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RESEARCH ARTICLE

Humboldt Squid Optimization Algorithm (HSOA): A Novel Nature-Inspired Technique for Solving Optimization Problems

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ABSTRACT This study presents a new natural-based algorithm called the Humboldt Squid Optimization Algorithm (HSOA). HSOA is inspired by Humboldt squids hunting, moving, and mating behavior. The HSOA search procedure involves an attack on fish schools, a fish's escape, a successful attack, an attack of bigger squids on smaller ones, and mating, which is the inspiration for creating an algorithm to address existing issues. In HSOA, half of the best populations are Humboldt squid, and the rest are school fish. Individuals connect with each other and cooperate to achieve the optimal response. HSOA is versatile and applicable to mathematical and engineering problems. Solving eighty-four benchmark function problems (twenty-three classic functions, twenty-nine CEC-BC-2017 with 10, 30, 50, and 100 dimensions, ten CEC-C06 2019, ten CEC2020 with 5, 10, 15, and 20 dimensions, and twelve CEC2022 with 10 and 20 dimensions) and twenty-four engineering problems (six CEC2006 and eighteen CEC2011) shows that our proposed algorithm provides proper and acceptable answers to nine algorithms, including well-known (PSO, DE, and WOA), recent (AVOA, RW_GWO, HHO, and GBO), and state-of-the-art algorithms (LSHADE and EBOwithCMAR). Friedman's rank from HSOA for one hundred and eight problems was 16.45% and 7.45% lower than LSHADE and EBOwithCMAR. Thus, HSOA has the potential to solve various complex problems in the sciences and engineering fields.

INDEX TERMS Optimization, nature-inspired, Humboldt squid, swarm intelligence, mathematical functions, engineering problems.

Abbreviation list

XS	Humboldt squid's current position.
XF	Current position of school fish.
XS_{new}	Updated position of Humboldt squid.
XF_{new}	Updated position of school fish.
Xb	The best global position.
V_{jet}	Velocity parameter.
V_{jet2}	The second velocity parameter.
PopAll	The memory to save the position of Humboldt squid and fish school.
Pbest	The memory to save the top best individuals (10%).
ArchiveX	The archive to save unique positions during the search process.
Fb	The best fitness function.
F_{fi}	Fitness function of the i^{th} fish.
N	Population size.
Eggs	Humboldt squid eggs mass.
ω	Adaptive weight.
γ	Adaptive weight.
W	Adaptive weight.
μ_ω	Average value for ω, which has been updated over. increasing generations.
μ_γ	Average value for γ, which has been updated over. increasing generations.
c_1	Constant parameters.
c_2	Constant parameters.

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M_{cycle}	Maximum number of mating repetitions.
a1, a2, a3, a4	Parameters to control the shape of V_{jet1} and V_{jet2}.
$Diff_F$	Difference between fitness of Humboldt squids and their Eggs
I	The index that determines whether Humboldt squids have more fitness than their eggs.
nfes	Current generation counter.
Max_{nfes}	Maximum number of generations.
X	The ratio between nfes and Max_{nfes}.

I. INTRODUCTION

Natural phenomena can inspire new optimization methods for solving scientific and engineering problems. These problems are complex because of various factors such as nonlinearity, high variable dispersion, multiple decision variables, and many constraints. As problems become more complex, traditional optimization methods become inefficient. AI methods are important in these situations because they can handle problems with many constraints and variables. By studying natural phenomena like animal life, we can create more advanced methods than standard intelligent methods. If we address this issue, we can find faster and accurate engineering solutions as well as practical ways to solve global challenges. Intelligent methods compared to mathematical, physical, and numerical methods have high popularity and acceptance [1], [2], [3], because of advantages such as not requiring boundary conditions, requiring fewer data, not searching the entire problem space, the ability to solve any type of problems, even non-derivative problems, the ability to work with many dimensions and any number limitation [2]. Therefore, these methods can be ideal for solving scientific and engineering problems. Nature-based algorithms is category of these methods that are inspired by nature or physical phenomena. These algorithms are divided into four classes: evolutionary, population-based, physical and chemical-based, sport-based and human-based algorithms.

So far, various algorithms have been presented; however, efforts to provide newer algorithms are still ongoing. The reason is the theory that "there is no free lunch in search and optimization" [4]. According to this theory, there is currently no universally applicable optimization technique that can solve all optimization problems. Instead, each technique can provide an optimal solution within a specific problem range. This theory indicates that the excellent accuracy of one algorithm in solving a particular problem does not guarantee that it can solve other classes of problems efficiently. This theory encourages researchers to develop new algorithms for solving problems in different classes [5], [6].

Therefore, the aim of this research is to propose a new and powerful global optimization algorithm for complex problems. The proposed algorithm called Humboldt Squid Optimization Algorithm (HSOA) is inspired by the hunting, moving, and mating behavior of Humboldt squids. Unlike most similar algorithms, the HSOA uses two interacting populations (artificial squids and fish schools) instead of just one. This enhances the diversity of the search. The HSOA utilizes swarm-oriented operators, evolution-oriented operators, and adaptive parameter control mechanisms, resulting in a smooth transition from exploration to exploitation. The main contributions of this study can be summarized as follows:

1) This study introduces a new natural algorithm called the Humboldt squid optimization algorithm (HSOA) that is based on the hunting, moving, and mating behavior of Humboldt squids.
2) In contrast to other optimization algorithms that solely consider one type of population, HSOA is made up of two populations of squids and schools of fish, and both populations interact with each other.
3) HSOA takes into account the cannibalistic behavior of squid to distinguish it from other similar algorithms and enhance optimization accuracy.

The rest of the paper is arranged in the following way: Section II presents the literature review of nature-inspired algorithms, Section III describes the hunting and matting behaviors of the Humboldt squids, Section IV presents the HSOA, which consists of an introduced algorithm theory and its formulation and flowchart, and real-world problems, and discussion, Section V deals with testing the HSOA's performance using standard classic and new benchmark functions and engineering problems, Section VI provides conclusions and future scopes.

II. RELATED WORK

In recent decades, nature-based algorithms have been widely welcomed by researchers. These algorithms are divided into evolutionary, physics or chemistry-based, sport-based, human-based, and population-based. The Genetic Algorithm (GA) [7] is one of the most famous evolutionary algorithms developed in 1975. The Evolution Strategy (ES) [8] (Developed in 1978), Differential Evolution (DE) [9] (Developed in 1997), Harmony Search (HS) [10] (Developed in 1997), triple distinct search dynamics (TDSD) [11] (Developed in 2020), and Black Widow Optimization Algorithm (BWO) [12] (Developed in 2020), are other famous evolutionary algorithms. Evolutionary algorithms use the laws of natural evolution, such as selection, mating, mutation, and reproduction, to find the optimal solution to problems. The Simulated Annealing Algorithm [13] (Developed in 1983), Gravitational Search (GSA) Algorithm [14] (Developed in 2009), Ion Motion Algorithm [15] (Developed in 2015), Heat Transfer Search (HTS) [16] (Developed in 2015), Henry Gas Solubility Optimization (HGSO) [17] (Developed in 2019), Flow Direction Algorithm (FDA) [1] (Developed in 2021), Transit Search (TS) [18] (Developed in 2022), and Young's Double-Slit Experiment (YDSE) optimizer [19] (Developed in 2023) are the most well known physical and chemical-based algorithms. Some of well-known sport-inspired algorithms include Tug of War Optimization (TWO) [20] (Developed in 2017), Volleyball

Premier League Algorithm (VPL) [21] (Developed in 2018), Football Game Based Optimization (FGBO) [22] (Developed in 2020), Puzzle Optimization Algorithm (POA) [23] (Developed in 2022). Human-based algorithms are another group of nature-based algorithms. The Tabu Search (TS) [24] (Developed in 1989), Human Mental Search (HMS) [25] (Developed in 2017), Poor and Rich Optimization Algorithm (PRO) [26] (Developed in 2019), Doctor and Patient Optimization Algorithm (DPO) [27] (Developed in 2020) and Incomprehensible But Intelligible-In-Time Logics Optimization Algorithm (ILA) [28] (Developed in 2023) are popular human-based algorithms.

Swarm-based algorithms are a crucial category of nature-based algorithms. PSO is a swarm-based algorithm developed in 1995 that mimics the behavior of particles, birds, or fish [29]. In the following, some well-known swarm-based algorithms are reviewed. In 1996, the Ant Colony Optimization (ACO) algorithm was inspired by the foraging behavior of ants [30]. In 2007, the Artificial Bee Colony (ABC) algorithm was developed from honey bee foraging [31]. In 2007, the Firefly Algorithm (FA) is inspired by the firefly light emitting behaviour [31]. In 2007, the Bat Algorithm (BA) was inspired by the method of sound generation and echo reception by fruit bats [31]. In 2012, the Flowers Pollination Algorithm (FPA) was developed based on plant reproduction through pollination [32]. In 2014, the Ggray Wolf Optimizer (GWO) was designed inspired by gray wolves hunting method [2]. In 2021, the Aquila Optimizer (AO) was proposed by [33] and draws inspiration from the natural hunting behaviours of Aquila. In 2021, [33] developed the Vultures Optimization Algorithm (AVOA) that was worked based on the African vultures' food-seeking and navigational behaviors. In 2021, the Golden Eagle Optimizer (GEO) is proposed by [34] and inspired by the Golden Eagles' method of tuning at different stages of their spiral trajectory for hunting. In 2023, the Termite Life Cycle Optimizer (TLCO), which was worked based on both the life cycle of a termite colony and the modulation of movement strategies employed by animals in nature [6]. Figure 1 illustrates the mind map plot for the classification of nature-based optimization algorithms.

In addition, recently, different swarm-based algorithms have been introduced based on the behavior of marine animals. The Whale Optimization Algorithm (WOA) was created in 2016 by imitating the hunting strategy of humpback whales [35]. In 2016, the Salp Swarm Algorithm (SSA) is inspired by the swarming behavior of salps when navigating and foraging in oceans [35]. In 2019, the Harris Hawks Optimization (HHO) was suggested. It was inspired by the hunting technique of Harris' Hawks known as surprise pounce [36]. In 2020, the Manta Ray Foraging Optimization (MRFO) is proposed based on the foraging strategies of manta rays [37]. The Marine Predators Algorithm (MPA) was created in 2020 using the foraging strategy of ocean predators and the optimal encounter rate policy they use when hunting prey [38]. The Tunicate Swarm Algorithm (TSA) was made in 2020. It was inspired by how tunicates move and find food [39]. The Jellyfish Search (JS) optimizer in 2021 was inspired by how real jellyfish move and swarm in the ocean, including their active and passive motions and how they come together in a "jellyfish bloom." It also included a way to switch between these movements over time [40]. In 2021, the Northern Goshawks Optimization (NGO) algorithm was developed, taking inspiration from the hunting behavior of northern goshawks [41]. The parasitic behavior of remoras inspired the creation of the ROA algorithm [42]. The Golden Jackal Optimization (GJO) was created in 2022, inspired by the hunting behavior of the golden jackal [43]. In 2022, the Snake Optimizer (SO) was proposed by mimicking the unique mating behavior of snakes [44]. The White Shark Optimizer (WSO) was created in 2022 based on the exceptional senses of hearing and smell that great white sharks use to navigate and forage [45]. The Orca Predation Algorithm (OPA) was proposed in 2022. It simulates the hunting behavior of orcas and turns it into mathematical models [46]. In 2022, the Artificial Hummingbird Algorithm (AHH) was created based on the simulation of the unique flight skills and intelligent foraging strategies of hummingbirds in nature [47]. The Reptile Search Algorithm (RSA) was developed in 2022, inspired by the hunting behavior of crocodiles [48]. The Cheetah Optimizer (CO) was developed in 2022, inspired by the hunting strategies of cheetahs [49]. The social life and hierarchy of wild mountain gazelles were the basis for the design of the Gazelle Optimizer (MGO) in 2022 [50]. By mimicking the behaviors of the Fennec Fox animal, researchers developed the Fennec Fox Algorithm (FFA) in 2022 [51]. The Coati Optimization Algorithm (COA) was created in 2023. It was based on the coati's behavior of hunting iguanas and escaping from predators [52]. In 2023, the Leopard Seal Optimization (LSO) was introduced by inspiration from hunting strategy of the leopard seals including searching, encircling, and attacking [53]. In 2023, the Termite Life Cycle Optimizer (TLCO) was proposed by [6] and operates based on the life cycle of a termite colony in nature. Furthermore, efforts are being made to improve the existing algorithms, and Random Walk Gray Wolf Optimizer (In 2019) [54], Improved Harris Hawks Optimization (In 2020) [55], an Improved Firefly Algorithm with Dynamic Self-Adaptive Adjustment (In 2021) [56], a Hybrid of Opposition Learning and Spiral Modelling based Arithmetic Optimization Algorithm (AOA), called OSAOA (In 2022) [57], a Modified version of the Seahorse Optimization Algorithm integrated with Chaotic Maps (CSHO) (In 2023) [58], and a Quasi-Opposition based Learning and Q-Learning Based Marine Predators Algorithm (QQLMPA) (In 2023) [59], can be mentioned in this field. These efforts show the importance of designing and developing optimizers to solve optimization problems.

According to the authors' best knowledge, well-known algorithms like GA, PSO, and ACO can only perform well in either global or local searches. The performance

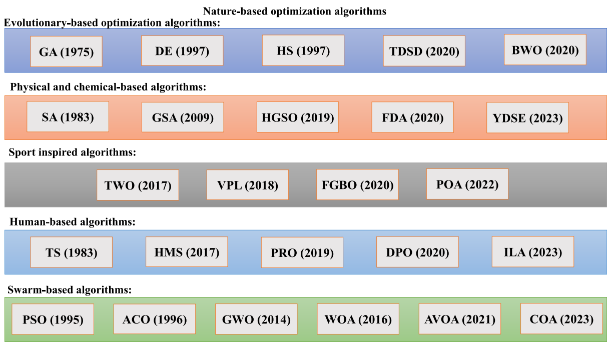


FIGURE 1. **Classification of nature-based optimization algorithms.**

of these algorithms is poor in balancing global and local search. Recent algorithms such as HHO and WSO also mainly benefit from the ability of one type of operator, such as evolution, physics-based or population-based operators. While using a combination of different operators can increase their accuracy. By utilizing adaptive weights that are updated when approaching the better fitness function, a better balance can be achieved between global and local search ability. FDA algorithms employ adaptive weights, but the weights are only updated based on the number of iterations and not on the fitness function. Such algorithms are incapable of balancing global and local search in some conditions. In addition, the reviewed algorithms such as FA, BA, GWO, WOA, and AO are designed based on the creation of one type of population. While in nature, the predator and the prey can each have their own behavior.

The mentioned gaps can be addressed by designing an optimization algorithm that is based on the various aspects of aquatic animal life, due to the unique surprises of these animals. Moreover, previous studies have been inspired by marine animal research and prey behavior in the wild. However, there is still no research to mimic the lifestyles of Humboldt squid to design and develop a nature-based algorithm. The problem prompted us to create a mathematical model of the life behavior of Humboldt squid and to present the Humboldt Squid Optimization Algorithm (HSOA). It should be noted that in most previous studies, only one type of population is taken into account for optimization. However, HSOA is designed in relation to two populations of squids and schools of fish, and both populations interact with each other. Furthermore, there is a kind of cannibalistic behavior in the squid, which taking into account in HSOA, leads to distinguishing HSOA from other similar algorithms and increase the accuracy of optimization. We first investigate the unique aspect of the Humboldt squid and then present the proposed HSOA.

III. HUMBOLDT SQUID OPTIMIZATION ALGORITHM (HSOA)

A. HUMBOLDT SQUID CHARACTERISTICS

Humboldt squid are large squid found east of the Pacific Ocean. They have substantial economic and ecological

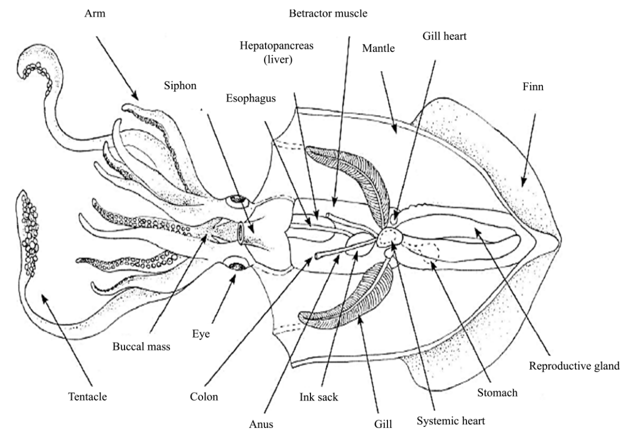


FIGURE 2. **Scheme of the squid anatomy [64].**

importance in the eastern Pacific Ocean [60]. These fish are 1.5m long and are the largest members of their family. They swim at a velocity of 24 km/h. Humboldt's squid is a fast-growing species, ranging from 1 mm at birth to more than 1 m at the age of 1 to 2 years [61]. The common prey species are copepods, hyperiid amphipods, euphausiids, pelagic shrimps, red crabs (Pleuroncodes planipes), heteropod molluscs, squid, pelagic octopus, and various fish [61]. Moreover, Humboldt squid east heir fellows [62]. The larger the Humboldt Squid, the more it eats other Humboldts. All squid can move at an incredible speed [63]. Figure 2 shows the schematic anatomy of a squid.

It is worth mentioning that Humboldt squid are on flying squids. In general, the motion of the multi-modal flying squid includes roaming, accelerating, launching, jetting, gliding and diving, which is clearly illustrated in Figure 3. The other aspect of Humboldt squid life is mating. Humboldt squid grow fast and probably only live for one year. During this time, they reach their maximum size, reproduce several times, and die. Humboldt squid mate via internal fertilization and lay large egg masses of at least one million eggs. Over their short lifespan, females can lay 20 million eggs [65]. Figure 4 depicts a swimming Humboldt squid, a swarm of Humboldt squid and a mass of squid eggs.

The way in which a squid feeds (hunting fish and other squid), swims, locomotion and mating can inspire an optimization algorithm. All of these behaviors of Humboldt squids differ from those of other animals. The behaviors of this animal make it a unique predator. Also, the method of movement, hunting, and mating of this animal is very similar to finding the optimal solution to an optimization problem. The ocean environment is like the search domain of the problem, and the school fishes and weaker squids are like the optimal solution. Hence, in the present research, a new optimizer named HSOA inspired by the hunting, locomotion and reproduction behaviors of squid is designed to solve optimization problems, which will be discussed in more detail in the next sections.

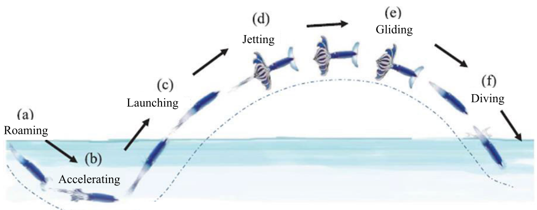


FIGURE 3. Illustration of Humboldt squid locomotion [66].

B. BASIC STEPS OF THE ALGORITHM

Key steps in HSOA included hunting, moving and mating. To mathematically model this process, five mechanisms are defined for the search operation. These mechanisms are composed of attack of fish schools, escape of fish, successful attack, attack of stronger squid on smaller squids, and mating of Humboldt squid. In HSOA, in increasing iterations, the searching process by the attack of fish schools, attack of stronger squids on smallest squids, and mating is converted from exploration to exploitation. However, fish escape handles exploration through every iteration. Figure 5 demonstrates the different mechanisms of Humboldt squid.

C. GENERATING INITIAL POPULATION

The population of HSOA comprises Humboldt squid and fish swarms. HSOA uses the following pseudocode (Algorithm 1) to generate the initial population. As seen, the best individuals in the population are considered being Humboldt squid, and the rest are fish. This issue is consistent with nature because Hublot squid has a larger body and greater fitness than school fish.

Algorithm 1 Initial Population Generation

Input: Population size (N), upper bound (ub) and lower bound (lb) of decision variables
Output: Initial population for both Humboldt squid and school fish
Initialization :
for $i = 1$ to $2.N$ **do**
    $X_{init} \leftarrow lb + (ub - lb).\overrightarrow{rnd}$;
    Evaluate fitness function;
end for
Save merged population in $PopAll$;
Sort the population according to the fitness function;
Select the first NS members of the population to be Humboldt squids (XS); *and the other NF members to be fish*(XF);
$XS_{new} \leftarrow XS$;
$XF_{new} \leftarrow XF$;
Select the best individual as the global best (X_b);
Save the N top best individuals;

D. ATTACK OF FISH SCHOOLS

In HSOA, equation 1 is used to simulate the attack of fish schools.

$$XS^d_{new,\,i} = X_b + V_{jet}.(-XF^d_{new,\,r_1} - PopAll^d_{r_2}). \tag{1}$$

In Eq. (1), $XS^d_{new,\,i}$ is the new position of i^{th} Humboldt squid in d^{th} dimension, V_{jet} is the locomotion velocity parameter, $XF^d_{new,\,r_1}$ is the position of r_1^{th} fish in d^{th} dimension, and $PopAll^d_{r_2}$, is the saved r_2^{th} position in the HSOA memory. Furthermore, r_1 is a random integer number between 1 and population size of fish, and r_2 is a random integer number between 1 and size of $PopAll$. Responsibility for V_{jet} will be set out in section 4.7.

E. SUCCESSFUL ATTACK

After updating the new position for Humboldt squid and fish, the current position for Humboldt squid is replaced with the new position for Humboldt squid (XS_i).

$$XS^d_i = \begin{cases} XS_i = XS_{new,\,i} & if F_{S,new\,i} < F_{S,\,i} \\ Successful\ escape, & otherwise. \end{cases} \tag{2}$$

In Eq. (2), $F_{S,new\,i}$ and $F_{S,\,i}$ are new fitness functions and current fitness functions of the i^{th} Humboldt squid.

F. SUCCESSFUL ESCAPE

After the squid attacks the fish school, the fish escape to a randomly located location. In this escape, the velocity and position of the fish are updated based on the following equation:

$$XF_{new,i} = \begin{cases} XF_i + \overrightarrow{rn}. & \\ (pbest - XF_i).wf, & if nfes < 0.1.max_{nfes}, \\ XS_i + \overrightarrow{rn}.(ArchiveX_{r1} & \\ -PopAll_{r2}).wf, & otherwise. \end{cases} \tag{3}$$

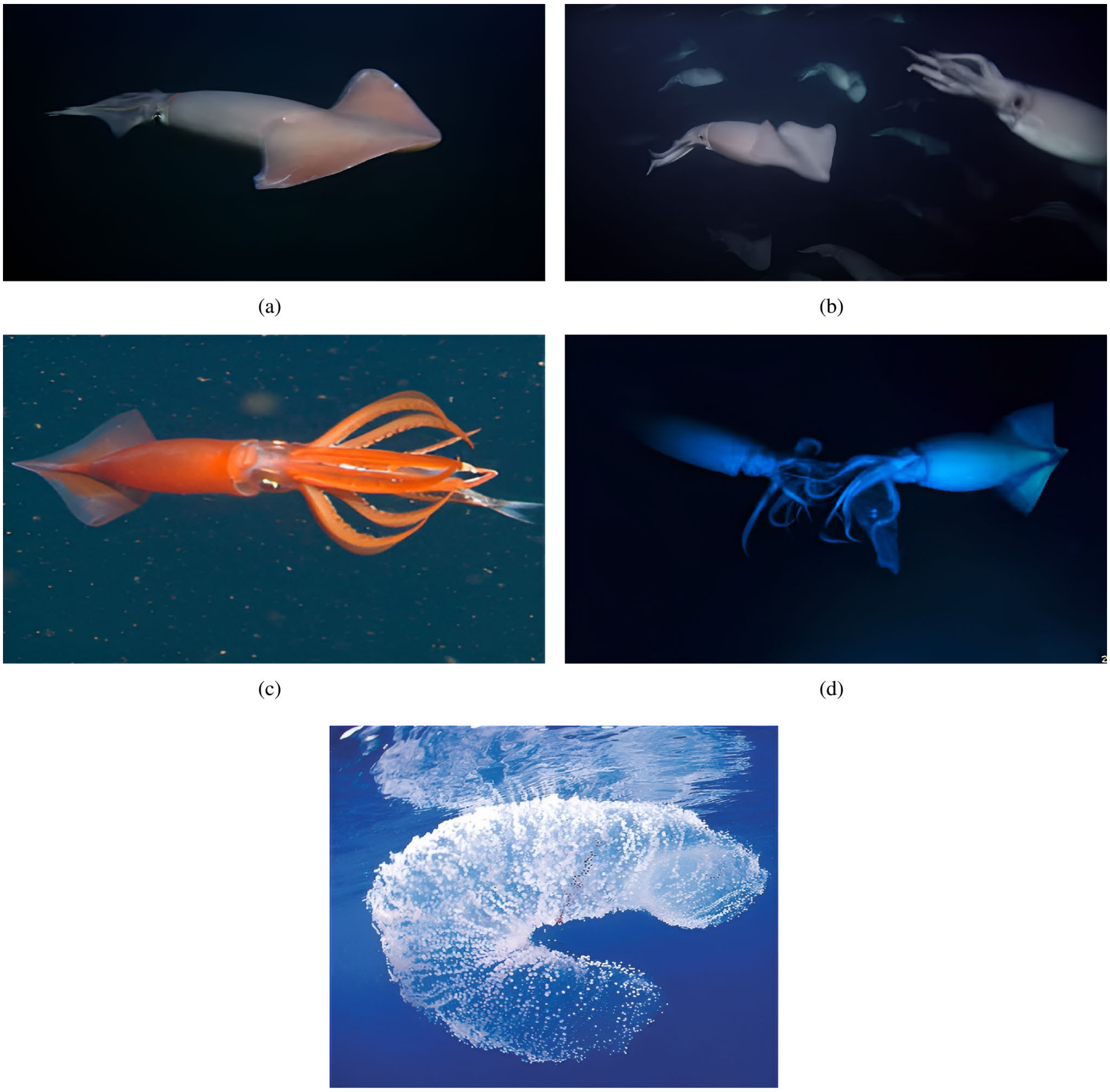
In Eq. (3), $nfes$ is the current number of function evaluations, max_{nfes} is the maximum number of function evaluations, $XF_{new,i}$ is a new position of i^{th} fish, XF_i is the current position of i^{th} fish, $pbest$ is N top of the best positions, $ArchiveX_{r1}$ is r_1^{th} position in archive of the best results, XS_i is $i^t h$ position of Humboldt squid, $\overrightarrow{rn}$ is the normal random vector, $wf = \frac{F_b}{F_{fi}}$, F_b is the best fitness function, and F_{fi} is the fitness function of i^{th} fish. In this equation, if the counter of function evaluations is in the initial generations, the fish moves towards one of the N best solutions. Otherwise, it moves to a random position.

G. ATTACK OF STRONGER SQUIDS TO SMALLEST SQUIDS

If fish and Humboldt squid do not find a better position in the previous steps, it is assumed that there are no longer any fish to hunt. Consequently, the larger Humboldt squid eats the smaller ones. In this stage, the position of the Humboldt squid is derived from the following equation:

$$XS^d_{new,i} = XS^d_{new,i} + V_{jet2}.(XS^d_{new,i} - X^d_b) \tag{4}$$

In Eq. (4), V_{jet2} is the second velocity parameter. To find the best solutions, it is assumed that the smaller



(a) (b) (c) (d) (e)

FIGURE 4. **Photographs of squid a) one Humboldt squid in swimming, b) swarm of Humboldt squid, c) Squid in hunting a fish [67] d) Humboldt squid in eating other Humboldts, and e) squid species' egg mass [68].**

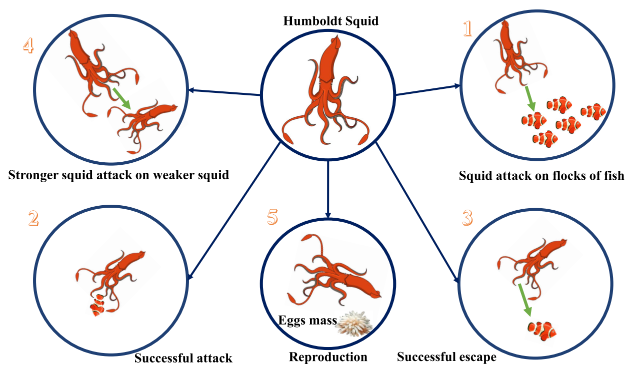


FIGURE 5. **Different mechanisms of HSOA.**

Humboldt squid is in a better position and the larger one moves toward it based on this relationship. Therefore, it is assumed that smaller Humboldt squid is in the best position (X_b).

H. HUMBOLDT SQUID MATING

Equation 4 is utilized in HSOA to generate the egg position. It was previously employed by [69] for enhancing the deferential evolutionary (DE) algorithm.

$$Eggs = (\omega.XS + (1 - \omega.pbest)).\gamma + (1 - \gamma) .pop(r_1, :) + W.(pop(r_3, :) - popAll(r_2, :)) \quad (5)$$

In Eq. (5), *Eggs* is position of Humboldt squid eggs mass, ω, γ and W are adaptive weights which control search process. ω and γ are between 0 and 1. W can be estimated using the following equation:

$$W = max\{\omega.\gamma, (1 - \omega).\gamma, 1 - \gamma\} \quad (6)$$

In the present work, for estimating ω and γ following equations are defined:

$$\omega = \mu_\omega + c_1.x \tag{7}$$

$$\gamma = \mu_\gamma + c_2.\overrightarrow{rn} \tag{8}$$

where c_1 and c_2 are constant parameters which are determined by the user. Moreover, μ_ω and μ_γ in the first generation are vectors with a value of 0.5, and updated in other generations as follows:

$$\mu_\omega = \frac{[Diff_F(I)].[\omega(I)^2]}{[Diff_F(I)].[\omega(I)]} \tag{9}$$

$$\mu_\gamma = \frac{[Diff_F(I)].[\gamma(I)^2]}{[Diff_F(I)].[\gamma(I)]} \tag{10}$$

In Eq. (9) and Eq. (10), I is the index of which Humboldt squids that have more fitness than their Eggs, $Diff_F$ is the difference between the fitness of Humboldt squids and their Eggs. Since each Humboldt squid mate several times in its lifetime, at each generation, the mating action in the HSOA is repeated several times.

It should be noted that the γ must be greater than zero. Thus, if the γ value becomes less than zero, it is corrected using the following equation:

$$\gamma = \mu_\gamma + 0.1.tan(\pi.rnd) \tag{11}$$

In Eq. (11), rnd is a normal random number which is between [0, 1]. The x in equation 6 is computed as follows:

$$x = \frac{nfes}{max_{nfes}}.\overrightarrow{rnd}^{r.10} \tag{12}$$

In Eq. (12), *overrightarrowrnd* and r are a normal random vector and a normal random number, which are between 0 and 1,respectively.

The mating process in HSOA is presented in Algorithm 2.

Algorithm 2 Mating Process in HSOA

Define initial parameters: N, M_{cycle}, $memory_{size}$,$Pbest_{rate}$, $Archive_{size}$
Output: Updated Humboldt squid's position
Main loop :
while $M < M_{cycle}$ **do**
  $M \leftarrow M + 1$;
  Sort the population according to the fitness function;
  $Memory_{Index,rand} \leftarrow \lfloor dim.\overrightarrow{rnd} \rfloor$
  $\mu_\omega = Memory_\omega(Memory_{Index,rand})$
  $\mu_\gamma = Memory_\gamma(Memory_{Index,rand})$
  Calculate x using equation 11;
  Calculate ω and γ using equation 6 and 7;
  Generate three random integer number (r_1, r_2, r_3);
  Choose $Pbest_{rate}$% of best solutions and save them on $Pbest$;
  Compute W using equation 5;
  Generate the Position of eggs using equation 4;
  Evaluate eggs position;
  if $fitness_{eggs} < fitness_{Pop}$ **then**
    $Pop = eggs$%If the eggs had a better position than the parent, the position of the egg should be considered as the position of the parent;
  end if
  Update Archive (Add new solution to archive and remove duplicate or randomly remove some solutions to maintain the archive size);
  Update μ_ω and μ_γ
  if $\gamma < 0$ **then**
    γ updated using equation 10;
  end if
  $\omega \leftarrow min(\omega, 1)$; $\gamma \leftarrow min(\gamma, 1)$;
  Merge Pop and Archive and save on PopAll;
  $XS_i \leftarrow lb + (ub - lb).\overrightarrow{rnd}$
  Evaluate fitness function;
end while
return Humboldt squid's position

I. CONTROL SEARCH PROCESS

The HSOA search process is controlled by various parameters, including V_{jet}, V_{jet2}, x, wf, W, ω, and γ. V_{jet} and V_{jet2} are employed to simulate the shape of locomotion of Humboldt squids. For this purpose, a polynomial function is used. The power of this polynomial function for V_{jet} and V_{jet2} is considered being 3rd and 4th degree, respectively. V_{jet} and V_{jet2} are calculated using the following equations:

$$V_{jet} = (X - a1).(X - a2).(X - a3) \tag{13}$$

$$V_{jet2} = (X - a1).(X - a2).(X - a3).(X - a4) \tag{14}$$

In Eq. (13) and Eq. (14), $a1$, $a2$, $a3$, and $a4$ are parameters of polynomial function which define its shape, and X can be calculated using the following equation:

$$X = \frac{nfes}{max_{nfes}} \tag{15}$$

wf modifies the fish's escape radius as a function of the ratio between the value of the best objective function and the current value of the fish's objective function. At the beginning of the search, when there is a wide range of solutions, this parameter limits the extent of fish escapement. However, with the increase in the number of generations, this parameter becomes close to one and its effect is neutralized.

In equation 8, by increasing the generations, the value of x is increased, and therefore in equation 4, the impacts of XS are increased compared with pbest. These parameters help HSOA overcome the local optima trap. wf, W, */omega* and */gamma* are responsible for avoiding premature convergence of the resulting responses in the mating section. These parameters change according to the value of the objective function and increasing the number of generations, thereby changing the search range and balancing exploration and exploitation. For example, using equations 10 and 11, more changes apply to the position of eggs that did not result in better responses.

This increases the chances of getting out of the local optima trap.

J. FLOWCHART OF HSOA

HSOA follows the following principles when it comes to problem solving:

1) The HSOA algorithm's search space contains three types of solutions, including the current position of Humboldt squids (XS_{new}), the current position of fish (XF_{new}), the best positions in memory of Humboldt squids (XS) and fish (XF), the best position of Humboldt squids and fishes in the current generation (X_b).
2) The Humboldt squids have a better position (less fitness function in minimizing) than fishes.
3) Humboldt squid with a smaller fitness function are larger than other Humboldt squid.
4) Humboldt squid attack schools of fish and smaller Humboldt squid.
5) The location of Humboldt squid and fish is updated if a better solution is found.
6) In each generation, mating is performed multiple times.
7) Adaptive weights control HSOA's exploration and exploitation ability so that as generations increase, the exploration phase becomes exploitation.

For a better understanding, every stage of HSOA is fully illustrated in Figure 6. HSOA uses a variety of operators for optimization, as described in the flowchart.

K. TIME COMPLEXITY AND SPACE COMPLEXITY

In the section, the operating efficiency of the algorithm is evaluated by analyzing the computational complexity of the HSOA. Time and space complexity are the two components of the algorithm's computational complexity. The amount of storage space required by the algorithm is space complexity. In HSOA, the complexity of the algorithm is dependent only on the population number, the dimensions of the optimization problem, and the required memory for PopAll, Pbest, and ArchiveX. Therefore, the HSOA complexity can be given by:

$$Space\ complexity = O((N + PopAll_{size} + Pbest_{size} + Archive_{size}).D) \quad (16)$$

The time complexity in HSOA depends on the Max_{nfes}, and the time taken to calculate the fitness value (f). As a result, the time complexity is calculated as follows:

$$Time\ complexity = O(Max_{nfes}.D + f) \quad (17)$$

The first term in Eq. (17) is for initialization, the second is for updating solutions, and the third is for fitness function evaluation.

IV. EXPERIMENTAL STUDY SETTINGS

HSOA is examined using 23 classic benchmark functions, 29 CEC-BC-2017 benchmarks with 10, 30, 50, and 100 dimensions [70], 10 CEC-C06 2019 benchmark functions [71],

Algorithm 3 HSOA Algorithm

Define initial parameters: N, c_1, c_2, $a1$, $a2$, $a3$, $a4$, M, and Max_{nfes}
Output: Best solution

Generate initial population (Algorithm 1);s
%%Main loop :
while $nfes < Max_{nfes}$ **do**
  Update x using equation 11;
  for $i = 1$ to N **do**
    for $d = 1$ to dim **do**
      Calculate V_{jet} using equation 12;
      Attack of fish schools
      Update the new position of Humboldt squid using equation 1 (XS_{new});
      Check the feasibility of the solution;
    end for
    Evaluate new position of Humboldt squid; Update $nfes$;
    if $f_{S_{new}} < f_S$ **then**
      Update XS
    end if
    Update X_b, P_{best}, $Archive$;
  end for
  %%Escape
  Update new position of school fish using equation 2;
  %%Successful escape
  for $i = 1$ to N **do**
    Check the feasibility of the solution XF_{new};
    Evaluate new position of the fish; Update $nfes$
    if $f_{F_{new}} < f_S$ **then**
      Update XF;
    end if
    Update X_b, P_{best};
    %%Larger Humboldt squid attack on smaller Humboldt squid
    if $f_{F_{new}} > f_F \ || \ f_{S_{new}} > f_S$ **then**
      for $d = 1$ to dim **do**
        Update V_{jet2} using equation 13;
        Update new position of Humboldt squid using equation 3 (X_{Snew});
        Evaluate new position of Humboldt squid; Update $nfes$
        if $f_{S_{new}} < f_S$ **then**
          Update X_S
        end if
      end for
    end if
  end for
  Run mating Algorithm (Algorithm 2); Update $nfes$
  Merge Humboldt squids and fish population;
  Sort the population according to the fitness function
  Select the first N members of the population to be Humboldt squids and the other N members to be fish
end while
return Best solution

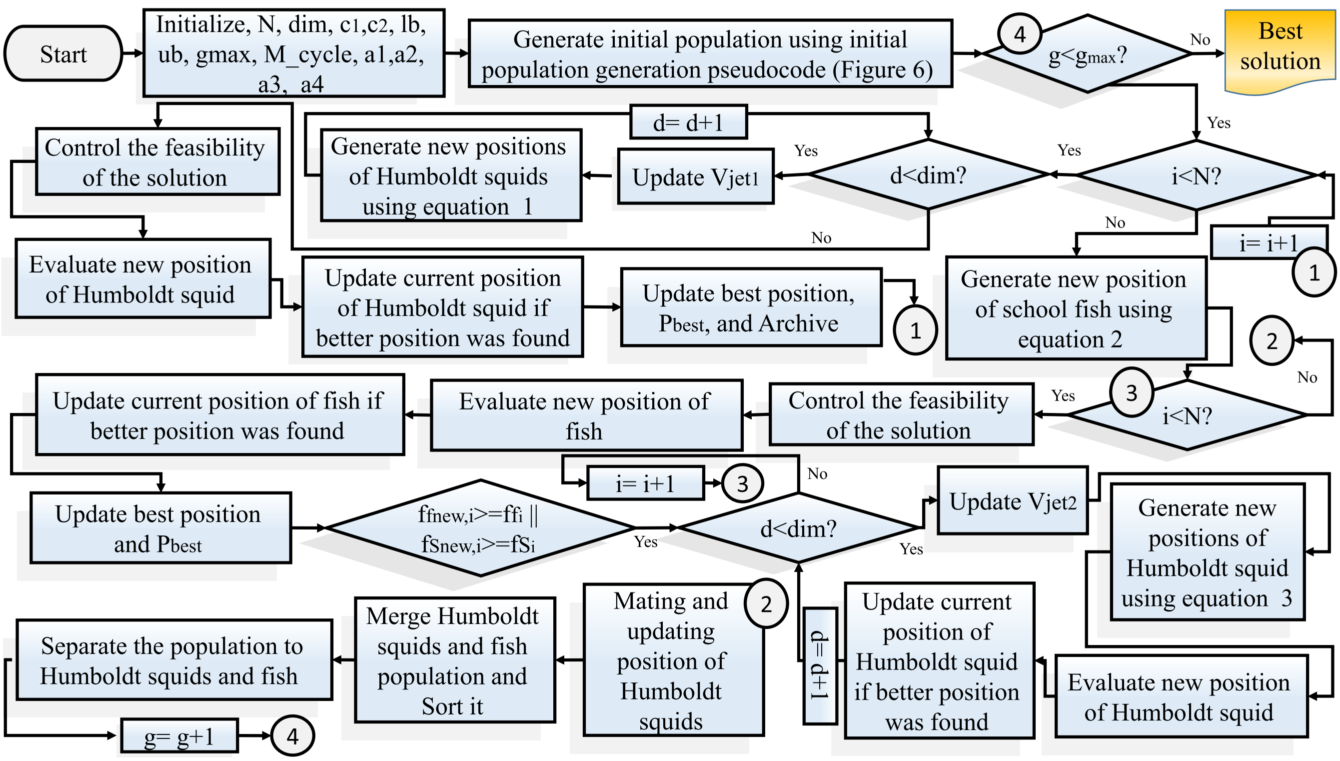


FIGURE 6. Flowchart of HSOA.

