IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 8 September 2023, accepted 22 October 2023, date of publication 26 October 2023, date of current version 1 November 2023.
Digital Object Identifier 10.1109/ACCESS.2023.3327741

THEORY

Tracking Control With Zero Steady State for UAV Flight Platform

WANG JIANHONG AND WANG YANXIANG
School of Electronic Automation, Jiangxi University of Science and Technology, Ganzhou 341000, China
Corresponding author: Wang Yanxiang (9120180007@jxust.edu.cn)

This work was supported in part by the Jiangxi Provincial National Science Foundation under Grant 20232BAB201015.

ABSTRACT Consider one controller design problem for UAV flight control system, while guaranteeing the closed loop output track one expected output, this new paper derives this optimal controller through our own mathematical theories. After introducing the practical UAV flight control and formulating this practical problem into one closed loop control problem, the concept about tracking control with zero steady state is defined. Then from two aspects of tracking mission, two optimal controllers are yielded through solving two different optimization problems. Although their forms are different, we establish the equivalent properties between them together. Moreover, the detailed algorithm if designing the tracking controller within the case of zero steady state is shown. Finally, a platform about the practical UAV flight system is constructed to do some simulations so that our theoretical analysis is proven.

INDEX TERMS Tracking control, zero steady state, UAV, optimal controller, equivalence.

I. INTRODUCTION

During recent years, unmanned aerial vehicle (UAV) has been extensively deployed in the military and civilian industries due to its tremendous performance improvement. Compared with other vehicle, multi UAVs offer greater dependability and safety, and are more likely to execute difficult tasks under complex circumstance. One more interesting topic of coordinated strikes by multi UAVs has sparked significant concern in the military sphere, meaning that multi UAVs concurrently attack single or multiple targets from time varying locations and angles. Taking an example about UAV as one life, the engine is its heart, and the sensor systems correspond to eye, ear or nose. The important part of flight control system means its brain or mind, as the obvious difference between UAV and manned vehicle lies in its autonomous flight capability, being the most basic and important function of flight control system. All above tell us the performance of UAV depends on the flight controller, which is the core content of designing or devising one UAV. Strictly speaking, UAV is a coupling of motion with multi body system, rotor, body and lift surface, and also UAV includes inertial coupling, structural coupling, aerodynamic coupling. But some factors, existing in nonlinear characteristic, bring difficulty for modeling and analyzing UAV. To the best of our knowledge, the mathematical modeling for UAV is a long term research topic within aviation field. More specifically, whatever physical principle or system identification is used, the mathematic model for UAV is hard to construct due to lots of uncertainties and nonlinearities. In practice, empirical formula and data fitting of blowing test, used in modeling UAV, often lead to inaccurate model. For example, when only the degrees of freedom about aerodynamics components are considered, the order of mathematical model for UAV will be at least 25. Moreover if we take into account all dynamic characteristics, such as engine, power system, sensor system and actuator together, the corresponding order will be more higher, thus increasing the system complexity and model uncertainty. Therefore, the resulting flight controller must be strongly robust for external or internal uncertainties or nonlinearities.

The development of UAV experiences three stages, manual flight mode, remote control and autonomous control. Roughly speaking, manual flight mode is a transition form, used for test in the early stage of development, then the second remote control is a common form. Now most modern UAVs combine manual, real time, remote control with autonomous

The associate editor coordinating the review of this manuscript and approving it for publication was Haibin Sun.

flight, and have richer control modes, for example, visual control mode, instrument control mode, autonomous navigation control mode, autonomous take off and autonomous landing control mode etc. these special multiple control modes guarantee the ability to accomplish various complex tasks within different flight environment. Without loss of generally, the flight control system can not only achieve the expected robust stabilization, but also manage various flight control modes without any time delay. When to switch the flight control modes freely, a good switching mechanism is needed to ensure the considered UAV maintain good flight quality during the mode switch process, so the design and realization of flight control modes are the ongoing problem, studied through the whole control system for UAV. On the other hand, some considerations, such as balance, compensation and decoupling process used by the pilot in manned UAV, must be accurately reflected in the flight controller design, so that UAV has the same maneuverability with the manned vehicle, and has other additional properties, for example, complete fault tolerant, error correcting and emergency handling, guaranteeing the reliability of UAV in various fault situations.

From above detailed description about UAV in engineering, lots of research about UAV exist, for example, controller design, anomaly detection, fault isolation, trajectory planning, UAV identification, structure design etc. due to the heart as flight control design, this paper still concentrates on the flight controller design for UAV, thus ensuring the perfect tracking control performance with zero steady state condition. As the number of papers about UAV flight controller design is vast, here we only list some related papers. Reference [1] decomposes the dynamic task allocation problem in air warfare into a game problem between single UAV via dynamic game theory, and solves it with Nash equilibrium. Additional, the influence graph game theory is adopted to examine multi controllers design [2]. Then more researchers tend to deal with small scale cluster rather than large scale cluster. Reference [3] applies expert system into our air combat decision making simulation, where a rule base is established according to the potential environment, but this technique only deals with known difficulties and can not make entirely independent judgment [4]. Reference [5] links genetic algorithm, fuzzy control algorithm and decision tree algorithm to construct a high security vacuum warfare simulation environment. For simplicity of discussion, the classical linear quadratic regulator is applied to design the tolerant flight controller [6], which describes the control performance requirement as a weighted quadratic cost with respect to system state and control input [7]. Through selecting the weighted matrix appropriately, and solving one Riccati equation [8], the optimal controller is yielded to satisfy closed loop stable and expected performance cost. The interesting combination with UAV identification and UAV control is proposed in [9], where a new concept about identification for control is also put forth. Rough speaking, there are two control strategies for UAV flight control system [10], i.e. model based control and data driven control, such as iterative feedback tuning [11] and virtual reference feedback tuning [12] etc. During these five years, data driven control is widely studied from the theoretical perspective and an applied point of view respectively. Nonlinear data driven control is considered by using reference generic terminal ingredient [13] and its robust form is studied from the computational complexity [14]. Additional, direct data driven control is firstly used in ship control [15], then its corresponding asymptotically guaranteed stability is analyzed through differential geometry [16] Recently, new research develops a Bell 205 nonlinear model with high confidence based on the classical state space model, being dependent on the persistency of excitation [17]. Direct and indirect data driven control are bridged together to design one distributed robust controller [18], corresponding to distributed remote control. When given one reference model or the desired flight trajectory, data driven model reference control [19] is formulated as one prediction error identification problem, solving via a matrix S-lemma [20]. Generally, the detailed theoretical analysis and engineering application in practice for data driven control is seen in [21].

