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RESEARCH ARTICLE

Diversification of Energy Resources for Electricity Generation in Pakistan via Portfolio Optimization

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ABSTRACT The aim of this paper is to investigate Pakistan's efficient generation portfolios by comparing the portfolio costs, risks, efficient frontiers, and diversification levels under different cases and scenarios. Uncertainty in the energy sector of Pakistan has contributed to the necessity of effective electricity planning that can minimize given market risks and discourage the use of stand-alone generating costs. This paper presents a novel energy production optimization empirically using mean-variance portfolio theory. The proposed research work leverages the mean-variance portfolio theory to design an efficient electricity generation mix to diversify its energy mix by investing in renewable energy sources. The portfolio optimization for the energy sector is driven by the minimization of either the cost or risk of the energy resource portfolio subject to various budget constraints. In the context of decentralized energy sources, the current work presents an optimal portfolio design that takes into account average costs, risks, and diversity of the energy resources for electricity generation. This is followed by the computation of optimal efficient frontier for energy portfolio based on extensive statistical analysis over data collected for eight different technologies, namely coal, high-speed diesel, fuel oil, gas, solar, wind, and hydro. Numerical results reveal that diversification of energy sources, which consists of a mixture of all technologies, yields a solution that guarantees reliability of more than 90% for the given data.

INDEX TERMS Mean-variance portfolio, energy resources planning, diversification, optimal efficient frontier.

I. INTRODUCTION

Energy is an indispensable entity for any society due to its important role in social and economic development [1]. Global energy safety concerns have been raised due to the increasing growth in energy consumption. With each passing day, the issue of fulfilling the energy demand is becoming crucial and is the center of many public and political discussions. Some of the key factors include uncertainties in oil prices, their economic entanglements, and environmental and social impacts [2], [3]. Today, analysts and researchers from several domains concentrate on the long-term evaluation of energy needs by creating plausible scenarios based on energy factors and the community's historical energy consumption. In the future, energy demand in developing countries will be more likely to increase as population and social, economic development will increase [4].

Energy raises [5] the standard of living by playing a major part in socio-economic success. With the growth in the economy, the energy demand enhances which as well not only changes the consumption pattern but also affects the source of energy and points out losses and efficiency [5]. Wind speed, availability, and flow of water in hydro generation, and sun-light availability in solar generation are the different uncertainties that overwhelm the modern energy system. There is a need to tackle these uncertainties as they are becoming important because of the revolution in energy generation by the addition of renewable energy sources.

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Using renewable energy technology also helps maintain environmental concerns that are produced by conventional sources such as air and water pollution, land use, community values, and public opposition, among others [6]. Assigning capital over various resources to increase the return and reduce the risk level is referred to as portfolio selection [7]. Risks in investing in renewable energy sources are becoming increasingly important due to the renewable energy revolution in recent years. Various tools are available for the optimization of portfolio dealing with vulnerabilities directs that optimization can only be found by evaluating all the plans and strategies together in the form of a project. It is difficult to evaluate the costs, risks, and returns of stand-alone projects [8]. The major hurdle in dealing with these uncertainties in a project is to balance costs with socio-economic impacts [9].

A. LITERATURE REVIEW

Portfolio theory was first proposed by Markowitz in 1952 [10] for addressing problems in financial research. Bar-Lev and Katz [11] pioneered its application to the power industry, and they mainly analyzed fossil fuel procurement in the United States. Since then, numerous modeling techniques have been designed to quantify energy services and supply resources, many of which are based on mathematical modeling. Energy systems can be made more efficient by addressing issues such as new energy sources discovery, greenhouse gas mitigation, energy conservation, and the introduction of efficient and renewable energy technologies [12]. If the techniques of energy planning have been effectively used, the efficiency of the system can be enhanced further. Forecasting of energy, supply and demand management, assessment of future investments in the power sector, and evaluation of optimal growth strategy are all the nominal features of energy planning [13]. Towards this end, a least-cost methodology is a traditional tool used for energy planning [14]. In this approach, the levelized cost of electricity of each technology is evaluated and after comparing costs among these technologies, the selected technology would be that which has the lowest cost. However, this methodology has been met with criticism due to inefficient results in policymaking and supporting investment decisions. Selection of the technology based on the lowest individual cost brings this approach into action. Therefore, the usage of this methodology is more appropriate for the technologies that use fossil fuels over renewable energy sources [15], [16].