10 CEC2020 benchmarks with 5, 10, 15, and 20 dimensions [72], 12 CEC2022 benchmarks with 10 and 20 dimensions [73], six cec2006 and eighteen cec2011 engineering design problems [74]. HSOA is compared to various competitors in benchmark functions and engineering problems. All algorithms and problems were implemented using MATLAB R2020b; they were run on Windows 10, Intel Xeon 2.2GHz CPU, 40G RAM. The following assumptions are considered in comparing HSOA with other optimization algorithms:

1) To ensure a fair evaluation of optimization algorithms, the maximum number of fitness evaluations (Max_{nfes}) is set to be the same. For this purpose, the Max_{nfes} for classic benchmark, CEC2017, CEC2006 engineering design problem, and CEC2011 engineering design problem is set to 150000, 10000.*Dim* [70], 15000, and 150000 [74], respectively. The Max_{nfes} for the first to the tenth CEC2019 are set to 90000, 320000, 1800000, 1000000, 100000, 100000, 1000000, 1000000, 1000000, and 600000, respectively. The Max_{nfes} for CEC2020 in 5, 10 15 and 20 dimensions is equal to 50000, 1000000, 3000000 and 10000000 [72]. For CEC2022 in 10 and 20 dimensions, the Max_{nfes} is considered by value of 200000 and 1000000, respectively [73].
2) The algorithm runs 51 times for CEC2017, CEC2019, and CEC2006 benchmarks. For CEC2020 and CEC2022, it runs 30 times, and for CEC2011, it runs 25 times. These values are determined according to the studies of [70], [72], [73], and [74].
3) The application of optimization algorithms is evaluated in different benchmark and engineering problems.
4) More details about the experimental study setting are presented in the following sub-section.

A. BENCHMARK FUNCTIONS

Fifty-two benchmark functions are employed to evaluate HSOA and its competitors. Details on benchmark functions are presented in Table 1. Eighty-three benchmark functions are employed to evaluate HSOA and its competitors. Details on benchmark functions are presented in Table 1.

The classic benchmark functions include seven unimodal (1 to 7), six multimodal (8-13) and ten multimodal-based fixed-dimension (14 to 23). CEC2017 set comprises two unimodal functions (1 and 3), seven multimodal functions (4-10), ten hybrid functions (11-20), and ten composite functions (21-30). The second benchmark function of CEC2017 is removed for unstable behavior. CEC2020 set includes one unimodal function (1), three multimodal functions (2-4), three hybrid functions (5-7), and three composite functions (8-10). The CEC2022 set consists of one unimodal function (1), four multimodal functions (2-5), three hybrid functions (6-8), and four composite functions (9-12). Better results in unimodal, multimodal benchmark functions show a more remarkable ability for exploitation and exploration. The excellent performance in solving hybrid and composite functions shows a good ability to escape from the local optimum. The CEC2019 characteristics are also tabulated in Table 2.

The global optimal values for $CEC2020$ in $CEC-20/01-CEC-20/010$ are equal to 100, 1100, 700, 1900, 1700, 1600, 2100, 2200, 2400 and 2500, respectively. Additionally, the global optimal values for $CEC2022$ in $CEC-22/01-CEC-22/12$ are 300, 400, 600, 800, 900, 1800, 2000, 2200, 2300, 2400 2600 and 2700.

B. REAL-WORLD PROBLEMS

In addition, the accuracy of the HSOA is evaluated in terms of the six CEC2006 real-world problems and the eighteen CEC2011 real-world problems. The CEC2006 problems include tension/compression spring [75], speed reducer [37], and three models of 25-bar truss [35], [76]. Details of CEC2011 functions can be found in [74]. These problems have several linear and nonlinear constraints that increase their complexity. As a result, the use of these problems can demonstrate the capability of HSOA in the real-world. The CEC2011 problems which employed in this study consist of Parameter Estimation for Frequency-Modulated (FM) Sound Waves (F1), Lennard-Jones Potential Problem (F2), Optimal Control of a Non-Linear Stirred Tank Reactor (F3), Spread Spectrum Radar Polly phase Code Design (F4), Transmission Network Expansion Planning (TNEP) Problem (F5), Large Scale Transmission Pricing Problem (F6), Circular Antenna Array Design Problem (F7), Static Economic Load Dispatch (ELD) Problem (F8), different models of Dynamic Economic Dispatch (DED) Problem (F9 to F13), different models of Hydrothermal Scheduling Problem (F14 to F16), Messenger: Spacecraft Trajectory Optimization Problem (F17), Cassini 2: Spacecraft Trajectory Optimization Problem (F18). The results of 51 random HSOA implementations in designing real-world problems are compared with other metaheuristic optimization algorithms. In CEC2011 real-world problems, HSOA results are presented over 150,000 generations [74]. Thus, the Lagrangian method applies to violate the fitness function to solve constraint problems.

C. HSOA COMPETITORS

The performance of HSOA was compared with nine (9) different optimization algorithms, namely, AVOA, PSO, DE, RW_GWO, WOA, HHO, DE variants with linear population size reduction (LSHADE) [77], Gradient based optimizer (GBO) [78] and Effective Butterfly Optimizer with Covariance Matrix Adapted Retreat Phase (EBOwithCMAR) [79]. PSO, DE and WOA are regarded as well-known optimizing algorithms. AVOA, RW_GWO, GBO and HHO are considered as new optimizing algorithms. LSHADE and EBOwithCMAR are employed as state-of-the-art algorithms.

D. PARAMETER SETTINGS OF HSOA COMPETITORS

The parameter settings for the investigated algorithms are given in Table 3. These parameters were chosen based on earlier studies, trials, and experience. The population size and the maximum number of iterations for all algorithms were chosen so that the Max_{nfes} is the same for them.

E. STATISTICAL ANALYSIS

This research uses two nonparametric tests to compare HSOA with other competitors. These tests include the Friedman and Wilcoxon signed-rank. To determine each algorithm's mean rank, many studies use Friedman tests [81], [82]. The Wilcoxon signed-rank test is commonly used for pairwise comparisons in studies like [83] based on [84] recommendation. Wilcoxon determines the number of victories and defeats of one algorithm compared to the other competitors.

V. RESULTS AND DISCUSSION

The robustness and convergence efficiency of the proposed Humboldt squid optimization algorithm (HSOA) was tested on 84 benchmark functions and 21 real-world problems. The 84 benchmark functions include 23 classical benchmark functions, 29 CEC2017 on dimensions 10, 30, 50 and 100, 10 CEC2019, 10 CEC2020 on dimensions 5, 10, 15 and 20, 12 CEC2022 on dimensions 10 and 20. The 21 real-world problems consist of 18 CEC2011 and 6 CEC2006.

A. SENSITIVITY ANALYSIS

The HSOA parameters are determined using the sensitivity analysis. In this regard, HSOA is executed multiple times by changing the values of the setting parameters. In the next step, the mean fitness functions related to each setting parameter are evaluated. The parameter setting with a minimum mean fitness function is considered being a desired value. In other studies, such as [85] and [86], this technique is employed. Figures 7 to 13 show the HSOA sensitivity analysis results for classic, CEC2017, CEC2019, CEC2020, CEC2022, CEC2006 and CEC2011. In this study, the normalized fitness function is used for sensitivity analysis to show the results more clearly. In this method, the evaluation results for each function are scaled between 0 and 1. This purpose is achieved by using the min-max normalization method. Values closer to zero (white color) represent the minimum value of the objective function, whereas values closer to one (blue color) indicate the maximum value. The sensitivity analysis of HSOA parameters has assigned the values 0.001, 0.1, 0.5, 0.6, 0.8, and 0.9 to c1 and c2. The values of 50, 24, 20, 14, 10, and 4 are used for sensitivity analysis of N. The sensitivity analysis of M_cycle considers the values of 5, 10, 50, 100, 150, and 300. Sensitivity analysis of [a1, a2, a3, a4] is performed using four sets: [1, -0.8, -1, -0.1], [0.9, -0.9, -1.1, -0.2], [0.8, -0.95, -1.2, -0.3], and [0.5, -1, -1.4, -0.4].

Figures 7 to 13 show the best parameters for each function based on sensitivity analysis. The optimal values for c1 and c2 were 0.001 and 0.5, respectively. For N, the best values were 4 or 10. The results for M_cycle indicated that 50 was a desirable value for low dimensions problems, while 300 was a better value for high dimensions problems. According to

TABLE 1. Description of classic benchmark functions.

Function	Range	Global optimum	Dim
$F_1(x) = \sum_{i=1}^{n} x_i^2$	$[-100, 100]^n$	0	30
$F_2(x) = \sum_{i=1}^{n} \lvert x_i\rvert + \prod_{i=1}^{n} \lvert x_i\rvert$	$[-10, 10]^n$	0	30
$F_3(x) = \sum_{i=1}^{n}(\sum_{j=1}^{i}(x_j))^2$	$[-100, 100]^n$	0	30
$F_4(x) = \max_i\{\lvert x_i\rvert, 1 \leq i \leq n\}$	$[-100, 100]^n$	0	30
$F_5(x) = \sum_{i=1}^{n} -1[100.(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	$[-30, 30]^n$	0	30
$F_6(x) = \sum_{i=1}^{n}([x_i + 0.5])^2$	$[-100, 100]^n$	0	30
$F_7(x) = \sum_{i=1}^{n}(i.x_i^4 + random[0, 1)])$	$[-1.28, 1.28]^n$	0	30
$F_8(x) = \sum_{i=1}^{n}(-x_i.\sin\sqrt{\lvert X_i\rvert})$	$[-500, 500]^n$	428.9829.n	30
$F_9(x) = \sum_{i=1}^{n}[x_i^2 - 10.\cos(2.\pi.x_i) + 10]$	$[-5.12, 5.12]^n$	0	30
$F_{10}(x) = -20.\exp(-0.2.\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i^2)}) - \exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2.\pi.x_i)) + 20$	$[-32, 32]^n$	0	30
$F_{11}(x) = \frac{1}{4000}\cdot\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}}) + 1$	$[-600, 600]^n$	0	30
$F_{12}(x) = \frac{\pi}{n}\{10.\sin(\pi.y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10.\sin^2(\pi.y_{i+1})] + (y_n - 1)^2\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{x_{i+4}u(x_i,10,100,4)}{4} = \begin{cases} k.(x_i - a)^m & x_i > a \\ 0 & 0 < x_i < a \\ k.(-x_i - a)^m & x_i < -a \end{cases}$	$[-50, 50]^n$	0	30
$F_{13}(x) = 0.1.\{\sin^2(3.\pi.x_1)+\sum_{i=1}^{n}(x_i-1)^2[1+\sin^2(3.\pi.x_i+1)]+(x_n-1)^2.[1+\sin^2(2.\pi.x_n)]\}+\sum_{i=1}^{n} u(x_i, 5, 100, 4)$	$[-50, 50]^n$	0	30
$F_{14}(x) = (\frac{1}{500} + \sum_{j=1}^{25}\frac{1}{j+(x_i-a_{ij})^6})^{-1}$	$[-65.53, 65.53]^2$	1	2
$F_{15}(x) = \sum_{i=1}^{11}[a_i - \frac{x_1.(b_i^2+b_i.x_2)}{b_i^2+b_i.x_3+x_4}]^2$	$[-5, 5]^4$	0.00030	4
$F_{16}(x) = 4.x_1^2 - 2.1.x_1^4 + \frac{1}{3}.x_1^6 + x_1.x_2 - 4.x_2^2 + 4.x_2^4$	$[-5, 5]^2$	−1.0316	2
$F_{17}(x) = (x_2 - \frac{-5.1}{4.\pi^2}.x_1^2 + \frac{5}{\pi}.x_1 - 6)^2 + 10.(1 - \frac{1}{8.\pi}).\cos(x_1) + 10$	$[-5, 5]^2$	0.398	2
$F_{18}(x) = [1 + (x_1 + x_2 + 1)^2.(19 - 14.x_1 + 3.x_1^2 - 14.x_2 + 6.x_1.x_2 + 3.x_2^2)].[30 + (2.x_1 - 3.x_2)^2.(18 - 32.x_1 + 12.x_1^2 + 48.x_2 - 36.x_1.x_2 + 27x_2^2)]$	$[-2, 2]^2$	3	2
$F_{19}(x) = -\sum_{i=1}^{4} c_i.\exp(-\sum_{j=1}^{3} a_{ij}.(x_j - p_{ij}))$	$[1, 3]^3$	−3.86	3
$F_{20}(x) = -\sum_{i=1}^{4} c_i.\exp(-\sum_{j=1}^{6} a_{ij}.(x_j - p_{ij}))^2$	$[0, 1]^6$	−3.32	6
$F_{21}(x) = -\sum_{i=1}^{5}[(X - a_i).(X - a_i)^T + c_i]^{-1}$	$[0, 10]^n$	−10.1532	4
$F_{22}(x) = -\sum_{i=1}^{7}[(X - a_i).(X - a_i)^T + c_i]^{-1}$	$[0, 10]^n$	−10.4028	4
$F_{23}(x) = -\sum_{i=1}^{10}[(X - a_i).(X - a_i)^T + c_i]^{-1}$	$[0, 10]^n$	−10.5363	4

TABLE 2. Characteristic of CEC2019 benchmark functions.

Name	Benchmark Function Name	Dim	Range	Global optimum
CEC – 19/01	STORN'S CHEBYSHEV POLYNOMIAL FITTING PROBLEM	9	[–8192, 8192]	1
CEC – 19/02	INVERSE HILBERT MATRIX PROBLEM	16	[–16384, 16384]	1
CEC – 19/03	LENNARD-JONES MINIMUM ENERGY CLUSTER	18	[–4, 4]	1
CEC – 19/04	RASTRIGIN'S FUNCTION	10	[–100, 100]	1
CEC – 19/05	GRIEWANGK'S FUNCTION	10	[–100, 100]	1
CEC – 19/06	WEIERSTRASS FUNCTION	10	[–100, 100]	1
CEC – 19/07	MODIFIED SCHWEFEL'S FUNCTION	10	[–100, 100]	1
CEC – 19/08	EXPANDED SCHAFFER'S CEC06 FUNCTION	10	[–100, 100]	1
CEC – 19/09	HAPPY CAT FUNCTION	10	[–100, 100]	1
CEC – 19/10	ACKLEY FUNCTION	10	[–100, 100]	1

TABLE 3. Parameter setting of investigated algorithms.

Algorithm	Parameters	Values
AVOA	L_1, L_2, K, P_1, P_2, P_3	0.8, 0.2, 2.5, 0.6, 0.4 and 0.6 [81]
PSO	c_1, c_2, inertia weight	2, 2, linear decreasing from 0.9 to 0.1 [82]
DE	Lower bound of scaling factor (β_{min}), Upper bound of scaling factor (β_{max}), Crossover probability	0.2, 0.8, 0.2
RW_GWO	Convergence parameter (a)	Linear decreasing from 2 to 0 [2]
WOA	Convergence parameter (a)	Linear decreasing from 2 to 0 [35]
HHO	beta	1.5
LSHADE	Archive size rate, initial population size (N_{init}), top best individuals present (p), memory size (H)	2, 20, 0.1, 5 [78]
GBO	B_{min}, B_{max}, Pr	0.2, 1.2, 0.5 [79]
EBOwithCMAR	Max primary population size ($PS_{1,max}$), minimum primary population size ($PS_{1,min}$), max secondary population size ($PS_{2,max}$), minimum secondary population size ($PS_{2,min}$), memory size, PS_3, local search probability (probls)	18.Dim, 4, 46.8.Dim, 10, 6, 4+3log(Dim), 0.1 [79]

the got results, using the first set [a1, a2, a3, a4] led to better performance. For the running HSOA, the mentioned parameters can be considered as the default.

B. COMPARISON WITH POPULAR METAHEURISTICS ALGORITHMS

Table 4 lists the results of HSOA algorithms and other optimization algorithms for solving classic benchmark functions over 150000 function evaluations. In this table, the min, mean, median, max, and std results from 51 independent runs are displayed. The min, mean, and median results indicate the accuracy of the proposed algorithm and the std results and the difference between the max and min indicate the quality of the results. The lower the mentioned criteria, the higher the accuracy and quality of the algorithm under review. As seen, HSOA had a reasonable accuracy in finding the global optimum of classic benchmark functions. The proposed algorithm got better results than other selected algorithms in F6, F11, F14, F16, F17, F19, F20 and F21. Although sometimes the precision of other algorithms was superior to that of HSOA, most times the HSOA was more accurate than other algorithms. Additionally, HSOA had competitive performance with popular algorithms (PSO, DE, WOA), new optimization algorithms (AVOA, RW_GWO, HHO, GBO), and stat-of-the-art algorithms (LSHADE and EBOwithCMAR). Although in the fifth and eighth functions, the value of the std and the difference between min and max were not very small. However, in others, the value of std and the difference between min and max were close to zero.

As there are many results and evaluation criteria for their verification, the use of statistical tests can help to more clearly analyze and compare HSOA results with other algorithms. Hence, the Wilcoxon test and the Friedman test were used as a post huc test to analyze optimization results. Table 5 shows the results of the Wilcoxon post huc test with a significant level of 0.01. HSOA won 9 out of 23 benchmark functions compared to AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR. HSOA won in functions 13, 19, 18, 20, 17, 14, 7, 12, and 6. HSOA's equality with AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR was equal to 2, 4, 5, 2, 4, 3, 11, 6 and 12, respectively. Friedman's ranking (Figure 14) for the solving of classic benchmark functions shows that HSOA was in 1st place and EBOwithCMAR and LSHADE in 2nd and 3rd place.

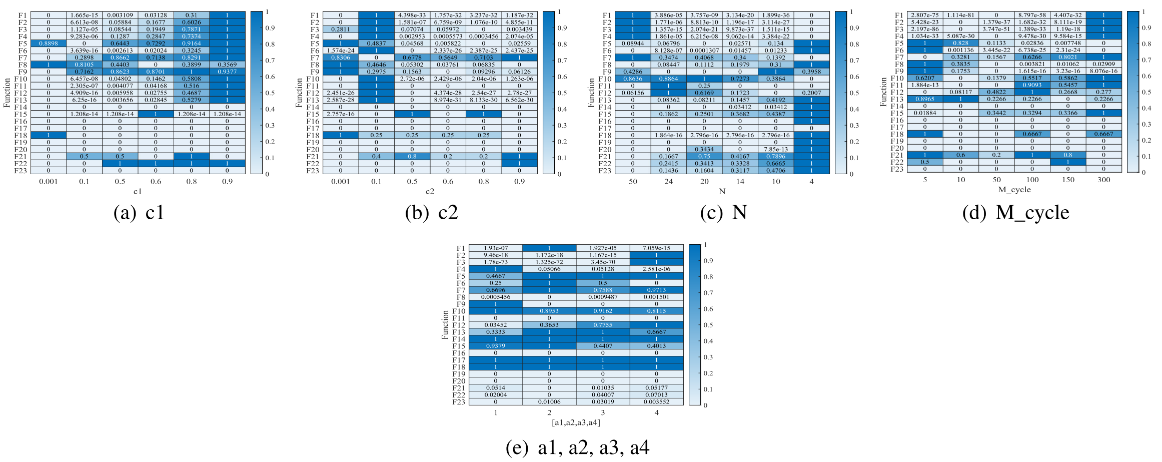
Tables 6 to 9 present the min, mean, median, max, and std results of solving CEC2017 benchmark functions using HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR. Based on the findings of these tables, the proposed algorithm was superior to AVOA, PSO, DE, RW_GWO, WOA, HHO and GBO. In dimensions 10, HSOA has found the global optimum of $CEC-17/01$ (Min = 100.00), $CEC-17/03$ (Min = 300.00), $CEC-17/04$ (Min = 400.000), $CEC-17/05$

TABLE 4. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the classic benchmark functions over 51 runs and 150000 number function evaluations.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
	Min	1.41E-181	0.00E+00	3.76E-24	4.92E-12	6.74E-97	1.04E-216	3.70E-231	5.49E-76	1.11E-306	0.00E+00
	Mean	1.57E-169	0.00E+00	3.89E-21	8.46E-12	4.75E-94	5.56E-199	1.31E-207	1.25E-73	1.66E-298	8.88E-65
F1	Median	1.80E-174	0.00E+00	1.15E-21	7.94E-12	7.48E-95	1.17E-206	1.21E-217	3.86E-74	5.89E-302	1.96E-66
	Max	3.68E-168	0.00E+00	4.84E-20	1.37E-11	4.52E-93	2.62E-197	6.06E-206	2.78E-72	7.10E-297	3.65E-63
	Std	0.00E+00	0.00E+00	7.92E-21	2.21E-12	9.74E-94	0.00E+00	0.00E+00	3.96E-73	0.00E+00	5.10E-64
	Min	1.57E-111	0.00E+00	1.66E-15	4.87E-08	3.34E-55	4.60E-125	8.52E-125	6.54E-37	6.57E-156	0.00E+00
	Mean	1.73E-77	0.00E+00	6.04E-14	6.92E-08	5.90E-54	3.74E-115	2.17E-110	1.82E-35	5.41E-151	1.02E-31
F2	Median	4.76E-104	0.00E+00	3.06E-14	6.74E-08	4.19E-54	1.08E-118	6.66E-115	1.21E-35	2.69E-153	9.14E-34
	Max	8.34E-76	0.00E+00	3.44E-13	9.95E-08	2.43E-53	1.40E-113	6.65E-109	8.67E-35	2.63E-149	4.33E-30
	Std	1.17E-76	0.00E+00	8.42E-14	1.19E-08	5.50E-54	2.02E-114	1.04E-109	1.80E-35	3.68E-150	6.11E-31
	Min	5.85E-249	0.00E+00	2.75E+00	1.28E+04	1.59E-36	4.47E+01	7.80E-209	7.35E-28	4.39E-255	9.62E-18
	Mean	3.27E-131	0.00E+00	1.21E+01	2.12E+04	2.31E-30	2.50E+03	3.91E-189	2.50E-22	1.55E-247	1.49E-15
F3	Median	1.29E-234	0.00E+00	1.11E+01	2.19E+04	5.91E-33	1.69E+03	1.50E-199	7.59E-24	1.17E-250	1.68E-17
	Max	1.66E-129	0.00E+00	3.41E+01	2.82E+04	8.70E-29	8.39E+03	1.16E-187	5.41E-21	7.30E-246	2.55E-14
	Std	2.33E-130	0.00E+00	7.30E+00	3.73E+03	1.22E-29	2.26E+03	0.00E+00	8.52E-22	0.00E+00	4.29E-15
	Min	2.11E-92	0.00E+00	2.21E-01	1.55E+00	3.78E-26	2.73E-15	2.84E-118	2.65E-20	1.85E-142	5.00E-13
	Mean	7.72E-81	0.00E+00	4.63E-01	2.02E+00	3.28E-24	1.15E+01	3.24E-104	1.77E-18	4.71E-139	1.55E-11
F4	Median	1.28E-85	0.00E+00	4.30E-01	2.02E+00	1.95E-24	1.96E-01	4.04E-109	6.34E-19	1.10E-139	8.55E-12
	Max	1.83E-79	0.00E+00	1.17E+00	2.48E+00	1.43E-23	8.26E+01	1.39E-102	2.66E-17	3.49E-138	2.31E-10
	Std	2.83E-80	0.00E+00	2.11E-01	2.35E-01	3.34E-24	2.20E+01	1.95E-103	3.84E-18	8.34E-139	3.34E-11
	Min	2.60E-17	8.57E-09	4.28E+00	2.65E+01	2.46E+01	1.61E-02	1.39E-11	1.01E-24	1.79E-02	0.00E+00
	Mean	5.47E-01	4.13E-07	3.98E+01	3.58E+01	2.61E+01	2.50E+01	1.29E-04	1.56E-01	3.55E+00	0.00E+00
F5	Median	2.84E-11	1.48E-07	2.49E+01	3.39E+01	2.62E+01	2.55E+01	5.68E-05	1.06E-20	3.28E+00	0.00E+00
	Max	3.99E+00	6.96E-06	1.34E+02	6.08E+01	2.71E+01	2.60E+01	7.88E-04	3.99E+00	9.19E+00	0.00E+00
	Std	1.39E+00	1.02E-06	2.98E+01	7.33E+00	7.38E-01	3.58E+00	1.80E-04	7.82E-01	2.60E+00	0.00E+00
	Min	0.00E+00	6.68E-14	3.30E-25	4.13E-12	3.66E-06	2.28E-05	1.78E-09	0.00E+00	0.00E+00	0.00E+00
	Mean	0.00E+00	9.20E-13	6.58E-21	8.10E-12	1.20E-01	8.05E-05	1.03E-06	0.00E+00	8.86E-31	0.00E+00
F6	Median	0.00E+00	6.68E-13	9.35E-22	7.78E-12	7.06E-06	7.45E-05	8.20E-07	0.00E+00	1.97E-31	0.00E+00
	Max	0.00E+00	4.89E-12	7.05E-20	1.33E-11	6.58E-01	1.75E-04	4.06E-06	0.00E+00	1.61E-29	0.00E+00
	Std	0.00E+00	7.91E-13	1.32E-20	2.19E-12	1.76E-01	3.53E-05	1.00E-06	0.00E+00	2.45E-30	0.00E+00
	Min	4.94E-06	7.11E-07	2.87E-03	1.29E-02	3.50E-05	1.50E-05	5.79E-07	4.33E-04	1.37E-05	2.75E-04
	Mean	7.09E-05	1.92E-05	6.13E-03	2.13E-02	1.76E-04	5.71E-04	1.81E-05	1.27E-03	1.18E-04	1.07E-03
F7	Median	4.34E-05	1.63E-05	5.74E-03	2.04E-02	1.52E-04	3.12E-04	1.21E-05	1.20E-03	1.01E-04	9.55E-04
	Max	2.97E-04	9.81E-05	9.76E-03	3.34E-02	4.53E-04	2.88E-03	1.65E-04	2.40E-03	4.37E-04	2.38E-03
	Std	6.45E-05	1.73E-05	1.87E-03	4.29E-03	1.04E-04	6.53E-04	2.63E-05	4.40E-04	8.10E-05	4.75E-04
	Min	-1.26E+04	-1.26E+04	-8.66E+03	-1.26E+04	-2.18E+06	-1.26E+04	-1.26E+04	-1.26E+04	-1.22E+04	-9.55E+03
	Mean	-1.26E+04	-1.25E+04	-7.04E+03	-1.23E+04	-1.01E+05	-1.19E+04	-1.26E+04	-1.26E+04	-9.56E+03	-7.26E+03
F8	Median	-1.26E+04	-1.26E+04	-7.12E+03	-1.24E+04	-3.98E+04	-1.24E+04	-1.26E+04	-1.26E+04	-9.39E+03	-7.22E+03
	Max	-1.25E+04	-1.16E+04	-5.74E+03	-1.15E+04	-1.53E+04	-8.21E+03	-1.26E+04	-1.25E+04	-8.28E+03	-5.84E+03
	Std	3.56E+01	1.55E+02	7.12E+02	2.78E+02	3.03E+05	1.08E+03	8.86E-03	1.66E+01	7.94E+02	7.70E+02
	Min	0.00E+00	0.00E+00	1.99E+01	4.62E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	-2.90E+02
	Mean	0.00E+00	0.00E+00	3.86E+01	5.88E+01	0.00E+00	1.11E-15	0.00E+00	3.34E-15	0.00E+00	-2.90E+02
F9	Median	0.00E+00	0.00E+00	3.58E+01	5.94E+01	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	-2.90E+02
	Max	0.00E+00	0.00E+00	8.66E+01	6.99E+01	0.00E+00	5.68E-14	0.00E+00	1.14E-13	0.00E+00	-2.90E+02
	Std	0.00E+00	0.00E+00	1.17E+01	4.91E+00	0.00E+00	7.96E-15	0.00E+00	1.77E-14	0.00E+00	1.23E-10
	Min	8.88E-16	8.88E-16	2.00E-13	5.12E-07	7.99E-15	8.88E-16	8.88E-16	4.44E-15	8.88E-16	-1.07E+13
	Mean	2.70E-15	8.88E-16	8.25E-12	7.67E-07	9.60E-15	3.60E-15	8.88E-16	5.69E-15	8.88E-16	-1.07E+13
F10	Median	4.44E-15	8.88E-16	4.92E-12	7.64E-07	7.99E-15	4.44E-15	8.88E-16	4.44E-15	8.88E-16	-1.07E+13
	Max	4.44E-15	8.88E-16	3.54E-11	1.01E-06	1.51E-14	7.99E-15	8.88E-16	7.99E-15	8.88E-16	-1.07E+13
	Std	1.79E-15	0.00E+00	8.34E-12	9.95E-08	2.69E-15	2.42E-15	0.00E+00	1.71E-15	0.00E+00	1.97E-03
	Min	0.00E+00	0.00E+00	0.00E+00	1.77E-11	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Mean	0.00E+00	0.00E+00	1.36E-02	7.01E-11	1.07E-03	1.70E-03	0.00E+00	0.00E+00	0.00E+00	9.67E-05
F11	Median	0.00E+00	0.00E+00	9.86E-03	6.09E-11	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Max	0.00E+00	0.00E+00	7.12E-02	2.07E-10	3.46E-02	2.44E-02	0.00E+00	0.00E+00	0.00E+00	4.93E-03
	Std	0.00E+00	0.00E+00	1.47E-02	4.10E-11	5.18E-03	5.49E-03	0.00E+00	0.00E+00	0.00E+00	6.91E-04
	Min	1.57E-32	1.33E-14	2.34E-26	6.27E-13	2.09E-07	4.69E-06	1.30E-10	1.57E-32	1.57E-32	1.57E-32
	Mean	1.57E-32	7.99E-14	7.13E-22	1.05E-12	8.90E-03	1.28E-05	1.47E-07	1.57E-32	3.77E-31	1.57E-32
F12	Median	1.57E-32	7.08E-14	4.53E-24	1.03E-12	6.58E-03	1.20E-05	5.30E-08	1.57E-32	2.86E-32	1.57E-32
	Max	1.59E-32	2.20E-13	3.21E-20	2.00E-12	3.34E-02	2.89E-05	1.32E-06	1.57E-32	1.32E-29	1.57E-32
	Std	5.42E-35	4.62E-14	4.50E-21	2.95E-13	8.04E-03	5.53E-06	2.63E-07	2.76E-48	1.86E-30	2.76E-48
	Min	1.35E-32	1.75E-12	1.53E-25	2.16E-12	2.88E-06	6.56E-05	1.17E-09	1.35E-32	1.35E-32	1.35E-32
	Mean	1.36E-32	1.10E-11	8.62E-04	5.14E-12	1.12E-01	1.95E-03	1.35E-06	1.35E-32	7.50E-03	1.35E-32
F13	Median	1.35E-32	1.08E-11	2.38E-22	4.72E-12	1.00E-01	2.13E-04	5.19E-07	1.35E-32	9.87E-30	1.35E-32
	Max	1.47E-32	2.78E-11	1.10E-02	8.83E-12	4.14E-01	1.13E-02	9.86E-06	1.35E-32	4.39E-02	1.35E-32
	Std	2.93E-34	7.23E-12	2.98E-03	1.56E-12	1.03E-01	4.01E-03	1.91E-06	2.76E-48	1.10E-02	2.76E-48
	Min	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	Mean	9.98E-01	9.98E-01	9.98E-01	9.98E-01	1.02E+00	1.04E+00	9.98E-01	9.98E-01	9.98E-01	9.98E-01
F14	Median	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	Max	9.98E-01	9.98E-01	9.98E-01	9.98E-01	1.99E+00	2.98E+00	9.98E-01	9.98E-01	9.98E-01	9.98E-01
	Std	0.00E+00	2.11E-16	0.00E+00	0.00E+00	1.39E-01	2.78E-01	1.60E-14	0.00E+00	0.00E+00	0.00E+00

TABLE 4. *(Continued.)* **Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the classic benchmark functions over 51 runs and 150000 number function evaluations.**

	Min	3.07E-04	3.07E-04	3.07E-04	3.52E-04	3.07E-04	3.08E-04	3.07E-04	3.07E-04	3.07E-04	3.07E-04
	Mean	3.07E-04	3.07E-04	4.17E-04	6.00E-04	3.76E-04	5.26E-04	3.12E-04	3.07E-04	4.40E-04	3.07E-04
F15	Median	3.07E-04	3.07E-04	3.07E-04	6.04E-04	3.07E-04	3.55E-04	3.09E-04	3.07E-04	3.07E-04	3.07E-04
	Max	3.07E-04	3.07E-04	1.30E-03	7.46E-04	1.80E-03	1.23E-03	4.26E-04	3.07E-04	1.59E-03	3.07E-04
	Std	9.85E-20	6.87E-10	3.02E-04	8.53E-05	2.84E-04	3.27E-04	1.66E-05	9.14E-20	3.40E-04	1.03E-19
	Min	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	Mean	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
F16	Median	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	Max	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00	-1.03E+00
	Std	0.00E+00	1.94E-16	0.00E+00	0.00E+00	1.16E-09	1.07E-13	7.56E-16	0.00E+00	0.00E+00	0.00E+00
	Min	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Mean	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
F17	Median	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Max	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01
	Std	5.61E-17	5.61E-17	5.61E-17	5.61E-17	1.49E-06	1.19E-08	4.93E-11	5.61E-17	5.61E-17	5.61E-17
	Min	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Mean	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
F18	Median	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Max	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00	3.00E+00
	Std	6.31E-16	2.76E-09	6.65E-16	7.59E-16	4.01E-07	6.35E-08	3.85E-14	1.07E-15	8.72E-16	1.18E-15
	Min	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	Mean	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
F19	Median	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	Max	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.85E+00	-3.85E+00	-3.86E+00	-3.86E+00	-3.86E+00	-3.86E+00
	Std	1.35E-15	9.50E-16	1.35E-15	1.35E-15	1.66E-03	1.49E-03	1.34E-08	1.35E-15	1.35E-15	1.35E-15
	Min	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00
	Mean	-3.32E+00	-3.26E+00	-3.27E+00	-3.32E+00	-3.24E+00	-3.25E+00	-3.25E+00	-3.32E+00	-3.25E+00	-3.32E+00
F20	Median	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.20E+00	-3.20E+00	-3.20E+00	-3.32E+00	-3.20E+00	-3.32E+00
	Max	-3.32E+00	-3.20E+00	-3.20E+00	-3.32E+00	-3.08E+00	-3.14E+00	-3.14E+00	-3.20E+00	-3.20E+00	-3.32E+00
	Std	5.86E-16	6.00E-02	5.99E-02	4.44E-16	7.28E-02	6.18E-02	6.62E-02	2.83E-02	5.91E-02	4.98E-16
	Min	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01	-1.02E+01
	Mean	-1.02E+01	-1.02E+01	-6.76E+00	-1.02E+01	-9.58E+00	-1.02E+01	-5.75E+00	-1.01E+01	-9.35E+00	-1.02E+01
F21	Median	-1.02E+01	-1.02E+01	-5.10E+00	-1.02E+01	-1.02E+01	-1.02E+01	-5.06E+00	-1.02E+01	-1.02E+01	-1.02E+01
	Max	-1.02E+01	-1.02E+01	-2.63E+00	-1.02E+01	-5.06E+00	-1.02E+01	-5.06E+00	-5.10E+00	-5.06E+00	-1.02E+01
	Std	0.00E+00	2.47E-15	3.24E+00	2.27E-12	1.60E+00	1.03E-05	1.77E+00	7.07E-01	1.87E+00	0.00E+00
	Min	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01
	Mean	-1.04E+01	-1.04E+01	-8.12E+00	-1.04E+01	-1.04E+01	-1.00E+01	-6.13E+00	-1.04E+01	-9.88E+00	-1.04E+01
F22	Median	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-5.09E+00	-1.04E+01	-1.04E+01	-1.04E+01
	Max	-1.04E+01	-1.04E+01	-2.75E+00	-1.04E+01	-1.04E+01	-2.77E+00	-5.09E+00	-1.04E+01	-5.09E+00	-1.04E+01
	Std	1.85E-15	1.44E-15	3.20E+00	1.31E-15	7.67E-05	1.47E+00	2.13E+00	3.55E-16	1.60E+00	1.79E-15
	Min	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01
	Mean	-1.05E+01	-1.05E+01	-8.10E+00	-1.05E+01	-1.03E+01	-1.05E+01	-5.87E+00	-1.05E+01	-9.69E+00	-1.05E+01
F23	Median	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-1.05E+01	-5.13E+00	-1.05E+01	-1.05E+01	-1.05E+01
	Max	-1.05E+01	-1.05E+01	-2.42E+00	-1.05E+01	-5.13E+00	-1.05E+01	-5.13E+00	-1.05E+01	-5.13E+00	-1.05E+01
	Std	1.85E-15	1.76E-15	3.41E+00	1.79E-15	1.06E+00	1.80E-03	1.88E+00	2.04E-15	1.99E+00	2.04E-15



(a) c1 (b) c2 (c) N (d) M_cycle

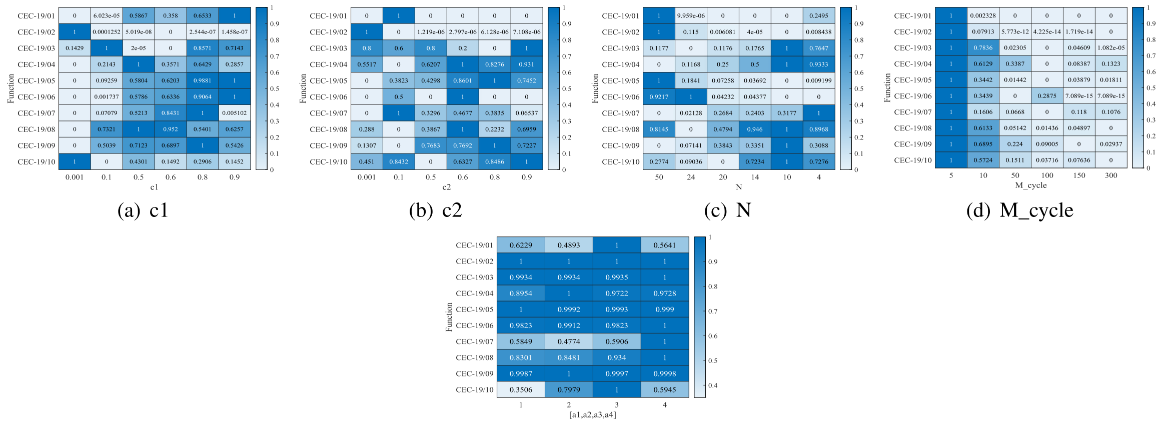
(e) a1, a2, a3, a4

FIGURE 7. **Sensitivity analysis of HSOA using classic benchmark functions.**

(a) c1 (dim=10) (b) c2 (dim=10) (c) N (dim=10) (d) M_cycle (dim=10)
(e) a1, a2, a3, a4 (dim=10) (f) c1 (dim=30) (g) c2 (dim=30) (h) N (dim=30)
(i) M_cycle (dim=30) (j) a1, a2, a3, a4 (dim=30) (k) c1 (dim=50) (l) c2 (dim=50)
(m) N (dim=50) (n) M_cycle (dim=50) (o) a1, a2, a3, a4 (dim=50) (p) c1 (dim=100)
(q) c2 (dim=100) (r) N (dim=100) (s) M_cycle (dim=100) (t) a1, a2, a3, a4 (dim=100)

FIGURE 8. Sensitivity analysis of HSOA using 2017 benchmark functions over different dimensions.

