}^2+1}{1+\xi}\right)^{\varpi_1}-1\right)\frac{1}{\xi}} \end{pmatrix} \oplus_{min} \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\vartheta_2^2(\frac{\xi}{1+\xi})\right)^{\varpi_2}\right)}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\iota_2^2+1}{1+\xi}\right)^{\varpi_2}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\kappa_2^2+1}{1+\xi}\right)^{\varpi_2}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\breve{r}_2{}^2+1}{1+\xi}\right)^{\varpi_2}-1\right)\frac{1}{\xi}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\vartheta_1^2(\frac{\xi}{1+\xi})\right)^{\varpi_1}\left(1-\vartheta_2^2(\frac{\xi}{1+\xi})\right)^{\varpi_2}\right)}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\iota_1^2+1}{1+\xi}\right)^{\varpi_1}\left(\frac{\xi\iota_2^2+1}{1+\xi}\right)^{\varpi_2}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\kappa_1^2+1}{1+\xi}\right)^{\varpi_1}\left(\frac{\xi\kappa_2^2+1}{1+\xi}\right)^{\varpi_2}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\frac{\xi\breve{r}_1{}^2+1}{1+\xi}\right)^{\varpi_1}\left(\frac{\xi\breve{r}_2{}^2+1}{1+\xi}\right)^{\varpi_2}-1\right)\frac{1}{\xi}} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{2}\left(1-\vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{2}\left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{2}\left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{2}\left(\frac{\xi\breve{r}_i{}^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}} \end{pmatrix}$$

This indicates that the outcome works properly for n=2.

Make the assumption that the equation is hold true for $n = \pi$, then

$$C-SFSWA_{min}(\alpha_1, \alpha_2, \ldots, \alpha_\pi) = \sum_{i=1}^{\pi} \varpi_i\alpha_i$$

$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{\pi}\left(1-\vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi}\left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi}\left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi}\left(\frac{\xi\breve{r}_i{}^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}} \end{pmatrix}$$

Now, for $n = \pi + 1$, we have as shown in the equation at the next page,

$$C-SFSWA_{min}(\alpha_1,\alpha_2,\ldots,\alpha_{\pi+1})=\sum_{i=1}^{\pi+1}\varpi_i\alpha_i$$

$$C-SFSWA_{min}(\alpha_1,\alpha_2,\ldots,\alpha_{\pi+1})=\sum_{i=1}^{\pi}\varpi_i\alpha_i\oplus\varpi_{\pi+1}\alpha_{\pi+1}$$

$$=\begin{pmatrix}\sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{\pi}\left(1-\vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)},\\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi}\left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}},\\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi}\left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}},\\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi}\left(\frac{\xi\breve{r}_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}}\end{pmatrix}\oplus_{min}$$

$$\begin{pmatrix}\sqrt{\frac{1+\xi}{\xi}\left(1-\left(1-\vartheta_{\pi+1}^2(\frac{\xi}{1+\xi})\right)^{\varpi_{\pi+1}}\right)},\\ \sqrt{\left((1+\xi)\left(\frac{\xi\iota_{\pi+1}^2+1}{1+\xi}\right)^{\varpi_{\pi+1}}-1\right)\frac{1}{\xi}},\\ \sqrt{\left((1+\xi)\left(\frac{\xi\kappa_{\pi+1}^2+1}{1+\xi}\right)^{\varpi_{\pi+1}}-1\right)\frac{1}{\xi}},\\ \sqrt{\left((1+\xi)\left(\frac{\xi\breve{r_{\pi+1}}^2+1}{1+\xi}\right)^{\varpi_{\pi+1}}-1\right)\frac{1}{\xi}}\end{pmatrix}$$

$$=\begin{pmatrix}\sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{\pi}\left(1-\vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)\left(1-\vartheta_{\pi+1}^2(\frac{\xi}{1+\xi})\right)^{\varpi_{\pi+1}}},\\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi}\left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\varpi_i}\left(\frac{\xi\iota_{\pi+1}^2+1}{1+\xi}\right)^{\varpi_{\pi+1}}-1\right)\frac{1}{\xi}},\\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi}\left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\varpi_i}\left(\frac{\xi\kappa_{\pi+1}^2+1}{1+\xi}\right)^{\varpi_{\pi+1}}-1\right)\frac{1}{\xi}},\\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi}\left(\frac{\xi\breve{r}_i^2+1}{1+\xi}\right)^{\varpi_i}\left(\frac{\xi\breve{r_{\pi+1}}^2+1}{1+\xi}\right)^{\varpi_{\pi+1}}-1\right)\frac{1}{\xi}}\end{pmatrix}$$

