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RESEARCH ARTICLE

Output Feedback Control for Switched Systems With Unmatched Uncertainties Based on the Common Robust Integral Sliding Mode

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ABSTRACT The output feedback robust control design is required for switched systems (SSs) without the state feedback information. For the sliding mode control (SMC) based on state observers, the unmatched uncertainty and disturbance cannot be rejected completely. In this paper, an output feedback sliding mode control (SMC) based on a robust integral sliding mode (RISM) is presented for switched systems with unmatched uncertainties. First, the control task of the output feedback stabilization based on the state observer is given. Then, in order to reject the unmatched uncertainty, the RISM is constructed on the space of the estimated state, whose parameters are selected ensuring that the system in the sliding mode is robust exponentially stable. Linear matrix inequality conditions for the parameter design and the stabilization switching rule are achieved, which make the sliding mode can reject the unmatched uncertainty and suppress the unmatched disturbance both at the designed control switching rule and the arbitrary switching rule. Consequently, the corresponding SMC controller is designed based on the state estimation and the propsoed RISM. Finally, the application simulation results to a one-link manipulator with the load change validated the effectiveness and feasibility.

INDEX TERMS Switched systems, unmatched uncertainty, sliding mode control, output feedback, integral sliding mode.

I. INTRODUCTION

A switched system (SS) is a hybrid system composed of a group of continuous or discrete dynamic subsystems that switch between each subsystem according to a certain switching rule [1]. It is a very active branch in the research of hybrid system theory and application. Due to the fact that many practical engineering systems, such as chemical processes, transportation transmission processes, computer control systems, and power systems etc., can be modeled as switched systems, switched systems have received widespread attention from scholars in the past few decades [2], [3].

The state information of the control system is sometimes not directly measurable. In this way, for switched systems, it is necessary to study how to achieve the stabilization or trajectory tracking without state feedback information. At this point, the complexity of control design and switching strategy design increases. Therefore, the control problem of switched systems based on output feedback or observers has become one of the main research topics in the field of switched systems [4], [5], [6], [7], [8].

In recent years, the sliding mode control (SMC) based on observers for uncertain switched systems has achieved many beneficial results [9], [10], [11], due to its strong

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robustness against the matched uncertainties/perturbations [12], [13]. Gao et al. presented a reduced-order observer-based sliding mode control scheme for switched descriptor systems to guarantee the exponentially asymptotic stability of the sliding motions [14]. Kao et al. investigated an asynchronous adaptive observer-based sliding mode control for a class of nonlinear stochastic switched systems with Markovian switching signal [15]. Some fixed-time or finite-time observers was proposed. Qi et al. studied the sliding mode control for semi-Markov switching systems with quantized measurement and gave the corresponding finite-time observer design [16]. Likely, Zhang et al. designed a nonfragile finite-time bounded sliding-mode observer for stochastic Markovian jump systems with a deterministic switching chain [17]. Zhang et al. discussed the issue of observer-based sliding mode control for fuzzy stochastic semi-Markov switching systems under cyber attacks [18]. Sakthivel et al. studied the robust stochastic stabilization problem for a class of fuzzy Markovian jump systems with time-varying delay and external disturbances, and presented a SMC scheme with an online disturbance estimator to reject the unknown disturbance [19]. On other hands, Meng et al. dealt with the problem of observer-based event-triggered sliding mode control for fractional-order uncertain switched systems [20]. However, these results didn't consider the case that the uncertainty or disturbance of the SS is unmatched for the sliding mode.

The integral sliding mode (ISM) method is an efficient approach for increasing the robustness [21], [22], because the ISM method eliminates the reaching phase that is not robust with respect to uncertainty and disturbance. As a result, throughout the entire control process, the robustness of the system is confirmed from the initial point in time. The ISM method has been successfully applied to the uncertain SS to compensate exactly the matched uncertainties or perturbations [15], [23], [24], [25], [26], [27]. For example, Kchaou and Ahmadi extended an adaptive SMC design based on the ISM for a class of uncertain switched descriptor systems with state delay and nonlinear input [23]. Chen et al. investigated the robust exponential stabilization $\mathcal{H}_\infty$-based integral sliding mode controller design for uncertain stochastic Takagi-Sugeno (T-S) fuzzy switched time-delay systems [24]. Qi et al. studied the issue of SMC design for a class of nonlinear semi-Markovian switching systems via T-S fuzzy approach by designing a novel ISM surface [25]. Zhang presented a robust integral sliding mode control under arbitrary time-dependent switching rules for uncertain switched systems [26]. Kao et al. concerned the non-fragile observer-based integral sliding mode control for a class of uncertain switched hyperbolic systems [15]. Wang et al. presented an output feedback SMC controller based on the ISM for Markovian jump systems [27]. However, these studies still did not take into account the unmatched uncertainty and disturbance during the ISM design.

Inspired by the application of the ISM method to SSs, this paper presents a robust integral sliding mode (RISM) design for the switched system with unmatched uncertainties, while the existing output feedback control designs based on ISM methods cannot deal with the unmatched uncertainty and disturbance. First, the control task is described as a general output feedback stabilization based on the observer. And then, the robust integral sliding mode (RISM) is designed in the estimated state space, on which the system sliding motion can guarantee a exponentially stability with the robustness to the unmatched uncertainties. Then, the linear matrix inequality conditions for the RISM parameters and the switching rule are achieved. Afterwards, the SMC will be devised and the reachability of the RISM will be analyzed.

The contributions of this paper are as follows.

(1) The RISM designed on the observed state space is proposed. Compared with other existing ISM designs, the proposed RISM has the robustness to the structured unmatched uncertainty and can suppress the unmatched disturbance. In addition, the observing error and the state converge simultaneously, and the reaching phase of the sliding mode is eliminated.

(2) The stability of the overall SS on the RISM surface is analyzed in two cases. One is that the switching rule can be designed to ensure the overall stability. The other one is that the overall SS on the RISM surface can be stabilized with arbitrary rules. The LMI stability conditions for the two cases are provided.

The rest of the paper is organized as follows. Section II states the problem formulation. Section III gives the definition of the RISM sliding surface, and the design of the sliding mode parameters with the stability analysis. Then, the reachability of the sliding mode is proved by the SMC controller design in Section IV. An application simulation results are described in Section V. Finally, Section VI presents a brief conclusion of this paper.

II. PROBLEM FORMULATION

Consider the following uncertain switched system

$$\begin{aligned} \dot{x}(t) &= (A_\sigma + \Delta A_\sigma)x(t) + (B_\sigma + \Delta B_\sigma)u(t) + G_\sigma \omega_\sigma(t) \\ y(t) &= C_\sigma x(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the system output; ΔA_σ,ΔB_σ represent the parameter uncertainties of the parameter matrices A_σ,B_σ, respectively; $\omega_\sigma(t)$ denotes the bounded exogenous disturbance, and $\sigma(t) : \mathbb{R} \to \mathbb{N} \cong \{1, 2, \cdots, N\}$ is a piecewise constant function of the time t, also referred as the switching signal (rule). Then, we define a switching sequence:

$$Q := x_{t_0}; (i_0, t_0), (i_1, t_1), \cdots, (i_k, t_k), \cdots, \forall i_k \in \mathbb{N}, k \in \mathbb{Z}^+$$

where x_{t_0} is the state value at the initial time t_0. Namely, the i_k-th subsystem runs when $t \in [t_k\ ,\ t_{k+1})$.

