IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 7 August 2023, accepted 5 October 2023, date of publication 11 October 2023, date of current version 16 October 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3323203

RESEARCH ARTICLE

Roadside Units Optimization Considering Path Flow Uncertainty

ZIJIAN BAI[1], ZIXUAN BAI[2], HENGBO ZHU[3], SHUIPING KE[4], AND YAO SUN[1]
[1]Tianjin Municipal Engineering Design & Research Institute, Tianjin 300072, China
[2]College of Xuanhuai, Tianjin University, Tianjin 300072, China
[3]College of Management and Economy, Tianjin University, Tianjin 300072, China
[4]College of Future Technology, Tianjin University, Tianjin 300072, China

Corresponding author: Zixuan Bai (baizixuan@tju.edu.cn)

This work was supported in part by the Tianjin Science and Technology Planning Project 20YFZCSN01110.

ABSTRACT Traffic flow is crucial for the efficient and safe operation of transportation systems. Understanding and managing traffic flow can help alleviate congestion, reduce travel time, and enhance transportation safety. In order to better identify traffic flow in a traffic network, we propose a new method that uses roadside units (RSUs) for path flow reconstruction. Roadside units (RSUs) are vital transportation facilities in cooperative vehicle infrastructure systems. They utilize modern communication technologies to exchange information directly with intelligent connected vehicles and their influence on accurate path flow reconstruction and average travel time are respectively analyzed. Considering the path flow uncertainty in traffic networks, a two-stage stochastic model is formulated, which aims to balance RSU deployment cost and value of reduced travel time. On the first stage, we solve a fully path flow reconstruction problem; On the second stage, we calculates the reduction on average travel time under different scenarios. To effectively handle the characteristics of the second stage model, we employ the integer L-shaped algorithm for solution. Numerical experiments suggest that (1) Expanding the size of scenarios has little impact on experimental results, which indicating that this model has good applicability; (2) some links play important roles in path flow reconstruction.

INDEX TERMS Urban traffic, location decision, two-stage stochastic programming, roadside unit, path flow reconstruction.

I. INTRODUCTION

In recent years, with the development of mobile communication technology, Vehicular ad hoc network (VANET) has received great attention from the academic and industry. Intelligent cooperative vehicle infrastructure systems(CVIS), characterized by high reliability and low delay, are beneficial for the efficient and safe operation of transportation systems by reducing the reliance on people and improving the perception of global information.

In a VANET, each vehicle is defined as a node of the network and is equipped with a unit of on-board communication OBU (On-Board Unit), the function of which is to exchange information with other vehicles or stationary access points located on the roads called Roadside Units [1]. Furthermore, The nodes collect and transmit real-time road traffic information through collision rules avoidance and safety warning systems to improve the driver's safe driving environment [2]. Roadside unit (RSU), a type of infrastructure, is a network element located along the road or at the roadside. RSU consists of wireless communication equipment, storage and processing unit, power supply, and environmental monitoring system. It is capable of providing real-time traffic information, data transmission, and data storage services.

In the context of the Internet of vehicles (IoV) and the cooperative vehicle infrastructure system(CVIS), the perception information of surrounding objects can be obtained by various types of sensors or communication networks [3]. Thus, the perception scope of vehicles is defined by the detection range of the various on-board sensors. These on-board sensors can only collect local information, while RSUs have the functions of long-range, blind area information

The associate editor coordinating the review of this manuscript and approving it for publication was Giovanni Pau.

sensing, and edge computing. It broadcasts the blind area motion information to help vehicles respond proactively, which improves the safety of vehicles and assists vehicles to achieve a global view in the transportation system [4].

Using wireless network, RSUs can increase the connectivity of the entire transportation system, but related technologies have not yet matured, and the stability of remote communication, reliability of signal connection, and resource consumption must be further improved. Guerna et al. [5] thoroughly categorized the literature based on the objectives they addressed. These objectives included maximizing the transmission coverage area, enhancing network connectivity, and minimizing deployment costs. From our perspective, the optimization of deployment with RSUs as crucial communication equipment has been examined through three primary facets: enlarging the communication coverage range, improving the connectivity of transportation system and enhancing the performance of RSU.

