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RESEARCH ARTICLE

PlaciFi: Orchestrating Optimal 3D Access Point Placement for LiFi-WiFi Heterogeneous Networks

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ABSTRACT This paper presents PlaciFi, a comprehensive framework for orchestrating optimal 3D Access Point (AP) placement in LiFi-WiFi heterogeneous networks. Integrating LiFi and WiFi technologies allows leveraging of their strengths in different use cases and environments. This has resulted in new challenges in the effective deployment of APs. To address these challenges, PlaciFi extends traditional network planning approaches by incorporating the three-dimensional positioning of APs. The framework formulates a multi-objective optimization problem to minimize the AP count and maximize the rate coverage while considering illumination uniformity constraints for consistent illumination throughout the coverage area. PlaciFi introduces the consideration of user technology occurrence probability to tailor AP placement based on user requirements, improving network performance and user satisfaction. Various solution methods, including heuristics, meta-heuristics, and off-the-shelf solvers, are explored to offer flexibility to network planners. Through extensive evaluations and simulations, PlaciFi demonstrates its effectiveness in achieving optimal AP placement, balancing infrastructure costs and network performance. The contributions of PlaciFi advance the field of network planning in LiFi-WiFi heterogeneous networks, enabling the deployment of efficient and reliable networks.

INDEX TERMS Access point placement, light fidelity (LiFi), multi-objective optimization, wireless fidelity (WiFi).

I. INTRODUCTION

Wireless communication technologies have undergone significant advancements owing to the increasing demand for high-speed and reliable connectivity. While Wireless-Fidelity (WiFi) networks have been the dominant means of communication, the emergence of Visible Light Communication (VLC) and Infrared (IR) communication networks, such as Light-Fidelity (LiFi) [1], has introduced a promising alternative. LiFi offers advantages such as higher bandwidth and enhanced security compared to WiFi-based communication [2].

In today's wireless communication, providing seamless connectivity and meeting the diverse needs of users have become crucial challenges. The increasing demand for high-speed data transmission, low latency, and ubiquitous coverage necessitates the integration of multiple wireless technologies. One such emerging combination is the integration of LiFi and WiFi technologies in a heterogeneous network [3], commonly referred to as a LiFi-WiFi network. Such a network efficiently utilizes the available resources, enhances coverage and capacity, and provides flexibility in accommodating diverse user demands. However, the optimal design of such networks poses challenges owing to various factors, including cost constraints, rate coverage requirements, and the need for adequate illumination.

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Network design in indoor heterogeneous wireless environments is vital for ensuring optimal performance and user satisfaction. A critical aspect of designing a LiFi-WiFi heterogeneous network is the placement of Access Points (APs). However, the placement of APs in such a heterogeneous network is a complex task due to the distinct characteristics of the LiFi and WiFi technologies. Existing research on AP placement predominantly focuses on either WiFi or LiFi networks separately [4], [5], [6], [7], [8], [9], with limited attention given to the integration of both technologies. Therefore, there is a need for a systematic and efficient approach to determine the optimal placement of APs in a LiFi-WiFi heterogeneous network, considering the unique requirements and interactions of both systems.

In this paper, we present a novel framework for the optimal 3D placement of APs in a LiFi-WiFi heterogeneous network, considering various aspects such as cost, rate coverage, user occurrence, and illumination. This provides a comprehensive network plan that considers the layout of the indoor space and strategically places the APs.

A. CONTRIBUTION

This paper builds upon our previous work in [10], where we investigated the network planning of a LiFi-only communication and illumination network in an indoor environment. In our previous paper, we formulated an optimization problem with multiple objectives: minimizing the number of APs and maximizing the sum rate while satisfying constraints on the minimum guaranteed achievable rate and minimum illumination level. The optimization variables included the height of each AP, which we allowed to be freely positioned (3D free-height) or constrained to a uniform height (3D fixed-height). To tackle this LiFi AP placement problem, we used a genetic Multi-Objective Optimization (MOO) algorithm.

Building on this foundation, the present paper extends and expands our research in several key directions to comprehensively address the design of LiFi-WiFi heterogeneous networks. Our contributions are outlined below.

1) **Holistic Network Optimization:** We propose a comprehensive framework for the optimal placement of APs in a LiFi-WiFi heterogeneous network. We consider both VLC and IR communication for LiFi, enabling adaptability to evolving LiFi hardware generations and diverse usage scenarios.
2) **Multi-Objective Optimization:** We address the challenge of optimal AP placement in three dimensions by formulating a MOO problem, yielding significantly higher average rates compared to the state-of-the-art 2D power optimization models. The objective is to minimize the cost of placing APs while maximizing the rate coverage of the network. The results clearly showed that MOO techniques are highly effective in dealing with multiple objectives. Additionally, we incorporate illumination uniformity constraints to ensure a consistent and adequate illumination level throughout the coverage area. We introduce a refined objective of minimizing the cost associated with placement, which recognizes the varying costs of APs for LiFi and WiFi, contributing to a more economically optimized network. To cater to the diverse needs of users, we incorporate distinct user technology occurrence probabilities for each technology (LiFi and WiFi). Thus, we can tailor the placement of APs to accommodate the specific requirements of each user category.
3) **Versatile Solution Methods:** We explore various solution methods, including heuristics, meta-heuristics, and off-the-shelf solvers, which consistently outperform the baseline random approach. We also investigate various options to combine multiple objectives to get a single optimal solution. The flexibility in solution approaches empowers network planners to choose the most suitable approach based on the specific requirements of their deployment scenario.

B. ORGANIZATION

The remainder of this paper is organized as follows. Section II provides an overview of related work in AP placement for LiFi networks, WiFi networks and general 3D placement. Section III presents the system models for LiFi and WiFi technologies in a heterogeneous network. In Section IV, we formulate the optimization problem as a mathematical model. The solution methods, including the heuristic and meta-heuristic algorithms, are described in Section V. Section VI presents the simulation setup and evaluation results. Section VII concludes the paper, summarizes the findings, discusses the implications, and Section VIII provides guidelines for planning a LiFi-WiFi heterogeneous network in indoor environments.

II. RELATED WORK

In this section, we provide an overview of relevant literature related to the placement of APs in LiFi and Radio Frequency (RF) networks. We also identify the gaps in the existing literature that have motivated our current research.

A. PLACEMENT IN LiFi NETWORKS

The authors in [4] addressed the optimal placement of APs in LiFi networks by considering the stationary distribution of users' mobility. They examine the stationary distribution of users following a Random Waypoint model in an indoor environment. This initial study demonstrated feasibility using a small Fixed-cell Single-user setup with four APs.

This work is extended in [5] where they also optimize the average throughput in an indoor environment while considering the stationary distribution of users and, as a result, place APs in a 2D space using an adaptive gradient projection algorithm. In contrast to these works, we do not assume the position of actual users but rather consider the coverage on a user plane and look into different patterns of user occurrence in these indoor environments. This is beneficial when we do not know the actual position or number of users in the planning stage. Moreover, these works do not address the optimal number of APs to be placed.

While the study in [6] considers minimizing the number of Light Emitting Diodes (LEDs) for the expected user distribution in a room, they do not optimize the system for network performance in terms of throughput but rather try to maximize the number of users served or minimize the number of APs placed. They solve their problem with an exhaustive search, which is highly infeasible for our problem. The works described above only guarantee a minimum illumination level in the room. However, this might result in spotty illumination due to optimizing the positions or powers of the APs for maximizing rate. Therefore, we aim to guarantee a minimum uniformity of illumination.

The authors in [7] focus on improving the arrangement of a fixed number of LEDs in LiFi communication systems by minimizing the outage probability. They only look into 2D placement and solve the problem using a heuristic.

Similarly, [8] goes beyond exhaustive search and solves the 2D placement problem for an LED array by solving successive convex sub-problems. They present a power-efficient LED placement algorithm for indoor VLC networks. Their approach aims to minimize power consumption while ensuring reliable communication. However, they only consider placement in a 2D plane with a fixed number of LED. Both works detailed above also consider providing uniformity in illumination.

In contrast, our research significantly extends the existing literature by considering the unique characteristics and constraints of both LiFi and WiFi technologies. We propose an optimization framework that places APs in a 3D space and considers multiple user occurrence patterns.

B. PLACEMENT IN WiFi AND HETEROGENEOUS RF NETWORKS

In the domain of RF placement research, [9] introduced a modified vector quantization approach for small-cell WiFi networks. Their optimization strategy aimed to minimize interference and optimize the placement of APs within the network. They perform this investigation for single-user networks and discuss the limitations of it in a small-scale network.