For the switching signal $\sigma(t) = i$, $i \in \mathbb{N}$, the i-th subsystem matrices are denoted as:

$$A_\sigma \doteq A_i,\ B_\sigma \doteq B_i,\ G_\sigma \doteq G_i,\ C_\sigma \doteq C_i,$$
$$\Delta A_\sigma \doteq \Delta A_i,\ \Delta B_\sigma \doteq \Delta B_i,\ \omega_\sigma(t) \doteq \omega_i(t).$$

Accordingly, for the k-th switching, when $t_k \le t < t_{k+1}$, we can set $\sigma(t) = i$, i.e., $i_k = i \in \mathbb{N}$. As a result, the system (1) can be described as:

$$\begin{aligned}\dot{x}(t) &= (A_i+\Delta A_i)\,x(t)+(B_i+\Delta B_i)\,u(t)+G_i\omega_i(t)\\ y(t) &= C_i x(t).\end{aligned} \tag{2}$$

The following assumptions are satisfied and the lemmas described below are used in this work.

Assumption 1: (A_i, B_i) is stabilizable and B_i is required to be full column rank; $(A_i.C_i)$ is observable.

Assumption 2: The uncertainty parameters, ΔA_i and ΔB_i, satisfy the following relationships:

$$\Delta A_i = H_{a,i}F_{a,i}(t)E_{a,i},\ \Delta B_i = H_{b,i}F_{b,i}(t)E_{b,i}, \tag{3}$$

where $H_{a,i} \in \mathbb{R}^{n\times r_a}$, $H_{b,i} \in \mathbb{R}^{n\times r_b}$, $E_{a,i} \in \mathbb{R}^{r_a\times n}$ and $E_{b,i} \in \mathbb{R}^{r_b\times m}$ are all of the known-constant matrices, and the unknown time-varying matrices $F_{a,i}(t)$ and $F_{b,i}(t)$ satisfy:

$$F_{a,i}^T(t)F_{a,i}(t) \le I,\ F_{b,i}^T(t)F_{b,i}(t) \le I.$$

Assumption 3: The bounded disturbance $\omega_i(t)$ satisfies $\|\omega_i(t)\| < d_i$, in which d_i is a positive scalar parameter.

Lemma 1: Assuming that H and E are the matrices consisting of real constants with appropriate dimensions, $F(t)$ satisfies $F^T(t)F(t) \le I$. The following relation is true for any positive constant $\varepsilon > 0$ [28]:

$$HF(t)E + E^TF^T(t)H^T \le \varepsilon^{-1}HH^T + \varepsilon E^TE. \tag{4}$$

When the system state $x(t)$ cannot be measured directly, the following state observer is constructed

$$\dot{\hat{x}}(t) = A_i\hat{x}(t) + B_iu(t) + L_i\left(y(t) - C_i\hat{x}(t)\right), \tag{5}$$

where $\hat{x}(t)$ is the observation of the system state $x(t)$, $L_i \in \mathbb{R}^{n\times p}$ is the designed observer feedback gain, which can make the matrices

$$\bar{A}_i = A_i - L_iC_i, \forall i \in \mathbb{N} \tag{6}$$

to be *Hurwitz*.

Define the state observation error as $\tilde{x}(t) = x(t) - \hat{x}(t)$. By (2),(5) and (6), we get

$$\dot{\tilde{x}}(t) = \left(\bar{A}_i+\Delta A_i\right)\tilde{x}(t)+\Delta A_i\hat{x}(t)+\Delta B_iu(t)+G_i\omega_i(t). \tag{7}$$

The control task is to design an SMC control for guaranteeing the asymptotic stability of the switched system as described by (1),(2) with the unmatched uncertainties to realize the robust output feedback stabilization based on the state observer (5).

Remark 1: The uncertainties and disturbances in the SS (2) cover a wide range of structured uncertainties and disturbances, because they are essentially described as the time-varying $\Delta A_i(t)$, $\Delta B_i(t)$,$G_i\omega_i(t)$, and can be decomposed as (3). They include the matched which occurs at the control input channel, and the unmatched which cannot be treated in the common sliding mode designs.

III. ROBUST INTEGRAL SLIDING MODE DESIGN VIA THE STATE OBSERVER

Design a robust integral sliding mode (RISM) as

$$\hat{S}(t) = D\left[\hat{x}(t) - \hat{x}(t_0)\right] + \int_{t_0}^{t} K\hat{x}(t)\,dt \tag{8}$$

in which the parameter $D \in \mathbb{R}^{m\times n}$ satisfies

$$\begin{aligned}&\forall i \in \mathbb{N}, \text{rank}(DB_i) = m, \text{ and}\\ &\exists\, 0 \le \gamma_i < 1, \gamma_i \in \mathbb{R},\ \ D\Delta B_i \le \gamma_i DB_i,\end{aligned} \tag{9}$$

another parameter K is the common-state-feedback stabilization coefficient to be designed and essentially ensures the matrices

$$\hat{A}_i = A_i - B_i(DB_i)^{-1}(DA_i + K), \forall i \in \mathbb{N} \tag{10}$$

are *Hurwitz*.

Based on (2) and (8), we can write

$$\dot{\hat{S}}(t) = (DA_i+K)\hat{x}(t)+DB_iu(t)+DL_i\left(y(t)-C_i\hat{x}(t)\right).$$

According to the sliding mode control theory, when the system state reaches the sliding surface and remains there, $\hat{S}_i(t) = 0$, $\dot{\hat{S}}_i(t) = 0$. Thus, the equivalent control can be written as

$$u_{eq}(t) = -[DB_i]^{-1}\left[(DA_i+K)\hat{x}(t)+DL_i\left(y(t)-C_i\hat{x}(t)\right)\right]. \tag{11}$$

By substituting (11) into (2) and using (10), the sliding mode equation can be written as:

$$\begin{aligned}\dot{\hat{x}}(t) &= \hat{A}_i\hat{x}(t) + H_{y,i}\left(y(t) - C_i\hat{x}(t)\right),\\ H_{y,i} &= L_i - B_i(DB_i)^{-1}DL_i.\end{aligned} \tag{12}$$

The design criterion of the parameters D, K for the RISM (8) is presented as follows.

Theorem 1: Given positive scalars $\alpha_i > 0$ for the switched system (1), if $\forall i \in \mathbb{N}$ there exist the matrices $P_i > 0$, $Q > 0$ and any positive scalars $\varepsilon_{1,i}$, $\varepsilon_{2,i}$, $\varepsilon_{3,i}$, $\varepsilon_i > 0$ that satisfy

$$\begin{bmatrix} \Theta_{1,i} & 0 & P_iH_{y,i} & 0 & 0 & 0\\ * & \Theta_{2,i} & 0 & QH_{a,i} & QH_{b,i} & QG_i\\ * & * & -\varepsilon_{1,i}I & 0 & 0 & 0\\ * & * & * & -0.5\varepsilon_{2,i}I & 0 & 0\\ * & * & * & * & -0.5\varepsilon_{3,i}I & 0\\ * & * & * & * & * & -\varepsilon_iI \end{bmatrix} < 0, \tag{13}$$

where

$$\begin{aligned}\Theta_{1,i} &= P_i\hat{A}_i+\hat{A}_i^TP_i+\alpha_iP_i+\varepsilon_{2,i}E_{a,i}^TE_{a,i}+\varepsilon_{3,i}\bar{E}_{b,i}^T\bar{E}_{b,i},\\ \Theta_{2,i} &= Q\bar{A}_i+\bar{A}_i^TQ+\alpha_iQ+\varepsilon_{1,i}C_i^TC_i\\ &\quad+\varepsilon_{2,i}E_{a,i}^TE_{a,i}+\varepsilon_{3,i}\tilde{E}_{b,i}^T\tilde{E}_{b,i},\end{aligned}$$

$$\bar{E}_{b,i} = E_{b,i}(DB_i)^{-1}(DA_i + K),$$
$$\tilde{E}_{b,i} = E_{b,i}(DB_i)^{-1}DL_iC_i,$$

then, the system state under the sliding mode $\hat{S}(t) = 0$ will be stable by the control of the switching rule

$$\sigma(t_0) = \underset{i\in\mathbb{N}}{\operatorname{argmin}}\left\{\hat{x}^T(t_0)P_i\hat{x}(t_0)\right\} \tag{14}$$

$$\sigma(t_k) = \begin{cases} i, \quad \hat{x}(t_k) \in \Lambda_i \text{ and } \sigma(t_{k-1}) = i, \\ \underset{i\in\mathbb{N}}{\operatorname{argmin}}\left\{\hat{x}^T(t)\left(P_i - P_j\right)\hat{x}(t)\right\}, \\ \qquad \hat{x}(t_k) \in \Lambda_i \text{ and } \sigma(t_{k-1}) = j, j \neq i \end{cases} \tag{15}$$

$$\Lambda_i = \left\{\hat{x}(t)\middle|\hat{x}^T(t)(P_i - P_j)\hat{x}(t) \leq 0, \forall j \in \mathbb{N}, j \neq i\right\}. \tag{16}$$

Furthermore, the closed-loop system satisfies the following:

1) When $\omega_i(t) \equiv 0$, the system state will be exponentially stable.