Enlarging the communication coverage range generally refers to the coverage rate in a VANET. Xue et al. [6] identified the potential locations of RSU with a modified k-means clustering algorithm and proposed a multi-objective optimization problem to maximize RSU connectivity and the number of covered vehicles. Kim et al. [7]. investigated a new strategy to maximize the spatiotemporal coverage of RSU under a limited budget. Lehsaini et al. [8] adopted genetic algorithm, standard version of simulated annealing and their improved versions in order to reduce the number of RSU by choosing locations at intersections that maximize the surface covered of the urban area and minimize the area of overlapping zones. Anbalagan et al. [9] proposed an efficient memetic-based RSU (M-RSU) placement algorithm for Software-defined-IoV to reduce communication delay and increase the coverage area among IoV devices. Magsino et al. [10] proposed an new scheme to capture and share the environment data in the vehicular network. And effective positions (EPs) are located based on the amount of information of an area and the average road speed between EPs to ensure urban-wide connectivity and wider coverage. Guerna et al. [11] propose a new formulation of RSUs deployment issue as a maximum intersection coverage problem through a graph-based modeling and developed an Ant colony optimization system to discover the minimum number of RSU intersections that ensures the maximum network connectivity.

As for the second category, considering the uncertainty of the selection of intersection control strategies, Liang et al. [12] proposed a two-stage stochastic mixed-integer nonlinear program which aims to minimize the sum of the cost associated with RSU investment and the expectation of the penalty cost associated with V2R communication delay exceeding a pre-determined threshold. Wang et al. [13] proposed an analytical model for analyzing the connectivity probability, taking into account the existence of the entry and exit and the deployment of multiple RSUs. Fogue et al. [14] proposed the genetic algorithm, which is capable of automatically providing an RSU deployment suitable for any given road map layout, in order to overcome the signal propagation problem and delayed warning notification time issues. Ni et al. [15] investigated the RSU deployment scheme in 2-D IoV networks with limited capacity. Considering the expected delivery delay requirements and task assignment, they developed a utility-based RSU deployment problem and proposed a linear programming-based clustering algorithm to solve it. Ahmed et al. [16] investigated the problem of RSU placement on a highway-like roadway and proposed an integer linear programming model with the objective of minimizing network latency that depicts the network under consideration.

The third class of literature concentrates on the efficiency or energy consumption. Yang et al. [17] proposed an energy harvest roadside unit (EHRSU) deployed on the roadside with a dynamic service radius and formulated a model taking into account the stochastic properties of the traffic flow, the energy harvesting process and the energy consumption of the EHRSU. For flexible deployment, energy-saving operation and low-latency services, Zhang et al. [18] proposed a cache-enabled green RSU which can store popular contents locally and harvest renewable energy as power source and developed a model to minimize network deployment cost, under the constraints of quality of service requirements and limited backhaul capacities. Shareeda et al. [19] provided the basic simulation environment of this work, such as OSM to download real map data, GatcomSUMO to generate car mobility, SUMO to simulate road traffic and so on. Then they chose genetic algorithm to find an optimal or near optimal location of RSU, which proves to enhance the reception of basic safety message delivered from the vehicles. Heo et al. [20] investigated the performance-cost tradeoff and viability of using buses as mobile RSUs (mRSUs). They also show how mRSUs can replace static RSUs while maintaining the same level of throughput, contact time, and inter-contact time. Lee et al. [21] considered the environment with densely deployed mobile RSUs (mRSUs), in which multiple active mRSUs generate lots of control messages to form the mRSU backbone network. And they propose a mechanism in which each mRSU adaptively and effectively determines its own state, active or inactive, according to the states of its neighboring mRSUs and vehicles.

