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RESEARCH ARTICLE

Availability Evaluation and Design Optimization of Multi-State k-out-of-n: G Systems With Random Performance Requirements

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ABSTRACT As an important aspect of reliability theory, availability has now been considered a very meaningful design criterion of repairable system. This paper investigates the availability evaluation and design optimization of the multi-state *k-out-of-n: G* systems considering random weight threshold. The system availability is evaluated by extending the recursive algorithm (RA) and universal generating function (UGF) technique. Based on the traditional recursive algorithm, the total probability theorem is used to solve the discrete random weight threshold. Another better UGF method combines a new stochastic joint operator, which is suitable for both continuous and discrete random weight thresholds. Furthermore, we constructed two system design optimization models under availability or cost constraint respectively, and genetic algorithm (GA) programming can be applied to obtain the optimal state probability distribution and weight distribution of multi-state components of the suggested system. Finally, through numerical examples, the flexibility and effectiveness of the proposed methods for design optimization are demonstrated. In addition, two evaluation methods are compared to show that the customized UGF method features higher generality than RA in the case of continuous stochastic weight threshold, and higher operational efficiency in the case of increasing component quantity and state. The results can be helpful for engineers to optimize the design of complex systems.

INDEX TERMS Multi-state *k-out-of-n: G* system, availability, design optimization, universal generating function, recursive algorithm.

I. INTRODUCTION

In traditional binary reliability/availability studies, a system or component is typically assumed to have two states: completely working or totally failed. However, complex systems in the real-world, such as power systems, communication systems, and production systems [1], [2], [3] etc., often present multiple states during operation, and different states have different performance rates, which are called multi-state system (MSS). As a more flexible and accurate tool for complex system analysis, the MSS model has been widely studied, because it can characterize the multi-state deteriorating nature of complex systems [4].

The *k-out-of-n* structure is widely used in both industrial and military systems [5]. An n-component system is said to be a *k-out-of-n* system, $1 \leq k \leq n$, if it operates as long as at least k components out of n operate [6], [7]. Two general types of generalized MSS models are obtained from the binary *k-out-of-n: G* systems. The first category comprises component-based models, such as multi-state *k-out-of-n: G* systems [8], [9] and multi-state consecutive-*k-out-of-n: G* systems [10], where the state of the system depends on the number of components in a specified state space. Here, k represents the threshold of the number of components when the system is working. The second type is weighted-based

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systems, such as multi-state weighted *k-out-of-n: G* systems [7], [11], where the state of the system depends on the weight of the components. Here, k is the weight threshold for the system work. The "weight" (or performance rate) of each component represents its utility. When the weight of each component is equal to one, the multi-state weighted *k-out-of-n: G* system can be simplified to a multi-state *k-out-of-n: G* system. However, Levitin [12] pointed out that these two types of models cannot be mapped to each other, this means that the two elements of quantity and weight cannot be substituted for each other in the definition of system state.

In most studies involving generalized multi-state *k-out-of-n: G* systems, the quantity threshold of the component and the performance threshold of the system are usually considered individually, and assumed to be constant [13], [14], [15], [16]. In fact, system states may be affected by both quantitative and weight elements in real-world applications. For example, a rope transportation system used to transfer ships coming to the shipyard for repairs from platform to the repair post and back from repair post to the platform [17]. The mission of taking the ships coming to the shipyard for repairs can be divided into five stages, which are ship docking, ship's transportation to the repair post, the repair measures, ship's transportation to the platform, and ship undocking. Depending on its mission stage and operating environment, the requirement of system transport capacity will change constantly. Moreover, the system is composed of three broaching machines working independently equipped in steel ropes. Depending on the weight and length of the vessel and the repair station at which the vessel should be trans-shipped, at least one broaching machine will be used to transport the ships on the traverse, and in the extreme case of trans-shipping large vessels of more than 1,800 tons, all three pulling machines are operated. We consider broaching machine as basic components of the system. The broaching machine presents a variety of states corresponding to different performance levels depending on the degree of corrosion. The system can successfully complete the transport mission only when the transport capacity of the system reaches the required value, and at least a fixed number of broaching machines transport the ship on the wire rope simultaneously.

Both quantity requirement of and random performance requirement should be considered in the availability definition for above system. The existing MSS availability evaluation model can no longer accurately describe the state of this complex system and evaluate its availability. Considering both quantity threshold and weight threshold, Eryilmaz [18] proposed a recursive formula for the state probability of a *k-out-of-n: G* system. However, the multi-state characteristic of system and the randomness of performance threshold were not considered in the analysis of system reliability. To address above problem, this paper proposes an availability evaluation method for MSS that comprehensively considers quantity and random weight threshold. Furthermore, the reliability design optimization problem of the components in the system is studied.

As the main availability evaluation methods of MSSs, Universal generating function (UGF) technique and recursive algorithm (RA) are widely used in both component-based systems, such as multi-state *k-out-of-n: G* systems [19], and weighted-based systems, such as multi-state weighted *k-out-of-n: G* systems [20], [21], [22]. The UGF method is a simple and efficient method for discrete random variable combination operations. By designing combination operators and filter operators, the availability of systems with different topologies, interactions between different elements, and different physical performance indicators can be evaluated [4]. More information on the UGF technique can be found in [23]. Compared with the UGF method, RA is more efficient in reliability evaluation for some classical topological systems [24], [25]. However, the traditional RA is based on the total probability theorem and is suitable for the case where the weight threshold follows a discrete random distribution. It is difficult to evaluate the availability of the system accurately when the system weight variables obey the continuous random distribution. In view of the above characteristics of the two methods, this paper presents the availability model and algorithm of MSS with quantity threshold and random weight threshold based on the UGF method and RA respectively.

Previous design optimization studies of MSS mainly focus on the problem of determining the optimal redundancy level for each component in the system, where the reliability value of the component is known [26], [27]. However, when we want to determine what components are optimal to use in a system, the reliability characteristics of the components themselves are more of a concern. Li and Zuo [28] proposed two optimization models of a multi-state weighted *k-out-of-n: G* system with a fixed weight threshold, which considered system availability constraints or cost constraints respectively. The optimal reliability distributions and performance (weight) distributions of multi-state components are obtained by running the GA program. Faghih-Roohi et al. [17] developed an optimization model to minimize the expected total system cost subject to system availability requirements. The above optimization model only considers the weight threshold of the system, and the value of the threshold is fixed. To the best of our knowledge, few studies on availability optimization problems consider the quantity threshold and random weight threshold.

