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RESEARCH ARTICLE

Some Aczel–Alsina Power Aggregation Operators Based on Complex q-Rung Orthopair Fuzzy Set and Their Application in Multi-Attribute Group Decision-Making

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ABSTRACT The complex q-Rung orthopair fuzzy (Cq-ROF) sets are an extended version of the fuzzy sets and q-rung orthopair fuzzy circumstances. The Cq-ROF set carried extensive information about an object with two components: membership grade (MG) and non-membership grade (NMG). The MG and NMG enlarged the unit circle to a complex plane. The decision maker evaluates complex and complicated information more accurately considering the Cq-ROF domain. This article aims to present the importance of energy sources, and we discuss the application for the selection of a suitable company selection for dam construction. Dams are the most appropriate and cheap sources to produce electricity. Now, the day's importance of electricity is not ignorable. So, construct the multi-attribute group decision-making (MAGDM) methodology by utilizing Aczel-Alsina (AA) t-norm (TN) and t-conorm (TCN) operational laws with power aggregation operators (PAO) to solve the appropriate company selection problem for dam construction. PAO provides more accurate aggregation results between the several attributes. Finally, we define the Cq-ROF Aczel-Alsina power-weighted averaging (Cq-ROFAAPWA) and aggregation Cq-ROF Aczel-Alsina power-weighted geometric (Cq-ROFAAPWG) operators. Also, some fundamental axioms are discussed. By applying the proposed Cq-ROFAAPWA and Cq-ROFAAPWG operators, solve the real-life MAGDM problematic issue through a numerical example. For the superiority and applicability of proposed techniques, provide a comprehensive comparative analysis with other prevailing aggregation operators (AOs). In the end, provide solid conclusions.

INDEX TERMS q-rung orthopair fuzzy set, complex q-rung orthopair fuzzy set, multi-attribute group decision-making, Aczel-Alsina t-norm and t-conorm, aggregation operators.

I. INTRODUCTION

Crisp set theory has failed to dispose of unknown data in the field of decision-making science. To overcome this gap, in 1965, Zadeh [1] presented the innovative idea of a fuzzy set (FS), which the MG characterizes with a range [0, 1]. He has proved that the concept of FS is a powerful tool for aggregating uncertain data. Many mathematicians offered several extensions of FS theory, such as Ramot et al. [2] pioneered complex FS theory, and Gehrke et al. [3] delivered the advanced idea of interval-valued FS. In a recent discussion, although FS has many advantages in decision-making science. But for some complex situations, FS for MG fails to aggregate the information. To overcome this gap,

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Atanassov provided the idea of intuitionistic FS (IFS) with the accumulation of NMG. Its range also lies between the interval [0, 1]. IFS is also a powerful tool for expressing uncertain and fuzzy information. Furthermore, IFS has drawn much attention from mathematicians and has been effectively applied in MAGDM and engineering information decision-making sciences [4], [5], [6], [7], [8]. The IFS theory plays a vital role in solving the MAGDM problems.

When the decision maker provided such type of data that does not fall in the interval [0, 1] such as whose MG is 0.7 and NMG is 0.6, i.e., $0 \leq 0.7+0.6 = 1.31 \nleq 1$, and it will cross the limitation of the IFS, then mathematicians need some new ideas for aggregating the information. To handle this shortcoming, Yager [9] presents the theory of Pythagorean FS (PyFS). This is the extension of IFS by taking the square of MG (ɯ) and NMG (ɥ) such as $0 \leq ɯ^2 + ɥ^2 \geq 1$. Is the suitable technique for aggregating information where the FS and IFS concept has failed? However, for some more complex situations, like when the decision-maker wants to aggregate the information such as MG is 0.9 and NMG is 0.7, then the concept of PyFS fails to provide precise results, i.e., $0.9^2 + 0.7^2 = 1.30 > 1$ because it crosses the limitation of FS theory. To overcome these problems, Yager [10] presented the thought of q-Rung orthopair FS (q-ROFS) by taking the qth power of MG and NMG. The structure of q-ROFS is the generalized shape of PyFS and IFS and a more powerful tool for handling complicated and uncertain opinions of human beings.

However, with the further development of FS theory, questions were circulating among mathematicians. When we change the co-domain of the FS with complex numbers (CNs) instead of interval [0, 1] then what type of changes occur? The solution to this problem Ramot [2] provided by giving the theory of complex FS (CFS) in 2002, in which he presented the MG in the form of CNs. In addition, Salleh and Alkouri [11] also developed the idea of complex IFS (CIFS) by involving the NMG in CFS. CIFS failed for those situations in which the value of MG and NMG exceeded the interval [0, 1]. Ullah et al. [12] proposed the complex PyFS (CPyFS) concept to remove this gap. The CPyFS is defined as taking the sum of the squares of the MG and NMG. CPyFS is a more suitable and appropriate tool than IFS due to its comprehensive range for aggregating human opinions. Although the PyFS is a valuable tool for aggregating ambiguous information for some complex problems, CPyFS theory fails where the sum of the square of MG and NMG exceeds the range [0, 1]. To cover this problem, Liu et al. [12] proposed a complex Cq-ROF set (q-ROFS) by pleasing the function's q^{th} power on NMG and MG. The q-ROFS is the most important and generalized form of CIFS and CPyFS.

MAGDM is a trending research topic nowadays when the decision-maker tries to select the most suitable option from the list of groups. Many researchers proposed several MAGDM techniques using power AOs (PAOs) based on TNs and TCN. For example, CIF utilizing the idea of PAOs depends upon the TN and TCN operations was presented by Ali et al. [13], and complex cubic q-ROFS was developed for decision-making sciences by Ren et al. [14]. Rani and Garg [15] propose the idea of trigonometric AOs to select the best alternative through MAGDM, and Mahmood and Ali [16] defined the concept of CIFS through an AA power operator. The Cq-ROF set for two tuples linguistic information using MAGDM methodology described by Akram et al. [17], and confidence levels Cq-ROF set for MAGDM is diagnosed by Qiyas et al. [18]. Khan et al. [19] proposed complex T-spherical FS for PAOs. The AOs for decision-making sciences are based on confidence levels PyFS defined by Mahmood et al. [20].

Several mathematicians defined many TN and TCNs. Initially, the first concept of TN and TCNs was presented by Menger [21]. In addition, many researchers proposed several TN and TCNs under multiple fuzzy environments, such as Aczel-Alsina (AATN) and Aczel-Alsina (AATCNs) were defined by Aczel-Alsina [22], named AATN and AATCN in 1982. Archimedean TN and TCN for complex IFSs proposed by Khan et al. [23]. The PAOs based on TCN and TN operations for complex IFS are presented by Ali et al. [13]. For Cq-ROF sets, AATN and AATCN were developed by Ali and Naeem [24], and AATN and AATCN for complex IFS were proposed by Mahmood et al. [25]. Lukasiewicz TCN and TN operations were presented by Venkatesan and Sriram [26], and some operations for Einstein TCN and TN were suggested by Riaz et al. [27]. Sarkar [28] provided the dual hesitant Dombi (DHD) TCN and TN theory. Hussain et al. [29] also proposed Frank TN and TCN operations and construct AOs data aggregation.

AATN and AATCN operations play a crucial role in data aggregation through AOs because it gave high priority on variability with the movements of variable quantity. After deep studing and discussing these mentioned examples, we have clearly concluded that the structureb of Cq-ROFS is a well-organized and truthful platform to deal with the controversial and credible statistics that manifest in real-world problems. Our inspiration for writing the presented work came from the above-discussed complications facing MAGDM experts for different uncertain and fuzzy aggregation (FA) contexts for TCN and TNs. AA operations give more truthful and crucial reliability in outcomes than the other prevailing approaches.

The suggested Cq-ROFS has a significant approach to handling complicated information and applying it to solve the MAGDM problems with better preciseness in results. The structure of Cq-ROFS has the more generalized form of C-PyFS and CIFS. Suppose the imaginary part is zero from MG and NMG. The suggested structure is turned into the q-ROFS. If we consider the value of parameter $q = 2$ in our proposed q-ROFS, then it will be converted into the PyFS. Also, when we take the value of $q = 1$, the suggested q-ROFS change into the IFS. So, we concluded that the IFS and PyFS are the generalized versions of q-ROFS.

The framework of Cq-ROFS has a more reliable and suitable structure for the following reasons: the Cq-ROFS can deal with two-dimensional (2-D) uncertain and fuzzy information, including amplitude and phase term. On the other hand, the simple structure of q-ROFS has failed to aggregate information, including complex values. Also, by combining the concept of Cq-ROFS with AATCN and AATN operations, the worth of proposed AOs increases because it enhances the accuracy of aggregated data and provides more suitable results than previously existing AOs. In AATN and AATCN, the operational rule emphasizes parametric values during data aggregation, a keynote quality of these operations.

The proposed algorithm is based on the suitable site selection for constructing a water dam on a river. Dams play a vital role in recreation, flood control, water supply, hydroelectric power, waste management, river navigation, and wildlife habitat. In this regard, many researchers proposed several thoughts, such as the best suitable location selection through the MAGDM technique given by Rezaei et al. [30] and the concept for assessment of appropriate site selection for water dam selection by using fuzzy logic offered by Noori et al. [31]. The optimal place selection for building a dam through the undefined TOPSIS method provided by Noori et al. [32] and construction material selection using FS information discussed by Khan et al. [33].

The article is organized as follows: Section II offers essential definitions that would make it easy to understand the paper. Elementary operations are discussed in Section III. Section IV presents the concept of PAOs for Cq-ROFAAPWA and Cq-ROFAAPWG AOs and discusses some desirable properties. Also, the algorithm is based on the Cq-ROFSs addressed and a numerical problem for illustrating the worth of the proposed work. Section V offers a comparative analysis with other prevailing AOs and discusses the sensitivity analysis. A concrete conclusion is provided in Section VI.

II. PRELIMINARIES

This segment contains the fundamental vital concepts such as PyFS and CPyFS, q-ROFS, and Cq-ROFS that will help us understand the manuscript, also for our convenience, in this manuscript. X is to denote the non-empty set, and $ш_\eta(x)$ and $ч_\eta(x)$ represents the MG and NMG, respectively.

Definition 1 [9]: Let X be the universe of discourse, then PyFS is specified by two functions called MG and NMG such as $\eta = \left\{x, ш_\eta^2(x), ч_\eta^2(x) \,|x \in X\right\}$ here $ш_\eta^2 : X \to [0,1], ч_\eta^2 : X \to [0,1]$ also it satisfy the condition $0 \le ш_\eta^2(x) + ч_\eta^2(x) \ge 1$. And hesitancy grade X is defined as $H_\eta = 1 - \left(ш_\eta^2(x) + ч_\eta^2(x)\right)^{\frac{1}{2}}$.

Definition 2 [34]: A CPyFS $\eta = \left\{x, ш_\eta^2(x), ч_\eta^2(x) / x \in X\right\}$ is defined as in which $ш_\eta^2 : X \to \{s_1 : s_1 \in \eta, |s_1| \le 1\}$ and $ш_\eta^2 : X \to \{s_2 : s_2 \in \eta, |s_2| \le 1\}$ such as $ч_\eta^2(x) = s_1 = x_1 + iy_1$ and $ч_\eta^2(x) = s_2 = x_2 + iy_2$ given that $0 \le |s_1|^2 + |s_2|^2 \le 1$ or $ш_\eta^2(x) = \check{T}_\eta(x).e^{i2\pi W_{\check{T}_\eta(x)}}$ and $ч_\eta^2(x) = {}^\circ F_\eta(x).e^{i2\pi W_{{}^\circ F_\eta(x)}}$ satisfying the condition $0 \le \check{T}_\eta^2(x) + {}^\circ F_\eta^2(x) \le 1$ and $0 \le W_{\check{T}_\eta}^2(x) + W_{{}^\circ F_\eta}^2(x) \le 1$.

Furthermore, the hesitancy grade is defined as $H_\eta(x) = S.e^{i2\pi Z_s(x)}$ such that $\mathcal{R} = \left(1 - \left(\check{T}_\eta^2(x) + {}^\circ F_\eta^2(x)\right)\right)^{\frac{1}{2}}$ and $W_{\mathcal{R}}(x) = \left(1 - \left(P_{\check{T}_\eta}^2(x) + P_{{}^\circ F_\eta}^2(x)\right)\right)^{\frac{1}{2}}$. Then $\eta = \left(\check{T}.e^{i2\pi W_{\check{T}}}, {}^\circ F.e^{i2\pi W_{{}^\circ F}}\right)$ is known as complex Pythagorean fuzzy value (CPyFV).

Definition 3 [10]: Let X be the universe of discourse, then q-ROFS is specified by two functions called MG and NMG such as $\eta = \left\{x, ш_\eta^q(x), ч_\eta^q(x) \,|x \in X\right\}$ here $ш_\eta^q : X \to [0,1], ч_\eta^q : X \to [0,1]$ also, it satisfy the condition $0 \le ш_\eta^q(x) + ч_\eta^q(x) \ge 1$. And hesitancy grade X is defined as $H_\eta = \sqrt[q]{1 - \left(ш_\eta^q(x) + ч_\eta^q(x)\right)}$.

