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RESEARCH ARTICLE

Decision Aid Algorithm for Kidney Transplants Under Disc Spherical Fuzzy Sets With Distinctive Radii Information

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ABSTRACT The Circular Spherical Fuzzy Set (C-SFS) and Disc Spherical Fuzzy Set (D-SFS), an innovation of the well-known Spherical Fuzzy Set, are introduced in this research study. It is intended to improve the representation and processing of uncertainty in decision-making scenarios. The study establishes the C-SFS's basic relations and operations, giving it a solid foundation for use in a variety of fields. An ELECTRE method utilising the C-SFS framework is suggested to properly quantify the decision-making process. This methodology combines circular spherical fuzzy concordant and discordant matrices, making it easier to evaluate performance thoroughly in terms of many different factors. The orderly structure of this strategy is demonstrated by a structured flow chart. Real-world applications of the C-SFS ELECTRE approach, particularly in the critical setting of kidney transplant selection, serve to demonstrate its effectiveness. A comparative section is included to demonstrate the suggested method's accuracy.

INDEX TERMS ELECTRE method, circular spherical fuzzy sets, disc spherical fuzzy set, extension of spherical fuzzy set, decision-making.

I. INTRODUCTION

Decision-making (DM) is a complex handle that involves making choices among multiple alternatives based on various factors and criteria. The application of fuzzy set theory has been prevalent in decision making to address uncertainty and imprecision associated with the DM process [1]. Fuzzy set theory enables decision-makers to deal with qualitative and quantitative data, and it provides a more flexible and robust framework for decision making. In particular, fuzzy analysis provides a systematic approach to assessing alternatives based on multiple criteria, taking into account both subjective and objective information [2]. In the past, many research papers have included decision-making, which includes drug selection to treat COVID-19 [3], hydrogen power plant selection [4], urinary diseases and other medical problems [5], solar power plant [6] and many more. This paper aims to establish a new concept called the circular spherical fuzzy set, which extends the existing fuzzy set theory, and to demonstrate its applicability in a case study on kidney transplant selection. The proposed approach combines circular spherical fuzzy set theory with the ELECTRE a way to provide a comprehensive and efficient approach to DM.

Fuzzy-sets (FS) theory was developed by Zadeh [7] to represent ambiguous and incomplete data. It generalises the idea of the characteristic function that determines if a component belongs to a universal set to the idea of the belonging value of a fuzzy set. FSs have been widely used to address the shortcomings and constraints of traditional approaches. FSs are used to deal with occurrences when it is impossible to determine the risk priority using conventional models. The ratings and importance of triangular fuzzy

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numbers were used to convey linguistic variables in the first adaptation of the TOPSIS approach under an uncertain environment [8]. In this sense, a straightforward and effective approach may be used to analyse the risks of future failures [9]. To calculate the decision making process, a fuzzy TOPSIS method may be utilised [10]. To characterise the importance of risk variables, it mixed the advantages of fuzzy TOPSIS and analytic hierarchy process (AHP) approaches. Employing fuzzy evidence-based reasoning the grey theory, traditional failure mode and effective analysis (FMEA) approaches were made more effective [11]. Boral et al suggested an combined MCDM approach that integrates the fuzzy analytical hierarchy process with the modified fuzzy MAIRCA [12]. With the use of a case study involving a steam valve system, [13] provided a type-2 interval fuzzy evidential reasoning approach for FMEA and highlighted the advantages of the suggested threat model.

Following this, several scholars question why the non-membership component is not included in the fuzzy sets? To fulfil this idea, Atanassov [14] introduced the intuitionistic FS (IFS), which includes non-membership with membership. After this, a radius value is attached to the IFS, and the name of this set is the circular intuitionistic fuzzy set (C-IFS) [15], [16]. Then, time by time, further kinds of fuzzy sets come to help the DM problems. Interval-valued fuzzy sets (IVFS) are an extension of the fuzzy set theory utilizes a number from an interval to symbolize the degree of membership and highlights the ambiguity in the membership degrees that have been assigned [17]. Torra [18] introduced the hesitant fuzzy sets (HFSs) which are an growth of fuzzy sets in which we include more than one membership. In some cases, it may not be possible to determine whether a proposition is true or false, and it may be only partially true or partially false. Neutrosophic fuzzy [19] sets allow for the representation of such degrees of indeterminacy, which is not possible using traditional crisp sets or fuzzy sets. A neutrosophic set, especially a FS, is a potent formal framework that generalises the idea of a set. Yager [20] introduced Pythagorean fuzzy sets (PyFSs) with a bigger region for belonging and non-belonging degrees. After this [49] is attached the circular and disc value with PyFSs. As a generalisation of IFSs and PyFSs, Yager [21] presented q-rung orthopair fuzzy sets (q-ROFSs). A spherical fuzzy set (SFS) [22] is a sort of fuzzy set that is defined on a sphere in n-dimensional space, which is the extension of a neutrosophic fuzzy set (NFS) because in a NFS the range of all three tuples lies between 0 and 3, but in a spherical fuzzy set its range is restricted to 1. A complemental fuzzy set is introduced by Alcantud, J.C.R. which is the justification of q-Rung orthopair fuzzy set [23]. In this paper, we introduce the novel idea of a circular and disc spherical fuzzy set, which is an extension of SFS. Figure 1 shows the extension of fuzzy set theory annually.

There are several techniques to solve daily life problems, such as TODIM [24], VIKOR [25], ELECTRE method [26] and much more. In this paper, we implicated the

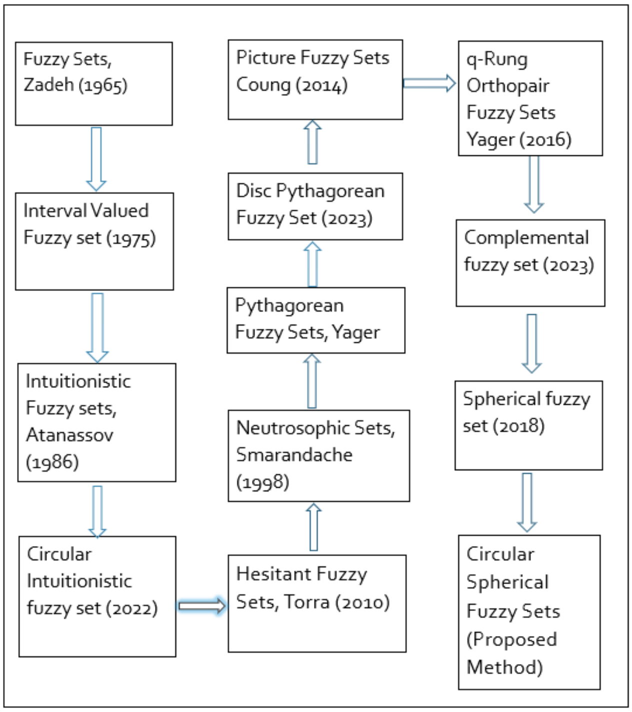

FIGURE 1. Developmentation in fuzzy set theory.

ELECTRE method for multi criteria decision-making problems (MCDM) due to the use of the proposed definition. Previously, researchers have used the ELECTRE method for different purposes. The group of performance evaluation is significantly impacted by the ELECTRE approach, [27]. It was first suggested by Siskgs [28] and Roy [29] under the name ELECTRE-I. The concept becomes more developed into the ELECTRE-II [30], "III," and "IV" ranked problem-solving procedures as well as the ELECTRE-A [31], [32] sorting problem-solving approaches. Pythagorean fuzzy set and the ELECTRE-I approach in MCDM, according to [33]. Several industries have seen considerable application of various ELECTRE approaches, including internet business [34], online purchasing [35] and choosing a dentist [36]. Likewise, a number of authors have employed the ELECTRE method when it comes to group decision-making [37]. Akram et al. [38] uses ELECTRE-I with an m-polar fuzzy soft set in MCDM. Using incomplete data, which simulates the circumstance when assessments of the values that each parameter should have are unknown, Dias and Climaco derived the validity indices for ELECTRE [39]. Different priorities among a group of decision-makers might cause this dilemma, as might inadequate, inconsistent, or insufficient information. To resolve conflicts between limits on the parameters, Mousseau [40] applies the accumulation technique for the ELECTRE TRI method. Fernandez and Olmedo [41] offered an action a model that the ideas of concordant and discordant for problems with group ranting. Using an integrated fuzzy AHP-ELECTRE tackle, Kaya and Kahraman [42] demonstrated an evaluation of the environment's influence method for urban economic planning. Also, the ELECTRE technique has recently been applied to solve the circumstance where the assessment data of the judgement issues may be unclear and fuzzy, as a result of the judgement's low knowledge and genuine real-fuzziness [43]. To account for the confusing, incorrect, and subjective judgements made by a group of DM, Hatami-Marbini and Tavana [44] suggested the enlarged ELECTRE I technique and utilised the mean value to aggregate all of the evaluations. In intuitionistic fuzzy (IF) ecosystems, Wu and Chen [45] employed the tackle to tackle the MCDM issues. The ELECTRE technique compares possibilities pair by pair using the evaluated data supplied by the decision maker. Connections that are concordant, discordant, or outranking are all significant in this technique. Using concordant and discordant indices, the decision-maker approach outranks correlations between a variety of possibilities before employing the crisp data to choose the best option.

A. MOTIVATION

C-SFSs and the ELECTRE approach to MCDM are two appealing branches of literature that served as inspiration for the research that is proposed in this paper. Our ability to carry out relevant evaluations for decision-making is made possible by a revolutionary technique known as circular spherical fuzzy ELECTRE (C-SF ELECTRE), which is the outcome of their combination. Thus, the following are the primary motivational factors behind this article:

1) C-SFS provides for many occurrences of belonging, non-belonging, and indeterminacy degrees with radius to evaluate the ratings of prospective failures and danger signs in the representation of assessment data.
2) In order to address more complex scenarios, C-SFSs include both the capabilities of SFS and radius of the given value.
3) Also, the ELECTRE approach is a successful structure in several areas where the decision-makers need to take into account three or more risks variables and when there is also a variety between these criteria that is essential to the nature of assessments.
4) When addressing programs that contain multiple evaluation data values, the new C-SF ELECTRE method makes it possible to produce findings that are more accurate and trustworthy.

If we want to check the radius of a circle in SFS, we are unable to find it. As a result, authors are thrilled to be able to meet this need. As a result, we must employ C-SFS to deal with this sort of issue. This is a transition to all predicting algorithms capable of handling any form of membership, indeterminacy, and non-membership includes circular radius. They are useful when we need to calculate the radius of SFS. The question arises: why do we calculate the radius of any set? The answer is that after finding the radius, we know to check where the values of oversetting lie in this radius, which is helpful in observing our results. It is the only set in fuzzy set theory which gives the radius including three degrees like membership, indeterminacy and non-membership.

Example 1: Let $\Re = \{\hat{x_1}, \hat{x_2}, \hat{x_3}\}$. An example of C-SFS on $\Re$ can be given as:

$$\underset{\sim}{A}_{0.3} = \{\langle \hat{x_1}, 0.1, 0.4, 0.3; 0.3\rangle, \langle \hat{x_2}, 0.6, 0.1, 0.1; 0.3\rangle, \langle \hat{x_3}, 0.3, 0.1, 0.4; 0.3\rangle\}$$

B. DESIGN/PROCEDURE

To be able to assess risks in a wider context, the ELECTRE technique is expanded to include the C-SF ELECTRE approach in this article. The C-SF ELECTRE approach suggests pairwise comparisons between all failures that have been found and are related to each risk factor. By removing failures with low-risk priorities, the suggested model produces a collection of solutions.

C. FINDINGS

A correlation of the model presented in this article with the current methods shows its applicability and viability. Our computational findings show that the generated model has a higher reference value and is better suited to the project's real condition. Our findings show that C-SFSs are capable of correctly interpreting the decision-making issues and the differences in views among multiple decision-makers.

D. PRACTICAL APPLICATION

The results of the study might be used as the basis for supporting information for decisions on risk management, and the recommended model offers a framework for developing risk assessments of failure modes, such as in medical settings. In this article, we use a kidney transplant as an example. The goal of kidney transplant research is to identify strategies to lower the risks and difficulties connected with this treatment while also increasing the availability and success of kidney transplantation. After reading this study report, we can tackle the question of what stage of kidney transplantation is most in demand.