Based on above descriptions about theoretic and engineering application of UAV, this new paper continues to design the controller for UAV, while guaranteeing the considered UAV fly with perfectly tracking, i.e. zero steady state error. After introducing the basic knowledge of UAV flight system to make our analysis result understand well, the concept of tracking control with zero steady state error is defined, i.e. the closed loop output will approach to one desired value. This requirement is proven to be equivalent with the variance of closed loop output converge to its minimum value or zero. In case of our defined tracking control with zero steady state error, the corresponding tracking controller is derived through our mathematical analysis and some considerations about the derive controller are also given to complete our analysis. It is well known that theoretical analysis serves for the latter engineering application, so to prove the efficiency of our derived results, we construct one platform about UAV flight system and do some simulation example.

Generally, the main contributions in this new paper are formulated as follows.

(1) Concept of tracking with zero steady state error is defined.

(2) The tracking controller is designed to satisfy zero steady state error.

(3) A practical platform is constructed to combine the theory and practice.

II. UAV FLIGHT SYSTEM

As UAV is a multi variable coupling system, and it is difficult to establish an accurate nonlinear mathematical model, so closed control scheme is more suitable to solve this control problem. It means the closed control scheme is adopted to design and control a certain type of UAV layer by layer,

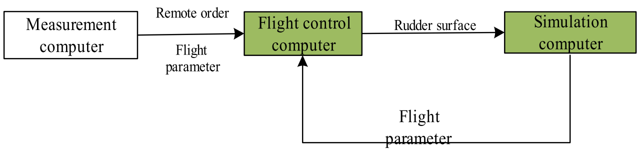

FIGURE 1. **UAV formation flight structure.**

then finally to complete the whole control process. The flight control computer is called dual machine simulation in the circuit simulation, and it is also a physical object or a test device. The circuit structure of the flight control computer is shown in Figure 1.

Where in Figure 1, the simulation computer, the measurement computer and the flight control computer together constitute the flight control computer in closed-loop system. Each part in the system is connected through a digital interface [22]. After the flight control computer receives the flight parameters sent by the simulation computer, it calculates the control value of the rudder surface according to the control law, and transmits it to the simulation computer through the digital interface. The parameters are sent to the measurement computer for display monitoring, and the measurement and control computer will also send necessary continuous or discrete instructions to the simulator or flight control machine, and finally form the flight control computer in-loop simulation system. The flight control computer in-loop simulation is a real-time simulation, the main purpose is to evaluate the correctness and performance of the flight control computer.

The flight control computer has a simple loop structure and requires less equipment, but it can more completely verify the correctness of the flight control law and the reliability of the flight control computer, and is more targeted. However, it should also be noted that since the values of each parameter in the loop are basically solved by mathematical models, the system simulation effect will be directly affected by the model, and the impact of hardware devices on system performance cannot be evaluated, so there are certain defects. The characteristics of the sensor in the loop system are: the flight parameters of the helicopter are not calculated by the mathematical model, but measured by sensors such as inertial navigation, gyroscope, GPS, atmospheric computer, etc. and transmitted to the simulation computer or flight control computer through the data link. Proceed to the next step of model and control law solution. The sensor loop can be seen in Figure 2.

Where in Figure 2, after receiving the instructions of the measurement computer, the simulation computer performs the model calculation, and sends the parameters required by UAV to the control computer. The control computer then controls the turntable to move. The physical quantity is then sent to the flight control computer, and the flight control machine sends the control quantity to the simulation computer according to the control law to form a closed loop. It can be seen from this that UAV is not only the carrier of the sensor but also the exciter of the sensor. The steering gear

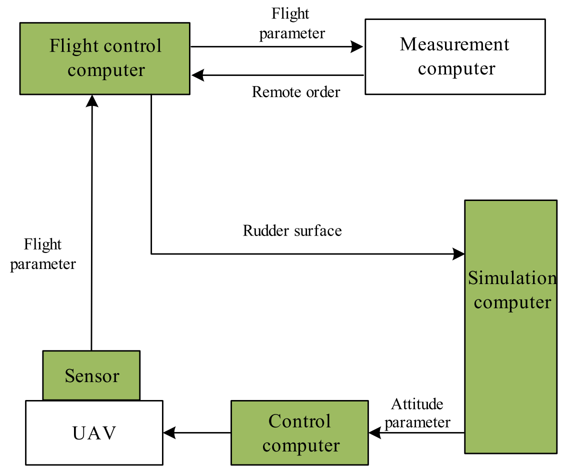

FIGURE 2. **Sensor loop.**

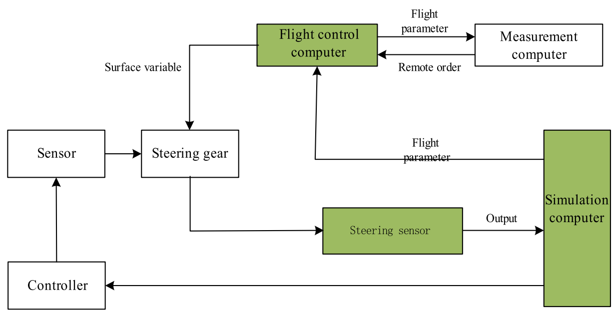

FIGURE 3. **Steering gear structure.**

can affect the flight aerodynamics of the unmanned helicopter and control the flight attitude of the unmanned helicopter. The servo loop is formed by adding a servo system to the flight control computer loop. The structure diagram is shown in Figure 3.