Definition 4 [35]: A Cq-ROFS $\eta = \left\{x, ш_\eta^q(x), ч_\eta^q(x) \,|x \in X\right\}$ is defined as in which $ш_\eta^q : X \to \{s_1 : s_1 \in \eta, |s_1| \le 1\}$ and $ш_\eta^q : X \to \{s_2 : s_2 \in \eta, |s_2| \le 1\}$ such as $ч_\eta^q(x) = s_1 = x_1 + iy_1$ and $ч_\eta^q(x) = s_2 = x_2 + iy_2$ given that $0 \le |s_1|^q + |s_2|^q \le 1$ or $ш_\eta^q(x) = \check{T}_\eta(x).e^{i2\pi W_{\check{T}_\eta(x)}}$ and $ч_\eta^q(x) = {}^\circ F_\eta(x).e^{i2\pi W_{{}^\circ F_\eta(x)}}$ satisfying the condition $0 \le \check{T}_\eta^q(x) + {}^\circ F_\eta^q(x) \le 1$ and $0 \le W_{\check{T}_\eta}^q(x) + W_{{}^\circ F_\eta}^q(x) \le 1$.

Furthermore, the hesitancy grade is defined as $H_\eta(x) = \mathcal{R}.e^{i2\pi Z_{\mathcal{R}}(x)}$ such that $\mathcal{R} = \sqrt[q]{\left(1 - \left(\check{T}_\eta^q(x) + {}^\circ F_\eta^q(x)\right)\right)}$ and $W_{\mathcal{R}}(x) = \sqrt[q]{1 - \left(W_{\check{T}_\eta}^q(x) + W_{{}^\circ F_\eta}^q(x)\right)}$. Then $\eta = \left(\check{T}.e^{i2\pi W_{\check{T}}}, {}^\circ F.e^{i2\pi W_{{}^\circ F}}\right)$ is said to be complex q-Rung orthopair fuzzy values (Cq-ROFVs). Where $ш_\eta^q(x)$ and $ч_\eta^q(x)$ are complex values in cartesian/polar format. We take $\check{T}_\eta(x) = {}^\circ F_\eta = r$ and $2\pi.W_{\check{T}_\eta}(x) = \vartheta_1, 2\pi; W_{{}^\circ F_\eta}(x) = \vartheta_2$ then each term can be interchangeable with the other.

$$
\begin{aligned}
ш_\eta^q(x) &= \check{T}_\eta(x).e^{i2\pi \check{T}_\eta(x)} \\
&= \check{T}_\eta(x).\left(\cos 2\pi W_{\check{T}_\eta}(x) + isin2\pi W_{\check{T}_\eta}(x)\right) \\
&= \check{T}_\eta(x).\cos 2\pi W_{\check{T}_\eta}(x) + i\check{T}_\eta(x).\sin 2\pi W_{\check{T}_\eta}(x) \\
&= rcos\vartheta_1 + isin\vartheta_1 = x_1 + iy_1 = s_1 \\
ч_\eta^q(x) &= {}^\circ F_\eta(x).e^{i2\pi\, {}^\circ F_\eta(x)} \\
&= {}^\circ F_\eta(x).\left(\cos 2\pi W_{{}^\circ F_\eta}(x) + isin2\pi W_{{}^\circ F_\eta}(x)\right) \\
&= {}^\circ F_\eta(x).\cos 2\pi W_{{}^\circ F_\eta}(x) \\
&\quad + i\,{}^\circ F_\eta(x).\sin 2\pi W_{{}^\circ F_\eta}(x) \\
&= rcos\vartheta_2 + isin\vartheta_2 = x_2 + iy_2 = s_2
\end{aligned}
$$

Here, we will explain how Cq-ROFS is superior to CIFS and show the advantages of Cq-ROFS with some numerical examples.

Let us take a CIFS of the form. $\{x, (0.488 + 0.017i), (0.277 + 0.037i)\}$. This set satisfies the condition of CIFS as $|0.488 + 0.017i| = 0.488$ and $|0.277 + 0.037i| = 0.279$ and $0 \le 0.488 + 0.279 \le 1$. The polar form of complex IFS values (CIFVs) is $\{x, 0.488e^{i(0.017)}, 0.277e^{i(0.037)}\}$. Whenever we take $\{x, 0.688848 + 0.038463i, 0.58883 + 0.05378i\}$ then $|0.688848 + 0.038463i| = 0.78895$ and $|0.58883 +$

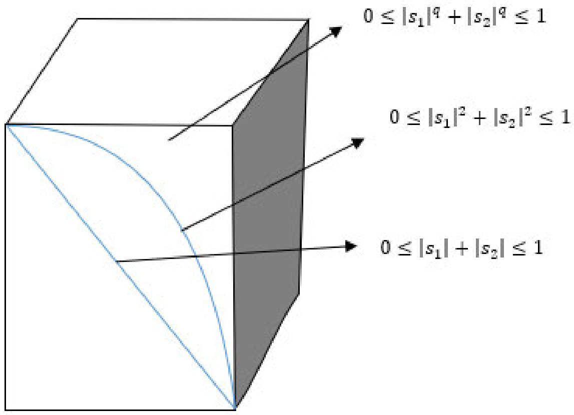

FIGURE 1. **Graphical representation shows that Cq-ROFS is superior to CIFS and CPyFS.**

$0.05378i| = 0.05912$ and $0 \le 0.78895 + 0.05912 \nleq 1$. This means the structure of CIFS cannot deal with such information. However, the thought of Cq-ROFS is allowed to deal with this type of problem and give precise results under the range of $[0, 1]$. Thus, the values in the form of $\{x, (0.688848 + 0.038463i), (0.58883 + 0.05378i)\}$ can be written as $\{x, 0.688848e^{i(0.038463)}, 0.58883e^{i(0.05378)}\}$ is called Cq-ROFVs.

Definition 5 [12]: A score value (SV) and accuracy values of function $\ddot{\Upsilon}$ on $\eta = \left(\check{T}.e^{i2\pi W_{\check{T}_\eta}}, {}^{\circ}F.e^{i2\pi W_{{}^{\circ}F_\eta}}\right)$

$$S(\eta) = \left[\left(\check{T}^q - {}^{\circ}F^q\right) + \left(W_{\check{T}^q_\eta} - W_{{}^{\circ}F^q_\eta}\right)\right] \tag{1}$$

where $SV(\eta) \in [-1, 1]$.

$$A(\eta) = \left[\left(\check{T}^q + {}^{\circ}F^q\right) + \left(W_{\check{T}^q_\eta} + W_{{}^{\circ}F^q_\eta}\right)\right] \tag{2}$$

where $A(\eta) \in [0, 1]$.

Definition 6: The ordering relation between two Cq-ROFVs η and $\dot{\eta}$ is defined as follows:

1. If $S(\eta) > S(\dot{\eta})$ then $\eta > \dot{\eta}$
2. If $S(\eta) = S(\dot{\eta})$ then
   a) If $A(\eta) > A(\dot{\eta})$ then $\eta > \dot{\eta}$
   b) If $A(\eta) = A(\dot{\eta})$ then $\eta = \dot{\eta}$

Definition 7 [36]: Consider the collection of two Cq-ROFVs $\alpha = \left(\check{T}.e^{i2\pi W_{\check{T}_\alpha}}, {}^{\circ}F.e^{i2\pi W_{{}^{\circ}F_\alpha}}\right)$ and $\beta = \left(\check{T}.e^{i2\pi W_{\check{T}_\beta}}, {}^{\circ}F.e^{i2\pi W_{{}^{\circ}F_\beta}}\right)$ then the normalized hamming distance between numbers can be described as:

$$\text{Ɗ}(\alpha, \beta) = \frac{1}{2}\begin{bmatrix} \left\{\left(\check{T}^q_\alpha - \check{T}^q_\beta\right) + \left(W_{\check{T}^q_\alpha} - W_{\check{T}^q_\beta}\right)\right\} \\ + \left\{\left({}^{\circ}F^q_\alpha - {}^{\circ}F^q_\beta\right) + \left(W_{{}^{\circ}F^q_\alpha} - W_{{}^{\circ}F^q_\beta}\right)\right\} \end{bmatrix} \tag{3}$$

Definition 8 [36]: The PAO was first introduced by Yager [36], and it is defined as

$$\text{þ}(\text{à}_1, \text{à}_2, \ldots, \text{à}_n) = \underset{\text{ȷ}=1}{\overset{n}{\oplus}} \frac{\sum_{\text{ȷ}=1}^{n}\left(1 + \check{T}(\text{đ}_\text{ȷ})\right).\text{đ}_\text{ȷ}}{\sum_{\text{ȷ}=1}^{n}\left(1 + \check{T}(\text{đ}_\text{ȷ})\right)} \tag{4}$$

where

$$\check{T}(\text{đ}_\text{ȷ}) = \sum_{\text{ȷ}=1}^{n} Sup(\text{đ}_\text{ȷ}, \text{đ}_\text{j}) \tag{5}$$

Here $\check{T}(\text{đ}_\text{ȷ}, \text{đ}_\text{j})$ is said to support of $\text{đ}_\text{ȷ}$ and đ_j that will must satisfy the following conditions discussed below:

1. $Sup(\text{đ}_\text{ȷ}, \text{đ}_\text{j}) \in [0, 1]$
2. $Sup(\text{đ}_\text{ȷ}, \text{đ}_\text{j}) = Sup(\text{đ}_\text{j}, \text{đ}_\text{ȷ})$
3. $Sup(\text{đ}_\text{ȷ}, \text{đ}_\text{j}) \ge 2.Sup(\text{đ}_\text{ȷ}, \text{đ}_\text{j})$

The PAO is the relationship among the weight vectors and aggregated values, and it depends upon the argument and makes the values combine and support one another.

Definition 9 [21]: Consider a function $Z : [0, 1] \times [0, 1] \to [0, 1]$ and assume that æ, ç, ë be three fuzzy values (FV), such that æ, ç, ë $\in [0, 1]$ if function Z satisfy the following axioms, then Z is said to be TN of the function. Such that identity element exists $Z(\text{æ}, 1) = \text{æ}$; monotonicity hold $Z(\text{æ}, \text{ç}) = Z(\text{ç}, \text{ë}), \text{æ} \le \text{ç}, \text{ç} \le \text{ë}$; associativity holds $Z((\text{æ}, \text{ç}), \text{ë}) = Z(\text{æ}, (\text{ç}, \text{ë}))$; commutativity holds $Z(\text{æ}, \text{ç}) = Z(\text{ç}, \text{æ})$.

Example 1 [21]: The example of the product of TN can be presented as $Z_{\text{æ}}(\text{ç}, \text{ë}) = \text{ç}.\text{ë}$; minimum of TN is denoted as $Z_M(\text{ç}, \text{ë}) = \min(\text{ç}, \text{ë})$; Lukasiewicz TN is defined as $Z_L(\text{ç}, \text{ë}) - \min(\text{ç} + \text{ë} - 1, 0)$; and Drastic TN is provided below as follows:

$$Z_D = (r, \mathfrak{s}) = \begin{cases} \text{ç} & if\ \text{ë} = 1 \\ \mathfrak{s} & if\ \text{ë} = 1 \\ 0 & \text{otherwise} \end{cases} \quad \forall \text{ç}, \text{ë} \in [0, 1] \tag{6}$$

Definition 10 [37]: Consider a function $Z: [0, 1] \times [0, 1] \to [0, 1]$ and assume that æ, ç, ë be three fuzzy values (FV), such that æ, ç, ë $\in [0, 1]$ if function Z satisfy the following axioms, then Z is said to be TCN of the function. Such that identity element exists $Z(\text{æ}, 0) = \text{æ}$; monotonicity holds $Z(\text{æ}, \text{ç}) = Z(\text{ç}, \text{ë}), \text{æ} \le \text{ç}, \text{ç} \le \text{ë}$; associativity holds $Z((\text{æ}, \text{ç}), \text{ë}) = Z(\text{æ}, (\text{ç}, \text{ë}))$; commutativity holds $Z(\text{æ}, \text{ç}) = Z(\text{ç}, \text{æ})$.

Example 2 [37]: The example of a product of TN can be presented as $Z_{\text{æ}}(\text{ç}, \text{ë}) = \text{ç}.\text{ë}$; maximum of TCN is denoted as $Z_M(\text{ç}, \text{ë}) = \max(\text{ç}, \text{ë})$; Lukasiewicz TCN is defined as $Z_L(\text{ç}, \text{ë}) - \max(\text{ç} + \text{ë} - 1)$; and Drastic TCN is provided as follows:

$$Z_D = (r, \mathfrak{s}) = \begin{cases} \text{ç} & if\ \text{ë} = 0 \\ \mathfrak{s} & if\ \text{ë} = 0 \\ 1 & \text{otherwise} \end{cases} \quad \forall \text{ç}, \text{ë} \in [0, 1] \tag{7}$$

Definition 11: Aczel et al. [22] 1980s diagnosed the classification of TCNs and TNs for functional equations.

The AATN can be written as

$$\left(Z_A^\lambda\right) = \begin{cases} Z_D(\text{ç}, \text{ë}) & \text{if } \lambda = 0 \\ min(\text{ç}, \text{ë}) & \text{if } \lambda = \infty \\ e^{-\left((-\ln(\text{ç}))^\lambda + (-\ln(\text{ë}))^\lambda\right)^{\frac{1}{\lambda}}} & \text{otherwise} \end{cases} \tag{8}$$

The AATCN can be written as

$$\left(Z_A^{\lambda}\right)=\begin{cases} Z_D\,(\text{ç},\text{ë}) & \text{if } \lambda=0\\ max\,(\text{ç},\text{ë}) & \text{if } \lambda=\infty\\ e^{-\left((-\ln(1-\text{ç}))^{\lambda}+(-\ln(1-\text{ë}))^{\lambda}\right)^{\frac{1}{\lambda}}} & \text{otherwise}\end{cases} \tag{9}$$

Definition 12 [10]: Let $\eta_{\jmath}=\left(\text{ɯ}_{\jmath},\text{ɥ}_{\jmath}\right),(\jmath=1,2)$ be the assembly of two q-ROFVs, here $\ddot{\Upsilon}>0$. Then

1. $\eta_1\oplus\eta_2=\left(\sqrt[q]{\text{ɯ}_1^q+\text{ɯ}_2^q-\text{ɯ}_1^q.\text{ɯ}_2^q},\text{ɥ}_1.\text{ɥ}_2\right)$
2. $\eta_1\otimes\eta_2=\left(\text{ɯ}_1.\text{ɯ}_2,\sqrt[q]{\text{ɥ}_1^q+\text{ɥ}_2^q-\text{ɥ}_1^q.\text{ɥ}_2^q}\right)$
3. $\ddot{\Upsilon}.\eta=\left(\sqrt[q]{1-(1-\text{ɯ}^q)^{\ddot{\Upsilon}}},\text{ɥ}^{\ddot{\Upsilon}}\right)$
4. $\eta^{\ddot{\Upsilon}}=\left(\text{ɯ}^{\ddot{\Upsilon}},\sqrt[q]{1-(1-\text{ɥ}^q)^{\ddot{\Upsilon}}}\right)$

III. OPERATIONAL LAWS BASED ON AATN AND AATCN

This section contains Aczel-Alsina [10], operational laws based on Cq-ROFS philosophy, and some elementary properties.