2) When $\omega_i(t) \neq 0$, the system state trajectory will exponentially converge to

$$\lim_{t\to\infty}\|x(t)\| \leq \max_{i\in\mathbb{N}}\left\{\sqrt{\frac{\varepsilon_i d_i^2}{\alpha_i\lambda_{\min}(P_i)}} + \sqrt{\frac{\varepsilon_i d_i^2}{\alpha_i\lambda_{\min}(Q)}}\right\} \tag{17}$$

Proof: Select the Lyapunov function as

$$V_i(t) = \hat{x}^T(t)P_i\hat{x}(t) + \tilde{x}^T(t)Q\tilde{x}(t) \tag{18}$$

for the i-th subsystem and find its derivative with respect to the time along with (7) and (12), one gets

$$\begin{aligned}\dot{V}_i(t) =& \hat{x}^T(t)(P_i\hat{A}_i + \hat{A}_i^TP_i)\hat{x}(t) + 2\tilde{x}^T(t)Q[(\bar{A}_i + \Delta A_i)\tilde{x}(t) \\ &+ \Delta A_i\hat{x}(t)] + 2\tilde{x}^T(t)Q[\Delta B_iu(t) + G_i\omega_i(t)] \\ &+ 2\hat{x}^T(t)P_iH_{y,i}(y(t) - C_i\hat{x}(t)).\end{aligned}$$

Because the system reaches into the sliding mode $\hat{S}(t) = 0$, $\dot{\hat{S}}(t) = 0$, and $u(t) = u_{eq}(t)$. Substitute (11) into the above equation, one gets

$$\begin{aligned}\dot{V}_i(t) =& \hat{x}^T(t)(P_i\hat{A}_i + \hat{A}_i^TP_i)\hat{x}(t) + 2\tilde{x}^T(t)Q[(\bar{A}_i + \Delta A_i)\tilde{x}(t) \\ &+ \Delta A_i\hat{x}(t)] - 2\tilde{x}^T(t)Q[\Delta B_i(DB_i)^{-1}(DA_i + K)\hat{x}(t) \\ &+ \Delta B_i(DB_i)^{-1}DL_iC_i\tilde{x}(t)] + 2\tilde{x}^T(t)QG_i\omega_i(t) \\ &+ 2\hat{x}^T(t)P_iH_{y,i}C_i\tilde{x}(t).\end{aligned} \tag{19}$$

Based on Assumption 2 and Lemma 1, for any positive scalars $\varepsilon_{1,i}, \varepsilon_{2,i}, \varepsilon_{3,i}, \varepsilon_i$, the following inequalities can be obtained,

$$\begin{aligned}2\hat{x}^T(t)P_iH_{y,i}C_i\tilde{x}(t) \leq& \varepsilon_{1,i}^{-1}\hat{x}^T(t)P_iH_{y,i}H_{y,i}^TP_i\hat{x}(t) \\ &+ \varepsilon_{1,i}\tilde{x}^T(t)C_i^TC_i\tilde{x}(t), \\ 2\tilde{x}^T(t)Q\Delta A_i\tilde{x}(t) \leq& \varepsilon_{2,i}^{-1}\tilde{x}^T(t)H_{a,i}\left(QH_{a,i}\right)^T\tilde{x}(t) \\ &+ \varepsilon_{2,i}\tilde{x}^T(t)E_{a,i}^TE_{a,i}\tilde{x}(t), \\ 2\tilde{x}^T(t)Q\Delta A_i\hat{x}(t) \leq& \varepsilon_{2,i}^{-1}\tilde{x}^T(t)QH_{a,i}\left(QH_{a,i}\right)^T\tilde{x}(t) \\ &+ \varepsilon_{2,i}\hat{x}^T(t)E_{a,i}^TE_{a,i}\hat{x}(t), \\ 2\tilde{x}^T(t)Q\Delta B_i(DB_i)^{-1}&DL_iC_i\tilde{x}(t) \\ \leq& \varepsilon_{3,i}^{-1}\tilde{x}^T(t)QH_{b,i}\left(QH_{b,i}\right)^T\tilde{x}(t) \\ &+ \varepsilon_{3,i}\tilde{x}^T(t)\tilde{E}_{b,i}^T\tilde{E}_{b,i}\tilde{x}(t), \\ 2\tilde{x}^T(t)Q\Delta B_i(DB_i)^{-1}&(DA_i + K)\hat{x}(t) \\ \leq& \varepsilon_{3,i}^{-1}\tilde{x}^T(t)QH_{b,i}\left(QH_{b,i}\right)^T\tilde{x}(t) \\ &+ \varepsilon_{3,i}\hat{x}^T(t)\bar{E}_{b,i}^T\bar{E}_{b,i}\hat{x}(t), \\ 2\tilde{x}(t)^TQG_i\omega_i(t) \leq& \varepsilon_i^{-1}\tilde{x}^T(t)QG_i(QG_i)^T\tilde{x}(t) \\ &+ \varepsilon_i\omega_i^T(t)\omega_i(t).\end{aligned} \tag{20}$$

From (19) and (20), it can be deduced that

$$\dot{V}_i(t) \leq \zeta^T(t)W_i\zeta(t) + \varepsilon_i\omega_i^T(t)\omega_i(t), \tag{21}$$

where

$$\begin{aligned}\zeta(t) =& \left[\hat{x}^T(t)\ \ \tilde{x}^T(t)\right]^T, \quad W_i = \operatorname{diag}\left(\Sigma_{i,1}, \Sigma_{i,2}\right), \\ \Sigma_{i,1} =& P_i\hat{A}_i + \hat{A}_i^TP_i + \varepsilon_{1,i}^{-1}P_iH_{y,i}H_{y,i}^TP_i \\ &+ \varepsilon_{2,i}E_{a,i}^TE_{a,i} + \varepsilon_{3,i}\bar{E}_{b,i}^T\bar{E}_{b,i}, \\ \Sigma_{i,2} =& Q\bar{A}_i + \bar{A}_i^TQ + 2\varepsilon_{2,i}^{-1}QH_{a,i}H_{a,i}^TQ + 2\varepsilon_{3,i}^{-1}QH_{b,i}H_{b,i}^TQ \\ &+ \varepsilon_i^{-1}QG_iG_i^TQ + \varepsilon_{1,i}C_i^TC_i + \varepsilon_{2,i}E_{a,i}^TE_{a,i} \\ &+ \varepsilon_{3,i}\tilde{E}_{b,i}^T\tilde{E}_{b,i}.\end{aligned} \tag{22}$$

Then, by (21) and (22), the following inequality holds,

$$\begin{aligned}\dot{V}_i(t) &+ \alpha_iV_i(t) - \varepsilon_i\omega_i^T(t)\omega_i(t) \\ &\leq \zeta^T(t)\left[W_i + \alpha_i\operatorname{diag}(P_i, Q)\right]\zeta(t).\end{aligned}$$

By Schur Lemma, $W_i + \alpha_i\operatorname{diag}(P_i, Q) < 0$ is equivalent to the LMI (13). Consequently, the following inequality holds,

$$\dot{V}_i(t) + \alpha_iV_i(t) - \varepsilon_i\omega_i^T(t)\omega_i(t) \leq 0.$$

Two cases are to be discussed for the system stability.