Where in Figure 3, during the simulation of the steering gear in the loop, after the flight control computer receives the instruction of the measurement and control computer, it no longer directly transmits the control amount to the simulation computer, but transmits it to the steering gear to drive its movement, and measures the angular displacement of the steering gear or the linear displacement signal is sent to the simulation computer [23]. After the simulation computer receives the output of the steering gear, the current flight parameters are calculated by the mathematical model, and then sent to the flight control computer for the calculation of the control law at the next moment.

Before putting the steering gear into the loop simulation, the steering gear must be calibrated first. The so-called calibration of the manipulated variable refers to the corresponding relationship between the given control variable and the actual manipulated variable. The reason for the calibration of the manipulated variable is as follows:

(1) Make the given variable correspond to the actual variable distance, which is convenient for the control solution.

(2) The size of the manipulation amount can be calculated from the position feedback information of the steering gear, which is convenient for real-time monitoring;

(3) The output of the manipulation amount is electrically limited by the calibration result to prevent the mechanical jamming.

From Figure 3, we can see that in addition to the steering gear, the hardware equipment in the circuit also includes the steering gear measurement device and the force loader device. Among them, the steering gear measurement device is to obtain a certain precision of the rudder deflection signal in the simulation. The steering gear measuring device is a device that measures the angular displacement or linear displacement of the electric servo steering gear. At present, the main methods of measuring the angle of the steering gear include mechanical dials, potentiometer feedback and induction synchronization. The mechanical dial has the defects of low position accuracy and large human reading error; the measurement accuracy of the potentiometer feedback is limited by the potentiometer accuracy, and the accuracy is not high; the rotor inertia of the induction synchronizer is large and the accuracy is low. The function of the steering gear loading device is to perform static loading and dynamic loading tests on the electric steering gear, detect and test the static and dynamic output force of the electric servo device, and comprehensively test the static characteristics and control of the steering gear in an environment similar to the actual load. precision.

III. TRACKING CONTROL WITH ZERO STEADY STATE ERROR

A. PRELIMINARY

From above basic knowledge about UAV flight control system, the most three parts are UAV model, controller and measurement sensor, which are simplified in the following Figure 4.

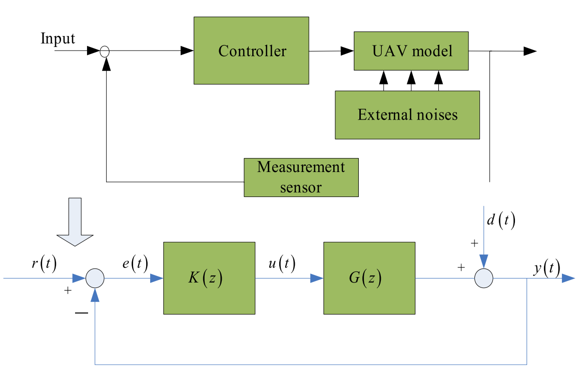

FIGURE 4. The considered flight control system and its simplified form.

Where in Figure 4, the considered UAV flight control system is also simplified into one closed loop system structure. Moreover the relations among all physical variables and their corresponding mathematical forms are given too. For example, $r(t)$ is the external excitation signal, $G(z)$ is one UAV model, and $K(z)$ is one known feed forward controller. $y(t)$ is the closed loop output signal, and $u(t)$ is the input signal for that unknown UAV model $G(z)$. $e(t)$ is the error signal, i.e. $e(t) = r(t) - y(t)$.Variable z is the shift operator.

Figure 4 shows the corresponding process about transforming one UAV controller design problem into designing that unknown feed forward controller $K(z)$ through our own control strategy. For the sake of completeness, that concept about tracking control with zero steady state is needed.

(Definition 1): Consider Figure 4 with nonzero input $r(t)$ and un-avoided disturbance $d(t)$, given one desired output signal $y_d(t)$, if there exists one feed forward controller $K(z)$ such that

$$\lim_{t\to\infty} y(t) = y_d(t) \tag{1}$$

Then the considered UAV closed loop output $y(t)$ tracks the desired output $y_d(t)$ with zero steady state. As in above Definition 1, $y_d(t)$ is the desired output signal, given by us in priori, so to simplify our latter mathematical derivation, we always set $y_d(t) = 0$, then equation (1) is reduced to that

$$\lim_{t\to\infty} y(t) = 0 \tag{2}$$

Furthermore, the external disturbance $d(t)$ is considered as follows.

(Definition 2): The priori assumption about that external disturbance $d(t)$ is zero mean white Gaussian and mutually uncorrelated with variance value σ.

Bearing in mind that, the mission in this paper is to devise one feed forward controller $K(z)$ while guaranteeing above equation (2), i.e. achieving that goal of tracking control with zero steady state.