$$=\begin{pmatrix}\sqrt{\frac{1+\xi}{\xi}\left(1-\prod_{i=1}^{\pi+1}\left(1-\vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)},\\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi+1}\left(\frac{\xi\iota_1^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}},\\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi+1}\left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}},\\ \sqrt{\left((1+\xi)\prod_{i=1}^{\pi+1}\left(\frac{\xi\breve{r}_i^2+1}{1+\xi}\right)^{\varpi_i}-1\right)\frac{1}{\xi}}\end{pmatrix}$$

The most intriguing feature is that the aggregated values deduced by the C-SFSWWA operator are likewise C-SFSN. Consider the following scenario that $\chi = \{(\vartheta_i, \iota_i, \kappa_i; \breve{r}_i)\}$ is a CFS where $i = 1, 2, \ldots, n$ $0 \leq \vartheta_i, \iota_i, \kappa_i \leq 1$ and satisfies $0 \leq \vartheta_\chi^2 + \iota_\chi^2 + \kappa_\chi^2 \leq 1$. And $\sum_{i=1}^n \varpi_i = 1, \varpi_i > 0$.

$$\begin{aligned}
&0 \leq \vartheta_i \leq 1 \Rightarrow 0 \leq \vartheta_i^2 \leq 1 \Rightarrow 0 \\
&\leq \left(1 - \vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i} \leq 1 \\
&\Rightarrow 0 \leq \prod_{i=1}^n \left(1 - \vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i} \leq 1 \\
&\Rightarrow 0 \leq \sqrt{\prod_{i=1}^n \left(1 - \vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}} \leq 1
\end{aligned}$$

Simlarly,

$$\begin{aligned}
&\Rightarrow 0 \leq \sqrt{\prod_{i=1}^n \left(1 - \iota_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}} \leq 1 \\
&\Rightarrow 0 \leq \sqrt{\prod_{i=1}^n \left(1 - \kappa_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}} \leq 1 \\
&\Rightarrow 0 \leq \sqrt{\prod_{i=1}^n \left(1 - \breve{r}_i{}^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}} \leq 1
\end{aligned}$$

Thus,

$$0 \leq \left(\begin{array}{l}
\sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^n \left(1 - \vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\varpi_i} - 1\right)\frac{1}{\xi}}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\varpi_i} - 1\right)\frac{1}{\xi}}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\breve{r}_i{}^2+1}{1+\xi}\right)^{\varpi_i} - 1\right)\frac{1}{\xi}},
\end{array}\right) \leq 1$$

So, the consequence of utilizing the C-SFSWWA operator, what emerges is also a C-SFSN.

APPENDIX D

Assume $\alpha_i = \alpha = (\vartheta_i, \iota_i, \kappa_i, \breve{r}_i)$ where $i = 1, 2 \ldots, n$ be a set of C-SFSNs. Then,

$$C - SFSWWA_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n)$$

$$= \left(\begin{array}{l}
\sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^n \left(1 - \vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\varpi_i} - 1\right)\frac{1}{\xi}}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\varpi_i} - 1\right)\frac{1}{\xi}}, \\
\sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^n \left(1 - \breve{r}_i{}^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)}
\end{array}\right)$$

$$= \left(\begin{array}{l}
\sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^n \left(1 - \vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\sum_{i=1}^n \varpi_i}\right)}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\sum_{i=1}^n \varpi_i} - 1\right)\frac{1}{\xi}}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\sum_{i=1}^n \varpi_i} - 1\right)\frac{1}{\xi}}, \\
\sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^n \left(1 - \breve{r}_i{}^2(\frac{\xi}{1+\xi})\right)^{\sum_{i=1}^n \varpi_i}\right)}
\end{array}\right)$$