The above-mentioned studies demonstrate a strong necessity for the construction of IoV environment based on the features of RSUs. However, some studies also focus on the improvement of the overall efficiency of the traffic system, and propose the optimization method for the location of RSUs in terms of social benefits. Li et al. [22] used a calculation method on travel time regarding the communication range of RSUs and developed a network equilibrium model to measure the influence of RSUs on travelers' route choice. Actually in the traffic system, the path flow is of particular important. On the one hand, the path flow itself contains information on road segment flow and OD pair flow, which is of vital importance for traffic management and control and is the data basis for a number of traffic applications. For example, the path flow information can improve the accuracy of highway tolls.

To measure the path flow in the road network, that is, the path flow reconstruction problem, most of the existing studies assumed the usage of automatic vehicle identification (AVI) sensors. Fu et al. [23] used both passive and active AVI sensors to solve path flow reconstruction problem. Passive sensors simply count vehicles, while active sensors can recognize vehicle plates but are more expensive. Considering the dynamic nature of the mobility and the uncertain knowledge of traffic conditions, Fu et al. [24] propose a scenario based two stage stochastic programming framework and Álvarez-Bazo et al. [25] proposed a genetic algorithm to determine the deployment of AVI sensors on a traffic network. AVI sensors can identify vehicle plates or infrared tags attached to vehicles, and they can use this information to provide travel times for specific paths in addition to data on the speed and flow of traffic [26]. Once the vehicle passes through the device, this detector is enabled to identify the vehicle and obtain the traffic flow, vehicle speed, and other data. The roadside unit differs from the AVI sensors in that, in addition to the above functions, the RSU is allowed to be notified of the intelligent connected vehicle's travel origin, historical travel trajectory, and possible routes to the destination through CVIS technology [27], [28], [29]. As the vehicle is within the communication range of RSU and completes the information interaction, the RSU is empowered to capture the complete driving trajectory of the vehicle from the origin to the current position. This also means that if RSU is installed near the vehicle destination, the path of the vehicle can be identified as precisely as possible, thus improving the accuracy of path flow reconstruction.

As for the end users, traffic information would largely influence travelers' routes and improve the average travel time of each road segment; for governments, traffic management is the key to the efficient and safe operation of the transportation system, and vehicle path flow reconstruction will bring more traffic flow information to effectively guide optimal decision-making.

In this paper, we develop a deterministic model and a stochastic model to find the optimal RSU locations. This work provides several theoretical contributions. First, this study aims to make balance between deployment cost and value of reduction on average travel time, on the basis of fully path flow reconstruction. Second, this study explores the reduction of average travel time under the uncertainty of path flow in a mixed traffic flow. Thus, a two-stage stochastic programming formulation is proposed and the integer L-shaped method is employed for solution. Third, this study analyses the influence of unit price of RSU, value of time, uncertainty in path flow and penetration of intelligent connected vehicles on optimal deployment of RSU. Numerical experiments not only demonstrate the results of proposed model but also present the role of different links in path flow reconstruction.

The remainder of this paper is organized as follows. In Section II, we present an illustrative network to demonstrate the influence of RSUs on path flow reconstruction. We calculate the average travel time within and outside the range of RSUs in a mixed traffic flow scenario. In Section III, we provide a description of both the deterministic model and the stochastic model that address the problem at hand. Section IV outlines the integer L-shaped method, which is utilized to solve the two-stage stochastic programming model. In Section V, we present the numerical results obtained by applying the stochastic model to two example networks. Finally, in Section VI, we conclude the paper by summarizing the key highlights and major findings of this research.

II. PROBLEM DESCRIPTION

This study makes RSU location decisions with two objectives: to realize accurate path flow reconstruction and to balance the RSU deployment cost and the overall traffic capacity of the transportation system. For path flow reconstruction, in contrast to AVI sensors, RSUs can identify vehicle's historical trajectories via V2I communication. To increase the overall traffic capacity of the transportation system, RSUs can reduce the travel times of intelligent connected vehicles by providing remote information perception. Next, we will elaborate on these two objectives.