(1) In case $\omega_i(t) \equiv 0$

The inequality $\dot{V}_i(t) + \alpha_iV_i(t) \leq 0$ holds. Therefore, the trajectories of $\hat{x}(t)$ and the observation error $\tilde{x}(t)$ of the subsystem are exponentially stable. The subsystem state of the switched system (2) is hence exponentially stable.

(2) In case $\omega_i(t) \neq 0$

By Assumption 3, the Lyapunov function $V_i(t)$ satisfies the inequality

$$V_i(t) = V_i(0)e^{-\alpha_it} + \frac{\varepsilon_id_i^2}{\alpha_i},$$

which implies that the trajectories of $\hat{x}(t)$ and the observation error $\tilde{x}(t)$ satisfy

$$\left\|\hat{x}(t)\right\|^2 \leq \lambda_{\min}^{-1}(P_i)V_i(0)e^{-\alpha_it} + \frac{\varepsilon_id_i^2}{\alpha_i\lambda_{\min}(P_i)},$$
$$\|\tilde{x}(t)\|^2 \leq \lambda_{\min}^{-1}(Q_i)V_i(0)e^{-\alpha_it} + \frac{\varepsilon_id_i^2}{\alpha_i\lambda_{\min}(Q)}.$$

Namely,

$$\lim_{t\to\infty}\left\|\hat{x}(t)\right\| \leq \sqrt{\frac{\varepsilon_id_i^2}{\alpha_i\lambda_{\min}(P_i)}},$$
$$\lim_{t\to\infty}\|\tilde{x}(t)\| \leq \sqrt{\frac{\varepsilon_id_i^2}{\alpha_i\lambda_{\min}(Q)}}.$$

Therefore, the subsystem state will exponentially converge to a small neibourgood of the origin.

From the switching rule (14)-(16), at the switching time point t_k, the Lyapunov function of the corresponding subsystem is non-increasing. The following is true,

$$V_i(t_k) \le V_j(t_k).$$

According to the multiple Lyapunov function (MLF) theory, the overall system state converges to the origin exponentially when $\omega_i(t) \equiv 0$, or satisfies (17) when $\omega_i(t) \neq 0$. ■

Remark 2: The RISM (8) is a common sliding mode surface for every subsystem of the SS (2). The condition (9) of the parameter D is to guarantee the uncertainty ΔB does not affect its conrtollability. The RISM is based on the state observer (5) and the conditions of Theorem 1 make the RISM is robust to the unmatched uncertainty.

Based on Theorem 1, the following corollaries can be achieved.

Corollary 1: For the switched system (1), if $\forall i \in \mathbb{N}$ there exist the matrices $P_i > 0, Q > 0$ and the positive scalars $\varepsilon_{1,i}, \varepsilon_{2,i}, \varepsilon_{3,i}, \varepsilon_i > 0$ that satisfy

$$\begin{bmatrix} \bar{\Theta}_{1,i} & 0 & P_iH_{y,i} & 0 & 0 & 0 \\ * & \bar{\Theta}_{2,i} & 0 & QH_{a,i} & QH_{b,i} & QG_i \\ * & * & -\varepsilon_{1,i}I & 0 & 0 & 0 \\ * & * & * & -0.5\varepsilon_{2,i}I & 0 & 0 \\ * & * & * & * & -0.5\varepsilon_{3,i}I & 0 \\ * & * & * & * & * & -\varepsilon_i I \end{bmatrix} < 0, \tag{23}$$

where

$$\begin{aligned} \bar{\Theta}_{1,i} &= P_i\hat{A}_i + \hat{A}_i^T P_i + I + \varepsilon_{2,i}E_{a,i}^T E_{a,i} + \varepsilon_{3,i}\bar{E}_{b,i}^T \bar{E}_{b,i}, \\ \bar{\Theta}_{2,i} &= Q\bar{A}_i + \bar{A}_i^T Q + I + \varepsilon_{1,i}C_i^T C_i \\ &\quad + \varepsilon_{2,i}E_{a,i}^T E_{a,i} + \varepsilon_{3,i}\tilde{E}_{b,i}^T \tilde{E}_{b,i}, \end{aligned}$$

then, the subsystem state under the sliding mode $\hat{S}(t) = 0$ will be stable under the switching rule (14)-(17). Furthermore, the closed-loop system satisfies the following:

1) When $\omega_i(t) \equiv 0$, the system state will be asymptotically stable, i.e., $\lim_{t\to\infty} \|x(t)\| = 0$.

2) When $\omega_i(t) \neq 0$, the following inequality

$$\int_{t_0}^{\infty} x^T(\tau)x(\tau)\,d\tau \le \gamma \int_{t_0}^{\infty} \max\left\{\omega_i^T(\tau)\,\omega_i(\tau)\right\} d\tau. \tag{24}$$

holds under the initial condition of zero, where $\gamma = \max\{\varepsilon_i\}$ is the L2 performance index.

Proof: Select the Lyapunov function as (18) and find its time derivative, one gets the relationship (21),(22) according to the proof of Theorem 1. Consequently, the following inequality holds,

$$\begin{aligned} &\dot{V}_i(t) + x^T(t)\,x(t) - \gamma\,\omega_i^T(t)\,\omega_i(t) \\ &\le \dot{V}_i(t) + \zeta^T(t)\,\zeta(t) - \gamma\,\omega_i^T(t)\,\omega_i(t) \\ &\le \zeta^T(t)\,(W_i+I)\,\zeta(t) + (\varepsilon_i-\gamma)\omega_i^T(t)\,\omega_i(t). \end{aligned} \tag{25}$$

By Schur Lemma, $W_i + I < 0$ is equivalent to the LMI (23). Obviously, if the condition (23) holds, one can get the following inequality,

$$\dot{V}_i(t) + x^T(t)\,x(t) - \gamma\omega_i^T(t)\,\omega_i(t) < 0.$$

And the inequality (24) hold. ■

Corollary 2: Given positive scalars $\alpha_i > 0$ for the switched system (1), if $\forall i \in \mathbb{N}$ there exist the matrices $P > 0, Q > 0$ and any positive scalars $\varepsilon_{1,i}, \varepsilon_{2,i}, \varepsilon_{3,i}, \varepsilon_i > 0$ that satisfy

$$\begin{bmatrix} \Theta'_{1,i} & 0 & PH_{y,i} & 0 & 0 & 0 \\ * & \Theta'_{2,i} & 0 & QH_{a,i} & QH_{b,i} & QG_i \\ * & * & -\varepsilon_{1,i}I & 0 & 0 & 0 \\ * & * & * & -0.5\varepsilon_{2,i}I & 0 & 0 \\ * & * & * & * & -0.5\varepsilon_{3,i}I & 0 \\ * & * & * & * & * & -\varepsilon_i I \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned} \Theta'_{1,i} &= P\hat{A}_i + \hat{A}_i^T P + \alpha_i P + \varepsilon_{2,i}E_{a,i}^T E_{a,i} + \varepsilon_{3,i}\bar{E}_{b,i}^T \bar{E}_{b,i}, \\ \Theta'_{2,i} &= Q\bar{A}_i + \bar{A}_i^T Q + \alpha_i Q + \varepsilon_{1,i}C_i^T C_i \\ &\quad + \varepsilon_{2,i}E_{a,i}^T E_{a,i} + \varepsilon_{3,i}\tilde{E}_{b,i}^T \tilde{E}_{b,i}, \end{aligned}$$

then, the system state under the sliding mode $\hat{S}(t) = 0$ will be stable under arbitrary switching rules. Furthermore, the closed-loop system satisfies the following:

1) When $\omega_i(t) \equiv 0$, the system state will be exponentially stable.