E. ORIGINALITY

The flexible C-SF environment is used to design a unique methodology utilising the ELECTRE method. It weighs the opinions of experts and risk factors in relation to C-SFSs. We provide proof that the suggested method has improved the robustness of the findings and considerably improved the integrity of the data used in expert assessment.

The remainder of the article is arranged as follows: There are four terms defined: SFS, C-IFS, and circular pythagorean fuzzy set (C-PFS) and Disc-PFS, which are used to help understand the other part of the article. The next step is to create a C-SFS and an example. After this, we introduced some operations and relations of C-SFS that are useful for calculating the results. Then define the C-SF decision matrix. Following this, we described an ELECTRE algorithm along with concordant and discordant sets. Just after that, we gave a detailed explanation of the C-SFS

ELECTRE method. We described the algorithm's step-by-step application in detail. We developed a method for determining which types of conditions are more likely to emerge after kidney transplantation. To further illustrate our concept using the comparative approach, we provide a second example. We provided the paper's conclusion in the last section.

II. PRELIMINARIES

This section provides a detailed explanation of the essential ideas that are connected with SFS, C-IFS, C-PFS and D-PFS.

Definition 1 [46]: Let $\Re$ be the universe set. Afterwards, the set

$$\underset{\sim}{A} = \{\langle \hat{x}, T^*_{\underset{\sim}{A}}(\hat{x}), I^*_{\underset{\sim}{A}}(\hat{x}), F^*_{\underset{\sim}{A}}(\hat{x}) | \hat{x} \in \Re\rangle\} \tag{1}$$

is reportedly a spherical fuzzy set, where $T^*_{\underset{\sim}{A}}(\hat{x}) : \Re \to [0, 1]$, $I^*_{\underset{\sim}{A}}(\hat{x}) : \Re \to [0, 1]$ *and* $F^*_{\underset{\sim}{A}}(\hat{x}) : \Re \to [0, 1]$ *are said to be degrees belonging of $\hat{x}$ in $\Re$, neutral-belonging degree of $\hat{x}$ in $\Re$ and non-belonging degree of $\hat{x}$ in $\Re$ respectively. Also,* $T^*_{\underset{\sim}{A}}, I^*_{\underset{\sim}{A}}$ *and* $F^*_{\underset{\sim}{A}}$ *fulfill the criteria below:*

$$(\forall \hat{x} \in \Re)(0 \leq (T^*_{\underset{\sim}{A}}(\hat{x}))^2 + I^*_{\underset{\sim}{A}}(\hat{x}))^2 + F^*_{\underset{\sim}{A}}(\hat{x}))^2 \leq 1) \tag{2}$$

For SFS $\{\langle \hat{x}, T^*_{\underset{\sim}{A}}(\hat{x}), I^*_{\underset{\sim}{A}}(\hat{x}), F^*_{\underset{\sim}{A}}(\hat{x}) | \hat{x} \in \Re\rangle\}$, *which is a triple component.*

$$\langle T^*_{\underset{\sim}{A}}(\hat{x}), I^*_{\underset{\sim}{A}}(\hat{x}), F^*_{\underset{\sim}{A}}(\hat{x})\rangle$$

are considered SFN, and every spherical number is indicated by $e = \langle T^*_e, I^*_e, F^*_e\rangle$ *where* $T^*_e, I^*_e, F^*_e \in [0, 1]$, *with the circumstance that* $0 \leq T^{2*}_e + I^{2*}_e + F^{2*}_e \leq 1$

Definition 2 [47]: Let's assume that $\Re$ is a fixed universe, and $\underset{\sim}{A}$ is its subset. This is

$$\underset{\sim}{A} = \{\langle \hat{x}, T^*_{\underset{\sim}{A}}(\hat{x}), F^*_{\underset{\sim}{A}}(\hat{x}); r\rangle | \hat{x} \in \Re\}$$

where $0 \leq T^*_{\underset{\sim}{A}}(\hat{x}) + F^*_{\underset{\sim}{A}}(\hat{x}) \leq 1$ *and* $r \in [0, 1]$ *is the radius of each set sphere.* $\hat{x} \in \Re$, *is said to be C-IFS and functions* $T^*_{\underset{\sim}{A}}(\hat{x}) : \Re \to [0, 1]$ *and* $F^*_{\underset{\sim}{A}}(\hat{x}) : \Re \to [0, 1]$ *indicate the degree of belonging and non-belonging degree of component* $\hat{x} \in \Re$ *to a fixed set* $\underset{\sim}{A} \subseteq \Re$.

Here, each element is symbolized by a sphere with a center $\langle T^*_{\underset{\sim}{A}}(\hat{x}), F^*_{\underset{\sim}{A}}(\hat{x})\rangle$ *and radius r as opposed to the normal IFSs where each element is represented by a point in the intuitionistic fuzzy interpretation triangle.*

Definition 3 [48]: Let $r \in [0, 1]$. *A C-PFS $\underset{\sim}{A}$ in $\Re$ is defined by:*

$$\underset{\sim}{A} = \{\langle \hat{x}, T^*_{\underset{\sim}{A}}(\hat{x}), F^*_{\underset{\sim}{A}}(\hat{x}); r\rangle | \hat{x} \in \Re\}$$

where $T^*_{\underset{\sim}{A}}(\hat{x}), F^*_{\underset{\sim}{A}}(\hat{x}) :\to [0, 1]$ *are functions like that*

$$(\forall \hat{x} \in \Re)(0 \leq (T^*_{\underset{\sim}{A}}(\hat{x}))^2 + F^*_{\underset{\sim}{A}}(\hat{x}))^2 \leq 1)$$

The radius of the circle is r is the point $(T^*_{\underset{\sim}{A}}(\hat{x}), F^*_{\underset{\sim}{A}}(\hat{x}))$ *on the sphere. This circle symbolizes the belonging degree and non-belonging degree of* $\hat{x} \in \Re$.

Definition 4 [49]: Let $r(\hat{x}) \in [0, 1]$. *A D-PFS $\underset{\sim}{A}$ in $\Re$ is defined by:*

$$\underset{\sim}{A} = \{\langle \hat{x}, T^*_{\underset{\sim}{A}}(\hat{x}), F^*_{\underset{\sim}{A}}(\hat{x}); r(\hat{x})\rangle | \hat{x} \in \Re\}$$

where $T^*_{\underset{\sim}{A}}(\hat{x}), F^*_{\underset{\sim}{A}}(\hat{x}) :\to [0, 1]$ *are functions like that*

$$(\forall \hat{x} \in \Re)(0 \leq (T^*_{\underset{\sim}{A}}(\hat{x}))^2 + F^*_{\underset{\sim}{A}}(\hat{x}))^2 \leq 1)$$

The radius of the circle is $r(\hat{x})$ *is the point* $(T^*_{\underset{\sim}{A}}(\hat{x}), F^*_{\underset{\sim}{A}}(\hat{x}))$ *on the plane. This circle symbolizes the belonging degree and non-belonging degree of* $\hat{x} \in \Re$.

Example 2: Let $\Re = \{\hat{x_1}, \hat{x_2}, \hat{x_3}\}$. *An example of C-PFS on $\Re$ can be given as:*

$$\underset{\sim}{A}_{0.4} = \{\langle \hat{x_1}, 0.1, 0.3; 0.4\rangle, \langle \hat{x_2}, 0.6, 0.1; 0.4\rangle, \langle \hat{x_3}, 0.3, 0.4; 0.4\rangle\}$$

Example 3: Let $\Re = \{\hat{x_1}, \hat{x_2}, \hat{x_3}\}$. *An example of D-PFS on $\Re$ can be given as:*

$$\underset{\sim}{A} = \{\langle \hat{x_1}, 0.1, 0.3; 0.4\rangle, \langle \hat{x_2}, 0.6, 0.1; 0.1\rangle, \langle \hat{x_3}, 0.3, 0.4; 0.2\rangle\}$$

III. CIRCULAR AND DISC SPHERICAL FUZZY SETS

In this section, the idea of circular spherical fuzzy set (C-SFS) and disc spherical fuzzy set (D-SFS) were introduced, which is an elongation of a SFS.

Definition 5: Let's assume that $\Re$ is a fixed universe, and $\underset{\sim}{A}$ is its subset. This is

$$\underset{\sim}{A} = \{\langle \hat{x}, T_{\underset{\sim}{A}}(\hat{x}), I_{\underset{\sim}{A}}(\hat{x}), F_{\underset{\sim}{A}}(\hat{x}); \hat{r}_{\underset{\sim}{A}} | \hat{x} \in \Re\rangle\} \tag{3}$$

is allegedly a C-SFS, where $T_{\underset{\sim}{A}}(\hat{x}) : \Re \to [0, 1]$, $I_{\underset{\sim}{A}}(\hat{x}) : \Re \to [0, 1]$, $F_{\underset{\sim}{A}}(\hat{x}) : \Re \to [0, 1]$. *are apparently degrees of positive-belonging of $\hat{x}$ in $\Re$, neutral-belonging degree of $\hat{x}$ in $\Re$ and non-belonging degree of $\hat{x}$ in $\Re$ respectively. Also* $T_{\underset{\sim}{A}}, I_{\underset{\sim}{A}}$ *and* $F_{\underset{\sim}{A}}$ *satisfy the following conditions:*

$$(\forall \hat{x} \in \Re)(0 \leq (T_{\underset{\sim}{A}}(\hat{x}))^2 + (I_{\underset{\sim}{A}}(\hat{x}))^2 + (F_{\underset{\sim}{A}}(\hat{x}))^2 \leq 1). \tag{4}$$

The radius of the circle around is $\hat{r}$ the point $(T_{\underset{\sim}{A}}(\hat{x}), I_{\underset{\sim}{A}}(\hat{x}), F_{\underset{\sim}{A}}(\hat{x}))$ *on the sphere. This circle represents the belonging degree, non-belonging degree, and indeterminacy of* $\hat{x} \in \Re$.

In this C-SFS, each element is denoted by a circle with a center $(T_{\underset{\sim}{A}}(\hat{x}), I_{\underset{\sim}{A}}(\hat{x}), F_{\underset{\sim}{A}}(\hat{x}))$ *and a radius $\hat{r}$ instead of a point in the spherical fuzzy interpretation triangle as in typical SFSs.*

Due to the fact that every standard SFS has the form, the new kind of sets is an upgrade to the standard SFS.

$$\underset{\sim}{A} = \underset{\sim}{A}_o = \{\langle \hat{x}, T_{\underset{\sim}{A}}(\hat{x}), I_{\underset{\sim}{A}}(\hat{x}), F_{\underset{\sim}{A}}(\hat{x}); 0\rangle\}$$

but the C-SFS with $\hat{r} > 0$ *is not compatible with a normal SFS.*

Definition 6: Let's assume that $\Re$ is a fixed universe, and $\underset{\sim}{A}$ is its subset. This is

$$\underset{\sim}{A} = \{\langle \hat{x}, T_{\underset{\sim}{A}}(\hat{x}), I_{\underset{\sim}{A}}(\hat{x}), F_{\underset{\sim}{A}}(\hat{x}); \hat{r}_{\underset{\sim}{A}}(\hat{x}) | \hat{x} \in \Re\rangle\} \tag{5}$$

is allegedly a D-SFS, where $T_{\underset{\sim}{A}}(\hat{x}) : \Re \to [0, 1]$, $I_{\underset{\sim}{A}}(\hat{x}) : \Re \to [0, 1]$, $F_{\underset{\sim}{A}}(\hat{x}) : \Re \to [0, 1]$. *are apparently degrees of positive-belonging of $\hat{x}$ in $\Re$, neutral-belonging degree of $\hat{x}$*

in $\Re$ *and non-belonging degree of* $\hat{x}$ *in* $\Re$ *respectively. Also* $T_{\underset{\sim}{A}}, I_{\underset{\sim}{A}}$ *and* $F_{\underset{\sim}{A}}$ *satisfy the following conditions*:

$$(\forall \hat{x} \in \Re)(0 \leq (T_{\underset{\sim}{A}}(\hat{x}))^2 + (I_{\underset{\sim}{A}}(\hat{x}))^2 + (F_{\underset{\sim}{A}}(\hat{x}))^2 \leq 1). \quad (6)$$

The radius of the circle around is $\hat{r}(\hat{x})$ *the point* $(T_{\underset{\sim}{A}}(\hat{x}), I_{\underset{\sim}{A}}(\hat{x}), F_{\underset{\sim}{A}}(\hat{x}))$ *on the sphere. This circle represents the belonging degree, non-belonging degree, and indeterminacy of* $\hat{x} \in \Re$.