To address the above problem, two design optimization models for MSS that consider both the quantity threshold and random weight threshold of components are proposed in this paper. The first model minimizes the expected total cost of the system while satisfying the system availability requirements. The other is to maximize the system availability for a given budget. Due to the low differentiability and continuity requirements of the objective function and fast global convergence of GA, it is widely used to solve various optimization problems of MSSs [29], [30]. The optimal component reliability distributions and component performance (weight) distributions in this paper are obtained using GA.

The main contributions of this paper are summarized below.

- The quantity requirement and random performance requirements are first considered comprehensively in the MSS model. Comparing to the classical multi-state *k-out-of-n: G* systems [19], and the multi-state weighted *k-out-of-n: G* systems [20], [21], [22], the proposed model is more general and realistic.
- The customized UGF method and RA are proposed respectively for evaluation the system availability. Furthermore, we verify the effectiveness of the proposed algorithms in different scenarios and compare the efficiency of the two algorithms.
- The system design optimization models are proposed respectively to access the optimal state probability distribution and weight distribution of multi-state components of the suggested system.

The remainder of this paper is organized as follows. In Section II, the availability definition of the system considering the quantity threshold and random weight threshold are presented. In Section III, based on the UGF and RA respectively, two evaluation methods for assessing system availability are proposed. In Section IV, considering the constraints of cost or availability, system reliability design optimization models are established, and the GA are presented to optimize the component reliability distributions and performance (weight) distributions. Section V shows a numerical example of a marine transportation system. Finally, conclusions and further work are given in Section VI.

II. SYSTEM DESCRIPTION

The considered system consists of $N(N \geq 2)$ multi-state components. The states of system and components are defined as discrete states from perfectly functioning to complete failure. More specifically, the system and components maybe in $M+1$ possible states: $0, 1, 2, \ldots, M$, where "M" represents the perfect functioning state and "0" represents the complete failure state. The performance rate of component i in state $j(j=0, 1, 2, .., M)$ is denoted as $g_{i,j}$. When component fails completely, $g_{i,j} = 0$.

We assume that the system performance depends deterministically on the performance of each of the component. The deterioration of components would lead to a change of their states and the degradation of their performance rates, which may negatively affect system performance.

The proposed system is in state j or above, if and only if its performance rate G is greater than or equal to a predetermined value w_j and no less than k_j components are in state j or above simultaneously. Let ϕ denote the state of the system, then the state probability of the system in state j or above, that is, the availability of the system can be defined as

$$\begin{aligned} \mathrm{pr}\left(\phi \geq j\right) &= \mathrm{pr}\left(\left(\sum_{i=1}^{N} X_i \geq k_j\right) \cap \left(G \geq w_j\right)\right) \\ &= \mathrm{pr}\left(G \geq w_j\right) \mathrm{pr}\left(\sum_{i=1}^{N} X_i \geq k_j \,\middle|\, G \geq w_j\right), \end{aligned} \tag{1}$$

where w_j and k_j are dual requirements for the system to be in state j or above, k_j indicates the quantity requirement of components in state j and above, w_j denotes the weight requirement of the system, and w_j is a random variable greater than 0 and subject to an arbitrary distribution. When component i is in state j or above, $X_i = 1$; otherwise, $X_i = 0$. Due to state 0 is the worst performing state, we have $\mathrm{pr}\left(\phi \geq 0\right) = 1$.

The above state definition of the system is a function of the component quantity threshold and system random weight threshold. When $w_j = 0$, the proposed system is simplified to a multi-state *k-out-of-n: G* system, and when $k_j = 0$, the system is simplified to a multi-state weighted *k-out-of-n: G* system.

In (1), the first part of the conditional probability form represents the system state probability that satisfies the weight requirement, and the second part represents the conditional probability of satisfying the quantity requirement under the condition of satisfying the weight requirement. Because X_i and G are not independent and cannot be mapped to each other, it is difficult to calculate the value of the second part based on the one-to-one correspondence between the component state probability and the state performance rate. In particular, when the system scale is expanded, this mapping relationship would be more complicated.

The concerned system is further described as follows:

- The components in the system are independent.
- The component failure can be found immediately and repaired in time, and the repair time of the components is exponentially distributed.
- The degradation of each component appears as a homogeneous continuous time Markov process. The state transition rate matrix of component i is

$$\boldsymbol{E}_i = \begin{bmatrix} \lambda^i_{M,M} & \lambda^i_{M,M-1} & \lambda^i_{M,M-2} & \cdots & \lambda^i_{M,1} & \lambda^i_{M,0} \\ 0 & \lambda^i_{M-1,M-1} & \lambda^i_{M-1,M-2} & \cdots & \lambda^i_{M-1,1} & \lambda^i_{M-1,0} \\ 0 & 0 & \lambda^i_{M-2,M-2} & \cdots & \lambda^i_{M-2,1} & \lambda^i_{M-2,0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \lambda^i_{1,1} & \lambda^i_{1,0} \\ \mu^i_{0,M} & 0 & 0 & 0 & 0 & \lambda^i_{0,0} \end{bmatrix}, \tag{2}$$

where $\lambda^i_{M,0}$ indicate the failure rate from a better state M to a worse state 0 and $\mu^i_{0,M}$ indicate the repair rate.

III. SYSTEM AVAILABILITY EVALUATION

In this section, we propose two availability evaluation methods for the suggested system using RA and UGF technique.

A. RECURSIVE ALGORITHM

In this section, the RA of the binary weighted *k-out-of-n: G* system is extended to multi-state cases.

Eryilmaz [18] gave the recursive equation to compute the binary weighted *k-out-of-n: G* system state probabilities. For $N>0$ and $k\leq N$, by condition the values of X_N and g_N (g_N is the performance rate of component N when it is working),

$$\begin{aligned} &Q(w,k,N)\\ &=\mathrm{pr}\left(\left(\sum_{i=1}^{N-1}X_i\geq k-1\right)\cap\left(\sum_{i=1}^{N-1}g_iX_i\geq w-g_N\right)\right)p_N\\ &\quad+\mathrm{pr}\left(\left(\sum_{i=1}^{N-1}X_i\geq k\right)\cap\left(\sum_{i=1}^{N-1}g_iX_i\geq w\right)\right)(1-p_N). \end{aligned} \tag{3}$$

with $Q(w,k,N)=1$ for $w\leq 0$ and $k\leq 0$; $Q(w,k,0)=0$ for $w>0$, $k>0$. In (3), w is the weight threshold of system, k represent the quantity requirement of working components. $p_N=\mathrm{pr}(X_N=1)$ is the probability that component N is in normal working condition.