(Min = 500.00), $CEC-17/06$ (Min = 600.00), $CEC-17/8$ (Min = 800.00), $CEC-17/09$ (Min = 900.00), $CEC-17/11$ (Min = 1100.00), $CEC-17/13$ (Min = 1300.00), $CEC-17/14$ (Min = 1400.00), $CEC-17/19$ (Min = 1900.00), $CEC-17/20$ (Min = 2000.00). For the other CECs with dimension 10, the HSOA results are close to the global optimum. For the higher dimensions (Dim = 30, 50 and 100), HSOA got competitive results with CEC2017 winners (LSHADE and EBOwith CMAR). In dimensions 10 for $CEC-17/01$, $CEC-17/03$, $CEC-17/04$, $CEC-17/06$ and $CEC-17/09$, HSOA results were equal with those of LSHADE and EBOwithCMAR and were better than them in $CEC-17/22$, $CEC-17/25$ and $CEC-17/27$. In dimension 30 for $CEC-17/01$, $CEC-17/03$, $CEC-17/18$ and $CEC-17/20$, HSOA demonstrated a very close performance compared to LSHADE and EBOwithCMAR. In contrast, the HSOA results were better than LSHADE in $CEC-17/04$, $CEC-17/05$, $CEC-17/06$, $CEC-17/08$, $CEC-17/10$, $CEC-17/11$, $CEC-17/12$, $CEC-17/16$, $CEC-17/17$, $CEC-17/21$ to $CEC-17/27$, $CEC-17/29$ and $CEC-17/30$. Moreover, HSOA was more accurate than EBOwithCMAR in $CEC-17/04$, $CEC-17/05$, $CEC-17/08$, $CEC-17/12$, $CEC-17/27$, $CEC-17/29$ and $CEC-17/30$. Regarding dimensions 50, HSOA achieved competitive results compared to LSHADE and EBOwithCMAR in $CEC-17/01$, $CEC-17/07$, $CEC-$



(a) c1 (b) c2 (c) N (d) M_cycle

(e) a1, a2, a3, a4

FIGURE 9. Sensitivity analysis of HSOA using CEC2019 benchmark functions.

TABLE 5. Comparison of victory(+), equality(=) and defeat(−) for HSOA in compression with AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the classic benchmark functions over 51 runs and 150000 number function evaluations.

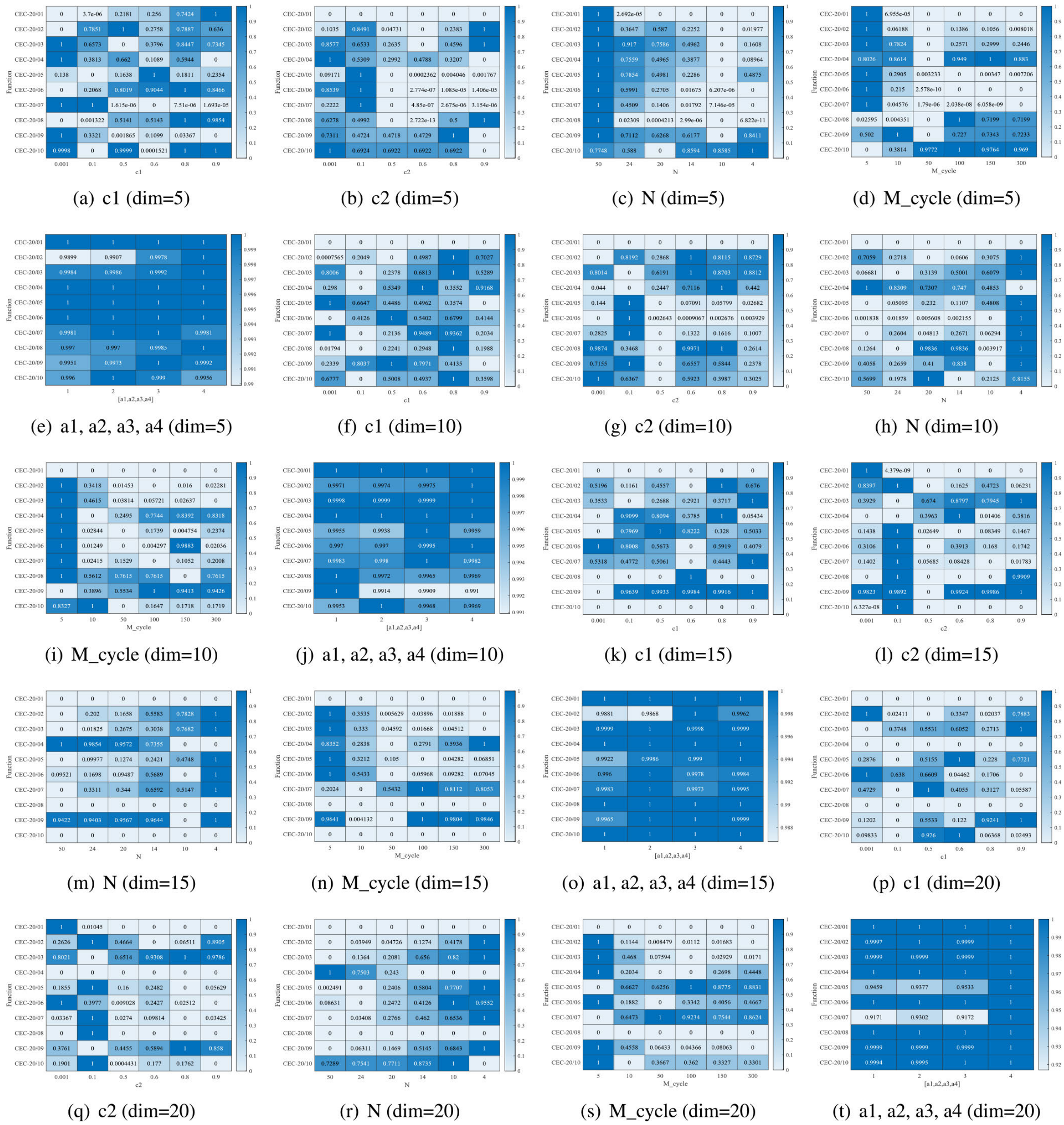
Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
F1	-1	1	1	1	-1	-1	1	-1	1
F2	-1	1	1	1	-1	-1	1	-1	1
F3	-1	1	1	1	1	0	1	-1	1
F4	-1	1	1	1	1	-1	1	-1	1
F5	1	1	1	1	1	1	-1	1	-1
F6	1	1	1	1	1	1	0	1	0
F7	-1	1	1	1	1	-1	1	1	1
F8	-1	1	1	-1	0	-1	-1	1	1
F9	0	1	1	0	0	0	0	0	-1
F10	-1	1	1	1	0	-1	1	-1	-1
F11	0	1	1	0	0	0	0	0	0
F12	1	1	1	1	1	1	-1	1	-1
F13	1	1	1	1	1	1	0	1	0
F14	1	0	0	1	1	1	0	0	0
F15	1	1	1	1	1	1	-1	1	-1
F16	1	0	0	1	1	1	0	0	0
F17	0	0	0	1	1	1	0	0	0
F18	1	1	1	1	1	1	-1	1	0
F19	1	0	0	1	1	1	0	0	0
F20	1	1	1	1	1	1	1	1	0
F21	1	1	1	1	1	1	0	1	0
F22	1	1	1	1	1	1	0	1	0
F23	1	1	0	1	1	1	0	1	0
+	13	19	18	20	17	14	7	12	6
=	3	4	5	2	4	3	11	6	12
-	7	0	0	1	2	6	5	5	5

17/08, $CEC-17/12$, $CEC-17/20$, $CEC-17/22$ to $CEC-17/24$, $CEC-17/26$ and $CEC-17/29$. The accuracy of HSOA was greater than LSHADE in $CEC-17/01$, $CEC-17/06$, $CEC-17/09$ through $CEC-17/11$, $CEC-17/13$ through $CEC-17/19$, $CEC-17/21$, $CEC-17/27$, $CEC-17/28$ and $CEC-17/30$. HSOA in comparison with EBOwithCMAR performed accurate in $CEC-17/04$, $CEC-17/05$, $CEC-17/09$, $CEC-17/11$, $CEC-17/14$ to $CEC-17/16$, $CEC-17/18$, $CEC-17/19$, $CEC-17/21$, $CEC-17/27$, $CEC-17/28$ and $CEC-17/30$. Based on dimensions 100, HSOA competed for LSHADE and EBOwithCMAR in $CEC-17/01$, $CEC-17/04$, $CEC-17/05$, $CEC-17/08$, $CEC-17/10$, $CEC-17/12$, $CEC-17/13$, $CEC-17/15$, $CEC-17/17$, $CEC-17/20$ to $CEC-17/21$, $CEC-17/23$, $CEC-17/24$, $CEC-17/28$ and $CEC-17/30$. HSOA was superior to LSHADE in $CEC-17/06$, $CEC-17/07$, $CEC-17/11$, $CEC-17/14$, $CEC-17/16$, $CEC-17/18$, $CEC-17/22$, $CEC-17/25$ to $CEC-17/27$ and $CEC-17/29$. Based on other results, HSOA demonstrated a better performance than EBOwithCMAR in $CEC-17/06$, $CEC-17/09$, $CEC-17/14$ and $CEC-17/27$.

It is worth's noting that HSOA in all dimensions and $CEC2017$ had superiority over AVOA, PSO, DE, RW_GWO, WO, HHO, and GBO. In terms of the quality of the HSOA results, it can be concluded that the HSOA results were comparable to those of other investigated algorithms.

The Wilcoxon test significant level of 0.01 (Tables 10 to 13), determined that HSOA had 28, 26, 25, 28, 29, 29, 5, 27 and 1 victories on 29 $CEC2017$ in 10 dimensions compared to AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR, respectively. In 30 dimensions, Wilcoxon recorded 29, 27, 29, 28, 29, 29, 12, 28 and 4 victories for HSOA in competition with AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR, respectively. HSOA in 50 dimensions was wine in 29, 27, 26, 28, 29, 29, 10, 2 and 8 $CEC2017$ of 29 $CEC2017$ in comparison with AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR, respectively. According to other reported results of the Wilcoxon test, in 100 dimensions, HSOA had 29, 26, 28, 27, 29, 29, 6, 28 and 4 victories of 29 $CEC2017$ compared to AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR, respectively. The number of equalities between HSOA and LSHADE and EBOwithCMAR, in dimensions 10, were 12, in dimensions 30 were 8 and 10, in dimension 50 were 14 and 11, and in dimensions 100 were 12 and 2, respectively.

The ranking of HSOA and competitors in the $CEC-17$ optimization is conducted using the Friedman test (Figures 15 through 18). As shown, in dimensions 10 and 100, the proposed algorithm ranked first after EBOwithCMAR and LSHADE. In dimensions 30 and 50, HSOA was ranked



(a) c1 (dim=5) (b) c2 (dim=5) (c) N (dim=5) (d) M_cycle (dim=5)
(e) a1, a2, a3, a4 (dim=5) (f) c1 (dim=10) (g) c2 (dim=10) (h) N (dim=10)
(i) M_cycle (dim=10) (j) a1, a2, a3, a4 (dim=10) (k) c1 (dim=15) (l) c2 (dim=15)
(m) N (dim=15) (n) M_cycle (dim=15) (o) a1, a2, a3, a4 (dim=15) (p) c1 (dim=20)
(q) c2 (dim=20) (r) N (dim=20) (s) M_cycle (dim=20) (t) a1, a2, a3, a4 (dim=20)

FIGURE 10. Sensitivity analysis of HSOA using CEC2020 benchmark functions over different dimensions.

first after EBOwithCMAR. The ranking results for the proposed algorithm and the $CEC-17$ winners are close. The competitive results of HSOA were for using two swarms in this algorithm (artificial Humboldt squids and school fish) and adaptive parameters that led to a good balance between exploration and exploitation.

Table 14 tabulates the results of solving CEC2019 benchmark functions by HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR. Considering the results, HSOA, in terms of min, mean, median, max, and std, had a competitive accuracy than other investigated algorithms. It is clear that HSOA performed well for $CEC-19$ problems, achieving the global optimum for $CEC-19/01$ to $CEC-19/06$ and finding optimal solutions for others with reasonable accuracy.

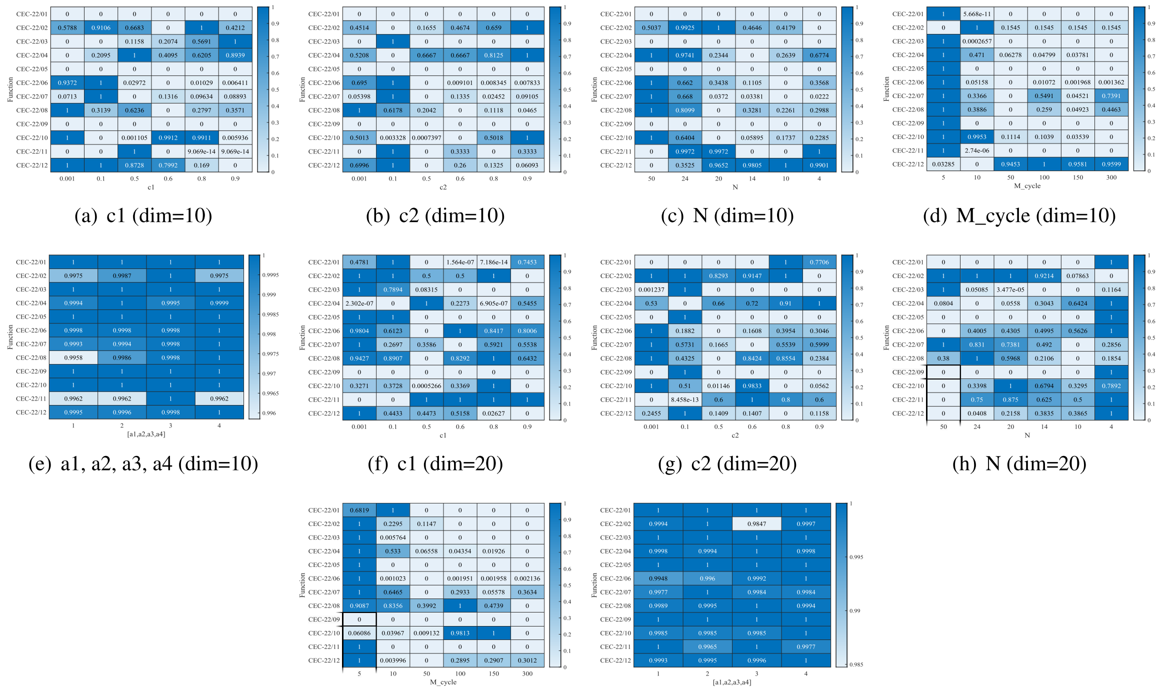
Using the Wilcoxon test of Table 15 revealed that HSOA had 10 wins out of 10 $CEC2019$ over PSO, DE, RW_GWO, WOA and GBO. According to the Wilcoxon test, HSOA won 9 out of 10 $CEC2019$ against AVOA and HHO, and also won 7 and 3 out of 10 $CEC2019$ against LSHADE and EBOwithCMAR, respectively. The number of HSOA equalities with HHO, LSHADE and EBOwithCMAR was estimated at 1, 2 and 2, respectively. In Figure 19, the Friedman test results indicate that HSOA ranked first with 2.06, followed by EBOwithCMAR and LSHADE with 2.49 and 2.78, respectively. The Friedman ranks of AVOA,

TABLE 6. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 10 dimensions, 51 runs and 100000 number function evaluations.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
	Min	100.00	11987.03	100.96	135.13	3403.96	7915.27	36460.41	100.00	100.10	100.00
	Mean	100.00	63713.27	1723.10	1371.99	3019083.03	295915.24	233033.39	100.00	1004.74	100.00
CEC-17/01	Median	100.00	57458.56	785.93	1230.12	15670.33	70542.61	218064.13	100.00	476.65	100.00
	Max	100.00	175235.90	12252.30	4280.86	43037788.62	2676940.69	577133.76	100.00	6084.59	100.00
	Std	0.00	40686.35	2329.07	1010.83	10231243.39	579476.71	117529.75	0.00	1235.43	0.00
	Min	300.00	690.82	300.00	879.94	305.21	310.19	300.20	300.00	300.00	300.00
	Mean	300.00	2533.03	300.00	1936.66	837.18	511.89	300.78	300.00	300.00	300.00
CEC-17/03	Median	300.00	2372.91	300.00	1922.24	456.75	468.70	300.73	300.00	300.00	300.00
	Max	300.00	6509.46	300.00	3465.34	4707.67	1241.93	301.66	300.00	300.00	300.00
	Std	0.00	1190.62	0.00	564.41	1002.97	187.70	0.35	0.00	0.00	0.00
	Min	400.00	400.34	400.03	404.94	402.18	400.12	400.06	400.00	400.00	400.00
	Mean	400.00	408.95	400.43	406.01	409.56	426.11	421.85	400.00	400.00	400.00
CEC-17/04	Median	400.00	406.65	400.37	406.10	407.43	407.60	407.04	400.00	400.00	400.00
	Max	400.00	467.71	401.52	406.57	463.78	550.26	500.39	400.00	400.00	400.00
	Std	0.00	14.04	0.32	0.36	10.97	38.06	30.38	0.00	0.00	0.00
	Min	500.00	507.97	508.95	505.94	503.99	517.04	513.32	501.03	503.98	500.00
	Mean	501.34	520.93	519.00	509.53	513.79	548.42	539.51	503.45	521.18	500.04
CEC-17/05	Median	501.01	519.92	517.91	509.57	509.09	545.77	539.87	503.18	520.89	500.00
	Max	503.14	535.87	544.77	513.22	530.69	584.60	567.02	505.16	543.78	500.99
	Std	0.71	6.36	7.20	1.64	7.79	17.64	14.66	1.04	8.67	0.20
	Min	600.00	600.01	600.00	600.00	600.02	606.41	601.79	600.00	600.00	600.00
	Mean	600.00	600.12	600.66	600.00	600.28	629.83	626.74	600.00	600.05	600.00
CEC-17/06	Median	600.00	600.11	600.00	600.00	600.11	629.63	625.50	600.00	600.00	600.00
	Max	600.00	600.50	608.72	600.00	601.42	660.34	654.49	600.00	601.01	600.00
	Std	0.00	0.09	1.59	0.00	0.33	12.25	12.64	0.00	0.18	0.00
	Min	710.66	712.49	707.96	714.85	706.58	724.62	725.78	710.74	716.88	710.37
	Mean	711.78	719.87	720.26	721.29	727.68	775.21	771.76	712.91	734.61	710.62
CEC-17/07	Median	711.77	719.50	719.26	721.55	726.95	772.32	775.75	712.87	733.43	710.60
	Max	713.40	730.20	736.39	724.17	751.85	849.95	811.68	714.86	767.46	711.33
	Std	0.71	3.78	5.69	1.68	8.99	25.87	19.81	0.82	10.55	0.21
	Min	800.00	804.01	804.97	806.37	803.98	811.95	808.04	801.10	806.96	800.00
	Mean	801.32	810.08	812.95	810.14	810.95	839.82	828.80	803.09	821.10	800.04
CEC-17/08	Median	800.99	809.95	812.93	810.45	810.01	839.80	828.95	803.11	820.89	800.00
	Max	802.98	819.90	824.87	814.72	830.64	872.64	853.80	805.13	837.81	800.99
	Std	0.73	3.75	4.66	1.96	4.93	14.67	10.09	0.99	6.99	0.18
	Min	900.00	900.03	900.00	900.00	900.04	955.95	936.51	900.00	900.00	900.00
	Mean	900.00	903.75	900.00	900.00	904.90	1329.27	1280.98	900.00	902.40	900.00
CEC-17/09	Median	900.00	903.34	900.00	900.00	900.63	1240.42	1251.71	900.00	900.54	900.00
	Max	900.00	913.52	900.09	900.00	953.84	2230.91	1988.21	900.00	917.77	900.00
	Std	0.00	3.10	0.02	0.00	10.45	284.21	246.47	0.00	4.24	0.00
	Min	1000.25	1000.49	1122.10	1224.47	1012.54	1462.16	1364.00	1002.32	1033.47	1000.12
	Mean	1063.21	1776.63	1725.69	1499.66	1496.61	2035.03	1964.97	1053.54	1796.73	1040.74
CEC-17/10	Median	1015.71	1754.28	1731.51	1488.98	1511.07	2003.27	1955.32	1020.27	1758.07	1010.31
	Max	1257.31	2540.95	2288.46	1655.37	2069.71	2790.73	2509.85	1156.71	2472.82	1140.59
	Std	74.72	294.11	267.92	89.18	266.41	328.16	317.63	53.66	382.81	52.59
	Min	1100.00	1110.32	1101.00	1101.84	1106.90	1110.62	1106.76	1100.00	1100.99	1100.00
	Mean	1100.94	1129.15	1117.56	1103.23	1127.58	1213.21	1174.19	1100.26	1111.44	1100.00
CEC-17/11	Median	1100.00	1128.02	1116.64	1103.29	1123.97	1182.55	1148.59	1100.00	1109.95	1100.00
	Max	1102.91	1157.86	1136.53	1104.62	1259.30	1474.54	1438.76	1102.54	1140.51	1100.00
	Std	1.12	10.51	9.99	0.69	22.76	94.62	78.49	0.67	7.39	0.00
	Min	1200.25	12991.09	1477.84	36299.73	12356.91	5248.95	5698.61	1200.00	1756.71	1200.00
	Mean	1295.56	1074586.63	11755.29	200376.87	664869.48	4206287.96	1162284.97	1254.76	9008.62	1305.65
CEC-17/12	Median	1304.12	1003315.58	7636.71	194254.38	275606.81	1128022.07	693823.69	1200.42	5114.42	1318.65
	Max	1539.80	2979315.99	42196.42	412382.29	3418609.14	16104189.76	5643740.41	1437.08	48238.37	1438.18
	Std	76.86	720576.58	9941.89	83886.38	797510.67	5158085.66	1369978.67	70.71	9667.51	54.85
	Min	1300.00	4665.38	1519.67	1399.24	1659.59	2001.09	1430.90	1300.00	1315.37	1300.00
	Mean	1303.67	8816.86	7000.24	2543.82	10274.57	17137.51	14957.44	1303.71	1617.12	1302.92
CEC-17/13	Median	1304.84	8673.94	6368.89	2251.27	8643.73	13227.39	11656.41	1304.84	1486.43	1304.84
	Max	1308.51	14256.86	25692.23	5957.97	27993.95	44508.74	36610.82	1307.95	4386.19	1308.37
	Std	2.82	2489.17	4786.33	1041.33	5919.59	12861.08	10314.04	2.24	481.00	2.88
	Min	1400.00	1429.47	1410.22	1402.10	1441.83	1461.81	1449.90	1400.00	1427.16	1400.00
	Mean	1400.87	3075.02	1493.21	1423.75	2099.85	1590.80	1516.01	1400.09	1479.00	1400.03
CEC-17/14	Median	1400.42	2892.64	1465.62	1415.58	1479.99	1532.57	1515.97	1400.00	1477.81	1400.00
	Max	1407.52	7457.80	2017.12	1617.52	5259.43	3587.22	1610.16	1401.08	1574.65	1401.00
	Std	1.37	1487.48	96.73	31.55	1366.23	294.60	29.84	0.25	33.86	0.14
	Min	1500.00	1520.94	1510.37	1503.15	1517.21	1645.24	1552.72	1500.00	1502.51	1500.00
	Mean	1500.26	3468.49	1667.28	1534.60	3196.52	4243.55	1935.86	1500.12	1576.95	1500.19
CEC-17/15	Median	1500.23	2797.66	1591.87	1522.00	2174.45	2885.60	1754.63	1500.01	1555.35	1500.04
	Max	1500.50	13610.89	2707.72	1625.71	7784.29	13038.27	3637.64	1500.50	1795.85	1500.50
	Std	0.21	2442.92	211.38	30.78	1804.85	2871.28	493.98	0.18	68.30	0.22

TABLE 6. ***(Continued.)*** **Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 10 dimensions, 51 runs and 100000 number function evaluations.**

	Min	1600.42	1600.35	1600.61	1600.84	1604.44	1611.55	1604.36	1600.09	1600.95	1600.02
	Mean	1601.40	1784.65	1817.30	1602.59	1668.38	1806.85	1845.21	1600.55	1727.66	1600.41
CEC-17/16	Median	1601.33	1839.06	1839.48	1602.26	1642.51	1782.08	1859.80	1600.57	1723.56	1600.42
	Max	1602.98	1988.83	2019.73	1609.01	1869.89	2118.66	2105.10	1600.99	2005.07	1600.86
	Std	0.50	110.99	124.51	1.33	59.85	115.27	145.92	0.21	116.57	0.20
	Min	1700.16	1719.26	1706.33	1700.49	1721.71	1730.66	1727.06	1700.06	1702.09	1700.01
	Mean	1701.18	1743.37	1752.92	1701.89	1743.94	1809.73	1767.20	1700.42	1733.61	1700.15
CEC-17/17	Median	1701.18	1744.98	1746.15	1701.79	1741.84	1786.60	1761.39	1700.41	1733.00	1700.07
	Max	1703.74	1796.19	1864.15	1705.56	1795.45	1957.83	1856.01	1701.13	1784.32	1700.64
	Std	0.66	14.13	29.38	0.95	13.08	57.23	26.69	0.23	18.64	0.16
	Min	1800.03	2234.93	1904.43	1970.86	4775.40	2123.21	2023.05	1800.00	1844.02	1800.00
	Mean	1800.92	5037.20	8241.89	2780.07	27613.23	17612.72	14585.47	1800.98	2022.97	1800.31
CEC-17/18	Median	1800.48	4460.64	4850.99	2591.89	32755.16	15656.72	12745.97	1800.06	1937.78	1800.39
	Max	1804.62	14523.83	29748.03	4465.11	55367.11	42302.15	48567.29	1820.14	3156.85	1800.50
	Std	1.11	2789.79	7398.50	641.46	14975.38	10840.98	11808.48	3.91	255.17	0.18
	Min	1900.00	1904.11	1907.17	1900.43	1909.39	1987.16	1960.31	1900.00	1903.82	1900.00
	Mean	1900.10	4792.24	2965.90	1934.34	5632.55	19300.84	9184.00	1900.02	1945.88	1900.02
CEC-17/19	Median	1900.05	4489.05	2051.55	1913.50	1940.65	9366.37	6255.76	1900.01	1933.75	1900.02
	Max	1901.56	11291.47	10403.01	2353.13	13889.48	231816.56	28920.82	1900.07	2077.16	1900.04
	Std	0.25	2276.28	1815.59	66.95	5233.10	33453.35	7683.22	0.02	40.65	0.01
	Min	2000.00	2002.51	2002.30	2000.00	2021.49	2039.55	2038.79	2000.00	2000.31	2000.00
	Mean	2000.08	2037.44	2080.23	2000.00	2055.00	2140.01	2148.19	2000.01	2047.36	2000.16
CEC-17/20	Median	2000.02	2027.97	2049.09	2000.00	2038.53	2124.64	2147.36	2000.00	2033.25	2000.31
	Max	2001.13	2148.85	2191.00	2000.00	2215.38	2290.38	2318.93	2000.31	2196.48	2000.31
	Std	0.20	37.74	57.84	0.00	46.04	62.09	71.40	0.04	47.01	0.16
	Min	2200.19	2205.55	2200.00	2212.26	2200.48	2206.74	2200.10	2200.00	2200.00	2200.00
	Mean	2202.47	2286.73	2259.97	2264.83	2298.08	2305.83	2303.38	2273.53	2241.59	2228.10
CEC-17/21	Median	2202.55	2312.48	2203.24	2255.08	2311.62	2332.06	2333.67	2303.90	2202.11	2200.00
	Max	2205.24	2326.63	2349.48	2319.04	2328.85	2398.64	2404.68	2307.78	2336.21	2302.84
	Std	1.25	46.86	61.84	37.49	38.12	61.37	70.63	47.08	58.33	45.68
	Min	2200.00	2301.99	2211.20	2258.63	2211.37	2247.75	2231.19	2200.00	2219.29	2300.00
	Mean	2289.72	2306.72	2298.89	2296.80	2303.43	2313.36	2308.94	2298.07	2299.93	2300.00
CEC-17/22	Median	2300.00	2306.11	2301.92	2300.57	2306.60	2313.40	2313.52	2300.00	2302.45	2300.00
	Max	2300.45	2313.54	2303.37	2301.42	2322.04	2336.06	2330.27	2300.35	2309.11	2300.00
	Std	28.98	2.90	15.49	9.53	18.77	11.94	20.33	14.01	14.93	0.00
	Min	2302.25	2619.61	2605.14	2607.02	2603.13	2615.62	2610.56	2602.86	2604.32	2600.00
	Mean	2596.28	2642.89	2624.95	2611.41	2617.00	2644.45	2657.81	2604.60	2624.02	2600.44
CEC-17/23	Median	2602.89	2642.49	2624.94	2611.60	2618.20	2642.83	2658.93	2604.60	2621.87	2600.00
	Max	2604.77	2667.43	2665.34	2615.19	2634.70	2695.53	2695.42	2607.27	2656.68	2604.11
	Std	42.02	11.36	12.33	1.86	8.26	16.34	20.39	1.02	10.90	1.13
	Min	2505.90	2500.09	2500.00	2620.00	2503.58	2501.92	2500.56	2500.00	2500.00	2500.00
	Mean	2645.00	2700.29	2726.34	2717.77	2734.55	2753.22	2771.01	2717.68	2689.53	2554.74
CEC-17/24	Median	2740.70	2753.70	2748.54	2743.72	2737.43	2777.74	2793.75	2732.03	2745.95	2500.00
	Max	2765.36	2797.30	2800.40	2750.26	2760.20	2856.97	2882.39	2735.65	2775.27	2729.54
	Std	114.41	112.85	79.72	40.92	33.91	85.54	94.40	50.33	112.35	91.60
	Min	2897.77	2899.16	2897.74	2899.84	2898.16	2606.80	2609.28	2897.74	2897.74	2897.74
	Mean	2902.09	2933.00	2923.15	2911.91	2934.27	2941.23	2926.75	2915.07	2925.92	2919.48
CEC-17/25	Median	2898.66	2946.84	2943.47	2909.79	2945.87	2951.52	2945.58	2898.03	2944.18	2899.58
	Max	2943.88	2952.23	2949.15	2944.47	2949.65	3052.13	2972.85	2945.80	2971.76	2943.37
	Std	12.30	22.38	23.30	10.71	16.68	66.69	51.10	22.42	25.43	22.73
	Min	2800.00	2601.09	2600.00	2715.72	2774.68	2601.90	2811.59	2900.00	2600.00	2800.00
	Mean	2898.04	3053.20	2885.47	2896.64	2989.49	3332.97	3316.84	2900.00	2953.87	2884.61
CEC-17/26	Median	2900.00	3059.49	2900.00	2918.33	2900.18	3137.58	3206.68	2900.00	2900.00	2900.00
	Max	2900.00	3987.34	3044.26	2952.48	3941.82	4801.41	4552.35	2900.00	3857.35	2900.00
	Std	14.00	247.10	81.83	54.60	268.82	487.08	426.84	0.00	173.27	36.08
	Min	3086.89	3092.83	3089.01	3089.05	3089.52	3090.24	3097.14	3089.01	3089.64	3089.52
	Mean	3089.17	3140.75	3106.63	3089.69	3093.97	3122.82	3131.65	3089.45	3103.45	3091.80
CEC-17/27	Median	3089.01	3140.58	3097.80	3089.66	3094.09	3106.23	3120.99	3089.52	3095.74	3090.75
	Max	3094.23	3208.20	3208.63	3090.47	3104.11	3221.35	3219.83	3089.52	3193.05	3095.98
	Std	0.93	28.00	27.44	0.25	2.98	35.35	30.88	0.18	22.92	2.38
	Std	1.37	1487.48	96.73	31.55	1366.23	294.60	29.84	0.25	33.86	0.14
	Min	1500.00	1520.94	1510.37	1503.15	1517.21	1645.24	1552.72	1500.00	1502.51	1500.00
	Min	3100.03	2803.55	3100.00	3172.85	3166.70	3100.40	3101.60	3100.00	3100.00	3100.00
	Mean	3159.16	3288.83	3199.33	3236.69	3357.60	3403.55	3337.01	3214.03	3330.47	3100.00
CEC-17/28	Median	3100.10	3383.78	3100.00	3210.61	3399.98	3411.82	3383.87	3100.00	3402.69	3100.00
	Max	3411.82	3453.72	3444.13	3411.82	3433.98	3749.37	3501.44	3411.82	3411.82	3100.00
	Std	120.58	144.63	135.72	66.54	88.44	185.04	115.05	145.87	120.05	0.00
	Min	3129.52	3172.32	3148.21	3145.35	3139.43	3156.68	3160.31	3128.05	3145.56	3128.28
	Mean	3138.12	3217.23	3212.09	3161.97	3174.87	3308.50	3300.66	3138.34	3234.63	3132.96
CEC-17/29	Median	3136.98	3213.83	3199.64	3161.65	3170.35	3300.99	3294.46	3138.01	3224.14	3132.98
	Max	3159.40	3273.37	3331.33	3178.83	3295.69	3473.89	3479.31	3154.30	3359.96	3139.27
	Std	5.56	25.36	45.48	7.60	25.95	77.33	75.93	5.89	55.10	2.43
	Min	3440.37	8095.55	3762.14	4657.82	3710.83	6296.85	4838.12	3394.50	3416.36	3394.50
	Mean	3507.34	345495.71	167688.20	17934.90	614499.05	564983.34	527231.40	99555.88	395498.50	3409.27
CEC-17/30	Median	3497.63	220353.52	6759.34	13574.83	114555.27	220962.22	121932.13	3442.66	3729.03	3407.44
	Max	3735.20	3020926.86	1261343.43	67896.82	3598161.84	2136927.89	4219938.92	820578.06	1827680.48	3600.49
	Std	59.50	490959.92	359033.99	12874.06	808891.79	630800.00	851685.44	265899.84	540967.78	31.24

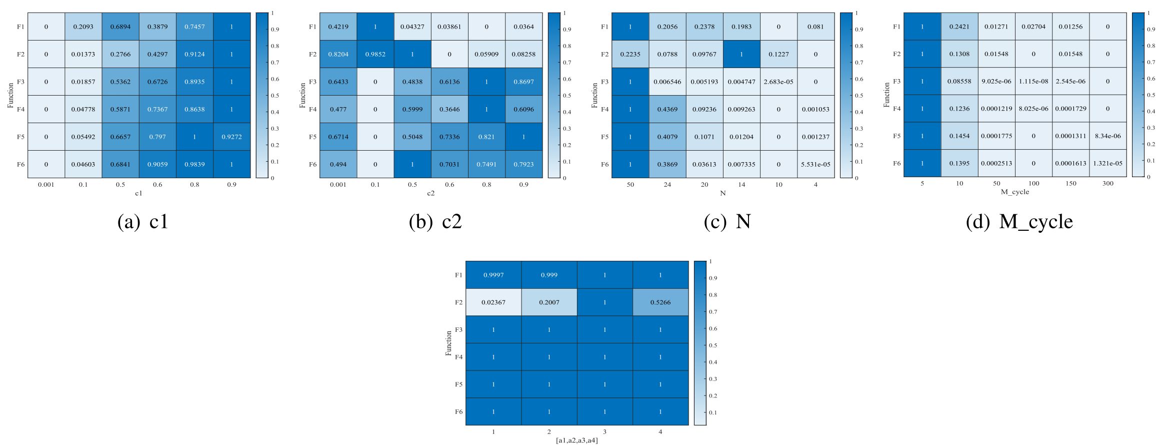


(a) c1 (dim=10) (b) c2 (dim=10) (c) N (dim=10) (d) M_cycle (dim=10)

(e) a1, a2, a3, a4 (dim=10) (f) c1 (dim=20) (g) c2 (dim=20) (h) N (dim=20)

(i) M_cycle (dim=20) (j) a1, a2, a3, a4 (dim=20)

FIGURE 11. **Sensitivity analysis of HSOA using CEC2022 benchmark functions over different dimensions.**



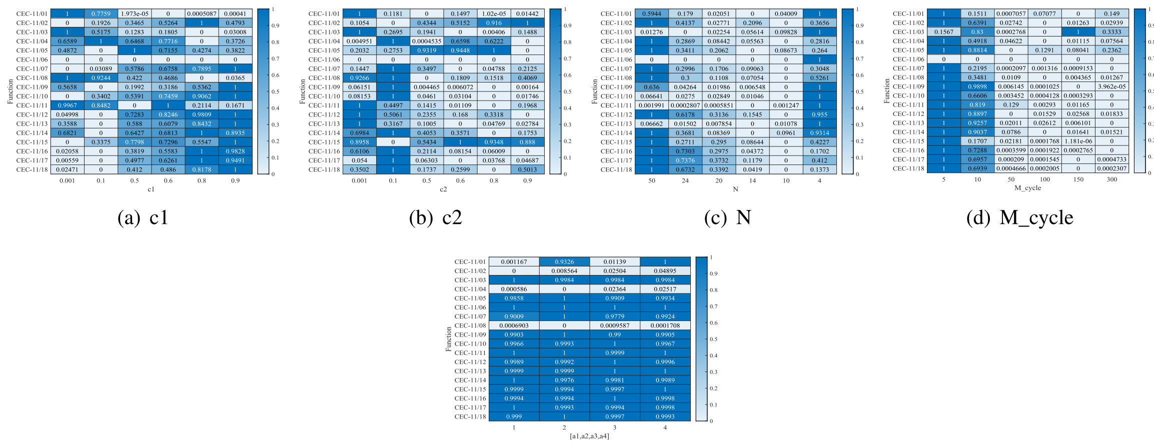
(a) c1 (b) c2 (c) N (d) M_cycle

(e) a1, a2, a3, a4

FIGURE 12. **Sensitivity analysis of HSOA using CEC2006 benchmark functions.**

PSO, DE, RW_GWO, WOA, HHO and, GBO were equal to 6.20, 7.59, 5.73, 6.30, 8.55, 7.53 and 5.77, respectively.