B. TRACKING CONTROLLER

Observing Figure 4 again, some explicit relations hold obviously

$$\begin{cases} y(t) = \dfrac{G(z)}{1+G(z)K(z)} r(t) + \dfrac{1}{1+G(z)K(z)} d(t) \\ u(t) = \dfrac{K(z)}{1+G(z)K(z)} r(t) + \dfrac{K(z)}{1+G(z)K(z)} d(t) \end{cases} \tag{3}$$

Condition $\lim_{t\to\infty} y(t) = 0$ means the closed loop output will converge to zero with time increases, being equivalent to equity $E\left[y^2(t)\right] = 0$ from our previous paper [22], where notation E denotes the mathematical operation. From equation (3), the mathematical operation $E\left[y^2(t)\right] = 0$ is given as follows in detail.

$$\begin{aligned} E[y(t)]^2 &= \left[\frac{G(z)}{1+G(z)K(z)}\right]^2 r^2(t) \\ &+ \left[\frac{1}{1+G(z)K(z)}\right]^2 d^2(t) \\ &+ 2\frac{G(z)}{[1+G(z)K(z)]^2} r(t)d(t) \end{aligned}$$

$$= \left[\frac{G(z)}{1+G(z)K(z)}\right]^2 r^2(t) + \left[\frac{1}{1+G(z)K(z)}\right]^2 d^2(t)$$
$$= \left[\frac{G(z)}{1+G(z)K(z)}\right]^2 \phi_r(w) + \left[\frac{1}{1+G(z)K(z)}\right]^2 \sigma \quad (4)$$

where in deriving equation (4), the condition that about $r(t)$ and $d(t)$ are uncorrelated, is used, and $\phi_r(w)$ denotes the auto power spectrum.

The requirement about tracking control with zero steady state is reduced to minimize the variance of that closed loop output signal, i.e. the idea case means $E[y(t)]^2 = 0$, being solved through the following optimization problem.

$$\min_{K(z)} \frac{G^2(z)\phi_r(w)+\sigma}{[1+G(z)K(z)]^2} \quad (5)$$

As that unknown controller $K(z)$ does not exist in numerator, so equation (5) is changed to differentiate with respect to $K(z)$ and set the derivative equal to zero.

$$G(z) + G^2(z)K(z) = 0 \rightarrow 1 + G(z)K(z) = 0 \quad (6)$$

i.e.

$$K(z) = -\frac{1}{G(z)} \quad (7)$$

Equation (7), denotes the optimal controller, guarantees our expected mission, i.e. $\lim_{t\to\infty} y(t) = 0$ or $E[y(t)]^2 = 0$. In addition, based on the derived controller $K(z)$, the variance of the closed loop output signal approaches to zero.

C. EQUIVALENCE

Due to the equivalent property between equation (1), and (2), here this section turns back to show the unknown feed forward controller for the requirement (1), i.e. the closed loop output tracks the desired output.

Rewrite that desired closed loop output as that

$$y(t) = \frac{G(z)}{1+G(z)K(z)}r(t) + \frac{1}{1+G(z)K(z)}d(t) \quad (8)$$

Applying the prediction theory, one step ahead prediction $\hat{y}(t)$ is defined as that

$$\hat{y}(t) = \frac{1+G(z)K(z)}{1}\frac{G(z)}{1+G(z)K(z)}r(t) + \left[1-\frac{1+G(z)K(z)}{1}\right]y(t)$$
$$= G(z)r(t) - G(z)K(z)y(t) \quad (9)$$

Then one step ahead prediction error $\varepsilon(t)$ is obtained as follows.

$$\varepsilon(t) = y(t) - \hat{y}(t)$$
$$= y(t) - G(z)r(t) + G(z)K(z)y(t)$$
$$= [1+G(z)K(z)]y(t) - G(z)r(t) \quad (10)$$

Then optimal controller is solved through the following optimization problem, i.e.

$$\arg\min_{K(z)} \sum_{t=1}^{N} \varepsilon^2(t)$$
$$[1+G(z)K(z)]^2 y^2(t)$$
$$= \sum_{t=1}^{N} -2G(z)[1+G(z)K(z)]y(t)r(t)$$
$$+ G^2(z)r^2(t)$$
$$= [1+G(z)K(z)]^2\phi_y(w)$$
$$- 2G(z)[1+G(z)K(z)]\phi_{yr}(w)$$
$$+ G^2(z)\phi_r(w) \quad (11)$$

where spectral theory is applied in above equation (11), $\{\phi_y(w), \phi_{yr}(w), \phi_r(w)\}$ are the three spectrums for closed loop input $r(t)$ and output $y(t)$.

By differentiating with respect to $K(z)$ and by setting the derivative equation to zero, we have

$$K(z) = \frac{G(z)\phi_{yr}(w) - \phi_y(w)}{G(z)\phi_y(w)} \quad (12)$$

Comparing controller (7) and (12), although their forms are different, but their intrinsic meanings are the same to each other, being formulated as the following Theorem 1.

(Theorem 1): Consider the controller design problem in Figure 4, our mission is to devise one controller to guarantee the closed loop output track the given output or zero respectively, leading to two derived optimal controllers through solving two optimization problem. We find these two forms are equivalent, i.e.

$$K(z) = \frac{G(z)\phi_{yr}(w) - \phi_y(w)}{G(z)\phi_y(w)} = -\frac{1}{G(z)} \quad (13)$$

Proof: Observing that

$$\begin{cases} \phi_y(w) = \left[\frac{G(z)}{1+G(z)K(z)}\right]^2 \phi_r(w) \\ + \left[\frac{1}{1+G(z)K(z)}\right]^2 \sigma \\ \phi_{yr}(w) = \frac{G(z)}{1+G(z)K(z)}\phi_r(w) \end{cases} \quad (14)$$

Then after simple but tedious calculations, we have.

$$G(z)\phi_{yr}(w) - \phi_y(w) = \frac{G^3(z)K(z)\phi_r(w) - \sigma}{[1+G(z)K(z)]^2};$$
$$G(z)\phi_y(w) = \frac{G^3(z)\phi_r(w) + G(z)}{[1+G(z)K(z)]^2};$$
$$K(z) = \frac{G^2(z)}{1+G(z)K(z)}\phi_r(w) - \left[\frac{G(z)}{1+G(z)K(z)}\right]^2\phi_r(w)$$

Algorithm 1 Tracking Controller Design

Step 1. Use one known input $r(t)$ to excite the closed loop system, and collect the closed loop input-output sequence $\{r(t), y(t)\}_{t=1}^{N}$.