2) When $\omega_i(t) \neq 0$, the system state trajectory will exponentially converge to

$$\lim_{t\to\infty} \|x(t)\| \le \max_{i\in\mathbb{N}} \left\{ \sqrt{\frac{\varepsilon_i d_i^2}{\alpha_i \lambda_{\min}(P)}} + \sqrt{\frac{\varepsilon_i d_i^2}{\alpha_i \lambda_{\min}(Q)}} \right\} \tag{27}$$

Select

Proof: Select the common Lyapunov function (CLF) as

$$V(t) = \hat{x}^T(t)\,P\hat{x}(t) + \tilde{x}^T(t)\,Q\tilde{x}(t) \tag{28}$$

for the SS (2) and find its derivative with respect to the time along with (7) and (12), one easily gets

$$\dot{V}(t) + \alpha_i V(t) - \varepsilon_i\omega_i^T(t)\,\omega_i(t) \le 0$$

according to the proof of Theorem 1. Therefore, the similar conclusions will be achieved. ■

IV. SMC CONTROLLER

Design the SMC controller as

$$\begin{aligned} u(t) &= u_0(t) + u_1(t), \\ u_0(t) &= -(DB_i)^{-1}\left[(DA_i+K)\,\hat{x}(t) + DL_i\left(y(t) - C_i\hat{x}(t)\right)\right] \\ u_1(t) &= -(DB_i)^{-1}\left(\eta_i\hat{S}(t) + \rho_i\mathrm{sgn}\hat{S}(t)\right), \end{aligned} \tag{29}$$

where the parameters are designed as $\eta_i > 0$, $\rho_i > 0$.

Remark 3: The SMC controller (29) is comprised of two parts: the output feedback equivalent control part u_0 and the discontinuous part u_1. The robust control term u_1 will lead to the higher gain and higher amplitude of chattering with the increasing of the uncertainty and disturbance. In practice, to reduce the higher amplitude of chattering, a simple way is the boundary layer method, namely, to replace the sign functions sgn(·) by the saturation function

$$\mathrm{sat}(\hat{S}(t)) = \begin{cases} 1, & \hat{S}(t) > \epsilon, \\ \epsilon^{-1}\hat{S}(t), & -\epsilon \leq \hat{S}(t) \leq \epsilon, \\ -1, & \hat{S}(t) < -\epsilon, \end{cases}$$

where $\epsilon > 0$ is the boundary layer parameter.

Remark 4: Another way to reduce the high gain of the switching signal is to construct a disturbance observer to measure the large disturbance and use the estimated disturbance to compensate. If the estimation is accurate, then the robust control term can be reduced so that the high gain chatting is smaller. The typical research hot topics include the robust control based on the disturbance observer (DO), the extended state observer (ESO), etc [29], [30].

Theorem 2: For the switched system (2), the state observer (5), and the RISM (8), the observation state trajectory will keep on the sliding mode surface $\hat{S}(t) = 0$ from the initial time instant and retain there keeping the movement on the sliding mode under the SMC control (29).

Proof: Select the Lyapunov function as

$$V(t) = 0.5\hat{S}^T(t)\hat{S}(t).$$

According to (8), the time derivative of $V(t)$ is

$$\begin{aligned}\dot{V}(t) =& \hat{S}^T(t)\left[(DA_i + K)\hat{x}(t) + DB_i u(t)\right. \\ &\left.+ DL_i\left(y(t) - C_i\hat{x}(t)\right)\right].\end{aligned}$$

Substitute the SMC controller (29) into the above equation, one gets

$$\dot{V}(t) = -\hat{S}^T(t)\,\eta_i\hat{S}(t) - \hat{S}^T(t)\,\rho_i\mathrm{sgn}\hat{S}(t). \tag{30}$$

It means that the observer state $\hat{x}(t)$ will converge to the sliding mode surface $\hat{S}(t) = 0$ within the limited time. From the RISM (8), $\hat{S}(t_0) = 0$ is easily satisfied only if the initial observation state $\hat{x}(t_0)$ is known. Therefore, the observation state retains there keeping the movement on the sliding mode surface. ■

V. ILLUSTRATIVE EXAMPLE

Consider a one-link robotic manipulator whose joint angle can only be acquired directly. Referring to [31], its mathematical description is as follows,

$$M\ddot{y} + R\dot{y} + T\sin y = u(t) + \phi(t), \tag{31}$$

where y is the joint angle, $\dot{y}$ is the angular velocity, $x = [y, \dot{y}]^T$ is the state vector, M is the inertia coefficient of the manipulator, R is the damp coefficient, T represents the gravitational coefficient. The control torque is $u(t)$ and $\phi(t)$ indicates the disturbance. It is considered that the operation point is $y = 0$; and only the angle y can be measured. In this way, by applying the approximation $\sin y \approx y$, the mathematical model (31) is transformed into a linear form

$$\dot{x} = (A + \Delta A)x + Bu(t) + Gw(t),\ \ y(t) = Cx, \tag{32}$$

where $x = [y, \dot{y}]^T$ is the transformed state vector, $w(t) = \phi(t)$ denotes the disturbance, Δx_1 represents the small angle deviation. The parameter matrices are as follows,

$$A = \begin{bmatrix} 0 & 1 \\ -M^{-1}T & -R \end{bmatrix}, \qquad \Delta A = \begin{bmatrix} 0 & 1 \\ M^{-1}\Delta T & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 & M^{-1} \end{bmatrix}^T, \quad G = \begin{bmatrix} 0 & 1 \end{bmatrix}^T, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}^T.$$

When the manipulator grasps different parts in industrial operations, the physical parameters M, G will change along with its operation conditions. Therefore, two cases with different loads lead to the switched system with two sets of parameters of the system (32),

$$A_1 = \begin{bmatrix} 0 & 1 \\ -M_1^{-1}T_1 & -R \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ -M_2^{-1}T_2 & -R \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0 \\ -M_1^{-1} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -M_2^{-1} \end{bmatrix},$$
$$\Delta A_1 = \begin{bmatrix} 0 & 0 \\ \frac{\Delta T_1 \cos\Delta z_1}{M_1} & 0 \end{bmatrix}, \Delta A_2 = \begin{bmatrix} 0 & 0 \\ \frac{\Delta T_2 \cos\Delta z_1}{M_2} & 0 \end{bmatrix},$$
$$\Delta B_1 = \begin{bmatrix} 0 & \frac{\Delta M}{M_1 M_2} \end{bmatrix}^T, \Delta B_2 = \begin{bmatrix} 0 & \frac{-\Delta M}{M_1 M_2} \end{bmatrix}^T.$$

Refering to [31] and [32], the physical parameters are given as $M_1 = 0.1852$kgm, $M_2 = 0.2216$kgm, $T_1 = 2.6$kgm/s^2, $T_2 = 2.8$kgm/s^2, $R = 0.019$kgm/s. It can be proved that $\forall i \in \mathbb{N}$, (A_i, B_i) can be stabilizable and (A_i, C_i) is observable. Considering the real situation in practice, we give the bigger uncertainties. According to Assumption 2, the uncertainties are decomposed as the following matrices.