Modern portfolio theory helps the planners to analyze the technologies from a cost-risk or return-risk perspective and when risk is considered, priority is given to renewable energy sources over non-renewable energy sources. The classical treatment of portfolio theory for energy mixed addressed in [17], considers renewable energy sources as fixed-cost technologies which in turn reduces the risk as compared to the risk attached to the cost of non-renewable energy sources. Similar work was then reported in [18], [19] to evaluate the optimal energy mix. Due to the change in the portfolio methodology opted, planners and investors are more interested in technology portfolio rather than in individual technology and now, both the investors and regulators are part of this long-term vision. In comparison with the least-cost methodology, analysis of both cost and risk in the mean-variance approach has enhanced its potential and conceptual richness. Similarly, the portfolio model includes the key elements of energy analysis: the advantage of portfolio effect in reducing the risk, the risk associated with the variations in the cost of electricity generating units, and the risks of irregularity that are related to renewable energy sources.

Several works have reported the use of a framework based on the cost and risk of the technologies from which efficient frontier is produced [20]. Authors in [21] analyzed wind energy and proposed a minimum cost framework. In this framework, both the installed capacity for each plant in the portfolio and the mode of plant use have been optimized. A previous study [22], [23] used the Shannon-Wiener index and portfolio theory to analyze the diversification of the portfolio. According to the authors, the index enriches the robustness of the model because in terms of probability any forecast of the future is no longer required. Authors in [24] developed an algorithm to maximize the expected return which is calculated as the least increase in cost per unit of energy production. The production of shale gas is based on the interplay between CO2 and water by exploiting portfolios optimization for cost-effective solution [25]. A recent work [26] on solar based micro-grid has utilized portfolio theory to yield better planning for economical production of renewable energy.

Fuel prices and investments are the major uncertainties faced by the planners and in the long run, these uncertainties are unpredictable [27]. In addition, hydroelectricity generation, availability of solar irradiance, and speed of wind are also important factors of variability [28], [29]. These key factors have an impact on the planning and operation of the electricity system. All of the above-mentioned uncertainties, including unavailability of electricity storage, variable demand, and the complex supply function, produce an imbalance between prices and quantities in the energy sector which finally affects the cost and risk exposed by the market agents. Storage technologies are still not efficient enough to remove the coupling between supply and demand in markets, but with technology development and reduction in cost, this paradigm can be changed [30], [31]. Another study [32] targets the problem of district heating by utilizing portfolio optimization in conjunction with integer based programming.

To solve the industrial and economic problems related to expected returns and risks associated with them, portfolio optimization is used. The resulting diversification helps study the combination of renewable energy sources and conventional energy sources or the combination of all renewable sources if a carbon-free environment is concerned. Positive effects exist for a non-fossil generation goal or a policy target package from the perspectives of minimizing cost and risk, as well as for improving the diversification level. It is crucial to investigate empirically the diversification possibilities in a country like Pakistan where the energy crisis is rampant. To the best of the authors' knowledge, no such study has been done so far.

B. RESEARCH OBJECTIVES

The energy crisis in Pakistan refers to the shortage of electricity, which is of prime importance for the efficient functioning of the industries and factories of the country to progress economically. There are many problems and weaknesses in Pakistan's energy sector [33]. It's worth noting that Pakistan faces significant energy shortages, particularly during peak demand periods. This has led to load shedding and power outages in many parts of the country. To address this issue, the Pakistani government has launched several initiatives to increase energy generation capacity and improve energy efficiency. This research aims to create an appropriate framework that will guide the policymakers and investors for investments and policy making of energy plants in the desired direction. Firstly, the electricity planning problem is formulated by leveraging the mean-variance optimization approach. This is followed by the numerical assessment of current energy cost along with a standard deviation on that cost (risk) after calculating correlations between different types of costs (e.g., investment costs, fuel costs, O&M costs) extracted from publicly available energy data. Finally, the resulting portfolio is mapped onto the optimal frontier to evaluate the efficiency of the energy mix of diversified sources. In summary, the following are the main objectives of this research work:

- Development of framework consisting of the risk-return tradeoff for energy planning.
- Apply mean-variance portfolio theory to find an efficient frontier of all energy resource mixtures.
- To find an energy portfolio based on minimum cost covariance risk, subject to a set level of return.
- Analysis of the publicly available data on energy generation to calculate efficient frontier and then accordingly propose the diversification of energy sources that efficiently provides electricity.