The statistical results of the HSOA of the benchmarks with 5, 10, 15, and 20 dimensions are summarized in Table 16 to Table 19. It includes the got min, mean, median, max and the std values over 30 random runs for all 10 *CEC*2020 benchmark functions. The HSOA could consistently get the optimal solution for all dimensions over 30 runs for unimodal functions, *CEC* − 20/01. HSOA performs well in all dimensions for multimodal functions (*CEC* − 20/02-*CEC* − 20/04), except for *CEC* − 20/03. Despite the many local optimal for *CEC* − 20/03, HSOA could approximate the global optimal with low error. The difference between the best obtained solutions of HSOA and the global optimal for *CEC* − 20/02 in 5,10, 15 and 20 dimensions was low and equal to 0.12, 0.12, 0.21, and 0.09, respectively. About *CEC* − 20/03, HSOA gets trapped in local optima in dimensions 10, 15 and 20, because *CEC* − 20/03 was a challenging problem. In terms of *CEC* − 20/04, the HSOA results were very close to the optimal. Regarding hybrid functions, *CEC* − 20/05 to *CEC* − 20/07, HSOA can consistently find the global optimal solution for the



(a) c1 (b) c2 (c) N (d) M_cycle

(e) a1, a2, a3, a4

FIGURE 13. Sensitivity analysis of HSOA using CEC2011 benchmark functions.

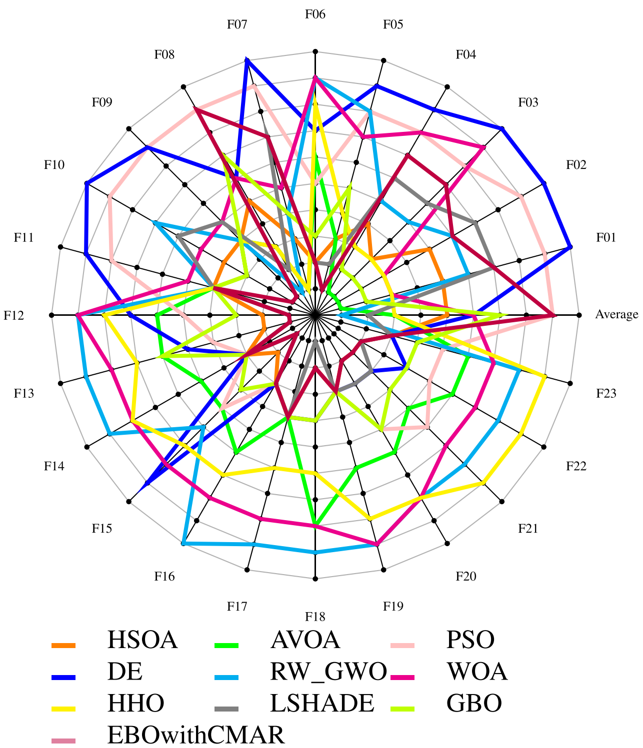


FIGURE 14. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the classic benchmark functions over 51 runs and 150000 number function evaluations.

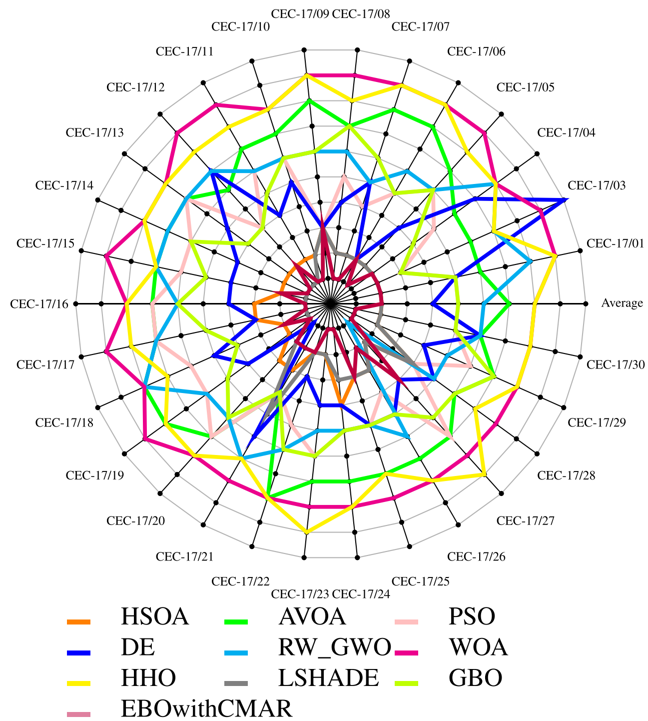


FIGURE 15. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 10 dimensions, 51 runs and 100000 number function evaluations.

three problems over 30 runs in 5 dimensions. In addition, HSOA got very close to the optimum in all functions for all the remaining three dimensionality except for $CEC - 20/05$. As the size of the problem increased, performing the mentioned problem slightly deteriorated. Considering the composition functions, $CEC - 20/08$ to $CEC - 20/10$, in 5 dimensions, HSOA could find the global optimal solution consistently in $CEC - 20/08$ over 30 runs. HSOA could also find the reasonable optimal solution in $CEC - 20/08$ for remaining dimensions. In addition, for $CEC - 20/09$ and in $CEC - 20/10$, the results were competitive. These test functions are challenging because of the presence of many sub-functions and local optimal in each problem. In these functions, HSOA had competitive results with new and state-of-the algorithms. The accuracy of the HSOA algorithm is competitive through its comparison with other algorithms based on min, mean, median, max, and std results. According

TABLE 7. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 30 dimensions, 51 runs and 300000 number function evaluations.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-17/01	Min	100.00	102.98	106.52	212.81	38377.90	665661.68	4499846.11	100.00	100.01	100.00
	Mean	100.00	4224.24	4050.68	1000.45	5136068.49	2268995.66	8054047.76	100.00	357.29	100.00
	Median	100.00	2025.53	1986.28	587.59	3818964.65	1898323.36	8009196.91	100.00	156.76	100.00
	Max	100.00	19388.31	18359.26	5040.67	33430312.04	6686501.55	12662974.38	100.00	3532.36	100.00
	Std	0.00	5616.02	4865.16	1056.94	5856386.97	1315520.78	1815136.59	0.00	575.82	0.00
CEC-17/03	Min	300.00	300.00	300.00	47264.28	405.55	39936.40	462.04	300.00	300.00	300.00
	Mean	300.00	302.89	300.00	77421.16	1925.99	167967.21	973.16	300.00	300.00	300.00
	Median	300.00	300.00	300.00	78391.28	1336.95	168353.99	906.63	300.00	300.00	300.00
	Max	300.00	373.48	300.00	98397.77	6154.85	367619.37	2557.15	300.00	300.00	300.00
	Std	0.00	14.40	0.00	10561.32	1520.83	75225.44	378.24	0.00	0.00	0.00
CEC-17/04	Min	400.00	400.73	400.00	487.35	440.77	483.63	440.26	400.00	400.03	400.00
	Mean	422.14	489.19	436.16	488.98	483.89	553.91	515.69	455.23	434.55	445.44
	Median	403.99	488.35	447.78	488.68	486.23	547.25	519.29	458.56	404.44	458.56
	Max	471.89	534.77	487.59	493.69	593.64	675.67	633.33	464.12	510.29	467.90
	Std	29.57	25.08	30.71	1.24	36.46	43.13	30.06	13.97	36.26	26.25
CEC-17/05	Min	506.98	621.38	552.73	607.68	561.60	630.76	649.15	508.25	577.61	503.98
	Mean	513.10	699.67	617.07	629.22	655.14	768.39	716.81	515.22	644.09	514.29
	Median	512.95	701.06	611.44	629.25	654.70	768.62	718.20	515.14	647.25	514.92
	Max	520.30	793.51	685.06	649.54	697.00	904.92	793.23	522.55	750.73	519.90
	Std	3.08	38.33	28.97	8.97	23.45	53.97	30.14	3.04	37.53	3.40
CEC-17/06	Min	600.00	617.78	600.35	600.00	600.09	642.00	632.14	600.00	601.49	600.00
	Mean	600.00	631.03	619.54	600.00	600.61	668.05	656.41	600.00	618.86	600.00
	Median	600.00	630.55	618.99	600.00	600.53	667.82	657.00	600.00	615.65	600.00
	Max	600.00	645.96	648.00	600.00	601.68	692.51	671.42	600.00	645.56	600.00
	Std	0.00	7.36	11.05	0.00	0.39	10.07	7.85	0.00	9.99	0.00
CEC-17/07	Min	737.78	890.89	782.91	832.55	771.27	970.15	1044.04	740.79	814.63	733.71
	Mean	743.53	1062.40	819.56	865.42	827.69	1219.07	1200.22	744.47	911.64	739.01
	Median	742.82	1066.84	815.21	867.17	808.97	1227.84	1205.24	744.41	903.90	737.41
	Max	753.32	1195.42	873.22	880.81	948.63	1437.82	1317.70	749.17	1049.18	747.06
	Std	2.91	69.18	22.38	8.85	47.23	113.93	58.39	2.11	53.20	4.08
CEC-17/08	Min	807.08	891.92	849.75	897.53	851.84	918.73	889.65	808.37	866.66	803.98
	Mean	811.51	952.39	898.01	931.39	950.07	1001.24	948.79	815.83	929.93	814.79
	Median	811.03	951.23	893.53	931.45	954.46	988.21	945.44	816.16	931.33	815.92
	Max	817.37	1029.83	948.25	949.73	1007.11	1127.57	1002.39	826.00	994.02	820.89
	Std	2.42	27.01	23.36	10.01	28.89	50.08	27.20	3.37	28.61	3.77
CEC-17/09	Min	900.00	3304.25	942.75	900.00	900.85	4045.01	4192.92	900.00	1269.23	900.00
	Mean	900.00	4960.05	2376.94	900.00	1072.20	7624.71	5741.65	900.04	2415.88	900.00
	Median	900.00	4965.89	2556.07	900.00	950.54	6605.28	5746.24	900.00	2223.02	900.00
	Max	900.00	8133.58	4931.76	900.00	3758.81	19864.97	7353.05	900.45	4728.04	900.00
	Std	0.00	875.03	798.98	0.00	428.97	3182.47	627.85	0.09	827.60	0.00
CEC-17/10	Min	1613.99	3600.94	3151.84	5570.60	5449.52	4456.07	3353.79	1959.27	3550.19	1915.75
	Mean	2636.78	5250.50	4603.72	6250.93	6555.74	5940.29	5328.90	2639.91	4875.87	2572.14
	Median	2662.18	5250.54	4570.15	6264.21	6562.91	6004.70	5343.33	2694.72	4835.05	2618.47
	Max	3287.43	7671.22	5857.98	6874.20	7815.49	7265.31	6930.96	3075.74	6876.76	2978.07
	Std	379.08	721.49	665.92	302.42	608.37	719.10	662.72	266.31	660.29	268.76
CEC-17/11	Min	1105.95	1149.29	1156.91	1172.03	1137.22	1291.20	1154.68	1105.97	1119.91	1101.99
	Mean	1126.93	1227.75	1211.95	1211.34	1236.15	1492.07	1251.09	1129.86	1228.93	1111.86
	Median	1118.40	1229.11	1206.54	1212.00	1227.70	1466.25	1256.38	1121.90	1229.36	1108.95
	Max	1176.83	1314.20	1331.11	1230.11	1349.63	1706.81	1391.59	1181.87	1336.77	1168.93
	Std	20.89	40.64	38.37	12.62	53.21	106.58	47.02	23.32	51.24	14.52
CEC-17/12	Min	1421.01	120736.43	3476.54	1928477.57	377318.88	2378332.84	1302535.90	1667.76	3327.10	1559.83
	Mean	2103.83	1100703.62	24088.16	4790126.13	9559114.35	44028601.18	8155343.18	2388.74	25753.76	2262.83
	Median	2011.66	809576.42	23307.24	4514042.73	4376168.29	38116682.71	8424103.30	2338.33	22029.89	2236.68
	Max	3079.09	3563762.94	54707.01	9741650.12	59516470.09	136707295.16	19384765.91	3368.97	61419.69	3222.54
	Std	399.52	748699.40	11032.90	1613361.15	12304373.82	29508877.67	4214263.45	399.49	15347.92	362.11
CEC-17/13	Min	1364.46	7660.82	1343.03	30473.74	35773.36	30091.76	35522.10	1305.97	1408.25	1303.00
	Mean	1389.52	28911.06	16171.10	146726.65	980345.93	144160.58	206240.87	1350.26	9108.23	1323.80
	Median	1387.75	25854.18	7967.07	133370.81	136741.35	123359.28	163679.27	1334.62	5430.03	1321.78
	Max	1441.73	80043.26	62326.73	354936.28	7774765.83	384604.94	784358.31	1910.22	36223.86	1391.54
	Std	14.02	14649.55	16821.97	69413.70	1848724.45	87239.35	151738.41	83.12	8514.52	13.01
CEC-17/14	Min	1427.86	2199.83	1638.88	11935.62	1715.83	23294.09	1947.64	1422.12	1520.90	1420.04
	Mean	1470.47	19239.67	4163.46	53701.50	31337.86	955063.45	29981.07	1428.79	1684.61	1427.89
	Median	1467.11	12833.49	3348.02	47578.41	31225.57	463613.62	15836.48	1427.00	1668.28	1427.27
	Max	1547.65	58848.90	11083.32	138042.59	113684.69	3824324.28	182969.26	1450.91	1893.95	1437.93
	Std	19.55	16640.26	2309.51	25009.51	25513.18	1088994.45	36465.09	5.08	77.56	3.77
CEC-17/15	Min	1511.77	2526.12	1671.24	4990.13	12331.29	6749.46	7755.33	1504.08	1609.74	1503.06
	Mean	1551.33	14482.50	11147.89	18124.53	54752.89	64357.96	49555.97	1525.09	4791.77	1511.50
	Median	1548.66	10159.23	7065.03	14642.30	38400.49	51815.01	36241.07	1518.17	2776.77	1510.30
	Max	1612.19	45212.56	42505.78	51425.50	174560.03	264670.50	330469.83	1620.85	34792.95	1534.86
	Std	18.34	11175.95	10247.63	11210.95	38705.50	47347.85	50979.42	22.68	5478.39	6.02
	Min	1619.15	2001.29	2080.19	1858.60	1757.82	2628.65	2342.21	1629.54	1967.64	1602.80
	Mean	1837.19	2766.62	2573.18	2219.25	2470.15	3470.23	3077.23	1845.49	2651.18	1728.31

TABLE 7. *(Continued.)* **Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 30 dimensions, 51 runs and 300000 number function evaluations.**

CEC-17/16	Median	1836.41	2775.27	2563.02	2207.27	2502.98	3499.70	3056.74	1866.02	2636.80	1729.03
	Max	2209.32	3560.38	3372.83	2461.32	3174.82	4992.73	3846.58	2092.32	3280.31	1978.39
	Std	169.83	350.94	292.69	128.78	335.57	503.71	350.49	123.12	316.06	107.19
	Min	1731.46	1785.89	1773.35	1804.48	1763.58	1996.83	1925.65	1723.09	1827.40	1710.19
	Mean	1752.22	2374.58	2160.49	1872.42	1944.33	2505.29	2479.08	1750.36	2267.10	1732.73
CEC-17/17	Median	1750.33	2409.20	2160.03	1869.52	1908.98	2522.54	2417.80	1748.21	2251.78	1735.06
	Max	1785.16	2974.73	2670.06	1957.60	2559.81	2965.75	3196.54	1862.02	2785.67	1748.07
	Std	11.48	248.37	186.86	37.72	137.17	260.12	297.26	19.68	203.65	9.05
	Min	1836.77	31369.81	9221.01	139286.37	58623.46	106357.24	44349.37	1821.42	2697.75	1823.29
	Mean	1846.48	196418.28	81372.01	434267.58	426071.81	2378285.00	551704.58	1870.87	17325.42	1843.23
CEC-17/18	Median	1846.85	151462.26	69280.04	394068.42	313723.60	1407405.99	418531.59	1851.22	14203.73	1836.22
	Max	1856.98	700979.12	205596.08	1030494.17	2123401.96	11153514.86	2187907.17	2228.37	65205.56	1931.80
	Std	3.55	153670.44	41025.70	198408.22	378599.12	2670246.23	522709.93	66.95	14492.49	22.00
	Min	1918.70	2198.05	2004.66	6791.08	8940.79	305811.87	20034.00	1904.56	1937.65	1904.79
	Mean	1933.33	13104.75	8094.25	20373.55	119739.32	3042004.00	167480.21	1914.71	2476.18	1913.38
CEC-17/19	Median	1933.60	9274.25	4551.45	18876.17	74442.16	2511558.92	159769.80	1911.43	2105.42	1911.50
	Max	1945.33	52272.64	35267.60	64664.69	663763.90	12760968.24	406243.80	1951.43	8195.16	1937.99
	Std	4.53	12669.47	8101.85	10908.20	123601.86	2539552.67	107615.39	9.71	1142.17	6.30
	Min	2022.10	2114.61	2207.68	2049.38	2131.39	2342.07	2263.56	2030.19	2123.39	2017.40
	Mean	2080.64	2545.71	2463.99	2147.24	2370.87	2697.81	2664.92	2078.53	2431.03	2062.60
CEC-17/20	Median	2053.32	2542.39	2458.83	2128.83	2344.58	2694.95	2671.11	2048.39	2437.53	2041.63
	Max	2182.55	2923.51	3070.79	2268.86	2730.48	3198.92	3144.70	2175.35	2818.73	2163.91
	Std	55.33	183.45	184.22	55.23	137.49	201.51	206.20	54.18	178.13	45.71
	Min	2306.42	2392.67	2345.60	2404.06	2342.50	2468.86	2398.43	2310.30	2356.23	2302.99
	Mean	2313.54	2480.64	2406.29	2432.32	2441.72	2559.22	2522.68	2316.63	2415.63	2312.41
CEC-17/21	Median	2312.83	2486.59	2398.34	2434.47	2443.02	2548.67	2519.62	2316.61	2417.02	2313.57
	Max	2324.94	2594.28	2488.09	2449.46	2481.53	2686.51	2667.02	2322.08	2495.22	2320.12
	Std	4.21	50.71	34.36	9.60	22.67	57.94	51.96	2.64	33.89	4.77
	Min	2300.00	2300.00	2300.00	2559.60	2310.52	2314.49	2316.49	2300.00	2300.00	2300.00
	Mean	2300.00	5217.29	3636.20	3254.76	6084.43	6341.02	4902.78	2300.00	3254.30	2300.00
CEC-17/22	Median	2300.00	6308.16	2302.45	3225.11	7258.64	7314.27	6042.55	2300.00	2302.46	2300.00
	Max	2300.00	8175.04	7606.95	4246.95	8508.85	8811.62	7858.86	2300.00	7977.74	2300.00
	Std	0.00	2129.19	2039.38	333.78	2506.43	2318.24	2321.01	0.00	1759.59	0.00
	Min	2645.55	2788.25	2715.95	2747.63	2704.46	2847.34	2914.39	2650.05	2713.95	2652.03
	Mean	2661.14	2897.02	2826.55	2779.46	2811.14	3055.07	3048.60	2665.11	2788.20	2660.83
CEC-17/23	Median	2661.73	2898.06	2827.39	2780.46	2811.92	3062.16	3032.12	2665.80	2789.27	2660.63
	Max	2673.82	3024.60	2983.14	2795.50	2863.25	3334.17	3247.26	2674.96	2914.16	2671.19
	Std	6.09	63.75	47.59	9.37	24.68	102.82	78.36	5.02	43.80	4.34
	Min	2827.70	2951.23	2895.97	2955.73	2924.29	2972.52	3041.84	2828.20	2880.57	2600.00
	Mean	2833.51	3114.91	2990.45	2983.34	2968.64	3155.93	3345.31	2835.97	2949.61	2826.84
CEC-17/24	Median	2832.95	3111.11	2985.80	2984.03	2968.28	3157.83	3364.12	2835.73	2943.68	2831.60
	Max	2840.15	3326.07	3134.56	3002.68	3018.77	3354.54	3641.83	2845.93	3084.74	2841.04
	Std	3.23	83.47	56.86	10.30	17.92	94.77	131.14	3.33	44.55	32.59
	Min	2886.69	2883.83	2883.48	2887.08	2884.17	2889.25	2884.33	2886.74	2883.72	2883.61
	Mean	2886.72	2900.64	2891.52	2887.33	2903.82	2938.48	2907.97	2886.84	2896.51	2886.65
CEC-17/25	Median	2886.72	2890.40	2887.94	2887.36	2904.20	2944.49	2908.69	2886.83	2889.71	2886.79
	Max	2886.76	2945.48	2941.42	2887.62	2940.71	2978.91	2950.68	2887.06	2940.78	2888.37
	Std	0.02	17.75	10.54	0.10	10.74	25.17	17.57	0.07	15.18	0.79
	Min	2800.00	2900.00	2800.00	4746.81	2901.70	3815.79	2864.07	3595.14	2800.00	2800.00
	Mean	3620.51	6193.00	4548.70	4931.23	5181.01	7397.92	6699.50	3691.92	4917.60	3544.95
CEC-17/26	Median	3629.18	6421.87	4980.99	4936.54	5244.56	7545.58	6956.79	3688.00	5152.93	3589.33
	Max	3737.94	8311.80	6911.57	5071.31	5868.04	9707.03	8755.10	3796.25	7554.15	3760.51
	Std	129.07	1185.95	1350.94	66.71	443.32	1282.68	1294.23	48.29	1152.94	221.23
	Min	3179.42	3209.05	3204.78	3201.77	3161.41	3251.25	3221.17	3181.59	3208.95	3182.53
	Mean	3202.32	3271.09	3249.76	3209.04	3198.05	3367.50	3312.66	3205.39	3248.64	3205.07
CEC-17/27	Median	3203.31	3259.92	3244.79	3209.00	3200.01	3342.96	3299.11	3206.41	3243.92	3205.45
	Max	3221.94	3416.33	3330.63	3213.64	3200.01	3696.48	3522.39	3224.62	3342.70	3226.91
	Std	8.31	39.14	28.08	2.47	8.11	90.82	64.17	8.24	30.46	6.98
	Min	3100.00	3100.00	3100.00	3208.26	3289.66	3236.60	3111.29	3100.00	3100.00	3100.00
	Min	2800.00	3100.00	3100.00	3172.85	3100.18	3100.40	3101.60	3100.00	3100.00	3100.00
	Mean	3200.01	3310.54	3199.33	3236.69	3235.88	3403.55	3337.01	3214.03	3330.47	3100.00
CEC-17/28	Median	3100.00	3383.75	3100.00	3210.61	3222.55	3411.82	3383.87	3100.00	3402.69	3100.00
	Max	3411.82	3411.82	3444.13	3411.82	3300.00	3749.37	3501.44	3411.82	3411.82	3100.00
	Std	158.46	124.69	135.72	66.54	64.23	185.04	115.05	145.87	120.05	0.00
	Min	3129.61	3145.87	3148.21	3145.35	3137.22	3156.68	3160.31	3128.05	3145.56	3128.28
	Mean	3140.45	3233.85	3212.09	3161.97	3175.42	3308.50	3300.66	3138.34	3234.63	3132.96
CEC-17/29	Median	3137.70	3226.35	3199.64	3161.65	3168.03	3300.99	3294.46	3138.01	3224.14	3132.98
	Max	3174.45	3412.72	3331.33	3178.83	3300.25	3473.89	3479.31	3154.30	3359.96	3139.27
	Std	9.88	58.82	45.48	7.60	30.91	77.33	75.93	5.89	55.10	2.43
	Min	3408.33	4296.06	3762.14	4657.82	4676.02	6296.85	4838.12	3394.50	3416.36	3394.50
	Mean	3451.00	150294.12	167688.20	17934.90	24185.38	564983.34	527231.40	99555.88	395498.50	3409.27
CEC-17/30	Median	3454.01	16180.83	6759.34	13574.83	15177.31	220962.22	121932.13	3442.66	3729.03	3407.44
	Max	3503.40	1237436.14	1261343.43	67896.82	168236.16	2136927.89	4219938.92	820578.06	1827680.48	3600.49
	Std	23.51	345469.25	359033.99	12874.06	27828.94	630800.00	851685.44	265899.84	540967.78	31.24

TABLE 8. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 50 dimensions, 51 runs and 500000 number function evaluations.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
	Min	100.00	100.17	100.00	605.87	1670157.63	1684606.05	22820942.68	100.00	100.17	100.00
	Mean	100.00	4577.60	4447.91	8518.81	47380363.48	8174316.10	33116463.10	100.00	3289.60	100.00
CEC-17/01	Median	100.00	3236.96	2371.44	6211.29	44898327.03	6977503.00	31911908.27	100.00	1947.05	100.00
	Max	100.00	18512.74	24284.22	49166.80	128007344.64	25573323.10	48631908.15	100.00	18347.79	100.00
	Std	0.00	4578.47	6185.88	8557.35	28212702.75	5530006.96	5719541.68	0.00	3661.13	0.00
	Min	300.00	306.20	1228.41	137505.56	1431.51	28954.53	1311.27	300.00	300.00	300.00
	Mean	300.11	998.55	5926.34	195430.08	14082.67	78789.73	2653.51	300.00	300.15	300.00
CEC-17/03	Median	300.06	684.57	5042.26	197005.47	12760.09	66596.15	2586.33	300.00	300.05	300.00
	Max	301.01	4739.99	13706.63	230895.63	27996.63	202623.84	5566.20	300.00	302.35	300.00
	Std	0.16	998.83	3439.55	18389.98	6070.63	38408.67	931.04	0.00	0.36	0.00
	Min	400.00	403.99	400.15	504.27	469.50	566.49	453.15	400.00	400.16	400.00
	Mean	432.91	523.82	482.14	538.89	595.10	685.88	622.07	455.45	475.90	434.26
CEC-17/04	Median	428.51	541.42	497.41	530.26	598.10	684.39	623.37	428.51	475.35	428.51
	Max	518.00	618.09	553.03	617.55	693.39	958.28	791.56	542.31	563.58	542.31
	Std	38.11	50.54	41.67	26.78	52.87	66.72	59.94	52.55	47.77	43.49
	Min	524.91	720.88	638.30	782.97	619.14	806.99	788.76	520.15	705.96	524.87
	Mean	533.68	830.73	721.98	816.85	848.03	920.24	861.84	532.98	793.92	534.24
CEC-17/05	Median	533.89	837.46	716.90	818.73	862.98	903.70	867.18	533.17	797.49	533.83
	Max	545.81	920.86	825.35	845.72	922.78	1160.48	920.32	548.38	891.01	548.75
	Std	4.83	41.21	41.75	15.95	70.25	75.42	30.98	4.91	39.88	4.70
	Min	600.00	628.26	612.56	600.00	600.42	657.25	648.98	600.00	618.75	600.00
	Mean	600.00	636.55	635.33	600.00	602.59	675.59	667.58	600.00	636.30	600.00
CEC-17/06	Median	600.00	635.10	636.33	600.00	602.01	672.95	667.40	600.00	636.14	600.00
	Max	600.00	653.38	649.77	600.00	609.63	699.53	678.32	600.00	662.69	600.00
	Std	0.00	6.11	8.05	0.00	1.85	10.19	5.48	0.00	9.58	0.00
	Min	765.61	1241.20	856.68	1010.87	875.15	1209.27	1525.98	771.21	1020.05	772.87
	Mean	785.54	1445.83	964.49	1073.10	946.70	1689.24	1714.36	782.10	1227.97	781.21
CEC-17/07	Median	784.31	1445.43	954.03	1074.41	937.93	1673.31	1718.57	781.79	1218.88	781.85
	Max	806.60	1638.29	1087.63	1096.50	1142.95	1998.46	1902.58	791.55	1463.82	792.71
	Std	9.74	106.54	47.99	13.89	49.87	133.97	96.50	4.46	111.88	4.02
	Min	822.95	1034.81	963.17	1090.66	909.96	1069.19	1067.37	824.24	968.15	825.87
	Mean	834.82	1139.65	1045.09	1116.78	1136.92	1207.07	1155.61	831.90	1097.32	834.57
CEC-17/08	Median	834.84	1140.27	1036.80	1116.47	1148.04	1205.91	1162.08	831.30	1092.52	833.83
	Max	842.93	1248.21	1151.22	1141.02	1223.58	1360.98	1230.03	845.41	1204.35	848.75
	Std	4.81	43.45	44.20	11.75	78.35	70.14	37.00	4.30	55.07	4.62
	Min	900.00	10019.08	2530.20	900.00	1045.18	11991.17	11841.15	900.00	2861.72	900.00
	Mean	900.00	12890.15	7678.15	900.48	3808.95	20535.21	16271.33	901.10	7060.11	900.64
CEC-17/09	Median	900.00	12782.68	7372.11	900.03	3013.66	19032.21	15672.13	900.81	6799.83	900.54
	Max	900.00	16395.42	14306.88	902.98	15843.34	45133.38	21972.05	906.90	12180.18	902.73
	Std	0.00	1472.61	2319.52	0.82	2634.10	6412.27	2072.99	1.29	2249.90	0.64
	Min	3311.31	6042.83	5546.07	11154.90	9856.37	7707.26	6220.21	3652.64	4825.42	3136.77
	Mean	4206.80	8087.00	7460.04	12207.68	12087.03	9816.14	8451.49	4353.74	7772.80	4069.16
CEC-17/10	Median	4186.49	8070.98	7405.04	12224.63	12122.56	10113.93	8472.94	4346.98	7710.24	4100.14
	Max	4910.69	10039.90	10672.11	12727.89	14207.41	12600.48	11624.49	4918.79	10282.92	4622.84
	Std	354.34	835.46	1039.71	312.87	753.10	1209.41	1105.83	283.20	961.84	387.44
	Min	1139.98	1215.29	1195.81	1264.32	1228.31	1357.63	1286.76	1166.02	1263.53	1146.76
	Mean	1162.84	1326.52	1271.23	1286.75	1324.45	1567.94	1401.06	1233.41	1371.14	1185.64
CEC-17/11	Median	1162.29	1324.32	1257.56	1287.50	1327.23	1550.88	1388.34	1226.71	1364.97	1182.88
	Max	1194.45	1502.39	1350.09	1309.98	1541.03	1837.24	1645.54	1317.20	1499.33	1236.66
	Std	13.22	60.99	39.84	11.89	49.60	111.42	77.61	37.52	59.55	21.55
	Min	3390.55	541664.81	22885.18	22183437.30	3945280.51	36435630.20	15666921.05	3198.00	41347.21	2155.25
	Mean	7593.06	4334805.02	167111.75	48841523.31	55849602.53	211041964.56	60461136.84	7080.84	133934.55	4241.58
CEC-17/12	Median	7043.83	3478776.80	167428.00	45793562.32	28795701.87	192956616.65	58738578.48	6750.50	126105.55	3533.97
	Max	17828.05	11233159.09	330719.18	90649117.45	248514634.63	544732228.03	139533682.69	16107.62	381628.58	16989.93
	Std	3418.40	2483709.01	75454.20	16916756.74	69329851.47	119281845.43	28163750.13	3047.10	74127.81	2356.40
	Min	1447.52	17950.58	1538.71	44047.62	60569.90	43545.93	353191.48	1336.81	1610.43	1357.71
	Mean	1538.09	49641.60	4604.70	195786.26	1470233.59	212953.29	1202608.24	1584.81	7795.81	1551.59
CEC-17/13	Median	1542.11	46756.82	3141.51	153888.96	260512.52	174262.04	992346.34	1519.43	4750.71	1498.09
	Max	1608.41	111187.29	21185.68	587126.47	26607924.61	853813.02	3601114.97	2730.35	31392.02	2425.29
	Std	40.00	21547.51	4332.33	125660.19	5003204.22	161046.90	642965.60	227.99	6572.06	189.65
	Min	1511.16	7829.86	2591.68	129784.28	7530.13	30026.58	30847.87	1510.58	1708.43	1467.78
	Mean	1535.46	73692.52	22866.05	501344.80	129492.00	562872.06	225388.93	1626.82	3160.28	1571.88
CEC-17/14	Median	1535.56	57312.01	17805.62	489629.74	108810.75	432352.25	183623.24	1603.09	2825.03	1562.28
	Max	1561.15	261793.38	74444.19	1085071.94	408864.93	2195056.48	759905.37	1844.88	8012.41	1708.56
	Std	8.97	61771.02	17749.84	203768.77	97343.30	456415.83	149083.37	76.73	1303.78	50.81
	Min	1600.11	6723.19	1841.29	12930.67	20479.62	19431.49	29535.92	1596.54	1650.06	1588.60
	Mean	1628.25	22042.42	7025.29	31395.13	330386.02	80865.55	178630.38	1823.88	9768.76	1732.96
CEC-17/15	Median	1625.78	20638.55	5305.91	28421.23	72135.47	61928.35	172952.17	1801.54	7890.76	1727.40
	Max	1706.24	39457.70	18283.74	103037.27	6284859.36	563154.26	482990.18	2151.67	32625.69	1923.92
	Std	17.22	8315.98	5075.64	17500.64	1139745.62	86140.80	93804.62	135.36	7511.56	83.44
	Min	1826.96	2743.41	2156.60	2761.71	2282.57	3211.69	2990.04	1891.16	2479.80	1739.65
	Mean	2083.93	3741.27	3236.92	3296.68	3417.32	4666.16	4049.99	2310.70	3585.17	2089.60
CEC-17/16	Median	2039.43	3605.40	3206.28	3293.88	3442.22	4582.45	4063.00	2307.25	3572.64	2075.95
	Max	2566.83	4626.45	4331.39	3695.49	4325.93	6396.89	5108.63	2601.15	4859.45	2406.47

TABLE 8. *(Continued.)* **Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 50 dimensions, 51 runs and 500000 number function evaluations.**