$$H_{a,1} = \begin{bmatrix} 0 \\ 14 \end{bmatrix}, H_{a,2} = \begin{bmatrix} 0 \\ 9.25 \end{bmatrix}, F_{a,1} = F_{a,2} = \cos\Delta x_1,$$
$$E_{a,1} = E_{a,2} = \begin{bmatrix} 0.2 & 0 \end{bmatrix},\ \ H_{b,1} = \begin{bmatrix} 0 & 6 \end{bmatrix}^T,\ \ H_{b,2} = \begin{bmatrix} 0 & -2 \end{bmatrix}^T,$$
$$F_{b,1} = F_{b,2} = 1, E_{b,1} = E_{b,2} = 0.1.$$

According to the parameter design conditions (9), the parameter $D = [1\ 5]$ is selected. It can be verified that $\forall i \in \mathbb{N}$, DB_i is of full rank. Moreover, the parameter design conditions (10) are verified. To select the matrices $L_1 = [28, 145.4]^T$, $L_2 = [28, 146.8]^T$, and $K = [450, 89]$, then by (6) and (10) we have

$$\bar{A}_1 = \bar{A}_2 = \begin{bmatrix} -28 & 1 \\ -159.5 & -0.02 \end{bmatrix}, \hat{A}_1 = \hat{A}_2 = \begin{bmatrix} 0 & 1 \\ -90 & -18 \end{bmatrix},$$

which eigenvalues are all negative.

After the above-described calculations, it can be verified whether D and K can make a RISM based on Theorem 1. Given the positive scalars $\alpha_1 = \alpha_2 = 0.5$, we can directly solve the LMI (13) using the LMI toolbox in Matlab. The matrices P_1, P_2, Q are found to be:

$$P_1 = \begin{bmatrix} 452.8 & 70.18 \\ 70.18 & 16.44 \end{bmatrix}, P_2 = \begin{bmatrix} 448.6 & 71.28 \\ 71.28 & 16.94 \end{bmatrix}, Q = \begin{bmatrix} 1873 & -176.4 \\ -176.4 & 38 \end{bmatrix}.$$

Thus, the RISM (8) is obtained.

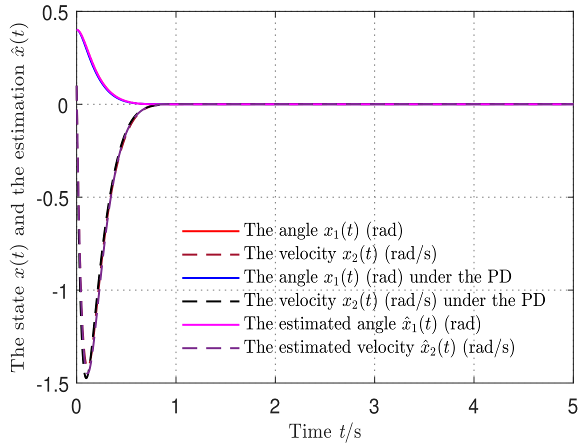

FIGURE 1. The angle and angular velocity stabilization curves.

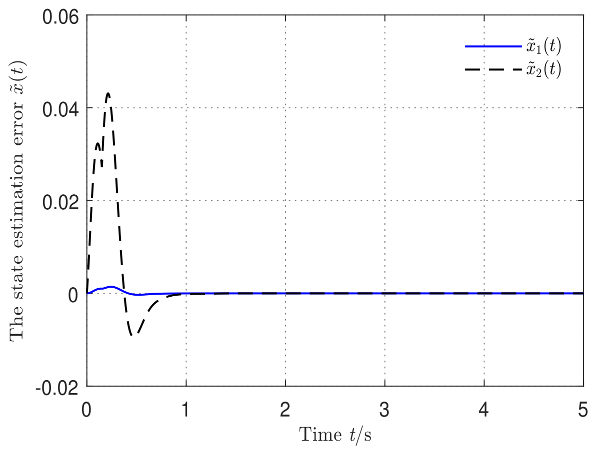

FIGURE 2. The state observation error.

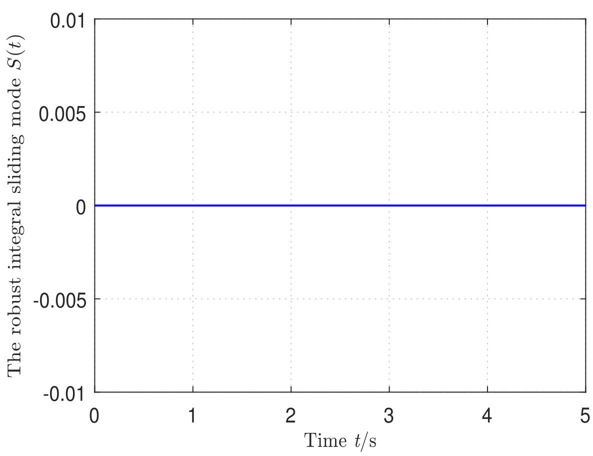

FIGURE 3. The RISM signal.

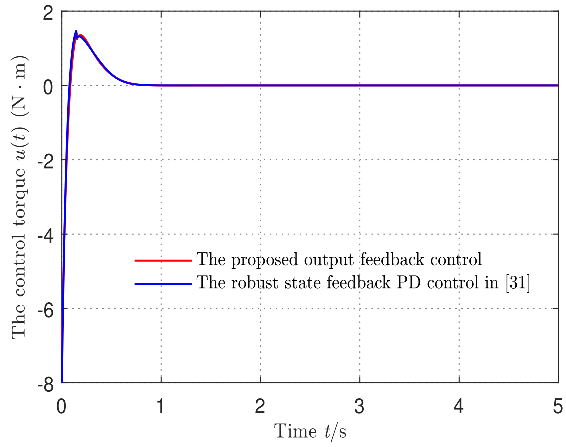

FIGURE 4. The control torque curve.

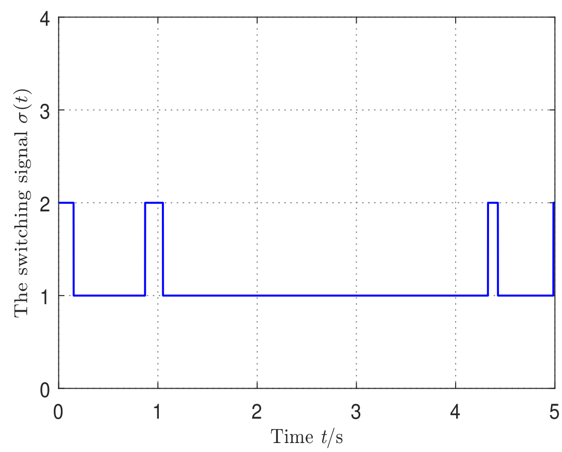

FIGURE 5. The switching signal.

First, the stabilization of the one-link manipulator was validated. We set the system state initial value as $x(t_0) = [0.4,\ 0.1]^T$. The sliding mode controller under the designed switching rules (14)-(16) can be used by Theorem 1, for which $\rho_i = 4$ and $\eta_i = 20$ are set. By substituting D, K, A_i, B_i and C_i into (29), the practical output feedback switched controller can be obtained.

To compare with other robust control designs, the robust PD control method in [31] was designed as $u_i = -K_{pi}x_1 - K_{di}x_2$; and $K_{p1} = 16$, $K_{p2} = 19.36$, $K_{d1} = 3.69$, $K_{d2} = 4.413$. Fig. 1-Fig. 5 display the simulation results of the switched system from which we can observe that under the existence of uncertain parameters, the system was stabilized. Fig. 1 shows that the system states under the presented control and the robust PD control had nearly the same performance. However, the robust PD control required the direct state feedback information but the presented control is based on the observer. Fig. 2 shows that the observation error of the system state is kept within the neighborhood of the origin and converged. Fig. 3 shows that the system state remains on the sliding surface with the designed switching rule from the initial time onward. Fig. 4 shows that the control torques of the two control methods were nearly the same, but the maximum torque of the presented control signal is smaller than the robust PD control. Fig. 5 shows the switching signal of the two sub-controllers.