Definition 13 [38]: Let $\alpha=\left(\check{T}.e^{i2\pi W_{\check{T}_\alpha}},{}^{\circ}F.e^{i2\pi W_{{}^{\circ}F_\alpha}}\right)$ and $\beta=\left(\check{T}.e^{i2\pi W_{\check{T}_\beta}},{}^{\circ}F.e^{i2\pi W_{{}^{\circ}F_\beta}}\right)$ two Cq-ROFVs and suppose the symbols Z and Z to signify the AATCN and AATN respectively. Then the term P is considered to be union, and term Q is consider to be intersection of Cq-ROFVs can be defined as:

$$\alpha\otimes\beta=\begin{pmatrix} Z_A\left\{\check{T}.e^{i2\pi W_{\check{T}_\alpha}},\check{T}.e^{i2\pi W_{\check{T}_\beta}}\right\},\\ Z_A\left\{{}^{\circ}F.e^{i2\pi W_{{}^{\circ}F_\alpha}},{}^{\circ}F.e^{i2\pi W_{{}^{\circ}F_\beta}}\right\}\end{pmatrix}$$

$$\alpha\oplus\beta=\begin{pmatrix} Z_A\left\{{}^{\circ}F.e^{i2\pi W_{{}^{\circ}F_\alpha}},{}^{\circ}F.e^{i2\pi W_{{}^{\circ}F_\beta}}\right\},\\ Z_A\left\{\check{T}.e^{i2\pi W_{\check{T}_\alpha}},\check{T}.e^{i2\pi W_{\check{T}_\beta}}\right\}\end{pmatrix}$$

Definition 14 [38]: Let $\alpha=\left(\check{T}.e^{i2\pi W_{\check{T}_\alpha}},{}^{\circ}F.e^{i2\pi W_{{}^{\circ}F_\alpha}}\right)$, $\alpha_1=\left(\check{T}_1.e^{i2\pi W_{\check{T}_{\alpha1}}},{}^{\circ}F_1.e^{i2\pi W_{{}^{\circ}F_{\alpha1}}}\right)$ and $\alpha_2=\left(\check{T}_2.e^{i2\pi W_{\check{T}_{\alpha2}}},{}^{\circ}F_2.e^{i2\pi W_{{}^{\circ}F_{\alpha2}}}\right)$ be three q-ROFVs, with conditions such as $\mathfrak{z}\geq 1$ and $\lambda\geq 0$. Then, the AATN and AATCN operations can be explained as follows:

i.

$$\alpha_1\oplus\alpha_2=\begin{pmatrix}\sqrt[q]{1-e^{-\left(\left(-\ln\left(1-(\check{T}_1)^q\right)\right)^{\mathfrak{z}}+\left(-\ln\left(1-(\check{T}_2)^q\right)\right)^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}}\; e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\left(-\ln\left(1-(\check{T}_1)^q\right)\right)^{\mathfrak{z}}+\left(-\ln\left(1-(\check{T}_2)^q\right)\right)^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}}\right)},\\ e^{-\left((-\ln({}^{\circ}F_1))^{\mathfrak{z}}+(-\ln({}^{\circ}F_2))^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}\; e^{i2\pi\left(e^{-\left((-\ln({}^{\circ}F_1))^{\mathfrak{z}}+(-\ln({}^{\circ}F_2))^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}\right)}\end{pmatrix}$$

ii.

$$\alpha_1\otimes\alpha_2=\begin{pmatrix} e^{-\left((-\ln(\check{T}_1))^{\mathfrak{z}}+(-\ln(\check{T}_2))^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}\; e^{i2\pi\left(e^{-\left((-\ln(\check{T}_1))^{\mathfrak{z}}+(-\ln(\check{T}_2))^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}\right)},\\ \sqrt[q]{1-e^{-\left((-\ln(1-({}^{\circ}F_2)^q))^{\mathfrak{z}}+(-\ln(1-({}^{\circ}F_2)^q))^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}}\; e^{i2\pi\left(\sqrt[q]{1-e^{-\left((-\ln(1-({}^{\circ}F_2)^q))^{\mathfrak{z}}+(-\ln(1-({}^{\circ}F_2)^q))^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}}\right)}\end{pmatrix}$$

iii.

$$\lambda\alpha=\begin{pmatrix}\sqrt[q]{1-e^{-\left(\lambda\left(-\ln\left(1-(\check{T})^q\right)\right)^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}}\; e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\lambda\left(-\ln\left(1-(\check{T})^q\right)\right)^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}}\right)},\\ e^{-\left(\lambda(-\ln({}^{\circ}F))^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}\; e^{i2\pi\left(e^{-\left(\lambda(-\ln({}^{\circ}F))^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}\right)}\end{pmatrix}$$

iv.

$$\alpha^{\lambda}=\begin{pmatrix} e^{-\left(\lambda\left(-\ln(\check{T})\right)^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}\; e^{i2\pi\left(e^{-\left(\lambda(-\ln(\check{T}))^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}\right)},\\ \sqrt[q]{1-e^{-\left(\lambda(-\ln(1-({}^{\circ}F)^q))^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}}\; e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\lambda(-\ln(1-({}^{\circ}F)^q))^{\mathfrak{z}}\right)^{\frac{1}{\mathfrak{z}}}}}\right)}\end{pmatrix}$$

IV. CQ-ROF POWER AGGREGATION OPERATOR

Applying the operational rules defined in Definition (14) Based on Aczel-Alsina TN and TCN, we establish new Cq-ROFAAWA and Cq-ROFAAWG operators.

Definition 15: Let $\text{à}=\left(\check{T}_{\jmath}.e^{i2\pi W_{\check{T}_{\jmath}}},{}^{\circ}F_{\jmath}.e^{i2\pi W_{{}^{\circ}F_{\jmath}}}\right),(\jmath=1,2,\ldots,n)$ be the group of Cq-ROFVs, and the Cq-ROFAAWA operator is defined as: $\mu^n\to\mu$, if

$$Cq-ROFAAPWA\,(\text{à}_1,\text{à}_2,\ldots,\text{à}_n)=\bigoplus_{\jmath=1}^{n}\frac{\sum_{\jmath=1}^{n}\left(1+\check{T}\left(\text{đ}_{\jmath}\right)\right)\text{đ}_{\jmath}}{\sum_{\jmath=1}^{n}\left(1+\check{T}\left(\text{đ}_{\jmath}\right)\right)} \tag{10}$$

where μ be the collection of all Cq-ROFVs and $\check{T}\left(\text{đ}_{\jmath}\right)=\sum_{\jmath=1}^{n}Sup\left(\text{đ}_{\jmath},\text{đ}_{\jmath}\right)$, then Cq-ROF AA power weighted averaging operator (AAPWAO). Suppose

$$\varepsilon_{\jmath}=\frac{\left(1+\check{T}\left(\text{đ}_{\jmath}\right)\right)}{\sum_{\jmath=1}^{n}\left(1+\check{T}\left(\text{đ}_{\jmath}\right)\right)}$$

then (4) will become as

$$Cq-ROFAAPWA\,(\text{à}_1,\text{à}_2,\ldots,\text{à}_n)=\bigoplus_{\jmath=1}^{n}\varepsilon_{\jmath}\text{đ}_{\jmath}$$

Theorem 1: Let $\text{à}_{\jmath}=\left(\check{T}_{\jmath}.e^{i2\pi W_{\check{T}_{\jmath}}},{}^{\circ}F_{\jmath}.e^{i2\pi W_{{}^{\circ}F_{\jmath}}}\right),(\jmath=1,2,\ldots,n)$ be the Cq-ROFVs group using Definition 4. aggregation results from Cq-ROFAAPWA is also Cq-ROFV.

$$Cq-ROFAAPWA\,(\text{à}_1,\text{à}_2,\ldots,\text{à}_n)=\bigoplus_{\jmath=1}^{n}\left(\alpha_{\jmath}\varpi_{\jmath}\right)$$

$$= \begin{pmatrix} \sqrt[q]{1-e^{-\left(\sum_{ȷ=1}^{n}\xi_ȷ\left(-\ln\left(1-Ť_ȷ^q\right)\right)^{z}\right)^{\frac{1}{z}}}} \\ e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\sum_{ȷ=1}^{n}\xi_ȷ\left(-\ln\left(1-Ť_ȷ^q\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)}, \\ e^{-\left(\sum_{ȷ=1}^{n}\xi_ȷ\left(-\ln\left(°F_ȷ\right)\right)^{z}\right)^{\frac{1}{z}}} \\ e^{i2\pi\left(e^{-\left(\sum_{ȷ=1}^{n}\xi_ȷ\left(-\ln\left(°F_ȷ\right)\right)^{z}\right)^{\frac{1}{z}}}\right)} \end{pmatrix}. \quad (11)$$

where $\xi_ȷ$ ($ȷ = 1, 2, \ldots, n$) be the group of integrated weights such as

$$\varepsilon_ȷ = \frac{\varpi_ȷ\left(1+Ť\left(đ_ȷ\right)\right)}{\sum_{ȷ=1}^{n}\varpi_ȷ\left(1+Ť\left(đ_ȷ\right)\right)}$$

And it is always. $\varepsilon_ȷ > 0$ and $\sum_{ȷ=1}^{n}\varpi_ȷ = 1$.

Proof: By using the AA operational rule and mathematical induction rule, take $n = 2$, then we have

$$à_1\varpi_1 = \left\{ \begin{matrix} \sqrt[q]{1-e^{-\left(\xi_1\left(-\ln\left(1-Ť_1^q\right)\right)^{z}\right)^{\frac{1}{z}}}} \\ e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\xi_1\left(-\ln\left(1-Ť_1^q\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)}, \\ e^{-\left(\xi_1\left(-\ln\left(°F_1\right)\right)^{z}\right)^{\frac{1}{z}}} \\ e^{i2\pi\left(e^{-\left(\xi_1\left(-\ln\left(°F_1\right)\right)^{z}\right)^{\frac{1}{z}}}\right)} \end{matrix} \right\}$$

$$à_2\varpi_2 = \left\{ \begin{matrix} \sqrt[q]{1-e^{-\left(\xi_2\left(-\ln\left(1-Ť_2^q\right)\right)^{z}\right)^{\frac{1}{z}}}} \\ e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\xi_2\left(-\ln\left(1-Ť_2^q\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)}, \\ e^{-\left(\xi_2\left(-\ln\left(°F_2\right)\right)^{z}\right)^{\frac{1}{z}}} \\ e^{i2\pi\left(e^{-\left(\xi_2\left(-\ln\left(°F_2\right)\right)^{z}\right)^{\frac{1}{z}}}\right)} \end{matrix} \right\}$$

Consider

$$v = \sqrt[q]{1-e^{-\left(\xi_1\left(-\ln\left(1-Ť_1^q\right)\right)^{z}+\xi_1\left(-\ln\left(1-Ť_2^q\right)\right)^{z}\right)^{\frac{1}{z}}}} \times e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\xi_1\left(-\ln\left(1-Ť_1^q\right)\right)^{z}+\xi_1\left(-\ln\left(1-Ť_2^q\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)}$$