In a separate study, [11] addressed the challenge of colocated and non-colocated node placement in Long Term Evolution (LTE)-WiFi aggregation networks. They formulated this problem as a Mixed Integer Nonlinear Programming (MINLP) problem. This work is closest to ours in the nature of the problem because it also investigates heterogeneous networks.

Similarly, [12] presented an optimal deployment strategy for heterogeneous wireless nodes in integrated LTE-WiFi networks. They formulated the problem as a mixed-integer nonlinear program and proposed a genetic algorithm to obtain near-optimal solutions. We also use a genetic algorithm to demonstrate its effectiveness in placement problems.

The cited studies focus on optimizing AP placement in either WiFi or LTE-WiFi networks, whereas our research addresses the specific challenges of integrating LiFi with WiFi in a heterogeneous network setting. This is challenging because the LiFi channel depends on the user's orientation. This makes it challenging to frame a mathematically convex optimization problem.

C. 3D PLACEMENT

The previously mentioned works only explored the 2D space for placing APs. Therefore, we expand our survey to the placement optimization of Unmanned Aerial Vehicles (UAVs) to learn about 3D placement.

In their survey, [13] discussed various approaches for optimizing UAV placement and designing their flight path. In one section, they investigate the various placement optimization methods. Their focus was on UAVs placement, which is related to our work due to the placement in a 3D space. It is also different due to having actual users in the network during the optimization.

Another study by [14] proposed an efficient 3D aerial base station placement strategy considering user mobility using reinforcement learning. Their work is around the start of more recent works that use machine learning.

The authors in [15] addressed joint trajectory and communication design for multi-UAV enabled wireless networks. Their mathematical formulation is interesting since they also have a non-convex problem, which they solve using approximate convex sub-problems.

An AP placement approach for UAV-terrestrial small-cell networks is proposed in [16]. Their work aimed to optimize the placement of APs considering UAVs as small cells while minimizing interference. They employ a vector quantization approach as in [9].

Prior research has explored various aspects of AP placement in homogeneous and heterogeneous networks. Several studies have focused on WiFi-based networks, while others have investigated LiFi-based networks. However, the integration of these technologies and the optimization of AP placement in a LiFi-WiFi heterogeneous network have received limited attention. Therefore, our work distinguishes itself by addressing the unique challenges of LiFi-WiFi heterogeneous networks, introducing novel optimization techniques, and taking a comprehensive approach that considers multiple objectives and three-dimensional AP placement.

III. SYSTEM MODELS

In this section, we present the system model for the optimal 3D placement of APs in a LiFi-WiFi heterogeneous network. We describe the network model, channel models for both LiFi and WiFi technologies, the illuminance model for LiFi, and specific application scenarios for our study. Table 1 summarizes the notation used throughout this paper. Specifically, we use bold lowercase letters for vectors, cursive capital letters for sets, and $\mathbb{1}$ to represent indicator functions.

TABLE 1. List of notations used in the 3D placement of LiFi and WiFi APs.

Notation	Description
l, M^L	index, size of LiFi APs
w, M^W	index, size of WiFi APs
$u, M^U, \mathcal{U}$	index, size, set of all user positions
v, M^V	index of, total positions on the illuminance grid
$\boldsymbol{c} = (x, y, z)$	vector of 3D coordinates
$\Delta\boldsymbol{c}$	grid spacing in 3 dimensions
α	binary existence variable for AP
C	cost of placing an AP
$\boldsymbol{\Theta}_u = (\Theta^Y, \Theta^P, \Theta^R)_u$	vector of yaw, pitch and roll of user device
$\hat{\boldsymbol{n}}_u$	normal vector of the rotated user device
p_u^L, p_u^W	probability of occurrence of LiFi, WiFi user
$\boldsymbol{d}_{u,l}, \boldsymbol{d}_{u,w}$	3D Euclidean distance between user and AP
ϕ	angle of irradiance of LiFi signal
θ	angle of incidence of LiFi signal
P_l, P_w	transmission power of the APs
P_u	received power at the user
$R_u, \tilde{R}_u$	achievable and normalized achievable rate
I_v	illuminance at the grid position
I	illumination uniformity

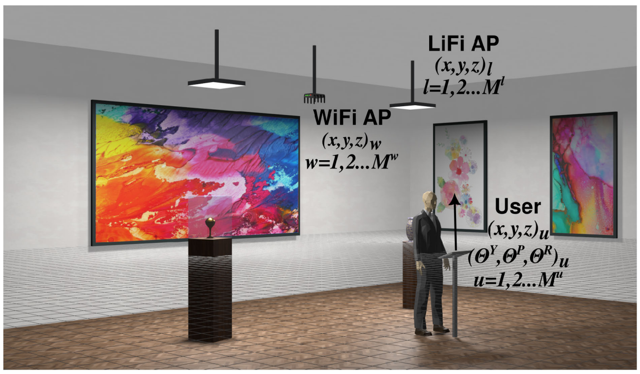


FIGURE 1. Example architecture of a LiFi-WiFi communication and illumination network in a Museum, which is one among several application scenarios addressed.

A. NETWORK MODEL

This work focuses on deploying a LiFi-WiFi heterogeneous network, combining wireless technologies for indoor communication. The network consists of a maximum of M^L LiFi APs and a maximum of M^W WiFi APs strategically placed throughout the environment. The LiFi APs, represented by the white LED panels in Fig. 1, operate on the visible light or Infrared spectrum and are mounted above the user plane facing downwards. They can provide both illumination and data transmission when operating on visible light. The three-dimensional coordinates of each LiFi and WiFi AP are denoted by $\boldsymbol{c}_l = (x, y, z)_l$ and $\boldsymbol{c}_w = (x, y, z)_w$, respectively. The maximum height of the APs is denoted by z_{maxdim}, which corresponds to the height of the ceiling in the indoor environment. The minimum height is denoted by z_{mindim} and is assumed to be at least one meter above the user plane to avoid saturating the receivers. As all LiFi APs operate at the same frequency, co-channel interference occurs in areas where the coverage area of cells overlaps, while this is not the case for the WiFi network.

The user plane is the region where users are expected to be located and is quantized by a grid with a spacing of $\Delta\boldsymbol{c}_u = 0.25$ m at a height of 1.4 m. Each position $u \in \mathcal{U}$ on this grid is represented by the coordinates $\boldsymbol{c}_u = (x, y, z)_u$, with M^U such positions. Each user in a specific position is equipped with LiFi photodiode and/or WiFi receivers for downlink traffic. The orientation of these receivers is denoted by $\boldsymbol{\Theta_u} = (\Theta^Y, \Theta^P, \Theta^R)_u$, representing the device's Yaw, Pitch, and Roll angles. A (0, 0, 0) value indicates that the user device is parallel to the floor and faces the ceiling. Since the exact positions of the users are not known and dynamically changing, each user grid position u is associated with a weight p_u^L or p_u^W proportional to the expected probability of occurrence of a LiFi or WiFi user in that position. Users are expected to connect to the AP with the highest offered signal strength within one technology, providing an additional degree of freedom to the model as there are no dedicated APs for users. If users can connect to both technologies, they choose the one that can offer the maximum link rate.

B. LiFi CHANNEL MODEL

The channel model for LiFi described in [17] serves as the basis for our work. It considers the positions of the APs and users in three dimensions and the orientation of the user devices. The three-dimensional distance between the user u and AP l is given by,

$$\mathrm{d}_{u,l} = \|\boldsymbol{c}_l - \boldsymbol{c}_u\|_2 \tag{1}$$

The cosines of the angles of irradiance ($\phi_{u,l}$) at the AP and angle of incidence ($\theta_{u,l}$) at the user can be expressed as,

$$\cos\phi_{u,l} = \frac{z_l - z_u}{d_{u,l}} \tag{2}$$

$$\cos\theta_{u,l} = \frac{(x_l - x_u)\hat{n}_{u,x} + (y_l - y_u)\hat{n}_{u,y} + (z_l - z_u)\hat{n}_{u,z}}{d_{u,l}} \tag{3}$$

where $\hat{\boldsymbol{n}}_u$ is the normal vector of the rotated user device. These values can be put together to give the Line-of-Sight (LoS) gain HLoS

$$\mathrm{HLoS}_{u,l} = \begin{cases} \dfrac{H_0}{d_{u,l}^2} \cdot \cos\phi_{u,l}^m \cdot \cos\theta_{u,l} & \text{if } \theta_{u,l} \le \Theta_f \\ & \text{and } \phi_{u,l} \le \Phi_f \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

where H_0 is a constant given by $((m+1)A_p\chi^2 T_s)/2\pi$. Here, m is the Lambertian order of the AP, A_p is the area of the photodiode receiver, χ is the refractive index, and T_s is the gain of the optical filter. The gain only exists when both the uplink and downlink signal are within the Field of View

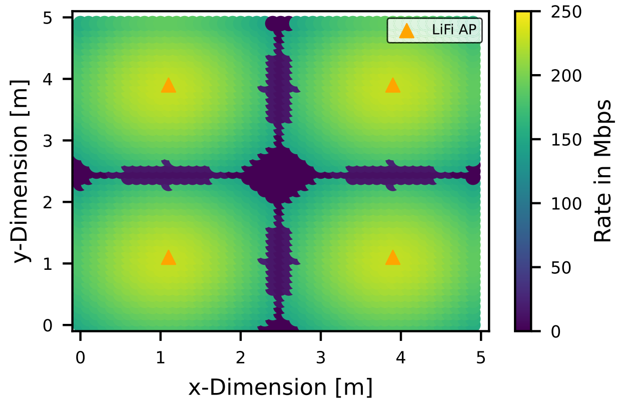


FIGURE 2. LiFi link rate with a lattice grid placement of 4 LEDs.