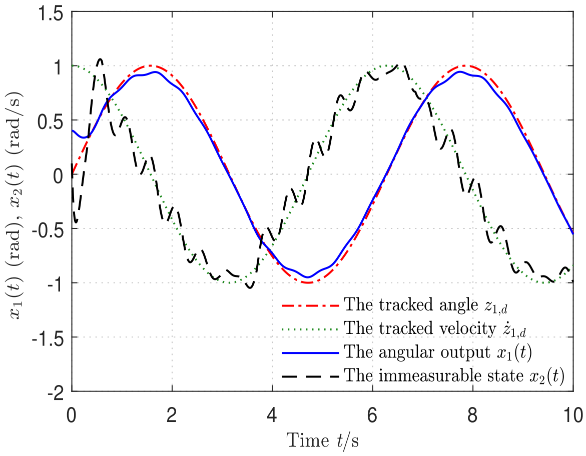

FIGURE 6. The state tracking curves.

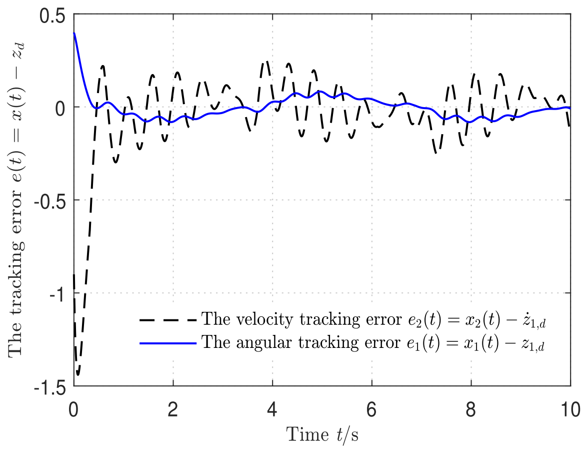

FIGURE 7. The state tracking error curves.

Then, the output tracking was considered for the one-linke manipulator. The tracked trajectory is $z_{1,d} = \sin t$. For the output tracking problem, the tracked state $z_d = [z_{1,d}, \dot{z}_{1,d}]^T = [\sin t, \cos t]^T$ was introduced into the RISM

$$\hat{S}(t) = D\left[\hat{e}(t) - \hat{e}(t_0)\right] + \int_{t_0}^{t} K\hat{e}(t)dt,$$

where $\hat{e}(t) = \hat{x}(t) - z_d$. And the corresponding output feedback equivalent control is

$$u_0 = -(DB_i)^{-1}\left[(DA_i + K)\,\hat{e}\,(t) + DA_i z_d - D\dot{z}_d + DL_i\big(y\,(t) - C_i\hat{z}\,(t)\big)\right].$$

The parameters of the tracking controller are set as the same as before.

Fig. 6 shows that the manipulator can track the reference signal although there are the uncertainties. The state tracking errors are limited in a bounded range, which are shown in Fig. 7. The RISM was reached from the initial time instant, shown in Fig. 8. The control torque signal is shown in Fig. 9 and the corresponding switching signal is shown in Fig. 10 by the switching rule in Theorem 1.

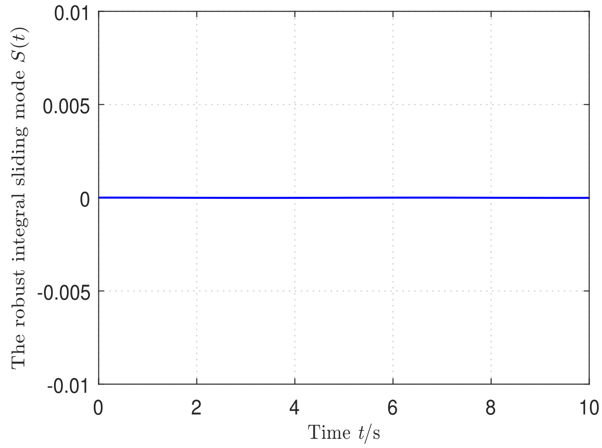

FIGURE 8. The RISM signal.

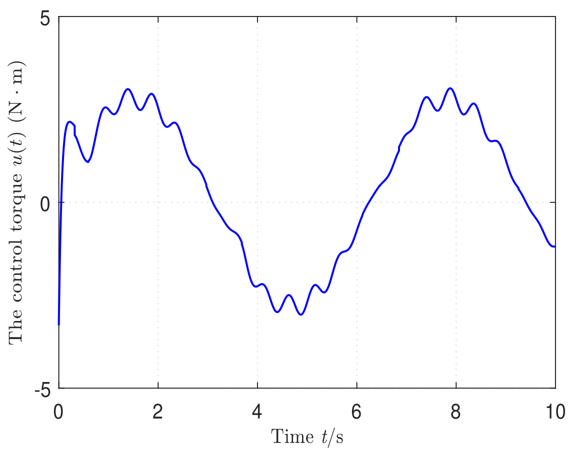

FIGURE 9. The control torque signal.

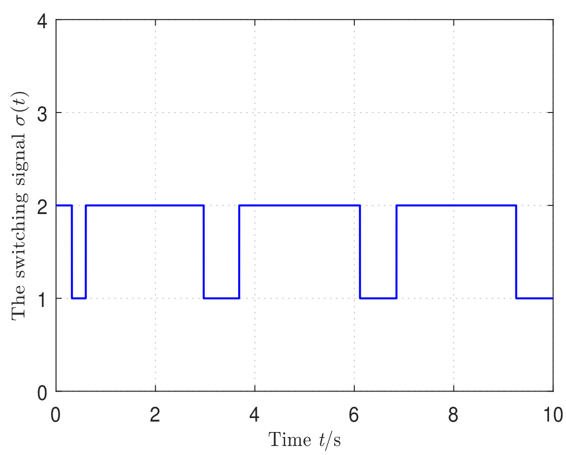

FIGURE 10. The switching signal.

Fig. 6-Fig. 10 clearly demonstrate that the output feedback control based on the observer (5) and the RISM (8) can track the reference signal with a bounded error when there are uncertainties.

Remark 5: The output tracking problem is resolved by the presented output feedback stabilization control design. This is only to certificate that the feasibility and effectiveness of the presented output feedback stabilization control design based on the observer for the switched system with unmatched uncertainty and disturbance. There must be the better tracking control designs to resolve it.

Fig. 1-Fig. 5 show the validation that the stabilization performance is better than the reference signal tracking. The essential reason is that its reference signal varies along the time t, which leads to the observation error increasing, especially near the peak or valley.

VI. CONCLUSION

The output feedback robust control design is presented for switched systems (SSs) without the state feedback information in the case that the unmatched uncertainty and disturbance exist. A robust integral sliding mode (RISM) designed on the estimated state space of the observer makes a robust stable sliding motion, in which the system has the ability to suppress the unmatched uncertainties. Linear matrix inequality (LMI) conditions for the parameter design and the stabilization criteria are achieved. The stability of the switched system in accordance with the proposed RISM is ensured by two standard approaches. The first is the common Lyapunov function method, and the second is the stabilization via the switching rule. The system on the RISM surface is ensured to be robust exponentially stable. The simulation results of the application to a one-link manipulator were proposed to illustrate the output feedback control design, which validated the effectiveness and the feasibility.

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REFERENCES

[1] L. Wu, P. Shi, and X. Su, *Sliding Mode Control of Uncertain Parameter-Switching Hybrid Systems*. Hoboken, NJ, USA: Wiley, 2014.

