IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 4 October 2023, accepted 19 October 2023, date of publication 25 October 2023, date of current version 31 October 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3327520

RESEARCH ARTICLE

Quaternion-Based Attitude Estimation Kalman Filter Using Global Optimization

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This work was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education under Grant RS-2023-00240093.

ABSTRACT This paper presents a modified Kalman filter for estimating quaternion using inertial and magnetic sensors. When the initial estimation error is large, the convergence rate of the multiplicative extended Kalman filter tends to be slow due to the assumption of small estimation errors. In this paper, a new measurement equation is proposed, in which a quaternion is directly estimated instead of estimating multiplicative estimation errors. Through simulation and experiment data, we demonstrate that the proposed algorithm is robust to large initial estimation errors.

INDEX TERMS Attitude estimation, inertial and magnetic sensors, quaternion estimation, Kalman filter.

I. INTRODUCTION

As inertial and magnetic sensor technologies advance, primarily due to MEMS technology [1], they become more affordable and compact. These sensors find extensive use in various applications, including the estimation of vibration, attitude, and position. In this paper, attitude estimation using inertial and magnetic sensors is considered. Attitude estimation plays a crucial role in applications such as drone control [2], manipulator joint angle estimation [3], and human joint angle estimation [4].

Attitude is usually represented by Euler angles, rotation matrix, and quaternion. Among these representations, quaternion is the most widely used due to its ease of computation and the straightforward maintenance of orthonormal properties through simple quaternion normalization. There are many different attitude estimation methods, which can be classified into complementary filter, nonlinear observer, and Kalman filter.

In complementary filters [5], [6], [7], high pass filtered accelerometer and magnetic sensor outputs are fused with low pass filtered gyroscope integrated output. This approach is widely adopted in practice because of its straightforward structure and relatively simple tuning. Additionally, it offers the primary advantage of light computational load compared to other filtering methods.

Nonlinear observers [8], [9] guarantee global estimation error convergence, making them particularly valuable when dealing with large initial errors or significant sensor errors. However, for small estimation errors, the use of nonlinear observers may not provide substantial benefits.

Kalman filters are widely used in attitude estimation because they not only provide attitude estimates but also offer attitude estimation error covariances. Furthermore, they naturally accommodate sensor bias estimation. In the standard Kalman filters [10], attitude is represented by using quaternion, and quaternion errors are estimated in the extended Kalman filter employing first-order approximation.

Various variations of the standard Kalman filter exist, which use higher-order approximation or non-Gaussian noise assumptions, including the unscented Kalman filter [11], the geometric extended Kalman filter [12], the cubature filter [13] and maximum correntropy filter [14]. There also exists a multiple-model adaptive filter [15] for systems with model uncertainty.

Among these methods, probably most widely used algorithms are Kalman filters based on multiplicative error representation [16]. The current estimate of quaternion is used as "global" attitude representation and a three-component vector is used as "local" representation of attitude errors (see

The associate editor coordinating the review of this manuscript and approving it for publication was Angel F. García-Fernández.

Chapter 6.1 in [17]). The main advantage of multiplicative error representation is its covariance is a well-conditioned 3×3 matrix, which has a clear physical interpretation.

The primary drawbacks of multiplicative error Kalman filters are their high computational load (compared with complementary filters) and slow convergence in the presence of large initial estimation errors [18]. Efforts have been made to partially reduce the computational load [19], but this paper primarily addresses the issue of slow convergence. In [20], a nonlinear observer is combined with a Kalman filter to address this problem.

The main reason of slow convergence is that attitude is computed near the current estimation using a three-component error vector. In the process, the measurement update is derived based on the assumption that the error vector is small. When there is large error in current estimation, this small error vector assumption is violated and measurement update could not provide optimal estimation. In this paper, we propose a new estimation algorithm modifying the measurement update algorithm. In the new measurement update equation, quaternion is directly estimated by solving global optimization problem instead of estimating three-dimensional error vector. The other parts of the proposed filter are the same with those of the standard Kalman filter. This optimization approach draws inspiration from an optimization problem used in quaternion averaging [21].

II. ATTITUDE ESTIMATION ALGORITHM

In this paper, two coordinate systems are used: the world coordinate system and the body coordinate system. In the world coordinate system, the x and z axes align with the north and (upward) local gravitational direction, respectively. Meanwhile, the x, y, and z axes of the body coordinate system correspond to the x, y, and z axes of the inertial sensors.

Let $q \in R^4$ be a quaternion that describes the rotation transformation from the world coordinate system to the body coordinate system, and let $C(q) \in SO(3)$ be the corresponding rotation matrix.