TABLE 2. LiFi channel and illumination parameters.

Parameter	Notation	Value
Optical Power of a LiFi AP	P_l	5 W
Half power beam width	$\theta_{1/2}$	60°
Physical area of the receiver	A_p	1 cm^2
FoV of the receiver	Θ_f	90°
FoV of the transmitter	Φ_f	90°
Optical filter gain	T_s	1
Refractive index	χ	1
Lambertian order	m	1
Modulation bandwidth of LED	B_L	20 MHz
Noise power spectral density	noise	10^{-21} A^2/Hz
Maximum capacity LiFi	$R^L_{\max}$	250 Mb/s
Illumination uniformity threshold	I_{thresh}	0.7
Luminous efficacy of LED	I_0	200 lm/Watt

(FoV) of the transmitter (Φ_f) and receiver (Θ_f). The received signal power can then be calculated as,

$$P_{u,l} = (\text{HLoS}_{u,l} \cdot P_l \cdot k)^2 \tag{5}$$

where P_l is the optical transmission power of the AP in Watts and k is the optical to electrical conversion efficiency. Since this is just in the network planning stage, we assume that the user connects to the AP offering the highest signal strength. Therefore, the signal at the receiver is given by

$$S^L_u = \max_l P_{u,l}. \tag{6}$$

Hence, the Signal to Interference and Noise Ratio (SINR) at the receiver is given by,

$$\text{SINR}^L_u = \frac{S^L_u}{\sum_l P_{u,l} - S^L_u + \text{noise}} \tag{7}$$

The link data rate between a user u and a LiFi AP l is calculated using the modified Shannon formula [18]:

$$R^L_u = \min\left(B^L \cdot \log_2(1 + \frac{e}{2\pi} \cdot \text{SINR}^L_u), R^L_{\max}\right) \tag{8}$$

where B^L denotes the LiFi modulation bandwidth of an LED. We assume a maximum data rate of 250 Mb/s for a LiFi AP which is denoted by $R^L_{\max}$. Fig. 2 illustrates the distribution of LiFi rate on the user plane, located at a height of 1.4 m when 4 LEDs are placed in a lattice grid at a height of 3 m. To enhance the generalizability of our approach to future technology generations with potentially higher capacities, we introduce the normalized rate $\tilde{R}^L_u$

$$\tilde{R}^L_u = \frac{R^L_u}{R^L_{\max}}. \tag{9}$$

This term represents the ratio of the achieved rate to the maximum capacity offered by the technology.

C. ILLUMINATION MODEL

When LiFi operates using VLC, the APs also provide illumination for the indoor area. Like the user grid, the room is also subdivided with an illuminance grid. The position of the illuminance grid point v is denoted as $\boldsymbol{c}_v = (x, y, z)_v$, with a total of M^V grid positions. The illuminance at grid position v from a single LiFi AP l is given by

$$I_{v,l} = I_0 \frac{1}{d^2_{v,l}} \cos\phi^m_{v,l} \cdot \cos\theta_{v,l} \tag{10}$$

where I_0 is the luminous efficacy of the LED in lumens/Watt. The total illuminance at grid position v from all LiFi APs is the sum of the individual contributions

$$I_v = \sum\nolimits_l I_{v,l}. \tag{11}$$

Furthermore, the illumination uniformity (I) is given by the ratio of the minimum and the average illumination intensity [19]

$$I = \frac{\min_v I_v}{\frac{\sum_v I_v}{M^V}}. \tag{12}$$

In our work, we aim to guarantee a minimum level of uniformity in illumination. Simulation parameters specific to the LiFi channel are summarized in Table 2.

D. WiFi CHANNEL MODEL

The IEEE 802.11n standard models the WiFi network. The channel bandwidth is 20 MHz, according to which the total capacity of a WiFi AP is 160 Mb/s denoted by $R^W_{\max}$. Similar to the LiFi channel model, the three-dimensional distance between user u and AP w is given by,

$$\text{d}_{u,w} = \|\boldsymbol{c}_w - \boldsymbol{c}_u\|_2 \tag{13}$$

The channel gain is as defined in [17]. We rewrite it here completely in linear terms and not in dB for ease of explanation and to compare it with the LiFi model. Hence, the gain is given by

$$H_{u,w} = \frac{1}{d^2_{u,w}} \cdot \frac{1}{f^2} \cdot 10^{14.45} \cdot h_r \tag{14}$$

where h_r is the small scale fading gain as described in [17] with an average power of 2.46 dB and f is the carrier frequency of transmission. The received signal power can then be calculated as,

$$P_{u,w} = (H_{u,w} \cdot P_w)^2 \tag{15}$$

TABLE 3. WiFi channel parameters.

Parameter	Notation	Value
Small-scale fading gain	h_r	2.46 dB
Transmission Power of a WiFi AP	P_w	0.1 W
Bandwidth of WiFi	B_W	20 MHz
Noise power spectral density	noise	10^{-15} A^2/Hz
Frequency of WiFi	f_W	2.45 GHz
Maximum capacity WiFi	$R^L_{\max}$	160 Mb/s

where P_w is the transmission power of the AP in Watts. We assume that the user connects to the AP offering the highest signal strength as with LiFi. Therefore, the signal at the receiver is given by

$$S_u^W = \max_w P_{u,w}. \tag{16}$$

We assume that frequency reuse is employed and there is no interference between WiFi APs. Hence, the Signal to Noise Ratio (SNR) at the receiver is given by,

$$\mathrm{SNR}_u^W = \frac{S_u^W}{\mathrm{noise}} \tag{17}$$

The link data rate between a user u and a WiFi AP w is calculated using the Shannon formula:

$$R_u^W = \min\left(B^W \cdot \log_2(1+\mathrm{SNR}_u^W), R_{\max}^W\right) \tag{18}$$

where B^W denotes the transmission bandwidth. Just as with LiFi we also introduce the normalized rate for WiFi as

$$\tilde{R}_u^W = \frac{R_u^W}{R_{\max}^W}. \tag{19}$$

Simulation parameters specific to the WiFi channel are summarized in Table 3.

IV. 3D PLACEMENT PROBLEM FORMULATION

The optimization variables in our 3D placement model are the amount and three-dimensional positions of LiFi and WiFi APs. The goal of the optimization is to minimize the cost of the APs placed and to maximize the network coverage in terms of the sum rate on the user grid weighted by the LiFi and WiFi user occurrence probabilities: p_u^L and p_u^W. The cost of placement is calculated with the number of placed APs and a cost unit, C^L and C^W, for each placed AP. We constrain the minimum guaranteed rate to ensure reliable coverage across the entire user plane. We define this constraint in terms of the normalized rate $\tilde{R}_{\mathrm{thresh}}$. The constraint is not placed on each technology but is rather dependent on the user's technology. If a particular user position u has 0 probability of a LiFi user, i.e., $p_u^L = 0$, then the guaranteed minimum rate ratio should be provided by WiFi. However, if u has a non-zero probability of both technologies, then at least one of the technologies should offer the guarantee. Additionally, we distinguish between using VLC or IR for LiFi. The only difference is that in the VLC model, we additionally impose a constraint I_{thresh} on the minimum illumination uniformity on the illumination grid in the indoor area.