[2] D. Liberzon, *Switching in Systems and Control*. Boston, MA, USA: Birkhauser, 2003.

[3] M. Margaliot, "Stability analysis of switched systems using variational principles: An introduction," *Automatica*, vol. 42, no. 12, pp. 2059–2077, Dec. 2006.

[4] J. Wang, S. Huo, J. Xia, J. H. Park, X. Huang, and H. Shen, "Generalised dissipative asynchronous output feedback control for Markov jump repeated scalar non-linear systems with time-varying delay," *IET Control Theory Appl.*, vol. 13, no. 13, pp. 2114–2121, Sep. 2019.

[5] J. Li, Z. Gao, and Z. Wang, "Robust output strictly passive H-infinity control of uncertain switched singular systems," *IEEE Access*, vol. 8, pp. 224260–224268, 2020.

[6] M. J. Mirzaei, E. Aslmostafa, M. Asadollahi, and M. A. Badamchizadeh, "Robust adaptive finite-time stabilization control for a class of nonlinear switched systems based on finite-time disturbance observer," *J. Franklin Inst.*, vol. 358, no. 7, pp. 3332–3352, May 2021.

[7] Y. Zhang, H. Sun, and C. Sun, "Fixed-time sliding mode output tracking control for second-order switched systems with power integrators," *Comput. Electr. Eng.*, vol. 96, Dec. 2021, Art. no. 107503.

[8] R. Yavari, S. Shamaghdari, and A. Sadeghzadeh, "Robust H_2 output-feedback bumpless transfer control of polytopic uncertain LPV systems," *Eur. J. Control*, vol. 63, pp. 277–289, Jan. 2022.

[9] Y. Qi and J. Hu, "Observer-based bumpless switching control for switched linear systems with sensor faults," *Trans. Inst. Meas. Control*, vol. 40, no. 5, pp. 1490–1498, Mar. 2018.

[10] H. Zhao, Y. Niu, and J. Song, "Finite-time output feedback control of uncertain switched systems via sliding mode design," *Int. J. Syst. Sci.*, vol. 49, no. 5, pp. 984–996, Apr. 2018.

[11] S. Yin, H. Gao, J. Qiu, and O. Kaynak, "Descriptor reduced-order sliding mode observers design for switched systems with sensor and actuator faults," *Automatica*, vol. 76, pp. 282–292, Feb. 2017.

[12] L. Wu and J. Lam, "Sliding mode control of switched hybrid systems with time-varying delay," *Int. J. Adapt. Control Signal Process.*, vol. 22, no. 10, pp. 909–931, Dec. 2008.

[13] L. Wu, D. W. C. Ho, and C. W. Li, "Sliding mode control of switched hybrid systems with stochastic perturbation," *Syst. Control Lett.*, vol. 60, no. 8, pp. 531–539, Aug. 2011.

[14] R. Gao, D. Zhai, and X. Xie, "On the design of output information-based sliding mode controllers for switched descriptor systems: Linear sliding variable approach," *Appl. Math. Comput.*, vol. 364, Jan. 2020, Art. no. 124680.

[15] Y. Kao, X. Liu, M. Song, L. Zhao, and Q. Zhang, "Non-fragile-observer-based integral sliding mode control for a class of uncertain switched hyperbolic systems," *IEEE Trans. Autom. Control*, vol. 68, no. 8, pp. 5059–5066, Aug. 2023.

[16] W. Qi, G. Zong, and H. R. Karimi, "Finite-time observer-based sliding mode control for quantized semi-Markov switching systems with application," *IEEE Trans. Ind. Informat.*, vol. 16, no. 2, pp. 1259–1271, Feb. 2020.

[17] P. Zhang, Y. Kao, J. Hu, B. Niu, H. Xia, and C. Wang, "Finite-time observer-based sliding-mode control for Markovian jump systems with switching chain: Average dwell-time method," *IEEE Trans. Cybern.*, vol. 53, no. 1, pp. 248–261, Jan. 2023.

[18] N. Zhang, W. Qi, G. Pang, J. Cheng, and K. Shi, "Observer-based sliding mode control for fuzzy stochastic switching systems with deception attacks," *Appl. Math. Comput.*, vol. 427, Aug. 2022, Art. no. 127153.

[19] R. Sakthivel, R. Sakthivel, V. Nithya, P. Selvaraj, and O. M. Kwon, "Fuzzy sliding mode control design of Markovian jump systems with time-varying delay," *J. Franklin Inst.*, vol. 355, no. 14, pp. 6353–6370, Sep. 2018.

[20] X. Meng, B. Jiang, H. R. Karimi, and C. Gao, "An event-triggered mechanism to observer-based sliding mode control of fractional-order uncertain switched systems," *ISA Trans.*, vol. 135, pp. 115–129, Apr. 2023.

[21] V. Utkin and J. Shi, "Integral sliding mode in systems operating under uncertainty conditions," in *Proc. 35th IEEE Conf. Decis. Control*, Kobe, Japan, Dec. 1996, pp. 4591–4596.

[22] W.-J. Cao and J.-X. Xu, "Nonlinear integral-type sliding surface for both matched and unmatched uncertain systems," *IEEE Trans. Autom. Control*, vol. 49, no. 8, pp. 1355–1360, Aug. 2004.

[23] M. Kchaou and S. A. Ahmadi, "Robust H_∞ control for nonlinear uncertain switched descriptor systems with time delay and nonlinear input: A sliding mode approach," *Complexity*, vol. 2017, pp. 1–14, Jun. 2017.

[24] H. Chen, C.-C. Lim, and P. Shi, "Robust $\mathcal{H}_\infty$-based control for uncertain stochastic fuzzy switched time-delay systems via integral sliding mode strategy," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 2, pp. 382–396, Feb. 2022.

[25] W. Qi, X. Gao, C. K. Ahn, J. Cao, and J. Cheng, "Fuzzy integral sliding-mode control for nonlinear semi-Markovian switching systems with application," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 52, no. 3, pp. 1674–1683, Mar. 2022.

[26] X. Zhang, "Robust integral sliding mode control for uncertain switched systems under arbitrary switching rules," *Nonlinear Anal., Hybrid Syst.*, vol. 37, Aug. 2020, Art. no. 100900.

[27] C. Wang, R. Li, X. Su, and P. Shi, "Output feedback sliding mode control of Markovian jump systems and its application to switched boost converter," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 68, no. 12, pp. 5134–5144, Dec. 2021.

[28] T. Wang, L. Xie, and C. E. D. Souza, "Robust control of a class of uncertain nonlinear systems," *Syst. Control Lett.*, vol. 19, no. 2, pp. 139–149, 1992.

[29] B. Kaviarasan, O.-M. Kwon, M. J. Park, S. Lee, and R. Sakthivel, "Combined H_∞ and anti-disturbance control for semi-Markovian jump systems via a nonlinear disturbance observer," *Int. J. Robust Nonlinear Control*, vol. 33, no. 13, pp. 7968–7985, 2023.
[30] M. Vijayakumar, R. Sakthivel, D. Almakhles, and S. M. Anthoni, "Observer based tracking control for fuzzy control systems with time delay and external disturbances," *Int. J. Control, Autom. Syst.*, vol. 21, no. 8, pp. 2760–2769, 2023.
[31] A. Perrusquía, "Robust state/output feedback linearization of direct drive robot manipulators: A controllability and observability analysis," *Eur. J. Control*, vol. 64, Mar. 2022, Art. no. 100612.
[32] Y. Shi, B. Qiu, D. Chen, J. Li, and Y. Zhang, "Proposing and validation of a new four-point finite-difference formula with manipulator application," *IEEE Trans. Ind. Informat.*, vol. 14, no. 4, pp. 1323–1333, Apr. 2018.

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