When (3) is extended to the component of the proposed multi-state weighted *k-out-of-n: G* system, p_N should be replaced by the sums of all the probabilities that the component is in states j and above, that is states $j, j+1, \ldots,$ through M, and $(1-p_N)$ should be replaced by the sums of all the probabilities that the component is in other states, that is states $0, 1, \ldots,$ through $j-1$. And accordingly, g_N should be replaced by the performance rate of component N when it is in state j and above. Therefore, (3) should be revised to read (4), as shown at the bottom of the page.

Since the state of each component corresponds to the performance rate individually, there is $\mathrm{pr}\left(w=g_{N,j}\right)=p_{N,j}$. Then, the recursive equation for the system availability can be expressed as follows

$$\begin{aligned} A\left(w_j,k_j,N\right)&=Q_j\left(w_j,k_j,N\right)\\ &=\sum_{l=j}^{M}A\left(w_j-g_{N,l},k_j-1,N-1\right)p_{N,l}\\ &\quad+\sum_{l=0}^{j-1}A\left(w_j,k_j,N-1\right)p_{N,l}. \end{aligned} \tag{5}$$

In (5), the system weight threshold w_j is constant. The boundary conditions are: $A\left(w_j,k_j,N\right)=1$ when $w_j\leq 0$, and $k_j\leq 0$; $A\left(w_j,k_j,0\right)=0$ when $w_j>0$, $k_j>0$.

When the system weight threshold w_j follows a discrete random distribution (such as a two-point distribution, binomial distribution, Poisson distribution, geometric distribution, etc.), the system availability can be obtained based on the above equation. All possible values of w_j form collectively exhaustive events, according to the total probability theorem, the availability of the system is given by (6), as shown at the bottom of the page, where H is the number of possible values of the random variable w_j. w_j^h is the hth ($h=1,2,\ldots,H$) possible value of w_j.

B. UGF METHOD

In this section, system availability with different thresholds is evaluated by designing the operators in the UGF. There are usually two types of operators in the UGF methods. One is the combinatorial operator, which essentially defines the operational rules of discrete random variables and is used to describe the structure of the system in this paper. The other operator is used to filter the random variables that meet the filter conditions, and it is used to define the requirements that the system needs to satisfy for normal operation in this paper. The key of availability evaluation is to provide a random joint operator that combines the quantity and weight threshold.

The specific implementation process is as follows: the system state probability distribution meeting the random weight threshold is calculated according to the cumulative distribution of the system weight threshold firstly, and then, the system state probability meeting the quantity threshold is extracted through screening.

Let the vector $\boldsymbol{p}_i(t)=\left(p_{i,0}(t),p_{i,1}(t),\ldots,p_{i,M}(t)\right)$ denote the state probability of component i in states $0,1,2,\ldots,M$ at time t. Then, the Kolmogorov differential equation for the multi-state component i is

$$\frac{\mathbf{d}\boldsymbol{p}_i(t)}{\mathbf{d}t}=\boldsymbol{p}_i(t)\,\boldsymbol{E}_i. \tag{7}$$

$$\begin{aligned} Q_j\left(w_j,k_j,N\right)=&\sum_{l=j}^{M}\left[\mathrm{pr}\left(\left(\sum_{i=1}^{N-1}X_i\geq k_j-1\right)\cap\left(\sum_{i=1}^{N-1}\boldsymbol{g}_iX_i\geq w_j-g_{N,l}\right)\right)\mathrm{pr}\left(w=g_{N,l}\right)\right]\\ &+\sum_{l=0}^{j-1}\left[\mathrm{pr}\left(\left(\sum_{i=1}^{N-1}X_i\geq k_j\right)\cap\left(\sum_{i=1}^{N-1}\boldsymbol{g}_iX_i\geq w_j\right)\right)\mathrm{pr}\left(w=g_{N,l}\right)\right] \end{aligned} \tag{4}$$

$$A\left(w_j,k_j,N\right)=\sum_{h=1}^{H}\left[\left(\sum_{l=0}^{j-1}p_{N,l}A\left(w_j^h,k_j,N-1\right)+\sum_{l=j}^{M}p_{N,l}A\left(w_j^h-g_{N,l},k_j-1,N-1\right)\right)\mathrm{pr}\left(w_j=w_j^h\right)\right] \tag{6}$$

The sum of the probabilities of all the states of component i at time t is 1,

$$\boldsymbol{p}_i(t)^{\mathbf{T}} \cdot \mathbf{1} = 1, \tag{8}$$

where $\mathbf{1}$ is an $M+1$ column vector in which all elements are 1. Assume that all components are in the working condition at $t=0$, that is

$$p_{i,0}(0) = p_{i,1}(0) = \ldots = p_{i,M-1}(0) = 0, \tag{9}$$
$$p_{i,M}(0) = 1. \tag{10}$$

The state probability of component i can be obtained by solving (7), (8), (9) and (10). The steady state probability can be calculated using $t \to \infty$.