Step 2. Apply the existed knowledge about closed loop system identification to identify the model $G(z)$.

Step 3. Set

$$K(z) = -\frac{\sigma}{G(z)\sigma} = -\frac{1}{G(z)}$$

Step 4. Check if $y(t) = 0$ or $y(t) \to y_d(t)$, then terminate, or turn to step 1 again.

$$-\left[\frac{1}{1+G(z)K(z)}\right]^2 \sigma = \frac{G^3(z)K(z)\phi_r(w) - \sigma}{G^3(z)\phi_r(w) + G(z)\sigma} \tag{15}$$

i.e.

$$\begin{aligned} K(z)\left[G^3(z)\phi_r(w) + G(z)\sigma\right] &= G^3(z)K(z)\phi_r(w) - \sigma; \\ K(z)\left[\begin{array}{l} G^3(z)\phi_r(w) + G(z)\sigma \\ -G^3(z)\phi_r(w) \end{array}\right] &= -\sigma \end{aligned} \tag{16}$$

It holds that

$$K(z) = -\frac{\sigma}{G(z)\sigma} = -\frac{1}{G(z)} \tag{17}$$

This completes the proof.

From the optimal controller

$$K(z) = -\frac{\sigma}{G(z)\sigma} = -\frac{1}{G(z)}$$

then optimal controller is related with UAV model, so firstly the premise step is to apply the closed loop input-output data $\{r(t), y(t)\}_{t=1}^{N}$ to identify the unknown UAV model. i.e. system identification or modeling process, where N is the total number of the observed data. The above whole tracking controller design processes are formulated as follows.

IV. ROBUST ANALYSIS

From equation (17), we see the optimal controller $K(z)$ is related with UAV model $G(z)$. Within the case of robustness, external noise $d(t)$ affects the controller $K(z)$ through the identified model $G(z)$. Specifically, set $G_0(z)$ be the nominal model, and $\Delta G(z)$ is the error term, causing by some disturbances, i.e.

$$G(z) = G_0(z) + \Delta G(z) \tag{18}$$

To see the effect on controller $K(z)$ from disturbance, we have

$$\begin{aligned} K(z) &= -\frac{1}{G(z)} = -\frac{1}{G_0(z) + \Delta G(z)} \\ &= -\frac{1}{G_0(z)} + \frac{1}{G_0^2(z)}\Delta G(z) - \frac{2}{G_0^3(z)}\Delta^2 G(z) \\ &= K_0(z) + \Delta K(z) \end{aligned} \tag{19}$$

where $K_0(z)$ is called the nominal controller, and $\Delta K(z)$ is error term, i.e.

$$\begin{cases} K_0(z) = -\dfrac{1}{G_0(z)}; \\ \Delta K(z) = \dfrac{1}{G_0^2(z)}\Delta G(z) - \dfrac{2}{G_0^3(z)}\Delta^2 G(z) \end{cases} \tag{20}$$

So if the accuracy about UAV model $G(z)$ is acceptable or tolerable, for example, if $|\Delta G(z)| \leq 0.2$, then

$$|\Delta K(z)| \leq \frac{0.2}{G_0^2(z)} - \frac{0.08}{G_0^3(z)} \tag{21}$$

meaning that $K(z) \approx K_0(z)$, which tells the accuracy of model $G(z)$ will affect the accuracy of controller $K(z)$. But the accuracy of model $G(z)$ is guaranteed by system identification process.

To show the effect from disturbance $d(t)$, we rewrite those two equations as follows.

$$\begin{cases} y(t) = \dfrac{G(z)}{1+G(z)K(z)}r(t) + \dfrac{1}{1+G(z)K(z)}d(t) \\ u(t) = \dfrac{K(z)}{1+G(z)K(z)}r(t) + \dfrac{K(z)}{1+G(z)K(z)}d(t) \end{cases} \tag{22}$$

Define output sensitivity function and input sensitivity function as

$$\begin{aligned} s_{yd}(z) &= \frac{1}{1+G(z)K(z)}; \\ s_{ud}(z) &= \frac{K(z)}{1+G(z)K(z)} \end{aligned} \tag{23}$$

And define closed loop transfer functions as

$$\begin{aligned} s_{yr}(z) &= \frac{G(z)}{1+G(z)K(z)}; \\ s_{ur}(z) &= \frac{K(z)}{1+G(z)K(z)} \end{aligned} \tag{24}$$

Denote by $s_1(z)$ as the change of closed loop transfer function $s_{yr}(z)$ with the change of plant $G(z)$. Then it is derived by that.

$$s_1(z) = \frac{\partial s_{yr}(z)}{\partial G(z)} \times \frac{G(z)}{s_{yr}(z)} = \frac{1}{1+G(z)K(z)} \tag{25}$$

Similarly, $s_1(z)$, denoting the change of closed loop transfer function $s_{yr}(z)$ with the change of plant $G(z)$, is yielded by that.

$$s_2(z) = \frac{\partial s_{ur}(z)}{\partial G(z)} \times \frac{G(z)}{s_{ur}(z)} = -\frac{G(z)K(z)}{1+G(z)K(z)} \tag{26}$$

Combining equation (25) and (26), we see whatever plant $G(z)$ changes, the relation about the change with plant holds, i.e.