Let $y_a \in R^3$, $y_m \in R^3$, and $y_g \in R^3$ be the accelerometer, magnetic sensor, and gyroscope outputs, respectively:

$$\begin{aligned} y_a &= C(q)\tilde{g} + b_a + v_a \\ y_m &= C(q)\tilde{m} + v_m \\ y_g &= \omega + b_g + v_g \end{aligned} \tag{1}$$

where $v_a \in R^3$, $v_m \in R^3$ and $v_g \in R^3$ are the sensor noises. These noises are assumed to be uncorrelated white Gaussian, and their covariances are given by

$$E\{v_a v_a'\} = r_a I_3, E\{v_m v_m'\} = r_m I_3, E\{v_g v_g'\} = r_g I_3. \tag{2}$$

Let $\tilde{g} \in R^3$ and $\tilde{m} \in R^3$ be the local gravity vector and the earth magnetic field vector, respectively:

$$\tilde{g} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}, \tilde{m} = \begin{bmatrix} \cos(\mu) \\ 0 \\ \sin(\mu) \end{bmatrix} \tag{3}$$

where g is the gravitational acceleration, and μ is the dip angle [22]. Additionally, let $b_a \in R^3$ represent the accelerometer biases, and $b_g \in R^3$ represent the gyroscope biases, satisfying

$$\dot{b}_a = w_{b_a}, \quad \dot{b}_g = w_{b_g} \tag{4}$$

where w_{b_a} and w_{b_g} are white Gaussian noises:

$$Q_{b_a} = E\{w_{b_a} w_{b_a}'\}, \quad Q_{b_g} = E\{w_{b_g} w_{b_g}'\}. \tag{5}$$

The goal of this paper is to find attitude (i.e., the quaternion q) from sensor outputs. A standard Kalman filter based on multiplicative quaternion error [16] is first introduced. The proposed algorithm is derived by modifying the measurement update part.

Let $\hat{q} \in R^4$ be the estimated value of q and its multiplicative error $q_e \in R^4$ is defined as

$$q = \hat{q} \otimes q_e \tag{6}$$

where $\otimes$ represents the quaternion multiplication. There are two different definitions of quaternion multiplication, as defined in [23] and [17], where $a \otimes b$ in [23] corresponds to $b \otimes a$ in [17]. In this paper, we adopt the definition from [23]. With this definition, the rotation relationship of (6) represents

$$C(q) = C(q_e)C(\hat{q}). \tag{7}$$

In a standard Kalman filter, the error q_e is assumed to be small and approximated by

$$q_e \approx \begin{bmatrix} 1 \\ \bar{q}_e \end{bmatrix} \in \begin{bmatrix} R \\ R^3 \end{bmatrix}. \tag{8}$$

The small error assumption in (6) enables the representation of quaternion error using a three-dimensional vector, thereby avoiding the singularity of the corresponding estimation error covariance [16].

Let $\hat{b}_a \in R^3$ and $\hat{b}_g \in R^3$ be the estimated value of b_a and b_g, respectively. The bias estimation errors $b_{a,e} \in R^3$ and $b_{g,e} \in R^3$ are defined by

$$\begin{aligned} b_{a,e} &= b_a - \hat{b}_a \\ b_{g,e} &= b_g - \hat{b}_g. \end{aligned} \tag{9}$$

Instead of directly estimating b_a, b_g, and q, the Kalman filter commonly estimates $\bar{b}_{a,e}$, $\bar{b}_{g,e}$, and $\bar{q}$ in attitude estimation [10]. The state of a Kalman filter is defined by

$$x = \begin{bmatrix} \bar{q}_e \\ b_a \\ b_g \end{bmatrix} \in R^9. \tag{10}$$

Let x_k be a discrete-time signal of x, sampled with a sampling period T. The dynamic equation of x_k is given by

$$x_{k+1} = \exp(A_k T) x_k + w_k \tag{11}$$

where $A_k \in R^{9\times 9}$ and $w_k \in R^3$ are given by

$$A_k = \begin{bmatrix} -\left[(y_{g,k-1} - \hat{b}_g)\times\right] & -0.5\, I_3 & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} \end{bmatrix}$$

$$w_k = \int_0^T \exp(A_k r) \begin{bmatrix} 0_{3\times 1} \\ w_{b_a}(r) \\ w_{b_g}(r) \end{bmatrix} dr.$$

Let $Q = E\{w_k w_k'\} \in R^{9\times 9}$ be the covariance matrix of w_k.

The measurement equation is given by

$$z_k = H x_k + \tilde{v}_k \tag{12}$$

where

$$z_k = \begin{bmatrix} y_{a,k} - C(\hat{q}_k^-)\tilde{g} - \hat{b}_{a,k} \\ y_{m,k} - C(\hat{q}_k^-)\tilde{m} \end{bmatrix} \in R^{6\times 1}$$

$$H_k = \begin{bmatrix} 2[C(\hat{q}_k)\tilde{g}\times] & I_3 & 0_{3\times 3} \\ 2[C(\hat{q}_k)\tilde{m}\times] & 0_{3\times 3} & 0_{3\times 3} \end{bmatrix} \in R^{6\times 9}$$

$$\tilde{v}_k = \begin{bmatrix} v_{a,k} \\ v_{m,k} \end{bmatrix}.$$

The covariance of $\tilde{v}_k$ is given by

$$R_v = E\{\tilde{v}_k \tilde{v}_k'\} = \begin{bmatrix} r_a I_3 & 0_{3\times 3} \\ 0_{3\times 3} & r_m I_3 \end{bmatrix}.$$