II. MEAN-VARIANCE TRADEOFF

In the cost-risk framework, the expected cost of the portfolio is the result of the addition of the total cost per technology, weighted by the percentage share of each technology in the portfolio. The risk of the portfolio is considered as a function of the risk for each technology which includes correlation factors among the various costs (like O&M, fuel cost) associated with different technologies and the percentage share of each technology. Risk is expressed through the variability of the returns/costs for the set of technologies. The technologies with the greatest risks are usually based on fossil fuels such as natural gas, petroleum, and their derivatives which are subject to a high degree of price variability. There are many ways to classify the usage of portfolio optimization applications in the electricity sector. One of the major tasks of investors is to allocate the resource in different projects that result in a commercial strategy in which electricity sales and purchases are allocated to different trading mechanisms. According to the public perspective, portfolio optimization is inclined towards making a policy design by facilitating the regulators and planners.

On the other hand, according to investors, the purpose of using portfolio theory is to maximize the profits and mitigate the risks in comparison to energy planners whose agenda is to reduce social and environmental costs. Thus, portfolio optimization applications are different for investors because from an investor perspective the focus is on financial profit while in the planner's perspective sustainability including environmental, social, and economic factors are more important.

In liberalized markets, investors invest in generation projects rather than government and therefore investments are influenced by the expected profit while paying little attention to sustainability solutions [34]. Investors analyses each available project by analyzing risk and their relationship with other projects. These measures involve risk in prices, technical risk, and financial risk. To investigate these risks, net present value, internal rate of return, and multiple cash flows has been analyzed [35].

The major hurdle for policymakers is to design a regulatory strategy in which the private incentives will be aligned with sustainability. The minimum investment is not a constraint for the investor, so they are suggested to consider the value of waiting in investment analysis [36]. Nonetheless, investments in the electricity sector are characterized by an irreversible attitude, due to the high sunken costs involved with the production of electricity and the purchase of property, the use of handling licenses, and the availability of grid connections.

TABLE 1. List of symbols and abbreviations.

Symbol	Meaning
O & M	operation and maintenance
MW	mega watt
MWh	mega watt hour
GWh	giga watt hour
NTDC	National Transmission & Dispatch Company
HSD	high-speed diesel
PKR	Pakistan Rupee
USD	United States Dollar
$\boldsymbol{\omega}$	weight vector
$\boldsymbol{\Omega}$	feasible set of weight vectors
$\omega_i^{\min}, \omega_i^{\max}$	min and max weight for the i-th
$\boldsymbol{c}$	random cost vector with mean $\boldsymbol{r}$ and covariance matrix $\boldsymbol{\Psi}$
γ	maximum allowed expected cost
NEPRA	National Electric Power Regulatory Authority

III. ENERGY PROFILE OF PAKISTAN

The percentage share of electricity generation in Pakistan produced by different sources is shown in Figure 1. According to the data, 68.89% of the energy profile is contributed by thermal sources, 21.00% by hydel, 6.77% by nuclear, 2.92%

by renewables, and the rest was imported [37]. The total electricity generation at that time was 133.669 GWh. With each passing year, an increase in thermal and nuclear power generation can be observed. But the percentage production of electricity from hydel has gradually reduced. As of June 2022, the percentage share by the public and private sector was 39.05 and 60.95 respectively [38]. More than 7,000 MW of generation systems connected to the National Transmission & Dispatch Company (NTDC) grid have been installed since 2013.

The data as provided by the NTDC shows the installed capacity, generation capability, and surplus/deficit. It may be noted from Figure 2 that is a clear difference between installed capacity and electricity generation capacity of technologies owing to a multitude of economic and environmental factors. Surplus electricity may not be available from 2022 to 2025 due to the unsure completion of hydro projects [38]. Figure 3 shows the data of the energy purchased from different sources. The percentage share of High-Speed Diesel (HSD) is maximum, and energy sold by the coal has minimum contribution. A concept paper has been prepared by the national electric power regulatory authority to review the rate of returns. It will provide the guidelines to the power generation companies to determine the internal rate of return of various technologies and tariff regimes.