In this D-SFS, each element is denoted by a circle with a center $(T_{\underset{\sim}{A}}(\hat{x}), I_{\underset{\sim}{A}}(\hat{x}), F_{\underset{\sim}{A}}(\hat{x}))$ *and a radius* $\hat{r}(\hat{x})$ *instead of a point in the spherical fuzzy interpretation triangle as in typical SFSs.*

The circular spherical fuzzy set has a fixed radius through all elements, but in the case of the disc spherical fuzzy set, the radius associated with each element is distinct.

A. CONSTRUCTION OF DISC SPHERICAL FUZZY SETS

In this part, we will discuss the method that is used to calculate the radius of D-SFS in order to convert SFS to D-SFS. Find the radius of a SFS by using equations (7) and (8).

In an SFS N_i, let spherical fuzzy pairs possess a shape $\{\langle \mu_{i,1}, \pi_{i,1}, \nu_{i,1}\rangle \langle \mu_{i,2}, \pi_{i,1}, \nu_{i,2}\rangle, \ldots.\}$, where i is a numeric value of SFS N_i each containing λ_i. Initially, the arithmetic average of the spherical fuzzy pairs is determined as:

$$\langle \mu_{(N_i)}, \pi_{(N_i)}, \nu_{(N_i)}\rangle = \langle \sqrt{\frac{\Sigma_{j=1}^{\lambda_i}\mu_{i,j}^2}{\lambda_i}}, \sqrt{\frac{\Sigma_{j=1}^{\lambda_i}\pi_{i,j}^2}{\lambda_i}}, \sqrt{\frac{\Sigma_{j=1}^{\lambda_i}\nu_{i,j}^2}{\lambda_i}}\rangle \quad (7)$$

where λ_i is the number of spherical fuzzy pairs N_i.

The radius of the $\langle \mu_{(N_i)}, \pi_{(N_i)}, \nu_{(N_i)}\rangle$ is the highest of the Euclidian distances.

$$r_i = \max_{1\leq j\leq \lambda_i} \sqrt{(\mu_{(N_i)} - \mu_{i,j})^2 + (\pi_{(N_i)} - \pi_{i,j})^2 + (\nu_{(N_i)} - \nu_{i,j})^2} \quad (8)$$

After finding radius our SFS is converted into D-SFS.

B. SEMANTIC INTERPRETATION

It is generally known that an IV-IFS provides for considerable latitude in the definition of MDs and NMDs for each alternative. The alternative is characterized by a pair of intervals rather than an orthopair. These intervals might be of incredibly diverse sizes. Additionally, their lengths may change in accordance with the options to take measurement mistakes, uncertainty, etc. into consideration.

There is a fixed slackness $\hat{r}$ around the orthopair produced by $(T_a(\alpha), I_a(\alpha), F_a(\alpha))$ in a C-SFS for the description of $\alpha \in \Xi$. Evaluations of α are acceptable for any authorized orthopairs whose separation from $(T_a(\alpha), I_a(\alpha), F_a(\alpha))$ is less than this radius. But in a D-SFS, there is a specific slackness for the description of α in Ξ, thus permissible orthopairs whose separation from $(T_a(\alpha), I_a(\alpha), F_a(\alpha))$ is under $\hat{r}(\alpha)$ are allowed to evaluate α. It is clear that the radii indicate a margin of error in terms of the description of the orthopairs in both C-SFSs and D-SFSs. Each choice in a C-SFS has the same error margin. We should utilize a D-SFS whenever we believe that certain alternatives must have lower margins of error (for example, because they have been assessed using more accurate equipment, better statistical methods, or more dependable samples).

Example 4: Let $\Xi = \{\hat{\alpha_1}, \hat{\alpha_2}, \hat{\alpha_3}\}$. *An example of C-SFS on* Ξ *can be given as:*

$$A_{0.3} = \{\langle \hat{\alpha_1}, 0.1, 0.4, 0.3; 0.3\rangle, \langle \hat{\alpha_2}, 0.6, 0.1, 0.1; 0.3\rangle, \langle \hat{\alpha_3}, 0.3, 0.1, 0.4; 0.3\rangle\}$$

Example 5: Let $\Xi = \{\hat{\alpha_1}, \hat{\alpha_2}, \hat{\alpha_3}\}$ *be a collection of SFV on* Ξ *can be given as. Then the D-SFS can be given as:*

$$A = \{\langle \hat{\alpha_1}, 0.1, 0.4, 0.3; 0.4\rangle, \langle \hat{\alpha_2}, 0.6, 0.1, 0.1; 0.3\rangle, \langle \hat{\alpha_3}, 0.3, 0.1, 0.4; 0.1\rangle\}$$

IV. OPERATIONS AND RELATIONS ON CIRCULAR SPHERICAL FUZZY SETS

In this section, we are going to work on developing certain relations and operations involving the distance formula of the circular spherical fuzzy set. The operations and relations of disc spherical fuzzy set is same as C-SFS so we define only C-SFS. In addition to that, we are going to go over its theorem along with proof. In addition, we are going to look at certain criteria for comparison that will be used for ranking.

The following is a description of the relations that exist between the two C-SFSs $\underset{\sim}{A}_{\hat{r}_1}$ and $\underset{\sim}{B}_{\hat{r}_2}$:

$$\underset{\sim}{A}_{\hat{r}_1} = \{\langle \hat{a}, T_{\underset{\sim}{A}}(\hat{a}), I_{\underset{\sim}{A}}(\hat{a}), F_{\underset{\sim}{A}}(\hat{a}); \hat{r}_1 | \hat{a} \in \Re\rangle\}$$
$$\underset{\sim}{B}_{\hat{r}_2} = \{\langle \hat{a}, T_{\underset{\sim}{B}}(\hat{a}), I_{\underset{\sim}{B}}(\hat{a}), F_{\underset{\sim}{B}}(\hat{a}); \hat{r}_2 | \hat{a} \in \Re\rangle\}$$

are two C-SFSs in $\Re$. The following is a definition of several set operations that may be performed on C-SFSs:

1) $\underset{\sim}{A}_{\hat{r}_1} \subset \underset{\sim}{B}_{\hat{r}_2}$ iff $\hat{r}_1 \leq \hat{r}_2$ and $T_{\underset{\sim}{A}}(\hat{x}) \leq T_{\underset{\sim}{B}}(\hat{a}), I_{\underset{\sim}{A}}(\hat{a}) \leq I_{\underset{\sim}{B}}(\hat{a})$ and $F_{\underset{\sim}{A}}(\hat{a}) \geq F_{\underset{\sim}{B}}(\hat{a})$
2) $\underset{\sim}{A}_{\hat{r}_1} = \underset{\sim}{B}_{\hat{r}_2}$ iff $\hat{r}_1 = \hat{r}_2$ and $T_{\underset{\sim}{A}}(\hat{a}) = T_{\underset{\sim}{B}}(\hat{a}), I_{\underset{\sim}{A}}(\hat{a}) = I_{\underset{\sim}{B}}(\hat{a})$ and $F_{\underset{\sim}{A}}(\hat{a}) = F_{\underset{\sim}{B}}(\hat{a})$
3) Complement $\underset{\sim}{A}^c_{\hat{r}_1}$ of $\underset{\sim}{A}_{\hat{r}_1}$ is defined as: $\underset{\sim}{A}^c_{\hat{r}_1} = \{\langle \hat{a}, F_{\underset{\sim}{A}}(\hat{a}), I_{\underset{\sim}{A}}(\hat{a}), T_{\underset{\sim}{A}}(\hat{a}); \hat{r}_1 | \hat{a} \in \Re\rangle\}$
4) $\underset{\sim}{A}_{\hat{r}_1} \cup_{\min} \underset{\sim}{B}_{\hat{r}_2} = \{\langle \hat{a}, \max(T_{\underset{\sim}{A}}(\hat{a}), T_{\underset{\sim}{B}}(\hat{a})), \min(I_{\underset{\sim}{A}}(\hat{a}), I_{\underset{\sim}{B}}(\hat{a})), \min(F_{\underset{\sim}{A}}(\hat{a}), F_{\underset{\sim}{B}}(\hat{a})); \min(\hat{r}_1, \hat{r}_2) | \hat{a} \in \Re\rangle\}$
5) $\underset{\sim}{A}_{\hat{r}_1} \cup_{\max} \underset{\sim}{B}_{\hat{r}_2} = \{\langle \hat{a}, \max(T_{\underset{\sim}{A}}(\hat{a}), T_{\underset{\sim}{B}}(\hat{a})), \min(I_{\underset{\sim}{A}}(\hat{a}), I_{\underset{\sim}{B}}(\hat{a})), \min(F_{\underset{\sim}{A}}(\hat{a}), F_{\underset{\sim}{B}}(\hat{a})); \max(\hat{r}_1, \hat{r}_2) | \hat{a} \in \Re\rangle\}$
6) $\underset{\sim}{A}_{\hat{r}_1} \cap_{\min} \underset{\sim}{B}_{\hat{r}_2} = \{\langle \hat{a}, \min(T_{\underset{\sim}{A}}(\hat{a}), T_{\underset{\sim}{B}}(\hat{a})), \min(I_{\underset{\sim}{A}}(\hat{a}), I_{\underset{\sim}{B}}(\hat{a})), \max(F_{\underset{\sim}{A}}(\hat{a}), F_{\underset{\sim}{B}}(\hat{a})); \min(\hat{r}_1, \hat{r}_2) | \hat{x} \in \Re\rangle\}$
7) $\underset{\sim}{A}_{\hat{r}_1} \cap_{\max} \underset{\sim}{B}_{\hat{r}_2} = \{\langle \hat{a}, \min(T_{\underset{\sim}{A}}(\hat{a}), T_{\underset{\sim}{B}}(\hat{a})), \min(I_{\underset{\sim}{A}}(\hat{a}), I_{\underset{\sim}{B}}(\hat{a})), \max(F_{\underset{\sim}{A}}(\hat{a}), F_{\underset{\sim}{B}}(\hat{a})); \max(\hat{r}_1, \hat{r}_2) | \hat{a} \in \Re\rangle\}$

Example 6: If we have two D-SFS $\underset{\sim}{A} = \langle 0.4, 0.3, 0.2; 0.3\rangle$ *and* $\underset{\sim}{B} = \langle 0.2, 0.7, 0.1; 0.2\rangle$ *then*

$$\begin{aligned}
\underset{\sim}{A}^c &= \langle 0.2, 0.3, 0.4; 0.3\rangle \\
\underset{\sim}{B}^c &= \langle 0.1, 0.7, 0.2; 0.2\rangle \\
\underset{\sim}{A} \cup_{\min} \underset{\sim}{B} &= \langle 0.4, 0.3, 0.1; 0.2\rangle \\
\underset{\sim}{A} \cup_{\max} \underset{\sim}{B} &= \langle 0.4, 0.3, 0.1; 0.3\rangle \\
\underset{\sim}{A} \cap_{\min} \underset{\sim}{B} &= \langle 0.2, 0.3, 0.2; 0.2\rangle \\
\underset{\sim}{A} \cap_{\max} \underset{\sim}{B} &= \langle 0.2, 0.3, 0.2; 0.3\rangle
\end{aligned}$$

Definition 7: The normalized Euclidean distance for two C-SFSs and D-SFSs $\underset{\sim}{A}_{\hat{r}_1}$ *and* $\underset{\sim}{B}_{\hat{r}_2}$ *specified above in* $\hat{a} \in \Re$ *is defined as shown in the equation at the bottom of the page.*

A. ALGEBRAIC OPERATIONS

Let $\underset{\sim}{A} = \langle T_{\underset{\sim}{A}}, I_{\underset{\sim}{A}}, F_{\underset{\sim}{A}}; \hat{r}_{\underset{\sim}{A}}\rangle$ and $\underset{\sim}{B} = \langle T_{\underset{\sim}{B}}, I_{\underset{\sim}{B}}, F_{\underset{\sim}{B}}; \hat{r}_{\underset{\sim}{B}}\rangle$ be two C-SFSs. The following definitions apply to several algebraic operations among C-SFSs:

1) $\underset{\sim}{A} +_{\min} \underset{\sim}{B} = \langle \sqrt{T_{\underset{\sim}{A}}^2 + T_{\underset{\sim}{B}}^2 - T_{\underset{\sim}{A}}^2 T_{\underset{\sim}{B}}^2}, I_{\underset{\sim}{A}} I_{\underset{\sim}{B}}, F_{\underset{\sim}{A}} F_{\underset{\sim}{B}}; \min(\hat{r}_{\underset{\sim}{A}}, \hat{r}_{\underset{\sim}{B}})\rangle$;
2) $\underset{\sim}{A} +_{\max} \underset{\sim}{B} = \langle \sqrt{T_{\underset{\sim}{A}}^2 + T_{\underset{\sim}{B}}^2 - T_{\underset{\sim}{A}}^2 T_{\underset{\sim}{B}}^2}, I_{\underset{\sim}{A}} I_{\underset{\sim}{B}}, F_{\underset{\sim}{A}} F_{\underset{\sim}{B}}; \max(\hat{r}_{\underset{\sim}{A}}, \hat{r}_{\underset{\sim}{B}})\rangle$;
3) $\underset{\sim}{A} \times_{\min} \underset{\sim}{B} = \langle T_{\underset{\sim}{A}} T_{\underset{\sim}{B}}, I_{\underset{\sim}{A}} I_{\underset{\sim}{B}}, \sqrt{F_{\underset{\sim}{A}}^2 + F_{\underset{\sim}{B}}^2 - F_{\underset{\sim}{A}}^2 F_{\underset{\sim}{B}}^2}; \min(\hat{r}_{\underset{\sim}{A}}, \hat{r}_{\underset{\sim}{B}})\rangle$;
4) $\underset{\sim}{A} \times_{\max} \underset{\sim}{B} = \langle T_{\underset{\sim}{A}} T_{\underset{\sim}{B}}, I_{\underset{\sim}{A}} I_{\underset{\sim}{B}}, \sqrt{F_{\underset{\sim}{A}}^2 + F_{\underset{\sim}{B}}^2 - F_{\underset{\sim}{A}}^2 F_{\underset{\sim}{B}}^2}; \max(\hat{r}_{\underset{\sim}{A}}, \hat{r}_{\underset{\sim}{B}})\rangle$;
5) $\underset{\sim}{t}\underset{\sim}{A} = \langle \sqrt{1 - (1 - T_{\underset{\sim}{A}}^2)^{\underset{\sim}{t}}}, (I_{\underset{\sim}{A}})^{\underset{\sim}{t}}, (F_{\underset{\sim}{A}})^{\underset{\sim}{t}}; (\hat{r}_{\underset{\sim}{A}})\rangle$
6) $\underset{\sim}{A}^{\underset{\sim}{t}} = \langle (T_{\underset{\sim}{A}})^{\underset{\sim}{t}}, (I_{\underset{\sim}{A}})^{\underset{\sim}{t}}, \sqrt{1 - (1 - N_{\underset{\sim}{A}}^2)^{\underset{\sim}{t}}}; (\hat{r}_{\underset{\sim}{A}})\rangle$

Example 7: Let the two D-SFS $\underset{\sim}{A} = \langle 0.4, 0.3, 0.2; 0.3\rangle$ *and* $\underset{\sim}{B} = \langle 0.2, 0.7, 0.1; 0.2\rangle$ *and* $\underset{\sim}{t} = 0.3$ *then*

$$\begin{aligned}
\underset{\sim}{A} +_{\min} \underset{\sim}{B} &= \langle 0.4, 0.3, 0.2; 0.3\rangle +_{\min} \langle 0.2, 0.7, 0.1; 0.2\rangle \\
&= \langle 0.44, 0.21, 0.02; 0.2\rangle \\
\underset{\sim}{A} +_{\max} \underset{\sim}{B} &= \langle 0.4, 0.3, 0.2; 0.3\rangle +_{\max} \langle 0.2, 0.7, 0.1; 0.2\rangle \\
&= \langle 0.44, 0.21, 0.02; 0.3\rangle \\
\underset{\sim}{A} \times_{\min} \underset{\sim}{B} &= \langle 0.4, 0.3, 0.2; 0.3\rangle \times_{\min} \langle 0.2, 0.7, 0.1; 0.2\rangle \\
&= \langle 0.08, 0.21, 0.22; 0.2\rangle \\
\underset{\sim}{A} \times_{\max} \underset{\sim}{B} &= \langle 0.4, 0.3, 0.2; 0.3\rangle \times_{\max} \langle 0.2, 0.7, 0.1; 0.2\rangle \\
&= \langle 0.08, 0.21, 0.22; 0.3\rangle \\
\underset{\sim}{t}\underset{\sim}{A} &= 0.3\langle 0.4, 0.3, 0.2; 0.3\rangle = \langle 0.23, 0.70, 0.62; 0.3\rangle \\
\underset{\sim}{A}^{\underset{\sim}{t}} &= \langle 0.4, 0.3, 0.2; 0.3\rangle^{0.3} = \langle 0.76, 0.70, 0.11; 0.3\rangle
\end{aligned}$$

Theorem 1: Assuming that $\underset{\sim}{A} = \langle T_{\underset{\sim}{A}}, I_{\underset{\sim}{A}}, F_{\underset{\sim}{A}}; \hat{r}_{\underset{\sim}{A}}\rangle$, $\underset{\sim}{B} = \langle T_{\underset{\sim}{B}}, I_{\underset{\sim}{B}}, F_{\underset{\sim}{B}}; \hat{r}_{\underset{\sim}{B}}\rangle$ *and* $\underset{\sim}{C} = \langle T_{\underset{\sim}{C}}, I_{\underset{\sim}{C}}, F_{\underset{\sim}{C}}; \hat{r}_{\underset{\sim}{C}}\rangle$ *be any three C-SFSs and* $\underset{\sim}{t} \geq 1$. *Then the following identities are satisfies.*

1) $\underset{\sim}{A} + \underset{\sim}{B} = \underset{\sim}{B} + \underset{\sim}{A}$;
2) $\underset{\sim}{A} \times \underset{\sim}{B} = \underset{\sim}{B} \times \underset{\sim}{A}$;
3) $(\underset{\sim}{A} + \underset{\sim}{B}) + \underset{\sim}{C} = \underset{\sim}{A} + (\underset{\sim}{B} + \underset{\sim}{C})$;
4) $(\underset{\sim}{A} \times \underset{\sim}{B}) \times \underset{\sim}{C} = \underset{\sim}{A} \times (\underset{\sim}{B} \times \underset{\sim}{C})$;
5) $\underset{\sim}{t}\underset{\sim}{A} + \underset{\sim}{t}\underset{\sim}{B} = \underset{\sim}{t}(\underset{\sim}{A} + \underset{\sim}{B}), \underset{\sim}{t} \geq 0$;
6) $\underset{\sim}{t}_{\underset{\sim}{A}}\underset{\sim}{A} + \underset{\sim}{t}_{\underset{\sim}{B}}\underset{\sim}{A} = (\underset{\sim}{t}_{\underset{\sim}{A}} + \underset{\sim}{t}_{\underset{\sim}{B}})\underset{\sim}{A}, \underset{\sim}{t}_{\underset{\sim}{A}}$ *and* $\underset{\sim}{t}_{\underset{\sim}{B}} \geq 0$;
7) $(\underset{\sim}{A} \times \underset{\sim}{B})^{\underset{\sim}{t}} = \underset{\sim}{A}^{\underset{\sim}{t}} \times \underset{\sim}{B}^{\underset{\sim}{t}}, \underset{\sim}{t} \geq 0$;
8) $\underset{\sim}{A}^{\underset{\sim}{t}_{\underset{\sim}{A}}} \times \underset{\sim}{A}^{\underset{\sim}{t}_{\underset{\sim}{B}}} = \underset{\sim}{A}^{\underset{\sim}{t}_{\underset{\sim}{A}} + \underset{\sim}{t}_{\underset{\sim}{B}}}, \underset{\sim}{t}_{\underset{\sim}{A}}$ *and* $\underset{\sim}{t}_{\underset{\sim}{B}} \geq 0$;

Proof: (1). To show this, $\underset{\sim}{A} +_{\min} \underset{\sim}{B} = \underset{\sim}{B} +_{\min} \underset{\sim}{A}$. Consider.

$$\begin{aligned}
&L.H.S \\
&= \underset{\sim}{A} +_{\min} \underset{\sim}{B} \\
&= \langle T_{\underset{\sim}{A}}, I_{\underset{\sim}{A}}, F_{\underset{\sim}{A}}; \hat{r}_{\underset{\sim}{A}}\rangle + \langle T_{\underset{\sim}{B}}, I_{\underset{\sim}{B}}, F_{\underset{\sim}{B}}; \hat{r}_{\underset{\sim}{B}}\rangle \\
&= \langle \sqrt{T_{\underset{\sim}{A}}^2 + T_{\underset{\sim}{B}}^2 - T_{\underset{\sim}{A}}^2 T_{\underset{\sim}{B}}^2}, I_{\underset{\sim}{A}} I_{\underset{\sim}{B}}, F_{\underset{\sim}{A}} F_{\underset{\sim}{B}}; \min(\hat{r}_{\underset{\sim}{A}}, \hat{r}_{\underset{\sim}{B}})\rangle \\
&= \langle \sqrt{T_{\underset{\sim}{B}}^2 + T_{\underset{\sim}{A}}^2 - T_{\underset{\sim}{B}}^2 T_{\underset{\sim}{A}}^2}, I_{\underset{\sim}{B}} I_{\underset{\sim}{A}}, F_{\underset{\sim}{B}} F_{\underset{\sim}{A}}; \min(\hat{r}_{\underset{\sim}{B}}, \hat{r}_{\underset{\sim}{A}})\rangle \\
&R.H.S \\
&= \underset{\sim}{B} +_{\min} \underset{\sim}{A}
\end{aligned}$$

Hence, we prove this.

(2). To show this, $\underset{\sim}{A} \times_{\min} \underset{\sim}{B} = \underset{\sim}{B} \times_{\min} \underset{\sim}{A}$. Consider.

$$\begin{aligned}
L.H.S &= \underset{\sim}{A} \times_{\min} \underset{\sim}{B} \\
&= \langle T_{\underset{\sim}{A}}, I_{\underset{\sim}{A}}, F_{\underset{\sim}{A}}; \hat{r}_{\underset{\sim}{A}}\rangle \times \langle T_{\underset{\sim}{B}}, I_{\underset{\sim}{B}}, F_{\underset{\sim}{B}}; \hat{r}_{\underset{\sim}{B}}\rangle \\
&= \langle T_{\underset{\sim}{A}} T_{\underset{\sim}{B}}, I_{\underset{\sim}{A}} I_{\underset{\sim}{B}}, \sqrt{F_{\underset{\sim}{A}}^2 + F_{\underset{\sim}{B}}^2 - F_{\underset{\sim}{A}}^2 F_{\underset{\sim}{B}}^2}; \min(\hat{r}_{\underset{\sim}{A}}, \hat{r}_{\underset{\sim}{B}})\rangle \\
&= \langle T_{\underset{\sim}{B}} T_{\underset{\sim}{A}}, I_{\underset{\sim}{B}} I_{\underset{\sim}{A}}, \sqrt{F_{\underset{\sim}{B}}^2 + F_{\underset{\sim}{A}}^2 - F_{\underset{\sim}{B}}^2 F_{\underset{\sim}{A}}^2}; \min(\hat{r}_{\underset{\sim}{B}}, \hat{r}_{\underset{\sim}{A}})\rangle \\
R.H.S &= \underset{\sim}{B} \times_{\min} \underset{\sim}{A}
\end{aligned}$$

Hence, we prove this.