Now, the standard Kalman filter algorithm is provided in Algorithm 1. Let $\hat{x}_k \in R^9$ and $P_k \in R^{9\times 9}$ be the estimated value of x_k and the estimation error covariance of $\hat{x}_k$. We also use the standard notation for the Kalman filter [24], where $\hat{x}_k^-$ and P_k^- denote the prior estimate and the prior estimation error covariance.

Algorithm 1 Standard Kalman Filter

Initialization: compute $\hat{q}_1$ and P_1

Time Update:

compute $\hat{q}_k^-$ by integrating the gyroscope

error covariance time update

$$P_k^- = \exp(A_{k-1}T) P_{k-1} \exp(A_{k-1}T)' + Q \tag{13}$$

Measurement Update:

error covariance measurement update

$$K_k = P_k^- H'(H P_k^- H' + R)^{-1}$$
$$P_k = (I - K_k H) P_k^- (I - K_k H)' + K_k R K_k' \tag{14}$$

$\hat{x}_k$ estimation

$$\hat{x}_k = K_k z_k \tag{15}$$

quaternion update using $\hat{x}_k$

$$\hat{q}_k = \hat{q}_k^- \otimes \begin{bmatrix} 1 \\ \hat{\bar{q}}_{e,k} \end{bmatrix}. \tag{16}$$

bias $\hat{b}_a$ and $\hat{b}_g$ update using $\hat{x}_k$

In Algorithm 1, there is no $\hat{x}_k^-$ term in the measurement update equation (15) since $\hat{x}_k^-$ is set to zero after the quaternion update (16) and bias terms update.

One of the primary limitations of a standard Kalman filter is its slow convergence when dealing with a large initial estimation error. This sluggishness primarily results from the assumption of small estimation errors in the multiplicative error model (8). Due to this assumption, the measurement equation is essentially a local optimization problem near the current quaternion $\hat{q}_k^-$ (see (16)): $\bar{q}_{e,k}$ is estimated near $\hat{q}_k^-$, and then $\hat{q}_k$ is updated. If $\hat{q}_k^-$ is not correct, the update process could give wrong estimation $\bar{q}_{e,k}$.

III. GLOBAL ATTITUDE ESTIMATION

In this paper, the measurement equation is modified to tackle a global optimization problem when a significant estimation error is present. Conversely, the standard measurement update (local optimization) is used when the estimation error is small.

A. MAGNETIC MEASUREMENT MODIFICATION

As the true attitude is unknown, it is challenging to determine the exact magnitude of the estimation error. In this paper, we determine the magnitude using the measurement (12), where a large z_k implies a large estimation error. To compute z_k (see (12)), the dip angle μ is required. However, in the proposed algorithm, we modify z_k to eliminate the requirement for the dip angle μ. It's worth noting that a similar approach is used in the complementary filter [5]. Let c_k be defined by

$$c_k = C(\hat{q}_k^-)' \frac{y_{m,k}}{\|y_{m,k}\|}$$

When $\hat{q}_k^-$ is correct, and there is no sensor noise, we have $c_k = \tilde{m}$. However, this is not the case due to incorrect prior estimated attitude and the presence of sensor noise. A magnetic field vector $\bar{m}(\hat{q}_k^-) \in R^3$ is defined by

$$\bar{m}(\hat{q}_k^-) = \begin{bmatrix} \cos\bar{\mu} \\ 0 \\ \sin\bar{\mu} \end{bmatrix}.$$

The dip angle $\bar{\mu}$ is estimated so that $\bar{m}(\hat{q}_k^-)$ and c_k have the same inclination angle. Let θ be the angle between a vector and the world coordinate z axis, then we have

$$\cos\theta = e_3' \bar{m}(\hat{q}_k^-) = e_3' c_k. \tag{17}$$

where

$$e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

From (17), the modified magnetic field vector $\bar{m}(\hat{q}_k^-) \in R^3$ is given by

$$\bar{m}(\hat{q}_k^-) = \begin{bmatrix} \|c_k(1{:}2)\| \\ 0 \\ c_k(3) \end{bmatrix} \tag{18}$$

where $c_k(1{:}2) \in R^2$ is the first and second element of c_k, and $c_k(3)$ is the third element of c_k.