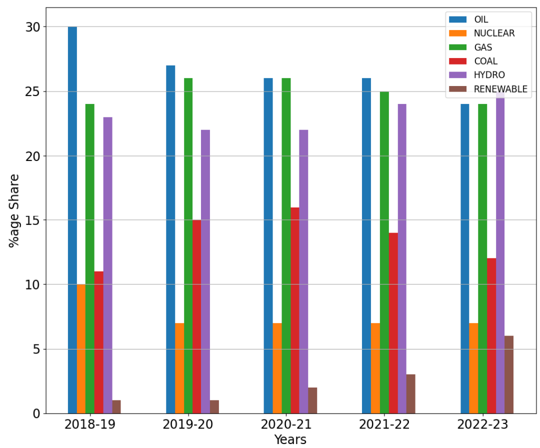

FIGURE 1. **Percentage share of electricity generation.**

IV. METHODOLOGY

This paper leverages modern portfolio theory for the calculation of the efficient frontier of electricity-producing technologies. To this end, a portfolio of electricity mix is prepared by assigning an optimal set of nonnegative weights. Certain technical constraints decide the range of variation of cost of each energy technology. This approach unifies the data of expected cost and risk related to each feasible portfolio. For a given energy source, the corresponding risk can be calculated from the available cost dispersion (standard deviation). On the other hand, to estimate the risk of the energy portfolio, it is compulsory to examine

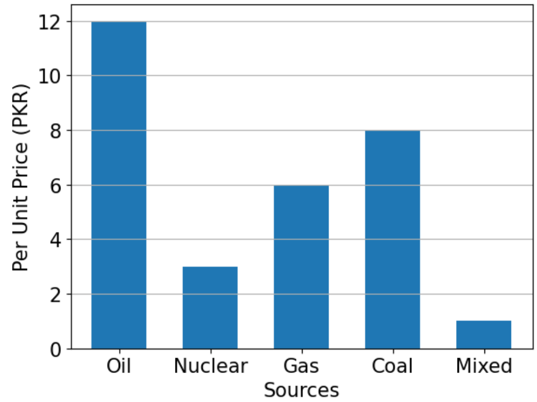

FIGURE 2. **Surplus and deficit of energy production.**

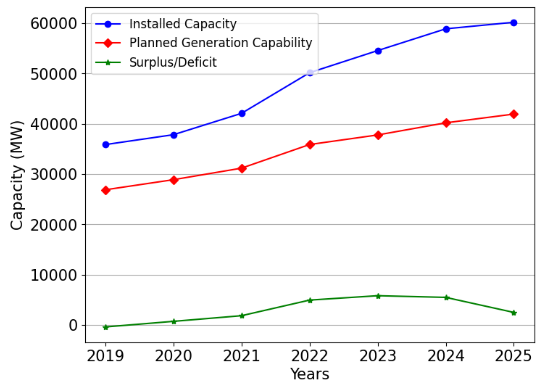

FIGURE 3. **Source wise per unit energy purchase price (PKR).**

the cross-correlations cost among all distinct technologies. Once the risk and expected cost of all feasible portfolios are calculated, an efficient mixture/portfolio, which minimizes cost covariance risk subject to a set level of return, is obtained.

We consider that we have m energy technologies to build an energy portfolio. Components of the vector $\boldsymbol{\omega} = [\omega_1, \omega_2, \cdots, \omega_m]^\top$ indicate the weights of each technology. We have used bold-faced letters to denote vector quantities. As $i = 1, \ldots, m$, the proportion of energy portfolio produced by the i-th technology is $\omega_i \geq 0$. Let $\boldsymbol{\Omega}$ denote the set of all possible mixes of electricity generation, thus we have $\boldsymbol{\omega} \in \boldsymbol{\Omega}$. Sum of the components of each portfolio is equal to one, and the maximum and the minimum values for each technology's contributions ($\omega_i^{\min}$ and $\omega_i^{\max}$) are 0 and 1, respectively. Formally, we have

$$\sum_{i=1}^{m} \omega_i = 1, \quad \omega_i^{\min} \leq \omega_i \leq \omega_i^{\max}, \; i = 1, \ldots, m. \tag{1}$$

It is observed that the cost vector of most of the energy sources present high uncertainty and can vary according to the region in a country and across time due to technologic evolution and fuel prices movements. This is the reason why it is more realistic to consider a random cost vector. Let the random cost vector $\boldsymbol{c} = [c_1, c_2, \cdots, c_m]^\top$ represent the per unit cost (USD kWh) of the m energy resources with mean value of $\mathbb{E}[\boldsymbol{c}] = \boldsymbol{r}$ and covariance matrix $\mathbb{E}\left[(\boldsymbol{c}-\boldsymbol{r})(\boldsymbol{c}-\boldsymbol{r})^\top\right] = \boldsymbol{\psi}$. For a given energy mix, parameterized by the weight vector $\boldsymbol{\omega}$, the total random cost is

$$C = \sum_{i=1}^{m} \omega_i c_i = \boldsymbol{\omega}^\top \boldsymbol{c} \tag{2}$$