A. PATH FLOW RECONSTRUCTION

Using a wireless network, RSUs can obtain information about historical trajectories and future possible routes of intelligent connected vehicles via V2I communication. After collecting vehicle data, RSUs positioned at different locations can integrate these data to identify vehicle paths. However, when faced with complex traffic conditions, it is highly probable that the driver of an intelligent connected vehicle may deviate from the preset route. As a result, the future routes of intelligent connected vehicles are inherently uncertain. Given this context, this subsection focuses solely on the capability of RSUs to identify historical vehicle trajectories, which greatly contributes to path flow reconstruction. To further explain the role of RSUs, we assume four origin-destination (OD) pairs in an illustrative network, each containing a path, as shown in Figure 1.

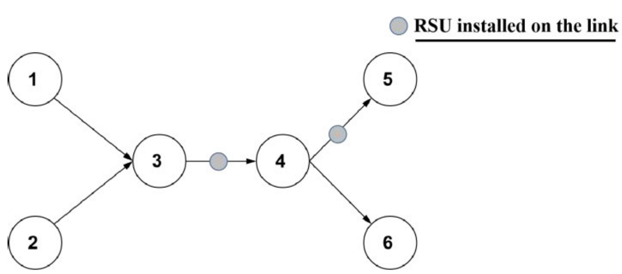


FIGURE 1. Illustrative network.

TABLE 1. **OD pair and path information for the illustrative network.**

OD pair	Start	End	paths
(1，5)	1	5	1-3-4-5
(1，6)	1	6	1-3-4-6
(2，4)	2	4	2-3-4
(2，5)	2	5	2-3-4-5

The specific path information is shown in the following Table 1.

Now, the following assumptions are made for the problem of path flow reconstruction:

1) In practice, RSUs are generally installed along road segments.
2) On each path, there is at least one segment that has an RSU.

With these assumptions, there are two ways to distinguish different paths:

1) If at least one RSU is installed in a non-shared road segment of any two paths, this RSU can identify these two paths. For example, for paths 1-3-4-5 and 1-3-4-6, segment 4-5 is a part of path 1-3-4-5 but not of path 1-3-4-6. Thus, it is not shared by these two paths. If an RSU is installed here, it can monitor the traffic flow of path 1-3-4-5, and the undetected traffic flow corresponds to path 1-3-4-6. Likewise, for paths 2-3-4 and 2-3-4-5, if an RSU is installed in nonshared segment 4-5, it can distinguish these two paths ($\sigma_l^{p,p'} = 1$).

2) If an RSU is installed in a shared segment of any two paths, there should be at least one nonshared segment in front of this shared segment. Otherwise, this RSU cannot identify these two paths. For example, segment 3-4 is shared by paths 1-3-4-5 and 2-3-4. After an RSU is installed in this segment, since this RSU can identify the historical trajectory of each vehicle, it knows that the traffic flow of segment 3-4 is from segment 1-3 or 2-3, and it can thus identify these two paths. Likewise, for paths 1-3-4-5 and 2-3-4-5, there are two nonshared segments in front of segment 3-4 or 4-5, i.e., segments 1-3 and 2-3. An RSU installed in shared segment 3-4 or 4-5 can obtain the historical trajectories of vehicles to identify these two paths.($\rho_l^{p,p'} = 1$)

B. EFFECT OF RSUS ON SEGMENT TRAVEL

In addition to obtaining the historical trajectories of vehicles, RSUs can shorten the overall travel time of the transportation system, thereby improving its overall traffic capacity. As shown in Figure 2, using vehicle sensors, intelligent connected vehicles can only sense the vehicles nearby. RSUs can also detect vehicles in the distance and in blind spots from a global perspective to obtain information on vehicles in the distance. To ensure the safety of intelligent connected vehicles, the manufacturer often sets a long safe distance. Nevertheless, wireless networks make real-time communication between RSUs and intelligent connected vehicles