Then

$$\ln\left(1-v^q\right) = -\begin{pmatrix} \xi_1\left(-\ln\left(1-Ť_1^q\right)\right)^{z} \\ +\xi_1\left(-\ln\left(1-Ť_2^q\right)\right)^{z} \end{pmatrix}^{\frac{1}{z}} \times e^{i2\pi\left(-\begin{pmatrix} \xi_1\left(-\ln\left(1-Ť_1^q\right)\right)^{z} \\ +\xi_1\left(-\ln\left(1-Ť_2^q\right)\right)^{z} \end{pmatrix}^{\frac{1}{z}}\right)}$$

by using this, we obtain the following:

$$Cq-ROFAAPWA\left(à_1, à_2, \ldots, à_n\right) = à_1\varpi_1 \oplus à_2\varpi_2$$

$$= \left\{ \begin{matrix} \sqrt[q]{1-e^{-\left(\xi_1\left(-\ln\left(1-Ť_1^q\right)\right)^{z}\right)^{\frac{1}{z}}}} \\ e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\xi_1\left(-\ln\left(1-Ť_1^q\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)}, \\ e^{-\left(\xi_1\left(-\ln\left(°F_1\right)\right)^{z}\right)^{\frac{1}{z}}} e^{i2\pi\left(e^{-\left(\xi_1\left(-\ln\left(°F_1\right)\right)^{z}\right)^{\frac{1}{z}}}\right)} \end{matrix} \right\} \oplus \left\{ \begin{matrix} \sqrt[q]{1-e^{-\left(\xi_2\left(-\ln\left(1-Ť_2^q\right)\right)^{z}\right)^{\frac{1}{z}}}} \\ e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\xi_2\left(-\ln\left(1-Ť_2^q\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)}, \\ e^{-\left(\xi_2\left(-\ln\left(°F_2\right)\right)^{z}\right)^{\frac{1}{z}}} e^{i2\pi\left(e^{-\left(\xi_2\left(-\ln\left(°F_2\right)\right)^{z}\right)^{\frac{1}{z}}}\right)} \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} \sqrt[q]{1-e^{-\left(\xi_1\left(-\ln\left(1-Ť_1^q\right)\right)^{z}+\xi_2\left(-\ln\left(1-Ť_2^q\right)\right)\right)^{\frac{1}{z}}}} \\ e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\xi_1\left(-\ln\left(1-Ť_1^q\right)\right)^{z}+\xi_2\left(-\ln\left(1-Ť_2^q\right)\right)\right)^{\frac{1}{z}}}}\right)}, \\ e^{-\left(\xi_1\left(-\ln\left(°F_1\right)\right)^{z}+\xi_2\left(-\ln\left(°F_2\right)\right)^{z}\right)^{\frac{1}{z}}} \\ e^{i2\pi\left(e^{-\left(\xi_1\left(-\ln\left(°F_1\right)\right)^{z}+\xi_2\left(-\ln\left(°F_2\right)\right)^{z}\right)^{\frac{1}{z}}}\right)} \end{matrix} \right\}$$

$$= \begin{pmatrix} \sqrt[q]{1-e^{-\left(\sum_{ȷ=1}^{2}\xi_2\left(-\ln\left(1-Ť_2^q\right)\right)^{z}\right)^{\frac{1}{z}}}} \\ i2\pi\left(\sqrt[q]{1-e^{-\left(\sum_{ȷ=1}^{2}\xi_2\left(-\ln\left(1-Ť_2^q\right)\right)^{z}\right)^{\frac{1}{z}}}}\right), \\ e^{-\left(\sum_{ȷ=1}^{2}\xi_2\left(-\ln\left(°F_2\right)\right)^{z}\right)^{\frac{1}{z}}} e^{2\pi i\left(e^{-\left(\sum_{ȷ=1}^{2}\xi_2\left(-\ln\left(°F_2\right)\right)^{z}\right)^{\frac{1}{z}}}\right)} \end{pmatrix}$$

Hence the statement is true for $n = 2$.

Now we take $n = k$, then we have

$$Cq-ROFAAPWA\left(à_1, à_2, \ldots, à_k\right) = \bigoplus_{ȷ=1}^{k}\left(à_ȷ\varpi_ȷ\right)$$

$$= \begin{pmatrix} \sqrt[q]{1-e^{-\left(\sum_{ȷ=1}^{k}\xi_ȷ\left(-\ln\left(1-Ť_ȷ^q\right)\right)^{z}\right)^{\frac{1}{z}}}} \\ e^{2\pi i\left(\sqrt[q]{1-e^{-\left(\sum_{ȷ=1}^{k}\xi_ȷ\left(-\ln\left(1-Ť_ȷ^q\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)}, \\ e^{-\left(\sum_{ȷ=1}^{k}\xi_ȷ\left(-\ln\left(°F_ȷ\right)\right)^{z}\right)^{\frac{1}{z}}} e^{2\pi i\left(e^{-\left(\sum_{ȷ=1}^{k}\xi_ȷ\left(-\ln\left(°F_ȷ\right)\right)^{z}\right)^{\frac{1}{z}}}\right)} \end{pmatrix}$$

Hence, the declaration is true for $n = k$.

Consider the declaration is true for $n = k + 1$, then.

$$Cq-ROFAAPWA\left(à_1, à_2, \ldots, à_k, à_{k+1}\right) = \bigoplus_{ȷ=1}^{k}\left(à_ȷ\varpi_ȷ\right) \oplus \left(à_{k+1}\varpi_{k+1}\right)$$

$$=\left(\begin{array}{c}\sqrt[q]{1-e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left(1-\check{T}_j^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\,e^{\sqrt[q]{1-e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left(1-\check{T}_j^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}},\\ e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left({}^{\circ}F_j\right)\right)^{z}\right)^{\frac{1}{z}}}\,e^{e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left({}^{\circ}F_j\right)\right)^{z}\right)^{\frac{1}{z}}}}\end{array}\right)$$

Hence, the declaration is true for $n=k+1$.

Theorem 2 (Idempotency): If $\grave{a}_j=\left(\check{T}_j.e^{i2\pi W_{\check{T}_j}},{}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right)$, $(j=1,2,\ldots,n)$ be the group of Cq-ROFVs that $\grave{a}_j=\grave{a}$ then we have

$$Cq-ROFAAPWA\,(\grave{a}_1,\grave{a}_2,\ldots,\grave{a}_n)=\grave{a}$$

Proof: Since $\grave{a}_j=\left(\check{T}_j.e^{i2\pi W_{\check{T}_j}},{}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right)$, $(j=1,2,\ldots,n)$, then by using (11), we have

$$\begin{aligned}&Cq-ROFAAPWA\,(\grave{a}_1,\grave{a}_2,\ldots,\grave{a}_n)\\&=\underset{j=1}{\overset{n}{\oplus}}\left(\alpha_j\varpi_j\right)\\&=\left(\begin{array}{c}\sqrt[q]{1-e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-\check{T}_j^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\,e^{2\pi i\left(\sqrt[q]{1-e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-\check{T}_j^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)},\\ e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left({}^{\circ}F_j\right)\right)^{z}\right)^{\frac{1}{z}}}\,e^{2\pi i\left(-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left({}^{\circ}F_j\right)\right)^{z}\right)^{\frac{1}{z}}\right)}\end{array}\right)\\&=\left(\begin{array}{c}\sqrt[q]{1-e^{\ln\left(1-\check{T}_j^{q}\right)}}\,i2\pi\left(\sqrt[q]{1-e^{\ln\left(1-\check{T}_j^{q}\right)}}\right),\\ e^{\ln\left({}^{\circ}F_j\right)}e^{i2\pi\left(e^{\ln\left({}^{\circ}F_j\right)}\right)}\end{array}\right)\\&=\sqrt[q]{\check{T}_j^{q}}\,e^{i2\pi\left(\sqrt[q]{\check{T}_j^{q}}\right)},\ {}^{\circ}Fe^{i2\pi\left({}^{\circ}F_j\right)}\\&=\left(\check{T}_j.e^{i2\pi W_{\check{T}_j}},{}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right)=\grave{a}\end{aligned}$$

Thus, $Cq-ROFAAPWA\,(\grave{a}_1,\grave{a}_2,\ldots,\grave{a}_n)$ satisfied.

Theorem 3 (Boundedness): If $\grave{a}_j=\left(\check{T}_j.e^{i2\pi W_{\check{T}_j}},{}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right)$, $(j=1,2,\ldots,n)$ be the group of Cq-ROFVs that is $\grave{a}^-=\min(\grave{a}_1,\grave{a}_2,\ldots,\grave{a}_n)$ and $\grave{a}^+=\max(\grave{a}_1,\grave{a}_2,\ldots,\grave{a}_n)$ then condition holds for boundedness $\grave{a}^-\leq Cq-ROFAAPWA(\grave{a}_1,\grave{a}_2,\ldots,\grave{a}_n)\leq\grave{a}^+$.

Proof: Suppose that $\grave{a}_j=\left(\check{T}_j.e^{i2\pi W_{\check{T}_j}},{}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right)$, $(j=1,2,\ldots,n)$ be the group of Cq-ROFVs. Let $\grave{a}^-=\min(\grave{a}_1,\grave{a}_2,\ldots,\grave{a}_n)=\left(\check{T}_j^-.e^{i2\pi W_{\check{T}_j^-}},{}^{\circ}F_j^-.e^{i2\pi W_{{}^{\circ}F_j^-}}\right)$ and $\grave{a}^-=\max(\grave{a}_1,\grave{a}_2,\ldots,\grave{a}_n)=\left(\check{T}_j^+.e^{i2\pi W_{\check{T}_j^+}},{}^{\circ}F_j^+.e^{i2\pi W_{{}^{\circ}F_j^+}}\right)$ then we have $\grave{a}^-=\min\left(\check{T}_j.e^{i2\pi W_{\check{T}_j}}\right)$, $\grave{a}^-=\max\left({}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right)$, $\grave{a}^+=\max\left(\check{T}_j.e^{i2\pi W_{\check{T}_j}}\right)$, $\grave{a}^+=\min\left({}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right)$

$$\left(\sqrt[q]{1-e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-\left(\check{T}_j^-\right)^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\,e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-\left(\check{T}_j^-\right)^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)}\right)$$

$$\leq\left(\sqrt[q]{1-e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-\left(\check{T}_j\right)^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\,e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-\left(\check{T}_j\right)^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)}\right)$$

$$\leq\left(\sqrt[q]{1-e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-\left(\check{T}_j^+\right)^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\,e^{i2\pi\left(\sqrt[q]{1-e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-\left(\check{T}_j^+\right)^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)}\right)$$

$$\left(e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln{}^{\circ}F_j^-\right)^{z}\right)^{\frac{1}{z}}}\,e^{2\pi i\left(e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln{}^{\circ}F_j^-\right)^{z}\right)^{\frac{1}{z}}}\right)}\right)$$

$$\leq\left(e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln{}^{\circ}F_j^-\right)^{z}\right)^{\frac{1}{z}}}\,e^{i2\pi\left(e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln{}^{\circ}F_j^-\right)^{z}\right)^{\frac{1}{z}}}\right)}\right)$$

$$\leq\left(e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln{}^{\circ}F_j^+\right)^{z}\right)^{\frac{1}{z}}}\,e^{i2\pi\left(e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln{}^{\circ}F_j^+\right)^{z}\right)^{\frac{1}{z}}}\right)}\right)$$

Thus $\grave{a}^-\leq Cq-ROFAAPWA\,(\grave{a}_1,\grave{a}_2,\ldots,\grave{a}_n)\leq\grave{a}^+$ satisfied.

Theorem 4 (Monotonicity): Let $\grave{a}_j=\left(\check{T}_j.e^{i2\pi W_{\check{T}_j}},{}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right)$, $(j=1,2,\ldots,n)$ be the group of Cq-ROFVs, if $\grave{a}\leq\grave{a}'$ then $Cq-ROFAAPWA(\grave{a}_1,\grave{a}_2,\ldots,\grave{a}_n)\leq Cq-ROFAAPWA(\grave{a}_1',\grave{a}_2',\ldots,\grave{a}_n')$.

Theorem 5: Let $\grave{a}_j=\left(\check{T}_j.e^{i2\pi W_{\check{T}_j}},{}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right)$, $(j=1,2,\ldots,n)$ be the Cq-ROFVs group using the Definition (4). aggregation results from Cq-ROFAAPWG are also Cq-ROFV.

$$\begin{aligned}&Cq-ROFAAPWG\,(\grave{a}_1,\grave{a}_2,\ldots,\grave{a}_n)\\&=\underset{j=1}{\overset{n}{\oplus}}\left(\alpha_j\varpi_j\right)\\&=\left(\begin{array}{c}\sqrt[q]{e^{-\left(\sum_{i=1}^{n}\xi_j\left(-\ln\left(\check{T}_j^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\,e^{2\pi i\left(\sqrt[q]{e^{-\left(\sum_{i=1}^{n}\xi_j\left(-\ln\left(\check{T}_j^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)},\\ 1-e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-{}^{\circ}F_j\right)\right)^{z}\right)^{\frac{1}{z}}}\,e^{2\pi i\left(1-e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-{}^{\circ}F_j\right)\right)^{z}\right)^{\frac{1}{z}}}\right)}\end{array}\right)\end{aligned}\qquad(12)$$

where ξ_j $(j = 1, 2, \ldots, n)$ be the group of integrated weights such as

$$\varepsilon_j = \frac{\varpi_j\left(1 + \check{T}\left(đ_j\right)\right)}{\sum_{j=1}^{n} \varpi_j\left(1 + \check{T}\left(đ_j\right)\right)}$$

And it is always. $\varepsilon_j > 0$ and $\sum_{j=1}^{n} \varpi_j = 1$.