$$s_1(z) - s_2(z) = 1 \tag{27}$$

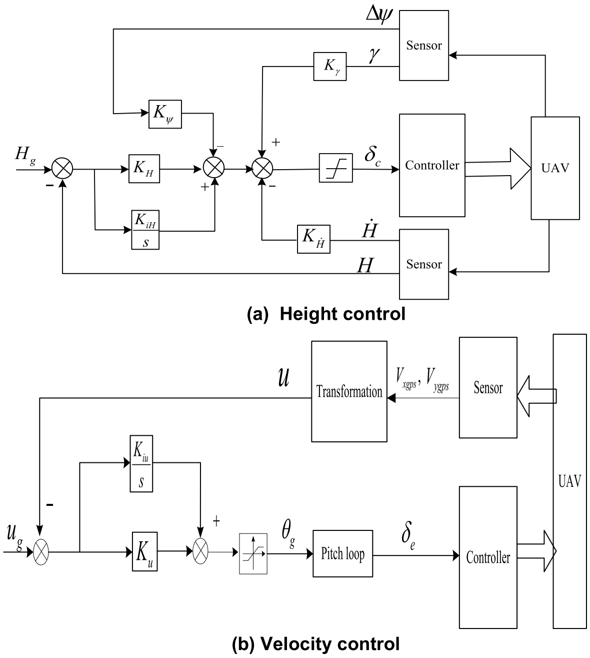

FIGURE 5. **Flight control structure for height and velocity.**

V. PLATFORM AND SIMULATION

In this simulation example, a quadratic UAV is used, as it can take off and land vertically. It has strong air control capability, good static flight and low speed flight characteristics. The altitude control of UAV is achieved through collective pitch control. The size of the collective pitch determines the size of the main rotor lift. In fact, height control is to compare the real height fed back by the height sensor with the set height, and adjust the size of the collective distance according to the deviation value. In order to increase the control damping, the feedback of the rate of change of the height is introduced to form a cascade control system, shown in following Figure 5(a).

Where in Figure 5(a), all physical variables are defined as our previous paper. The altitude control loop adds the effect of the heading channel to the control law, $\Delta\psi$ is the yaw angle. Due to the structural characteristics of the single-rotor helicopter with tail rotor, the lateral force required for left turning is greater than the lateral force required for right turning, so the power consumed by the tail rotor when turning left is relatively large. Velocity control for UAV refers to the control of the forward flying speed of the unmanned helicopter. To make the helicopter fly forward, it is generally necessary to change the longitudinal cyclic pitch of UAV, and use the pulling force generated by the rotor to pull UAV forward to fly. Structure of this velocity control system is seen in Figure 5(b).

Furthermore, in this velocity control loop for UAV in Figure 5(a), u_g is the given forward velocity, corresponding to UAV body. The eastward velocity and northward velocity are measured by GPS, then after comparing them with their given velocities, one velocity error is generated. During the

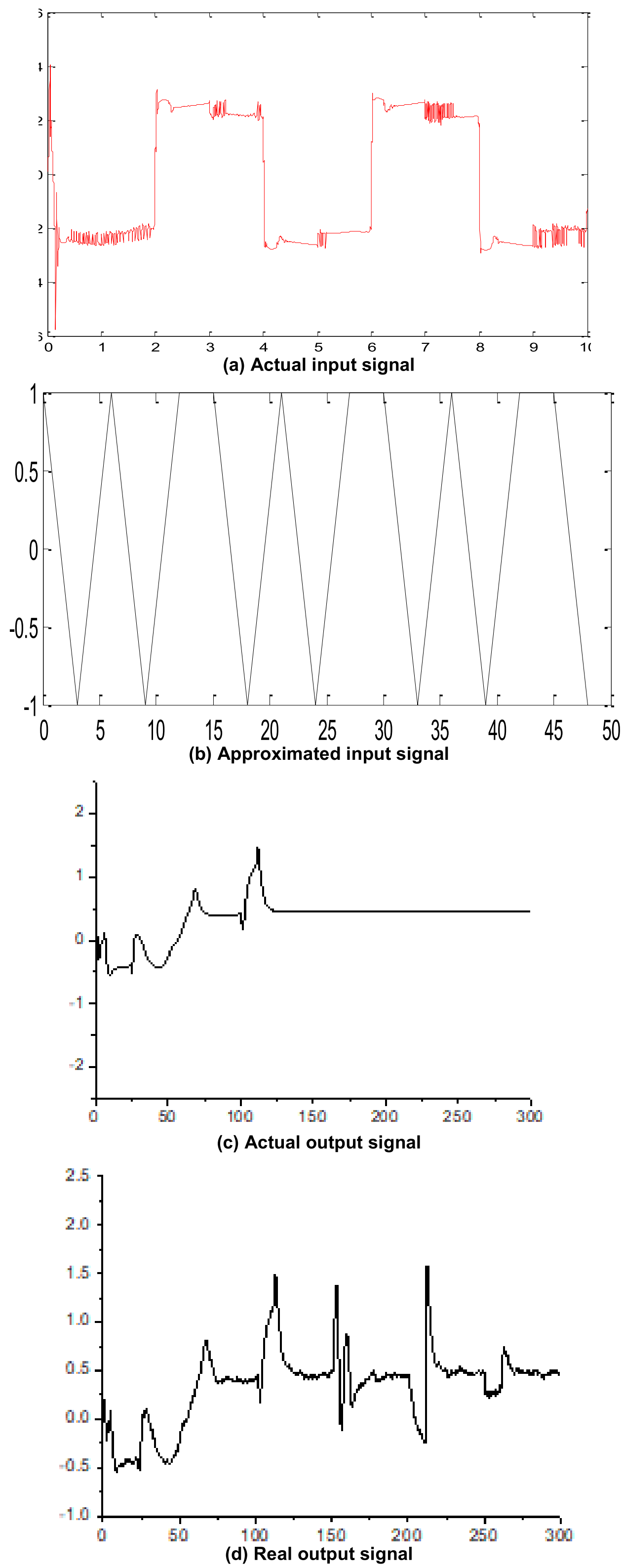

FIGURE 6. **Input-output data sequence.**

simulation process for cascade controller design in the velocity loop system structure, the two plants $G(z)$ is chosen as the following transfer functions, where the parameters can be estimated by using system identification, for example, least squares method, recursive identification algorithm etc [23].