Hence, the decision variables of this 3D placement problem are the existence and positions of the LiFi and WiFi APs. The existence of the LiFi AP $\alpha_l \in \{0,1\}$ is a binary variable denoting if the AP l is placed or not. Similarly, $\alpha_w \in \{0,1\}$ denotes the existence of the WiFi AP. The three-dimensional positions of WiFi APs can be defined with

$$x_w \in [0, x_{\mathrm{dim}}] \qquad y_w \in [0, y_{\mathrm{dim}}], \qquad w = 1, 2, \ldots, M^W$$
$$z_w \in [z_{\mathrm{mindim}}, z_{\mathrm{maxdim}}], \qquad w = 1, 2, \ldots, M^W$$

and the same for LiFi APs with

$$x_l \in [0, x_{\mathrm{dim}}] \qquad y_l \in [0, y_{\mathrm{dim}}], \qquad l = 1, 2, \ldots, M^L$$
$$z_l \in [z_{\mathrm{mindim}}, z_{\mathrm{maxdim}}], \qquad l = 1, 2, \ldots, M^L$$

where the room size constrains the x and y dimensions, and the z dimension is lower bounded by 2.5 m and upper bounded by the ceiling, which is at 3.5 m.

The optimization problem can be mathematically formulated as follows:

$$\min_{c_w, c_l, \alpha_w, \alpha_l} \quad C^W \sum_{w=1}^{M^W} \alpha_w + C^L \sum_{l=1}^{M^L} \alpha_l \tag{20}$$

$$\max_{c_w, c_l, \alpha_w, \alpha_l} \quad \sum_u \left(\tilde{R}_u^W p_u^W + \tilde{R}_u^L p_u^L\right) \tag{21}$$

$$\text{s.t.} \quad \max\left(\tilde{R}_u^L \lceil p_u^L \rceil, \tilde{R}_u^W \lceil p_u^W \rceil\right) \geq \tilde{R}_{\mathrm{thresh}}$$
$$\forall u \in \{\mathcal{U} \mid p_u^L > 0 \vee p_u^W > 0\} \tag{22}$$

$$\frac{\min_v I_v}{\frac{\sum_v I_v}{M^V}} \geq I_{thresh} \tag{23}$$

The first objective function, f_1, represented by (20), aims to minimize the cost. In contrast, the second objective function, f_2, in (21) aims to maximize the sum rate. The constraint given in (22) ensures that a minimum guaranteed rate is achieved at each position on the user grid, expressed as a ratio of the maximum supported rate. On the other hand, the constraint in (23) enforces a minimum illumination uniformity on the illumination grid, applicable only when LiFi operates in the VLC mode.

The optimization problem at hand is categorized as an MINLP problem due to the presence of both integer α and continuous (x, y, z) variables. This problem is also non-convex due to the nature of the second objective function, which makes this problem mathematically intractable and computationally complex. Moreover, this problem falls into the domain of MOO problems, where multiple conflicting objective functions are involved. In such scenarios, a unique solution does not exist, and the optimal outcome comprises a set of Pareto-optimal solutions, representing the best trade-offs between the different objective functions [20].

Apart from this 3D Placement formulation, we also look into the "2D Pow" formulation. In this problem, we consider the same optimization formulation except that instead of optimizing the height of the APs, we optimize their transmission power. This formulation can be considered as an extended State of Art. To the best of our knowledge, there is no related literature that considers placement in LiFi-WiFi

Heterogeneous networks, so we look into LiFi-only literature. The authors in [8] formulate a problem similar to our "2D Pow'' except that they aim to minimize power instead of our multi-objective formulation. We use this model to provide the readers with an alternative closer to the State of Art and discuss our model's advantages and limitations.

V. METHODS TO SOLVE THE 3D PLACEMENT PROBLEM

To solve the MINLP MOO problem described in Section IV, we propose various methods as part of a comprehensive orchestration framework. Specifically, we propose a random optimizer as a baseline, an analytical approach that can be solved by any off-the-shelf solver, and a novel heuristic Dynamic Grid Explorer (DGE). Additionally, we also utilize existing meta-heuristic and black-box optimization algorithms and adapt them to solve our problem.

A. RANDOM BASELINE

To benchmark and validate the performance of our optimization framework for AP placement in LiFi-WiFi heterogeneous networks, we implement a random optimizer as a baseline. We generate a set of random candidate solutions within the search space defined by the 3D coordinates of potential AP locations. These candidate solutions represent the network's possible placements of APs. This analysis allows us to establish a baseline performance for our optimization framework and understand how well it performs compared to a random search.

B. OPTIMIZER

To tackle the MINLP optimization problem, we employ the powerful optimization software Gurobi [21], which can efficiently find optimal solutions for relatively small and less complex instances of the problem. Gurobi specializes in handling Linear Programming (LP) problems efficiently, and while our problem is nonlinear, Gurobi can handle LP relaxations for MINLP problems.

The optimization formulation in its complete form is too complex to be solved directly, so we employ some transformations to convexify or linearize functions as much as possible. The first objective function is a linear sum of the binary variables. The second objective involves the rate achieved by the user, so this is a non-convex, nonlinear function of the optimization variables. The first step to calculating the rate for LiFi is to obtain the 3D distance, which can be done using simple auxiliary variables and the Euclidean norm function, which is convex. The next step is calculating the cosine of the angles of irradiance and incidence. This is also easily accomplished with auxiliary variables and bilinear transformations for the multiplications and divisions. Finally, these terms are combined with more bilinear constraints for multiplications to obtain the gain HLoS. Recalling (4), the gain is only non-zero within the FoV of the receiver. This involves adding IF-conditions to the problem. These can be modeled with indicator constraints in Gurobi. The indicator variables for the angle of incidence at the receiver and transmitter can be written as

$$\mathbb{1}_0 \leq \theta_{u,l} \leq \Theta_f \tag{24}$$

$$\mathbb{1}_0 \leq \phi_{u,l} \leq \Phi_f \tag{25}$$

Therefore, the gain can now be written as

$$\mathrm{HLoS}_{u,l} = \frac{H_0}{d_{u,l}^2} \cos \phi_{u,l}^m \cos \theta_{u,l} \mathbb{1}_{0\leq\theta_{u,l}\leq\Theta_f} \mathbb{1}_{0\leq\phi_{u,l}\leq\Phi_f} \tag{26}$$

This gain can easily be translated into power.

The SINR of the user can then be calculated by considering the AP offering the maximum signal strength as the connected AP. We replace the max function here with binary variables, which have the value of 1 for the AP connected to the user. For this to work as intended, an additional constraint is required to force the user to only connect to one AP by constraining the sum of the binary variables to 1. So the signal at the receiver is given by,

$$S_u^L = \sum\nolimits_l g_{u,l} P_{u,l}. \tag{27}$$

where $g_{u,l}$ is the binary association variable which is constrained by

$$\sum\nolimits_l g_{u,l} = 1 \qquad \forall u = 1, 2, \ldots, M^U \tag{28}$$

The next step to calculate the rate is the division involved in the SINR calculation. This division can be converted to subtraction using the $\log_2$ function, which converts the SINR to the rate as in (8). In contrast to the equation, we do not limit the rate to the maximum capacity of the technology since that would involve a Piecewise function. The WiFi rate for the user is calculated similarly. The second objective function is then calculated.

The constraints on the rate are achieved by fixing the lower bound of the auxiliary variable of the rate. However, this is only when the user can connect to a single technology. When the user can use both technologies, we only require that one offers the minimum guaranteed rate. So, we again introduce binary variables to select one of the two technologies and constrain the sum of the binary variables to 1 for every user.

Additionally, we have the illumination uniformity constraint in the VLC model. Recalling (23), we need to calculate the minimum illuminance in the room. For this, we introduce an auxiliary variable $I_{\min}$ representing the minimum illuminance and adding a constraint that this variable should be less than or equal to every grid position's illuminance, i.e.

$$I_{\min} \leq I_v \qquad \forall v = 1, 2, \ldots, M^V. \tag{29}$$

This simplification works due to the lower bound on the uniformity constraint, which attempts to increase the value of $I_{\min}$ as much as possible. In contrast, this constraint (29) forces it to be lower than or equal to every illuminance value I_v.

Applying these transformations allows us to simplify the problem. However, the non-convexity remains due to how the optimization variables interact in calculating the LiFi gain.

1) SINGLE OBJECTIVE OPTIMIZATION: WEIGHTED SUM

The described formulations are put together while combining the two objectives into a single objective function using the weighted sum method by weighting the cost objective with 60% and the rate maximization objective with 40%, resulting in the following representation

$$F = 0.6 \cdot f_1(\boldsymbol{c}_w, \boldsymbol{c}_l, \alpha_w, \alpha_l) + 0.4 \cdot f_2(\boldsymbol{c}_w, \boldsymbol{c}_l, \alpha_w, \alpha_l).$$

2) MULTI-OBJECTIVE OPTIMIZATION: EPSILON-CONSTRAINT

One practical approach to tackle MOO problems is the epsilon-constraint method. This method converts the multi-objective problem into a series of single-objective sub-problems by introducing an epsilon constraint for the first objective. The epsilon constraint acts as a threshold, imposing a minimum requirement on the value of the objective. By varying the value of epsilon, we can generate a set of solutions that represent trade-offs between the objectives.

$$\begin{aligned} \text{Maximize:} \quad & f_2(\boldsymbol{c}_w, \boldsymbol{c}_l, \alpha_w, \alpha_l) \\ \text{Subject to:} \quad & f_1(\boldsymbol{c}_w, \boldsymbol{c}_l, \alpha_w, \alpha_l) \leq \epsilon \end{aligned}$$

Solving these sub-problems independently yields Pareto-optimal solutions for their respective epsilon values. Finally, we select the solution that optimizes the combined objective.