	Std	175.98	481.71	471.81	189.67	452.79	681.30	553.28	187.47	488.68	158.76
	Min	1814.74	2994.78	2327.63	2305.85	2120.32	3043.46	2962.24	1915.93	2478.23	1912.16
	Mean	2220.06	3548.55	3063.94	2615.98	2969.23	3936.57	3636.28	2220.16	3142.14	2089.82
CEC-17/17	Median	2220.07	3608.18	3061.83	2629.85	3060.95	3939.66	3606.85	2224.63	3098.16	2093.00
	Max	2558.15	4228.98	3848.87	2909.34	3714.70	4991.35	4372.53	2502.21	3927.08	2276.62
	Std	162.96	300.37	326.63	141.65	400.26	441.69	360.51	130.62	343.61	88.32
	Min	1872.39	48011.63	28154.04	840139.45	237519.92	542752.40	194292.53	1838.56	4422.64	1843.20
	Mean	1902.85	354316.49	73927.78	2501584.71	2261881.46	6809268.87	2093474.45	2018.27	23812.73	1985.11
CEC-17/18	Median	1895.78	287513.31	66770.98	2421316.03	1705677.57	4958884.12	1708668.08	2003.72	17637.23	1978.10
	Max	2014.82	1106454.42	161628.10	5548910.83	11702982.76	23684567.30	7103693.87	2298.13	66047.00	2208.04
	Std	24.35	214449.43	30439.35	995450.01	1901410.71	5925562.18	1547116.39	127.19	16295.27	91.29
	Min	1945.54	3217.72	2029.38	12456.38	29290.32	25079.14	31026.66	1976.69	2303.36	1957.88
	Mean	1980.37	19193.86	14449.09	23141.82	642435.12	2274195.64	360263.60	2063.61	15394.75	2021.88
CEC-17/19	Median	1979.33	14505.56	9862.24	21944.33	343272.63	1977944.78	266742.62	2059.02	16293.59	2021.66
	Max	2026.94	45233.59	39089.58	42341.48	4979804.14	6803584.44	1689025.71	2195.55	38950.41	2093.70
	Std	11.78	12979.34	10236.87	6795.81	993844.02	1799909.54	320833.28	53.89	9665.91	35.16
	Min	2102.25	2631.72	2276.06	2396.19	2282.70	2924.69	2745.34	2094.55	2296.80	2068.02
	Mean	2345.34	3335.42	3047.48	2747.58	3010.75	3646.83	3382.18	2301.69	3156.51	2225.53
CEC-17/20	Median	2353.80	3373.39	3076.87	2752.59	3016.59	3665.40	3394.19	2317.75	3117.15	2235.82
	Max	2779.93	3954.94	3637.10	3030.08	3563.85	4253.41	3963.43	2573.23	3903.22	2422.09
	Std	164.44	323.72	311.47	159.36	285.64	319.59	280.25	129.31	324.58	99.48
	Min	2323.33	2579.89	2456.18	2579.22	2410.93	2708.49	2611.37	2322.16	2431.93	2327.30
	Mean	2333.95	2705.16	2535.43	2625.01	2618.72	2878.00	2784.42	2334.43	2568.09	2335.09
CEC-17/21	Median	2334.66	2700.11	2538.41	2628.95	2643.23	2874.29	2788.48	2335.33	2562.09	2335.39
	Max	2347.97	2810.69	2637.55	2654.08	2709.89	3114.15	2992.26	2344.39	2706.61	2345.97
	Std	5.67	61.10	44.90	17.26	80.99	96.67	78.67	4.54	57.59	3.95
	Min	2300.00	2302.95	6898.72	12342.48	11095.89	9227.51	8458.41	2300.00	2300.00	2300.00
	Mean	5155.63	9840.48	9365.78	13888.82	13516.13	11436.15	10582.39	5344.89	9337.36	3672.80
CEC-17/22	Median	6451.96	9989.38	9431.80	13975.88	13605.58	11395.87	10626.60	6009.33	9750.16	2300.00
	Max	7801.20	12161.63	11245.08	14616.39	14968.16	14998.30	12553.77	6789.42	11640.86	6610.96
	Std	2096.46	1411.89	1067.85	440.87	725.31	1182.00	957.25	1464.83	1690.30	1811.90
	Min	2748.51	2976.87	2944.22	3012.29	2811.09	3308.55	3288.83	2739.99	2887.14	2736.18
	Mean	2765.89	3285.08	3127.89	3048.33	3100.27	3618.39	3564.60	2756.57	3079.81	2755.07
CEC-17/23	Median	2764.30	3266.63	3115.82	3048.57	3110.65	3614.99	3542.66	2756.57	3091.27	2755.04
	Max	2793.01	3526.33	3388.63	3069.78	3189.85	3933.48	3916.35	2773.69	3269.47	2772.21
	Std	10.32	111.48	99.51	11.84	65.62	155.09	143.91	8.91	83.30	7.33
	Min	2919.20	3248.89	3052.41	3219.89	3049.09	3303.61	3814.43	2915.14	3047.76	2911.17
	Mean	2931.28	3548.02	3254.22	3260.76	3258.40	3657.97	4135.41	2930.63	3199.21	2928.06
CEC-17/24	Median	2931.88	3560.76	3241.33	3263.11	3257.62	3662.49	4161.98	2929.73	3181.23	2929.25
	Max	2947.74	4019.27	3439.42	3285.52	3354.84	3938.13	4507.13	2953.11	3492.76	2938.01
	Std	6.64	141.97	92.81	16.06	50.17	136.43	178.96	7.91	95.53	5.73
	Min	2960.23	2961.20	2956.81	2980.33	2972.44	3082.32	3055.85	2979.76	2961.73	2928.74
	Mean	3017.85	3066.44	3033.35	2982.35	3090.91	3139.43	3119.34	3003.70	3062.66	3000.42
CEC-17/25	Median	3027.21	3073.20	3031.90	2980.35	3086.77	3134.80	3116.36	2980.40	3068.95	2991.97
	Max	3068.29	3112.73	3089.69	3021.85	3197.86	3205.37	3195.94	3068.65	3115.97	3068.87
	Std	35.84	32.20	41.45	7.79	49.19	31.29	34.90	33.79	34.52	28.16
	Min	3897.66	2900.00	2900.00	6512.07	5062.07	4745.58	2941.17	3785.35	2900.00	3855.22
	Mean	4056.39	9177.14	6762.93	6850.55	7305.15	12708.12	9385.51	4004.13	6649.52	4000.98
CEC-17/26	Median	4041.11	9368.39	7616.96	6856.33	7480.17	12848.07	10417.31	3995.68	7404.67	3994.59
	Max	4311.35	11914.36	9896.62	7074.37	8433.33	18304.43	12448.88	4221.68	11620.56	4206.09
	Std	84.91	1616.64	2441.30	118.53	739.53	2020.60	2718.65	89.51	3405.48	80.87
	Min	3203.28	3429.40	3286.84	3231.31	3200.01	3511.87	3458.62	3211.41	3276.20	3217.63
	Mean	3227.57	3680.69	3511.52	3248.21	3200.01	4217.39	3940.30	3246.49	3613.03	3262.68
CEC-17/27	Median	3225.53	3646.34	3491.55	3246.92	3200.01	4121.60	3892.13	3238.28	3598.62	3255.13
	Max	3296.78	4180.75	3878.28	3290.33	3200.01	5268.64	4906.91	3299.94	4084.62	3386.20
	Std	15.21	175.00	126.57	11.95	0.00	395.00	263.44	24.46	160.98	30.88
	Min	3253.35	3258.85	3253.35	3258.85	3278.78	3325.47	3263.56	3258.85	3258.52	3251.96
	Mean	3283.08	3302.02	3289.99	3259.37	3299.60	3420.71	3365.27	3286.41	3288.23	3284.03
CEC-17/28	Median	3293.88	3307.71	3296.67	3258.85	3300.01	3417.56	3364.36	3307.69	3296.67	3295.20
	Max	3307.69	3323.69	3397.50	3270.47	3300.01	3519.42	3481.01	3307.69	3314.63	3333.39
	Std	21.45	15.66	30.19	2.26	2.97	54.03	38.77	24.29	19.64	24.13
	Min	3255.86	3914.07	3990.26	3688.92	3754.56	5654.72	4335.02	3264.19	3945.35	3222.71
	Mean	3345.92	4804.78	4443.09	4081.53	4450.64	7135.84	5296.51	3351.09	4664.58	3277.07
CEC-17/29	Median	3323.04	4790.17	4413.46	4095.93	4432.81	7014.21	5193.53	3321.21	4616.44	3264.44
	Max	3562.30	5568.79	5310.72	4374.06	5307.27	9444.32	6839.25	3650.91	5687.80	3435.50
	Std	72.91	385.55	296.77	145.98	326.37	943.42	525.59	85.70	437.31	46.92
	Min	582419.09	662354.91	696401.55	850585.87	3377929.99	42870693.14	5586342.07	582411.37	582446.49	582412.37
	Mean	602162.26	1407176.80	879583.04	1162219.88	9028706.16	94011085.54	12390619.55	676334.24	659661.01	670999.37
CEC-17/30	Median	582440.16	1376082.93	879433.83	1142131.95	8106247.96	90051617.53	12662209.20	654647.65	611533.00	664055.13
	Max	672679.83	2605124.91	1125762.23	1478477.88	22851833.55	178721186.09	19096625.20	955209.29	1162667.73	928188.22
	Std	26842.97	340680.34	104641.29	162242.13	3711188.45	30853888.68	3053384.13	90357.41	114279.47	73507.51

TABLE 9. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 100 dimensions, 51 runs and 1000000 number function evaluations.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
	Min	100.00	106.36	113.94	117.56	47597183.46	20394203.12	159317986.86	100.00	104.74	100.00
	Mean	100.00	7292.34	9396.68	6915.99	389024453.65	34942862.54	210098155.31	100.00	7066.34	100.00
CEC-17/01	Median	100.00	4314.88	6346.00	3694.66	312892414.16	32873854.80	208910955.17	100.00	3477.29	100.00
	Max	100.00	40027.33	53307.10	36982.74	1449521296.76	82459700.62	255597980.32	100.00	50189.20	100.00
	Std	0.00	8744.79	10688.93	8315.25	278111759.81	11030280.94	21593995.71	0.00	9615.04	0.00
	Min	351.58	8918.53	61770.16	503922.59	61255.51	198647.67	19415.13	300.00	301.65	300.00
	Mean	2217.32	20295.04	111828.82	597385.37	106248.26	608994.27	34573.96	300.00	311.64	300.00
CEC-17/03	Median	1549.00	19492.19	109143.15	597826.75	104786.41	583665.90	34386.47	300.00	310.19	300.00
	Max	10824.40	35773.26	177245.41	676095.86	185637.71	1075156.07	54832.82	300.00	342.99	300.00
	Std	1957.52	6439.85	25745.75	38176.92	23947.20	196928.93	6962.76	0.00	8.84	0.00
	Min	400.44	558.71	462.66	597.36	690.79	813.74	656.94	400.00	499.80	400.00
	Mean	540.70	649.66	577.58	626.55	759.28	1015.23	830.76	530.23	632.71	461.11
CEC-17/04	Median	550.21	643.02	587.52	628.40	759.57	1012.52	834.80	541.63	639.06	403.99
	Max	648.00	778.52	683.38	635.71	857.97	1247.64	947.87	625.55	712.64	678.37
	Std	54.08	47.40	52.05	5.41	40.57	94.58	65.52	57.87	42.38	87.32
	Min	590.65	1154.68	956.68	1345.19	830.70	1230.15	1324.37	566.56	1074.09	568.65
	Mean	616.90	1304.37	1120.69	1401.16	1173.62	1427.52	1416.68	600.34	1269.02	590.03
CEC-17/05	Median	616.49	1315.26	1123.83	1405.98	968.56	1413.57	1424.68	600.99	1278.05	590.54
	Max	657.01	1450.17	1311.88	1434.97	1581.45	1714.36	1504.74	627.14	1450.17	611.27
	Std	10.69	62.97	81.53	20.97	305.78	99.59	45.21	13.62	69.89	10.62
	Min	600.00	632.92	631.48	600.00	605.98	662.48	670.81	600.01	631.45	600.00
	Mean	600.00	642.44	645.94	600.00	613.41	680.16	676.44	600.08	647.02	600.00
CEC-17/06	Median	600.00	643.00	646.42	600.00	612.74	677.06	676.39	600.06	648.52	600.00
	Max	600.00	650.78	667.87	600.00	626.45	708.82	681.34	600.21	661.05	600.02
	Std	0.00	4.20	6.70	0.00	4.04	10.35	2.61	0.05	5.83	0.00
	Min	879.78	2376.48	1189.64	1642.50	1321.43	2972.41	2927.66	875.21	1996.15	864.61
	Mean	905.52	2774.21	1489.26	1724.85	1481.62	3243.23	3421.76	910.92	2436.84	881.74
CEC-17/07	Median	901.45	2782.91	1453.34	1724.26	1466.94	3245.97	3434.59	912.22	2346.11	880.44
	Max	939.09	3085.18	1966.92	1769.58	1697.51	3574.33	3644.18	940.91	3026.21	901.88
	Std	13.93	145.88	159.44	25.40	86.87	152.77	119.74	14.87	262.87	8.61
	Min	889.52	1489.50	1295.49	1608.26	1105.93	1688.20	1724.51	869.15	1383.04	870.64
	Mean	912.85	1696.14	1452.59	1698.16	1417.13	1878.96	1855.67	898.88	1627.56	887.28
CEC-17/08	Median	913.06	1692.47	1453.68	1704.44	1225.52	1858.74	1860.78	897.71	1625.81	887.56
	Max	935.86	1880.51	1612.87	1733.68	1916.19	2206.28	1972.05	926.93	1874.54	907.46
	Std	12.13	87.65	76.97	25.45	287.00	110.68	61.03	12.38	96.91	8.18
	Min	900.00	19188.23	13422.43	1342.00	10463.32	24489.16	24051.09	908.95	16066.52	900.91
	Mean	900.00	22622.90	19061.46	1932.65	22940.81	37080.47	29653.90	930.32	20616.59	908.37
CEC-17/09	Median	900.00	22484.84	19039.48	1992.69	17993.22	32944.74	29295.43	925.30	20480.79	906.81
	Max	900.09	27160.69	32497.28	2607.85	42231.28	74169.65	43704.81	1037.96	25898.33	939.60
	Std	0.01	1272.09	3290.08	289.40	9776.38	11443.15	3693.05	20.34	2142.49	7.24
	Min	8869.46	13759.83	11548.80	27479.20	25557.97	15580.23	15382.11	8821.63	12489.90	8966.89
	Mean	10528.48	16370.65	14782.49	29299.73	28340.45	19940.44	18241.74	10345.32	15433.88	9817.42
CEC-17/10	Median	10599.67	16343.15	14847.32	29193.75	28490.16	19838.72	18114.85	10436.24	15508.93	9835.91
	Max	11658.92	20275.97	17402.09	30231.43	30593.49	24937.57	21005.64	11332.06	20267.56	10573.39
	Std	605.23	1554.47	1285.06	535.42	967.79	2168.75	1207.48	537.98	1436.49	428.80
	Min	1716.57	1848.73	1627.09	6354.41	2393.80	4033.27	2478.72	1674.85	1959.72	1500.73
	Mean	1978.66	2277.43	2011.98	8516.60	3116.09	7357.08	2939.04	2208.34	2414.40	1823.79
CEC-17/11	Median	1940.00	2280.45	2031.04	8572.75	3006.41	6847.67	2940.03	2236.43	2353.27	1806.17
	Max	2675.24	2852.79	2340.73	10928.74	5965.62	13729.68	3806.82	2713.59	3097.51	2336.70
	Std	181.89	187.43	175.96	1020.04	579.45	1870.76	214.05	208.45	268.16	178.37
	Min	35794.16	2368449.11	265973.64	289971578.25	56695946.84	149956101.44	112685152.18	6147.37	194903.47	3890.60
	Mean	106317.81	11722791.80	867352.13	370007827.41	187415887.72	632541372.99	290321315.26	20769.66	740216.59	17708.18
CEC-17/12	Median	87155.58	10945699.15	753391.33	367093730.84	191858837.94	644395114.00	283908994.44	13614.30	737664.49	5760.74
	Max	321224.95	24198208.40	2322942.31	463054570.00	313940010.21	1297508667.15	429434170.74	142477.88	1285629.80	96045.21
	Std	62628.30	5633631.18	440810.05	39464063.10	71034897.79	246016502.73	79337205.20	19897.79	236865.55	21045.35
	Min	1571.31	16839.43	2019.83	2490.01	95682.29	42607.70	1393374.80	1399.01	2996.14	1483.15
	Mean	3584.57	41399.42	7633.80	12859.15	1915545.17	91569.22	3295965.04	2603.42	10365.71	1842.54
CEC-17/13	Median	2724.97	42419.99	6722.08	8068.77	1239717.51	83491.39	3361944.66	1765.89	8263.50	1702.86
	Max	9960.92	65687.61	23083.99	79267.35	10585325.12	207186.66	5176087.46	10061.48	23496.53	6071.14
	Std	2066.60	10696.09	5066.00	12796.95	2109607.46	37464.80	920283.72	1805.33	5435.91	640.58
	Min	1722.88	70094.27	30038.80	4437636.27	467851.83	550918.16	295485.38	1628.25	7359.90	1609.29
	Mean	1807.43	218220.30	159297.81	9187110.98	1402592.17	1656839.85	918039.58	2011.82	25606.63	1908.01
CEC-17/14	Median	1799.91	210613.90	130539.08	9316782.63	1269257.83	1483296.94	842315.29	1993.56	24862.04	1891.61
	Max	1912.08	463861.89	387781.94	16067469.75	3147744.34	5223854.35	2776266.87	2410.78	57903.67	2151.73
	Std	50.36	83524.19	85194.60	2446545.59	644714.34	879379.60	412681.78	188.25	11804.89	106.93
	Min	1689.30	12112.15	1891.81	6968.13	37615.84	21935.67	155297.84	1697.09	2057.17	1660.59
	Mean	2302.73	24360.70	4526.43	37311.25	342701.37	139637.00	1045892.81	1851.45	5765.32	1810.78
CEC-17/15	Median	1979.65	22521.24	3498.30	32551.03	170216.27	60483.32	819538.49	1856.24	4587.87	1812.82
	Max	4618.53	54190.55	16000.41	113427.34	1554972.74	2713486.55	8152670.75	2076.46	20219.62	1962.34
	Std	752.75	8672.17	3074.51	21323.51	377160.39	381475.15	1151919.14	93.86	4420.88	78.76
	Min	2867.73	5139.83	3635.13	7813.65	4365.70	6306.64	5159.01	2960.91	4435.34	2832.72
	Mean	3860.99	6366.10	5309.06	8928.68	7729.06	9425.19	7073.44	3882.19	5788.48	3420.55
CEC-17/16	Median	3892.85	6289.08	5404.45	8976.22	8028.66	9226.19	6966.66	3952.76	5837.70	3352.84
	Max	4542.64	8060.63	6978.37	9657.37	9268.35	13151.66	8906.61	4324.21	8027.44	4191.14
	Std	329.87	697.75	675.23	374.51	943.32	1474.99	785.09	331.77	786.76	299.53

TABLE 9. *(Continued.)* **Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 100 dimensions, 51 runs and 1000000 number function evaluations.**

	Min	2628.64	4515.06	3247.45	4730.31	3933.47	5873.63	4888.74	3065.85	4726.02	2834.67
	Mean	3498.27	5916.83	4733.13	5318.08	6177.91	6990.83	6114.33	3573.35	5446.05	3175.20
CEC-17/17	Median	3524.88	5856.80	4716.28	5347.45	6310.98	6964.48	6291.02	3590.64	5330.69	3140.11
	Max	4199.01	7209.78	6256.67	5718.73	7306.02	8629.20	7080.62	4038.46	7107.64	3574.36
	Std	316.36	593.55	631.29	271.42	782.59	702.85	609.64	219.99	489.66	183.62
	Min	2096.67	118075.90	94759.25	10002816.41	239153.08	965001.72	817917.29	2121.40	25727.05	1954.79
	Mean	2261.43	333355.02	265321.50	19578179.65	1996903.93	2096164.85	2016116.41	3694.52	96433.83	2078.29
CEC-17/18	Median	2227.25	320188.66	243642.40	19755450.13	1841653.27	1799820.74	2031950.54	3209.81	95332.61	2057.81
	Max	2586.05	633659.66	698160.60	28866206.78	4433191.06	5041024.10	3261813.33	8092.27	185330.19	2322.14
	Std	118.71	113252.74	115568.25	4963306.71	1063032.69	889773.69	602230.93	1505.49	37561.54	80.37
	Min	2062.59	3645.17	2119.22	5895.21	601831.57	999357.93	884570.46	2056.57	2194.12	2025.04
	Mean	2359.15	11795.75	4578.58	37475.39	2211734.62	14068547.09	3405104.51	2190.07	5837.60	2090.38
CEC-17/19	Median	2173.05	10516.47	3540.85	31933.62	2005437.21	14206280.01	3137545.84	2152.21	3985.29	2088.73
	Max	7813.51	24138.80	13747.13	116706.42	4918732.46	29381540.11	7211833.46	3376.88	19070.20	2235.52
	Std	816.88	5605.63	2817.99	26291.64	1052654.66	7005404.79	1438985.84	204.52	4486.09	41.66
	Min	3092.96	4354.79	3280.58	4929.70	4700.98	4693.24	4328.29	3230.59	3869.38	2923.86
	Mean	3773.48	5660.28	5028.53	5470.97	5795.40	6148.91	5745.00	3696.51	5078.62	3453.90
CEC-17/20	Median	3783.03	5787.09	5089.81	5497.14	5772.72	6019.97	5789.40	3658.06	5089.31	3470.12
	Max	4372.28	6649.63	6025.60	5906.01	6482.45	7460.43	6751.95	4076.48	6102.80	3997.64
	Std	269.00	550.89	553.37	219.39	400.51	600.53	492.82	190.42	443.77	232.17
	Min	2422.00	3134.36	2928.72	3160.61	2589.21	3361.45	3483.50	2402.65	2849.28	2399.60
	Mean	2448.67	3447.98	3119.81	3224.29	2936.03	3886.00	3832.42	2429.30	3082.30	2418.10
CEC-17/21	Median	2447.86	3465.01	3102.94	3222.04	2743.00	3856.07	3821.95	2428.17	3081.61	2417.00
	Max	2469.49	3925.50	3446.58	3264.50	3395.99	4385.72	4278.84	2459.77	3352.96	2440.19
	Std	11.58	155.37	109.21	23.44	308.29	229.76	184.59	13.00	131.12	9.80
	Min	10483.88	15322.46	14134.68	30040.40	28082.65	19041.26	19548.56	11390.79	15023.76	2300.00
	Mean	12638.03	19158.68	18258.87	31420.19	30300.50	24108.83	22444.44	12791.02	18645.42	12082.06
CEC-17/22	Median	12585.68	19179.59	18326.86	31510.94	30375.36	24242.82	22584.01	12975.87	18933.05	12343.47
	Max	14979.43	22760.70	21370.02	32329.56	32837.36	28973.57	27314.35	13655.74	22061.04	13375.30
	Std	875.44	1555.51	1728.11	461.90	1001.02	2245.82	1586.79	511.17	1582.70	1513.32
	Min	2919.14	3385.37	3541.98	3496.59	3660.48	4227.12	4170.77	2888.00	3383.84	2877.15
	Mean	2952.55	3761.69	3946.65	3547.74	3782.73	4800.32	4508.73	2911.06	3642.01	2906.42
CEC-17/23	Median	2951.29	3735.28	3935.77	3550.78	3786.83	4757.49	4483.89	2911.49	3624.90	2905.21
	Max	3001.05	4213.33	4463.11	3588.41	3927.03	5309.38	4922.31	2949.85	3994.87	2940.03
	Std	17.63	173.54	175.56	21.55	54.85	236.45	186.42	14.84	124.47	12.36
	Min	3359.57	4312.28	3912.05	4050.32	3494.07	5260.83	4866.99	3348.90	3950.78	3332.38
	Mean	3407.70	4716.47	4286.32	4098.86	3864.44	5973.98	5629.99	3399.84	4334.06	3373.11
CEC-17/24	Median	3404.19	4689.91	4261.24	4098.19	3679.54	5942.96	5653.54	3392.46	4317.62	3376.03
	Max	3456.63	5215.92	5012.06	4133.57	4309.87	6797.58	6633.74	3482.26	4952.77	3406.87
	Std	23.38	216.67	250.61	16.79	309.68	386.78	373.24	27.12	244.29	16.67
	Min	3123.07	3145.76	3120.85	3240.10	3436.59	3484.79	3344.22	3133.82	3198.20	3102.08
	Mean	3227.37	3291.65	3252.28	3328.44	3536.15	3616.48	3529.40	3241.06	3294.37	3224.33
CEC-17/25	Median	3218.58	3295.26	3258.19	3320.60	3528.77	3606.51	3525.23	3243.04	3287.75	3216.35
	Max	3415.90	3379.79	3345.70	3460.73	3820.87	3815.45	3712.70	3350.92	3476.38	3334.10
	Std	63.48	60.73	67.54	48.83	84.32	79.61	70.28	49.07	58.88	53.39
	Min	6253.63	15671.07	2900.00	14013.49	8777.62	25783.35	4660.26	6332.19	2900.00	6250.89
	Mean	6738.14	21071.83	14736.49	14553.56	11844.71	31617.05	23849.18	6792.19	18327.89	6564.92
CEC-17/26	Median	6729.62	20683.79	15842.00	14558.10	10076.84	31456.75	24353.22	6781.11	17959.83	6605.46
	Max	7262.22	26174.51	21962.55	15059.20	16794.81	39277.38	27392.53	7302.08	24343.20	6969.08
	Std	183.51	2353.12	5296.76	221.32	2950.54	3300.90	3089.12	203.27	3820.78	149.17
	Min	3282.95	3547.54	3484.85	3470.13	3200.02	3788.71	3662.10	3301.55	3445.33	3277.20
	Mean	3337.96	3905.64	3790.36	3543.66	3200.02	4974.13	4110.94	3373.20	3713.94	3353.90
CEC-17/27	Median	3340.72	3883.65	3773.45	3543.87	3200.02	4991.77	4086.12	3371.41	3702.54	3350.82
	Max	3383.68	4259.91	4090.91	3602.52	3200.02	6648.73	4819.35	3434.38	4172.46	3425.64
	Std	21.48	182.96	139.59	27.09	0.00	586.10	255.86	33.32	156.52	26.75
	Min	3278.22	3307.31	3288.49	3327.32	3300.02	3597.66	3428.13	3209.63	3301.39	3100.00
	Mean	3339.46	3366.18	3346.01	3347.40	3300.02	3718.60	3570.03	3324.66	3369.76	3283.82
CEC-17/28	Median	3344.23	3359.84	3340.80	3346.19	3300.02	3714.44	3568.69	3320.07	3363.29	3318.64
	Max	3416.79	3465.33	3422.81	3423.84	3300.02	3868.30	3690.78	3434.40	3436.87	3418.24
	Std	28.43	33.13	30.12	14.12	0.00	68.85	54.86	48.08	32.60	89.81
	Min	4450.14	6288.14	5572.02	6648.97	7001.12	9447.24	7363.53	4481.43	5471.30	3891.10
	Mean	4961.87	7887.23	6853.07	7269.34	8115.64	13591.33	8929.57	5025.78	7159.33	4447.44
CEC-17/29	Median	4910.46	7917.74	6745.60	7324.38	7955.99	13280.36	8823.02	5017.78	7143.42	4463.92
	Max	5961.27	9304.54	8091.26	7673.19	9897.92	18105.82	10486.00	5701.76	8218.33	5233.81
	Std	320.63	673.67	505.57	248.67	702.76	1731.80	654.17	304.74	509.78	246.88
	Min	5408.88	62774.26	5662.20	16891.33	40823905.93	70568462.27	10438810.60	5332.63	6649.44	5220.44
	Mean	6896.61	136331.10	9920.47	26961.79	64625903.41	220411229.62	19696989.78	5793.26	11182.19	5583.76
CEC-17/30	Median	6733.85	125436.83	8395.69	25904.40	64403338.35	191993438.42	19180090.35	5707.24	9618.69	5524.20
	Max	9743.50	291003.37	20481.54	42839.89	103310275.73	614413569.24	31214878.66	8781.00	21715.16	6321.60
	Std	1007.42	53295.39	3704.77	6026.26	14556025.50	117525290.23	4647662.07	517.73	4049.77	246.34

to the Wilcoxon results in Table 20 to Table 23, HSOA was the winner under almost conditions compared to well-known and new algorithms. In comparison with EBOwithCMAR as the state-of-the-art algorithm, HSOA had 1, 3, and 5 wins in 5, 10,

TABLE 10. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 10 dimensions, 51 runs and 100000 number function evaluations.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-17/01	1	1	1	1	1	1	0	1	0
CEC-17/03	1	1	1	1	1	1	0	1	0
CEC-17/04	1	1	1	1	1	1	0	1	0
CEC-17/05	1	1	1	1	1	1	1	1	-1
CEC-17/06	1	1	0	1	1	1	0	1	0
CEC-17/07	1	1	1	1	1	1	1	1	-1
CEC-17/08	1	1	1	1	1	1	1	1	-1
CEC-17/09	1	0	0	1	1	1	0	1	0
CEC-17/10	1	1	1	1	1	1	0	1	0
CEC-17/11	1	1	1	1	1	1	-1	1	-1
CEC-17/12	1	1	1	1	1	1	-1	1	0
CEC-17/13	1	1	1	1	1	1	-1	1	0
CEC-17/14	1	1	1	1	1	1	-1	1	-1
CEC-17/15	1	1	1	1	1	1	-1	1	0
CEC-17/16	1	1	1	1	1	1	-1	1	-1
CEC-17/17	1	1	1	1	1	1	-1	1	-1
CEC-17/18	1	1	1	1	1	1	-1	1	-1
CEC-17/19	1	1	1	1	1	1	-1	1	-1
CEC-17/20	1	1	-1	1	1	1	-1	1	0
CEC-17/21	0	0	1	1	1	1	1	0	-1
CEC-17/22	1	1	1	1	1	1	0	1	0
CEC-17/23	1	1	1	1	1	1	1	1	-1
CEC-17/24	1	1	0	1	1	1	0	0	-1
CEC-17/25	1	1	1	1	1	1	0	1	0
CEC-17/26	1	1	1	1	1	1	0	1	-1
CEC-17/27	1	1	1	-1	1	1	0	1	1
CEC-17/28	1	-1	1	1	1	1	-1	1	-1
CEC-17/29	1	1	1	1	1	1	0	1	-1
CEC-17/30	1	1	1	1	1	1	-1	1	-1
+	28	26	25	28	29	29	5	27	1
=	1	2	3	0	0	0	12	2	12
-	0	1	1	1	0	0	12	0	16

TABLE 11. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 30 dimensions, 51 runs and 300000 number function evaluations.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-17/01	1	1	1	1	1	1	0	1	-1
CEC-17/03	1	0	1	1	1	1	-1	1	-1
CEC-17/04	1	1	1	1	1	1	1	1	1
CEC-17/05	1	1	1	1	1	1	1	1	0
CEC-17/06	1	1	1	1	1	1	1	1	0
CEC-17/07	1	1	1	1	1	1	0	1	-1
CEC-17/08	1	1	1	1	1	1	1	1	1
CEC-17/09	1	1	1	1	1	1	0	1	-1
CEC-17/10	1	1	1	1	1	1	0	1	0
CEC-17/11	1	1	1	1	1	1	0	1	-1
CEC-17/12	1	1	1	1	1	1	1	1	0
CEC-17/13	1	1	1	1	1	1	-1	1	-1
CEC-17/14	1	1	1	1	1	1	-1	1	-1
CEC-17/15	1	1	1	1	1	1	-1	1	-1
CEC-17/16	1	1	1	1	1	1	0	1	-1
CEC-17/17	1	1	1	1	1	1	0	1	-1
CEC-17/18	1	1	1	1	1	1	0	1	-1
CEC-17/19	1	1	1	1	1	1	-1	1	-1
CEC-17/20	1	1	1	1	1	1	0	1	-1
CEC-17/21	1	1	1	1	1	1	1	1	0
CEC-17/22	1	1	1	1	1	1	-1	1	-1
CEC-17/23	1	1	1	1	1	1	1	1	0
CEC-17/24	1	1	1	1	1	1	1	1	-1
CEC-17/25	1	1	1	1	1	1	1	1	1
CEC-17/26	1	1	1	1	1	1	1	1	0
CEC-17/27	1	1	1	-1	1	1	0	1	0
CEC-17/28	1	-1	1	1	1	1	-1	-1	-1
CEC-17/29	1	1	1	1	1	1	1	1	-1
CEC-17/30	1	1	1	1	1	1	1	1	1
+	29	27	29	28	29	29	12	28	4
=	0	1	0	0	0	0	10	0	8
-	0	1	0	1	0	0	7	1	17

TABLE 12. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 50 dimensions, 51 runs and 500000 number function evaluations.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-17/01	1	1	1	1	1	1	0	1	-1
CEC-17/03	1	1	1	1	1	1	-1	0	-1
CEC-17/04	1	1	1	1	1	1	0	1	0
CEC-17/05	1	1	1	1	1	1	0	1	0
CEC-17/06	1	1	-1	1	1	1	1	1	-1
CEC-17/07	1	1	1	1	1	1	0	1	0
CEC-17/08	1	1	1	1	1	1	-1	1	0
CEC-17/09	1	1	1	1	1	1	1	1	1
CEC-17/10	1	1	1	1	1	1	0	1	0
CEC-17/11	1	1	1	1	1	1	1	1	1
CEC-17/12	1	1	1	1	1	1	0	1	-1
CEC-17/13	1	1	1	1	1	1	0	1	0
CEC-17/14	1	1	1	1	1	1	1	1	1
CEC-17/15	1	1	1	1	1	1	1	1	1
CEC-17/16	1	1	1	1	1	1	1	1	0
CEC-17/17	1	1	1	1	1	1	0	1	-1
CEC-17/18	1	1	1	1	1	1	1	1	1
CEC-17/19	1	1	1	1	1	1	1	1	1
CEC-17/20	1	1	1	1	1	1	0	1	-1
CEC-17/21	1	1	1	1	1	1	0	1	0
CEC-17/22	1	1	1	1	1	1	-1	1	-1
CEC-17/23	1	1	1	1	1	1	-1	1	-1
CEC-17/24	1	1	1	1	1	1	0	1	0
CEC-17/25	1	0	-1	1	1	1	0	1	0
CEC-17/26	1	1	1	1	1	1	-1	0	-1
CEC-17/27	1	1	1	-1	1	1	1	1	1
CEC-17/28	1	0	-1	1	1	1	0	1	0
CEC-17/29	1	1	1	1	1	1	0	1	-1
CEC-17/30	1	1	1	1	1	1	1	1	1
+	29	27	26	28	29	29	10	27	8
=	0	2	0	0	0	0	14	2	11
-	0	0	3	1	0	0	5	0	10

TABLE 13. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 100 dimensions, 51 runs and 1000000 number function evaluations.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-17/01	1	1	1	1	1	1	-1	1	-1
CEC-17/03	1	1	1	1	1	1	-1	-1	-1
CEC-17/04	1	1	1	1	1	1	0	1	-1
CEC-17/05	1	1	1	1	1	1	-1	1	-1
CEC-17/06	1	1	1	1	1	1	1	1	1
CEC-17/07	1	1	1	1	1	1	0	1	-1
CEC-17/08	1	1	1	1	1	1	-1	1	-1
CEC-17/09	1	1	1	1	1	1	1	1	1
CEC-17/10	1	1	1	1	1	1	0	1	-1
CEC-17/11	1	0	1	1	1	1	1	1	-1
CEC-17/12	1	1	1	1	1	1	-1	1	-1
CEC-17/13	1	1	1	1	1	1	-1	1	-1
CEC-17/14	1	1	1	1	1	1	1	1	1
CEC-17/15	1	1	1	1	1	1	-1	1	-1
CEC-17/16	1	1	1	1	1	1	0	1	-1
CEC-17/17	1	1	1	1	1	1	0	1	-1
CEC-17/18	1	1	1	1	1	1	1	1	-1
CEC-17/19	1	1	1	1	1	1	0	1	-1
CEC-17/20	1	1	1	1	1	1	0	1	-1
CEC-17/21	1	1	1	1	1	1	-1	1	-1
CEC-17/22	1	1	1	1	1	1	0	1	0
CEC-17/23	1	1	1	1	1	1	-1	1	-1
CEC-17/24	1	1	1	1	1	1	0	1	-1
CEC-17/25	1	0	1	1	1	1	0	1	0
CEC-17/26	1	1	1	1	1	1	0	1	-1
CEC-17/27	1	1	1	-1	1	1	1	1	1
CEC-17/28	1	0	0	-1	1	1	-1	1	-1
CEC-17/29	1	1	1	1	1	1	0	1	-1
CEC-17/30	1	1	1	1	1	1	-1	1	-1
+	29	26	28	27	29	29	6	28	4
=	0	3	1	0	0	0	12	0	2
-	0	0	0	2	0	0	11	1	23

15, and 20 dimensions, respectively. The Friedman ranking of HSOA and competitors was shown in Figure 20 to Figure 23. As seen, HSOA had a second rank in 5 dimensions and a first rank in the remaining dimensions. These results show that the stability of HSOA is even more than state-of-the-art algorithms such as LSHADE and EBOwithCMAR.

Table 24 and Table 25 show the results of optimizing CEC2022 by HSOA and competitors. According to CEC2022

TABLE 14. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2019 benchmark functions over 51 runs.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
	Min	1.00	1.00	15687.32	477575.81	1.00	8.79	1.00	1.00	1.00	1.00
	Mean	1.00	1.00	8542182.77	2006007.93	19824.68	3031265.52	1.00	1.00	1.00	18.48
CEC-19/01	Median	1.00	1.00	2578552.14	1985994.72	897.58	2210414.02	1.00	1.00	1.00	1.41
	Max	1.00	1.00	90408077.34	3695753.18	208490.41	8949464.63	1.00	1.00	1.00	263.71
	Std	0.00	0.00	14190821.68	824405.80	46856.90	2921171.39	0.00	0.00	0.00	44.42
	Min	1.00	4.22	259.20	746.97	16.92	168.77	4.64	1.00	3.73	85.98
	Mean	1.00	4.54	2041.36	1170.76	239.10	7125.43	4.97	1.00	4.26	301.17
CEC-19/02	Median	1.00	4.31	628.19	1165.37	231.73	6892.84	5.00	1.00	4.26	302.64
	Max	1.00	5.00	10311.45	1518.66	632.95	13211.22	5.00	1.00	4.52	564.38
	Std	0.00	0.35	2464.02	187.09	151.77	2648.86	0.09	0.00	0.10	106.03
	Min	1.00	1.00	1.41	1.60	1.00	1.41	1.00	1.00	1.00	1.00
	Mean	1.00	1.38	4.87	2.36	1.64	1.74	2.15	1.02	1.40	1.00
CEC-19/03	Median	1.00	1.41	4.61	2.40	1.41	1.41	1.69	1.00	1.41	1.00
	Max	1.00	1.41	9.71	2.87	6.71	6.71	4.38	1.41	1.41	1.00
	Std	0.00	0.11	2.64	0.27	1.11	1.05	1.03	0.10	0.06	0.00
	Min	1.00	6.97	9.04	1.00	4.98	12.94	12.05	1.99	6.97	1.00
	Mean	1.12	22.34	24.98	1.06	14.48	42.13	45.21	3.52	23.29	1.00
CEC-19/04	Median	1.00	19.90	24.52	1.01	13.46	39.81	43.94	2.99	21.89	1.00
	Max	2.00	46.77	44.58	2.03	27.63	73.63	80.82	5.97	45.41	1.00
	Std	0.32	9.30	8.72	0.20	4.89	14.34	13.45	1.10	9.01	0.00
	Min	1.00	1.08	1.03	1.03	1.08	1.17	1.59	1.00	1.03	1.00
	Mean	1.00	1.40	8.42	1.08	1.43	1.90	1.91	1.00	1.14	1.00
CEC-19/05	Median	1.00	1.32	9.84	1.07	1.41	1.81	1.90	1.00	1.12	1.00
	Max	1.01	2.56	28.14	1.13	1.84	3.25	2.33	1.02	1.48	1.00
	Std	0.00	0.29	7.65	0.02	0.22	0.44	0.17	0.01	0.09	0.00
	Min	1.00	3.18	1.00	1.00	1.10	3.28	4.86	1.00	1.00	1.00
	Mean	1.00	5.93	4.65	1.07	1.59	7.92	8.06	1.00	3.74	1.00
CEC-19/06	Median	1.00	5.99	4.67	1.01	1.25	7.74	7.92	1.00	3.55	1.00
	Max	1.00	9.32	9.03	1.48	4.31	11.76	11.26	1.00	8.08	1.00
	Std	0.00	1.41	2.08	0.13	0.78	1.54	1.52	0.00	1.85	0.00
	Min	1.08	245.49	238.45	4.12	105.53	482.69	369.43	1.19	300.26	1.06
	Mean	7.68	723.22	750.82	19.91	578.50	1059.58	983.63	33.65	797.86	46.17
CEC-19/07	Median	1.47	727.08	718.49	17.71	565.65	1086.83	969.22	4.79	773.41	7.89
	Max	126.39	1281.30	1443.48	63.32	1137.52	1595.21	1564.28	238.52	1468.09	241.42
	Std	24.19	264.11	264.11	12.05	239.93	266.34	265.48	55.15	288.32	63.88
	Min	1.10	2.77	2.24	1.56	1.64	2.87	3.76	1.58	2.50	1.06
	Mean	1.33	3.76	3.59	2.21	3.00	4.19	4.40	2.20	3.59	1.21
CEC-19/08	Median	1.23	3.76	3.60	2.20	3.05	4.17	4.43	2.22	3.61	1.21
	Max	2.90	4.51	4.52	2.80	4.04	4.98	4.98	2.65	4.55	1.83
	Std	0.32	0.37	0.54	0.28	0.61	0.35	0.30	0.24	0.43	0.15
	Min	1.02	1.09	1.04	1.07	1.04	1.07	1.10	1.02	1.04	1.00
	Mean	1.05	1.26	1.23	1.10	1.07	1.38	1.41	1.05	1.16	1.02
CEC-19/09	Median	1.05	1.23	1.17	1.10	1.07	1.37	1.38	1.05	1.15	1.02
	Max	1.06	1.51	1.55	1.13	1.19	1.88	1.78	1.07	1.34	1.04
	Std	0.01	0.11	0.14	0.02	0.03	0.21	0.19	0.01	0.07	0.01
	Min	1.00	1.00	21.00	8.96	1.03	6.45	20.92	1.02	1.00	1.00
	Mean	1.40	20.63	21.02	20.16	19.71	20.76	21.01	9.06	19.13	1.19
CEC-19/10	Median	1.01	21.00	21.01	21.04	21.25	21.00	21.00	3.02	21.00	1.00
	Max	21.02	21.25	21.14	21.07	21.37	21.34	21.10	21.00	21.22	3.01
	Std	2.80	2.80	0.04	2.73	5.34	2.05	0.03	9.15	5.86	0.54

results in 10 dimensions (Table 24), the HSOA achieved the global optimal solution for the min, mean, median, and max results in the unimodal function (CEC-22/01). The global optimal solutions for all multimodal functions (CEC-22/02- CEC-22/05) were obtained by HSOA. For the hybrid functions (CEC-22/06- CEC-22/08), HSOA got very close results to the global optimal value. Evaluation of the composition functions (CEC-22/09- CEC-22/12) revealed that HSOA could achieve the global optimal value for CEC-22/11. However, its performance suffered for CEC-22/09, CEC-22/10 and CEC-22/12, as it remained stuck in the local optimum. The comparison of the results of HSOA with other well-known algorithms, new algorithms, and state-of-the-art algorithms showed that the proposed algorithm is competitively accurate.