$$G(z) = \frac{14}{z^2 + 5.8z + 14}$$

All input-output data sequence $\{r(t), y(t)\}_{t=1}^{N}$ are measured by some devices, and they are recorded in Figure 6, where the actual input and its approximated input is used to excited the considered control system, then the actual output and its corrupted real output are all measured. Two controllers, proposed by our prediction error method, are applied to control UAV flight trajectory. After takeoff, UAV flies according to the predetermined track. We take the simulation process of 0∼500s for analysis, and the simulation process curve is shown in Figure 7(a), where during 0∼100s, the height of UAV gradually rises, and after reaching 1000m, it maintains a constant altitude flight, and completes the turning flight between 100 and 210s. In Figure 7(b), the throttle rudder value is increased by 50 times, and the forward flying speed value is increased by 100 times, reflecting the changing relationship between the three, which verifies the relationship between the throttle control and the forward flying speed.

During robustness analysis, external noise $d(t)$ is assumed as one bounded noise, i.e. its amplitude satisfies $|d(t)| \leq 0.1$. three simulation results are given in Figure 8, where the first one corresponds to an idea case, i.e. no external noise, and the second one chooses the maximum external noise. And the second simulation is equivalent to be sum of input and noise, i.e. $r(t) + d(t)$. The third simulation is our named approximated one, i.e. choosing $K(z) \approx K_0(z)$ for case of $G(z) \approx G_0(z)$. Three output curves, corresponding to above three cases are seen in Figure 8, where three output curves are closed with each other.

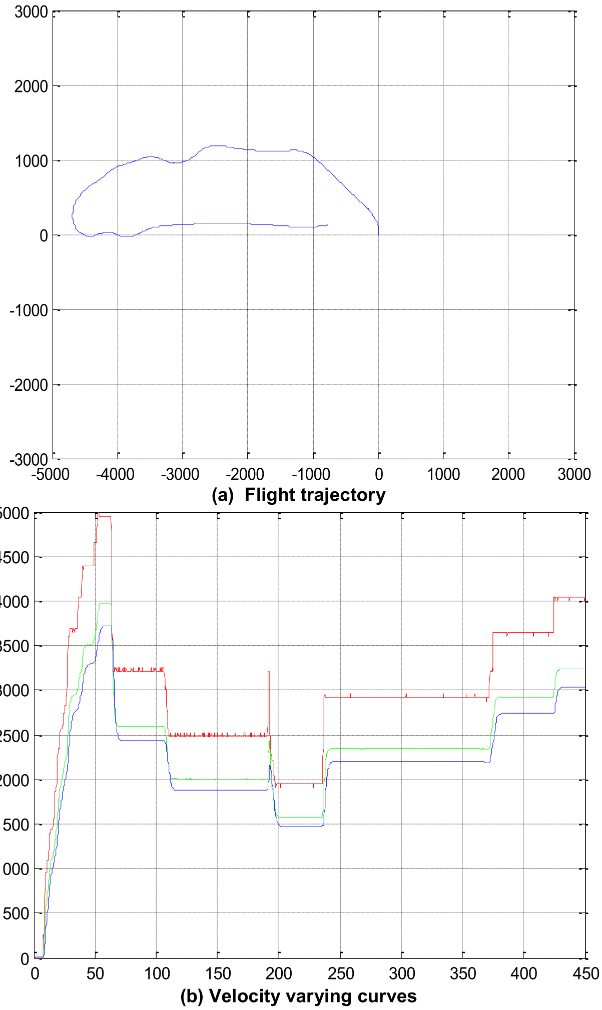

FIGURE 7. Flight simulation results.

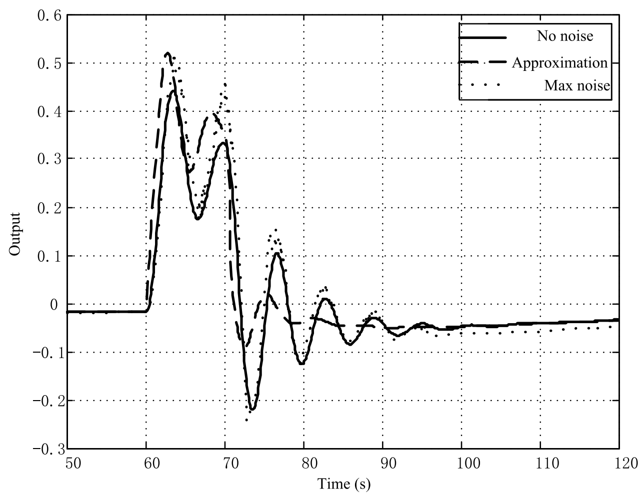

FIGURE 8. Robustness results.

VI. CONCLUSION

Perfectly tracking property is a necessary performance for UAV flight control system, so this new paper concentrates on designing one controller to achieve this good property. After given some basic knowledge about UAV flight control system and defined the idea of tracking control with zero steady state, the optimal controller is derived into two difference forms. Under the situation of zero steady state, we prove these two different forms are same with each other through our own theoretical analysis. In additional adaptation is another measurement index along with tracking property, so next paper will study the adaptive tracking control.

ACKNOWLEDGMENT

The authors are grateful to Prof. Eduardo F. Camacho for the warm invitation in his control laboratory at the University of Seville, Seville, Spain. Thanks for his assistance and advice on zonotopes in guaranteed state estimation and model predictive control.

REFERENCES

[1] A. Bemporad, M. Morari, V. Dua, and E. N. Pistikopoulos, "The explicit linear quadratic regulator for constrained systems," *Automatica*, vol. 38, no. 1, pp. 3–20, Jan. 2002.