Thus, the UGF of component i can be represented by the following polynomial function

$$U_i(z) = \sum_{j=0}^{M} p_{i,j} z^{g_{i,j}}. \tag{11}$$

Based on (11), the parallel structure composition operator Ωpar can be used to obtained the system UGF. For a system consisting of N components in a parallel structure, the UGF is

$$\begin{aligned} U(z) &= \Omega\mathrm{par}\left(U_1(z), U_2(z), \ldots, U_N(z)\right) \\ &= \sum_{j=0}^{M}\sum_{j=0}^{M}\cdots\sum_{j=0}^{M}\left(\prod_{i=1}^{N} p_{i,j} z^{\varphi(\boldsymbol{g}_1,\boldsymbol{g}_2,\ldots,\boldsymbol{g}_N)}\right), \end{aligned} \tag{12}$$

where $\varphi(\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_N)$ is the system structure function. $\boldsymbol{g}_i = (g_{i,0}, g_{i,1}, \ldots, g_{i,M})$ is the performance vector of component i corresponding to states $0, 1, 2, \ldots, M$. The system performance G is the sum of the performances of component i in state j or above, that is

$$\varphi(\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_N) = G = \sum_{i=1}^{N} X_i g_{i,j}. \tag{13}$$

Then, when the weight threshold is fixed, the probability that the system is in state j or above can be calculated by designing a joint operator δ to introduce the quantity threshold and weight threshold

$$\begin{aligned} Q_j&(w_j, k_j, N) \\ &= \delta\left(U(z), w_j, k_j, N\right) \\ &= \delta\left(\sum_{j=0}^{M}\sum_{j=0}^{M}\cdots\sum_{j=0}^{M}\left(\prod_{i=1}^{N} p_{i,j} z^{\varphi(\boldsymbol{g}_1,\boldsymbol{g}_2,\ldots,\boldsymbol{g}_N)}\right), w_j, k_j, N\right) \\ &= \sum_{j=0}^{M}\sum_{j=0}^{M}\cdots\sum_{j=0}^{M}\left(\prod_{i=1}^{N} p_{i,j} z^{\varphi(\boldsymbol{g}_1,\boldsymbol{g}_2,\ldots,\boldsymbol{g}_N)}\right) \cdot 1(\phi \geq j) \\ &= \sum_{j=0}^{M}\sum_{j=0}^{M}\cdots\sum_{j=0}^{M}\left(\prod_{i=1}^{N} p_{i,j} z^{G}\right) \cdot \\ &\quad \times 1\left(\left(\sum_{i=1}^{N} X_i \geq k_j\right) \cap (G \geq w_j)\right), \end{aligned} \tag{14}$$

where $1(\cdot)$ is the indicator function, 1(TURE)=1, 1 (FALSE)=0.

It is assumed that the system operates normally when it is in the j or above state. When the weight threshold w_j is constant, the availability of the system can be expressed as

$$\begin{aligned} A(w_j, k_j, N) &= Q_j(w_j, k_j, N) \\ &= U(z) \cdot 1\left(\left(\sum_{i=1}^{N} X_i \geq k_j\right) \cap (G \geq w_j)\right). \end{aligned} \tag{15}$$

Furthermore, when the weight threshold w_j changes randomly, it is difficult to directly screen out the system state probability that satisfies both the quantity and weight thresholds. In this paper, a new stochastic joint operator $\Delta\delta$ is proposed, and the probability of the system in state j and above with a random weight threshold is given as

$$\begin{aligned} Q_j&(w_j, k_j, N) \\ &= \Delta\delta\left(U(z), w_j, k_j, N\right) \\ &= \Delta\delta\left(\sum_{j=0}^{M}\sum_{j=0}^{M}\cdots\sum_{j=0}^{M}\left(\prod_{i=1}^{N} p_{i,j} z^{G}\right), w_j, k_j, N\right) \\ &= \sum_{j=0}^{M}\sum_{j=0}^{M}\cdots\sum_{j=0}^{M}\left(\prod_{i=1}^{N} p_{i,j}\mathbf{pr}(G \geq w_j)\right) \cdot 1\left(\sum_{i=1}^{N} X_i \geq k_j\right). \end{aligned} \tag{16}$$

If the cumulative distribution function of the system weight threshold w_j is $F(w_j)$, then the availability of a system with random weights is

$$\begin{aligned} A&(w_j, k_j, N) \\ &= \sum_{j=0}^{M}\sum_{j=0}^{M}\cdots\sum_{j=0}^{M}\left(\prod_{i=1}^{N} p_{i,j} F(w_j)\right) \cdot 1\left(\sum_{i=1}^{N} X_i \geq k_j\right). \end{aligned} \tag{17}$$

The above system availability evaluation method introduces quantitative and weight elements, and considers the randomness of the system weight threshold, which enhances the flexibility of the method and is more realistic than the existing availability model.

IV. OPTIMIZATION OF COMPONENT DESIGN

This section addresses the problem of optimizing the reliability distributions and performance (weight) distributions of multi-state components in the system. Considering the availability or total cost of the system as constraints, two optimization models are established.

A. OPTIMIZATION MODEL

The system optimization model with the system availability constraint is represented as Model 1, as shown below

$$\begin{aligned} &\mathrm{Min}\ C = \sum_{i=1}^{N} c_i + \left(1 - A(w_j, k_j, N)\right)c_j \\ &\text{s.t. } A(w_j, k_j, N) \geq \hat{A}, \end{aligned}$$

$$\sum_{j=0}^{M} p_{i,j} = 1,$$
$$0 \le p_{i,j} \le 1,$$
$$g_{i,0} = 0,$$
$$g_{i,j} \ge 0, j > 1, \tag{18}$$

where C denotes the total cost of the system. c_i denotes the design cost of component i.c_j denotes the penalty cost of the system failure and is a constant greater than 0. $\hat{A}$ is the design requirement for the system availability.

The system optimization model considering the total cost constraint is represented as Model 2, as shown below

$$\text{Max } A\left(w_j, k_j, N\right)$$
$$\text{s.t. } \sum\nolimits_{i=1}^{N} c_i + \left(1 - A\left(w_j, k_j, N\right)\right)c_j \ge \hat{C}$$
$$\sum\nolimits_{j=0}^{M} p_{i,j} = 1,$$
$$0 \le p_{i,j} \le 1,$$
$$g_{i,0} = 0,$$
$$g_{i,j} \ge 0, j > 1, \tag{19}$$

where $\hat{C}$ is the maximum acceptable values of total cost.