3) SINGLE OBJECTIVE OPTIMIZATION: SIMPLIFIED

While Gurobi is a valuable tool for finding accurate optimal solutions for small and less complex instances of the problem, it faces limitations when dealing with more extensive and highly complex optimization problems, which are common in real-world LiFi-WiFi network deployments. To simplify the problem, we quantize the three-dimensional AP space into a grid with half the spacing of the user grid in the x and y dimensions ($\Delta x = \Delta x_u/2$ and $\Delta y = \Delta y_u/2$) and one-fourth the spacing in the z dimension ($\Delta z = \Delta z_u/4$). Then we pre-calculate the signal powers $P_{u,\tilde{l}}$, $P_{u,\tilde{w}}$ between each AP grid $(\tilde{l}, \tilde{w})$ and user grid (u) position. Subsequently, during the optimization, we select the positions that should be occupied by an AP using binary variables $\alpha_{\tilde{l}}, \alpha_{\tilde{w}}$.

$$\alpha_{\tilde{l}} = \begin{cases} 1 & \text{if LiFi AP is placed at grid point } \tilde{l} \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_{\tilde{w}} = \begin{cases} 1 & \text{if WiFi AP is placed at grid point } \tilde{w} \\ 0 & \text{otherwise} \end{cases}$$

Thus, we modify the initial objective functions to

$$\begin{aligned} \min_{\alpha_{\tilde{w}}, \alpha_{\tilde{l}}} \quad & C^W \sum\nolimits_{\tilde{w}} \alpha_{\tilde{w}} + C^L \sum\nolimits_{\tilde{l}} \alpha_{\tilde{l}} \\ \max_{\alpha_{\tilde{w}}, \alpha_{\tilde{l}}} \quad & \sum\nolimits_u \left(\tilde{R}_u^W p_u^W + \tilde{R}_u^L p_u^L \right). \end{aligned}$$

However, this formulation is still computationally complex and has a high memory requirement due to the number of potential positions that can be occupied by APs. Nevertheless, this method works well in small scenarios up to a room size of 5 x 5 m with $\Delta \boldsymbol{c}_u$ =1 m.

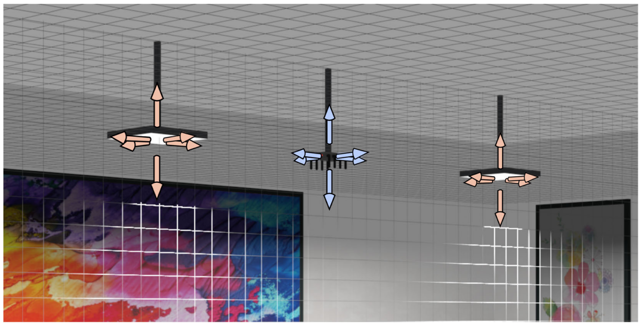


FIGURE 3. Possible movement directions for LiFi and WiFi APs in the DGE algorithm.

C. DYNAMIC GRID EXPLORER

The DGE algorithm aims to find the best configuration of LiFi and WiFi APs by exploring the three-dimensional placement space efficiently and effectively. This algorithm first fixes the number of APs like the epsilon-constraint method.

Once the number of APs is fixed, let N denote the total number of APs available for placement and let $\mathcal{C} = \{\boldsymbol{c}_0, \ldots, \boldsymbol{c}_N\}$ be the set of all coordinates. Initially, these APs are positioned randomly in the search space. In the next step, this set of positions is updated according to the following rules: Let Δ be the set of movement directions in which an AP moves

$$\Delta = \{(\pm d, 0, 0), (0, \pm d, 0), (0, 0, \pm d), (0, 0, 0)\} \tag{30}$$

with d being the multiple of the grid size, e.g., $d = 2$ denotes 2 steps on the grid. The possible movement directions are shown in Fig. 3. Then the coordinates of a single AP are updated by

$$\boldsymbol{c} = \boldsymbol{c} + \delta \mid \delta \in \Delta. \tag{31}$$

All the APs are moved in each one of these directions, and the value of the objective is calculated for all these possible movements. Let $f_{\text{dge}} : f(\mathcal{C}) \to \mathbb{R}$ denote a function that maps a set of coordinates to a real value representing our objective following the equation

$$f_{\text{dge}} = \sum\nolimits_u \left(\tilde{R}_u^W p_u^W + \tilde{R}_u^L p_u^L \right). \tag{32}$$

In order to uphold the constraints, both rate and illuminance, the objective is penalized for every constraint that has been violated. It is penalized by the worst of the constraint violations among all user positions if there exists even one user position that violates the rate constraint as described by

$$f_{\text{dge}}^{\text{corr}} = \begin{cases} f_{\text{dge}}(1 - \min\limits_u(\tilde{R}_{\text{thresh}} - \tilde{R}_u)) & \text{if } \exists u : \tilde{R}_u < \tilde{R}_{\text{thresh}} \\ f_{\text{dge}} & \text{otherwise} \end{cases} \tag{33}$$

A similar penalty is applied for violating the illumination uniformity constraint in the VLC model as described in

$$f_{\text{dge}}^{\text{corr}} = \begin{cases} f_{\text{dge}} \left(1 - \dfrac{I_{\text{thresh}} - I}{2}\right) & \text{if } I < I_{\text{thresh}} \\ f_{\text{dge}} & \text{otherwise} \end{cases} \tag{34}$$

with I representing the achieved illumination uniformity.

Then, the combination with the best objective is selected, and the APs are moved to this new position. Then, the entire process is repeated as long as the new objective is higher than the previous iteration's. When the objective stops increasing, we have either found the global optimum or are stuck in a local optimum. In order to avoid the local optimum, we increase the step size $d \leftarrow d + 1$ and continue the exploration until no more movements are possible and the grid has been explored. Once the solution for a certain number of APs has been found, we proceed to the next higher number of APs. After completing the entire process, the single optimum is selected using a weighted sum of the two objectives. Furthermore, we only select from the solutions with the least constraint violations.

D. META-HEURISTICS

The Non-dominated Sorting Genetic Algorithm (NSGA-II) [22] is a popular MOO technique that employs a genetic algorithm framework to efficiently find Pareto-optimal solutions. In MOO problems, different scalarization functions [23], [24] can be used to decompose the objectives and transform them into a single-objective problem that can be solved using standard single-objective optimization algorithms. The scalarization functions that we employ include:

- Weighted Sum: This function linearly combines the objectives with predefined weights, transforming the multi-objective problem into a single-objective problem. Given n objectives $f_i(x)$ with weights w_i, the weighted sum objective $F(x)$ can be represented as:
$$F(x) = \sum\nolimits_{i=1}^{n} w_i \cdot f_i(x)$$
- Tchebicheff: This function evaluates the objectives by considering the worst-case scenario and finding the maximum weighted deviation from the ideal point. Given n objectives $f_i(x)$ with weights w_i and an ideal point ideal(x), Tchebicheff can be represented as:
$$T(x) = \max_i \{w_i \cdot |f_i(x) - \text{ideal}_i(x)|\}$$
- Achievement Scalarization Function (ASF): ASF aims to minimize the weighted sum of deviations from the reference points, which are predefined aspiration levels for each objective. Given n objectives $f_i(x)$ with weights w_i and reference points ref$_i(x)$, ASF can be represented as:
$$\text{ASF}(x) = \sum\nolimits_{i=1}^{n} w_i \cdot |f_i(x) - \text{ref}_i(x)|$$
- Penalty-Based Boundary Intersection (PBI): PBI combines the objectives using a penalty function that encourages convergence to the Pareto front while penalizing solutions that deviate from it. Given n objectives $f_i(x)$ with weights w_i, and a penalty parameter ρ, PBI can be represented as:
$$\text{PBI}(x) = \sum\nolimits_{i=1}^{n} w_i \cdot f_i(x) + \rho \cdot \sqrt{\sum\nolimits_{i=1}^{n} (f_i(x))^2}$$

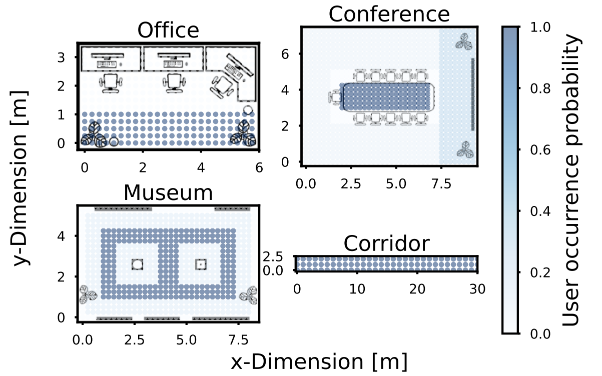


FIGURE 4. Floor plans of indoor scenarios with WiFi user occurrence probability. The pattern for LiFi probability is the same as WiFi in conference and corridor, while it is the opposite for office and museum.