$$= \left(\begin{array}{l}
\sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^n \left(1 - \vartheta_i^2(\frac{\xi}{1+\xi})\right)\right)}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\iota_i^2+1}{1+\xi}\right) - 1\right)\frac{1}{\xi}}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\kappa_i^2+1}{1+\xi}\right) - 1\right)\frac{1}{\xi}}, \\
\sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^n \left(1 - \breve{r}_i{}^2(\frac{\xi}{1+\xi})\right)\right)}
\end{array}\right)$$

$$= (\vartheta_i, \iota_i, \kappa_i, \breve{r}_i)$$

$$= \alpha$$

APPENDIX E

Assume α_i be a C-SFSNs and $\tau > 0$, we get

$$\tau\alpha_i$$

$$= \left(\begin{array}{l}
\sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^n \left(1 - \vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\tau}\right)}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\tau} - 1\right)\frac{1}{\xi}}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\tau} - 1\right)\frac{1}{\xi}}, \\
\sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^n \left(1 - \breve{r}_i{}^2(\frac{\xi}{1+\xi})\right)^{\tau}\right)}
\end{array}\right)$$

$$C - SFSWWA_{max}(\tau\alpha_1, \tau\alpha_2, \ldots, \tau\alpha_n) = \sum_{i=1}^n \tau\varpi_i\alpha_i$$

$$= \left(\begin{array}{l}
\sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^n \left(1 - \vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\tau\varpi_i}\right)}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\tau\varpi_i} - 1\right)\frac{1}{\xi}}, \\
\sqrt{\left((1+\xi)\prod_{i=1}^n \left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\tau\varpi_i} - 1\right)\frac{1}{\xi}}, \\
\sqrt{\frac{1+\xi}{\xi}\left(1 - \prod_{i=1}^n \left(1 - \breve{r}_i{}^2(\frac{\xi}{1+\xi})\right)^{\tau\varpi_i}\right)}
\end{array}\right)$$

$$= \begin{pmatrix} \sqrt{\frac{1+\xi}{\xi}\left(1-\left(\prod_{i=1}^{n}\left(1-\vartheta_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)^{\tau}\right)}, \\ \sqrt{\left((1+\xi)\left(\prod_{i=1}^{n}\left(\frac{\xi\iota_i^2+1}{1+\xi}\right)^{\varpi_i}\right)^{\tau}-1\right)\frac{1}{\xi}}, \\ \sqrt{\left((1+\xi)\left(\prod_{i=1}^{n}\left(\frac{\xi\kappa_i^2+1}{1+\xi}\right)^{\varpi_i}\right)^{\tau}-1\right)\frac{1}{\xi}}, \\ \sqrt{\frac{1+\xi}{\xi}\left(1-\left(\prod_{i=1}^{n}\left(1-\breve{r}_i^2(\frac{\xi}{1+\xi})\right)^{\varpi_i}\right)^{\tau}\right)} \end{pmatrix}$$

APPENDIX F

For all $i = 1, 2, \ldots, n$ we have

$$\max_i(\vartheta_i) \leq \vartheta_i \leq \min_i(\vartheta_i)$$

$$\Rightarrow 1-\max_i\left(\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right) \leq 1-\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)$$

$$\leq 1-\min_i\left(\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right)$$

$$\Leftrightarrow \left(1-\max_i\left(\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right)\right)^{\varpi} \leq \left(1-\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right)^{\varpi}$$

$$\leq \left(1-\min_i\left(\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right)\right)^{\varpi}$$

$$\Rightarrow \left(1-\max_i\left(\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right)\right)^{\sum_{i=1}^{n}\varpi}$$

$$\leq \prod_{i=1}^{n}\left(1-\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right)^{\varpi}$$

$$\leq \left(1-\min_i\left(\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right)\right)^{\sum_{i=1}^{n}\varpi}$$

$$\Rightarrow 1-\max_i\left(\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right) \leq \prod_{i=1}^{n}\left(1-\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right)^{\varpi}$$

$$\leq 1-\min_i \vartheta_i^2\left(\frac{\xi}{1+\xi}\right)$$

$$\Leftrightarrow \min_i\left(\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right) \leq 1-\prod_{i=1}^{n}\left(1-\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right)^{\varpi}$$