Proof: By using the AA operational rule and mathematical induction rule, take $n = 2$, then we have

$$\grave{a}_1\varpi_1 = \left\{\begin{array}{c} \sqrt[q]{e^{-\left(\xi_1\left(-\ln\left(\check{T}_1^q\right)\right)^z\right)^{\frac{1}{z}}}}\, e^{i2\pi\left(\sqrt[q]{e^{-\left(\xi_1\left(-\ln\left(\check{T}_1^q\right)\right)^z\right)^{\frac{1}{z}}}}\right)}, \\ 1 - e^{-\left(\xi_1\left(-\ln\left(1-{}^{\circ}F_1\right)\right)^z\right)^{\frac{1}{z}}}\, e^{i2\pi\left(1-e^{-\left(\xi_1\left(-\ln\left(1-{}^{\circ}F_1\right)\right)^z\right)^{\frac{1}{z}}}\right)} \end{array}\right\}$$

$$\grave{a}_2\varpi_2 = \left\{\begin{array}{c} \sqrt[q]{e^{-\left(\xi_2\left(-\ln\left(\check{T}_2^q\right)\right)^z\right)^{\frac{1}{z}}}}\, e^{i2\pi\left(\sqrt[q]{e^{-\left(\xi_2\left(-\ln\left(\check{T}_2^q\right)\right)^z\right)^{\frac{1}{z}}}}\right)}, \\ \sqrt[q]{1 - e^{-\left(\xi_2\left(-\ln\left(1-{}^{\circ}F_2\right)\right)^z\right)^{\frac{1}{z}}}}\, e^{i2\pi\left(1-e^{-\left(\xi_2\left(-\ln\left(1-{}^{\circ}F_2\right)\right)^z\right)^{\frac{1}{z}}}\right)} \end{array}\right\}$$

$$Cq-ROFAAPWG\left(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_n\right) = \grave{a}_1\varpi_1 \oplus \grave{a}_2\varpi_2$$

$$= \left\{\begin{array}{c} \sqrt[q]{e^{-\left(\xi_1\left(-\ln\left(\check{T}_1^q\right)\right)^z\right)^{\frac{1}{z}}}}\, e^{i2\pi\left(\sqrt[q]{e^{-\left(\xi_1\left(-\ln\left(\check{T}_1^q\right)\right)^z\right)^{\frac{1}{z}}}}\right)}, \\ 1 - e^{-\left(\xi_1\left(-\ln\left(1-{}^{\circ}F_1\right)\right)^z\right)^{\frac{1}{z}}}\, e^{i2\pi\left(1-e^{-\left(\xi_1\left(-\ln\left(1-{}^{\circ}F_1\right)\right)^z\right)^{\frac{1}{z}}}\right)} \end{array}\right\} \oplus \left\{\begin{array}{c} \sqrt[q]{e^{-\left(\xi_2\left(-\ln\left(\check{T}_2^q\right)\right)^z\right)^{\frac{1}{z}}}}\, e^{i2\pi\left(\sqrt[q]{e^{-\left(\xi_2\left(-\ln\left(\check{T}_2^q\right)\right)^z\right)^{\frac{1}{z}}}}\right)}, \\ 1 - e^{-\left(\xi_2\left(-\ln\left(1-{}^{\circ}F_2\right)\right)^z\right)^{\frac{1}{z}}}\, e^{i2\pi\left(1-e^{-\left(\xi_2\left(-\ln\left(1-{}^{\circ}F_2\right)\right)^z\right)^{\frac{1}{z}}}\right)} \end{array}\right\}$$

$$= \left\{\begin{array}{c} \sqrt[q]{e^{-\left(\xi_1\left(-\ln\left(\check{T}_1^q\right)\right)^z+\xi_2\left(-\ln\left(\check{T}_2^q\right)\right)\right)^{\frac{1}{z}}}} \\ e^{i2\pi\left(\sqrt[q]{e^{-\left(\xi_1\left(-\ln\left(\check{T}_1^q\right)\right)^z+\xi_2\left(-\ln\left(\check{T}_2^q\right)\right)\right)^{\frac{1}{z}}}}\right)}, \\ 1 - e^{-\left(\xi_1\left(-\ln\left(1-{}^{\circ}F_1\right)\right)^z+\xi_2\left(-\ln\left(1-{}^{\circ}F_2\right)\right)^z\right)^{\frac{1}{z}}} \\ e^{i2\pi\left(1-e^{-\left(\xi_1\left(-\ln\left(1-{}^{\circ}F_1\right)\right)^z+\xi_2\left(-\ln\left(1-{}^{\circ}F_2\right)\right)^z\right)^{\frac{1}{z}}}\right)} \end{array}\right\}$$

$$= \left(\begin{array}{c} \sqrt[q]{e^{-\left(\sum_{j=1}^{2}\xi_2\left(-\ln\left(\check{T}_2^q\right)\right)^z\right)^{\frac{1}{z}}}}\, e^{i2\pi\left(\sqrt[q]{e^{-\left(\sum_{j=1}^{2}\xi_2\left(-\ln\left(\check{T}_2^q\right)\right)^z\right)^{\frac{1}{z}}}}\right)}, \\ 1-e^{-\left(\sum_{j=1}^{2}\xi_2\left(-\ln\left(1-{}^{\circ}F_2\right)\right)^z\right)^{\frac{1}{z}}},\, e^{2\pi i\left(-\left(\sum_{j=1}^{2}\xi_2\left(-\ln\left(1-{}^{\circ}F_2\right)\right)^z\right)^{\frac{1}{z}}\right)} \end{array}\right)$$

Hence, the declaration is true for $n = 2$.

Now we take $n = k$, then we have

$$Cq-ROFAAPWG\left(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_k\right) = \bigoplus_{j=1}^{k}\left(\grave{a}_j\varpi_j\right)$$

$$= \left(\begin{array}{c} \sqrt[q]{e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left(\check{T}_j^q\right)\right)^z\right)^{\frac{1}{z}}}}\, e^{i2\pi\left(\sqrt[q]{e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left(\check{T}_j^q\right)\right)^z\right)^{\frac{1}{z}}}}\right)}, \\ 1-e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left(1-{}^{\circ}F_j\right)\right)^z\right)^{\frac{1}{z}}}\, e^{2\pi i\left(1-e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left(1-{}^{\circ}F_j\right)\right)^z\right)^{\frac{1}{z}}}\right)} \end{array}\right)$$

Hence, the declaration is true for $n = k$.

Consider the declaration is true for $n = k + 1$, then.

$$Cq-ROFAAPWG\left(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_k, \grave{a}_{k+1}\right) = \bigoplus_{j=1}^{k}\left(\grave{a}_j\varpi_j\right) \oplus \left(\grave{a}_{k+1}\varpi_{k+1}\right)$$

$$= \left(\begin{array}{c} \sqrt[q]{e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left(\check{T}_j^q\right)\right)^z\right)^{\frac{1}{z}}}} \\ e^{i2\pi\left(\sqrt[q]{e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left(\check{T}_j^q\right)\right)^z\right)^{\frac{1}{z}}}}\right)}, \\ 1 - e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left(1-{}^{\circ}F_j\right)\right)^z\right)^{\frac{1}{z}}} \\ e^{2\pi i\left(1-e^{-\left(\sum_{j=1}^{k}\xi_j\left(-\ln\left(1-{}^{\circ}F_j\right)\right)^z\right)^{\frac{1}{z}}}\right)} \end{array}\right)$$

Hence, the declaration is true for $n = k + 1$.

Theorem 6 (Idempotency): If $\grave{a}_j = \left(\check{T}_j.e^{i2\pi W_{\check{T}_j}}, {}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right)$, $(j = 1, 2, \ldots, n)$ be the group of Cq-ROFVs that $\grave{a}_j = \grave{a}$ then we have

$$Cq-ROFAAPWG\left(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_n\right) = \grave{a}$$

Proof: Since $\grave{a}_j = \left(\check{T}_j.e^{i2\pi W_{\check{T}_j}}, {}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right)$, $(j = 1, 2, \ldots, n)$, then by using Equation 11. we have

$$Cq-ROFAAPWG\left(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_n\right) = \bigoplus_{j=1}^{n}\left(\alpha_j\varpi_j\right)$$

$$= \left(\begin{array}{c} \sqrt[q]{e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(\check{T}_j^q\right)\right)^z\right)^{\frac{1}{z}}}} \\ e^{i2\pi\left(\sqrt[q]{e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(\check{T}_j^q\right)\right)^z\right)^{\frac{1}{z}}}}\right)}, \\ 1 - e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-{}^{\circ}F_j\right)\right)^z\right)^{\frac{1}{z}}} \\ e^{i2\pi\left(1-e^{-\left(\sum_{j=1}^{n}\xi_j\left(-\ln\left(1-{}^{\circ}F_j\right)\right)^z\right)^{\frac{1}{z}}}\right)} \end{array}\right)$$

$$= \left(\begin{array}{c} \sqrt[q]{e^{\ln\left(\check{T}_j^q\right)}}\, e^{i2\pi\left(\sqrt[q]{e^{\ln\left(\check{T}_j^q\right)}}\right)}, \\ 1 - e^{\ln\left(1-{}^{\circ}F_j\right)}\, e^{2\pi i\left(1-e^{\ln\left(1-{}^{\circ}F_j\right)}\right)} \end{array}\right)$$

$$= \left(\sqrt[q]{\check{T}_j^q}\, e^{i2\pi\left(\sqrt[q]{\check{T}_j^q}\right)}, \sqrt[q]{{}^{\circ}F_j}\, e^{i2\pi\left(\sqrt[q]{{}^{\circ}F_j}\right)}\right)$$

$$= \left(\check{T}_j.e^{i2\pi W_{\check{T}_j}}, {}^{\circ}F_j.e^{i2\pi W_{{}^{\circ}F_j}}\right) = \grave{a}$$

Thus, $Cq-ROFAAPWG\left(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_n\right)$ satisfied.

Theorem 7 (Boundedness): If $\grave{a}_{\jmath} = \left(\check{T}_{\jmath}.e^{i2\pi W_{\check{T}_{\jmath}}}, {}^{\circ}F_{\jmath}.e^{i2\pi W_{{}^{\circ}F_{\jmath}}}\right), (\jmath = 1, 2, \ldots, n)$ be the group of Cq-ROFVs that is $\grave{a}^{-} = \min(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_n)$ and $\grave{a}^{+} = \max(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_n)$ then condition holds for boundedness $\grave{a}^{-} \leq Cq - ROFAAPWG(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_n) \leq \grave{a}^{+}$.

Proof: Suppose that $\grave{a}_{\jmath} = \left(\check{T}_{\jmath}.e^{i2\pi W_{\check{T}_{\jmath}}}, {}^{\circ}F_{\jmath}.e^{i2\pi W_{{}^{\circ}F_{\jmath}}}\right)$, $(\jmath = 1, 2, \ldots, n)$ be the group of Cq-ROFVs. Let $\grave{a}^{-} = \min(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_n) = \left(\check{T}_{\jmath}^{-}.e^{i2\pi W_{\check{T}_{\jmath}^{-}}}, {}^{\circ}F_{\jmath}^{-}.e^{i2\pi W_{{}^{\circ}F_{\jmath}^{-}}}\right)$ and $\grave{a}^{-} = \max(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_n) = \left(\check{T}_{\jmath}^{+}.e^{i2\pi W_{\check{T}_{\jmath}^{+}}}, {}^{\circ}F_{\jmath}^{+}.e^{i2\pi W_{{}^{\circ}F_{\jmath}^{+}}}\right)$ then we have $\grave{a}^{-} = \min\left(\check{T}_{\jmath}.e^{i2\pi W_{\check{T}_{\jmath}}}\right)$, $\grave{a}^{-} = \max\left({}^{\circ}F_{\jmath}.e^{i2\pi W_{{}^{\circ}F_{\jmath}}}\right)$, $\grave{a}^{+} = \max\left(\check{T}_{\jmath}.e^{i2\pi W_{\check{T}_{\jmath}}}\right)$, $\grave{a}^{+} = \min\left({}^{\circ}F_{\jmath}.e^{i2\pi W_{{}^{\circ}F_{\jmath}}}\right)$

$$\begin{pmatrix} \sqrt[q]{1 - e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}\left(-\ln\left(1-(\check{T}_{\jmath}^{-})^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}} \\ e^{2\pi i\left(\sqrt[q]{1-e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}\left(-\ln\left(1-(\check{T}_{\jmath}^{-})^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)} \end{pmatrix}$$

$$\leq \begin{pmatrix} \sqrt[q]{1 - e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}\left(-\ln\left(1-(\check{T}_{\jmath})^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}} \\ e^{2\pi i\left(\sqrt[q]{1-e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}\left(-\ln\left(1-(\check{T}_{\jmath})^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)} \end{pmatrix}$$

$$\leq \begin{pmatrix} \sqrt[q]{1 - e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}\left(-\ln\left(1-(\check{T}_{\jmath}^{+})^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}} \\ e^{2\pi i\left(\sqrt[q]{1-e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}\left(-\ln\left(1-(\check{T}_{\jmath}^{+})^{q}\right)\right)^{z}\right)^{\frac{1}{z}}}}\right)} \end{pmatrix}$$

$$\leq \begin{pmatrix} \sqrt[q]{e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}\left(-\ln {}^{\circ}F_{\jmath}^{-}\right)^{z}\right)^{\frac{1}{z}}}} \\ e^{2\pi i\left(\sqrt[q]{e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}\left(-\ln {}^{\circ}F_{\jmath}^{-}\right)^{z}\right)^{\frac{1}{z}}}}\right)} \end{pmatrix}$$

$$\leq \begin{pmatrix} \sqrt[q]{e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}(-\ln {}^{\circ}F_{\jmath})^{z}\right)^{\frac{1}{z}}}} \\ e^{2\pi i\left(\sqrt[q]{e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}(-\ln {}^{\circ}F_{\jmath})^{z}\right)^{\frac{1}{z}}}}\right)} \end{pmatrix}$$

$$\leq \begin{pmatrix} \sqrt[q]{e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}\left(-\ln {}^{\circ}F_{\jmath}^{+}\right)^{z}\right)^{\frac{1}{z}}}} \\ e^{2\pi i\left(\sqrt[q]{e^{-\left(\sum_{\jmath=1}^{n} \xi_{\jmath}\left(-\ln {}^{\circ}F_{\jmath}^{+}\right)^{z}\right)^{\frac{1}{z}}}}\right)} \end{pmatrix}$$

Thus $\grave{a}^{-} \leq Cq - ROFAAPWG(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_n) \leq \grave{a}^{+}$ satisfied.

Theorem 8 (Monotonicity): Let $\grave{a}_{\jmath} = \left(\check{T}_{\jmath}.e^{i2\pi W_{\check{T}_{\jmath}}}, {}^{\circ}F_{\jmath}.e^{i2\pi W_{{}^{\circ}F_{\jmath}}}\right), (\jmath = 1, 2, \ldots, n)$ be the group of Cq-ROFVs, if $\grave{a} \leq \grave{a}'$ then $Cq - ROFAAPWG(\grave{a}_1, \grave{a}_2, \ldots, \grave{a}_n) \leq Cq - ROFAAPWG(\grave{a}'_1, \grave{a}'_2, \ldots, \grave{a}'_n)$

A. MAGM ALGORITHM BASED ON CQ-ROFS

In the Cq-ROFS system, we proposed a MAGDM algorithm using the derived Cq-ROFAAPWA and Cq-ROFAAPWG operators.