According to the optimizing results of CEC2022 in 20 dimensions (Table 25), the proposed HSOA could obtain the global optimal for CEC-22/01 of a unimodal function. In the case of multimodal functions, HSOA could attain the global optimal solution for CEC-22/03 and CEC-22/05, however, it encountered difficulty in achieving the global optimal for CEC-22/02 and produced a result that was in proximity. Additionally, HSOA was able to compute a solution very close to the global optimal for CEC-22/04. For the hybrid functions (CEC-22/06- CEC-22/08), HSOA estimated a good approximation of the optimal solution.

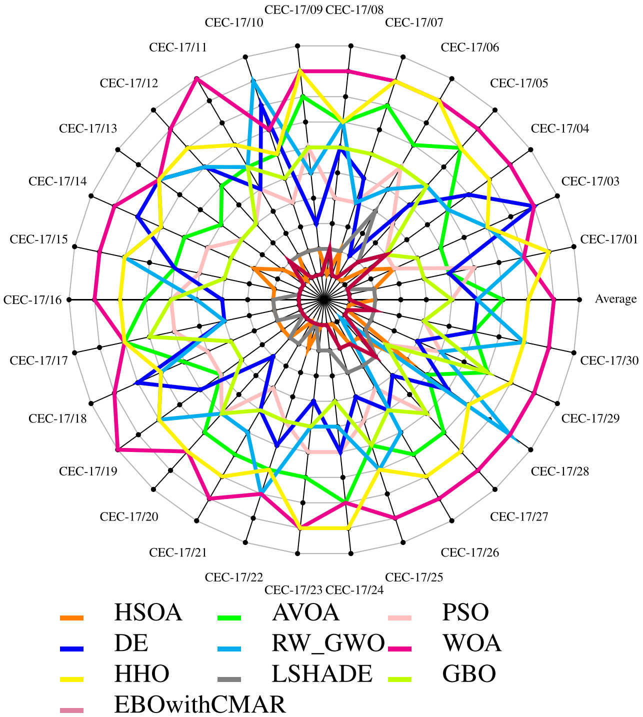


FIGURE 16. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 30 dimensions, 51 runs and 300000 number function evaluations.

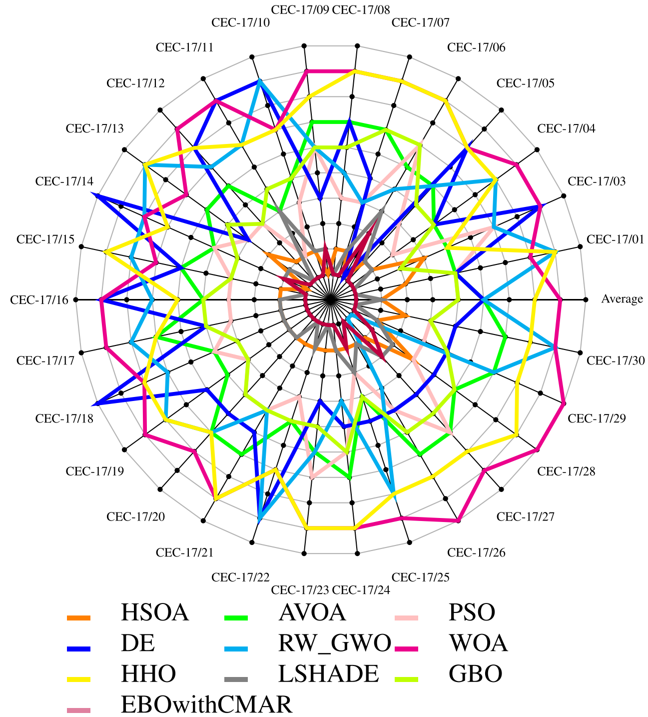


FIGURE 18. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 100 dimensions, 51 runs and 1000000 number function evaluations.

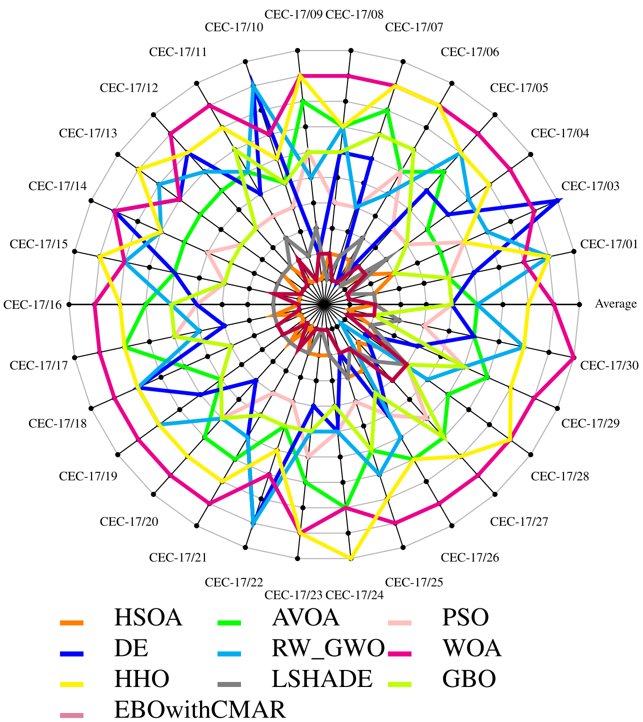


FIGURE 17. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2017 benchmark functions over 50 dimensions, 51 runs and 500000 number function evaluations.

TABLE 15. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2019 benchmark functions over 51 runs.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-19/01	0	1	1	1	1	0	0	1	1
CEC-19/02	1	1	1	1	1	1	-1	1	1
CEC-19/03	1	1	1	1	1	1	1	1	-1
CEC-19/04	1	1	1	1	1	1	1	1	-1
CEC-19/05	1	1	1	1	1	1	1	1	0
CEC-19/06	1	1	1	1	1	1	0	1	0
CEC-19/07	1	1	1	1	1	1	1	1	1
CEC-19/08	1	1	1	1	1	1	1	1	-1
CEC-19/09	1	1	1	1	1	1	1	1	-1
CEC-19/10	1	1	1	1	1	1	1	1	-1
+	9	10	10	10	10	9	7	10	3
=	1	0	0	0	0	1	2	0	2
-	0	0	0	0	0	0	1	0	5

By utilizing composition functions (CEC-22/06-CEC-22/08), HSOA was successful in obtaining the global optimal for CEC-22/11, however, it faced challenges with the remaining CEC2022 as it was limited to the local optimum. HSOA's solving ability for CEC2022 benchmark functions were comparable to AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR.

The Wilcoxon results for CEC2022 in 10 and 20 dimensions are shown in Table 26 and Table 27, respectively. It is evident that HSOA was the winner compared to well-known and new algorithms. In 10 dimensions, the number of wines and equality of HSOA compared to LSHADE were equal to 2 and 7, respectively. While those values compared to EBOwithCMAR were equal to 2 and 5. In 20 dimensions, HSOA had 2 wines and 9 equalities

TABLE 16. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 5 dimensions, 30 runs and 50000 number function evaluations.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
	Min	100.00	109.98	112.34	100.82	381.98	149.39	121.11	100.00	100.00	100.00
	Mean	100.00	3597.08	1965.96	131.61	3523.69	4645.39	4972.82	100.00	100.00	100.00
CEC-2020/01	Median	100.00	1887.22	1130.79	123.87	2233.46	3239.07	3803.38	100.00	100.00	100.00
	Max	100.00	13230.69	6796.25	237.04	13659.58	14595.42	16657.36	100.00	100.00	100.00
	Std	0.00	3833.31	1882.45	30.07	3315.83	4427.67	4733.42	0.00	0.00	0.00
	Min	1100.12	1106.95	1100.12	1103.69	1102.73	1107.09	1113.86	1100.47	1100.25	1100.00
	Mean	1102.31	1278.50	1259.28	1121.96	1180.92	1339.61	1409.47	1101.71	1223.15	1100.15
CEC-2020/02	Median	1100.37	1248.35	1232.09	1118.97	1138.49	1347.41	1455.53	1100.87	1185.20	1100.13
	Max	1113.66	1576.58	1456.09	1144.59	1467.69	1612.21	1709.16	1107.11	1632.98	1100.52
	Std	3.55	125.06	88.02	11.15	82.96	128.32	137.12	1.89	111.23	0.12
	Min	700.61	706.21	701.61	705.46	705.49	704.19	706.95	700.19	705.65	705.15
	Mean	705.11	712.02	705.51	706.22	707.80	716.96	714.29	705.44	707.90	705.20
CEC-2020/03	Median	705.68	711.65	705.88	706.09	707.57	717.33	714.30	705.73	708.12	705.15
	Max	707.48	720.39	707.31	707.29	713.21	737.35	731.29	705.90	712.28	705.40
	Std	1.86	3.56	1.57	0.45	1.86	7.44	5.57	1.08	1.34	0.09
	Min	1900.00	1900.00	1900.01	1900.02	1900.00	1900.00	1900.00	1900.00	1900.00	1900.01
	Mean	1900.05	1900.00	1900.17	1900.10	1900.15	1900.00	1900.00	1900.08	1900.00	1900.06
CEC-2020/04	Median	1900.04	1900.00	1900.17	1900.10	1900.18	1900.00	1900.00	1900.09	1900.00	1900.05
	Max	1900.14	1900.00	1900.32	1900.18	1900.35	1900.00	1900.00	1900.13	1900.00	1900.12
	Std	0.05	0.00	0.09	0.04	0.12	0.00	0.00	0.04	0.00	0.03
	Min	1700.00	1701.00	1700.64	1700.00	1705.32	1706.80	1708.65	1700.00	1700.00	1700.00
	Mean	1700.00	1773.02	1729.68	1700.00	1840.89	2461.76	2024.85	1700.00	1705.76	1700.03
CEC-2020/05	Median	1700.00	1739.44	1709.58	1700.00	1805.20	2075.74	1949.70	1700.00	1700.99	1700.00
	Max	1700.01	1973.34	1841.53	1700.00	2243.95	10643.33	3206.04	1700.00	1733.50	1700.99
	Std	0.00	76.04	41.44	0.00	126.02	1641.60	350.75	0.00	9.02	0.18
	Min	1600.00	1600.02	1600.00	1600.00	1600.05	1600.40	1600.21	1600.00	1600.00	1600.00
	Mean	1600.00	1600.88	1603.97	1600.00	1600.71	1603.45	1605.21	1600.00	1600.12	1600.00
CEC-2020/06	Median	1600.00	1600.77	1600.00	1600.00	1600.63	1602.42	1601.20	1600.00	1600.00	1600.00
	Max	1600.00	1602.21	1718.44	1600.00	1601.55	1613.92	1718.97	1600.00	1600.63	1600.00
	Std	0.00	0.55	21.62	0.00	0.33	3.07	21.50	0.00	0.25	0.00
	Min	2100.00	2100.00	2100.00	2100.00	2101.21	2241.59	2101.78	2100.00	2100.00	2100.00
	Mean	2100.00	2213.19	2187.76	2100.00	2321.10	3660.02	2341.52	2100.00	2105.71	2100.00
CEC-2020/07	Median	2100.00	2222.72	2218.44	2100.00	2225.07	2588.05	2315.18	2100.00	2100.00	2100.00
	Max	2100.00	2398.11	2222.33	2100.00	3815.45	10135.37	2818.00	2100.00	2267.05	2100.00
	Std	0.00	95.97	53.85	0.00	410.00	2145.07	159.19	0.00	30.47	0.00
	Min	2200.00	2200.00	2200.00	2200.00	2200.02	2200.04	2200.01	2200.00	2200.00	2200.00
	Mean	2200.00	2239.82	2243.90	2203.75	2232.06	2244.72	2234.54	2200.00	2209.02	2203.78
CEC-2020/08	Median	2200.00	2217.14	2200.00	2200.19	2200.04	2217.55	2217.89	2200.00	2206.61	2200.00
	Max	2200.00	2307.63	2301.46	2230.46	2302.50	2312.69	2307.59	2200.00	2301.25	2300.31
	Std	0.00	43.27	50.62	7.97	46.62	44.72	36.23	0.00	18.52	18.31
	Min	2400.01	2500.00	2400.00	2487.73	2400.45	2404.82	2500.04	2500.00	2400.00	2500.00
	Mean	2487.74	2500.00	2514.06	2508.19	2560.20	2542.99	2517.62	2506.67	2490.00	2500.00
CEC-2020/09	Median	2500.00	2500.00	2500.00	2506.67	2500.44	2500.39	2500.19	2500.00	2500.00	2500.00
	Max	2500.87	2500.00	2716.80	2527.37	2721.26	2737.36	2608.67	2700.00	2500.00	2500.00
	Std	32.19	0.00	87.51	8.90	102.17	96.97	39.72	36.51	30.51	0.00
	Min	2557.72	2800.00	2847.37	2818.12	2847.37	2847.39	2800.63	2800.00	2800.00	2800.00
	Mean	2831.46	2847.21	2847.37	2846.39	2847.39	2847.50	2848.63	2844.21	2844.22	2845.79
CEC-2020/10	Median	2847.41	2847.39	2847.37	2847.37	2847.39	2847.47	2847.41	2847.37	2847.37	2847.37
	Max	2847.46	2889.26	2847.40	2847.37	2847.42	2847.87	2889.30	2847.37	2847.44	2847.37
	Std	58.23	11.74	0.01	5.34	0.01	0.10	13.92	12.02	12.02	8.65

in comparison with LSHADE, and 1 wine and 7 equalities compared to EBOwithCMAR.

The Friedman ranking for CEC2022 results (Figure 24 and Figure 25) shows that EBOwithCMAR had first rank and HSOA was placed in the second rank. However, there was little difference between the Friedman's ranking for HSOA and EBOwithCMAR.

Table 28 shows the optimal results for six CEC2006 real-world problems using HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE and EBOwithCMAR over 51 random runs. As seen, HSOA worked with greater precision than other algorithms in $CEC - 06/02$ to $CEC - 06/06$ real-world problems, although in $CEC - 06/01$ LSHADE and EBOwithCMAR was better. The Wilcoxon test results in Table 29 indicated that HSOA had 6 wins over AVOA, PSO, DE, RW_GWO, WOA, HHO and GBO. The number of victories and defeats of HSOA, in contrast LSHADE and EBOwithCMAR, were 5 and 1, respectively. Based on the Figure 26, Friedman ranks for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR were 1.59, 6.92, 5.33, 6.75, 6.68, 9.21, 8.60, 3.13, 4.42 and 2.37, respectively.

In the following, the results of HSOA in CEC2011 optimization real-world problems are compared with

TABLE 17. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 10 dimensions, 30 runs and 1000000 number function evaluations.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
	Min	100.00	113.66	109.59	100.02	187.75	115.77	682.92	100.00	100.00	100.00
	Mean	100.00	4099.89	1470.16	108.98	207363.38	3592.90	20246.49	100.00	100.00	100.00
CEC-2020/01	Median	100.00	3431.72	771.86	103.54	3825.87	2720.34	14312.59	100.00	100.00	100.00
	Max	100.00	12723.42	6159.90	157.95	3089627.89	12977.37	85381.89	100.00	100.00	100.00
	Std	0.00	3650.86	1644.15	13.28	772185.89	3700.57	19333.30	0.00	0.00	0.00
	Min	1100.12	1235.26	1136.82	1111.59	1139.77	1388.95	1366.60	1100.19	1107.20	1100.25
	Mean	1104.06	1624.46	1340.00	1125.94	1410.57	1917.72	1745.04	1115.61	1703.53	1104.54
CEC-2020/02	Median	1103.66	1637.39	1346.37	1126.12	1389.44	1922.45	1727.99	1111.86	1702.29	1103.63
	Max	1110.24	2074.01	1712.18	1148.45	1978.89	2527.55	2071.91	1225.33	2237.13	1111.96
	Std	2.58	255.74	130.13	9.50	195.08	251.63	198.88	22.00	293.08	3.01
	Min	710.44	718.98	704.46	710.85	704.61	724.53	735.31	711.28	719.09	711.03
	Mean	710.86	737.03	713.82	712.16	719.80	761.16	753.30	712.31	734.47	711.80
CEC-2020/03	Median	710.90	736.48	714.03	712.16	717.78	760.73	750.01	712.38	734.93	711.72
	Max	711.37	756.80	718.46	713.47	735.34	834.38	790.71	714.25	753.58	713.43
	Std	0.25	9.22	2.64	0.63	6.22	23.53	13.13	0.71	8.28	0.54
	Min	1900.00	1900.00	1900.05	1900.07	1900.00	1900.00	1900.00	1900.10	1900.00	1900.10
	Mean	1900.00	1900.00	1900.42	1900.19	1900.22	1900.05	1900.00	1900.15	1900.00	1900.14
CEC-2020/04	Median	1900.00	1900.00	1900.39	1900.20	1900.03	1900.00	1900.00	1900.16	1900.00	1900.14
	Max	1900.00	1900.00	1900.68	1900.30	1901.39	1900.85	1900.00	1900.19	1900.00	1900.17
	Std	0.00	0.00	0.14	0.06	0.33	0.21	0.00	0.02	0.00	0.02
	Min	1702.32	2008.64	1776.13	1852.03	1958.54	2874.80	2071.39	1700.00	1756.71	1700.21
	Mean	1710.00	3855.20	2629.91	2759.21	5115.20	50883.80	5794.08	1701.07	2224.71	1718.87
CEC-2020/05	Median	1707.99	3368.99	2237.93	2463.20	4137.57	14328.39	3994.81	1700.42	2160.51	1706.40
	Max	1734.10	6316.44	4420.46	5412.36	10262.09	346957.21	16874.14	1722.55	3169.52	1829.82
	Std	6.35	1379.56	781.46	877.90	2671.07	89863.19	3961.88	4.06	328.93	36.56
	Min	1600.07	1601.41	1600.25	1600.07	1601.27	1601.96	1601.70	1600.02	1600.79	1600.00
	Mean	1600.16	1707.52	1751.70	1600.18	1677.31	1706.37	1739.37	1600.33	1710.95	1600.20
CEC-2020/06	Median	1600.13	1722.89	1726.01	1600.16	1719.77	1728.56	1726.79	1600.37	1731.79	1600.16
	Max	1600.43	1850.93	1877.20	1600.40	1842.32	1944.77	1878.90	1600.68	1894.51	1600.48
	Std	0.10	77.26	75.60	0.09	68.62	78.67	73.31	0.17	84.43	0.15
	Min	2100.01	2117.25	2100.32	2100.00	2151.68	3321.76	2250.63	2100.00	2100.50	2100.00
	Mean	2102.20	2497.09	2163.11	2100.05	2786.19	14196.67	5163.28	2100.51	2346.20	2100.18
CEC-2020/07	Median	2100.57	2407.19	2158.37	2100.01	2476.01	8283.95	4442.29	2100.50	2291.05	2100.02
	Max	2117.57	4064.98	2276.65	2100.40	6279.52	33564.44	13879.62	2101.12	2931.93	2100.81
	Std	5.08	401.78	51.77	0.09	943.10	10832.08	2773.45	0.23	189.20	0.22
	Min	2200.00	2239.06	2300.79	2212.43	2215.19	2222.32	2239.51	2200.00	2300.63	2300.00
	Mean	2287.05	2303.93	2301.82	2276.88	2300.27	2331.91	2307.97	2296.69	2302.03	2300.00
CEC-2020/08	Median	2300.00	2305.37	2301.63	2300.00	2301.94	2311.88	2312.36	2300.00	2301.88	2300.00
	Max	2300.00	2313.51	2303.11	2300.02	2307.09	3005.92	2320.90	2300.58	2305.40	2300.00
	Std	33.63	12.73	0.64	28.65	16.22	129.05	17.64	18.26	1.07	0.00
	Min	2500.00	2500.00	2500.00	2518.86	2732.75	2500.01	2500.05	2500.00	2500.00	2500.00
	Mean	2531.01	2659.68	2697.30	2671.49	2742.93	2733.02	2735.32	2662.80	2543.70	2651.99
CEC-2020/09	Median	2500.00	2756.81	2734.88	2711.13	2741.80	2766.35	2764.54	2731.96	2500.00	2727.09
	Max	2735.26	2785.99	2751.18	2738.63	2753.40	2800.40	2860.89	2734.81	2745.59	2731.61
	Std	80.43	132.81	89.92	74.17	5.29	93.74	103.08	108.39	92.02	109.31
	Min	2897.75	2897.94	2897.74	2897.80	2897.75	2897.91	2897.80	2897.74	2897.76	2897.74
	Mean	2922.19	2928.64	2921.23	2898.38	2929.37	2936.30	2934.99	2905.66	2920.08	2916.16
CEC-2020/10	Median	2943.33	2946.49	2921.51	2898.40	2943.49	2948.60	2945.14	2898.04	2899.61	2898.02
	Max	2943.38	2950.93	2945.87	2899.49	2946.47	2973.81	3024.53	2945.79	2946.53	2945.16
	Std	23.01	24.10	23.03	0.36	22.63	25.66	33.85	17.37	23.45	22.70

other optimization algorithms. Table 30 compares HSOA's CEC2011 results with AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR. It is apparent that HSOA, in terms of min, mean, median, max and std, performed better in $CEC-06/03$, $CEC-06/08$, $CEC-11/12$ and $CEC-11/16$ than all other selected algorithms. Compared to other investigated algorithms, HSOA had reasonable accuracy in $CEC-11/1$, $CEC-11/2$, $CEC-11/4$ to $CEC-11/7$, $CEC-11/10$, $CEC-11/11$, $CEC-11/12$, $CEC-11/13$ to $CEC-11/15$, $CEC-11/16$, $CEC-11/17$ and $CEC-11/18$.

The overall results show that HSOA comparatively performs than other algorithms. Based on the results in Table 31, the HSOA won in 14, 16, 17, 17, 18, 18, 4 and 14 of the 18 *CEC*2011 compared to the AVOA and PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR. Moreover, HSOA was tied with AVOA in 2 of the 18 CEC2011, was equal with PSO, DE and RW_GWO in 1 of the 18 CEC2011, and had equality with LSHADE, GBO and EBOwithCMAR in 6, 2 and 4, respectively. The Friedman ranks (Figure 27) of HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO,

TABLE 18. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 15 dimensions, 30 runs and 3000000 number function evaluations.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-2020/01	Min	100.00	107.42	148.29	104.82	275.47	100.73	6139.70	100.00	100.00	100.00
	Mean	100.00	6994.28	5540.46	482.31	1164627.62	6481.13	38012.95	100.00	100.00	100.00
	Median	100.00	4942.33	3917.25	206.29	8078.84	4670.73	26137.05	100.00	100.00	100.00
	Max	100.00	26127.12	18295.46	2755.91	8314078.77	25924.99	225172.07	100.00	100.00	100.00
	Std	0.00	6143.24	5325.50	681.29	2637889.93	7232.84	40151.06	0.00	0.00	0.00
CEC-2020/02	Min	1100.21	1412.93	1223.50	1100.04	1300.26	1737.20	1465.76	1102.40	1629.78	1100.13
	Mean	1123.92	1697.50	1403.20	1100.95	1785.93	2556.68	1854.73	1140.14	2192.05	1126.86
	Median	1107.95	1677.50	1397.16	1100.17	1777.42	2546.65	1824.64	1110.10	2204.39	1110.16
	Max	1231.76	2569.19	1712.45	1111.22	2430.13	3345.76	2382.96	1228.82	2865.53	1225.53
	Std	40.34	234.08	142.09	2.12	252.90	394.39	238.96	51.76	337.00	39.44
CEC-2020/03	Min	715.57	723.76	715.90	715.57	720.10	776.79	739.41	715.70	734.42	715.57
	Mean	715.80	747.15	720.85	715.58	744.62	833.43	780.18	717.31	762.82	716.46
	Median	715.73	749.72	720.59	715.57	748.43	831.55	778.18	717.34	762.66	716.44
	Max	716.33	777.90	729.36	715.74	775.12	880.59	819.43	718.45	789.58	717.22
	Std	0.21	15.18	2.67	0.04	16.23	28.58	18.09	0.64	14.58	0.46
CEC-2020/04	Min	1900.00	1900.00	1900.49	1900.34	1900.00	1900.00	1900.00	1900.18	1900.00	1900.15
	Mean	1900.00	1900.00	1900.74	1900.44	1900.15	1900.10	1900.00	1900.26	1900.00	1900.22
	Median	1900.00	1900.00	1900.74	1900.44	1900.00	1900.00	1900.00	1900.26	1900.00	1900.22
	Max	1900.00	1900.00	1901.09	1900.52	1901.59	1902.24	1900.00	1900.36	1900.00	1900.29
	Std	0.00	0.00	0.16	0.05	0.37	0.43	0.00	0.03	0.00	0.03
CEC-2020/05	Min	1726.86	1838.93	1740.34	1803.10	2864.33	24459.48	2974.06	1700.47	1983.17	1704.29
	Mean	1842.12	2304.23	2107.02	2186.94	11086.58	186529.51	10149.68	1744.21	2443.02	1804.44
	Median	1853.28	2267.73	2089.33	2109.41	9187.81	177741.34	10285.08	1712.33	2466.49	1825.71
	Max	1985.06	2940.71	2536.13	2913.97	30773.87	642751.17	18043.43	1850.66	3112.84	1959.32
	Std	59.27	227.51	213.94	319.79	7042.27	149551.46	4395.52	53.48	273.46	72.73
CEC-2020/06	Min	1600.07	1610.55	1610.18	1600.09	1604.01	1635.43	1602.50	1600.14	1601.50	1600.14
	Mean	1600.58	1740.87	1738.22	1600.36	1692.17	1861.22	1815.54	1600.53	1750.41	1600.42
	Median	1600.28	1745.31	1742.50	1600.39	1700.25	1823.04	1760.09	1600.43	1749.63	1600.39
	Max	1608.97	1917.86	1910.06	1600.68	1803.95	2143.57	2084.39	1601.24	1910.13	1601.30
	Std	1.59	88.64	84.07	0.15	70.62	137.57	134.28	0.28	72.35	0.21
CEC-2020/07	Min	2101.70	2126.66	2102.05	2101.20	2436.55	19312.18	2776.59	2100.24	2231.20	2100.42
	Mean	2105.88	2401.02	2258.86	2102.66	3197.94	229643.73	3713.12	2140.45	2526.31	2121.60
	Median	2104.76	2386.20	2233.23	2101.89	3090.95	190218.78	3569.53	2100.73	2450.67	2100.70
	Max	2119.62	2640.15	2632.57	2110.67	4965.43	538481.94	4814.34	2338.80	3077.53	2230.25
	Std	3.63	114.85	122.74	2.34	518.97	172242.71	563.27	65.22	209.09	45.71
CEC-2020/08	Min	2300.00	2300.00	2300.00	2248.40	2230.50	2300.00	2249.15	2300.00	2300.00	2300.00
	Mean	2300.00	2315.06	2321.24	2291.93	2323.80	2564.54	2357.50	2300.00	2300.99	2300.00
	Median	2300.00	2301.36	2300.93	2300.00	2303.22	2302.82	2304.36	2300.00	2300.89	2300.00
	Max	2300.00	2713.39	2907.09	2300.00	2979.98	4693.60	3356.84	2300.00	2303.35	2300.00
	Std	0.00	75.24	110.65	14.82	124.66	696.93	218.08	0.00	0.72	0.00
CEC-2020/09	Min	2500.00	2796.90	2500.00	2789.68	2509.86	2809.32	2797.69	2787.69	2797.23	2500.00
	Mean	2780.57	2812.19	2784.82	2789.95	2799.87	2857.21	2823.08	2790.69	2816.10	2760.26
	Median	2790.30	2812.46	2794.07	2789.69	2807.97	2854.87	2820.27	2790.94	2812.31	2789.68
	Max	2792.42	2831.93	2801.41	2790.90	2828.14	2920.48	2857.47	2791.66	2852.91	2790.21
	Std	52.99	8.10	53.89	0.48	55.38	28.88	14.75	0.70	11.66	88.15
CEC-2020/10	Min	2900.00	2900.00	2900.00	2900.00	2900.05	2900.00	2620.97	2900.00	2900.00	2900.00
	Mean	2900.00	2937.77	2900.00	2900.00	3003.12	3033.25	2923.48	2900.00	2916.42	2900.00
	Median	2900.00	2900.00	2900.00	2900.00	3084.92	3090.25	2900.75	2900.00	2900.00	2900.00
	Max	2900.00	3173.81	2900.00	2900.00	3087.18	3173.03	3175.07	2900.00	3104.93	2900.00
	Std	0.00	86.97	0.00	0.00	91.97	112.30	100.51	0.00	51.47	0.00

and EBOwithCMAR were computed equal to 2.66, 5.83, 6.1, 6.79, 7.00, 6.03, 8.28, 8.16, 2.66, 5.18 and 3.13.

Figure 28 shows the violin plots of 51 runs of HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, and EBOwithCMAR. Violin plot is a combination of boxplot and kernel density plot. This graph displays the distribution of the results got for each optimization algorithm. A broader chart suggests greater uncertainty. In addition, the blank points in this chart represent the mean results. As seen, HSOA most times has less uncertainty and more accurate results than other considered algorithms.

Figure 29 demonstrates the convergence curve plots for HSOA, RW_GWO, PSO, DE, GWO, WOA, HHO, LSHADE, and EBOwithCMAR. As illustrated in Figure 29, the convergence curves were very smooth and dropped quickly for AVOA, PSO, DE, RW_GWO, WOA, and HHO. This means that exploitation was dominant in comparison with exploration. Moreover, the convergence curves of AVOA, PSO, DE, RW_GWO, WOA, and HHO were stagnant and could not approach the global optimum at the end of the optimization process. However, the convergence curve of HSOA, LSHADE, GBO, and EBOwithCMAR was quite rough and slowly declining. This means that the balance

TABLE 19. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 20 dimensions, 30 runs and 10000000 number function evaluations.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-2020/01	Min	100.00	114.96	100.00	100.00	102.86	100.99	582.20	100.00	100.00	100.00
	Mean	100.00	3532.53	247.49	108.38	383378.35	2295.59	5918.74	100.00	100.00	100.00
	Median	100.00	1641.61	139.42	100.01	2582.01	1665.23	5475.04	100.00	100.00	100.00
	Max	100.00	11934.45	944.22	294.84	11392749.48	7826.13	13906.63	100.00	100.00	100.00
	Std	0.00	3900.38	219.43	35.81	2079341.19	2142.79	3353.51	0.00	0.00	0.00
CEC-2020/02	Min	1100.09	1262.74	1245.99	1100.03	1586.05	2404.90	1334.64	1100.16	1677.80	1100.12
	Mean	1101.10	1678.15	1528.68	1100.26	1965.16	3281.72	1791.04	1105.08	2636.95	1103.41
	Median	1100.25	1661.76	1473.06	1100.09	1857.01	3287.96	1790.38	1105.11	2615.71	1103.40
	Max	1105.09	2329.35	1930.15	1101.80	2759.74	4053.79	2301.79	1111.77	3609.01	1109.24
	Std	1.27	226.39	160.07	0.51	281.08	424.03	273.80	3.00	482.95	2.56
CEC-2020/03	Min	720.39	729.18	722.28	720.39	727.85	813.62	762.31	720.57	750.21	720.50
	Mean	720.68	747.65	728.10	720.44	754.48	880.28	796.67	722.56	794.02	721.53
	Median	720.67	744.84	727.82	720.39	745.45	875.61	798.93	722.74	794.86	721.41
	Max	721.13	775.78	735.26	720.77	807.77	965.86	844.80	724.51	829.08	722.40
	Std	0.22	10.71	3.59	0.13	24.36	35.07	22.83	0.83	22.76	0.54
CEC-2020/04	Min	1900.00	1900.00	1900.65	1900.54	1900.00	1900.00	1900.00	1900.29	1900.00	1900.28
	Mean	1900.00	1900.00	1901.03	1900.67	1900.21	1900.13	1900.00	1900.37	1900.00	1900.32
	Median	1900.00	1900.00	1901.00	1900.67	1900.00	1900.00	1900.00	1900.37	1900.00	1900.31
	Max	1900.00	1900.00	1901.81	1900.77	1906.36	1901.90	1900.00	1900.42	1900.00	1900.37
	Std	0.00	0.00	0.26	0.05	1.16	0.38	0.00	0.04	0.00	0.02
CEC-2020/05	Min	1731.49	2097.95	2676.03	16222.25	4248.19	12716.04	3464.15	1731.16	2216.15	1726.41
	Mean	1892.42	4039.78	4659.16	57534.05	39392.74	63640.46	20009.97	1938.59	2783.17	1885.59
	Median	1891.86	3665.81	4498.10	55760.88	32977.26	39858.07	16748.13	1940.75	2758.37	1861.60
	Max	2013.78	7275.51	9286.02	131203.26	162716.09	354913.05	60070.90	2208.95	3448.40	1991.48
	Std	75.41	1394.09	1309.73	26711.60	35783.61	71995.13	15412.56	118.54	314.86	77.60
CEC-2020/06	Min	1600.08	1601.19	1601.42	1600.08	1614.79	1646.07	1603.16	1600.21	1608.10	1600.16
	Mean	1600.25	1603.08	1820.60	1600.43	1634.45	1950.82	1615.52	1600.62	1755.17	1600.57
	Median	1600.27	1602.62	1838.72	1600.25	1629.75	1974.92	1610.30	1600.57	1738.03	1600.43
	Max	1600.46	1608.30	1963.53	1605.89	1725.93	2327.66	1723.62	1601.26	1925.11	1601.21
	Std	0.11	1.71	112.35	1.03	20.31	180.23	21.90	0.21	84.86	0.36
CEC-2020/07	Min	2104.26	2391.67	2109.43	2941.75	3474.29	6150.20	3717.77	2100.37	2269.16	2100.03
	Mean	2159.25	2962.70	2827.10	9529.74	6128.73	72315.81	6847.27	2158.66	2823.69	2171.38
	Median	2130.29	2837.90	2661.38	8615.26	6429.75	36881.31	6529.83	2136.43	2800.93	2147.89
	Max	2320.17	5395.95	4906.20	18825.37	9359.64	221583.23	12245.55	2290.77	3341.49	2254.61
	Std	64.79	622.97	598.78	4360.68	1455.90	71637.40	2338.05	59.53	279.30	64.39
CEC-2020/08	Min	2300.00	2300.00	2300.00	2274.19	2300.74	2300.00	2301.90	2300.00	2300.00	2300.00
	Mean	2300.00	2768.80	2361.98	2299.71	2978.36	3744.66	2785.29	2300.00	2300.98	2300.00
	Median	2300.00	2301.26	2300.00	2300.00	2304.60	2313.38	2303.61	2300.00	2301.13	2300.00
	Max	2300.00	4976.53	4142.52	2317.21	4875.96	6160.52	5923.73	2300.00	2304.27	2300.00
	Std	0.00	963.53	336.29	6.07	1006.92	1596.13	1117.48	0.00	1.13	0.00
CEC-2020/09	Min	2798.53	2848.91	2802.68	2808.94	2827.13	2880.35	2500.27	2800.00	2822.39	2797.68
	Mean	2800.68	2914.43	2819.97	2816.65	2846.06	2949.64	3006.57	2804.75	2867.00	2800.48
	Median	2800.65	2913.25	2820.62	2817.14	2841.92	2955.23	3021.56	2804.72	2869.10	2800.49
	Max	2802.62	2999.94	2840.85	2823.66	2885.34	3010.25	3152.60	2810.30	2902.45	2803.71
	Std	0.99	35.77	9.97	3.78	13.78	35.31	116.90	1.92	19.62	1.39
CEC-2020/10	Min	2910.12	2911.60	2900.39	2910.92	2907.18	2950.70	2910.56	2910.03	2913.95	2913.66
	Mean	2913.48	2965.50	2926.43	2913.44	2923.64	2994.78	2966.52	2913.44	2974.96	2913.67
	Median	2913.70	2969.18	2913.83	2913.66	2907.59	2995.78	2973.29	2913.67	2972.82	2913.67
	Max	2913.82	3010.11	2972.24	2913.66	2969.93	3072.34	3010.02	2913.76	3004.57	2913.69
	Std	0.91	36.39	22.97	0.66	27.02	28.46	33.09	0.93	23.85	0.01

between exploitation and the exploration operators was well established. In addition, HSOA in almost conditions had a better approximation of the global optimum than LSHADE, GBO, and EBOwithCMAR. As a result, HSOA's performance and accuracy towards the global optimum were better than other investigated algorithms.

Based on the results obtained in the previous stages, the proposed algorithm had better performance than both popular (PSO, DE, RW_GWO, WOA), new (AVOA, HHO, GBO) and LSHADE as a state-of-the-art algorithm. In addition, except for the CEC2017 and CEC2022 sets, the introduced algorithm outperformed EBOwithCMAR. Therefore, the HSOA can be a competitor to the stat-of-the-art algorithms. This is because HSOA examines the search space more accurately by using two population groups. Additionally, the HSOA reproduction algorithm improves the solutions obtained in every generation and mitigates the risk of falling into the trap of local optima. The utilization of self-adaptive parameters allows HSOA operators to transform the exploration phase of the search process into an exploitation phase.