By employing these scalarization functions, Genetic Algorithm (GA) can be adapted to handle the multi-objective AP placement problem. These decomposition techniques can also be employed as a Multi-Criteria Decision Making (MCDM) technique to select a single optimum from a Pareto set. In addition to the techniques mentioned above Pseudo-Weights [25] can also be used. All meta-heuristics in this paper are implemented with the help of Pymoo framework [25] adapted to suit our purpose.

E. BLACK-BOX

In this section, we explore the application of black-box optimization techniques, such as Bayesian optimization [26], Random Forest [27], and Extra Trees [28], to solve the AP placement problem. These techniques do not rely on explicit mathematical formulations of objective functions or constraints. Instead, they treat the optimization problem as a black box, where the objective function evaluations are performed without requiring gradient information or access to the underlying optimization problem structure.

VI. PERFORMANCE EVALUATIONS

A. EVALUATION METHODS

Our proposed PlaciFi framework for optimal placement of APs in LiFi-WiFi heterogeneous networks is evaluated through extensive simulations. We use a custom simulation environment implemented in Python.

To evaluate the performance of our proposed optimal 3D placement algorithm, we consider various application scenarios commonly encountered in indoor environments visualized in Fig. 4. The figure also depicts the user occurrence probability for WiFi users. It is to be noted that

TABLE 4. Simulation parameters used to evaluate our implementation of PlaciFi.

Parameter	Notation	Value
Normalized Rate threshold	$\tilde{R}_{\text{thresh}}$	1%
Illumination Uniformity threshold	I_{thresh}	0.7
Cost of placing WiFi AP	C^W	10 cost units
Cost of placing LiFi AP	C^L	5 cost units

the LiFi user probability is also considered and may not be the same as WiFi. For example, the pattern for LiFi and WiFi is the same in Conference and Corridor, while it is the opposite for Office and Museum.

- Office: This scenario represents a typical office environment where users require reliable and high-speed wireless connectivity. The office scenario includes multiple workstations where users rely on LiFi technologies for communication. While WiFi is valuable in areas of movement.
- Museum: Museums often employ wireless technologies to enhance visitor experiences. In this scenario, LiFi APs are strategically placed to provide information and interactive content to museum visitors. WiFi APs complement the coverage to support mobile users.
- Conference: Conferences and events require robust wireless connectivity to support many participants. The conference scenario involves deploying a dense network of LiFi and WiFi APs to ensure high-capacity coverage and seamless connectivity for attendees.
- Corridor: Corridors in buildings serve as critical pathways for users. In this scenario, we consider the deployment of LiFi APs and WiFi APs along corridors to provide continuous wireless connectivity for users moving through these areas.

Since we do not specifically look into a minimum illumination level and rather consider the uniformity in illumination, all these scenarios have the same requirement in terms of uniformity. Apart from these application-specific scenarios, we also look into a regular 5 x 5 m room with a uniform user occurrence probability across the area of the room. Additional simulation parameters that are used to generate results are listed in Table 4.

All collected results represent 1000 runs of the simulations. To test the validity of our claims, hypothesis testing is performed using the Mann–Whitney U test [29] with the alternative hypothesis that the distribution underlying x (left box plot) is stochastically less than the distribution underlying y (right box plot). The test results are annotated on the relevant figures with the following notation using [30].

$$\text{ns} : p > .05$$
$$* : .01 < p <= .05$$
$$** : .001 < p <= .01$$
$$*** : .0001 < p <= .001$$
$$**** : p <= .0001$$

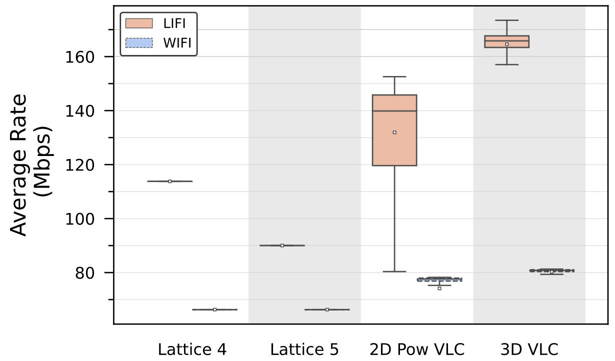


FIGURE 5. Average achievable rate for deterministic and optimized placements for the VLC model showing the need for optimization.

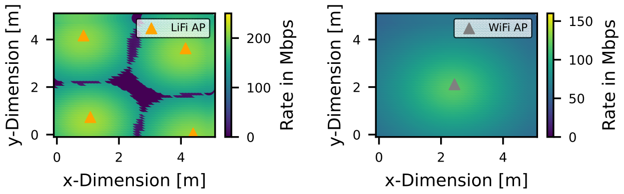
Since we perform multiple of these tests on the dataset, we also apply the Benjamini–Hochberg false discovery rate [31] controlling procedure.

B. RESULTS AND COMPARATIVE ANALYSIS

In the following section, we primarily focus on results obtained with meta-heuristics unless otherwise specified. We first study the regular room to grasp the algorithm's performance independent of real-world scenario-specific effects.

Fig. 5 illustrates a comparison of the average data rate obtained in a regular room using LiFi and WiFi technologies achieved by three different AP placement techniques: deterministic placement, 2D power optimization (2D Pow), and our proposed 3D optimized placement. The deterministic placement is accomplished using a fixed lattice grid of either 4 or 5 LiFi APs and one WiFi AP at the center of the room. All APs are placed at the same height of 3.5 m on the ceiling. In the deterministic placement scenario, APs are deployed based on predefined positions without considering any optimization strategy. As a result, the data rate coverage is sub-optimal, leading to areas with poor signal quality and lower average data rates. Compared to the deterministic placement, the 2D Pow optimized placement approach improves the overall data rate coverage on average. Deploying either 3 or 4 APs results in a larger rate spread relative to all other methods. In contrast, our suggested 3D optimized placement technique consistently utilizes 4 APs. This is due to the illumination uniformity constraint, which would be violated by placing fewer APs. Notably, this 3D technique also manages interference more effectively than other strategies. The total use of the three-dimensional space for AP placement allows this algorithm to surpass both the deterministic and 2D Pow methods in overall performance.

The reason for the better performance of our proposed 3D optimization technique is visualized in Fig. 6, where an exemplary result of the simulation can be seen. The APs leverage the height difference to achieve different coverage areas to further minimize regions of interference. Overall, the comparison emphasizes the importance of considering



(a) LiFi Rate (b) WiFi Rate

FIGURE 6. Rate coverage achieved by the meta-heuristic NSGA2 for the VLC model with our proposed optimized placement.

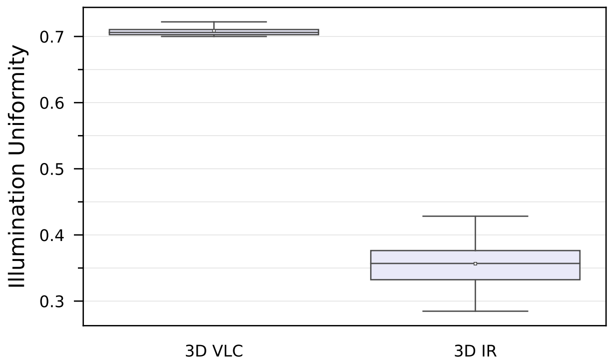


FIGURE 7. Illumination uniformity achieved using 3D VLC and 3D IR models showing the need to constrain the uniformity when using visible light.

the vertical dimension in AP placement for LiFi-WiFi heterogeneous networks in indoor spaces.