In the above models, the system availability $A\left(w_j, k_j, N\right)$ can be calculated from (6) and (17). The design cost of component ic_i mainly consists of two main parts. One part is the cost related to the state probability of the component, and the other part is the cost related to the component performance rate.

$$c_i = c_i^p + c_i^g, \tag{20}$$

where

$$c_i^p = \exp\left((1 - f_i) \frac{\sum_{j=1}^{M} p_{i,j} - p_{i\min}}{p_{i\max} - \sum\limits_{j=1}^{M} p_{i,j}} \right). \tag{21}$$

$$c_i^g = v_i \exp\left(\sum\nolimits_{j=0}^{M} p_{i,j} g_{i,j} - g_{i\min} \right). \tag{22}$$

Equation (21) is a function to describe the relationship between the component cost and component reliability [28]. c_i^p is the cost associated with the reliability (state probability) of component i. $\sum_{j=1}^{M} p_{i,j}$ represents the reliability of component i. There are three parameters in (21), namely, f_i, $p_{i\min}$ and $p_{i\max}$. The first parameter f_i, called the feasibility parameter, is a positive constant, which represents the difficulty in increasing the reliability of component i relative to the rest of the components in the system, and it assumes values between 0 and 1. $p_{i\min}$ is the initial (current) reliability value of the component i obtained from the failure distribution of that component and for the specified time. $p_{i\max}$ is the maximum achievable reliability of the component i. When $p_{i\min}$ and $p_{i\max}$ are fixed, the closer f_i is to 1, the higher the possibility of improving the component reliability and the lower the design cost.

Equation (22) is a function to describe the relationship between the component cost and component performance rate [28]. c_i^g is the cost of component i in terms of the relationship between component performance rate and component cost. $g_{i\min}$ indicates the minimum value for the weighted average of the component i's performances in all possible states. v_i is the feasibility parameter, indicate the feasibility of increasing the performance rate of component i. The higher the v_i, the lower the cost.

Furthermore, the sum of the state probabilities of each component and expected performance rate of each component should satisfy the following inequalities

$$p_{i\min} \le \sum_{j=1}^{M} p_{i,j} \le p_{i\max}, \tag{23}$$

$$\sum_{j=0}^{M} p_{i,j} g_{i,j} \ge g_{i\min}. \tag{24}$$

In optimization Model 1, the total cost of the system is minimized under the constraint of availability. Model 2 maximizes the system availability under the constraint of the total system cost. The decision variables of the two optimization models are the component state probability distribution $\boldsymbol{p} = \left(\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_N\right)$ and the component weight distribution $\boldsymbol{g} = \left(\boldsymbol{g}_1, \boldsymbol{g}_2, \ldots, \boldsymbol{g}_N\right)$. The number of decision variables is related to the number of components and the number of component states, and can be expressed as $2N(M+1)$. Both Model 1 and 2 are nonlinear optimization problems with discrete random decision variables, whose feasible solution space increases exponentially with an increase in the number of components and the number of states of components in the system.

B. OPTIMIZATION ALGORITHM

In this section, the Augmented Lagrange Genetic Algorithm is used to solve the above nonlinear constrained optimization models. The penalty function is used to transform the complex constrained optimization problem into a simple unconstrained optimization problem. At the same time, to avoid premature convergence, the population size is enlarged and the population diversity is enhanced.

The objective functions and nonlinear constrains in Model 1 are expressed in penalty function forms as follows

$$\text{Min } C = \sum_{i=1}^{N} c_i + \left(1 - A\left(w_j, k_j, N\right)\right)c_j$$
$$+ \eta \max\left(\left(\hat{A} - A\left(w_j, k_j, N\right)\right), 0\right)$$
$$\text{s.t. } \sum_{j=0}^{M} p_{i,j} = 1,$$
$$0 \le p_{i,j} \le 1,$$

$$g_{i,0} = 0,$$
$$g_{i,j} \geq 0, j > 1,$$
$$p_{i\min} \leq \sum_{j=1}^{M} p_{i,j} \leq p_{i\max},$$
$$\sum_{j=0}^{M} p_{i,j} g_{i,j} \geq g_{i\min}, \tag{25}$$

where η is the penalty factor, it is a large number.

The optimization function in the GA toolbox is to minimize the fitness function, subsequently, Model 2 is rewritten as

$$\begin{aligned} \text{Min} & -A\left(w_j, k_j, N\right) \\ & + \eta \max\left(\left(\sum_{i=1}^{N} c_i + \left(1 - A\left(w_j, k_j, N\right)\right) c_j - \hat{C}\right), 0\right) \\ \text{s.t.} & \sum_{j=0}^{M} p_{i,j} = 1, \\ & 0 \leq p_{i,j} \leq 1, \\ & g_{i,0} = 0, \\ & g_{i,j} \geq 0, j > 1, \\ & p_{i\min} \leq \sum_{j=1}^{M} p_{i,j} \leq p_{i\max}, \\ & \sum_{j=0}^{M} p_{i,j} g_{i,j} \geq g_{i\min}. \end{aligned} \tag{26}$$

V. NUMERICAL EXAMPLES

In this study, the data of the ship transportation system in [17] is used for example analysis. The transport system consists of $N = 3$ independent broaching machines, numbered M1, M2 and M3. The three broaching machines have different capacities and each has four states 0, 1, 2 and 3. State 3 corresponds to the best operating state, with the highest load capacity and the highest weight (performance rate); state 0 is the failure state, with a weight of 0 and no-load capacity; and states 1 and 2 are the degraded states between states 3 and 0. In some systems, even in the lowest state, the components can still contribute the basic capabilities to the system. However, in this example, the performance of each broaching machine in state 0 is assumed to be 0. The degradation of each broaching machine appears as a homogeneous continuous time Markov process, and maintenance is performed only when the broaching machines completely fails. The state transition rates of M1, M2 and M3 and the weights corresponding to different states are shown in Table 1.

At least two broaching machines are required to transport the ship during the transport mission, and the weight threshold of the system is a random variable with a distribution function of $F\left(w_j\right)$. According to the above description, the ship transportation system is a typical multi-state 2-out-of-3: G system with a random weight threshold.

TABLE 1. State transition rates and performances of each broaching machine.

Broaching machines		Transition rates (1/yr.)				Weights (performance rates)
Types	States	3	2	1	0	
M 1	3	0	2	1.3	0.7	2000
	2	0	0	0.9	0.5	1500
	1	0	0	0	0.3	1000
	0	4.2	0	0	0	0
M 2	3	0	1.8	1.1	0.8	2200
	2	0	0	0.8	0.4	1400
	1	0	0	0	0.2	1200
	0	7.2	0	0	0	0
M 3	3	0	2.2	1.6	0.9	2500
	2	0	0	1.2	0.7	2000
	1	0	0	0	0.5	1500
	0	5.4	0	0	0	0

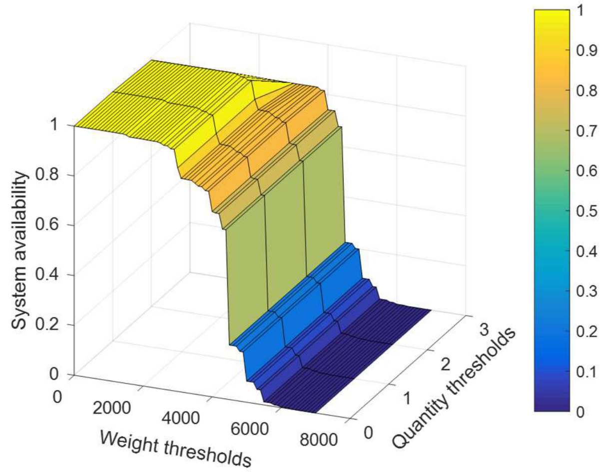

FIGURE 1. Case 1: System availability with different quantity thresholds and weight thresholds.