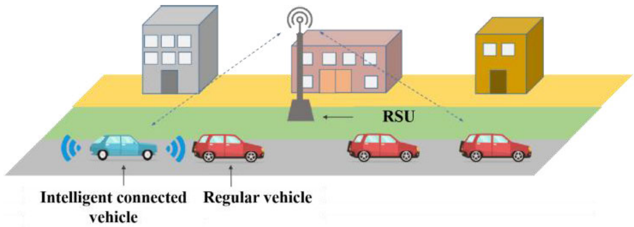


FIGURE 2. **Illustration of the function of RSU.**

possible. Then, the information on vehicles in the distance is transmitted to the intelligent connected vehicles to direct them to quickly respond to changes in the traffic flow. Therefore, the use of RSUs can reduce the original safe distance of intelligent connected vehicles while ensuring safety to improve road capacity.

To further assess the effects of RSUs on road capacity and travel time, we make the assumption that intelligent connected vehicles and regular vehicles are uniformly distributed within the traffic flow. This enables us to consider four distinct car-following scenarios, which are illustrated in Table 2.

TABLE 2. **Four Car-following scenarios.**

Car-following Scenario	Probability	Critical Headway
Intelligent connected vehicles follow regular vehicles	$p(1-p)$	L_A(without RSU's range) L_U(within RSU's range)
Intelligent connected vehicles follow intelligent connected vehicles	$p \cdot p$	L_C
Regular vehicle follow intelligent connected vehicles	$(1-p)p$	L_R
Regular vehicles follow regular vehicles	$(1-p)(1-p)$	L_R

In Table 2, p denote the ratio of intelligent connected vehicles to all vehicles in a mixed traffic flow; then, the ratio of regular vehicles to all vehicles in the mixed traffic flow is $1-p$. When an intelligent connected vehicle is within the range of RSU, it can obtains traffic information about vehicles in the distance. In such situations, the critical headway between an intelligent connected vehicle and the regular vehicle in front can be shortened, resulting in an increase in the link capacity. However, when intelligent connected vehicles are not within the range of an RSU, direct communication with regular vehicles is not feasible. Hence, to maintain the safety of intelligent connected vehicles, manufacturers often prescribe a longer safe distance, which means $L_A > L_U$. When an intelligent connected vehicle follows another intelligent connected vehicle, they can exchange information directly via V2V communication rather than V2I communication, which allows a relatively closer safe distance. On that basis, we assume $L_A \geq L_R > L_U > L_C$ (Milanés and Shladover [30]).

We sequentially place RSUs on the segments while considering the communication range of RSUs. Adjacent RSUs are placed at a fixed distance from each other to prevent overlapping coverage and avoid wastage of resources. We assume there is an uniform traffic flow and that RSUs only affect the critical headway for an intelligent connected vehicle following a regular vehicle (Li et al [22]). In this scenario, we can divide the segment into two sub-segments based on whether the segment falls within the communication coverage range of an RSU. Since the distances between RSUs are fixed and only influenced by the communication coverage range, the starting point of an RSU does not affect the total communication coverage length of the segment. The calculation of travel time is solely dependent on the communication coverage length of RSUs. Therefore, the starting point of an RSU does not impact the travel time. Assuming that the starting point of an RSU coincides with a node on the segment, we can easily understand the division of the segment into two sub-segments based on whether they fall within the communication coverage range of an RSU.