From (2), the total expected cost is given by $\mathbb{E}[C] = E\left[\boldsymbol{\omega}^\top \boldsymbol{c}\right] = \boldsymbol{\omega}^\top \boldsymbol{r}$, whereas the cost variance or risk is given by $\mathrm{Var}(C)) = \boldsymbol{\omega}^\top \boldsymbol{\psi} \boldsymbol{\omega}$. In our work, we seek the optimal weight vector $\boldsymbol{\omega}$ by minimizing the risk of cost subject to a maximum allowed expected cost γ provided by a policy planner. Mathematically, this can be formulated as an optimization problemÂ asÂ follows

$$\text{minimize} \quad \boldsymbol{\omega}^\top \boldsymbol{\psi} \boldsymbol{\omega} \tag{3a}$$

$$\text{subject to} \quad \boldsymbol{\omega}^\top \boldsymbol{r} \leq \gamma, \quad \boldsymbol{\omega} \in \boldsymbol{\Omega}. \tag{3b}$$

The above optimization problem is convex and thus can easily be solved by any convex solver to give a global optimal value for the weights of the optimized energy mix.

V. NUMERICAL RESULTS

Data has been collected from the annual report of NEPRA [38] and the international renewable energy agency assessment for Pakistan [35]. These reports have provided data for fuel costs along with operational and maintenance costs. Per unit (kWh) cost of electricity generation from each source has been taken from reports of central power purchasing agency. To formulate the objective function for the portfolio optimization, generation sources have been grouped into eight categories: coal, high-speed diesel, furnace oil, gas, mixed, solar, wind, hydel. To quantify the relation by which any two assets vary with each other, we need to calculate the correlation coefficient. It is used in the management of portfolios and its values lie between -1 and $+1$. We use the following relation to calculate the correlation coefficient

$$r = \frac{\sum_{n=1}^{N} \left(X_n - \bar{X}\right)\left(Y_n - \bar{Y}\right)}{\sqrt{\sum_{n=1}^{N} \left(X_n - \bar{X}\right)^2 \sum_{n=1}^{N} \left(Y_n - \bar{Y}\right)^2}} \tag{4}$$

of observation X and Y, respectively. The purpose of the computing correlation matrix is to observe the behavior between different sets of variables. In this matrix, the degree of correlation is found out between each variable. The correlation matrix consists of the correlation coefficient. For the diversification, the correlation matrix of both fuel cost and O&M costs have been calculated. Technologies that have been considered are coal, high-speed diesel, fuel oil, gas, mixed, solar, wind, and hydro.

Table 2 presents the correlation matrix of fuel cost for thermal power generation. It can be observed that coal is negatively correlated with high-speed diesel, fuel oil, gas, and other mixed technologies. As such, the remaining technologies have a positive correlation with each other. Just to illustrate this point, if the fuel price of coal increases, prices of other technologies will be reduced and because other technologies are positively correlated with each other, so increase in the price of one technology will increase the price of other technologies too. Table 3 describes the operational and maintenance cost correlation matrix of thermal power technologies with renewable technologies. This table shows that coal and fuel oil have a positive relationship with each other, and high-speed diesel and fuel oil are negatively correlated with mixed technologies. Solar has a positive correlation with all technologies except gas and mixed sources. While in the case of wind and hydro, the correlation coefficient is positive for coal and fuel oil only.

Standard deviation quantifies the dispersion of data from its mean. In portfolio theory, standard deviations are applied because they will describe the history of the variations in investment. Higher standard deviation means higher variance between price and its mean.

TABLE 2. Fuel cost correlation matrix.