The modified measurement equation is given by

$$\bar{z}_k = \bar{H}_k x_k + \tilde{v}_k \tag{19}$$

where

$$\bar{H}_k = \begin{bmatrix} 2[C(\hat{q}_k)\tilde{g}\times] & I_3 & 0_{3\times3} \\ 2[C(\hat{q}_k)\bar{m}\times] & 0_{3\times3} & 0_{3\times3} \end{bmatrix} \in R^{6\times9}$$
$$\bar{z}_k = \begin{bmatrix} y_{a,k} - C(\hat{q}_k^-)\tilde{g} - \bar{b}_{a,k} \\ y_{m,k} - C(\hat{q}_k^-)\bar{m}(\hat{q}_k^-) \end{bmatrix} \in R^{6\times1}.$$

Note that the dip angle is not required to compute $\bar{m}_k$. On the other hand, $\bar{m}_k$ should be computed at each time step.

B. GLOBAL OPTIMIZATION PROBLEM

Let f_k be defined by

$$\begin{aligned} f_k &= \bar{z}_k' R_v^{-1} \bar{z}_k' \\ &= \frac{1}{r_g}\|y_{a,k} - C(\hat{q}_k^-)\tilde{g}\|_2^2 \\ &\quad + \frac{1}{r_g}\|y_{m,k} - C(\hat{q}_k^-)\bar{m}(\hat{q}_k^-)\|_2^2 \end{aligned} \tag{20}$$

When the current attitude estimate $\hat{q}_k^-$ is accurate, the value of f_k tends to be small. In the proposed algorithm, a global attitude estimation is used in the measurement update if $f_k > f_{threshold}$ ($f_{threshold}$ is the threshold value). This approach allows for more rapid compensation of large estimation errors. Conversely, if $f_k \le f_{threshold}$, a standard measurement update is done using (19), as a local optimization algorithm is good enough for small errors. In the global attitude estimation, q is directly estimated instead of estimating $\bar{q}_{e,k}$ and updating $\hat{q}_k$ as in (16).

Consider the following optimization problem:

$$\begin{aligned} \min_q \Big\{ &(\hat{q}_k^- \otimes q)' S' (P_k^-)^{-1} S(\hat{q}_k^- \otimes q) \\ &+ \frac{1}{r_a}\|\tilde{y}_{a,k} - C(q)e_3\|_2^2 \\ &+ \frac{1}{r_m}\|y_{m,k} - C(q)\bar{m}(\hat{q}_k^-)\|_2^2 \Big\} \end{aligned} \tag{21}$$

subject to $\|q\| = 1$ where

$$S = \begin{bmatrix} 0_{3\times1} & I_3 \\ 0_{3\times1} & 0_{3\times3} \\ 0_{3\times1} & 0_{3\times3} \end{bmatrix} \in R^{9\times4}.$$

$\tilde{y}_{a,k} \in R^3$ is the normalized vector of $y_{a,k}$:

$$\tilde{y}_{a,k} = \frac{y_{a,k}}{\|y_{a,k}\|}.$$

The term $S(\hat{q}_k^- \otimes q) \in R^3$ represents the 3×1 vector part of the quaternion estimation error $q_e = \hat{q}_k^- \otimes q$. This can be expressed as follows:

$$S(\hat{q}_k^- \otimes q) = \Xi(\hat{q}_k^-)q \tag{22}$$

where

$$\Xi(p) = \begin{bmatrix} -\bar{p}' \\ p_0 I + [\bar{p}\times] \end{bmatrix} \in R^{4\times3}.$$

Also from [21], the second and third terms are given by

$$\begin{aligned} &\frac{g}{r_a}\|\tilde{y}_{a,k} - C(q)e_3\|_2^2 + \frac{1}{r_m}\|\tilde{y}_{m,k} - C(q)\bar{m}(\hat{q}_k^-)\|_2^2 \\ &= \lambda_0 - q'L(B)q \end{aligned} \tag{23}$$

where

$$\begin{aligned} \lambda_0 &= \frac{g}{r_a} + \frac{1}{r_m} \\ L(B) &= \begin{bmatrix} Tr\,B & z' \\ z & B + B' - Tr\,B I_3 \end{bmatrix} \\ z &= \begin{bmatrix} B_{23} - B_{32} \\ B_{31} - B_{13} \\ B_{12} - B_{21} \end{bmatrix} \\ B &= \frac{g}{r_a}\tilde{y}_{a,k}e_3' + \frac{1}{r_m}\tilde{y}_{m,k}\bar{m}(\hat{q}_k^-)'. \end{aligned}$$

Using (22) and (23), we can represent (21) as follows:

$$\min_q q'Mq \tag{24}$$

subject to $\|q\| = 1$ where

$$M = \Xi(\hat{q}_k^-)'S'(P_k^-)^{-1}S\,\Xi(\hat{q}_k^-) + \lambda_0 I_4 - L(B) \in R^{4\times4}. \tag{25}$$

Given the construction, M is a positive semi-definite matrix. The optimal solution to (24) can be obtained by computing the unit eigenvector corresponding to the smallest eigenvalue of M.