A. SYSTEM AVAILABILITY

Two methods of availability evaluation are proposed in this paper. Among these, the RA is applicable to the case where the weight threshold is a discrete random variable. The UGF method has a wider range of application, and it is applicable not only to the case of discrete random variables, but also to the case of continuous random variables. To illustrate the effectiveness of these two approaches, the availability of the system is evaluated for three cases.

In the first case, to show the availability with different quantity threshold and weight threshold, w_1 is taken as a fixed value and a three-dimensional plot showing in Fig.1 was used. We moved through the axis "quantity threshold" from 0 to 3. At the same time, we change the weight threshold values from 0 kg to 8000 kg.

TABLE 2. **Case 2: Distributions of weight threshold.**

Distribution types	Distribution law of w_1				
1	w_1	2000	2500	3000	3500
	Probability	1/4	1/4	1/4	1/4
2	w_1	2000	2500	3000	
	Probability	1/3	1/3	1/3	
3	w_1	2000	2500		
	Probability	1/2	1/2		

TABLE 3. **Case 2: System availability with different distributions of weight threshold.**

$A(w_1, k_1, N)$ $N = 3$	Distribution types		
	1	2	3
$k_1 = 0$	0.8905	0.9149	0.9559
$k_1 = 1$	0.8905	0.9149	0.9559
$k_1 = 2$	0.8903	0.9146	0.9554
$k_1 = 3$	0.8021	0.8021	0.8021

In the second case, w_1 is a discrete random variable. In order to verify the correctness and effectiveness of the two methods, the availability of the transportation system when $k_1 = 0, 1, 2, 3$ is evaluated separately for three different weight threshold distributions by using the RA and UGF method respectively. The different distributions of w_j are given in Table 2, and the results are shown in Table 3.

In the third case, w_1 is a continuous random variable. Let w_1 follows the uniform distribution of intervals $[a, b]$ kg and the Gaussian distribution with a mathematical expectation of μ and a standard variance of σ^2, respectively, and the system availability with different quantity thresholds is evaluated separately under different weight threshold distributions. The results are show in Fig. 2, 3, 4 and 5.

It is clear from above results of different cases that, for different quantality thresholds, there are $A(\mathrm{w}_1, k_1, N) \leq A(\mathrm{w}_1, k_1', N)$ when $k_1 > k_1'$. This indicates that an increase in the quantity threshold may lead to a decrease in system availability. The system availability is equal when $k_1 = 0$ and $k_1 = 1$, because there must be at least one broaching machine involved in the mission, regardless of whether there is a quantality threshold of the broaching machine. When $k_1 \geq 2$, system availability is unequal. In this case, the quantality threshold affects the value of the system availability, the larger the quantality threshold, the stricter the availability requirements, and the lower the availability. The above results show the influence of the weight threshold and quantity threshold on system availability, and verify the effectiveness of the availability evaluation method proposed in this paper under the condition that the quantity threshold and weight threshold cannot map each other.

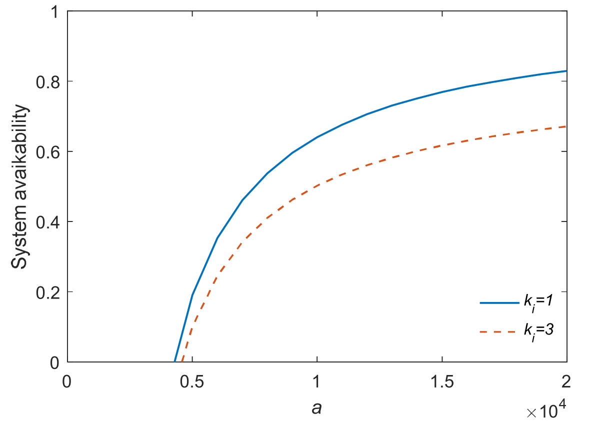

FIGURE 2. **Case 3: System availability with different quantity thresholds when weight threshold follows the uniform distribution of intervals [*a*, *b*] (*b*=1000).**

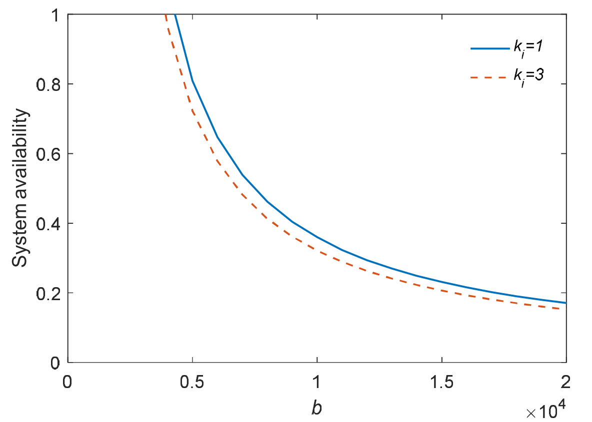

FIGURE 3. **Case 3: System availability with different quantity thresholds when weight threshold follows the uniform distribution of intervals [*a*, *b*] (*a*=1000).**