(5). To show this, $\underset{\sim}{t}\underset{\sim}{A} + \underset{\sim}{t}\underset{\sim}{B} = \underset{\sim}{t}(\underset{\sim}{A} + \underset{\sim}{B}), \underset{\sim}{t} \geq 1$. Consider.

$$\begin{aligned}
&L.H.S \\
&= \underset{\sim}{t}\underset{\sim}{A} +_{\min} \underset{\sim}{t}\underset{\sim}{B} \\
&= \underset{\sim}{t}\langle T_{\underset{\sim}{A}}, I_{\underset{\sim}{A}}, F_{\underset{\sim}{A}}; \hat{r}_{\underset{\sim}{A}}\rangle + \underset{\sim}{t}\langle T_{\underset{\sim}{B}}, I_{\underset{\sim}{B}}, F_{\underset{\sim}{B}}; \hat{r}_{\underset{\sim}{B}}\rangle \\
&= \langle \sqrt{1 - (1 - T_{\underset{\sim}{A}}{}^2)^{\underset{\sim}{t}}}, I_{\underset{\sim}{A}}{}^{\underset{\sim}{t}}, F_{\underset{\sim}{A}}{}^{\underset{\sim}{t}}; \hat{r}_{\underset{\sim}{A}}\rangle \\
&\quad + \langle \sqrt{1 - (1 - T_{\underset{\sim}{B}}{}^2)^{\underset{\sim}{t}}}, I_{\underset{\sim}{B}}{}^{\underset{\sim}{t}}, F_{\underset{\sim}{B}}{}^{\underset{\sim}{t}}; \hat{r}_{\underset{\sim}{B}}\rangle \\
&= \langle \sqrt{1 - (1 - T_{\underset{\sim}{A}}{}^2)^{\underset{\sim}{t}}(1 - T_{\underset{\sim}{B}}{}^2)^{\underset{\sim}{t}}}, (I_{\underset{\sim}{A}} I_{\underset{\sim}{B}})^{\underset{\sim}{t}}, \\
&\quad (F_{\underset{\sim}{A}} F_{\underset{\sim}{B}})^{\underset{\sim}{t}}; \min(\hat{r}_{\underset{\sim}{A}}, \hat{r}_{\underset{\sim}{B}})\rangle \\
&= \langle \sqrt{1 - (1 - (T_{\underset{\sim}{A}}{}^2 + T_{\underset{\sim}{B}}{}^2 - T_{\underset{\sim}{A}}{}^2 T_{\underset{\sim}{B}}{}^2))^{\underset{\sim}{t}}}, (I_{\underset{\sim}{A}} I_{\underset{\sim}{B}})^{\underset{\sim}{t}}, \\
&\quad (F_{\underset{\sim}{A}} F_{\underset{\sim}{B}})^{\underset{\sim}{t}}; \min(\hat{r}_{\underset{\sim}{A}}, \hat{r}_{\underset{\sim}{B}})\rangle
\end{aligned}$$

$$d(\underset{\sim}{A}_{\hat{r}_1}, \underset{\sim}{B}_{\hat{r}_2}) = \frac{|\hat{r}_1 - \hat{r}_2|}{\sqrt{2}} + \left(\sqrt{\frac{1}{h}\sum_{\hat{a}=1}^{h}(T_{\underset{\sim}{A}}(\hat{a}) - T_{\underset{\sim}{B}}(\hat{a}))^2 + (I_{\underset{\sim}{A}}(\hat{a}) - I_{\underset{\sim}{B}}(\hat{a}))^2 + (F_{\underset{\sim}{A}}(\hat{a}) - F_{\underset{\sim}{A}}(\hat{a}))^2}\right)$$

Next

$$R.H.S = \underline{t}(\underline{A} +_{\min} \underline{B})$$
$$= \underline{t}\langle\sqrt{T_{\underline{A}}{}^2 + T_{\underline{B}}{}^2 - T_{\underline{A}}{}^2 T_{\underline{B}}{}^2}, (I_{\underline{A}} I_{\underline{B}}), (F_{\underline{A}} F_{\underline{B}}); \min(\hat{r}_{\underline{A}}, \hat{r}_{\underline{B}})\rangle$$
$$= \langle\sqrt{1 - (1 - (T_{\underline{A}}{}^2 + T_{\underline{B}}{}^2 - T_{\underline{A}}{}^2 T_{\underline{B}}{}^2))^{\underline{t}}}, (I_{\underline{A}} I_{\underline{B}})^{\underline{t}}, (F_{\underline{A}} F_{\underline{B}})^{\underline{t}}; \min(\hat{r}_{\underline{A}}, \hat{r}_{\underline{B}})\rangle$$

Hence, we prove this.

The proofs of remaining are as follows by using algebraic operation which are given in subsection IV-A. □

Theorem 2: Let

$$\underline{D} = \langle T_{\underline{D}}, I_{\underline{D}}, F_{\underline{D}}; \hat{r}_{\underline{D}}\rangle and \underline{F} = \langle T_{\underline{D}}, I_{\underline{F}}, F_{\underline{F}}; \hat{r}_{\underline{F}}\rangle$$

be two C-SFSs in $\Re$. Then the following theorem shows the De Morgan's rule.

1) $(\underline{D} \cup_{\min} \underline{F})^c = \underline{D}^c \cap_{\min} \underline{F}^c$
2) $(\underline{D} \cup_{\max} \underline{F})^c = \underline{D}^c \cap_{\max} \underline{F}^c$
3) $(\underline{D} \cap_{\min} \underline{F})^c = \underline{D}^c \cup_{\min} \underline{F}^c$
4) $(\underline{D} \cap_{\max} \underline{F})^c = \underline{D}^c \cup_{\max} \underline{F}^c$

Proof: The proof from Section IV is obvious. □

B. COMPARISON RULES FOR C-SFS & D-SFS

The following are some functions that are crucial to the ranking of C-SFS and D-SFS that are introduced in this section:

Definition 8: Let $\underline{D} = \langle T_{\underline{D}}, I_{\underline{D}}, F_{\underline{D}}; \hat{r}_{\underline{D}}\rangle$ be any C-SFSs. Then

1) *Score function:- $\S(\underline{D}) = \frac{1}{4}(T_{\underline{D}} - I_{\underline{D}} - F_{\underline{D}} + \sqrt{2\hat{r}}(2p - 1))$ where $\S(\underline{D}) \in [-1, 1]$ and p is taking any value between [0, 1].*
2) *Accuracy function:- $\hat{D}(\underline{D}) = T^2_{\underline{D}} + I^2_{\underline{D}} + F^2_{\underline{D}}$ where $\hat{D}(\underline{D}) \in [0, 1]$*

Considering these two definitions for C-SFNs $\underline{D}$ and $\underline{F}$.

- *$\underline{D}$ is greater to $\underline{F}$ if $\S(\underline{D}) > \S(\underline{F})$*
- *$\underline{D}$ is less to $\underline{F}$ if $\S(\underline{D}) < \S(\underline{F})$*

If $\S(\underline{D}) = \S(\underline{F})$ for two C-SFNs. Then

- *$\underline{D}$ is greater to $\underline{F}$ if $\hat{D}(\underline{D}) > \hat{D}(\underline{F})$*
- *$\underline{D}$ is less to $\underline{F}$ if $\hat{D}(\underline{D}) < \hat{D}(\underline{F})$*
- *$\underline{D}$ is equivalent to $\underline{F}$ If $\hat{D}(\underline{D}) = \hat{D}(\underline{F})$*

Example 8: Let $\Re = \{a_1, a_2, a_3\}$ be a collection of SFV on $\Re$ can be given as

$$\{a_1, \langle 0.2, 0.4, 0.1\rangle, \langle 0.4, 0.2, 0.6\rangle, \langle 0.1, 0.4, 0.4\rangle\}$$
$$\{a_2, \langle 0.4, 0.1, 0.3\rangle, \langle 0.3, 0.1, 0.1\rangle, \langle 0.5, 0.3, 0.6\rangle\}$$
$$\{a_3, \langle 0.5, 0.2, 0.6\rangle, \langle 0.4, 0.1, 0.6\rangle, \langle 0.2, 0.6, 0.1\rangle\}$$

By using equation (7) and (8), we obtain D-SFS.

$$\{\langle a_1, 0.3, 0.3, 0.4; 0.3\rangle, \langle a_2, 0.4, 0.2, 0.4; 0.3\rangle, \langle a_3, 0.4, 0.4, 0.5; 0.5\rangle\}.$$

Figure 2, and 3 shows the geometrical representation of SFS and D-SFS respectively.

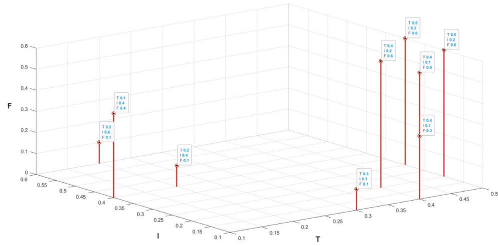

FIGURE 2. Geometrical representation of SFS.

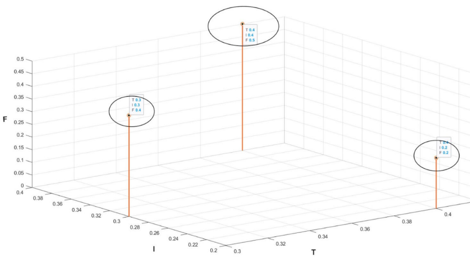

FIGURE 3. Geometrical representation of D-SFS.

V. CONSTRUCTION OF C-SF DECISION MATRIX

Let $\Re$ represent the universe that contains the MCDM problem's decision criteria. The whole set requirements is shown as $\Re = \tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n$. An C-SFS A_q is the qth parameter on $\Re$ is given by $A_q = \langle \tilde{x}_j, \Re_{qj}\rangle | \tilde{x}_j \in \Re$, where $\Re_{qj} = (\mu_{qj}, \nu_{qj})$.$\Re_{qj}$ shows the degree of non-belonging and belonging of the q th alternative with respect to the jth criterion, ν_{qj} and μ_{qj} are the respective degrees of non-belonging and belonging of $\Re_{qj}$, where $0 \le \mu_{qj} + \nu_{qj} \le 1$, $q = 1, 2, \ldots, m, j = 1, 2, \ldots, n$.

For calculating the degree of indeterminacy.

$$\pi_{qj} = 1 - \mu_{qj} - \nu_{qj} \tag{9}$$

The following is an expression for the decision matrix C-SF:

$$V = \begin{bmatrix} (\mu_{11}, \pi_{11}, \nu_{11}; \hat{r}_{11}) & . & . & (\mu_{1n}, \pi_{In}, \nu_{1n}; \hat{r}_{1n}) \\ . & . & & \\ . & & . & \\ . & & & . \\ (\mu_{m1}, \pi_{m1}, \nu_{m1}; \hat{r}_{m1}) & . & . & (\mu_{mn}, \pi_{mn}, \nu_{mn}; \hat{r}_{mn}) \end{bmatrix}$$

The judgement assigns a set of levels of importance because it is impossible to assume that all factors are equally important. In $\Re$, a C-SFS B is characterized as follows:

$$B = \{\langle \tilde{x}_j, \omega_j\rangle | \tilde{x}_j \in \Re\} \tag{10}$$

where $0 \le \omega_j \le 1$ and $\sum_{j=1}^{n} \omega_j = 1$, ω_j is the weights given to the various criteria, or the priority given to each one.

However, the usability of C-SFS data cannot be guaranteed; the sum of belonging, non-belonging, and indeterminacy degrees needs to be less than or equitable to one. The decision maker must spend twice as much to gather the assessed data including the degrees of belonging, non-belonging, and indeterminacy than they would with IVFS data in order to construct the C-SF matrix. The C-SFS theory and the IVFS theory are equivalent in mathematics.

The decision makers estimation with IVFS statistics is simpler than with C-SF statistics because of the restriction on the total of belonging and non-belonging degrees. Suppose Int ([0, 1]) represents the collection of all its closed subintervals. IVFS A_q of the qth alternative on $\Re$ is given by $A_q = \{\langle \tilde{x}_j, E_{qj}\rangle | \tilde{x}_j \in \Re\}$ where $E_{qj} : \Re \rightarrow Int([0, 1])$, such that $\tilde{x}_j \rightarrow E_{qj} = [E^-_{qj}, E^+_{qj}]$. E_{qj} designate the probable degree to which the parameter A_q fulfils the criteria $\tilde{x}_j$. E^+_{qj} and E^-_{qj} are the upper and lower bound, respectively of the interval E_{qj}.