We note that the measurement update equation in the standard Kalman filter solves the following optimization problem:

$$\begin{aligned} \min_{q_e} \Big\{ &q_e'(P_k^-)^{-1}q_e \\ &+ \frac{g}{r_a}\|\tilde{y}_{a,k} - C(\hat{q}_k^-)e_3 + 2[C(\hat{q}_k^-)e_3\times]q_e\|_2^2 \\ &+ \frac{1}{r_m}\|\tilde{y}_{m,k} - C(\hat{q}_k^-)\bar{m} + 2[C(\hat{q}_k^-)\bar{m}\times]q_e\|_2^2 \Big\} \end{aligned} \tag{26}$$

The optimization problem (26) is a local optimization problem near $\hat{q}_k^-$. However, due to the small value assumption in (8), the local optimization (26) may not provide an accurate estimate when $\hat{q}_k^-$ is inaccurate.

C. P_K^- UPDATE

When (21) is used in the measurement update, only the attitude estimation error component is updated. The estimation error covariance update equation (14) is modified as follows:

$$\begin{aligned} K_k &= P_k^- H'(HP_k^- H' + R)^{-1} \\ \bar{K} &= S_2 K_k \\ P_k &= (I - \bar{K}_k H)P_k^-(I - \bar{K}_k H)' + \bar{K}_k R \bar{K}_k' \end{aligned} \tag{27}$$

where

$$S_2 = \begin{bmatrix} I_3 & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix} \in R^{9\times9}.$$

Note that $\bar{K}_k \in R^{9\times6}$ has the following structure:

$$\bar{K}_k = \begin{bmatrix} K_k(1{:}3{,}3{:}6) \\ 0_{3\times6} \\ 0_{3\times6} \end{bmatrix}$$

where $K_k(1{:}3{,}3{:}6)$ denotes the first three rows of K_k. Consequently, only the covariance of the attitude estimation error is updated, while the covariance of the bias estimation error remains unchanged when the global update is applied. The Kalman gain K_k is only used in the computation of estimation error covariance and does not play a role in the quaternion computation.

D. SUBOPTIMAL SOLUTION

A suboptimal solution is proposed to solve (24), which offers computational simplicity compared to computing an eigenvector. The second and third terms in (21) correspond to a classic Wahba's problem [17]. Let $\hat{q}_{k,triad}$ be a solution using a triad algorithm [25]. It is assumed that the optimal solution lies on the linear interpolation between $\hat{q}_k^-$ and $\hat{q}_{k,triad}$:

$$q = t\hat{q}_k^- + (1-t)\hat{q}_{k,triad} \tag{28}$$

where $0 \leq t \leq 1$. There is no guarantee that the optimal solution can be represented by (28). Therefore, the proposed simple solution is considered suboptimal.

Let $d_1, d_2, d_3 \in R$ be defined by

$$\begin{aligned} d_1 &= (\hat{q}_k^-)' M \hat{q}_k^- \\ d_2 &= (\hat{q}_{k,triad})' M \hat{q}_{k,triad} \\ d_3 &= (\hat{q}_k^-)' M \hat{q}_{k,triad}, \end{aligned}$$

then we have

$$g(t) = q'Mq = (d_1 + d_2 - 2\,d_3)t^2 - 2(d_2 - d_3)t + d_2. \tag{29}$$

The derivative of $g(t)$ with respect to t is given by

$$\dot{g}(t) = 2(d_1 + d_2 - 2\,d_3)t - 2(d_2 - d_3).$$

The optimal t value minimizing (29) can be obtained from $\dot{g} = 0$:

$$t_{optimal} = \frac{d_2 - d_3}{d_1 + d_2 - 2d_3}. \tag{30}$$

By substituting (30) into (28) and subsequently normalizing q, we can derive a suboptimal solution.

E. PROPOSED ALGORITHM SUMMARY

The proposed algorithm is summarized in Algorithm 2.

Algorithm 2 Global Optimal Filter

Initialization: compute $\hat{q}_1$ and P_1
Time Update:
compute $\hat{q}_k^-$ by integrating the gyroscope
error covariance time update

$$P_k^- = \exp(A_{k-1}T)P_{k-1}\exp(A_{k-1}T)' + Q \tag{31}$$

Measurement Update:
if $f_k < f_{threshold}$ **then**
measurement update using (14), (15) and (16)
bias update
else
compute q_k by solving (21)
(computing an eigenvector or solving (28))
estimation error covariance update using (27)
end if

IV. SIMULATION AND EXPERIMENTS

A. SIMULATION

In the first simulation, the estimation error convergence rate is investigated. It is assumed that the attitude does not change, with the true attitude given by $q_{true} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}'$. To intentionally introduce an incorrect initial attitude, we choose it as follows:

$$\hat{q}_{initial} = \begin{bmatrix} \cos\frac{\delta}{2} \\ \sin\frac{\delta}{2}u \end{bmatrix} \in \begin{bmatrix} R \\ R^3 \end{bmatrix} \tag{32}$$

where $u \in R^3$ is a random unit vector. Since δ represents the rotation angle between the initial estimated attitude and the true attitude, a large δ implies a significant initial estimation error.