$$\leq \max_i \vartheta_i^2\left(\frac{\xi}{1+\xi}\right)$$

$$\Leftrightarrow \min_i\left(\vartheta_i^2\right) \leq \left(\frac{1+\xi}{\xi}\right)\left(1-\prod_{i=1}^{n}\left(1-\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right)^{\varpi}\right)$$

$$\leq \max_i(\vartheta_i^2) \Leftrightarrow \min_i\left(\vartheta_i^2\right)$$

$$\leq \sqrt{\left(\frac{1+\xi}{\xi}\right)\left(1-\prod_{i=1}^{n}\left(1-\vartheta_i^2\left(\frac{\xi}{1+\xi}\right)\right)^{\varpi}\right)}$$

$$\leq \max_i(\vartheta_i^2)$$

Similarly,

$$\Rightarrow \min_i\left(\iota_i^2\right)$$

$$\leq \sqrt{\left(\frac{1+\xi}{\xi}\right)\left(1-\prod_{i=1}^{n}\left(1-\iota_i^2\left(\frac{\xi}{1+\xi}\right)\right)^{\varpi}\right)}$$

$$\leq \max_i(\iota_i^2)$$

and

$$\Leftrightarrow \min_i\left(\kappa_i^2\right)$$

$$\leq \sqrt{\left(\frac{1+\xi}{\xi}\right)\left(1-\prod_{i=1}^{n}\left(1-\kappa_i^2\left(\frac{\xi}{1+\xi}\right)\right)^{\varpi}\right)}$$

$$\leq \max_i(\kappa_i^2)$$

and

$$\Leftrightarrow \min_i\left(\breve{r}_i^2\right)$$

$$\leq \sqrt{\left(\frac{1+\xi}{\xi}\right)\left(1-\prod_{i=1}^{n}\left(1-\breve{r}_i^2\left(\frac{\xi}{1+\xi}\right)\right)^{\varpi}\right)}$$

$$\leq \max_i(\breve{r}_i^2)$$

Then, by comparison we get

$$\alpha_i^- \leq C-SFSWWA_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \alpha_i^+$$

Similarly, for $\alpha_i^+ = (\max_i(\vartheta_i), \min_i(\iota_i), \min_i(\kappa_i), \min_i(\breve{r}_i))$

$\alpha_i^- = (\min_i(\vartheta_i), \min_i(\iota_i), \max_i(\kappa_i), \min_i(\breve{r}_i))$.

we can prove,

$$\alpha_i^- \leq C-SFSWWA_{max}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \alpha_i^+$$

REFERENCES

[1] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965.
[2] K. T. Atanassov, *Intuitionistic Fuzzy Sets*. Heidelberg, Germany: Physica-Verlag, 1999, pp. 1–137.
[3] Z. Xu, "Intuitionistic fuzzy aggregation operators," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 6, pp. 1179–1187, Dec. 2007.
[4] W. Wang and X. Xin, "Distance measure between intuitionistic fuzzy sets," *Pattern Recognit. Lett.*, vol. 26, no. 13, pp. 2063–2069, Oct. 2005.