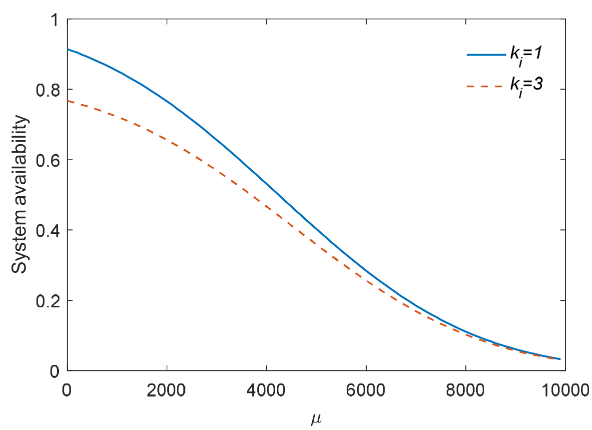

FIGURE 4. **Case 3: System availability with different quantity thresholds when weight threshold follows the Gaussian distribution with a mathematical expectation of μ and a standard variance of σ^2 (σ =3000).**

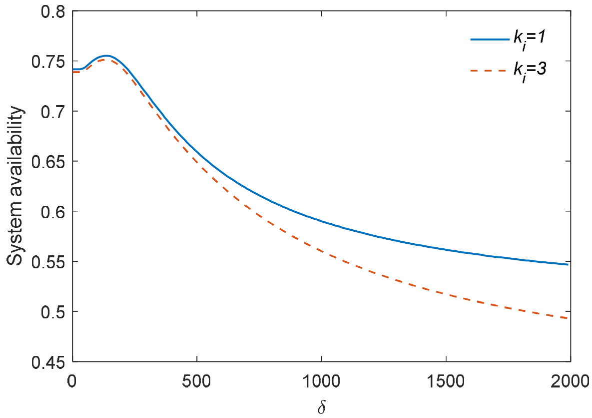

FIGURE 5. Case 3: System availability with different quantity thresholds when weight threshold follows the Gaussian distribution with a mathematical expectation of μ and a standard variance of σ^2 (μ = 4000).

In addition, using the method in [17], it can be observed that when the weight threshold is 3000 kg, the instantaneous availability of the system at a certain time is 0.8439. The method in this paper can be used to obtain a steady-state availability of the system is 0.8021 when the weight threshold follows the uniform distribution of the interval [2000,3000] kg. The method in this paper considers the quantity threshold of component and random weight threshold of system, which can better reflect the long-term state of the system, be closer to engineering practice, and provide a more valuable reference for the design optimization of the system.

B. COMPARISON OF METHODS

This section compares the computing efficiency of the RA and UGF method proposed in this article.

Computer programs for both methods were developed using MATLAB 2020a. Under the premise that the sum of the state probability of each component is 1, a random number between 0 and 1 is generated to represent the state probability value of the component. Under the premise that the weight of the component in the complete failure state is 0, a random number between 0 and 2500 is generated to represent the component weight. Assume that the weight threshold is 3000 kg. Then, the CPU times required by the two approaches for different system sizes and different number of states are given in Table 4 and 5.

As can be seen from Table 4 and 5, the CPU time required by the UGF method increases more slowly than that of RA as the number of components increases. When the values of the system parameters M and N are greater than 3 and 4, respectively, the CPU time required by the UGF method is shorter than that required by the RA. This is because that the UGF method filters out many items that do not satisfy the quantity threshold of the component. The computational complexity of the RA is the same as that of the recursive method in [31], which is $O\left((M+1)^N\right)$. The proposed UGF method requires $(M+1-j)^{k_j}(M+1)^{N-k_j}$ operations, the computational complexity of UGF can be expressed as $O\left((M+1-j)^{k_j}(M+1)^{N-k_j}\right)$. In general, the UGF approach is more computationally efficient than the RA for the system in this paper.

TABLE 4. CPU time comparison of two methods as a function of state number of the component M ($N=5, k_1=4$).

M	CPU(s) UGF method	CPU(s) RA	Iterations number of RA
3	0.0355	0.0224	1365
4	0.0116	0.0318	3906
5	0.0136	0.0753	9331
6	0.0249	0.1571	19608
7	0.0500	0.3019	37449
8	0.0901	0.5320	66430
9	0.1450	0.8981	111111
10	0.2455	1.4263	177156
11	0.3805	2.2023	271453
12	0.5539	3.2519	402234
13	0.8124	4.6592	579195
14	1.1350	6.6072	813616
15	1.6101	10.7861	1118481
16	2.6472	17.0817	1508598
17	3.0782	18.6452	2000719
18	3.8194	21.8844	2613660
19	4.9195	28.8928	3368421
20	6.8422	38.1399	4288306

TABLE 5. CPU time comparison of two methods as a function of system size N ($M=5, k_1=4$).

N	CPU(s) UGF method	CPU(s) RA	Iterations number of RA
3	0.0307	0.0083	259
4	0.0144	0.0130	1555
5	0.0182	0.0778	9331
6	0.0742	0.4626	55987
7	0.4543	2.7936	335923
8	2.8376	16.5404	2015539
9	25.6657	130.5668	12093235
10	Insufficient memory	664.9763	72559411

TABLE 6. Related parameters of the transport system.

f_i	v_i	$p_{imin}(t)$	$p_{imax}(t)$	$g_{imin}(t)$	c_j	$\hat{A}$	$\hat{C}$
0.99	1	0.9	0.9999	1000	3	0.9	10

C. OPTIMIZATION OF SYSTEM

Suppose that the shipyard decides to design a new transportation system. The new transport system is still a multi-state 2-out-of-3: G system, with a random weight threshold w_1, conforming to a uniform distribution of interval [2000,3000] kg. In this case, the probability distribution and weight distribution of the three broaching machines in

TABLE 7. Optimization results of model 1.

$A(w_1,k_1,N)$		$A(w_1,0,3)=0.9107$				$A(w_1,3,3)=0.9009$			
C		3.3398				5.0965			
j		0	1	2	3	0	1	2	3
g	$i=1$	0	1018	1060	1039	0	1065	1010	1005
	$i=2$	0	1055	1012	990	0	1004	988	1012
	$i=3$	0	1030	1034	997	0	1015	1011	998
p	$i=1$	0.0449	0.3099	0.3215	0.3237	0.0293	0.4000	0.2000	0.3707
	$i=2$	0.0267	0.3175	0.3319	0.3239	0.0375	0.4000	0.2000	0.3625
	$i=3$	0.0250	0.2773	0.2672	0.4305	0.0354	0.3900	0.2000	0.3746

TABLE 8. Optimization results of model 2.