The following sensor noise parameters are used:

$$r_g = 0.01^2, \quad r_a = 0.05^2, \quad r_m = 0.05^2. \tag{33}$$

Using the sampling period $T = 0.01$ seconds, a one-second simulation is conducted. Given two quaternions $q_{a,k}$ and $q_{b,k}$ ($1 \leq k \leq N$), the following measure [26] is used to compute attitude error between $q_{a,k}$ and $q_{b,k}$:

$$J(q_a, q_b) = \|C(q_{a,k}) - C(q_{b,k})\|_F \tag{34}$$

where $\|A\|_F$ denotes the Frobenius norm of a matrix A.

In Fig. 1, estimation errors of 4 simulations are plotted for the proposed filter and standard Kalman filter when $\delta =$ 10 deg: different random vector u is used for 4 simulations. In the graph, the estimation error is measured with the error equation in (34). It can be seen that both filters show similar convergence rate, which shows that the standard Kalman filter can handle small initial error.

In Fig. 2 and Fig. 3, attitude estimation errors are shown for $\delta = 40$ deg and $\delta = 100$ deg, respectively. It can be seen that convergence rate began to slow in Fig. 2 and Fig. 3.

An extreme case ($\delta = 180$) is also given in Fig. 4. It can be seen that the estimation error of the proposed filter converges quickly with very large initial estimation error. Also it can be

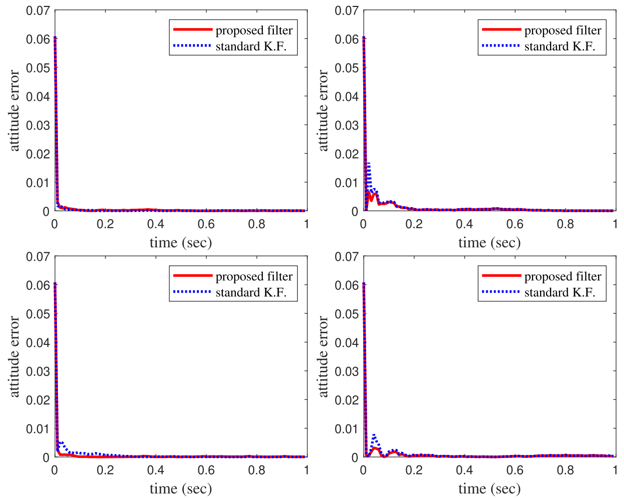

FIGURE 1. **Attitude estimation error convergence ($\delta = 10$ deg).**

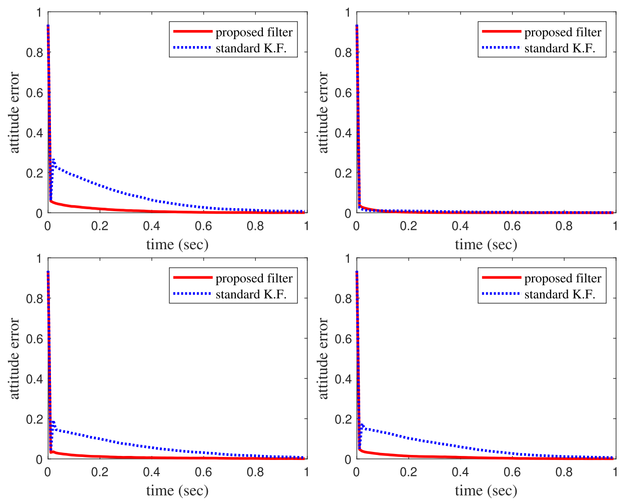

FIGURE 2. **Attitude estimation error convergence ($\delta = 40$ deg).**

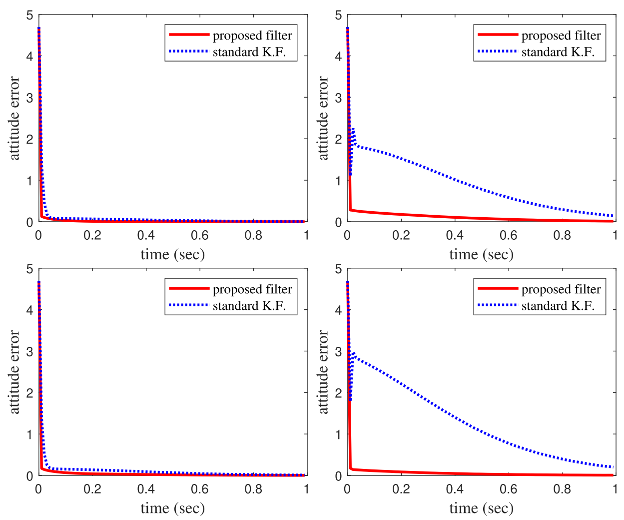

FIGURE 3. **Attitude estimation error convergence ($\delta = 100$ deg).**

observed that the convergence rate of the standard Kalman filter becomes very slow when there are large initial errors.