Figure 30 shows how algorithms performed on 108 optimization problems, according to the Friedman ranking. As this figure shows, HSOA with a value of 2.50 for

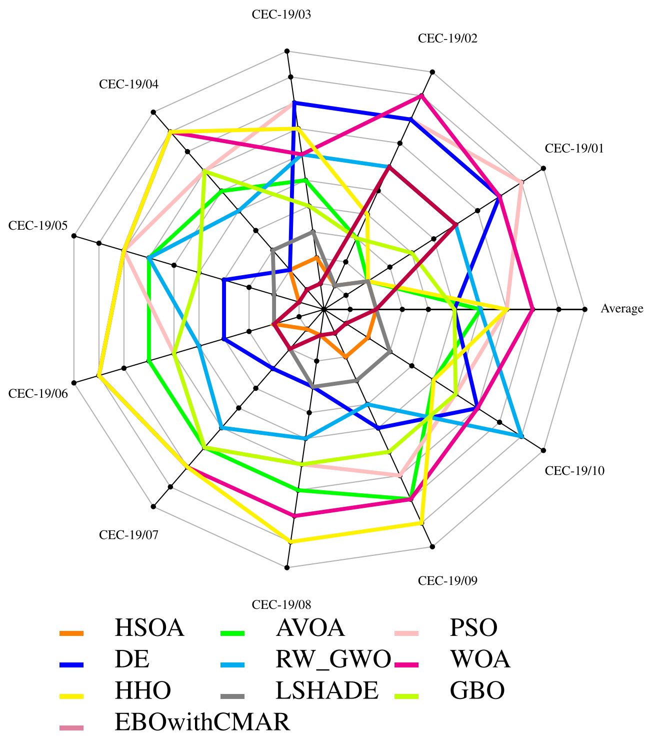


FIGURE 19. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2019 benchmark functions over 51 runs.

TABLE 20. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 5 dimensions, 30 runs and 50000 number function evaluations.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-20/01	1	1	1	1	1	1	0	1	0
CEC-20/02	1	1	1	1	1	1	1	1	-1
CEC-20/03	1	0	1	1	1	1	0	1	-1
CEC-20/04	-1	1	1	1	-1	-1	1	-1	1
CEC-20/05	1	1	-1	1	1	1	-1	1	-1
CEC-20/06	1	1	1	1	1	1	1	1	0
CEC-20/07	1	1	0	1	1	1	0	1	0
CEC-20/08	1	1	1	1	1	1	0	1	0
CEC-20/09	-1	-1	1	1	1	1	-1	-1	-1
CEC-20/10	0	-1	-1	-1	1	0	-1	-1	-1
+	7	7	7	9	9	8	3	7	1
=	1	1	1	0	0	1	4	0	4
-	2	2	2	1	1	1	3	3	5

the Friedman ranking was ranked first in relation to other comparison algorithms. The Friedman's ranking for AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR were equal to 7.30, 5.95, 5.81, 6.41, 8.44, 7.72, 2.99, 5.18 and 2.70, respectively. As observed, Friedman's ranking for HSOA was 65.79%, 58.05%, 57.00%, 61.00%, 70.40%, 67.63%, 16.45%, 51.78% and 7.45% less than AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO and EBOwithCMAR, respectively.

Another issue with the results is that the HSOA results are not the same as the other studied algorithms. This means that the HSOA is unique and distinct from other revised algorithms. The utilization of two distinct populations,

TABLE 21. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 10 dimensions, 30 runs and 1000000 number function evaluations.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-20/01	1	1	1	1	1	1	0	1	0
CEC-20/02	1	1	1	1	1	1	1	1	0
CEC-20/03	1	1	1	1	1	1	1	1	1
CEC-20/04	0	1	1	1	0	0	1	0	1
CEC-20/05	1	1	1	1	1	1	-1	1	0
CEC-20/06	1	1	0	1	1	1	1	1	0
CEC-20/07	1	1	-1	1	1	1	0	1	-1
CEC-20/08	1	1	0	1	1	1	0	1	0
CEC-20/09	1	1	1	1	1	1	1	0	1
CEC-20/10	1	0	0	1	1	1	0	0	0
+	9	9	6	10	9	9	5	7	3
=	1	1	3	0	1	1	4	3	6
-	0	0	1	0	0	0	1	0	1

TABLE 22. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 15 dimensions, 30 runs and 3000000 number function evaluations.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-20/01	1	1	1	1	1	1	0	1	0
CEC-20/02	1	1	-1	1	1	1	0	1	0
CEC-20/03	1	1	-1	1	1	1	1	1	1
CEC-20/04	0	1	1	1	0	0	1	0	1
CEC-20/05	1	1	1	1	1	1	-1	1	-1
CEC-20/06	1	1	0	1	1	1	1	1	1
CEC-20/07	1	1	-1	1	1	1	-1	1	-1
CEC-20/08	1	1	1	1	1	1	0	1	0
CEC-20/09	1	1	0	1	1	1	1	1	-1
CEC-20/10	1	1	0	1	1	1	0	1	0
+	9	10	4	10	9	9	4	9	3
=	1	0	3	0	1	1	4	1	4
-	0	0	3	0	0	0	2	0	3

TABLE 23. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 20 dimensions, 30 runs and 10000000 number function evaluations.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-20/01	1	1	1	1	1	1	0	1	0
CEC-20/02	1	1	-1	1	1	1	1	1	1
CEC-20/03	1	1	-1	1	1	1	1	1	1
CEC-20/04	0	1	1	0	1	0	1	0	1
CEC-20/05	1	1	1	1	1	1	0	1	0
CEC-20/06	1	1	0	1	1	1	1	1	1
CEC-20/07	1	1	1	1	1	1	0	1	0
CEC-20/08	1	1	1	1	1	1	0	1	1
CEC-20/09	1	1	1	1	1	1	1	1	0
CEC-20/10	1	1	-1	-1	1	1	-1	1	-1
+	9	10	6	8	10	9	5	9	5
=	1	0	1	1	0	1	4	1	4
-	0	0	3	1	0	0	1	0	1

integrating both swarm-oriented (updating the position of Humboldt squids and school fish) and evolution-oriented operators (Humboldt squid mating), and incorporating self-adaptive parameters to facilitate a shift from exploration to exploitation are among the defining features of HSOA that set it apart from other algorithms.

Nevertheless, the HSOA may have excellent and poor accuracy in some problems. One optimization algorithm can perform reasonably in a range of optimization problems, but not all because of the No Free Lunch Theorem (NFL). Hence, HSOA performed better in some problems than others [4].HSOA's superiority over other algorithms in certain

TABLE 24. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2022 benchmark functions over 10 dimensions, 30 runs and 200000 number function evaluations.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
	Min	300.00	300.00	300.00	316.65	300.04	1160.05	300.15	300.00	300.00	300.00
	Mean	300.00	300.00	300.00	356.61	300.62	4839.82	300.38	300.00	300.00	300.00
CEC-22/01	Median	300.00	300.00	300.00	351.08	300.24	3853.13	300.36	300.00	300.00	300.00
	Max	300.00	300.00	300.00	454.43	303.72	11269.51	300.75	300.00	300.00	300.00
	Std	0.00	0.00	0.00	29.44	0.83	2794.87	0.15	0.00	0.00	0.00
	Min	400.00	400.01	400.00	406.54	400.19	400.00	400.11	400.00	400.00	400.00
	Mean	402.74	407.34	402.01	407.50	410.25	427.75	418.31	404.74	405.92	402.96
CEC-22/02	Median	400.00	406.77	400.29	407.53	405.27	408.92	408.92	403.99	406.45	400.00
	Max	408.92	468.64	408.92	408.92	487.26	475.66	473.01	408.92	408.92	408.92
	Std	3.76	12.21	3.15	0.52	18.91	29.92	25.39	2.63	3.30	3.78
	Min	600.00	600.00	600.00	600.00	600.01	606.79	601.24	600.00	600.00	600.00
	Mean	600.00	603.47	600.00	600.00	600.04	628.38	619.76	600.00	600.01	600.00
CEC-22/03	Median	600.00	603.25	600.00	600.00	600.03	627.03	619.10	600.00	600.00	600.00
	Max	600.00	613.48	600.00	600.00	600.21	654.85	642.80	600.00	600.07	600.00
	Std	0.00	3.21	0.00	0.00	0.04	12.24	9.87	0.00	0.02	0.00
	Min	800.00	811.94	802.98	806.01	805.91	811.94	812.95	801.99	805.97	801.00
	Mean	800.63	825.82	810.78	811.44	814.00	831.46	823.43	803.32	818.46	801.99
CEC-22/04	Median	800.99	825.87	809.95	811.76	812.78	827.87	825.42	802.99	816.91	801.99
	Max	801.99	846.76	819.90	815.32	827.23	869.65	834.87	804.98	839.66	803.98
	Std	0.67	9.28	3.89	2.56	5.11	14.32	6.45	1.09	8.14	0.74
	Min	900.00	906.11	900.00	900.00	900.00	920.69	1107.29	900.00	900.00	900.00
	Mean	900.00	1131.60	900.02	900.00	900.10	1191.31	1303.74	900.00	901.29	900.00
CEC-22/05	Median	900.00	1052.23	900.00	900.00	900.08	1144.35	1293.81	900.00	900.95	900.00
	Max	900.00	1530.79	900.18	900.00	900.46	2096.12	1604.04	900.00	905.09	900.00
	Std	0.00	213.60	0.04	0.00	0.12	241.28	133.05	0.00	1.43	0.00
	Min	1800.18	1865.64	1820.11	1924.35	1906.25	1961.56	1877.05	1800.01	1832.04	1800.00
	Mean	1800.32	3730.14	3584.95	2353.08	5042.73	3793.29	3086.23	1800.38	1900.10	1800.18
CEC-22/06	Median	1800.33	3426.21	2919.82	2269.18	3450.43	2988.09	2203.38	1800.44	1882.24	1800.16
	Max	1800.48	8037.44	7942.31	3013.58	25018.42	8049.57	8087.37	1801.08	2061.60	1800.50
	Std	0.09	1654.50	1772.03	342.48	4425.24	1932.28	1936.34	0.21	62.02	0.13
	Min	2000.01	2004.67	2000.00	2000.00	2001.17	2004.73	2022.02	2000.00	2000.99	2000.00
	Mean	2000.07	2025.01	2014.27	2000.29	2023.16	2050.29	2034.39	2000.05	2019.46	2000.00
CEC-22/07	Median	2000.04	2024.30	2020.99	2000.10	2024.94	2048.51	2031.34	2000.00	2021.99	2000.00
	Max	2000.25	2042.57	2023.98	2002.50	2036.83	2106.15	2075.66	2000.34	2032.78	2000.00
	Std	0.05	6.78	9.59	0.52	8.24	21.95	12.89	0.09	8.23	0.00
	Min	2200.39	2203.86	2200.05	2205.88	2207.88	2220.09	2209.63	2200.26	2200.34	2200.12
	Mean	2201.67	2220.38	2222.34	2211.58	2223.82	2232.02	2228.07	2203.48	2218.05	2201.32
CEC-22/08	Median	2201.15	2221.01	2220.29	2210.57	2223.99	2231.08	2227.79	2201.60	2220.36	2200.31
	Max	2207.22	2224.72	2338.71	2220.77	2230.13	2251.85	2259.44	2220.34	2221.74	2220.22
	Std	1.53	4.44	22.83	3.94	4.06	6.30	7.43	4.14	6.67	3.84
	Min	2529.28	2529.28	2529.28	2529.28	2486.09	2529.29	2529.28	2529.28	2529.28	2529.28
	Mean	2529.28	2529.28	2534.18	2529.28	2488.39	2530.69	2529.30	2529.28	2534.18	2529.28
CEC-22/09	Median	2529.28	2529.28	2529.28	2529.28	2488.08	2529.33	2529.29	2529.28	2529.28	2529.28
	Max	2529.28	2529.28	2676.22	2529.28	2493.71	2565.69	2529.48	2529.28	2676.22	2529.28
	Std	0.00	0.00	26.83	0.00	1.83	6.62	0.04	0.00	26.83	0.00
	Min	2500.16	2500.32	2500.12	2439.45	2500.15	2500.30	2500.29	2500.16	2500.20	2500.14
	Mean	2500.25	2518.01	2554.78	2497.95	2541.71	2526.31	2558.08	2503.77	2500.35	2500.20
CEC-22/10	Median	2500.25	2500.55	2553.29	2500.33	2500.32	2500.67	2500.93	2500.22	2500.35	2500.20
	Max	2500.31	2641.23	2615.15	2500.42	2619.36	2644.32	2653.30	2606.78	2500.56	2500.26
	Std	0.03	45.49	55.53	11.21	55.45	52.32	67.27	19.46	0.08	0.03
	Min	2600.00	2600.00	2600.00	2600.00	2600.22	2600.48	2602.03	2600.00	2600.00	2600.00
	Mean	2600.00	2722.92	2610.00	2609.44	2658.42	2766.14	2735.81	2600.00	2676.67	2600.00
CEC-22/11	Median	2600.00	2600.00	2600.00	2600.00	2600.71	2751.09	2750.51	2600.00	2600.00	2600.00
	Max	2600.00	3183.57	2900.00	2748.85	3000.11	3000.25	3001.47	2600.00	3000.00	2600.00
	Std	0.00	165.93	54.77	31.13	132.54	139.69	138.43	0.00	156.87	0.00
	Min	2859.42	2861.40	2859.54	2858.62	2846.24	2862.57	2863.46	2858.62	2862.57	2862.70
	Mean	2861.99	2864.52	2864.13	2859.50	2848.65	2877.64	2877.19	2861.33	2863.79	2864.59
CEC-22/12	Median	2862.72	2864.79	2864.93	2859.37	2848.63	2870.43	2869.99	2861.44	2863.49	2864.92
	Max	2864.24	2868.11	2866.68	2861.44	2853.42	2947.29	2955.27	2864.93	2866.66	2864.93
	Std	1.33	1.56	1.57	0.87	1.65	17.78	20.90	1.75	1.35	0.72

problems can be attributed to its access to stored answers in its memory during the search process. While other algorithms only use previous iteration answers, HSOA saves all unique past generation answers in its memory. The probability of HSOA falling into the local optimal trap is reduced by this method. The domain of the problem is searched by HSOA using two populations, and the interaction of these two populations leads to more diverse answers. One additional rationale behind the superiority of HSOA is attributed to the utilization of multiple mating and self-adaptive weights, leading to the creation of a good between global and local search.

TABLE 25. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2022 benchmark functions over 20 dimensions, 30 runs and 1000000 number function evaluations.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
	Min	300.00	300.00	300.00	300.02	300.59	300.82	300.54	300.00	300.00	300.00
	Mean	300.00	300.00	300.00	300.70	307.72	318.10	301.81	300.00	300.00	1048.74
CEC-22/01	Median	300.00	300.00	300.00	300.50	304.10	311.77	301.77	300.00	300.00	300.00
	Max	300.00	300.00	300.00	302.91	372.99	359.08	303.78	300.00	300.00	14075.45
	Std	0.00	0.00	0.00	0.64	13.19	17.62	0.68	0.00	0.00	2926.71
	Min	400.00	400.03	400.01	447.75	414.80	406.17	408.73	444.90	400.00	400.00
	Mean	440.62	435.93	449.62	449.04	423.70	455.35	458.32	448.94	431.07	435.02
CEC-22/02	Median	449.08	449.08	449.08	449.08	419.20	449.14	458.44	449.08	444.90	448.60
	Max	449.08	471.77	473.53	449.08	494.64	478.95	479.53	449.08	449.08	449.08
	Std	18.51	24.08	15.87	0.24	17.40	15.41	17.19	0.76	22.66	21.58
	Min	600.00	601.47	600.00	600.00	600.01	632.86	613.60	600.00	600.21	600.00
	Mean	600.00	610.17	600.05	600.00	600.11	659.02	639.44	600.00	605.60	600.00
CEC-22/03	Median	600.00	609.76	600.00	600.00	600.03	660.18	641.63	600.00	602.47	600.00
	Max	600.00	635.30	600.92	600.00	601.03	684.81	655.80	600.00	621.46	600.00
	Std	0.00	7.14	0.18	0.00	0.21	13.01	8.74	0.00	6.12	0.00
	Min	800.99	821.89	818.90	832.78	828.60	853.73	848.03	805.97	829.85	803.98
	Mean	802.95	891.64	840.89	849.24	847.31	906.63	884.98	809.19	864.49	807.59
CEC-22/04	Median	802.99	895.02	835.82	849.89	846.80	909.53	885.70	808.96	865.81	807.46
	Max	805.66	945.26	896.51	858.25	874.24	975.97	939.57	812.93	894.52	809.95
	Std	1.16	27.53	15.88	6.25	10.01	27.57	16.76	2.09	15.31	1.44
	Min	900.00	1555.50	900.00	900.00	900.10	1786.10	1950.66	900.00	907.88	900.00
	Mean	900.00	2309.63	901.37	900.00	918.78	3396.97	2392.77	900.00	1147.51	900.00
CEC-22/05	Median	900.00	2318.32	900.81	900.00	900.49	3106.18	2398.49	900.00	1091.59	900.00
	Max	900.00	3195.54	911.06	900.00	1445.77	6607.63	2867.50	900.00	1628.37	900.00
	Std	0.00	336.08	2.00	0.00	99.53	1273.67	239.09	0.00	172.37	0.00
	Min	1801.79	1951.98	1855.79	7025.21	1952.19	1952.55	2146.06	1800.08	1821.92	1800.13
	Mean	1825.72	5630.19	4490.35	36648.99	35945.61	7616.87	8683.00	1814.08	2853.00	1806.74
CEC-22/06	Median	1825.89	3336.00	2897.17	34658.01	3853.75	5909.37	5377.70	1806.57	1922.81	1804.39
	Max	1852.15	20524.58	16019.52	93231.14	905546.76	20441.66	22545.43	1851.00	23344.85	1826.23
	Std	15.54	4699.44	3565.79	17235.98	164329.92	5466.21	6331.67	14.25	3932.94	6.40
	Min	2000.38	2030.33	2021.89	2014.38	2028.32	2065.74	2032.85	2002.04	2027.44	2000.75
	Mean	2012.13	2081.34	2032.22	2022.84	2050.97	2156.93	2095.22	2015.40	2062.83	2007.11
CEC-22/07	Median	2008.57	2074.67	2029.92	2023.80	2048.56	2148.13	2091.51	2019.38	2059.28	2006.00
	Max	2023.62	2165.55	2057.15	2025.96	2087.06	2270.39	2170.71	2022.17	2140.67	2020.67
	Std	8.24	33.16	8.33	2.61	13.42	46.26	34.46	6.98	25.10	5.71
	Min	2200.61	2221.49	2220.08	2222.70	2221.21	2226.36	2229.67	2219.31	2220.93	2200.38
	Mean	2218.81	2228.75	2244.79	2223.46	2236.31	2244.66	2241.76	2220.28	2226.98	2215.22
CEC-22/08	Median	2220.53	2224.18	2221.13	2223.46	2228.90	2241.65	2233.42	2220.43	2222.47	2220.25
	Max	2221.54	2243.97	2340.02	2224.12	2350.57	2311.71	2351.16	2220.72	2254.11	2220.97
	Std	5.63	8.32	48.12	0.33	30.28	16.75	24.61	0.39	8.56	8.59
	Min	2480.78	2480.78	2480.78	2480.78	2465.80	2480.79	2480.85	2480.78	2480.78	2480.78
	Mean	2480.78	2480.78	2480.78	2480.78	2467.02	2481.14	2481.33	2480.78	2480.78	2480.78
CEC-22/09	Median	2480.78	2480.78	2480.78	2480.78	2466.73	2480.85	2481.24	2480.78	2480.78	2480.78
	Max	2480.78	2480.78	2480.78	2480.78	2468.68	2482.49	2482.00	2480.78	2480.78	2480.78
	Std	0.00	0.00	0.00	0.00	0.87	0.46	0.31	0.00	0.00	0.00
	Min	2500.27	2439.29	2500.21	2400.00	2500.23	2500.55	2436.14	2500.27	2500.32	2500.24
	Mean	2500.32	2644.13	2684.75	2436.24	3109.11	3697.98	2805.02	2500.38	2551.64	2500.30
CEC-22/10	Median	2500.32	2541.36	2587.72	2417.50	3313.36	3945.18	2750.64	2500.38	2500.55	2500.30
	Max	2500.41	3154.11	3239.53	2500.39	4191.56	5511.50	3893.80	2500.46	3438.99	2500.36
	Std	0.03	202.77	225.00	40.21	594.51	1160.24	342.03	0.06	178.23	0.03
	Min	2600.00	2900.00	2600.00	2900.00	2604.19	2900.35	2923.73	2900.00	2600.00	2600.00
	Mean	2880.00	2923.33	2926.67	2900.00	2912.99	2955.87	2957.72	2906.67	2916.67	2896.67
CEC-22/11	Median	2900.00	2900.00	2900.00	2900.00	2917.24	2901.36	2933.85	2900.00	2900.00	2900.00
	Max	3000.00	3000.00	3000.00	2900.00	3000.14	3355.00	3353.80	3000.00	3000.00	3000.00
	Std	99.65	43.02	78.49	0.00	90.80	108.09	79.78	25.37	98.55	61.49
	Min	2930.81	2944.11	2933.39	2933.91	2900.00	2945.09	2953.95	2931.95	2938.62	2931.74
	Mean	2934.27	2964.87	2956.89	2934.67	2900.00	2985.54	3005.03	2935.00	2960.35	2935.47
CEC-22/12	Median	2932.82	2962.18	2954.97	2934.44	2900.00	2978.73	2991.40	2933.98	2958.90	2934.19
	Max	2944.43	3033.54	2998.52	2935.81	2900.00	3107.14	3084.39	2942.04	3011.33	2944.69
	Std	3.55	16.73	15.86	0.65	0.00	36.91	37.27	2.88	15.77	3.85

C. MATHEMATICAL FRAMEWORK FOR STABILITY ANALYSIS

In this study, the mathematical stability analysis of HSOA is performed based on the presented approach by study of [87]. In this approach, the actual solution of problem $x\_m(n, t)$ can be considered as a Fourier series solution. After replacing the solutions and components of HSOA equations with Fourier series and simplifying the resulting equations, it is possible to determine the range of parameters for stability of HSOA results. In this approach, the solution of the algorithm is

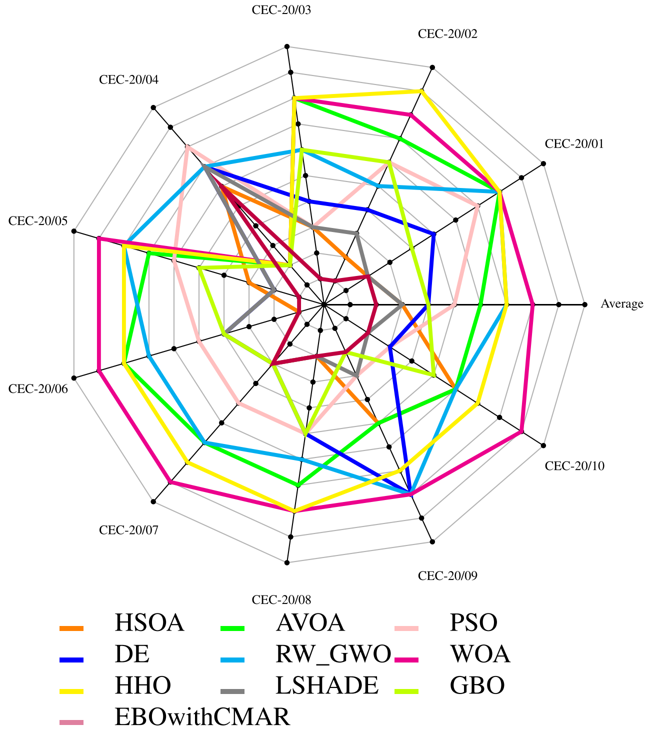


FIGURE 20. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 5 dimensions, 30 runs and 50000 number function evaluations.

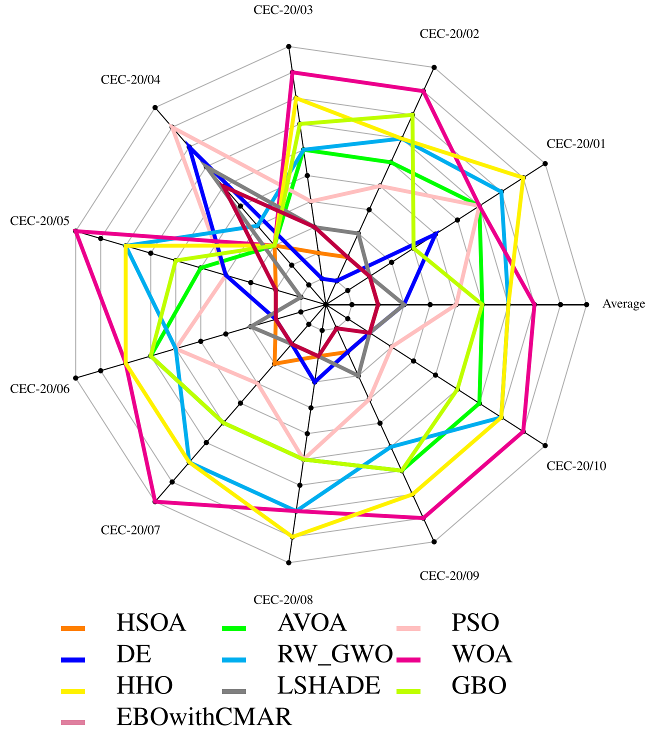


FIGURE 22. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 15 dimensions, 30 runs and 3000000 number function evaluations.

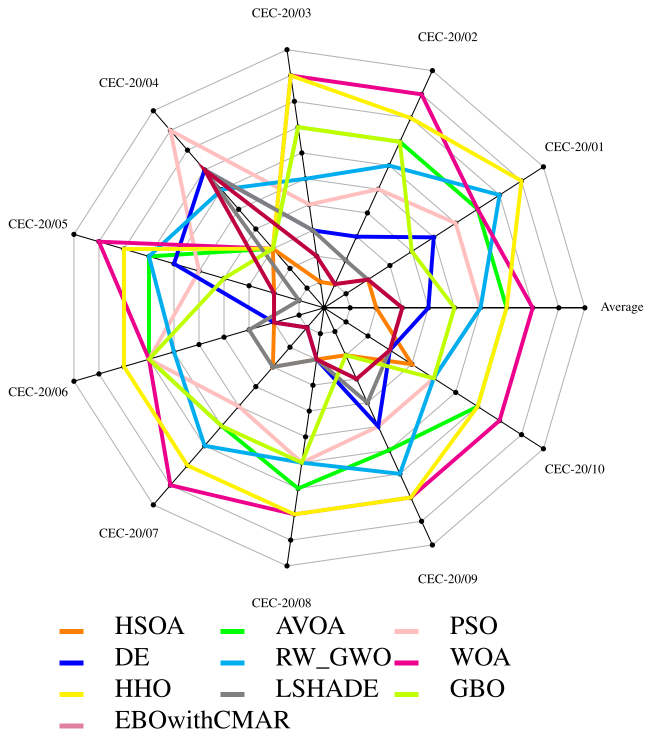


FIGURE 21. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 10 dimensions, 30 runs and 1000000 number function evaluations.

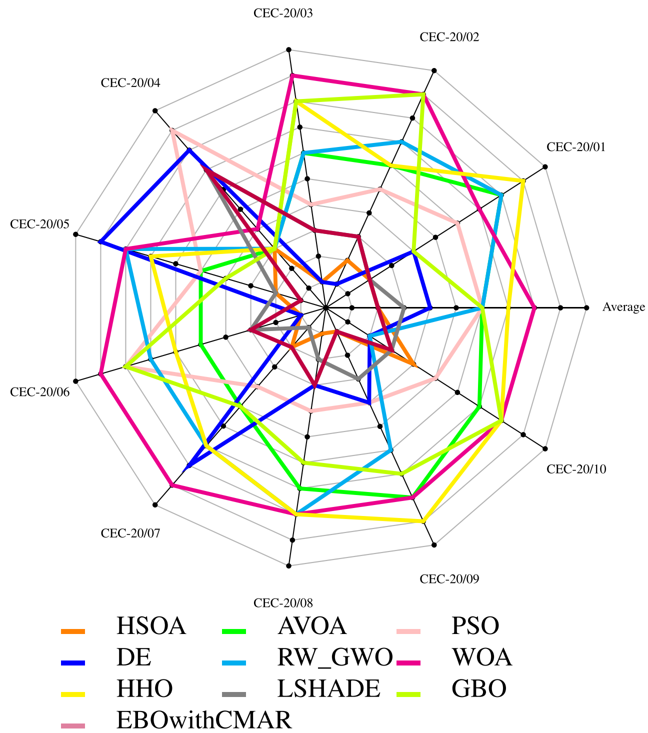


FIGURE 23. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2020 benchmark functions over 20 dimensions, 30 runs and 10000000 number function evaluations.

TABLE 26. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2022 benchmark functions over 10 dimensions, 30 runs and 200000 number function evaluations.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-22/01	1	0	1	1	1	1	0	1	0
CEC-22/02	1	0	1	1	1	1	0	0	-1
CEC-22/03	1	1	0	1	1	1	0	1	0
CEC-22/04	1	1	1	1	1	1	1	1	1
CEC-22/05	1	1	1	1	1	1	0	1	0
CEC-22/06	1	1	1	1	1	1	1	1	-1
CEC-22/07	1	1	0	1	1	1	-1	1	-1
CEC-22/08	1	1	1	1	1	1	0	1	-1
CEC-22/09	0	0	0	-1	1	1	0	0	0
CEC-22/10	1	0	1	1	1	1	-1	1	-1
CEC-22/11	1	1	1	1	1	1	0	1	0
CEC-22/12	1	1	-1	-1	1	1	-1	1	1
+	11	8	8	10	12	12	2	10	2
=	1	4	3	0	0	0	7	2	5
-	0	0	1	2	0	0	3	0	5

TABLE 27. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2022 benchmark functions over 10 dimensions, 30 runs and 200000 number function evaluations.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-22/01	1	1	1	1	1	1	0	1	0
CEC-22/02	1	1	1	-1	1	1	0	0	0
CEC-22/03	1	0	-1	1	1	1	0	1	-1
CEC-22/04	1	1	1	1	1	1	1	1	1
CEC-22/05	1	1	1	1	1	1	0	1	0
CEC-22/06	1	1	1	1	1	1	-1	1	-1
CEC-22/07	1	1	1	1	1	1	0	1	-1
CEC-22/08	1	1	1	1	1	1	0	1	-1
CEC-22/09	1	0	0	-1	1	1	0	1	0
CEC-22/10	1	0	-1	0	1	1	1	1	0
CEC-22/11	1	1	1	1	1	1	0	1	0
CEC-22/12	1	1	1	-1	1	1	0	1	0
+	12	9	9	8	12	12	2	11	1
=	0	3	1	1	0	0	9	1	7
-	0	0	2	3	0	0	1	0	4

given by:

$$x_m(n,t) = A_m.e^{r.(\sigma_m.n-\beta_m.t)} \tag{18}$$

In Eq. (18), $n - t$ is computation domain, $r = \sqrt{-1}$, A_m refer to m^{th} component, β_m known as the angular frequency of m^{th} component and σ_m is the wave number of m^{th} component. In HSOA, without loss of generality, the X_b and *pbest* can be taken as $x_{i\pm a,j}.XS_{i,j}$ and Eggs can be considered as $x_{i,j}.XF_{i,j}$, $PopAll_{r2}$, $ArchiveX$, pop_{r1} and pop_{r3} can take as $x_{i+N,j}$, $x_{i\pm b}, j$, $x_{i\pm c,j}$, $x_{i\pm d,j}$ and $x_{i\pm e,j}$, respectively. In term of grid points, Equation 1 (Attack of fish schools) can be rewritten as follows:

$$x_{i,j+1} = \theta.x_{i\pm a} + V_{jet}.(-x_{i+N,j} - x_{i\pm b,j}) \tag{19}$$

In Eq. (18), a is an integer number between 1 and N, N is Humboldt squids swarm's size, b is an integer number between 1 and archive size. The m^{th} component of Eq. (18) at point (n, t) is given by:

$$x_{m_{i,j}} = A_m.e^{-r.\beta_m.j.\Delta t}.e^{r.\sigma_m.i.\Delta n} \tag{20}$$

By substituting Eq. (20) in Eq. (19), the amplification factor is obtained in Eq. (21).

$$e^{-\beta_m.j.\Delta t} = \theta.e^{\pm\sigma_m.a.\Delta n} + V_{jet}.(-e^{\sigma_m.N.\Delta n} - e^{\pm\sigma_m.b.\Delta n}) \tag{21}$$

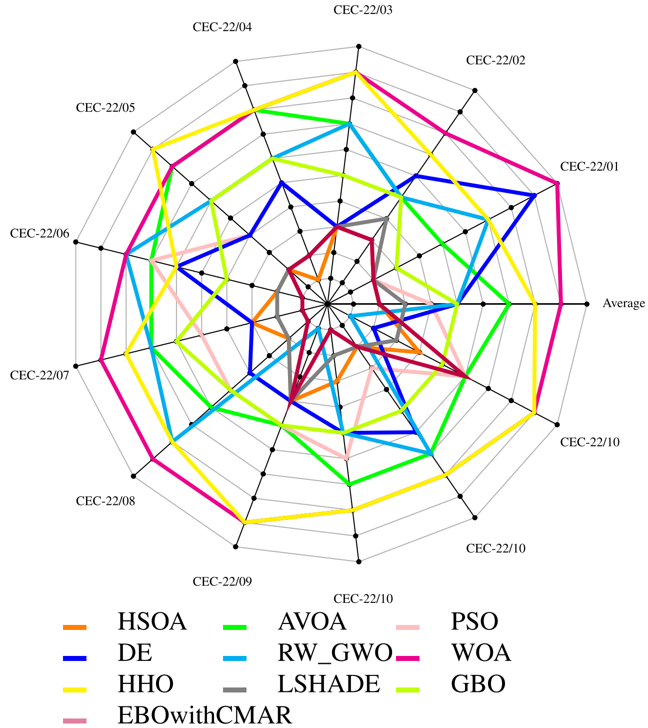


FIGURE 24. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2022 benchmark functions over 10 dimensions, 30 runs and 200000 number function evaluations.

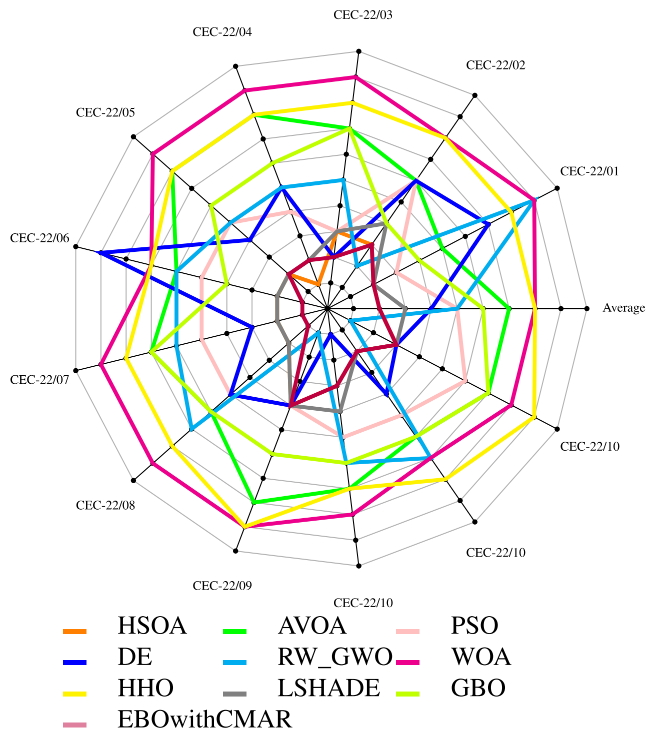


FIGURE 25. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2022 benchmark functions over 20 dimensions, 30 runs and 1000000 number function evaluations.