$A(w_1,k_1,N)$		$A(w_1,0,3)=0.9923$				$A(w_1,3,3)=0.9738$			
C		9.3315				9.9871			
j		0	1	2	3	0	1	2	3
g	$i=1$	0	995	994	1015	0	1001	1015	998
	$i=2$	0	999	1006	1008	0	1008	1019	1022
	$i=3$	0	990	1005	1011	0	1029	1020	968
p	$i=1$	0.0001	0.5531	0.1985	0.2683	0.0033	0.3763	0.2469	0.3735
	$i=2$	0.0033	0.2945	0.5000	0.2022	0.0157	0.2680	0.3241	0.3922
	$i=3$	0.0019	0.3126	0.3355	0.3500	0.0030	0.2914	0.3377	0.3679

different states are unknown. Therefore, it is necessary for designers to optimize the state probability and weight of the broaching machine.

In this section, the optimization models proposed in Section IV are used to optimize the state probability and weight of the broaching machine under the condition of satisfying the availability requirements of the transportation system or the total cost of system constraints respectively. The values of the relevant design parameters for the transportation system are shown in Table 6.

In order to study the influence of the quantity threshold on the system design optimization, when the system weight threshold follows a uniform distribution of interval [2000,3000] kg. the GA program of MATLAB 2020a is used to calculate the optimization results of the components quantity threshold $k_1 = 0$ and $k_1 = 3$, respectively. In the penalty functions, $\eta = 100000$. The population size (number of populations) is set to 100, simulated generation (stall generation) is 2000, crossover rate is 0.8, and mutation rate is 0.05.

In [32], the randomness of GA was addressed by running the optimization process 30 times and selecting the optimal fitness function value as the final optimization result. This method is used to determine the optimization results in this paper. First, the optimization results for $k_1 = 3$ in Model 1and 2 are calculated and selected. Then, considering that the increase of k_1 may leads to a decrease of system availability, and the penalty cost increases accordingly, the system cost at $k_1 = 0$ should be lower than the optimization results at $k_1 = 3$, and the system availability at $k_1 = 0$ should be higher than the optimization results at $k_1 = 3$. Finally, the optimization program is run 30 times, and the return values of multiple groups of fitness functions are compared and selected according to the above deductions, and the optimal fitness function value is selected as the final result. The results are given in Table 7 and 8, respectively.

As shown in Table 7 and 8, the optimal cost or optimal availability of the system differs when the k_1 takes different values. In Model 1, the optimal cost of the system is

$$Q(w,k,N)=\sum_{g\geq a_N}\left[\mathbf{pr}\left(\left(\sum_{i=1}^{N-1}X_i\geq k-1\right)\cap\left(\sum_{i=1}^{N-1}g_iX_i\geq w-g\right)\right)\mathbf{pr}\left(g_N=g\right)p_N\right.$$
$$\left.+\mathbf{pr}\left(\left(\sum_{i=1}^{N-1}X_i\geq k\right)\cap\left(\sum_{i=1}^{N-1}g_iX_i\geq w\right)\right)\mathbf{pr}\left(g_N=g\right)\left(1-p_N\right)\right]. \tag{27}$$

3.3398 when $k_1 = 0$, whereas when $k_1 = 3$, the optimal cost of the system increases to 5.0965. In Model 2, the optimal availability of the system is 0.9923 when $k_1 = 0$, whereas when $k_1 = 3$, the optimal availability of the system decreases to 0.9738. The above results indicate that the quantity threshold may affect the optimization results when considering the random weight threshold of the system. The reason is that the quantity threshold makes the definition of system availability more stringent, which leads to an increase in the system cost. It is clear that considering the quantity threshold of components when designing a system is necessary to achieve a better trade-off between the availability and cost of the system.

In addition to considering the weight threshold of the system, this paper introduces the quantity threshold of the component in the system, and takes into account the randomness of the system weight threshold. Compared with the optimization results in [17], the results in this article consider more reasonable optimization conditions and obtain more satisfactory optimization results. For example, when the availability constraints are all 0.9, the optimal system availability obtained in this paper is 0.9107, whereas the optimization result of [17] is 0.9677. The corresponding system cost is 7.354, which is also much higher than the cost value of 3.3398 in this article.

VI. CONCLUSION

Considering the random performance requirements and component quantity requirements during the execution of system missions, the availability evaluation and optimization problem of multi-state *k-out-of-n: G* system with random performance requirements is investigated in this paper. Two methods are proposed to evaluate system availability. Moreover, the performance distribution and probability distribution for each state of the component are optimized using two optimization models that consider the total cost constraint or availability constraint. A case study was conducted to consider the effect of the introduction of a quantity threshold on the system availability and optimization results. It revealed that the proposed methods and models enables the more accurate results when the system state be affected by both quantitative and random weight elements. Moreover, the proposed UGF method has higher computational efficiency and stronger universality than customized RA for the suggested system.

These results can be applied to model and optimize the availability of complex MSS with random performance requirements in practical engineering applications. In future research, different maintenance strategies can be introduced based on this study, and other complex mission scenarios can be taken into account to study the availability evaluation and optimization problems. In addition, the future direction will be considered as the multi-objective optimization of MSS considering random weight thresholds and quantity thresholds can also be further studied, such as the reliability-redundancy allocation optimization problem, join optimization of the design and maintenance, etc. Furthermore, some of the latest machine learning methods can be used to study the dynamic optimization problems related to the system in this study.

APPENDIX

The further explanation of (3) are as follows

Equation (3) refers to (27), as shown at the top of the page, in [18]. Equation (27) is designed to represent the probability in the working state of a *k-out-of-n: G* system with components having random weights. The basic idea of (3) is similar to that of the algorithm in (27): we enumerate the cases where component N is in different possible states, and thus evaluate a system with N components via evaluating several systems with $N-1$ components. For each certain state on the right-hand side of (27), we need to reorganize w and k. The idea is that if component N is in working state, the required number of working components should be decreased by one, and the required system performance should be decreased by g_N. On the contrary, the required number of working components and system performance remain the same.

In (27), the non-zero random weight g_i of component i is assumed to have a discrete probability distribution with the support $[a_i, b_i]$, i.e., $\mathrm{pr}\left(a_i \leq g_i \leq b_i\right) = 1$ for $0 < a_i < b_i < \infty$.

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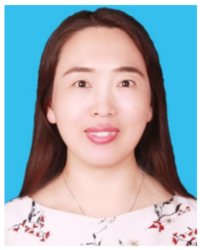

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