Fuel Cost USD cents/kWh					
	Coal	**HSD**	**F.O**	**Gas**	**Mixed**
Coal	1	-0.2	-0.43	-0.4	-0.27
HSD	-0.2	1	0.52	0.58	0.63
F.O	-0.43	0.52	1	0.61	0.559
Gas	-0.4	0.58	0.61	1	0.61
Mixed	-0.27	0.63	0.559	0.61	1

Following equation computes the standard deviation of the observations. Expected cost of the electricity generation and standard deviation presented in Table 4 is based on the data available from [37], [38]. Eight energy sources including coal, high-speed diesel, fuel oil, gas, mixed fuel, solar, wind and hydel have been considered against the period of 2018-2021 in terms of USD cents/kWh.

$$s = \sqrt{\frac{\sum_{i=1}^{N} \left(X_i - \bar{X}\right)^2}{N-1}} \tag{5}$$

VI. RESULTS AND DISCUSSIONS

As a risk-control methodology, mean-variance portfolio theory attempts to achieve the best diversification possible among the alternatives analyzed, and therefore, to find efficient portfolios. There are two main streams to work in the portfolio optimization perspective, economic criteria and production criteria. The studies that are based on the application of economic criteria produce both return-risk frontiers (return measured as the inverse of cost) and cost-risk frontiers. Risk is expressed through the variability of the returns/costs for the set of technologies. The technologies with the greatest risks are those that are based on the use of fossil fuels including natural gas, petroleum, and derivatives, and are usually subject to a high degree of price variability. The models based on production criteria include the expected value of average production and variability of electricity production. In the following research, economic criteria have been considered which in turn helped to develop the cost-risk

TABLE 3. O & M correlation matrix of all technologies.

O & M Cost USD(cents/kWh)								
	Coal	**HSD**	**F.O**	**Gas**	**Mixed**	**Solar**	**Hydro**	**Wind**
Coal	1	-0.038	0.51	-0.075	-0.11	0.42	0.59	0.64
HSD	-0.038	1	0.16	0.25	-0.27	0.019	-0.02	-0.036
F.O	0.51	0.16	1	0.025	-0.24	0.47	0.52	0.48
Gas	-0.075	0.25	0.025	1	0.41	-0.11	-0.088	-0.071
Mixed	-0.11	-0.27	-0.24	0.41	1	-0.2	-0.139	-0.1
Solar	0.42	0.019	0.47	-0.11	-0.2	1	0.51	0.42
Hydel	0.59	-0.02	0.52	-0.088	-0.139	0.51	1	0.59
Wind	0.64	-0.036	0.48	-0.071	-0.1	0.42	0.59	1

TABLE 4. Expected Cost Standard deviation of all technologies (USD Cents/kWh).

Technologies	2022	2020-21	2019-20	St. Deviation
Coal	2.2814	6.6429	7.3688	2.8
HSD	7.9422	10.0345	10.9556	1.6
Fuel Oil	6.5697	9.8576	6.7405	1.9
GAS	4.1236	5.4351	5.5327	8.1
Mixed	6.2098	5.5998	5.8072	3.2
Solar	1.5067	10.2663	10.3395	3.9
Wind	0.8235	9.9735	10.1443	1.3
Hydel	0.061	1.3542	1.3115	3.8

frontiers. Power plants must lean towards those with a lesser impact on the starting factor. The decision to incorporate renewable technologies in the portfolio implies giving the system greater flexibility.

First, the effects of five conventional sources including coal, high-speed diesel, fuel oil, gas, and mixed have been analyzed. The resulting efficient frontier consisting of ten different portfolios is shown as circled points along the curve in Figure 4. The distribution of these portfolios is given in Table 5. It can be observed that in portfolio 10, the risk is high because total electricity generation depends on only two technologies i.e., coal and gas, which lowers the system's reliability. In comparison to other technologies, percentage weight of coal and gas is increasing gradually from portfolio 1 to portfolio 10 for economic reasons due to low expected cost of coal and gas. Thus, the portfolio 1 consists of all technologies and has the lowest risk but high expected cost. From portfolio 1-5, the contribution by the high-speed diesel, fuel oil, and mixed technologies decreases.

TABLE 5. Characterization of the set of optimal portfolios of fossil fuel technologies.

Portfolios	Coal	HSD	Fuel Oil	Gas	Mixed
Portfolio 1	47%	8%	2%	31%	13%
Portfolio 2	47%	6%	2%	34%	11%
Portfolio 3	48%	4%	1%	38%	9%
Portfolio 4	50%	2%	0%	41%	7%
Portfolio 5	50%	1%	0%	44%	5%
Portfolio 6	52%	0%	0%	48%	0%
Portfolio 7	60%	0%	0%	40%	0%
Portfolio 8	67%	0%	0%	33%	0%
Portfolio 9	74%	0%	0%	26%	0%
Portfolio 10	82%	0%	0%	18%	0%