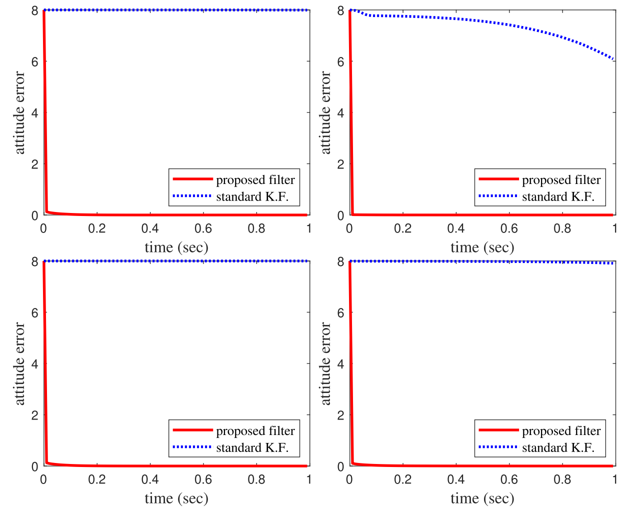

FIGURE 4. **Attitude estimation error convergence ($\delta = 180$ deg).**

TABLE 1. **Mean and worst estimation errors with respect to different initial errors (δ) out of 1000 simulation results.**

	proposed filter			standard K.F. filter		
δ	2.5 sec	2.5 sec	5 sec	2.5 sec	2.5 sec	5 sec
	mean	worst	worst	mean	worst	worst
10	0.0001	0.0007	0.0005	0.0001	0.0006	0.0006
40	0.0001	0.0008	0.0009	0.0001	0.0009	0.0009
100	0.0001	0.0005	0.0006	0.0001	0.0005	0.0007
150	0.0001	0.0006	0.0007	0.0001	0.0006	0.0006
180	0.0001	0.0157	0.0006	0.0331	7.8766	0.0007

To check convergence of the proposed algorithm, estimation errors with respect to different initial errors (δ) are investigated in Table 1. For each δ value, 1000 simulations are done. Worst (i.e., maximum) estimation errors at 2.5 and 5 seconds are given in Table 1. Also mean estimation errors at 2.5 seconds are given. In all simulations, it can be seen that estimation errors of both proposed and standard Kalman filters converge: see the worst case errors in 5 seconds. This is not suprising since estimation errors are bound to decrease during the measurement update while the standard Kalman filter requires more time for large initial estimation errors: see the last row ($\delta = 180$ case) in Table 1.

We note that the proposed algorithm requires more computation time than the standard Kalman filter. Recall that the main difference between the proposed algorithm and the standard Kalman filter is the measurement update: (21) for the proposed algorithm and (15) for the standard Kalman filter.

To solve (21), the eigenvector of a 4×4 in (25) needs to be computed. A suboptimal algorithm to solve (21) is also proposed in (28). In the standard Kalman filter, the measurement update is just a matrix multiplication Kz in (25).

Since it is not easy to derive analytic computation complexity, the computation time is compared in Matlab simulations. 100 simulations (one second interval) are performed for 18 different initial errors $\delta = 10, 20, \cdots, 180$ in (32): total $1800 = 100 \times 18$ simulations are done in Windows PC with

TABLE 2. Computation time comparison (one second 1800 simulation summation).

proposed filter		standard K.F.
eigenvector computation	suboptimal algorithm in (28)	
2.6275 sec	2.3474 sec	1.9601 sec

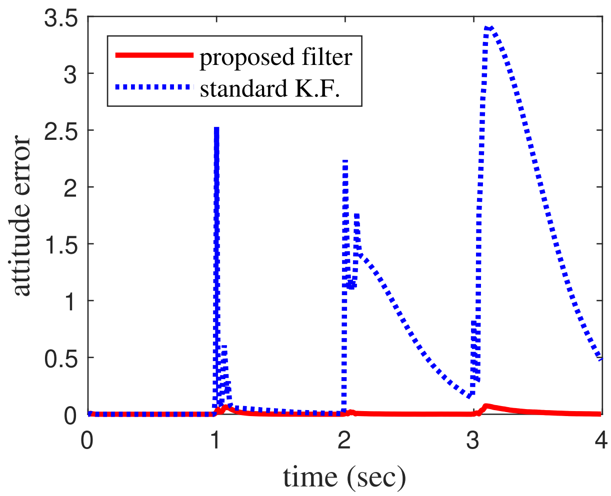

FIGURE 5. Attitude estimation error with intentional large sensor noises.

Intel i9-12900K 3.20GHz CPU. The computation time is given in Table 2.

The proposed filter requires 34.05% more computation time than the standard Kalman filter. Thus fast convergence is obtained at the sacrifice of more computation time in the proposed filter.