[5] M. Luo and R. Zhao, "A distance measure between intuitionistic fuzzy sets and its application in medical diagnosis," *Artif. Intell. Med.*, vol. 89, pp. 34–39, Jul. 2018.
[6] C. Jana, M. Dobrodolac, V. Simic, M. Pal, B. Sarkar, and Ž. Stević, "Evaluation of sustainable strategies for urban parcel delivery: Linguistic q-rung orthopair fuzzy choquet integral approach," *Eng. Appl. Artif. Intell.*, vol. 126, Nov. 2023, Art. no. 106811.
[7] C. Jana, H. Garg, M. Pal, B. Sarkar, and G. Wei, "MABAC framework for logarithmic bipolar fuzzy multiple attribute group decision-making for supplier selection," *Complex Intell. Syst.*, pp. 1–16, Jul. 2023.
[8] K. T. Atanassov, "Circular intuitionistic fuzzy sets," *J. Intell. Fuzzy Syst.*, vol. 39, no. 5, pp. 5981–5986, Nov. 2020.
[9] K. Atanassov and E. Marinov, "Four distances for circular intuitionistic fuzzy sets," *Mathematics*, vol. 9, no. 10, p. 1121, May 2021.
[10] E. Boltürk and C. Kahraman, "Interval-valued and circular intuitionistic fuzzy present worth analyses," *Informatica*, vol. 33, no. 4, pp. 693–711, 2022.
[11] E. Çakır and M. A. Taş, "Circular intuitionistic fuzzy decision making and its application," *Expert Syst. Appl.*, vol. 225, Sep. 2023, Art. no. 120076.
[12] C. Kahraman and N. Alkan, "Circular intuitionistic fuzzy TOPSIS method with vague membership functions: Supplier selection application context," *Notes Intuitionistic Fuzzy Sets*, vol. 27, no. 1, pp. 24–52, Jun. 2021.
[13] N. Alkan and C. Kahraman, "Circular intuitionistic fuzzy TOPSIS method: Pandemic hospital location selection," *J. Intell. Fuzzy Syst.*, vol. 42, no. 1, pp. 295–316, Dec. 2021.
[14] C. Kahraman and I. Otay, "Extension of VIKOR method using circular intuitionistic fuzzy sets," in *Proc. Int. Conf. Intell. Fuzzy Syst.* Cham, Switzerland: Springer, 2021, pp. 48–57.
[15] İ. Otay and C. Kahraman, "A novel circular intuitionistic fuzzy AHP-VIKOR methodology: An application to a multi-expert supplier evaluation problem," *Pamukkale Üniversitesi Mühendislik Bilimleri Dergisi*, vol. 28, no. 1, pp. 194–207, 2022.
[16] T.-Y. Chen, "A circular intuitionistic fuzzy evaluation method based on distances from the average solution to support multiple criteria intelligent decisions involving uncertainty," *Eng. Appl. Artif. Intell.*, vol. 117, Jan. 2023, Art. no. 105499.
[17] T.-Y. Chen, "Evolved distance measures for circular intuitionistic fuzzy sets and their exploitation in the technique for order preference by similarity to ideal solutions," *Artif. Intell. Rev.*, vol. 56, no. 7, pp. 7347–7401, Jul. 2023.
[18] M. J. Khan, W. Kumam, and N. A. Alreshidi, "Divergence measures for circular intuitionistic fuzzy sets and their applications," *Eng. Appl. Artif. Intell.*, vol. 116, Nov. 2022, Art. no. 105455.
[19] R. R. Yager, "Pythagorean fuzzy subsets," in *Proc. Joint IFSA World Congr. NAFIPS Annu. Meeting (IFSA/NAFIPS)*, Jun. 2013, pp. 57–61.
[20] R. R. Yager, "Pythagorean membership grades in multicriteria decision making," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 4, pp. 958–965, Aug. 2014.
[21] R. R. Yager and A. M. Abbasov, "Pythagorean membership grades, complex numbers, and decision making," *Int. J. Intell. Syst.*, vol. 28, no. 5, pp. 436–452, 2013.
[22] M. Olgun, M. Ünver, and Ş. Yardımcı, "Pythagorean fuzzy points and applications in pattern recognition and Pythagorean fuzzy topologies," *Soft Comput.*, vol. 25, no. 7, pp. 5225–5232, Apr. 2021.
[23] C. Peng and Y. Yang, "Some results for Pythagorean fuzzy sets," *Int. J. Intell. Syst.*, vol. 30, no. 11, pp. 1133–1160, 2015.
[24] H. Garg, "A new generalized Pythagorean fuzzy information aggregation using Einstein operations and its application to decision making," *Int. J. Intell. Syst.*, vol. 31, no. 9, pp. 886–920, Sep. 2016.
[25] H. Garg, "Generalized Pythagorean fuzzy geometric aggregation operators using Einstein t-norm and t-conorm for multicriteria decision-making process," *Int. J. Intell. Syst.*, vol. 32, no. 6, pp. 597–630, Jun. 2017.
[26] X. Zhang and Z. Xu, "Extension of TOPSIS to multiple criteria decision making with Pythagorean fuzzy sets," *Int. J. Intell. Syst.*, vol. 29, no. 12, pp. 1061–1078, Dec. 2014.
[27] P. Ren, Z. Xu, and X. Gou, "Pythagorean fuzzy TODIM approach to multi-criteria decision making," *Appl. Soft Comput.*, vol. 42, pp. 246–259, May 2016.