In the closed interval, the decision-maker assesses all alternatives $[E^-_{qj}, E^+_{qj}]$ beginning at the top. Let $E^-_{qj} = \mu_{qj}$ and $E^+_{qj} = 1 - \nu_{qj}$; therefore, $[E^-_{qj}, E^+_{qj}] = (\mu_{qj}, 1 - \nu_{qj})$. An interval $[E^-_{qj}, E^+_{qj}]$ can be mapped onto an C-SFS, $(\mu_{qj}, 1 - \nu_{qj})$. The idea of mathematical equality between C-SFS and IVFS may be used to convert IVFS data into C-SF data. Additionally, a decision-maker must assess a lot of information using IVFS data, making it difficult to compare all available options based on their understanding and experience. Ranking, partial, or missing data can be provided by decision makers and converted into C-SF data. The approach determines the number of alternatives that are categorically superior and inferior to a given option. Given that not all options may be rated in accordance with a criterion, it permits partial ordinal data. We define two functions for the case of incomplete data or non-comparable results, λ_{qj} and κ_{qj} for each A_q with respect to $\tilde{x}_j$. Let λ_{qj} indicate the number of choices $A_1, A_2, \ldots, A_{q1}, A_{q+1}, A_{q+2}, \ldots, A_m$ that are surely inferior than A_q, while κ_{qj} represents the number of alternatives $A_1, A_2, \ldots, A_{q1}, A_{q+1}, A_{q+2}, \ldots, A_m$ that is undoubtedly superior to A_q. Following are the levels of membership and non-membership, respectively:

$$\mu_{qj} = \frac{\lambda_{qj}}{m-1} \tag{11}$$

$$\nu_{qj} = \frac{\kappa_{qj}}{m-1} \tag{12}$$

VI. ELECTRE METHOD BASED ON C-SFS

This section covers the C-SF ELECTRE approach, concordant and discordant sets, and other ideas. We'll employ the C-SF ELECTRE method algorithm to provide numerical results.

Binary outranking relations are used to simulate ELECTRE procedures; the decision maker can construct the relationship, which does not need to be transitive. Non-dominant alternatives and incomplete ordering are enabled by the connection. For every pair of alternatives ι and $\kappa(\iota, \kappa = 1, 2, \ldots, m$ and $\iota \neq \kappa)$, the various options criteria may be split into two separate subsets. The concordant set $F_{\iota\kappa}$ of A_ι and A_κ its made up of all requirements for which A_ι is favored to A_κ. Alternatively put, $F_{\iota\kappa} = \{j | \tilde{x}_{\iota j} \geq \tilde{x}_{\kappa j}\}$, where $J = \{j | j = 1, 2, \ldots, n\}$. The complementary subset, which is the discordant set, is $F_{\iota\kappa} = \{j | \tilde{x}_{\iota j} \leq \tilde{x}_{\kappa j}\}$. The suggested C-SF ELECTRE technique uses the concepts of score and accuracy function, and intuitionistic indicator to categorize many kinds of concordant and discordant sets. Concordant and discordant sets are then utilized to build concordant and discordant matrices, respectively. Decision-makers may select the ideal course of action by utilizing the ideas of both negative and positive optimum points.

A. CONCORDANT AND DISCORDANT SETS

We may assess a number of parameters to their C-SF values by utilizing the ideas of scoring and accuracy function, and reluctant C-SF value. The superior choice has a higher score or is very accurate when two options are equal in score. A higher score indicates a bigger belonging degree or a smaller non-belonging degree, whereas a bigger accuracy degree indicates a lower hesitation degree. We classify a variety of concordant sets into concordant, medium concordant, and bad concordant sets using score function and accuracy function ideas. Additional names for the various types of discordant sets are the discordant set, moderate discordant set, and weak discordant set.

Chen and Tan [50] to measure how well an option meets a decision maker's needs. Let $\Re_{qj} = (\mu_{qj}, \nu_{qj})$ is a C-SF value, where $\mu_{qj} \in [0, 1]$, $\nu_{qj} \in [0, 1]$, $\mu_{qj} + \nu_{qj} \leq 1$. The score of $\Re_{qj}$ can be determined by the score function §, where score value is find by using definition 8, where $\S(\Re_{qj}) \in [-1, 1]$. A greater score $\S(\Re_{qj})$ associates with a greater C-SF value $\Re_{qj}$; When two options have the same score, we are unable to compare them using merely the score function. In order to assess the level of accuracy of ambiguous values, Hong and Choi [51] introduced the correctness function. The degree of exactness of $\Re_{qj}$ is able to be assessed using the accuracy function $\hat{A}$. Accuracy function $\hat{A}(\Re_{qj})$ can be calculated by using the definition 8 where $\Re_{qj} = (\mu_{qj}, \nu_{qj})$ is an C-SF value. A greater value of $\hat{A}(\Re_{qj})$ corresponds with a higher level of C-SF value membership grade correctness. According to equation (9) and the accuracy function, a smaller hesitation degree $\pi(\Re_{qj})$ and a greater accuracy degree $\hat{A}(\Re_{qj})$ are correlated.

As previously stated, $\Re_{qj} = (\mu_{qj}, \nu_{qj})$. The concordant set $C_{\iota\kappa}$ of A_ι and A_κ is made up of all requirements for which A_ι is better to A_κ. We apply the ideas of functions like scoring, accuracy, and hesitancy degree of the C-SF value to categorize concordant sets. $C_{\iota\kappa}$ is a concordant set that can be expressed as follows:

$$C_{\iota\kappa} = \{j | \mu_{\iota j} \geq \mu_{\kappa j}, \pi_{\iota j} < \pi_{\kappa j}, \nu_{\iota j} < \nu_{\kappa j} \; and \; \hat{r}_{\iota j} < \hat{r}_{\kappa j}\} \tag{13}$$

where $J = \{j | j = 1, 2, \ldots, n\}$, a higher score indicates a higher C-SF value, a higher accuracy degree refers to a degree of lower hesitancy, and equation (13) is more concordant than (14) or (15). The midrange concordant set $C^1_{\iota\kappa}$ is defined as:

$$C^1_{\iota\kappa} = \{j | \mu_{\iota j} \geq \mu_{\kappa j}, \pi_{\iota j} \geq \pi_{\kappa j} \; and \; \nu_{\iota j} < \nu_{\kappa j}\} \tag{14}$$

The main distinction between (13) and (14) is the amount of hesitation at the alternative kth with regard to the criteria

jth, which is more than the alternative lth with respect to the jth criterion in the median concordant set. Equation (13) is therefore more concordant than (14).
Weak concordant set $C^2_{\iota\kappa}$ is defined as:

$$C^2_{\iota\kappa} = \{j|\mu_{\iota j} \geq \mu_{lj}, \nu_{\iota j} \geq \nu_{\kappa j}\} \tag{15}$$

The degree of non-belonging at the alternative kth in relation to the criterion jth is higher than the alternative lth in relation to the criterion jth in the weak concordant set; thus, equation (14) is more concordant than (15). All criteria that meet the discordant set are included in A_ι is not favoured to A_κ. The discordant set $D_{\iota\kappa}$ using the preceding concepts can be calculated as follows:

$$D_{\iota\kappa} = \{j|\mu_{\iota j} < \mu_{\kappa j}, \pi_{\iota j} \geq \pi_{\kappa j}, \nu_{\iota j} \geq \nu_{\kappa j} \text{ and } \hat{r}_{\iota j} \geq \hat{r}_{\kappa j}\} \tag{16}$$

The formula also makes use of the same ideas, such as the ones that a higher score corresponds to a higher C-SF value and a greater accuracy degree corresponds to a lower hesitation degree. Midrange discordant set $D^1_{\iota\kappa}$ is defined as:

$$D^1_{\iota\kappa} = \{j|\mu_{\iota j} < \mu_{\kappa j}, \pi_{\iota j} < \pi_{\kappa j} \text{ and } \nu_{\iota j} \geq \nu_{\kappa j}\} \tag{17}$$

equation (16) is higher discordant than (17). Weak discordant set $D^2_{\iota\kappa}$ is defined as follows:

$$D^2_{\iota\kappa} = \{j|\mu_{\iota j} < \mu_{\kappa j}, \nu_{\iota j} < \nu_{\kappa j}\} \tag{18}$$

equation (17) is more discordant than (18) because the degrees of belonging and non-belonging at the ιth parameter with regard to the j th criteria are lower than those at the κth parameter in relation to the j th criterion in the weak discordant set.

Concordant and discordant matrices are calculated using the idea of concordant and discordant sets, and the aggregation dominance matrix is calculated using the suggested C-SF ELECTRE approach. Next, we select the best option.

B. C-SF ELECTRE METHOD

The evaluation data from the ELECTRE and C-SFS approaches are combined in the C-SF ELECTRE methodology. The proportional worth of the concordant set produced by the C-SF ELECTRE technique is calculated using the concordant index. The concordant index is produced by adding the weights corresponding to the criteria and relationships present in the concordant sets. As a consequence, the concordant index $c_{\iota\kappa}$ between A_ι and A_κ is defined as follows in this study:

$$g_{\iota\kappa} = \omega_C \times \sum_{j\in C_{\iota\kappa}} \omega_j + \omega_{C^1} \times \sum_{j\in C^1_{\iota\kappa}} \omega_j + \omega_{C^2} \times \sum_{j\in C^2_{\iota\kappa}} \omega_j \tag{19}$$

where the weight of the criteria is ω_j, as stated in (10), and ω_{C^1}, ω_{C^2}, and ω_C are the middling concordant and weak concordant sets and weights of the concordant, respectively. The concordant index demonstrates the relative dominance of one alternative over another based on the corresponding weights assigned to the subsequent choice criteria. The concordant matrix G is described as follows:

$$G = \begin{bmatrix} - & g_{12} & \cdots & \cdots & g_{1m} \\ g_{21} & - & g_{23} & \cdots & g_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ g_{(m-1)1} & \cdots & \cdots & - & g_{(m-1)m} \\ g_{m1} & g_{m2} & \cdots & g_{m(m-1)} & - \end{bmatrix}$$

where the highest component of $g_{\iota\kappa}$ is represented by g^*, which is the positive point of ideal, and a maximum value of $g_{\iota\kappa}$ indicates that A_ι is favouring A_κ.

Evaluations of a certain A_ι are inferior to evaluations of a rival A_κ. The discordant index is described as follows in this study:

$$h_{\iota\kappa} = \frac{\max_{j\in D_{\iota\kappa}} \omega^*_D \times d(X_{\iota j}, X_{\kappa j})}{\max_{j\in J} d(X_{\iota j}, X_{\kappa j})} \tag{20}$$

Additionally, $d(X_{\iota j}, X_{\iota\kappa})$ is specified in definition 7 and ω^*_D is equal to either ω_{D^1} or ω_{D^2} according to the various types of discordant sets. These sets, in that order, are the weak discordant, weight of discordant, and moderate discordant sets.

The following is a definition of the dissonant matrix H:

$$H = \begin{bmatrix} - & h_{12} & \cdots & \cdots & h_{1m} \\ h_{21} & - & h_{23} & \cdots & h_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ h_{(m-1)1} & \cdots & \cdots & - & h_{(m-1)m} \\ h_{m1} & h_{m2} & \cdots & h_{m(m-1)} & - \end{bmatrix}$$

A maximum value of $h_{\iota\kappa}$ demonstrates that A_ι is less favorable than A_κ, where the greatest value of $h_{\iota\kappa}$ is represented by h^*, which is the point of negative of ideal.