Consider $\mathcal{J} = (\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_{\mathfrak{x}})$ are $\mathfrak{x}^{th}$ attributes and $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_{\mathfrak{n}})$ be the $\mathfrak{n}^{th}$ alternatives for choosing the finest option among the multiple choices. Consider the weight vectors. $\varpi_{\jmath}$ in decision-makers' opinion, the sum of all weights must be 1. We represent the weight vector (WV) of decision-experts $\mathcal{D}_p$ is signified by γ_p $(\mathbb{K} = 1, 2, \ldots, p)$ and also the sum of all the weights must be 1. By utilizing the Cq-ROFS Information, construct the decision matrix which $\mathcal{R} = (\mathfrak{S}_{\mathbb{K}})_{\mathfrak{x}\times\mathfrak{n}}$. The information in the matrix is in the form of Cq-ROFS, such as $\grave{a}_{\jmath} = \left(\check{T}_{\jmath}.e^{i2\pi W_{\check{T}_{\jmath}}}, {}^{\circ}F_{\jmath}.e^{i2\pi W_{{}^{\circ}F_{\jmath}}}\right)$, where evaluate the alternatives within the range of $0 \leq \check{T}_{\jmath}.e^{i2\pi W_{\check{T}_{\jmath}}} + {}^{\circ}F_{\jmath}.e^{i2\pi W_{{}^{\circ}F_{\jmath}}} \leq 1$. Thus, construct the matrix $\mathcal{R} = (\mathfrak{S}_{\mathbb{K}})_{\mathfrak{x}\times\mathfrak{n}}$ using Cq-ROFS information.

In general, there are two types of alternatives: cost type and benefit type.

$$\bar{\ell}_i = \begin{cases} \ell_i & \text{benefit type attribute} \\ \ell_i^{c} & \text{cost type attribute} \end{cases}$$

Suppose that, ℓ_i^{c} be the cost type attribute and ℓ_i be the benefit type attribute of the matrix. If both facts are different, they must be interchanged with each other, while if they are the same, then there is no need to change during the aggregation of the decision matrix.

In this segment, we offer an algorithm by applying the developed Cq-ROFAAWA and Cq-ROFAAWG operators in the q-ROFS theory to select the finest alternative. By using the proposed AOs, solve the MAGDM problematic issues. The following steps are listed as follows:

Step 1: By using Definition (6). calculate the support values of the decision matrix.

$$Sup\left(b_{\jmath j}^{t}, b_{\jmath j}^{\mathcal{K}}\right) = 1 - d\left(b_{\jmath j}^{t}, b_{\jmath j}^{\mathcal{K}}\right)$$

where support function $t, \mathcal{K} = 1, 2, \ldots, p, \jmath = 1, 2, \ldots, \mathfrak{x}; j = 1, 2, \ldots, \mathfrak{n}$ satisfy the conditions discussed in Definition (6). and Definition (4).

Step 2: To calculating the values of $\check{T}\left(đ_{\jmath j}^{\mathcal{K}}\right)$

$$\check{T}\left(đ_{\jmath j}^{\mathcal{K}}\right) = \sum_{\jmath=1}^{p} Sup\left(đ_{\jmath j}^{\mathcal{K}}, đ_{\jmath j}^{t}\right)$$

where $t, \mathcal{K} = 1, 2, \ldots, p, \jmath = 1, 2, \ldots, \mathfrak{x}; j = 1, 2, \ldots, \mathfrak{n}$.

Step 3: To compute $\varepsilon_{\jmath}^{\mathcal{K}}$ weights of the attribute associated with Cq-ROFVs by using the following formula:

$$\varepsilon_{\jmath} = \frac{\gamma_p\left(1 + \check{T}\left(đ_{\jmath}\right)\right)}{\sum_{\jmath=1}^{n} \gamma_p\left(1 + \check{T}\left(đ_{\jmath}\right)\right)}$$

Step 4: Aggregate the decision matrices $\grave{a}_{\jmath}$ by using the developed Cq-ROFAAWA and Cq-ROFAAWG operators.

$$Cq - ROFAAPWA\left(\grave{a}_{\jmath j}^{1}, \grave{a}_{\jmath j}^{2}, \ldots, \grave{a}_{\jmath j}^{n}\right)$$

$$= \bigoplus_{\text{ȷj}=1}^{n} \left(\text{à}_{\text{ȷj}}\varpi_{\text{ȷj}}\right)$$

$$= \left(\begin{array}{c} \sqrt[q]{1-e^{-\left(\sum_{\text{ȷj}=1}^{n}\xi_{\text{ȷj}}\left(-\ln\left(1-\check{\text{T}}_{\text{ȷj}}^{q}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}} \\ e^{2\pi i\left(\sqrt[q]{1-e^{-\left(\sum_{\text{ȷj}=1}^{n}\xi_{\text{ȷj}}\left(-\ln\left(1-\check{\text{T}}_{\text{ȷj}}^{q}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}}\right)}, \\ e^{-\left(\sum_{\text{ȷj}=1}^{n}\xi_{\text{ȷj}}\left(-\ln {}^{\circ}\text{F}_{\text{ȷj}}\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}} \\ e^{2\pi i\left(e^{-\left(\sum_{\text{ȷj}=1}^{n}\xi_{\text{ȷj}}\left(-\ln {}^{\circ}\text{F}_{\text{ȷj}}\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}\right)} \end{array} \right)$$

and

$$Cq-ROFAAPWG\left(\text{à}_{\text{ȷj}}^{1}, \text{à}_{\text{ȷj}}^{2}, \ldots, \text{à}_{\text{ȷj}}^{n}\right)$$

$$= \bigotimes_{\text{ȷ}=1}^{n} \left(\text{à}_{\text{ȷj}}\varpi_{\text{ȷj}}\right)$$

$$= \left(\begin{array}{c} \sqrt[q]{e^{-\left(\sum_{\text{ȷj}=1}^{n}\xi_{\text{ȷj}}\left(-\ln\left(\check{\text{T}}_{\text{ȷj}}^{q}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}} \\ e^{2\pi i\left(\sqrt[q]{e^{-\left(\sum_{\text{ȷj}=1}^{n}\xi_{\text{ȷj}}\left(-\ln\left(\check{\text{T}}_{\text{ȷj}}^{q}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}}\right)}, \\ 1-e^{-\left(\sum_{\text{ȷj}=1}^{n}\xi_{\text{ȷj}}\left(-\ln\left(1-{}^{\circ}\text{F}_{\text{ȷj}}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}} \\ e^{2\pi i\left(1-e^{-\left(\sum_{\text{ȷj}=1}^{n}\xi_{\text{ȷj}}\left(-\ln\left(1-{}^{\circ}\text{F}_{\text{ȷj}}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}\right)} \end{array} \right)$$

Step 5: Calculate the values of $\check{\text{T}}\left(\text{đ}_{\text{ȷj}}^{\mathcal{K}}\right)$.

$$\check{\text{T}}\left(\text{đ}_{\text{ȷj}}^{\mathcal{K}}\right) = \sum_{\text{ȷ}=1}^{n} Sup\left(\text{đ}_{\text{ȷj}}^{\mathcal{K}}, \text{đ}_{\text{ȷh}}^{t}\right)$$

where $t, \mathcal{K} = 1, 2, \ldots, p$, ȷ, j, h $= 1, 2, \ldots, n$.

Step 6: To calculate $\varepsilon_{\text{ȷ}}^{\mathcal{K}}$ weights of the attribute associated with Cq-ROFVs by using the following formula:

$$\varepsilon_{\text{ȷ}} = \frac{\varpi_{\text{ȷ}}\left(1+\check{\text{T}}\left(\text{đ}_{\text{ȷ}}\right)\right)}{\sum_{\text{ȷ}=1}^{n}\varpi_{\text{ȷ}}\left(1+\check{\text{T}}\left(\text{đ}_{\text{ȷ}}\right)\right)}$$

And it is always. $\varepsilon_{\text{ȷ}} > 0$ and $\sum_{\text{ȷ}=1}^{n}\varpi_{\text{ȷ}} = 1$.

Step 7: Aggregate each attribute by applying the diagnosed Cq-ROFAAWA and Cq-ROFAAWG operators.

$$Cq-ROFAAPWA\left(\text{à}_1, \text{à}_2, \ldots, \text{à}_n\right)$$

$$= \bigoplus_{\varpi_{\text{ȷ}}=1}^{n} \left(\text{à}_{\text{ȷ}}\varpi_{\text{ȷ}}\right)$$

$$= \left(\begin{array}{c} \sqrt[q]{1-e^{-\left(\sum_{i=1}^{n}\xi_{\text{ȷ}}\left(-\ln\left(1-\check{\text{T}}_{\text{ȷ}}^{q}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}} \\ e^{2\pi i\left(\sqrt[q]{1-e^{-\left(\sum_{i=1}^{n}\xi_{\text{ȷ}}\left(-\ln\left(1-\check{\text{T}}_{\text{ȷ}}^{q}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}}\right)}, \\ e^{-\left(\sum_{i=1}^{n}\xi_{i}\left(-\ln\left({}^{\circ}\text{F}_{\text{ȷ}}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}} \\ e^{2\pi i\left(e^{-\left(\sum_{i=1}^{n}\xi_{i}\left(-\ln\left({}^{\circ}\text{F}_{\text{ȷ}}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}\right)} \end{array} \right)$$

and

$$Cq-ROFAAPWG\left(\text{à}_1, \text{à}_2, \ldots, \text{à}_n\right)$$

$$= \bigotimes_{\text{ȷ}=1}^{n} \left(\text{à}_{\text{ȷ}}\varpi_{\text{ȷ}}\right)$$

$$= \left(\begin{array}{c} \sqrt[q]{e^{-\left(\sum_{i=1}^{n}\xi_{\text{ȷ}}\left(-\ln\left(\check{\text{T}}_{\text{ȷ}}^{q}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}} \\ e^{i2\pi\left(\sqrt[q]{e^{-\left(\sum_{i=1}^{n}\xi_{\text{ȷ}}\left(-\ln\left(\check{\text{T}}_{\text{ȷ}}^{q}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}}\right)}, \\ 1-e^{-\left(\sum_{i=1}^{n}\xi_{i}\left(-\ln\left(1-{}^{\circ}\text{F}_{\text{ȷ}}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}} \\ e^{2\pi i\left(1-e^{-\left(\sum_{i=1}^{n}\xi_{i}\left(-\ln\left(1-{}^{\circ}\text{F}_{\text{ȷ}}\right)\right)^{\text{z}}\right)^{\frac{1}{\text{z}}}}\right)} \end{array} \right),$$

Step 8: Calculate the score value by using Definition (5).

$$S(\eta) = \left[\left(\check{\text{T}}^{q} - {}^{\circ}\text{F}^{q}\right) + \left(W_{\check{\text{T}}_{\eta}^{q}} - W_{{}^{\circ}\text{F}_{\eta}^{q}}\right)\right]$$

Step 9: We are arranging the alternatives to show the finest alternative.

B. NUMERICAL PROBLEM

Dam construction is a high-cost project, and it should be built at a location with more significant economic potential to cover the costs. In this instance, many researchers conducted several case studies to select a suitable company. In this article, we develop a case study of appropriate company selection for dam construction, and during the choice of a convenient company, consider the following factors as listed below:

1. Experience: It is the major factor before assigning the contract that the company should have extensive experience in the construction of dams, including a good track record of successful dam construction projects.
2. Expertise: The dam construction company must have a team of skilled professionals, including dam engineers and dam designers because the successful completion of the project depends on the company's expertise.
3. Resources: observe the dam construction company's necessary resources, such as advanced equipment and technology-based machinery.
4. Safety record: The dam construction company has a strong safety record and follows all safety regulations according to international standards.
5. Financial stability: The dam construction company has a stable financial budget to ensure the successful completion of the project without causing delays in the decided period.

TABLE 1. (Cq-ROFS decision matrix S^1).

	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$
$\bar{A}_1$	$\left(\binom{0.34,}{0.3},\binom{0.62,}{0.61}\right)$	$\left(\binom{0.55,}{0.54},\binom{0.26,}{0.31}\right)$	$\left(\binom{0.72,}{0.43},\binom{0.32,}{0.33}\right)$	$\left(\binom{0.71,}{0.71},\binom{0.69,}{0.321}\right)$
$\bar{A}_2$	$\left(\binom{0.23,}{0.22},\binom{0.65,}{0.73}\right)$	$\left(\binom{0.29,}{0.32},\binom{0.61,}{0.76}\right)$	$\left(\binom{0.52,}{0.53},\binom{0.36,}{0.37}\right)$	$\left(\binom{0.12,}{0.11},\binom{0.7,}{0.77}\right)$
$\bar{A}_2$	$\left(\binom{0.22,}{0.23},\binom{0.84,}{0.63}\right)$	$\left(\binom{0.41,}{0.42},\binom{0.3,}{0.32}\right)$	$\left(\binom{0.5,}{0.53},\binom{0.36,}{0.38}\right)$	$\left(\binom{0.32,}{0.35},\binom{0.48,}{0.54}\right)$
$\bar{A}_4$	$\left(\binom{0{,}23,}{0.33},\binom{0.82,}{0.52}\right)$	$\left(\binom{0.42,}{0.52},\binom{0.32,}{0.62}\right)$	$\left(\binom{0.16,}{0.61},\binom{0.35,}{0.37}\right)$	$\left(\binom{0.49,}{0.52},\binom{0.57,}{0.59}\right)$
$\bar{A}_5$	$\left(\binom{0.32,}{0.42},\binom{0.21,}{0.32}\right)$	$\left(\binom{0.43,}{0.46},\binom{0.31,}{0.21}\right)$	$\left(\binom{0.55,}{0.43},\binom{0.41,}{0.47}\right)$	$\left(\binom{0.51,}{0.21},\binom{0.31,}{0.32}\right)$

TABLE 2. (Cq-ROFS decision matrix S^2).