[28] X. Zhang, "A novel approach based on similarity measure for Pythagorean fuzzy multiple criteria group decision making," *Int. J. Intell. Syst.*, vol. 31, no. 6, pp. 593–611, Jun. 2016.
[29] A. Biswas and B. Sarkar, "Pythagorean fuzzy multicriteria group decision making through similarity measure based on point operators," *Int. J. Intell. Syst.*, vol. 33, no. 8, pp. 1731–1744, Aug. 2018.
[30] A. Biswas and B. Sarkar, "Pythagorean fuzzy TOPSIS for multicriteria group decision-making with unknown weight information through entropy measure," *Int. J. Intell. Syst.*, vol. 34, no. 6, pp. 1108–1128, Jun. 2019.
[31] K. Rahman, S. Abdullah, and F. Ghani, "Some new generalized interval-valued Pythagorean fuzzy aggregation operators using Einstein t-norm and t-conorm," *J. Intell. Fuzzy Syst.*, vol. 37, no. 3, pp. 3721–3742, 2019.
[32] M. Palanikumar, K. Arulmozhi, C. Jana, and M. Pal, "Multiple attribute decision-making Pythagorean vague normal operators and their applications for the medical robots process on surgical system," *Comput. Appl. Math.*, vol. 42, no. 6, p. 287, Sep. 2023.
[33] M. C. Bozyiğit, M. Olgun, and M. Ünver, "Circular Pythagorean fuzzy sets and applications to multi-criteria decision making," 2022, *arXiv:2210.15483*.
[34] M. J. Khan, J. C. R. Alcantud, W. Kumam, P. Kumam, and N. A. Alreshidi, "Expanding Pythagorean fuzzy sets with distinctive radii: Disc Pythagorean fuzzy sets," *Complex Intell. Syst.*, vol. 9, pp. 7037–7054, Dec. 2023.
[35] S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani, and T. Mahmood, "Spherical fuzzy sets and their applications in multi-attribute decision making problems," *J. Intell. Fuzzy Syst.*, vol. 36, no. 3, pp. 2829–2844, Mar. 2019.
[36] M. Riaz, H. M. A. Farid, C. Jana, M. Pal, and B. Sarkar, "Efficient city supply chain management through spherical fuzzy dynamic multistage decision analysis," *Eng. Appl. Artif. Intell.*, vol. 126, Nov. 2023, Art. no. 106712.
[37] S. Ashraf and S. Abdullah, "Spherical aggregation operators and their application in multiattribute group decision-making," *Int. J. Intell. Syst.*, vol. 34, no. 3, pp. 493–523, Mar. 2019, doi: 10.1002/int.22062.
[38] S. Ashraf, S. Abdullah, M. Aslam, M. Qiyas, and M. A. Kutbi, "Spherical fuzzy sets and its representation of spherical fuzzy t-norms and t-conorms," *J. Intell. Fuzzy Syst.*, vol. 36, no. 6, pp. 6089–6102, Jun. 2019, doi: 10.3233/JIFS-181941.
[39] F. K. Gundogdu and C. Kahraman, "Extension of WASPAS with spherical fuzzy sets," *Informatica*, vol. 30, no. 2, pp. 269–292, Jan. 2019.
[40] F. K. Gündoğdu and C. Kahraman, "Spherical fuzzy sets and spherical fuzzy TOPSIS method," *J. Intell. Fuzzy Syst.*, vol. 36, no. 1, pp. 337–352, Feb. 2019.
[41] T. Mahmood, K. Ullah, Q. Khan, and N. Jan, "An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets," *Neural Comput. Appl.*, vol. 31, no. 11, pp. 7041–7053, Nov. 2019.
[42] E. Ozceylan, B. Ozkan, M. Kabak, and M. Dagdeviren, "A state-of-the-art survey on spherical fuzzy sets," *J. Intell. Fuzzy Syst.*, vol. 42, no. 1, pp. 195–212, 2022.
[43] X. Peng and W. Li, "Spherical fuzzy decision making method based on combined compromise solution for IIoT industry evaluation," *Artif. Intell. Rev.*, vol. 55, pp. 1857–1886, Mar. 2022.
[44] M. J. Khan, P. Kumam, W. Deebani, W. Kumam, and Z. Shah, "Distance and similarity measures for spherical fuzzy sets and their applications in selecting mega projects," *Mathematics*, vol. 8, no. 4, p. 519, Apr. 2020.
[45] M. Sugeno, "Theory of fuzzy integrals and its applications," Doctoral thesis, Tokyo Inst. Technol., Tokyo, Japan, 1974.
[46] S. Weber, "A general concept of fuzzy connectives, negations and implications based on t-norms and t-conorms," *Fuzzy Sets Syst.*, vol. 11, nos. 1–3, pp. 115–134, 1983.
[47] A. Sarkar, T. Senapati, L. Jin, R. Mesiar, A. Biswas, and R. R. Yager, "Sugeno–Weber triangular norm-based aggregation operators under T-spherical fuzzy hypersoft context," *Inf. Sci.*, vol. 645, Oct. 2023, Art. no. 119305.
[48] Y. Jin, S. Ashraf, and S. Abdullah, "Spherical fuzzy logarithmic aggregation operators based on entropy and their application in decision support systems," *Entropy*, vol. 21, no. 7, p. 628, Jun. 2019.
[49] S. Ashraf, S. Abdullah, and R. Chinram, "Emergency decision support modeling under generalized spherical fuzzy Einstein aggregation information," *J. Ambient Intell. Hum. Comput.*, vol. 13, pp. 2091–2117, Apr. 2022.