The computation of the concordant dominance matrix is based on the assumption that the selected decision ought to be the one that comes closest to being perfect. For this reason, the concordant K dominance matrix is stated as follows:

$$K = \begin{bmatrix} - & k_{12} & \cdots & \cdots & k_{1m} \\ k_{21} & - & k_{23} & \cdots & k_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ k_{(m-1)1} & \cdots & \cdots & - & k_{(m-1)m} \\ k_{m1} & k_{m2} & \cdots & k_{m(m-1)} & - \end{bmatrix}$$

where,

$$k_{\iota\kappa} = g^* - g_{\iota\kappa}; \tag{21}$$

This has to do with separating each option from the wise, sensible course of action. A higher value of $k_\iota\kappa$ makes it very evident that A_ι is less advantageous than A_κ. The premise that the option selected should be the one that is farthest from the negative ideal answer is the foundation for how the discordant dominance matrix is constructed. Consequently, the following is the definition of the discordant dominance L matrix:

$$L = \begin{bmatrix} - & l_{12} & \cdots & \cdots & l_{1m} \\ l_{21} & - & l_{23} & \cdots & l_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ l_{(m-1)1} & \cdots & \cdots & - & l_{(m-1)m} \\ l_{m1} & l_{m2} & \cdots & l_{m(m-1)} & - \end{bmatrix}$$

where,

$$l_{\iota\kappa} = h^{*} - h_{\iota\kappa} \tag{22}$$

This explains how each option differs from the ideal negative result. A larger $l_{\iota\kappa}$ value denotes a preference for A_ι over A_κ. The aggregate dominance matrix determination approach may be used to rank the parameters by calculating the distance of each option from both negative and positive ideal points. The following is the definition of the aggregate dominance matrix R:

$$R = \begin{bmatrix} - & r_{12} & \cdots & \cdots & r_{1m} \\ r_{21} & - & r_{23} & \cdots & r_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ r_{(m-1)1} & \cdots & \cdots & - & r_{(m-1)m} \\ r_{m1} & r_{m2} & \cdots & r_{m(m-1)} & - \end{bmatrix}$$

where,

$$r_{\iota\kappa} = \frac{l_{\iota\kappa}}{k_{\iota\kappa} + l_{\iota\kappa}} \tag{23}$$

$k_{\iota\kappa}$ and $l_{\iota\kappa}$ are described in (21) and (22), respectively, and $r_{\iota\kappa}$ relates to how close the answer is to the perfect one, having a range of 0 to 1. A maximum value of $r_{\iota\kappa}$ shows that the choice A_ι is equally close to the positive perfect point and much further from the negative perfect point than the choice A_κ; thus, it is a better parameter. Picking the advantageous backup strategy, where;

$$T_\iota = \frac{1}{m-1} \sum_{\kappa=1,\kappa\neq\iota}^{m} r_{\iota\kappa}, \quad \iota = 1, 2, 3, \ldots\ldots, m \tag{24}$$

and T_ι is the result of the evaluation. Based on T_ι, all options may be graded. The best possible alternative K^*, which is also the one that is the most in the positive optimum solution and the least in the negative optimum solution, may be made and explained as follows:

$$K^* = \max\{T_\iota\} \tag{25}$$

where the ideal substitute is K^*. The whole C-SF ELECTRE method algorithm is covered by the definitions that follow.

C. ALGORITHM

This paragraph explains a unique MCDM methodology called the C-SF ELECTRE technique, which combines the C-SFS and ELECTRE procedures with evaluation data. The suggested algorithm's technique will be built using three key phases: *evaluation*, *aggregation*, and *selection*. Figure 4 provides a conceptual illustration of the recommended method.

The decision-makers characterize the options (according to the relative weights of various criteria) and build the decision matrix using the C-SFS during the evaluation process. We used the provided method to generate concordant and discordant dominance matrices, and we compared each choice to the other alternatives in the aggregation stage to affirm the relationship of dominance. Next, the aggregate dominance matrix is calculated. Using the C-SF ELECTRE algorithm, we rate every possibility during the selection step and choose the best one. The eight steps listed below can be used to summarize the algorithm and DM process of the C-SF ELECTRE method:

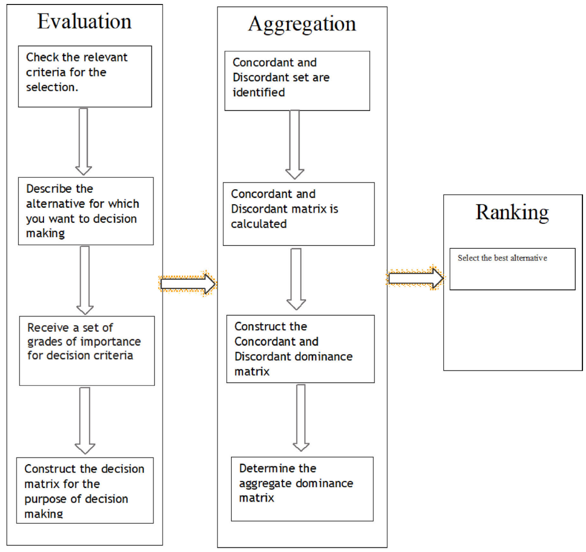

FIGURE 4. Flowchart of the circular spherical fuzzy ELECTRE method.

Step 1. Utilize the decision makers' evaluation data to build the choice matrix. Information is provided by the decision-makers in the form of C-SF values or comparison data between various alternatives. This phase consists of the following three phases.

1) Choose non-inferior options and pertinent standards that are suitable for the current circumstance. Different MCDM issues need have unique majority of the requirements may become split into two groups: subjective criteria, and objective criteria. It is the responsibility of the decision-makers to locate possible alternatives.
2) Obtain a set of significance ratings for the selection criteria. Criteria weights are defined as $\sum_{j=1}^{n} \omega_j = 1$.
3) Make a choice matrix: The creators of decisions create the C-SF decision matrix V using important information. Use the formals in (11) and (12) to convert if the decision matrix includes comparison data.

Step 2. In order to distinguish between the various types of concordant and discordant sets, we use the notions of scoring function, accuracy function, and the degree of hesitation of the C-SF value. Determine $C_{\iota\kappa}$, $C^1_{\iota\kappa}$, $C^2_{\iota\kappa}$, $D_{\iota\kappa}$, $D^1_{\iota\kappa}$, and $D^2_{\iota\kappa}$ for pair-wise compares of choices.

Step 3. The concordant G matrix should be known. A number of concordant sets their weights, as defined in (19) come together to form the concordant matrix index.

Step 4. The discordant matrix indices are the logical result of several discordant sets their weights, which are defined in (20).

Step 5. Create the K matrix for concordant dominance. According to (21), the concordant dominance matrix indices are the difference between the concordant matrix's maximal index and its own indices.

Step 6. The indices of the discordant dominance matrix are defined in (22). Create a discordant dominance L matrix.

Step 7. Utilize the concordant and discordant dominance matrices' indexes, which are listed in (23), to calculate R's matrix of collective dominance.

Step 8. Pick the best solution: determine the evaluation's final result using (24) and (25). When all options are rated in inverse order, the substitute with the highest value is regarded as the best choice.

VII. NUMERICAL EXAMPLE

We all know how important it is today to figure out the best time to select a kidney transplant. Assume that the selection issue takes into account four criteria: ψ_1 (Age compatibility between the donor and recipient), ψ_2 (donor-recipient size match), ψ_3 (health status compatibility between the donor and recipient), and ψ_4 (blood type compatibility between the donor and recipient). Decision makers choose the subjective value of criterion B;

$$B = [\omega_1, \omega_2, \omega_3, \omega_4] = [0.3, 0.1, 0.2, 0.4]$$

A. CASE STUDY

The decision to undergo a kidney transplant is a complex and important one that requires careful consideration of various factors. Let's take a look at a case study of decision-making in selecting a kidney transplant. Mark is a 55-year-old man who has been on dialysis for the past three years due to end-stage renal disease. His doctor has recommended a kidney transplant as the best treatment option for him. Mark is considering his options and needs to make a decision about whether to proceed with the transplant or continue with dialysis. The medical officer and doctor had to pitch the idea to their seniors about which time is more suitable for the selection of a kidney transplant. They have four parameters. $U = \{K_1, K_2, K_3, K_4\}$. These four parameters are: K_1 = Medical Condition, K_2 = Donor Availability, K_3 = Cost, K_4 = Lifestyle Changes. Assume the selection issue considers four criteria: ψ_1 (Age compatibility between the donor and recipient), ψ_2 (donor-recipient size match), ψ_3 (health status compatibility between the donor and recipient), and ψ_4 (blood type compatibility between the donor and recipient). Following are the details of the parameters:

1) MEDICAL CONDITION

The first and foremost parameter to consider is the patient's medical condition. In Mark's case, he has advanced kidney disease, which means that his kidneys are functioning at less than 10% of their normal capacity. A transplant would help him regain kidney function and improve his overall health.

2) DONOR AVAILABILITY

Another important factor to consider is the availability of a donor. Kidneys can be obtained from deceased donors or living donors, and there may be a waiting list for a deceased donor transplant. Mark may also have a family member or friend who is willing to donate a kidney. The availability of a donor is an important consideration when deciding whether to proceed with a transplant.

3) COST

The cost of a kidney transplant varies according on various factors, that is the type of transplant, hospital fees, and the cost of immunosuppressant medications. Mark will need to consider the financial implications of the procedure and whether he has adequate insurance coverage.

4) LIFESTYLE CHANGES

A kidney transplant requires significant lifestyle changes, such as adhering to a strict medication regimen, following a healthy diet, and avoiding certain activities that may put the new kidney at risk. Mark will need to consider whether he is willing and able to make these changes.

Similarly, details of the criteria that depend upon these parameters.

5) AGE COMPATIBILITY BETWEEN THE DONOR AND RECIPIENT

The first and foremost parameter to consider is the patient's medical condition. In Mark's case, he has chronic kidney disease, which means that his kidneys are functioning at less than 10% of their normal capacity. A transplant would help him regain kidney function and improve his overall health.

6) DONOR-RECIPIENT SIZE MATCH

This criterion assesses the degree of similarity between the size of the donor and recipient. A size match is important for the success of the transplant, as an organ that is too large or too small may not function properly. Therefore, we might consider assigning linguistic terms such as "highly compatible" if the donor and recipient are a good size match, "moderately compatible" if the size difference is somewhat larger, "slightly compatible" if the size difference is significant but not extreme, "neutral" if size is not a major factor, "slightly incompatible" if the size difference is large enough to be a concern, "moderately incompatible" if the size difference is very large, and "highly incompatible" if the size difference is extremely large.

7) HEALTH STATUS COMPATIBILITY BETWEEN THE DONOR AND RECIPIENT

This criterion assesses the degree of similarity between the health statuses of the donor and recipient. Health status compatibility may affect the success.

8) BLOOD TYPE COMPATIBILITY BETWEEN THE DONOR AND RECIPIENT

This criterion assesses the degree of similarity between the blood types of the donor and recipient. Blood type compatibility is crucial for the success of the transplant, as the recipient's immune system may reject the transplant if the blood types are incompatible. Therefore, we might consider assigning linguistic terms such as "highly compatible" if the donor and recipient have the same blood type, "moderately compatible" if the donor has a compatible blood type but not the same as the recipient, "slightly compatible" if the donor's blood type is somewhat compatible with the recipient, "neutral" if blood type is not a major factor, "slightly incompatible" if the donor's blood type is somewhat incompatible with the recipient, "moderately incompatible" if the donor's blood type is incompatible but can be managed with medical intervention, and "highly incompatible" if the donor's blood type is completely incompatible.

Additionally, decision-makers give the following relative weights:

$$B' = [\omega_C, \omega_{C^1}, \omega_{C^2}, \omega_D, \omega_{D^1}, \omega_{D^2}] = [1, \frac{2}{3}, \frac{1}{3}, 1, \frac{2}{3}, \frac{1}{3}]$$

The D-SF matrix choice with numerical data is changed given V, the IVF decision-making matrix, as shown in the equation at the bottom of the page.