	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$
$\bar{A}_1$	$\left(\binom{0.34,}{0.44},\binom{0.61,}{0.71}\right)$	$\left(\binom{0.55,}{0.65},\binom{0.26,}{0.36}\right)$	$\left(\binom{0.72,}{0.62},\binom{0.32,}{0.22}\right)$	$\left(\binom{0.55,}{0.34},\binom{0.78,}{0.88}\right)$
$\bar{A}_2$	$\left(\binom{0.24,}{0.34},\binom{0.78,}{0.88}\right)$	$\left(\binom{0.29,}{0.39},\binom{0.61,}{0.71}\right)$	$\left(\binom{0.52,}{0.42},\binom{0.41,}{0.32}\right)$	$\left(\binom{0.13,}{0.32},\binom{0.43,}{0.53}\right)$
$\bar{A}_2$	$\left(\binom{0.21,}{0.31},\binom{0.81,}{0.91}\right)$	$\left(\binom{0.33,}{0.44},\binom{0.31,}{0.41}\right)$	$\left(\binom{0.50,}{0.42},\binom{0.37,}{0.27}\right)$	$\left(\binom{0.45,}{0.32},\binom{0.65,}{0.46}\right)$
$\bar{A}_4$	$\left(\binom{0.22,}{0.33},\binom{0.82,}{0.82}\right)$	$\left(\binom{0.31,}{0.41},\binom{0.32,}{0.42}\right)$	$\left(\binom{0.16,}{0.82},\binom{0.56,}{0.46}\right)$	$\left(\binom{0.76,}{0.77},\binom{0.89,}{0.61}\right)$
$\bar{A}_5$	$\left(\binom{0.35,}{0.45},\binom{0.22,}{0.33}\right)$	$\left(\binom{0.43,}{0.53},\binom{0.31,}{0.41}\right)$	$\left(\binom{0.51,}{0.41},\binom{0.44,}{0.38}\right)$	$\left(\binom{0.76,}{0.67},\binom{0.12,}{0.19}\right)$

TABLE 3. (Cq-ROFS decision matrix S^3).

	$\aleph_1$	$\aleph_2$	$\aleph_3$	$\aleph_4$
$\bar{A}_1$	$\left(\binom{0.36,}{0.38},\binom{0.68,}{0.71}\right)$	$\left(\binom{0.57,}{0.61},\binom{0.39,}{0.33}\right)$	$\left(\binom{0.56,}{0.52},\binom{0.32,}{0.21}\right)$	$\left(\binom{0.71,}{0.74},\binom{0.63,}{0.67}\right)$
$\bar{A}_2$	$\left(\binom{0.25,}{0.27},\binom{0.67,}{0.72}\right)$	$\left(\binom{0.34,}{0.38},\binom{0.61,}{0.65}\right)$	$\left(\binom{0.52,}{0.57},\binom{0.36,}{0.31}\right)$	$\left(\binom{0.12,}{0.18},\binom{0.45,}{0.49}\right)$
$\bar{A}_2$	$\left(\binom{0.24,}{0.26},\binom{0.86,}{0.89}\right)$	$\left(\binom{0.44,}{0.48},\binom{0.56,}{0.33}\right)$	$\left(\binom{0.50,}{0.51},\binom{0.36,}{0.37}\right)$	$\left(\binom{0.34,}{0.34},\binom{0.56,}{0.69}\right)$
$\bar{A}_4$	$\left(\binom{0.25,}{0.27},\binom{0.86,}{0.91}\right)$	$\left(\binom{0.45,}{0.51},\binom{0.21,}{0.25}\right)$	$\left(\binom{0.16,}{0.31},\binom{0.35,}{0.37}\right)$	$\left(\binom{0.51,}{0.59},\binom{0.\ 56,}{0.69}\right)$
$\bar{A}_5$	$\left(\binom{0.34,}{0.36},\binom{0.25,}{0.27}\right)$	$\left(\binom{0.29,}{0.34},\binom{0.35,}{0.39}\right)$	$\left(\binom{0.55,}{0.21},\binom{0.14,}{0.44}\right)$	$\left(\binom{0.51,}{0.55},\binom{0.61,}{0.71}\right)$

6. Reputation: The company should have a good reputation within the industry and among its clients and stakeholders.

There are many multinational companies, such as China Gezhouba Group Company Limited (CGGC), Salini Impregilo S.p.A., China State Construction Engineering Corporation (CSCEC), Power Construction, Corporation of China (PowerChina), Vinci Construction Grands Projets, J-Power Systems Corporation, Hochtief Aktiengesellschaft, Larsen & Toubro Limited (L&T), Andritz Hydro, Strabag SE. These are some of the top-ranked dam construction companies in the world based on their experience, expertise, and reputation. It is problematic for decision-makers to find the finest company for dam construction. We construct the MAGDM algorithm depending on the C-q-ROFS methodology to solve this difficulty.

Consider a set of five different companies. ($\aleph_1$, $\aleph_2$, $\aleph_3$, $\aleph_4$, $\aleph_5$) as an alternative, and we also have a group of three decision-makers $\mathbb{D}_i$ $(i = 1, 2, 3)$ and their weight vector is $(0.24, 0.35, 41)^T$. Consider the attributes $\bar{A}_1$ is the experience of the company, $\bar{A}_2$ is resources, $\bar{A}_3$ is safety record, $\bar{A}_4$ is financial stability. The weight vectors of attributes kept in mind during the selection of the best alternative is (0.30, 0.20, 0.24, 0.26) respectively. Using the Cq-ROFS information, decision-makers evaluate the five companies with concerning attributes $\left(\bar{A}_1, \bar{A}_2, \bar{A}_3, \bar{A}_4, \bar{A}_5\right)$. Then, we built three decision matrices. $\left[S_{ij}^k\right]_{5\times 4}$ for group decision-making as given below in Tables 1-3.

The evaluation steps are discussed in detail as follows:

Step 1: Collect the fuzzy information from anonymous experts. Then, give weight to decision-makers ($\mathbb{D}_1$, $\mathbb{D}_2$, $\mathbb{D}_3$) is (0.24, 0.35, 0.41) respectively. We also assign a weight vector for attributes, 0.30 for the experience of the company ($\bar{A}_1$); 0.20 for resources ($\bar{A}_2$); 0.24 for safety record ($\bar{A}_3$); 0.26 for financial stability ($\bar{A}_4$). The collection of vague data is presented in Tables 1–3.

Step 2: Using the proposed Cq-ROFAAPWA and Cq-ROFAAPWG operators, aggregate the Cq-ROF Information. The aggregated findings are presented in Table 4. (When $\mathcal{Z} = 1$ and $q = 3$).

Step 3: To evaluate the SV from Definition 5. On aggregated findings, find the finest alternative. The results are given in Table 5.

Step 4: Arrange the alternatives based on the SV formula. It is noticed that $\aleph_1$ is the finest alternative by using the proposed Cq-ROFAAPWG AO while $\aleph_5$ be the finest alternative by using the proposed Cq-ROFAAPWA AO. The ordering of other options is shown in Table 6.

C. SENSITIVITY STUDY BY VARIATION OF PARAMETERS

In this section, we observe the sensitivity of parameters $\mathcal{Z}$ and q on our proposed Cq-ROFAAPWA and Cq-ROFAAPWG

TABLE 4. Aggregation results.

	Cq-ROFAAPWA	Cq-ROFAAPWG
$\aleph_1$	$\begin{pmatrix}(0.2260, 0.2561),\\(0.0164, 0.0065)\end{pmatrix}$	$\begin{pmatrix}(0.6325, 0.5667),\\(0.3096, 0.3506)\end{pmatrix}$
$\aleph_2$	$\begin{pmatrix}(0.1475, 0.1605),\\(0.0182, 0.0225)\end{pmatrix}$	$\begin{pmatrix}(0.5817, 0.5891),\\(0.3410, 0.3803)\end{pmatrix}$
$\aleph_3$	$\begin{pmatrix}(0.1718, 0.1755),\\(0.0185, 0.0183)\end{pmatrix}$	$\begin{pmatrix}(0.5970, 0.5987),\\(0.3378, 0.3331)\end{pmatrix}$
$\aleph_4$	$\begin{pmatrix}(0.1642, 0.3147),\\(0.0191, 0.0019)\end{pmatrix}$	$\begin{pmatrix}(0.5872, 0.4744),\\(0.3637, 0.4937)\end{pmatrix}$
$\aleph_5$	$\begin{pmatrix}(0.1981, 0.1839),\\(0.0101, 0.0119)\end{pmatrix}$	$\begin{pmatrix}(0.6185, 0.6158),\\(0.1845, 0.2092)\end{pmatrix}$

TABLE 5. (Score function of aggregated information).

	q-ROFAAPWA	q-ROFAAPWG
$\aleph_1$	−0.00524	0.05756
$\aleph_2$	−0.00093	−0.02301
$\aleph_3$	−0.00033	−0.00023
$\aleph_4$	−0.02673	0.01937
$\aleph_5$	0.00155	0.00022

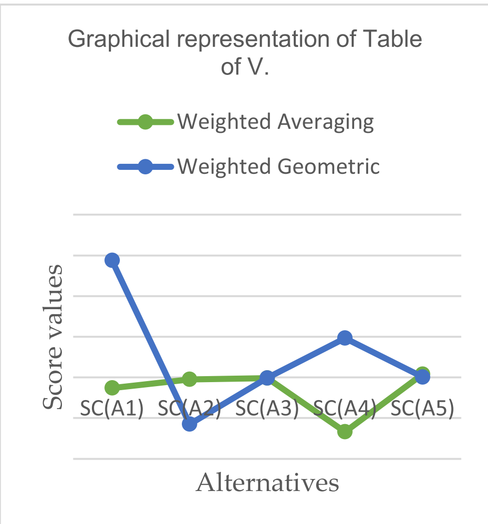

FIGURE 2. Represents the graphical depiction of Table 5, where green lines denote the ranking sequence of Cq-ROFAAPWA operators while blue lines denote the ranking sequence of Cq-ROFAAPWG operators.

AOs. For more clarity, we can see the effect of parameters through the geometrical representation and ranking order.

1) THE CONSEQUENCE OF PARAMETER Ƶ

The developed numerical example shows that ranking order varies when we vary the parameter Ƶ in our proposed AOs. For example, the variation in $Ƶ = 1, 3, 5, 7, 11$ in the proposed Cq-RFAAPWA operator, then the following changes in the ordering of alternatives can be observed in Table 7. Also, the variation in $Ƶ = 1, 3, 5, 7, 11$ in the proposed Cq-RFAAPWG operator, then the following changes in the ordering of alternatives can be observed in Table 8.

TABLE 6. Ranking of core values.

	Ordering of alternatives
Cq-ROFAAPWA	$\aleph_5 > \aleph_3 > \aleph_2 > \aleph_1 > \aleph_4$
Cq-ROFAAPWG	$\aleph_1 > \aleph_4 > \aleph_5 > \aleph_3 > \aleph_2$

TABLE 7. Ranking of the order of Cq-ROFAAPWA by variation in parameter Ƶ.

Ranking Order of SV using Cq-ROFAAPWA			
Ƶ	Ordering	Ƶ	Ordering
1	$\aleph_5 > \aleph_3 > \aleph_2 > \aleph_1 > \aleph_4$	2	Unworkable
3	$\aleph_1 > \aleph_5 > \aleph_2 > \aleph_3 > \aleph_4$	4	Unworkable
5	$\aleph_1 > \aleph_5 > \aleph_3 > \aleph_3 > \aleph_4$	6	Unworkable
7	$\aleph_5 > \aleph_1 > \aleph_3 > \aleph_2 > \aleph_4$	8	Unworkable
9	$\aleph_5 > \aleph_1 > \aleph_3 > \aleph_2 > \aleph_4$	10	Unworkable
11	$\aleph_5 > \aleph_3 > \aleph_2 > \aleph_1 > \aleph_4$	12	Unworkable

TABLE 8. The ranking sequence of Cq-ROFAAPWG by variation in parameter Ƶ.

Ranking Order of SV using Cq-ROFAAPWG			
Ƶ	Ordering	Ƶ	Ordering
1	$\aleph_1 > \aleph_4 > \aleph_5 > \aleph_3 > \aleph_2$	2	Unworkable
3	$\aleph_1 > \aleph_5 > \aleph_4 > \aleph_2 > \aleph_3$	4	Unworkable
5	$\aleph_4 > \aleph_5 > \aleph_2 > \aleph_1 > \aleph_3$	6	Unworkable
7	$\aleph_5 > \aleph_1 > \aleph_2 > \aleph_4 > \aleph_3$	8	Unworkable
9	$\aleph_5 > \aleph_1 > \aleph_2 > \aleph_4 > \aleph_3$	10	Unworkable
11	$\aleph_5 > \aleph_1 > \aleph_2 > \aleph_3 > \aleph_4$	12	Unworkable

For more clarity, the aggregation findings of Table 7 and Table 8 are presented in Figure 3 and Figure 4, respectively. We easily observe variation in the ranking sequence of Cq-ROFAAPWA operators by changing the value of Ƶ. Finally, when we place $Ƶ = 11$, the ranking of the alternative will be stable, which means $Ƶ = 11$ is the stability point for the Cq-ROFAAPWA operator. While in Cq-ROFAAPWG, operator ranking order also varies by the parameter variations in parameter Ƶ. In Cq-ROFAAPWA, operator $Ƶ = 11$ is the stability point because by putting $Ƶ = 11$, the following aggregation results will remain the same. It is also a noticeable feature, and no ranking outcome is obtained by placing an even number in Cq-ROFAAPWA and Cq-ROFAAPWG AOs.