In the second simulation, 4 seconds simulations are performed under a similar assumption with the first simulation. The difference is that there is no intentional initial estimation error. Instead, large sensor noises are added to accelerometers and magnetic sensors during time intervals [1.0, 1.1], [2.0, 2.1] and [3.0, 3.1] seconds. This simulation is to check tracking ability of the proposed algorithms.

In Fig. 5, the estimation error of the standard Kalman filter becomes large when there are sensor noises at 1,2 and 3 seconds. When large sensor noises are removed, the estimation error slowly decreases. In can be seen that estimation error increase of the proposed filter due to large sensor noises is small compared with the standard Kalman filter. Thus the proposed filter effectively rejects sensor noises and tracks the true attitude.

In the third simulation, we employ simulation data from [27] to evaluate the proposed algorithm. In this simulation, we assume the following angular velocity:

$$\omega(t) = \begin{cases} \begin{bmatrix} \tau_1 \sin \nu_1 t \\ \tau_2 \sin \nu_2 t \\ \tau_3 \sin \nu_3 t \end{bmatrix}, & 0 < t < 10 \\ 0_{3\times 1}, & 10 \le t \le 11 \end{cases}$$

TABLE 3. Estimation error with no intentional initial attitude error.

filter	(34) over 729 combinations
proposed filter	0.0105
proposed filter with (28)	0.0113
standard K.F.	0.0107
Madgwick ($\beta = 0.01$)	0.0711
Madgwick ($\beta = 0.1$)	0.0168
Madgwick ($\beta = 0.2$)	0.0184

TABLE 4. Estimation error with intentional initial attitude error.

filter	(34) over 729 combinations
proposed filter	0.0212
proposed filter with (28)	0.0174
standard K.F.	0.1045
Madgwick ($\beta = 0.01$)	1.3212
Madgwick ($\beta = 0.1$)	0.7762
Madgwick ($\beta = 0.2$)	0.5191

We select the parameter τ_i from the set of values $\{0.3, 0.4, 0.5\}$, and parameter ν_i from $\{0.4\pi, 0.8\pi, 1.6\pi\}$.

We test the proposed algorithm using 729 ($= 3^6$) possible parameter combinations, incorporating noise covariance parameters as defined in (33). For comparison, we also evaluate the performance of a standard Kalman filter and Madgwick's filter [5]. Madgwick's filter includes a parameter $\beta \in R$, which controls the mixing gain between the integrated attitude (using y_g) and the measurement values (using y_a and y_m).

First, in Table 3, we present the estimation errors when the initial attitude is correctly estimated.

In the first proposed filter, the quaternion is computed using an eigenvector of M (see (25)), while simplified interpolation (28) is used in the second proposed filter. We also present results from the standard Kalman filter and Madgwick's filter with different β values. In Table 3, all filter results are similar. Theoretically, all filters are solving almost the same optimization problems. Therefore, it cannot be asserted that one filter outperforms the others.

In the next simulation, we intentionally employ incorrect initial estimations in the filtering process to assess how quickly these incorrect initial estimations are removed.

In Table 4, it can be seen that the proposed filter demonstrates robustness to incorrect initial estimations.

B. EXPERIMENTS

We evaluate the proposed algorithm using an experimental dataset from [28], which provides 39 inertial and magnetic sensor data along with ground truth. This dataset includes a wide range of movement types and speeds, representing both undisturbed and disturbed (due to acceleration and magnetic field) environments.

In Table 5, we present estimation errors obtained from various filters when there are no intentionally large initial errors.

In Table 6, we provide estimation errors when intentional large initial errors are introduced. It can be seen that the proposed filter exhibits smaller estimation errors.

TABLE 5. Estimation error with no intentional initial attitude error (39 data).

filter	sum of (34)
proposed filter	0.7382
proposed filter with (28)	0.7253
standard K.F.	1.1256
Madgwick ($\beta = 0.01$)	0.7978
Madgwick ($\beta = 0.1$)	0.7355
Madgwick ($\beta = 0.2$)	0.6940

TABLE 6. Estimation error with intentional initial attitude errors (39 data).

filter	sum of (34)
proposed filter	0.7681
standard K.F.	1.3853
Madgwick ($\beta = 0.01$)	2.2808
Madgwick ($\beta = 0.1$)	2.1900
Madgwick ($\beta = 0.2$)	2.0831

V. CONCLUSION

In this paper, a new attitude estimation algorithm is proposed, where a global attitude estimation problem is used in the measurement update. Through simulation and experiment data, it is shown that the estimation error converges quickly even for very large initial estimation errors.

One disadvantage of the proposed algorithm is rather high computational load: it requires 34% more computation time compared with the standard Kalman filter. This additional computation time primarily arises from the need to calculate the eigenvector of a matrix associated with the smallest eigenvalue. Our future research will focus on devising an optimized algorithm for computing eigenvectors, capitalizing on the inherent problem structure. The aim is to significantly reduce the computation time required.

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