Fig. 7 compares the illumination uniformity achieved using two different techniques: 3D IR and 3D VLC. The primary difference between these two techniques lies in including the illumination uniformity constraint in the 3D VLC approach. The result demonstrates that the 3D VLC technique achieves better illumination uniformity compared to the 3D IR approach, highlighting how the 3D VLC technique effectively spreads the illumination throughout the coverage area, ensuring a more uniform distribution of light. This might seem obvious due to the missing illumination constraint in the 3D IR model, but this implies that it is necessary to consider including the constraint since just optimizing for the rate does not guarantee uniform illumination. However, the 3D IR model is still suitable when LiFi is based on IR communication, where illumination is not provided by LiFi APs. Therefore, the choice between the 3D VLC and 3D IR techniques depends on the specific requirements of the LiFi system. It is to be noted that the 3D IR model achieves a lower average rate without the illumination constraint. This is because it lowers the number of APs placed, thus still optimizing the objective function. However, this rate is still much higher than the minimum rate guarantee.

Knowing the difference between the two LiFi models, we look into how these models perform when comparing our proposed 3D optimization with the extended State-of-Art 2D Pow Model. As shown in Fig. 8, the average rate in our

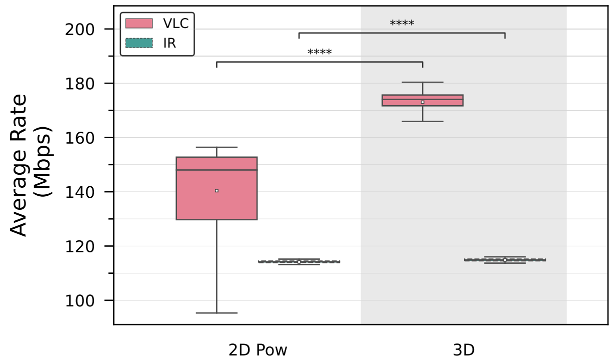


FIGURE 8. Comparing the proposed 3D model and extended State of Art 2D Pow model for both VLC and IR models of LiFi. The Figure is annotated with the significance levels of p-values achieved using hypothesis testing indicating the better performance of the proposed 3D optimization.

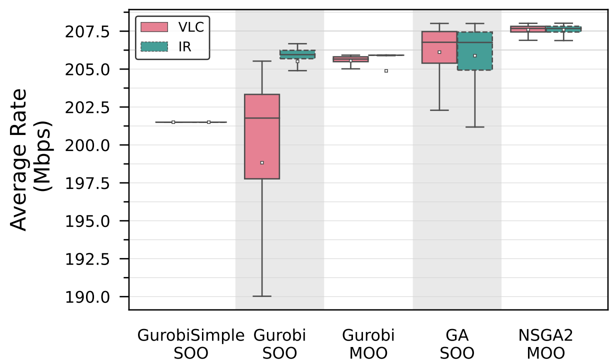
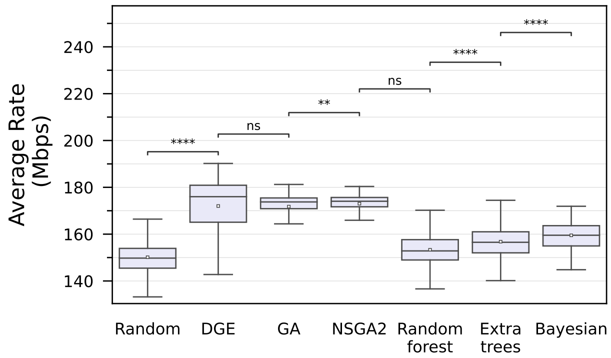


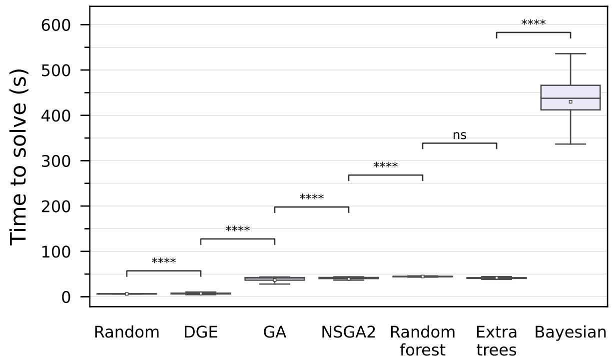
FIGURE 9. Comparing the solutions obtained with the Gurobi optimizer and meta-heuristic approaches for smaller rooms of size 3 x 2 m. The Figure displays close rate values indicating the competitive performance of the meta-heuristic solver.

proposed method is always significantly higher than that of the 2D Pow model for both LiFi models. The results of the significance tests are annotated on the figure in star notation.

Now that we have established the efficacy of our proposed 3D optimization approach, we look into the performance of the different solvers that we have proposed as part of this optimization framework. As a first step to establishing the optimality of the proposed meta-heuristic techniques, we compare them with solutions produced by various implementations with the Gurobi solver. The simulations are run for a smaller version of the regular room of size 3 x 2 m. The Fig. 9 reveals that the average rates achieved by the meta-heuristic techniques and the Gurobi solver exhibit a maximum difference of 7 Mb/s. This small range of rate differences is highly encouraging as it indicates that both approaches can deliver competitive results for the small scenario. Upon closer examination, the meta-heuristic results are slightly higher than those obtained with the Gurobi solver. This can be attributed to minor variations in the combination of the two objectives in the optimization process. Despite these slight differences, the overall performance of our proposed meta-heuristic techniques remains exceptionally close to the results obtained with the Gurobi solver.



(a) **Average Rate in Mb/s**



(b) **Time to solve in seconds**

FIGURE 10. **Comparing all proposed solvers of PlaciFi alongside a random solver as baseline focusing on the VLC model of LiFi. The Figure is annotated with the significance levels of p-values achieved using hypothesis testing, indicating the higher achieved values of each solver compared to the previous.**

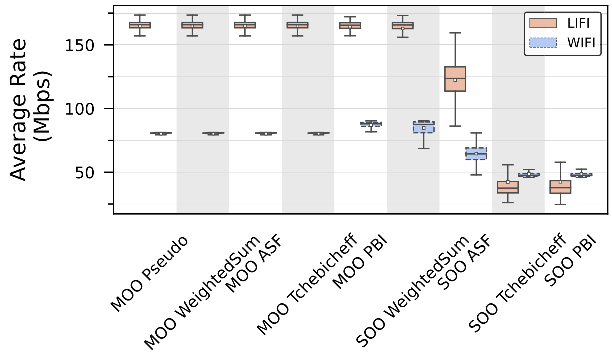


FIGURE 11. **Comparing single and multi-objective solution methods with the meta-heuristic solver focusing on the VLC model of LiFi. The Figure depicts the superior performance of the multi-objective methods.**

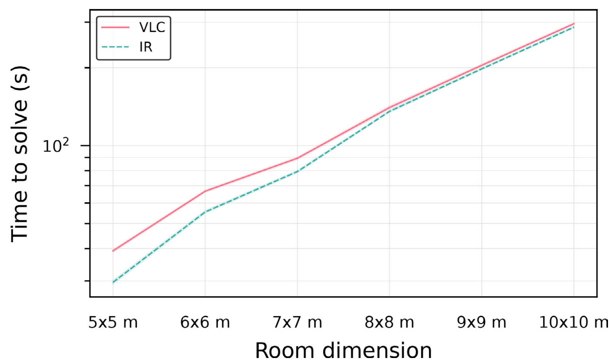


FIGURE 12. **Scalability of the meta-heuristic solvers represented by the time to solve with 95% confidence interval band for increasing room size.**

This observation is significant because it demonstrates the optimality of our results for both the VLC and IR models.

Encouraged by these results, we compared the average rates obtained using our proposed heuristic, meta-heuristic, and black-box solvers alongside a random solver as a baseline. However, we focus our attention on VLC, given the increased challenge introduced by the illumination constraint. The rate results are depicted in Fig. 10a, which shows the performance of each solver. The plot clearly illustrates that all the proposed solvers outperform the baseline random solver on average. Among the solvers, the DGE, NSGA-II, and GA stand out, achieving the highest average rates. The black-box optimization performs slightly worse than the meta-heuristics but still yields competitive results. The significance test results are annotated on the plot to validate the significance of these findings.

Additionally, we conducted a time to solve comparison for all the solvers, as depicted in Fig. 10b. The Bayesian optimization solver stands out with the longest time to solve, making the other solver times difficult to observe in the plot. Therefore, the significance test results effectively represent the comparison. The DGE exhibits very competitive performance, closely following the solving time of the random solver. However, it is essential to acknowledge that DGE has relatively high memory requirements, as it grows with a factorial complexity proportional to the number of APs, making it less scalable for larger scenarios. In conclusion, the DGE emerges as a highly effective solution for small scenarios, delivering outstanding results. For larger scenarios, the meta-heuristics, namely NSGA-II and GA, are the preferred choice, providing optimal performance with competitive solving times. The combined evaluation highlights the strengths and limitations of each solver, enabling network planners to select the most suitable approach based on their specific deployment requirements and constraints.