Determine the concordant and discordant sets by using Step 2. The concordant set is determined by using (13).

$$C_{\iota\kappa} = \begin{bmatrix} - & - & 4 & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

The midrange concordant set is determined by (14).

$$C^1_{\iota\kappa} = \begin{bmatrix} - & 2 & 3 & - \\ - & - & - & - \\ - & 2 & - & - \\ 2,4 & 2 & 3,4 & - \end{bmatrix}$$

The weak concordant set is determined by (15).

$$C^2_{\iota\kappa} = \begin{bmatrix} - & 1,4 & 1 & - \\ 3 & - & 1 & 3 \\ 2 & 4 & - & 2 \\ 3 & 1 & 1 & - \end{bmatrix}$$

The discordant set is determined by using (16).

$$D_{\iota\kappa} = \begin{bmatrix} - & - & - & - \\ - & - & - & - \\ 4 & - & - & - \\ - & - & - & - \end{bmatrix}$$

The midrange discordant set is determined by using (17).

$$D^1_{\iota\kappa} = \begin{bmatrix} - & - & - & 2,4 \\ 2 & - & 2 & 2 \\ 3 & - & - & 3,4 \\ 1 & - & - & - \end{bmatrix}$$

The weak discordant set is determined by using (18).

$$D^2_{\iota\kappa} = \begin{bmatrix} - & 3 & 2 & 3 \\ 1,4 & - & 4 & 1 \\ 1 & 1 & - & 1 \\ - & 3 & 2 & - \end{bmatrix}$$

Step 3 is used to calculate the concordant matrix.

$$G = \begin{bmatrix} - & 0.3 & 0.6333 & 0 \\ 0.0667 & - & 0.1 & 0.0667 \\ 0.3333 & 0.2 & - & 0.0333 \\ 0.4 & 0.1667 & 0.5 & - \end{bmatrix}$$

$$g_{13} = \omega_C \times \omega_4 + \omega_{C^1} \times \omega_3 + \omega_{C^2} \times \omega_1 = 1 \times 0.4 + \frac{2}{3} \times 0.2 + \frac{1}{3} \times 0.3 = 0.6333$$

$$V = \begin{array}{c} \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{array} \begin{array}{cccc} \psi_1 & \psi_2 & \psi_3 & \psi_4 \\ \hline [0.40, 0.65] & [0.22, 0.48] & [0.15, 0.75] & [0.23, 0.28] \\ [0.19, 0.71] & [0.17, 0.19] & [0.24, 0.67] & [0.11, 0.40] \\ [0.10, 0.88] & [0.44, 0.47] & [0.08, 0.61] & [0.15, 0.23] \\ [0.34, 0.53] & [0.24, 0.61] & [0.17, 0.74] & [0.25, 0.51] \end{array}$$

$$V = \begin{array}{c} \\ K_1 \\ K_2 \\ K_3 \\ K_4 \\ \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{array} \begin{bmatrix} \psi_1 & \psi_2 \\ ((0.40, 0.25, 0.35); 0.22) & ((0.22, 0.26, 0.52); 0.11) \\ ((0.19, 0.52, 0.29); 0.30) & ((0.17, 0.02, 0.81); 0.43) \\ ((0.10, 0.78, 0.12); 0.52) & ((0.44, 0.03, 0.53); 0.49) \\ ((0.34, 0.19, 0.47); 0.21) & ((0.24, 0.37, 0.39); 0.03) \\ \psi_3 & \psi_4 \\ ((0.15, 0.60, 0.25); 0.37) & ((0.23, 0.05, 0.72); 0.38) \\ ((0.24, 0.43, 0.33); 0.24) & ((0.11, 0.29, 0.60); 0.12) \\ ((0.08, 0.53, 0.39); 0.21) & ((0.15, 0.08, 0.77); 0.41) \\ ((0.17, 0.57, 0.26); 0.26) & ((0.25, 0.26, 0.49); 0.14) \end{bmatrix}$$

Apply step 4 to the calculation of the discordant matrix.

$$H = \begin{bmatrix} - & 0.1652 & 0.2267 & 0.6667 \\ 1 & - & 0.6006 & 0.0667 \\ 0.3333 & 0.3333 & - & 0.3795 \\ 0.2131 & 0.0742 & 0.2621 & - \end{bmatrix}$$

$$\omega_{D^2} \times d(X_{13}, X_{23}) = \frac{1}{3} \times 0.3 = 0.1$$

For example

$$h_{12} = \frac{\max_{j \in D_{12}} \omega^*_D \times d(X_{1j}, X_{2j})}{\max_{j \in J} d(X_{1j}, X_{2j})} = \frac{0.1}{0.606} = 0.1652$$

where, as shown in the equation at the bottom of the page.

Using step 5, the concordant dominance matrix is produced. The matrix of concordant dominance looks like this:

$$K = \begin{bmatrix} - & 0.3333 & 0 & 0.6333 \\ 0.5666 & - & 0.5333 & 0.5666 \\ 0.3003 & 0.2 & - & 0.6 \\ 0.2333 & 0.4666 & 0.1333 & - \end{bmatrix}$$

Using step 6, the discordant dominance matrix is produced. The matrix of discordant dominance looks like this:

$$L = \begin{bmatrix} - & 0.8348 & 0.7733 & 0.3333 \\ 0 & - & 0.3994 & 0.9333 \\ 0.6667 & 0.6667 & - & 0.6205 \\ 0.7869 & 0.9258 & 0.7379 & - \end{bmatrix}$$

Calculating the aggregate dominance matrix takes place in step 7.

$$R = \begin{bmatrix} - & 0.7147 & 1 & 0.3448 \\ 0 & - & 0.4282 & 0.6222 \\ 0.6895 & 0.7692 & - & 0.5084 \\ 0.7713 & 0.6649 & 0.8470 & - \end{bmatrix}$$

Pick the best one using step 8.

$$\top_1 = 0.6865 \quad \top_2 = 0.3502 \quad \top_3 = 0.6557 \quad \top_4 = 0.7611$$

The alternative is ranked in the following order:

$$K_4 > K_1 > K_3 > K_2$$

VIII. COMPARATIVE ANALYSIS

We now have a comparison between the suggested approach and the enhanced possibility degree method, [52]. Consider two options $\{K_1, K_2\}$ that are assessed using three criteria ψ_1, ψ_2, ψ_3 by allocating an equal proportion to these features. As a result, $B = [b_1, b_2, b_3] = [0.4, 0.5, 0.1]$.

$$B' = [b_C, b_{C^1}, b_{C^2}, b_D, b_{D^1}, b_{D^2}] = [1, \frac{2}{3}, \frac{1}{3}, 1, \frac{2}{3}, \frac{1}{3}]$$

Intuitionistic fuzzy decision matrix is as shown in the equation at the bottom of the next page.

Convert into D-SF decision matrix by using (7) and (8). V, as shown at the bottom of the next page.

Using step 2, recognize the concordant and discordant sets. Using (9) the concordant set is determined.

$$C_{\iota\kappa} = \begin{bmatrix} - & - \\ - & - \end{bmatrix}$$

By using (10), the midrange concordant set is determined.

$$C^1_{\iota\kappa} = \begin{bmatrix} - & - \\ 2 & - \end{bmatrix}$$

Using (11), the weak concordant set is calculated.

$$C^2_{\iota\kappa} = \begin{bmatrix} - & - \\ 1 & - \end{bmatrix}$$

Using (12), the discordant set is computed.

$$D_{\iota\kappa} = \begin{bmatrix} - & 1 \\ - & - \end{bmatrix}$$

Using (13), the midrange discordant set is determined.

$$D^1_{\iota\kappa} = \begin{bmatrix} - & 2 \\ - & - \end{bmatrix}$$

Using (14), the weak discordant set is determined.

$$D^2_{\iota\kappa} = \begin{bmatrix} - & - \\ - & - \end{bmatrix}$$

Step 3 is used to calculate the concordant matrix.

$$G = \begin{bmatrix} - & 0 \\ 0.4667 & - \end{bmatrix}$$

$$d(X_{11}, X_{21}) = \frac{|0.22 - 0.30|}{\sqrt{2}} + \left(\sqrt{\frac{1}{2}(0.40 - 0.19)^2 + (0.25 - 0.52) - (0.35 - 0.29)^2}\right) = 0.404$$

$$(X_{12}, X_{22}) = \frac{|0.11 - 0.43|}{\sqrt{2}} + \left(\sqrt{\frac{1}{2}(0.22 - 0.17)^2 + (0.26 - 0.02) - (0.52 - 0.81)^2}\right) = 0.606$$

$$(X_{13}, X_{23}) = \frac{|0.37 - 0.24|}{\sqrt{2}} + \left(\sqrt{\frac{1}{2}(0.15 - 0.24)^2 + (0.60 - 0.43) - (0.25 - 0.33)^2}\right) = 0.300$$

$$(X_{14}, X_{24}) = \frac{|0.38 - 0.12|}{\sqrt{2}} + \left(\sqrt{\frac{1}{2}(0.23 - 0.11)^2 + (0.05 - 0.29) - (0.72 - 0.60)^2}\right) = 0.478$$

TABLE 1. Ranking order of the alternative.

Authors	K_1	K_2	Ranking orders
Garg [52], Wei [53]	$K_1 = 0.25$	$K_2 = 0.75$	$K_2 > K_1$
Proposed method	$K_1 = 0$	$K_2 = 0.5$	$K_2 > K_1$

Apply step 4 to the computation of the discordant matrix.

$$H = \begin{bmatrix} - & 0.0735 \\ 0 & - \end{bmatrix}$$

Using step 5, the concordant dominance matrix is produced. The matrix of concordant dominance looks like this:

$$K = \begin{bmatrix} - & 0.4667 \\ 0 & - \end{bmatrix}$$

The discordant dominance matrix is created by employing step 6. The following is the discordant dominance matrix:

$$L = \begin{bmatrix} - & 0 \\ 0.0735 & - \end{bmatrix}$$

Calculating the aggregate dominance matrix takes place in step 7.

$$R = \begin{bmatrix} - & 0 \\ 1 & - \end{bmatrix}$$

Pick the best one using step 8.

$$\top_1 = 0 \ \ \top_2 = 0.5$$

Optimal ranking of the alternative is are:

$$K_2 > K_1$$

In Table 1, the recommended approach is contrasted with another approach.

The validity of our suggested technique, which is connected to the D-SF ELECTRE method, is displayed in the Table 1 below. We evaluate this technique in comparison to Garg's, which is [52] and uses an enhanced possibility degree method. The second method, [53] and developed by Wei, uses induced geometric aggregation operators and has the highest ranking. One of the benefits of utilising our methodology is that in addition to calculating the radius of the given values, which was not done in the previous iterations of the procedure.

IX. CONCLUSION

The primary objective of this work is to introduce the notion of C-SFS and D-SFS, which is symbolised by a circle of radius r and a pair in its centre, provided that the square of the total of the components is less than one. A circle is used to symbolise the degree, indeterminacy degree, and non-belonging degree in such a fuzzy collection. An D-SFS and C-SFS is a generalisation of both C-IFSs and C-PFSs, hence. Compared to both SFSs with C-SFSs and D-SFSs, C-SFSs and D-SFSs enable decision-makers or experts to assess things in a broader and more flexible region. In order to describe uncertainty, C-SFSs and D-SFSs can be used to manage changes D-SFSs belonging degree, indeterminacy degree, and non-belonging degree. More considerate choices can then be made. In this study, a technique for converting SFSs to C-SFSs and D-SFS is devised. Additionally, some basic algebraic operations and set theoretic procedures for C-SFSs and D-SFSs are provided. The C-SF ELECTRE approach is being introduced in this study as a means of resolving MCDM issues. We will use the C-SF ELECTRE approach in a later study to forecast customer behaviour utilising a questionnaire in an empirical investigation of market research using various items. Finally, using the aforementioned ideas, we present an MCDM approach in a circular and disc spherical fuzzy environment and use the suggested method to solve an MCDM issue concerning choosing the optimal timing for kidney transplantation. We determine the temporal complexity of the MCDM approach by comparing the results of the proposed method with those of the current methods. Different aggregation procedures and similarity metrics can be explored in further research. Additionally, alternative aggregation tools like fuzzy integrals or aggregation operators can be utilised when converting SFSs to C-SFSs. Additionally, MCDM issues including classification, pattern recognition, data mining, clustering, and medical diagnosis may be resolved using the suggested technique.

$$V = \begin{array}{c} \\ K_1 \\ K_2 \end{array} \begin{array}{c} \begin{array}{ccc} \quad\psi_1 \quad & \quad\psi_2\quad & \quad\psi_3\quad \end{array} \\ \begin{bmatrix} (0.0000, 0.9360) & (0.0010, 0.7840) & (0.0030, 0.6570) \\ (0.0080, 0.9360) & (0.0640, 0.8750) & (0.2160, 0.7840) \end{bmatrix} \end{array}$$

$$V = \begin{bmatrix} & \psi_1 & \psi_2 \\ K_1 & (0.000, 0.936, 0.064; 0.22) & (0.001, 0.783, 0.216; 0.03) \\ K_2 & (0.008, 0.928, 0.064; 0.21) & (0.064, 0.875, 0.125; 0.08) \\ & \psi_3 & \\ K_1 & (0.003, 0.654, 0.343; 0.18) & \\ K_2 & (0.216, 0.568, 0.216; 0.24) & \end{bmatrix}$$

COMPETING INTERESTS

All authors are here with confirm that there are no competing interests between them.

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