Solar technology was then added to analyze the efficient frontier. It can be observed from Table 6 that initially, the

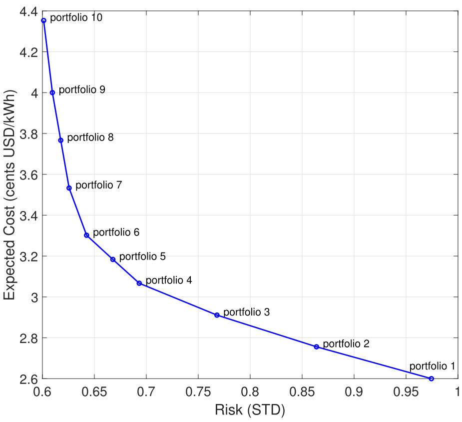

FIGURE 4. Efficient frontier for fossil energy.

contribution of solar is low which results in low risk but high expected cost. As the curve approaches portfolio 10, as shown in Figure 5, due to the addition of non-conventional sources the overall risk increases and the expected cost decreases.

TABLE 6. Characterization of the set of optimal portfolios including solar.

Portfolios	Coal	HSD	Fuel Oil	Gas	Mixed	Solar
Portfolio 1	33%	10%	0%	16%	20%	20%
Portfolio 2	34%	8%	0%	20%	17%	21%
Portfolio 3	35%	6%	0%	24%	15%	21%
Portfolio 4	35%	4%	0%	27%	12%	21%
Portfolio 5	36%	1%	0%	31%	10%	22%
Portfolio 6	36%	0%	0%	35%	6%	22%
Portfolio 7	37%	0%	0%	39%	0%	23%
Portfolio 8	36%	0%	0%	34%	0%	30%
Portfolio 9	35%	0%	0%	28%	0%	37%
Portfolio 10	34%	0%	0%	22%	0%	44%

To examine the combined effect of wind with current energy mix, wind is then included and a graph of efficient frontier consisting of both the fossil fuel technologies and wind is shown in Figure 6. It can be observed that the minimum variance portfolio has moved to the right effectively increasing the risk. Such a trend leads to a reduction in the reliability[1] of the source.

[1] The reliability of a particular energy resource is inversely related to its corresponding risk value. It means the lower the risk of a resource, the more reliable it will be for utilization in producing electricity.

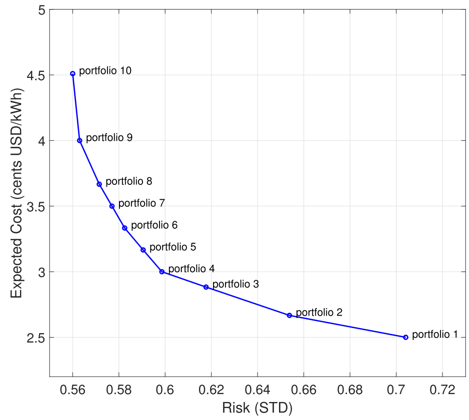

FIGURE 5. Efficient frontier for solar energy.

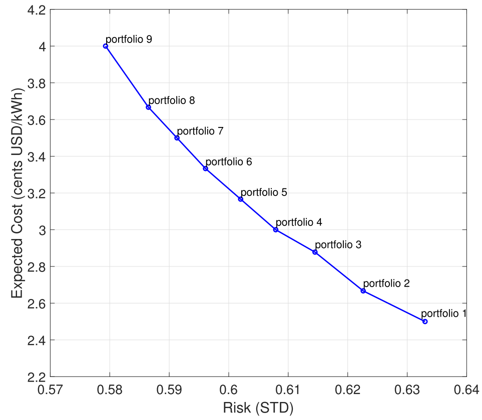

FIGURE 7. Efficient frontier for hydel energy.

TABLE 7. Characterization of the set of optimal portfolios including wind.

Portfolios	Coal	HSD	Fuel Oil	Gas	Mixed	Wind
Portfolio 1	36%	13%	0%	18%	18%	15%
Portfolio 2	36%	10%	0%	22%	16%	16%
Portfolio 3	36%	8%	0%	25%	13%	16%
Portfolio 4	37%	6%	0%	29%	11%	17%
Portfolio 5	37%	4%	0%	33%	9%	17%
Portfolio 6	38%	2%	0%	36%	6%	18%
Portfolio 7	38%	0%	0%	40%	3%	19%
Portfolio 8	37%	0%	0%	41%	0%	22%
Portfolio 9	34%	0%	0%	38%	0%	28%
Portfolio 10	31%	0%	0%	34%	0%	35%

TABLE 8. Characterization of the set of optimal portfolios including hydro.