Looking further into meta-heuristics, we compare the average rates of LiFi and WiFi obtained by different MOO and Single-Objective Optimization (SOO) techniques for the VLC model. The solution methods utilize various decomposition and MCDM approaches to handle the multiple objectives. The results, as illustrated in Fig. 11 demonstrate that, in general, the MOO methods outperform the SOO methods. Notably, the SOO Weighted Sum method performs close to the MOO techniques, indicating its effectiveness in achieving competitive results. The MOO methods stand out as superior in handling multiple objectives, while the SOO Weighted Sum method offers a compelling alternative with comparable performance.

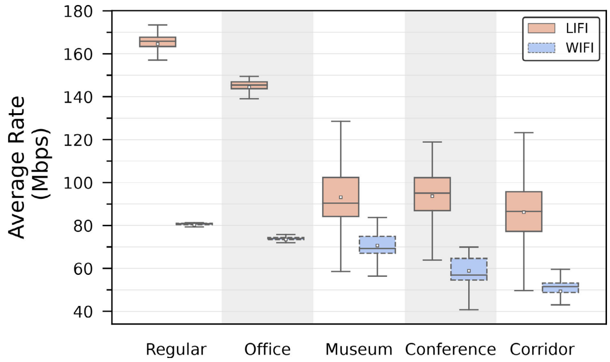
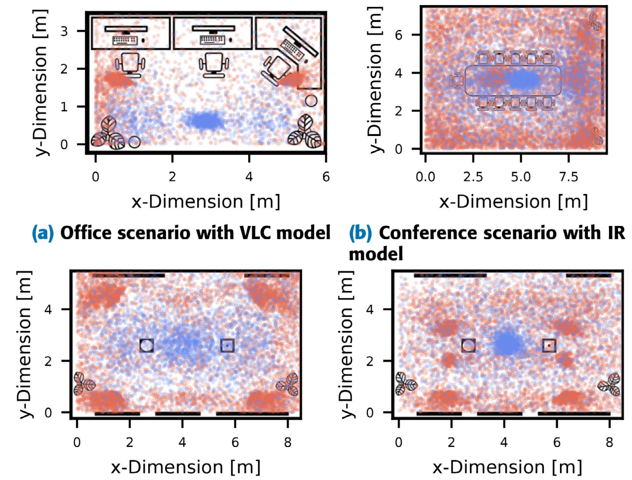


FIGURE 13. Average Rate that can be achieved with the NSGA2 solver for the VLC model of LiFi for various application scenarios.



(a) Office scenario with VLC model (b) Conference scenario with IR model

(c) Museum scenario with VLC model (d) Museum scenario with IR model

FIGURE 14. Distribution of LiFi (red) and WiFi (blue) AP positions obtained with the NSGA2 solver over 1000 runs for various application scenarios.

Finally, we investigate the scalability of our meta-heuristic solution by analyzing the time required to solve increasing room sizes in the regular scenario. We consider both the VLC and IR models to observe any differences in their performance. Fig. 12 showcases the result of this analysis. As expected, the trend of the average time to solve is primarily linear, indicating a steady increase in the computation time with the room size. However, an interesting observation is the presence of breakpoints in the slope. This is due to the change in multiple variables with changing room size, like the number of user positions and the maximum number of APs that can be placed. The VLC and IR models exhibit similar trends, indicating that the choice of communication technology does not significantly impact the scalability of the meta-heuristic solution. To provide a measure of uncertainty in the results, the figure also includes a 95% confidence interval band, which is not obviously visible in the figure due to the small width of the band. As the room size increases, the confidence interval band widens, reflecting the increasing variability in the time-to-solve for larger scenarios.

Now that we have shown the effectiveness of our optimization framework for the regular room, we assess the practical applicability of our framework in actual use cases. We conduct simulations in various indoor scenarios. In each scenario, we consider different user occurrence distributions for both LiFi and WiFi as described in Fig. 4. As seen in Fig. 13, the corridor scenario poses challenges in achieving uniform coverage and higher rates. As a result, the number of APs placed is relatively higher than in other use cases, and the range of the rates is also larger. The office layout is more amenable to the strategic placement of a smaller number of APs to provide adequate coverage for users.

Fig. 14 provides a more detailed look into the distribution of the positions of the LiFi and WiFi APs over the 1000 runs. The results clearly show that the solutions consider the users' probability patterns. In the Office scenario, the APs are predominantly placed where the occurrence of users of that technology is the highest. The LiFi APs are placed closer to the outer edges of the room to achieve uniformity in illumination for the VLC model. The Conference scenario shows a wider distribution of the APs since the probability patterns for both technologies are the same. The Museum scenario clearly shows the difference between the VLC and IR models. While both models place the APs closest to the high probability areas, the patterns are different to achieve uniformity in illumination. The LiFi APs are placed closest to the artifacts, and the WiFi APs are placed closer to the areas of user mobility. Overall, the performance of our framework across these diverse indoor scenarios demonstrates its versatility and effectiveness in tailoring AP placement to accommodate different user occurrence distributions for both LiFi and WiFi.

The presented results highlight the strengths and limitations of the proposed optimization framework, providing valuable insights for network planners and designers when considering deployment scenarios of different sizes.

VII. CONCLUSION

In this paper, we presented PlaciFi, a novel framework for orchestrating optimal 3D AP placement in LiFi-WiFi heterogeneous networks. We addressed the challenge of maximizing coverage and capacity while minimizing the cost of APs through their strategic placement.

The results of our simulations demonstrated the superiority of PlaciFi over the extended State of Art 2D Pow optimization model. By leveraging the third dimension for AP placement, PlaciFi achieved better interference management and data rate coverage, resulting in significantly higher average rates than those of the 2D models. Additionally, the heuristic, meta-heuristic, and black-box optimization techniques showcased the efficiency of PlaciFi across different scenarios, outperforming the baseline random solution method.

We explored various MOO and SOO techniques to handle the multiple objectives in our problem. The results indicated that MOO generally outperformed SOO techniques, with the

SOO Weighted Sum technique being the closest competitor to MOO methods.

Furthermore, we demonstrated the applicability of PlaciFi to real use cases, showing its performance in different indoor scenarios with varying user occurrence distributions for LiFi and WiFi. The results revealed that the AP placement is tailored to user occurrence patterns, proving the adaptability of PlaciFi to diverse deployment requirements.

A. FUTURE WORK

There are several directions to explore and expand on the findings of this paper. Firstly, while our current optimization framework assumes a stationary user distribution, future work could incorporate dynamic user mobility patterns to plan for outdoor scenarios. Energy efficiency is another crucial aspect of network design, especially in sustainable networks. While we consider this aspect by minimizing the number of APs, thereby reducing energy consumption, future research could focus on integrating energy consumption models into the optimization framework. In multi-operator scenarios, where multiple network operators coexist, the optimization framework could be extended to accommodate the objectives of different stakeholders. Our framework is adaptable to different objectives. Furthermore, the proposed framework can be readily extended to include other wireless technologies beyond LiFi and WiFi, such as emerging communication technologies like 5G or millimeter-wave.

Keeping the optimization problem the same, future work could still delve into the integration of advanced machine learning techniques to enhance the performance of the AP placement algorithm. By leveraging machine learning algorithms, the framework can learn from past deployment scenarios and make intelligent decisions to further optimize the placement process. Finally, practical implementation and validation of the proposed framework in real-world scenarios is a valuable direction for future work.

VIII. GUIDELINES FOR DEPLOYMENT

Our proposed optimization framework, PlaciFi, offers valuable tools and insights for network planners to efficiently plan rooms and deploy APs in LiFi-WiFi heterogeneous networks. The guidelines to assist in optimizing AP placement are as follows.

1) Conduct a thorough analysis of the indoor room to identify the specific requirements and constraints. Consider room size and user occurrence distribution for both LiFi and WiFi technologies.
2) Select appropriate objective functions based on the optimization goals. PlaciFi supports various MOO optimization techniques as well as SOO techniques. Consider using MOO methods to handle conflicting goals effectively.
3) Leverage the 3D AP placement capability of PlaciFi by allowing LiFi APs to be placed at varying heights. The third dimension allows for efficient interference management and better data rate coverage, thereby improving overall network quality.
4) PlaciFi can help determine the optimal number and placement of APs of different technologies to achieve a harmonious and efficient network.
5) PlaciFi supports the optimization of AP placement in various indoor scenarios, such as conference rooms, offices, corridors, and museums. So, customize the deployment strategy for each scenario, considering its specific needs.
6) While DGE offers excellent results for small to medium-sized scenarios, consider the computational complexity and memory requirements for larger rooms. For larger deployments, explore alternative solution methods, such as meta-heuristics.

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