SHAHZAIB ASHRAF received the B.S. degree in mathematics from the University of Sargodha, the M.S. degree in (fuzzy) mathematics from International Islamic University, Islamabad, and the Ph.D. degree from Abdul Wali Khan University, Mardan, Pakistan. He is an Assistant Professor with the Khawaja Fareed University of Engineering and Information Technology, Rahim Yar Khan, Pakistan. He served with Bacha Khan University, Charsadda, Pakistan, as an Assistant Professor. He has published 100 research articles with more than 3000 citations. He currently supervises four M.S. and four Ph.D. thesis and also supervised four M.S. thesis on his credit. His some papers have published in high impact journals, including *International Journal of Intelligent Systems*. Recently, he recognized as World's Top 2 % Scientists' list (STANFORD UNIVERSITY) from 2020 to 2023. He was Awarded for travel grant of Rs.2,28,362, in 2018 by the Higher Education Commission (HEC) of Pakistan to present a research paper in 5th International Conference on Recent Advances in Pure and Applied Mathematics to be held at Karadeniz Technical University, Trabzon, Turkey. His specialization is in fuzzy mathematics, spherical fuzzy sets, fuzzy decision support systems, computational intelligence, and soft computing.

WANIA IQBAL received the B.S. degree in mathematics from the Khwaja Fareed University of Engineering and Information Technology, Rahim Yar Khan, Pakistan, where she is currently pursuing the M.S. degree. Her target in M.S. degree is generalization of spherical fuzzy set. She is also an M.Phil. Scholar with the Khwaja Fareed University of Engineering and Information Technology. She is researching on the extension of spherical fuzzy sets on daily life problems. They are working on different types of aggregation operators by using many different types of norms. She received the Gold Medal in bachelor's degree by securing highest CPGA in batch.

SHAKOOR AHMAD received the B.S. degree in mathematics from the University of Sargodha, Sargodha, Pakistan, and the M.S. and Ph.D. degrees in mathematics from COMSATS University Islamabad, Pakistan. Currently, he is researching on the field of applied mathematics (numerical and fractional mathematics).

FAISAL KHAN received the B.S. degree in mathematics from the University of Malakand, Chankdara, Lower Dir, Pakistan, in 2015, and the M.Phil. degree in mathematics from Hazara University, Mansehra, Pakistan, in 2017. He is currently pursuing the Ph.D. degree with the National University of Ireland, Galway (NUIG). He is also with FotoNatioN/Xperi. His research interest includes machine learning using deep neural networks for tasks related to computer vision, including depth estimation and 3-D reconstruction.