Portfolios	Coal	HSD	Fuel Oil	Gas	Mixed	Hydro
Portfolio 1	32%	23%	0%	0%	30%	14%
Portfolio 2	32%	23%	0%	0%	30%	14%
Portfolio 3	32%	23%	0%	0%	30%	14%
Portfolio 4	32%	23%	0%	0%	30%	14%
Portfolio 5	32%	20%	0%	4%	28%	16%
Portfolio 6	33%	15%	0%	12%	22%	17%
Portfolio 7	34%	10%	0%	20%	17%	19%
Portfolio 8	35%	6%	0%	27%	12%	20%
Portfolio 9	36%	1%	0%	35%	7%	22%
Portfolio 10	34%	0%	0%	39%	0%	27%

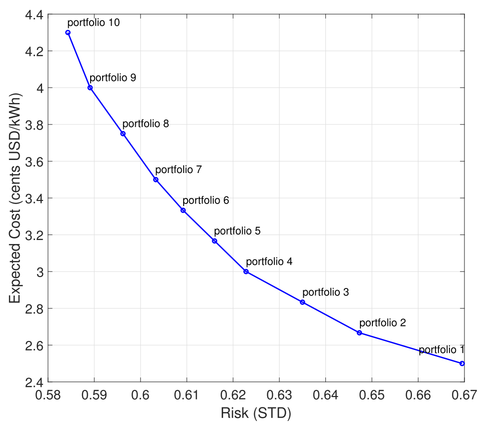

FIGURE 6. Efficient frontier with minimal risk for wind energy.

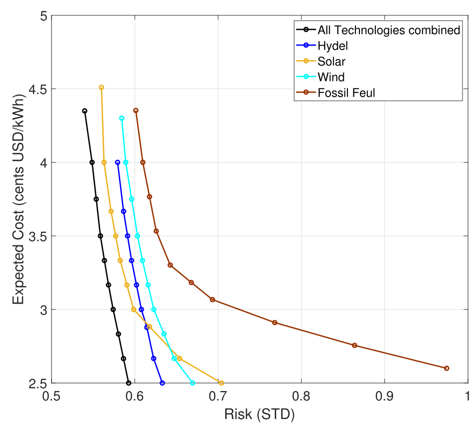

FIGURE 8. Efficient frontier with minimal risk for all technologies.

Hydro is added to the mix as a sixth technology, to evaluate the possibility of further diversification as indicated in Figure 7. Finally, Figure 8 shows the resulting practical efficient frontier for all six alternatives. Initially, the major share is of coal, HSD, mixed and hydel. Table 8 shows that with the gradual increase of the weight of hydro in the portfolios, contribution by the fossil fuel decreases and in portfolio 10, the major fossil fuels that are left behind are coal and gas. An interesting fact that can be observed is that contribution by gas technology in frontier starts at portfolio 5 and increases rapidly afterwards.

VII. CONCLUSION

Given the uncertainty in the energy market, this paper exploits the mean-variance tradeoff under optimal portfolio framework to advocate for an intelligent diversification of energy resources to achieve reliability and robustness while meeting the energy requirements. In the first part of this paper, we apply portfolio theory to obtain an optimal mix for the electricity generation through several resources. The second part focuses on computing portfolio results for Pakistan's energy system for the years 2019-2022. From the investor's perspective, the proposed model considers minimization of the expected cost. One of the key results of the paper is that a judicious mix of renewable energy sources with fossil fuel technologies yield less risky solutions. Specifically, more than 90% reliability is attained when using an optimal mixture of resources compared to the conventional setting of using standalone use of fossil fuel based energy production. Therefore, we suggest exploiting a combination of renewable energy sources and fossil fuel technologies instead of exclusive use of renewable energy scenarios or fossil fuel technologies. This is due to the highly variable prices of fossil fuel technologies leading to greater risks. Specifically, as a case study of Pakistan's energy profile, given the cost-risk perspective, the numerical results recommend excessive addition of renewable energy sources into the overall mix to achieve reduced cost (increased return).

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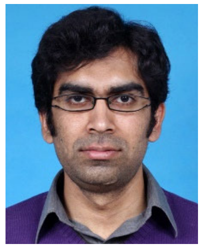

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