TABLE 28. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2006 real-world problems over 51 runs.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
	Min	0.012665	0.012665	0.012666	0.012779	0.012691	0.012687	0.012683	0.012665	0.012665	0.012665
	Mean	0.012689	0.013514	0.013362	0.013111	0.012814	0.014086	0.013978	0.012666	0.01281	0.012669
CEC-06/01	Median	0.012678	0.013202	0.013163	0.013037	0.012745	0.013545	0.013574	0.012666	0.012718	0.012667
	Max	0.01277	0.015374	0.015256	0.013612	0.014116	0.017776	0.017776	0.012667	0.013727	0.012691
	Std	2.62E-05	0.000853	0.00071	0.00022	0.000232	0.001339	0.00118	3.54E-07	0.000221	5.57E-06
	Min	2.7E-12	2.7E-12	2.7E-12	2.7E-12	2.7E-12	2.7E-12	2.7E-12	2.7E-12	2.7E-12	2.7E-12
	Mean	2.7E-12	2.29E-09	6.43E-10	8.72E-10	3.45E-10	4.6E-09	2.57E-09	6.3E-12	6.83E-10	5.9E-12
CEC-06/02	Median	2.7E-12	9.75E-10	1.17E-10	8.89E-10	2.31E-11	1.36E-09	9.92E-10	2.7E-12	1.17E-10	2.7E-12
	Max	2.7E-12	2.73E-08	4.5E-09	3.82E-09	2.36E-09	2.73E-08	1.83E-08	2.31E-11	4.5E-09	2.31E-11
	Std	0	5.36E-09	9.84E-10	8.22E-10	5.34E-10	7.34E-09	4.77E-09	7.85E-12	8.77E-10	7.48E-12
	Min	2749.583	2749.583	2749.583	2749.583	2751.004	2749.585	2749.628	2749.583	2749.583	2749.583
	Mean	2749.583	2750.277	2749.583	2749.583	2759.262	2886.487	2759.817	2749.583	2749.583	2749.583
CEC-06/03	Median	2749.583	2749.826	2749.583	2749.583	2759.271	2776.884	2756.177	2749.583	2749.583	2749.583
	Max	2749.583	2754.017	2749.584	2749.583	2770.045	4205.244	2808.028	2749.584	2749.583	2749.583
	Std	5.25E-08	0.991162	0.000132	1.06E-05	4.285455	284.0221	11.73525	0.000125	2.88E-07	1.25E-05
	Min	464.6511	464.7991	464.6766	466.0524	464.9195	478.7978	468.2258	464.6534	464.6795	464.6518
	Mean	464.6514	469.0961	464.8734	471.8558	467.4133	552.2649	479.4059	464.6646	465.0317	464.6575
CEC-06/04	Median	464.6512	467.0744	464.8436	471.3399	466.8249	550.9838	476.34	464.663	464.9094	464.655
	Max	464.6538	498.3122	465.3052	479.7259	474.2714	650.8336	528.1654	464.6869	467.2645	464.6854
	Std	0.000462	5.449119	0.140364	3.050324	2.296694	44.24396	10.28898	0.007829	0.441274	0.006145
	Min	384.7746	384.8836	384.8033	385.2214	385.0625	390.2523	386.5532	384.7792	384.7784	384.7758
	Mean	384.775	385.7153	385.004	386.0888	386.3423	471.0938	397.622	384.7895	384.988	384.7824
CEC-06/05	Median	384.7747	385.5891	384.9374	386.0461	386.4017	465.4706	396.082	384.7874	384.9142	384.7801
	Max	384.7784	387.5397	385.8221	387.535	388.3024	624.0185	418.6983	384.8128	385.4403	384.8071
	Std	0.000871	0.582077	0.195589	0.48061	0.729133	60.32003	7.569989	0.007404	0.184258	0.006799
	Min	455.3457	455.4967	455.3696	457.9681	455.7998	461.8094	462.4592	455.3505	455.359	455.3471
CEC-06/06	Mean	455.3459	459.9406	455.5531	460.955	457.7934	552.398	479.4522	455.3685	455.6375	455.3516
	Median	455.3458	457.9526	455.5522	460.8147	457.5453	545.452	475.3368	455.3664	455.5076	455.3505
	Max	455.3474	510.2169	455.875	467.1519	461.1604	682.9102	519.7142	455.4041	457.9322	455.3612
	Std	0.000304	8.381795	0.12376	1.92297	1.182555	49.8809	13.86532	0.012617	0.404846	0.003353

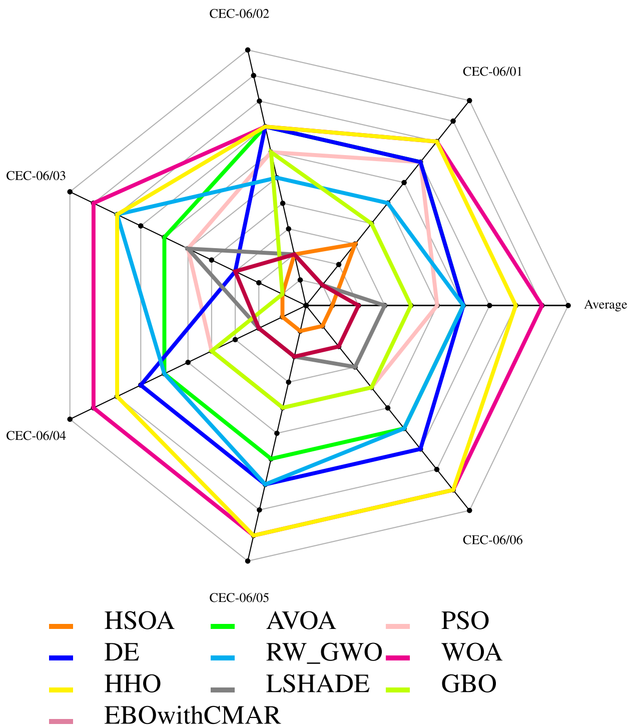


FIGURE 26. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2006 benchmark functions over 25 runs.

To achieve marginal stability, the magnitude of amplification factor $|e^{-r.\beta_m.j.\Delta t}|$ must be equal to one. Which

TABLE 29. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2006 benchmark functions over 51 runs.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-06/01	1	1	1	1	1	1	-1	1	-1
CEC-06/02	1	1	1	1	1	1	1	1	1
CEC-06/03	1	1	1	1	1	1	1	1	1
CEC-06/04	1	1	1	1	1	1	1	1	1
CEC-06/05	1	1	1	1	1	1	1	1	1
CEC-06/06	1	1	1	1	1	1	1	1	1
+	6	6	6	6	6	6	5	6	5
=	0	0	0	0	0	0	0	0	0
-	0	0	0	0	0	0	1	0	1

leads to:

$$|e^{-r.\beta_m.j.\Delta t}| = |\theta.e^{\pm r.\sigma_m.a.\Delta n} + V_{jet}.(-e^{\sigma_m.N.\Delta n} - e^{\pm\sigma_m.b.\Delta n})| \tag{22}$$

$$|e^{-r.\beta_m.j.\Delta t}| \leq |\theta.e^{\pm r.\sigma_m.a.\Delta n}| + |V_{jet}.(-e^{\sigma_m.N.\Delta n} - e^{\pm\sigma_m.b.\Delta n})| \tag{23}$$

$$|e^{-r.\beta_m.j.\Delta t}| \leq |\theta| + |2.V_{jet}| \tag{24}$$

The Eq. (1) is marginally stable if $|e^{-r.\beta_m.j.\Delta t}| = |\theta| + |2.V_{jet}| = 1$. Thus:

$$|\theta| = 1 - 2.|V_{jet}| \tag{25}$$

Since $|\theta| \geq 0$, we have $1 - 2.|V_{jet}| \geq 0$ or $|V_{jet}| \leq \frac{1}{2}$. In other words, Eq. (1) is marginally stable when $V_{jet} \in [\frac{-1}{2}, \frac{1}{2}]$. In HSOA, with increasing generation, V_{jet} will be close to zeros, which will therefore result in HSOA being stable according to Eq. (1). The Eq. (3) (Successful escape)

TABLE 30. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2011 real-world problems over 25 runs.

Function		HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
	Min	0.00	0.00	6.53	0.00	0.02	11.49	2.06	0.00	0.00	0.00
	Mean	0.00	15.73	16.48	4.60	13.46	19.05	19.41	0.02	14.66	0.00
CEC-11/01	Median	0.00	16.66	14.93	3.80	12.32	19.85	21.42	0.01	14.91	0.00
	Max	0.00	22.75	24.14	12.84	21.97	24.03	25.10	0.09	24.75	0.00
	Std	0.00	5.15	5.28	4.09	5.72	3.77	4.89	0.03	6.71	0.00
	Min	-27.99	-28.42	-27.52	-13.65	-27.40	-26.31	-22.27	-25.25	-28.42	-42.43
	Mean	-25.72	-25.89	-23.90	-11.86	-24.58	-19.28	-14.95	-23.72	-26.70	-28.98
CEC-11/02	Median	-25.87	-26.52	-25.33	-11.76	-25.36	-19.52	-15.17	-23.74	-26.52	-27.55
	Max	-24.11	-17.06	-11.11	-10.46	-20.27	-12.11	-9.73	-21.22	-25.34	-26.46
	Std	0.86	2.37	4.34	0.69	1.83	3.92	2.82	1.11	0.81	3.41
	Min	13.77	13.77	13.77	13.77	-	13.77	13.77	13.77	13.77	13.77
	Mean	13.77	13.93	15.74	19.04	-	13.86	14.06	14.08	13.96	14.56
CEC-11/03	Median	13.77	13.77	14.33	20.82	-	13.77	13.98	14.33	13.85	14.33
	Max	13.77	14.33	20.96	21.54	-	14.33	14.33	14.33	14.33	20.96
	Std	0.00	0.26	3.00	2.92	0.00	0.21	0.27	0.28	0.24	1.39
	Min	-29.13	-29.17	-21.07	-21.84	-29.03	-29.16	-26.33	-29.16	-22.71	-139.58
	Mean	-27.84	-22.77	68.96	639993.58	-23.22	-19.58	-17.44	-29.15	599992.22	119927.45
CEC-11/04	Median	-27.40	-23.01	28.67	999999.00	-23.01	-20.34	-16.84	-29.15	999999.00	-77.89
	Max	-22.91	-15.09	259.18	999999.00	-19.51	-12.42	-12.37	-29.09	999999.00	999999.00
	Std	1.38	3.64	89.24	489905.33	2.39	3.94	3.20	0.02	500008.48	331689.44
	Min	0.68	0.50	0.71	1.14	0.50	1.29	1.51	1.02	0.86	0.50
	Mean	1.07	0.92	0.94	1.45	1.12	1.84	1.79	1.24	1.12	0.61
CEC-11/05	Median	1.08	0.96	0.98	1.48	0.95	1.84	1.76	1.26	1.11	0.60
	Max	1.26	1.76	1.23	1.62	2.24	2.11	2.09	1.41	1.48	0.94
	Std	0.13	0.42	0.15	0.11	0.45	0.19	0.16	0.10	0.17	0.10
	Min	220.00	-	220.00	220.00	-	220.00	220.00	220.00	220.00	220.00
	Mean	220.00	-	276.86	220.00	-	260.60	253.60	220.00	221.24	220.00
CEC-11/06	Median	220.00	-	220.00	220.00	-	251.00	238.00	220.00	220.00	220.00
	Max	220.00	-	441.81	220.00	-	323.00	292.00	220.00	251.00	220.00
	Std	0.00	-	77.99	0.00	-	30.21	33.61	0.00	6.20	0.00
	Min	1415.76	19706.92	141739.85	70489.55	7043.95	85737.56	636544.57	1445.63	1650.06	951.89
	Mean	2507.69	46207.16	497339.93	77454.95	16423.15	329594.72	866109.74	2320.93	27312.16	2746.01
CEC-11/07	Median	2381.62	45322.32	490832.51	78416.59	11160.75	277460.83	829889.65	2172.22	6255.91	2486.81
	Max	3820.23	94640.56	937154.93	86909.05	40216.26	988037.86	1412178.14	3668.35	244113.32	6191.53
	Std	642.06	16504.72	191825.36	4452.09	9401.70	187525.11	172888.68	611.83	53248.10	1269.08
	Min	-21.81	-	-21.53	-20.47	-21.53	-13.85	-13.06	-21.76	-21.52	-21.63
	Mean	-21.64	-	-15.83	-18.20	-16.31	-11.08	-11.22	-21.54	-16.13	-21.52
CEC-11/08	Median	-21.60	-	-13.89	-17.91	-13.92	-11.05	-11.12	-21.58	-15.48	-21.58
	Max	-21.54	-	-10.69	-16.76	-10.71	-9.72	-10.17	-21.38	-10.49	-21.41
	Std	0.09	-	4.73	1.17	4.33	1.14	0.74	0.11	4.41	0.10
	Min	51884.18	50839.74	73657.95	2671395.85	179725.51	1079038.97	844051.65	51421.34	51516.81	49795.05
	Mean	53058.74	51931.42	977042.57	8108830.36	1226566.54	1283885.59	1077297.57	52340.32	52332.50	54633.15
CEC-11/09	Median	52993.60	51966.51	864259.83	7906327.72	1166252.02	1289379.36	1075151.78	52431.04	52272.64	51036.64
	Max	54005.53	52886.07	2162549.98	17822571.76	2791576.91	1517540.11	1307857.18	53085.25	53129.64	140531.31
	Std	549.92	584.81	567785.02	3554121.32	696306.91	127610.82	107536.82	443.53	416.53	17905.28
	Min	1071491.53	1064604.64	1085435.80	2167360.78	1081583.43	4754050.00	5055128.46	1070988.13	1066501.11	1078713.57
	Mean	1074664.08	1100215.05	1150668.14	2424708.11	1164355.05	5734290.01	5636371.99	1073658.22	1089573.42	1091056.62
CEC-11/10	Median	1074433.49	1076811.32	1125337.72	2409397.75	1145458.89	5705051.15	5732391.42	1073465.79	1076630.86	1089220.71
	Max	1079819.02	1234382.92	1432486.28	2592310.04	1305362.44	6836173.02	6136753.67	1077117.75	1170827.71	1116992.41
	Std	1862.76	42386.63	75513.32	108410.33	61487.58	517550.00	334879.67	1430.31	26038.43	8036.46
	Min	15444.19	15459.24	15447.53	15451.83	15444.85	15482.09	15448.95	15444.19	15451.72	15447.68
	Mean	15444.19	15509.64	15472.60	15466.50	15465.39	15542.90	15507.90	15444.19	15513.24	23783.56
CEC-11/11	Median	15444.19	15506.46	15467.74	15465.18	15464.02	15539.42	15502.58	15444.19	15489.84	15488.33
	Max	15444.24	15619.66	15514.91	15497.40	15517.96	15689.94	15713.47	15444.19	15603.33	82872.58
	Std	0.01	35.52	20.72	11.26	17.25	49.60	56.54	0.00	48.65	16470.80
	Min	18018.89	18751.11	18955.07	18864.00	18782.74	19045.79	18924.30	18018.81	18644.30	18117.65
	Mean	18101.34	19132.46	19236.57	19246.16	19162.64	19320.68	19331.89	18118.79	19010.07	18304.53
CEC-11/12	Median	18086.04	19148.63	19276.62	19229.57	19144.88	19328.97	19365.14	18113.04	19005.19	18288.00
	Max	18187.15	19417.13	19573.09	19571.02	19528.45	19535.33	19681.85	18253.92	19360.71	18639.64
	Std	48.92	141.26	170.96	206.68	214.78	138.79	187.60	56.40	162.56	134.20
	Min	32743.84	32878.03	32902.41	32820.39	32886.66	33017.61	33011.09	32740.27	32748.88	33221.98
	Mean	32766.14	33179.87	33065.87	32908.11	33028.46	68765.72	79767.09	32740.79	32976.34	231364.35
CEC-11/13	Median	32758.22	33180.67	33058.16	32896.03	33033.86	33330.44	33276.02	32740.56	32976.63	157042.72
	Max	32799.19	33444.90	33281.82	33017.64	33288.78	378760.69	351923.61	32743.42	33343.51	834765.88
	Std	19.34	131.17	94.80	44.91	93.90	81088.81	86648.34	0.69	118.44	235751.07
	Min	124160.72	128436.81	131901.91	132868.82	130831.47	136215.66	131465.71	124321.46	128658.44	123741.57
	Mean	125131.62	140780.47	138260.66	137048.59	138449.56	147430.49	147439.78	126600.93	142175.72	125055.74
CEC-11/14	Median	125180.00	140744.76	137783.90	136949.60	137405.52	148383.66	147541.42	126430.83	143535.65	124859.69
	Max	126155.52	163201.86	146011.73	142049.26	152004.76	156292.50	160652.12	128586.34	153778.42	127272.45
	Std	533.77	8359.81	4299.43	2319.69	5202.79	5251.41	5928.64	1066.73	6792.22	859.40
	Min	1896220.83	1930428.28	1945623.48	1944161.27	1945720.36	7726884358.46	2005470362.16	1861887.69	1927117.66	1855197.95
	Mean	1918521.45	2561734.09	2494342.73	2186020.42	2503682.90	11903068077.39	3941812140.22	1885476.33	2395618.07	1971252.82
CEC-11/15	Median	1918515.69	2284701.14	2427551.13	2153785.96	2200330.35	11447819359.27	3797990141.83	1886486.24	2353480.53	1939378.23
	Max	1937155.60	4399930.03	3962075.04	2521246.03	3890218.00	17528363172.57	6381060003.82	1913332.70	4529421.02	2283288.76
	Std	9953.36	675718.43	538192.41	169472.29	572426.41	2835687898.14	1140520813.39	12151.17	583799.27	103990.59
	Min	932621.23	941849.49	954638.61	1495756.99	951416.11	1126296.44	1143797.77	931788.50	940925.15	932554.77
	Mean	936843.55	1274993.49	978836.56	2264687.02	969450.84	3778221.29	2518310.61	937371.26	1184095.29	936770.93
CEC-11/16	Median	936753.36	955856.60	974694.73	2228997.20	968567.12	2657983.25	2600770.02	937265.55	955554.68	936959.84
	Max	939863.22	3168699.21	1113024.24	2956030.55	992366.83	14272534.86	3905498.51	941670.57	3140878.67	940695.62
	Std	1844.19	609751.60	29245.71	391035.75	9412.84	3045108.32	698322.05	2701.65	516519.40	2119.25

TABLE 30. Comparison of minimum, mean, median, maximum and standard deviation values for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2011 real-world problems over 25 runs.

CEC-11/17	Min	938530.45	1327658.14	1032135.13	2056007.54	1069126.57	1724267.96	1905637.22	937089.50	998182.72	937455.09
	Mean	941665.38	2046687.30	1258767.56	2828111.86	1309575.11	6264590.00	3297726.23	941015.11	1344472.06	942472.21
	Median	942079.88	1786470.00	1249938.18	2844296.39	1303127.52	4314947.66	2886428.18	940552.14	1279200.17	942014.10
	Max	944723.84	3751699.50	1487934.82	3416317.88	1663842.81	23442234.17	7092458.85	948157.10	2801185.32	953980.50
	Std	1591.32	676209.07	134492.73	327779.35	140301.90	5275163.04	1255903.48	2589.49	421179.08	3497.43
CEC-11/18	Min	933500.08	945322.70	957289.68	1533893.63	966100.40	1123529.06	1280726.03	933470.94	942804.32	932637.67
	Mean	936916.06	1316757.53	972938.11	2195326.57	987737.54	4475588.85	2367518.16	937300.70	1106441.65	936145.02
	Median	937665.00	1068889.56	972340.12	2109765.22	977394.59	2643035.90	1867048.70	937066.15	957536.60	936002.39
	Max	940271.94	2556828.08	987884.90	3014580.73	1097222.42	16686744.65	5039477.25	942275.84	2437343.25	939918.90
	Std	1740.81	506756.23	7858.08	398425.84	30763.56	4492618.92	1026801.36	2477.72	391839.26	1578.15

TABLE 31. Comparison of victory(+), equality(=) and defeat(−) for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2011 benchmark functions over 25 runs.

Function	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC-11/01	1	1	1	1	1	1	1	1	-1
CEC-11/02	-1	0	1	1	1	1	1	-1	-1
CEC-11/03	0	1	1	1	1	1	0	1	1
CEC-11/04	1	1	1	1	1	1	-1	1	-1
CEC-11/05	0	-1	1	0	1	1	1	0	-1
CEC-11/06	1	1	0	1	1	1	0	0	0
CEC-11/07	1	1	1	1	1	1	0	1	0
CEC-11/08	1	1	1	1	1	1	0	1	1
CEC-11/09	-1	1	1	1	1	1	-1	-1	-1
CEC-11/10	1	1	1	1	1	1	-1	1	1
CEC-11/11	1	1	1	1	1	1	-1	1	1
CEC-11/12	1	1	1	1	1	1	0	1	1
CEC-11/13	1	1	1	1	1	1	-1	1	1
CEC-11/14	1	1	1	1	1	1	1	1	0
CEC-11/15	1	1	1	1	1	1	-1	1	1
CEC-11/16	1	1	1	1	1	1	0	1	0
CEC-11/17	1	1	1	1	1	1	0	1	0
CEC-11/18	1	1	1	1	1	1	0	1	0
+	14	16	17	17	18	18	4	14	7
=	2	1	1	1	0	0	8	2	6
-	2	1	0	0	0	0	6	2	5

can be rewritten as follows:

$$x_{i+N,j+1} = \begin{cases} \theta.x_{i+N,j} + \overrightarrow{rn}. \\ (x_{i\pm a,j} - x_{i+N,j}).wf, & ifnfes < 0.1.max_{nfes}, \\ \theta.x_{i,j} + \overrightarrow{rn}.(x_{i\pm a,j} \\ -x_{i\pm b,j}).wf, & otherwise. \end{cases} \tag{26}$$

By putting Eq. (19) in Eq. (26), the amplification factor is computed in Eq. (27).

$$e^{-r.\beta_m j.\Delta t} = \begin{cases} \theta + \overrightarrow{rn}. \\ (e^{\pm r.\sigma_m.a.\Delta n} \\ -e^{r.\sigma_m.N.\Delta n}).wf, & ifnfes < 0.1.max_{nfes}, \\ \theta + \overrightarrow{rn}.(e^{\pm r.\sigma_m.a.\Delta n} \\ -e^{\pm r.\sigma_m.b.\Delta n}).wf, & otherwise. \end{cases} \tag{27}$$

For marginal stability, the magnitude of amplification factor $|e^{-r.\beta_m j.\Delta t}|$ must be equal to one. Simplifying the Eq. (27) gives the following equation:

$$1 = |\theta| + |2.\overrightarrow{rn}.wf| \tag{28}$$

$|\theta|$ is greater or equal to zero, thus $1 - 2.|\overrightarrow{rn}.wf| \geq 0$ or $|\overrightarrow{rn}.wf| \leq \frac{1}{2}$. In following, the stability of Eq. (6) (Humboldt squid mating) is analyzed as Eq. (1) and Eq. (3). Rewriting Eq. (6) get following equation:

$$x_{i,j+1} = (\omega.x_{i,j} + (1 - \omega.x_{i\pm a,j})).\gamma + (1-\gamma).x_{i\pm a,j} + W.(x_{i\pm e,j} - x_{i\pm b,j}) \tag{29}$$

By integrating Eq. (5) and Eq. (29), the amplification factor is given by:

$$e^{-r.\beta_m j.\Delta t} = (\omega + (1 - \omega.e^{\pm r.\sigma_m.a.\Delta n})).\gamma + (1-\gamma).e^{\pm r.\sigma_m.d.\Delta n} + W.(e^{\pm r.\sigma_m.e.\Delta n} - e^{\pm r.\sigma_m.b.\Delta n}) \tag{30}$$

As mentioned, for marginal stability, the magnitude of amplification factor $|e^{-r.\beta_m j.\Delta t)}|$ must be equal to one.

$$1 = |(\omega + (1 - \omega.e^{\pm r.\sigma_m.a.\Delta n})).\gamma| + |(1-\gamma).e^{\pm r.\sigma_m.d.\Delta n}| + |W.(e^{\pm r.\sigma_m.e.\Delta n} - e^{\pm r.\sigma_m.b.\Delta n})| \tag{31}$$

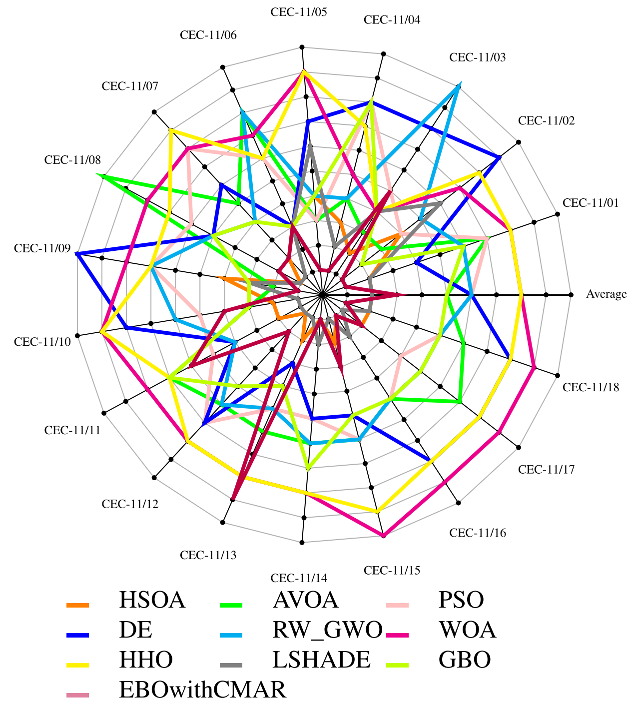


FIGURE 27. Comparison of Friedman ranking for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the CEC2011 benchmark functions over 25 runs.

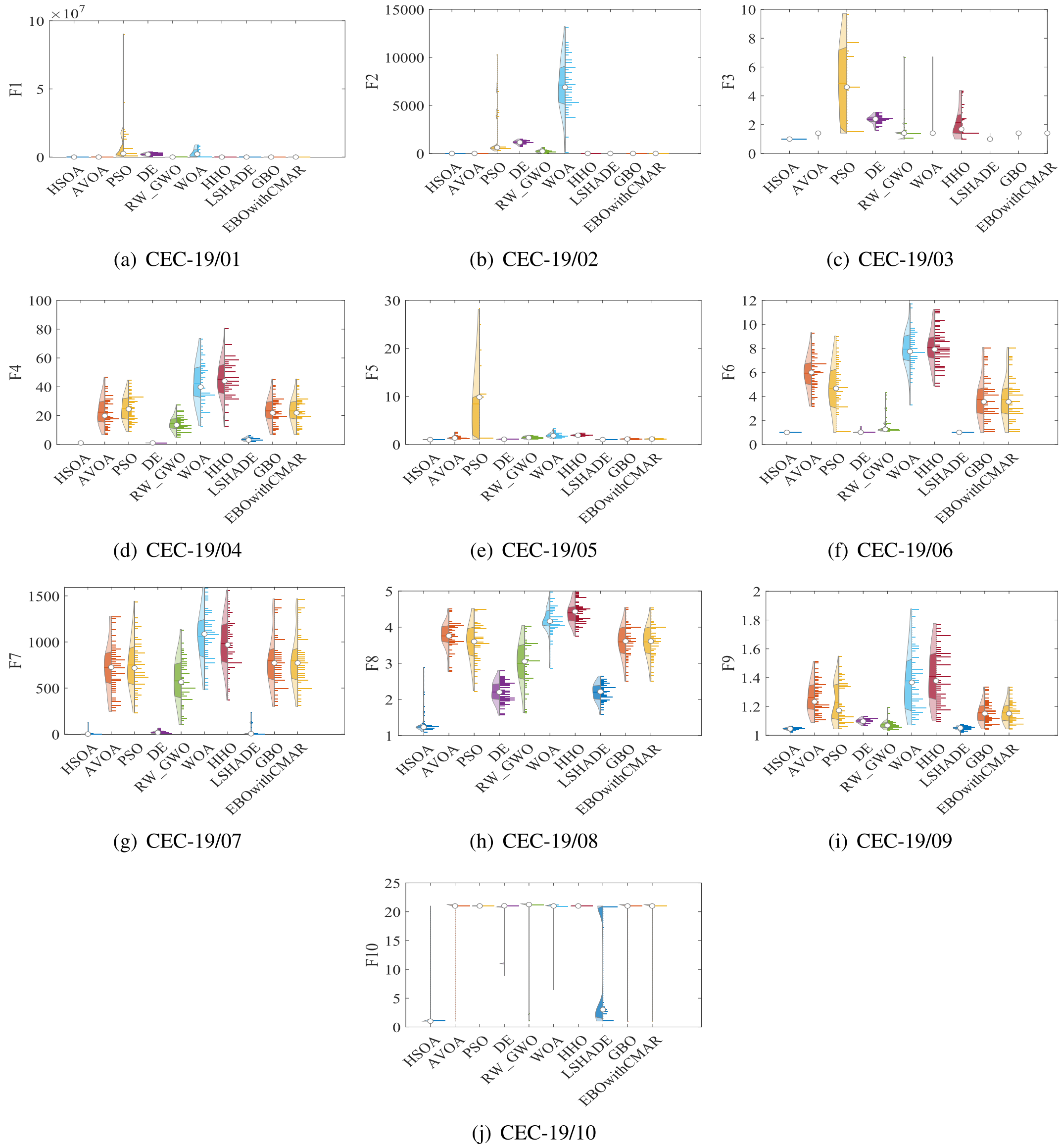


FIGURE 28. Violin plot of CEC2019 estimated by HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR.

By simplifying the Eq. 31 the following equation is obtained.

$$1 = |\omega.\gamma| + |\gamma| + |(\omega.e^{\pm r.\sigma_m.a.\Delta n}.\gamma| + |(1-\gamma).e^{\pm r.\sigma_m.d.\Delta n}| + |W.(e^{\pm r.\sigma_m.e.\Delta n} - e^{\pm r.\sigma_m.b.\Delta n})| \quad (32)$$

Or simply:

$$1 = |\omega.\gamma| + |\gamma| + |(1-\gamma)| + 2.|W| \quad (33)$$

The Humboldt squid mating operator will be marginally stable when:

$$1 - |\gamma| - |(1-\gamma)| - 2.|W| = |\omega.\gamma| \quad (34)$$

Sine $|\omega.\gamma| \geq 0$, we have $1-|\gamma|-|(1-\gamma)|-2.|W| \geq 0$ or $1-\gamma-1+\gamma-2.W \geq 0$. Which gives following equations:

$$2.|W| \leq 0 \quad (35)$$

Since $0 \geq W, \gamma, \omega \leq 1$, the HSOA mating operator is stable when W is equal to zero. In HSOA, γ and ω approach to zero with increasing generations. Therefore, W is close to zero in the last generations, which leads to the stability of the HSOA mating operator.

D. CONVERGENCE ANALYSIS

The study uses a criterion called average convergence rate, introduced by [88], to assess the performance of the investigated algorithms. This criterion is computed

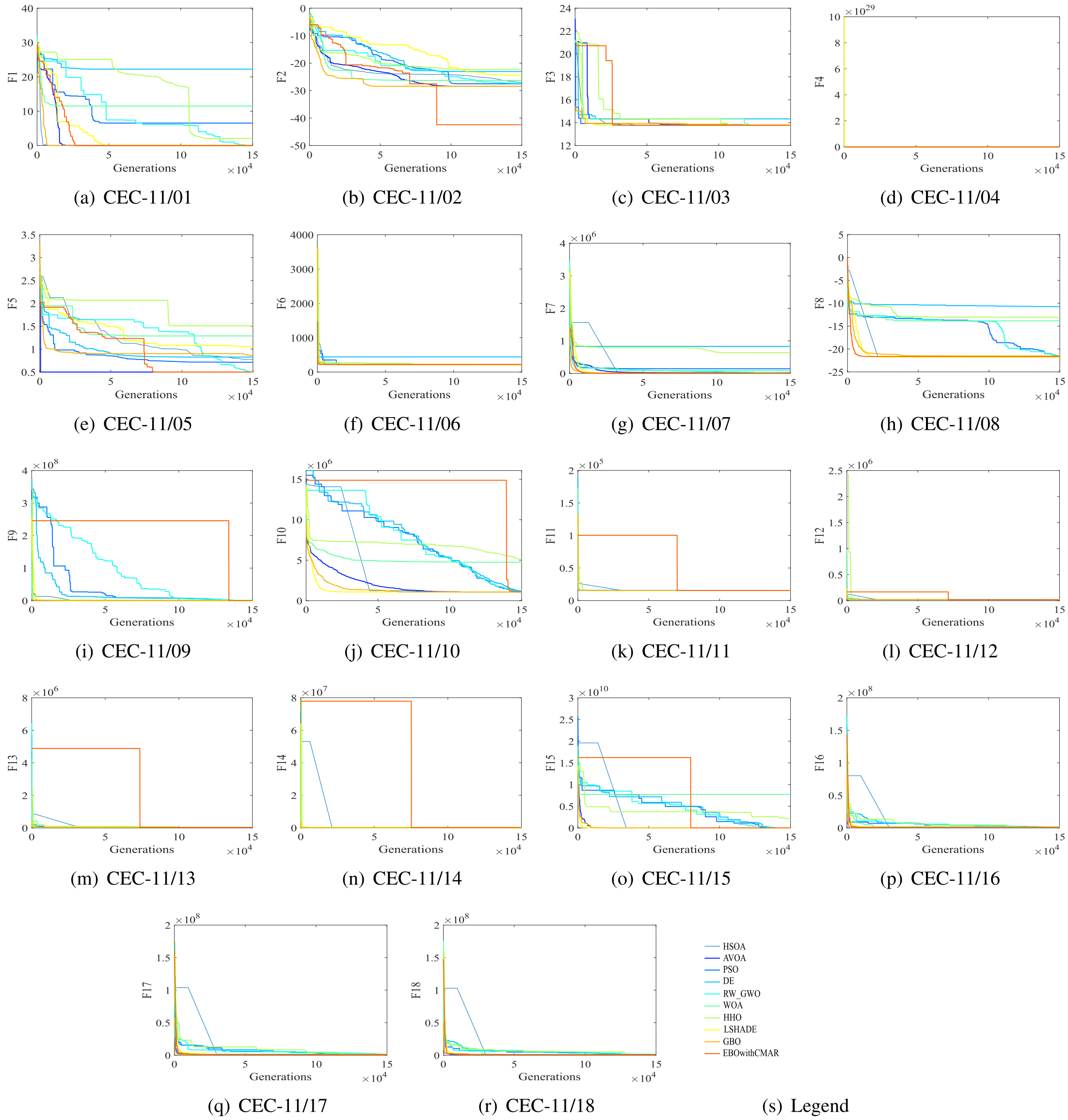


FIGURE 29. Convergence curve plot of CEC2011 estimated by HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR.

as follows:

$$CR = \left|\frac{fit_{opt} - \overrightarrow{fit}}{fit_{opt} - fit_0}\right| \tag{36}$$

In Eq. (36), fit_{opt}, $\overrightarrow{fit}$ and fit_0 are known as the global optimal, average fitness, and fitness at the beginning of the search.

Table 32 displays the results of the convergence rate analysis. According to the results, HSOA had a competitive performance in terms of convergence rate with other competitors. The convergence rate of HSOA was highest when it comes to CEC2006, classic, and CEC2019 functions. CEC2020 and CEC2022 functions also relate to the lowest value of the convergence rate's HSOA. The lower speed of convergence of HSOA in these two sets of benchmark functions was only related to some of their functions, and in general HSOA had good accuracy and speed.

E. COMPLEXITY ANALYSIS

The algorithm's complexity is determined by the computation of T0, T1 and T2 values [89]. T0 is estimated by evaluating mathematical functions. T1 is calculated by conducting 200000 evaluations of the 18th function in CEC2017, 1st function in CEC2020 [90], and CEC2022 [91] for all dimensions. T2 is obtained by averaging 5 algorithms executed on CEC18 (in CEC2017) and CEC01 (CEC2020 and CEC2022) with 200000 evaluations. The time complexity can be calculated as follows:

$$Time\ comlexity = \frac{T_2 - T_1}{T_0} \tag{37}$$

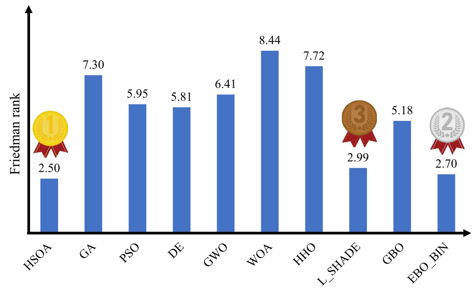


FIGURE 30. Average friedman ranking for on hundred and two benchmark functions and real-world problems.

TABLE 32. Convergence rate for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the classic, CEC2017, CEC2019, CEC2020, CEC2022, CEC2006, and CEC2011 benchmark functions.

Function	HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
Classic	0.94	0.96	0.94	0.91	0.93	0.96	0.98	0.96	0.97	0.96
CEC2017	0.89	0.90	0.91	0.86	0.87	0.87	0.82	0.92	0.90	0.92
CEC2019	0.92	0.75	0.72	0.80	0.79	0.67	0.68	0.93	0.77	0.89
CEC2020	0.71	0.91	0.93	0.92	0.92	0.91	0.90	0.92	0.90	0.93
CEC2022	0.79	0.90	0.90	0.89	0.90	0.88	0.84	0.92	0.87	0.92
CEC2006	1.00	0.92	0.89	0.88	0.97	0.94	0.87	0.92	0.91	0.89
CEC2011	0.82	0.83	0.83	0.75	0.71	0.82	0.79	0.93	0.91	0.69

TABLE 33. Algorithm complexity for HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR metaheuristic algorithms using the classic, CEC2017, CEC2019, CEC2020, CEC2022, CEC2006, and CEC2011 benchmark functions.

Function	HSOA	AVOA	PSO	DE	RW_GWO	WOA	HHO	LSHADE	GBO	EBOwithCMAR
CEC2017 (dim=10)	0.55	0.09	5.58	1.94	1.05	0.17	31.21	6.63	1.57	45.77
CEC2017 (dim=30)	0.89	12.20	4.31	0.93	1.60	2.60	49.30	9.02	0.55	113.91
CEC2017 (dim=50)	3.57	6.99	9.77	2.09	1.43	6.47	70.31	32.78	12.92	214.50
CEC2017 (dim=100)	1.58	21.63	6.37	5.93	8.11	12.55	136.15	24.19	21.44	373.97
CEC2020 (dim=5)	1.98	0.35	1.87	6.75	3.65	1.08	1988.45	1.79	1.89	52.69
CEC2020 (dim=10)	4.53	0.17	0.05	9.19	5.83	1.37	1670.09	19.21	1.42	39.29
CEC2020 (dim=15)	1.72	3.73	8.38	13.69	3.47	0.89	1910.49	7.49	0.10	33.69
CEC2020 (dim=20)	12.30	5.56	8.20	1.19	1.55	0.08	2007.30	2.00	0.61	34.42
CEC2022 (dim=10)	5.95	4.69	2.15	6.16	0.25	1.12	1467.61	6.18	2.56	53.95
CEC2022 (dim=20)	0.87	9.04	9.68	5.64	1.58	1.08	2339.70	0.00	1.37	57.20

Table 33 presents the time complexity values for different dimensions of CEC2017, CEC2020, and CEC2022. The results of HSOA's computation complexity were satisfactory compared to other investigated algorithms, as observed. The computational complexity analysis revealed that HSOA was less complex than EBOwithCMAR and can compete with LSHADE. Meanwhile, EBOwithCMAR and LSHADE had the second and third rank in accuracy. Therefore, it can be concluded that the proposed algorithm had a reasonable time complexity and acceptable accuracy.

VI. CONCLUSION

This study introduces a new nature-based optimization algorithm based on the Humboldt squid hunting behaviors called the HSOA. HSOA employs different operators for the attack of fish schools, escape of fish, successful attack, attack of stronger squid on smaller squids, and mating to update the population's position in the search domain. To change exploitation to exploration, adaptive weights with nonlinear and oscillation natural were defined. Eighty-two mathematical benchmark functions and twenty-four real-world problems were employed to validate the HSOA algorithm's efficiency in exploitation, exploration, escaping local optimum, and convergence speed. The results of HSOA in benchmark functions were superior to those of other well-known and recent natural-based optimization algorithms, such as AVOA, PSO, DE, RW_GWO, WOA, HHO, and GBO, and were competitors with state-of-the-art algorithms. Moreover, in the real-world optimization problem, HSOA had better results than well-known, recent natural-based optimization algorithms and state-of-the-art algorithms. HSOA's stability, convergence, and robustness were confirmed by the statistical analysis conducted. The Friedman ranking of HSOA, AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR for optimization all investigated problems were equal to 7.30, 5.95, 5.81, 6.41, 8.44, 7.72, 2.99, 5.18 and 2.70, respectively. Therefore, Friedman's ranking for HSOA was 65.79%, 58.05%, 57.00%, 61.00%, 70.40%, 67.63%, 16.45%, 51.78% and 7.45% less than AVOA, PSO, DE, RW_GWO, WOA, HHO, LSHADE, GBO, and EBOwithCMAR, respectively.

Future studies should employ HSOA to address problems such as data mining, structure design, water resource management, and image processing. Despite the proposed algorithm's competitive performance, it requires sensitivity analysis to determine the parameters. However, this issue can be solved using self-tuning approaches, such as the chaotic map or fuzzy method. Furthermore, to solve other optimization problems, different improvements will be made to HSOA, such as adding hybridization with other algorithms to solve other optimization problems.

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