[2] C. De Persis and P. Tesi, "Formulas for data-driven control: Stabilization, optimality, and robustness," *IEEE Trans. Autom. Control*, vol. 65, no. 3, pp. 909–924, Mar. 2020.

[3] C. D. Peris and P. Tesi, "Low-complexity learning of linear quadratic regulators from noisy data," *Automatica*, vol. 128, no. 3, pp. 241–250, 2021.

[4] D. Piga, S. Formentin, and A. Bemporad, "Direct data-driven control of constrained systems," *IEEE Trans. Control Syst. Technol.*, vol. 26, no. 4, pp. 1422–1429, Jul. 2018.

[5] F. Dorfler, J. Coulson, and I. Markovsky, "Bridging direct and indirect data driven control formulating via regularizations and relaxations," *IEEE Trans. Autom. Control*, vol. 68, no. 2, pp. 883–897, Jul. 2023.

[6] H. Hjalmarsson, "Iterative feedback tuning—An overview," *Int. J. Adapt. Control Signal Process.*, vol. 16, no. 5, pp. 373–395, 2002.

[7] H. J. van Waarde, M. K. Camlibel, and M. Mesbahi, "From noisy data to feedback controllers: Nonconservative design via a matrix S-lemma," *IEEE Trans. Autom. Control*, vol. 67, no. 1, pp. 162–175, Jan. 2022.

[8] H. J. van Waarde, J. Eising, H. L. Trentelman, and M. K. Camlibel, "Data informativity: A new perspective on data-driven analysis and control," *IEEE Trans. Autom. Control*, vol. 65, no. 11, pp. 4753–4768, Nov. 2020.

[9] I. Markovsky and P. Rapisarda, "Data-driven simulation and control," *Int. J. Control*, vol. 81, no. 12, pp. 1946–1959, 2008.

[10] J. Berberich, J. Kohler, F. Allgower, and M. A. Müller, "Dissipativity properties in constrained optimal control: A computational approach," *Automatica*, vol. 114, Apr. 2020, Art. no. 108840.

[11] J. Berberich, J. Köhler, M. A. Müller, and F. Allgöwer, "Data-driven model predictive control with stability and robustness guarantees," *IEEE Trans. Autom. Control*, vol. 66, no. 4, pp. 1702–1717, Apr. 2021.

[12] J. C. Willems, P. Rapisarda, I. Markovsky, and B. L. M. De Moor, "A note on persistency of excitation," *Syst. Control Lett.*, vol. 54, no. 4, pp. 325–329, Apr. 2005.

[13] J. Köhler, M. A. Müller, and F. Allgöwer, "A nonlinear model predictive control framework using reference generic terminal ingredients," *IEEE Trans. Autom. Control*, vol. 65, no. 8, pp. 3576–3583, Aug. 2020.

[14] J. Köhler, R. Soloperto, M. A. Müller, and F. Allgöwer, "A computationally efficient robust model predictive control framework for uncertain nonlinear systems," *IEEE Trans. Autom. Control*, vol. 66, no. 2, pp. 794–801, Feb. 2021.

[15] K. van Heusden, A. Karimi, and D. Bonvin, "Data-driven model reference control with asymptotically guaranteed stability," *Int. J. Adapt. Control Signal Process.*, vol. 25, no. 4, pp. 331–351, Apr. 2011.

[16] L. Campestrini, D. Eckhard, A. S. Bazanella, and M. Gevers, "Data-driven model reference control design by prediction error identification," *J. Franklin Inst.*, vol. 354, no. 6, pp. 2628–2647, Apr. 2017.

[17] M. Gevers, "Identification for control: From the early achievements to the revival of experiment design," *Eur. J. Control*, vol. 11, nos. 4–5, pp. 335–352, 2005.

[18] M. Tanaskovic, L. Fagiano, C. Novara, and M. Morari, "Data-driven control of nonlinear systems: An on-line direct approach," *Automatica*, vol. 75, pp. 1–10, Jan. 2017.

[19] S. Formentin, M. C. Campi, A. Carè, and S. M. Savaresi, "Deterministic continuous-time virtual reference feedback tuning (VRFT) with application to PID design," *Syst. Control Lett.*, vol. 127, pp. 25–34, May 2019.

[20] S. Formentin, S. M. Savaresi, and L. Del Re, "Non-iterative direct data-driven controller tuning for multivariable systems: Theory and application," *IET control Theory Appl.*, vol. 6, no. 9, pp. 1250–1257, 2012.

[21] V. Breschi and S. Formentin, "Direct data-driven design of switching controllers," *Int. J. Robust Nonlinear Control*, vol. 30, no. 15, pp. 1–31, 2019.

[22] W. Jianhong, "Minimum variance control strategy for closed loop linear time invariant system," *Int. J. Syst. Syst. Eng.*, vol. 9, no. 1, pp. 62–74, 2019.

[23] W. Jianhong, R. A. Ramirez-Mendoza, and R. Morales-Menendez, *Data Driven Strategies: Theory and Application*. Boca Raton, FL, USA: CRC Press, 2023.

WANG JIANHONG received the Diploma degree in engineering cybernetics from Yunnan University, China, in 2007, the Dr.Sc. degree from the College of Automation Engineer, Nanjing University of Aeronautics and Astronautics, China. From 2013 to 2015, he was a Postdoctoral Fellow in informazione with Politecnico di Milano. From 2016 to 2018, he was a Professor with the University of Seville. Currently, he is a full-time Professor with Tecnologico de Monterrey and a part-time Professor with the Jiangxi University of Science and Technology. His current research interests include real-time and distributed control and optimization and system identification.

WANG YANXIANG received the bachelor's degree from the Jiangxi University of Science and Technology, China, in 2020. She is currently a Lecturer with the Jiangxi University of Science and Technology. Her research interests include data driven control and system identification.