2) THE CONSEQUENCE OF PARAMETER q

In our proposed numerical example, we take the parameter $q = 3$ and aggregate the q-ROF data by applying the established Cq-ROFAAPWA and Cq-ROFAAPWG AOs. In this scenario, we also observe the consequences of ranking order by variation in parameter q. It is observed that the Cq-ROFAAPWA ranking order varies by increasing the value of parameter q. When we take $q = 7$ ranking order is $\aleph_5 > \aleph_2 > \aleph_3 > \aleph_1 > \aleph_4$ after $q = 7$, the ranking sequence will be constant for all higher terms of q. While in Cq-ROFAAPWG, there are also ranking order

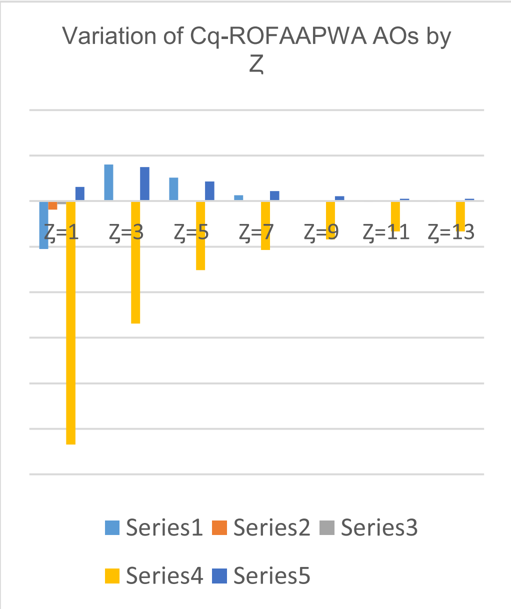

FIGURE 3. The geometrical depiction of SVs of Cq-ROFAAPWA operator, variation by ʐ.

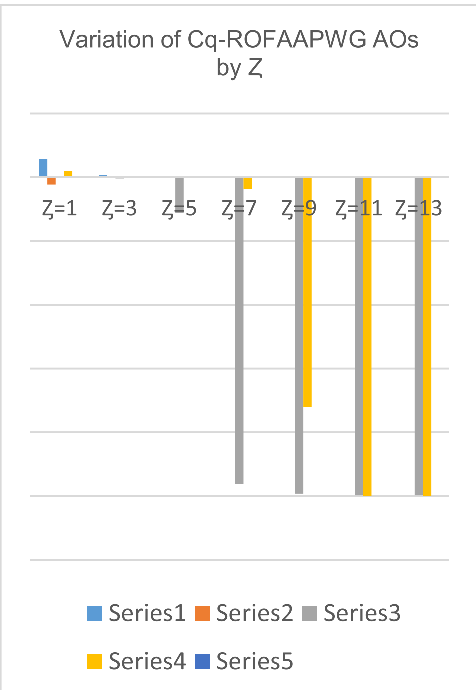

FIGURE 4. The geometrical depiction of score value of Cq-ROFAAPWG operator, variation by ʐ.

variations by changing the parameter q. When we take $q = 9$ ranking order, it is $\aleph_1 > \aleph_4 > \aleph_5 > \aleph_3 > \aleph_2$ after $q = 9$, the ranking sequence will be constant for all higher terms of q.

The graphical representation of Table 9. and Table 10 in Figure 6 and Figure 6, respectively. It is also noticed that by increasing the value of parameter q, the value of the score function gradually decreases for Cq-ROFAAPWA and Cq-ROFAAPWG operators.

TABLE 9. The ranking sequence of Cq-ROFAAPWA by variation in parameter *q*.

The ranking sequence of SV using Cq-ROFAAPWA		
q	**Ranking Ordering**	
3	$\aleph_5 > \aleph_3 > \aleph_2 > \aleph_1 > \aleph_4$	ʐ = 1
4	$\aleph_5 > \aleph_3 > \aleph_2 > \aleph_1 > \aleph_4$	ʐ = 1
5	$\aleph_5 > \aleph_3 > \aleph_2 > \aleph_1 > \aleph_4$	ʐ = 1
6	$\aleph_1 > \aleph_5 > \aleph_3 > \aleph_2 > \aleph_4$	ʐ = 1
7	$\aleph_5 > \aleph_2 > \aleph_3 > \aleph_1 > \aleph_4$	ʐ = 1
8	$\aleph_5 > \aleph_2 > \aleph_3 > \aleph_1 > \aleph_4$	ʐ = 1
9	$\aleph_5 > \aleph_2 > \aleph_3 > \aleph_1 > \aleph_4$	ʐ = 1

TABLE 10. The ranking sequence of Cq-ROFAAPWG by variation in parameter *q*.

Ranking sequence SV using Cq-ROFAAPWG		
q	**Ranking Ordering**	
3	$\aleph_1 > \aleph_4 > \aleph_5 > \aleph_3 > \aleph_2$	ʐ = 1
4	$\aleph_4 > \aleph_1 > \aleph_5 > \aleph_3 > \aleph_2$	ʐ = 1
5	$\aleph_4 > \aleph_1 > \aleph_5 > \aleph_3 > \aleph_2$	ʐ = 1
6	$\aleph_4 > \aleph_1 > \aleph_5 > \aleph_3 > \aleph_2$	ʐ = 1
7	$\aleph_4 > \aleph_1 > \aleph_5 > \aleph_3 > \aleph_2$	ʐ = 1
8	$\aleph_4 > \aleph_1 > \aleph_5 > \aleph_3 > \aleph_2$	ʐ = 1
9	$\aleph_1 > \aleph_4 > \aleph_5 > \aleph_3 > \aleph_2$	ʐ = 1

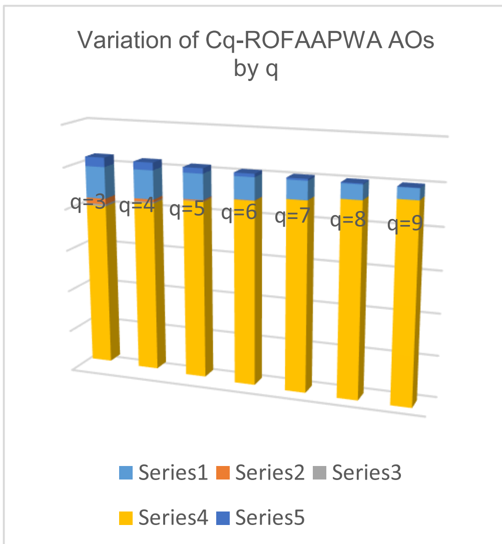

FIGURE 5. The geometrical depiction of the score value of the Cq-ROFAAPWA operator, variation by q.

V. COMPARATIVE STUDY

In this segment, we will compare our projected Cq-ROFAAPWG and Cq-ROFAAPWA operators with the presence of AOs. Also, we discussed the significance of developed AOs. In this scenario, we compare our proposed AOs with Cq-ROF weighted averaging (WA) (Cq-ROFWA) and Cq-ROF weighted geometric (WG) (Cq-ROFWG) given by Garg et al. [39], Cq-ROF Dombi WA (Cq-ROFDWA) and Cq-ROF Dombi WG (Cq-ROFDWG) presented by Ali and Mahmood [40], and the concept of Cq-ROF frank WA

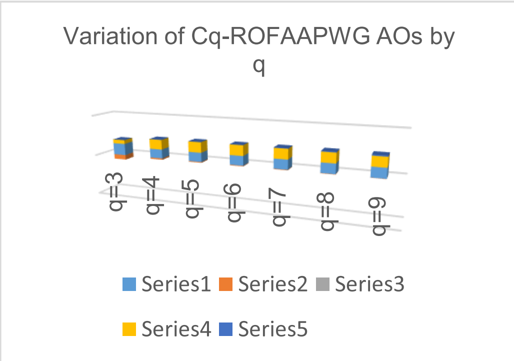

FIGURE 6. The geometrical depiction of the score value of the Cq-ROFAAPWG operator, variation by q.

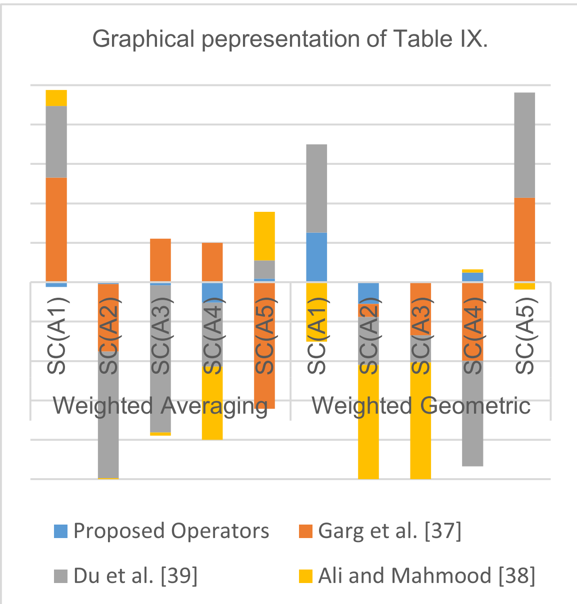

FIGURE 7. Represents the graphical view of developed AOs with existing operators in the comparative study section, where lines denote the alternatives' SV.

TABLE 11. Comparative analysis.

AOs	Fuzzy Frameworks	Ranking
Cq-ROFAAPWA ($q = 3, Z = 1$)	Cq-ROFSs	$\aleph_5 > \aleph_3 > \aleph_2 > \aleph_1 > \aleph_4$
Cq-ROFAAPWG ($q = 3, Z = 1$)	Cq-ROFSs	$\aleph_1 > \aleph_4 > \aleph_5 > \aleph_3 > \aleph_2$
Cq-ROFWA **[39]** ($q = 4, Z = 1$)	Cq-ROFSs	$\aleph_1 > \aleph_4 > \aleph_3 > \aleph_2 > \aleph_5$
Cq-ROFWG **[39]** ($q = 4, Z = 1$)	Cq-ROFSs	$\aleph_5 > \aleph_5 > \aleph_2 > \aleph_3 > \aleph_4$
Cq-ROFFWA **[41]** ($q = 3, Z = 2$)	Cq-ROFSs	$\aleph_1 > \aleph_5 > \aleph_3 > \aleph_2 > \aleph_4$
Cq-ROFFWG **[41]** ($q = 3, Z = 2$)	Cq-ROFSs	$\aleph_5 > \aleph_1 > \aleph_3 > \aleph_2 > \aleph_4$
Cq-ROFDWA **[40]** ($q = 3, Z = 2$)	Cq-ROFSs	$\aleph_5 > \aleph_1 > \aleph_3 > \aleph_2 > \aleph_4$
Cq-ROFDWG **[40]** ($q = 3, Z = 2$)	Cq-ROFSs	$\aleph_4 > \aleph_5 > \aleph_3 > \aleph_1 > \aleph_2$
TIFWA, TIFWG **[42]**	IFSs	Failed
CIFPWACIFPWG **[13]**	CIFSs	Failed
IFAA AOs **[43]**	IFSs	Failed
PIFHFS **[44]**	IFSs	Failed

(Cq-ROFFWA) and Cq-ROF Frank WG (Cq-ROFFWG) AOs given by Du et al. [41]. It is also noticed that many AOs are unable to handle Cq-ROF information due to limitations in their structures, such as triangular IF weighted averaging (TIFWA) and triangular IF weighted geometric (TIFWG) AOs given by Mahmood et al. [42], complex IF power WA (CIFPWA) and complex IF power WG (CIFPWG) AOs developed by Ali et al. [13], IF Aczel Alsina (IFAA) AOs Senapati et al. [43], and prioritized IF Hesitant set (PIFHFS) presented by Liu et al. [44]. These AOs failed to deal with the given information in Table 1-3 because the IFS structure is designed for only dealing with minimal values of MG and NMG. On the other hand, our q-ROS is the generalization of IFS and PyFS. So, the structure of q-ROFS allows us to deal with the bigger value of MG and NMG. For a better understanding and to discuss the superiority of proposed AOs, offer a comparative study in Table 10. as given below:

Some AOs cannot aggregate information because of their structural limitation. However, they cannot handle Cq-ROFS information due to lake of ability to deal with complex values. A brief review of our developed work with other present AOs is discussed in Table 10. For more simplicity, a geometric depiction is also provided in Figure 7.

VI. CONCLUSION

MAGDM is considered a trading tool for selecting the best alternative among the list of other options in decision-making sciences. The solid conclusion of the developed theory is as follows: construct the Cq-ROFAAPWA and ROFAAPWG operators and satisfy the fundamental axioms of AOs, such as boundedness, monotonicity, and idempotency. For better understanding, build the MAGMD algorithm and solve problematic real-life issues using the developed algorithm. The influence of changing the parameters q and Z is discussed with the geometrical representation of data. To demonstrate the significance of proposed AOs, give a comparative study and compare the results with present AO results. The geometrical model of comparative study is also part of the article.

Soon, our objective to apply our proposed technique in bipolar soft FS (BSFS) [45], Spherical FS (SFS) with Bonferroni mean AOs (SFSBAOs) [46], application for improved SFS (ISFS) [43], AATN and AATNS for IFS [48], interval-valued T-SFS do Dombi TN and TCN [49], and cubic q-ROFS with linguistic terms [50]. Also, we will explain our developed methodology in the picture FS (PFS) presented by Ullah [51] environment and rough IFS for Aczel-Alsina operations defined by Khan et al. [52].

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