We let d_l denote the length of link l, H denote the communication range of the RSUs, x_l denote the number of RSUs installed on link l. Therefore, the length of the segment within the communication range is $x_l H$, and that of the segment without the communication range is $d_l - x_l H$. Then, the capacity of the segment without communication range can be represented as follows (Chen et al [31]):

$$q_1 = \frac{v_M}{(1-p)L_R + p^2 L_C + p(1-p)L_A} = \frac{1}{(1-p)h_R + p^2 h_C + p(1-p)h_A} \tag{1}$$

where v_M denotes the critical speed, which does not change regardless of whether the segment is covered by RSUs, and $h_R = \frac{L_R}{v_M}$ refers to the critical time headway in corresponding car-following conditions; the other quantities are defined similarly. Equation (1) is used to calculate the capacity of link l. The numerator represents the critical speed of vehicles, which is treated as a constant in this paper. The denominator represents the average distance between vehicles, as detailed in Table 2. Therefore, dividing the numerator by the denominator allows us to determine the maximum number of vehicles that can pass through the link per unit of time.

Noting that capacity is distinct from density, road capacity refers to the maximum number of vehicles that a road or a specific road segment can handle during a specified time period. It is typically measured in terms of vehicles per hour (veh/h) or vehicles per day (veh/day). Road capacity represents the ability of the roadway infrastructure to efficiently and smoothly accommodate traffic flow, ensuring optimal utilization of the road.

Likewise, the capacity within the RSU communication range can be represented as follows:

$$q_2 = \frac{v_M}{(1-p)L_R + p^2 L_C + p(1-p)L_U} = \frac{1}{(1-p)h_R + p^2 h_C + p(1-p)h_U} \tag{2}$$

In order to calculate the travel time of one link, we assume it follows Bureau of Public Road (BPR) function.

$$t_i = t_{i0}(1 + \alpha(\frac{U}{Q})^{\beta}) \tag{3}$$

where t_{i0} denotes free-flow travel time, U denotes traffic flow, Q denotes capacity, α and β are regression coefficients depending on traffic condition.

By BPR function and equation (2), we can calculate the travel time of a link l without RSU.

$$\overline{t_l} - t_l = \frac{\alpha u_l^{\beta} x_l H}{v_0}[((1-p)h_R + p^2 h_C + p(1-p)h_A)^{\beta} \tag{4}$$

where v_0 denotes the free-flow speed and u_l indicates the traffic flow on link l.

If x_l RSUs are installed on link l, the whole link can be regarded as two segments, within and without communication range of RSU. Therefore, the travel time of link l is the sum of the travel time of these two segments.

$$t_l = \frac{d_l - x_l H}{v_0}(1 + \alpha(u_l((1-p)h_R + p^2 h_C + p(1-p)h_A))^{\beta}) + \frac{x_l H}{v_0}(1 + \alpha(u_l((1-p)h_R + p^2 h_C + p(1-p)h_U))^{\beta}) \tag{5}$$

Back to Table 2, it can be seen that RSU only influence the critical headway when intelligent connected vehicles follow regular vehicles, which means the reduced travel time only occurs within the communication range of RSU and can be calculated as follows:

$$\overline{t_l} = \frac{d_l}{v_0}(1 + \alpha(u_l((1-p)h_R + p^2 h_C + p(1-p)h_A))^{\beta}) - ((1-p)h_R + p^2 h_C + p(1-p)h_U)^{\beta}] \tag{6}$$

III. MATHEMATICAL MODEL

The model notations are defined in Table 3.

A. DETERMINISTIC MODEL

Via V2I communication, RSU can not only identify the historical trajectories of intelligent connected vehicles but also reduce the travel time on specific links by increasing the capacity. However, in a particular link with less traffic flow, RSU can barely improve the overall capacity of transportation system. Deploying costly RSU on these links may cause a waste of resources. Moreover, path flow reconstruction imposes new requirements on RSU deployment. Therefore, this paper focuses on balancing deployment cost and reduced travel time based on path flow reconstruction.

The deterministic model built on this basis is as follows:

$$\min \sum_{l \in L} c x_l - \frac{\gamma}{3600} \sum_{l \in L} (\overline{t_l} - t_l) \tag{7}$$

$$s.t. \sum_{l \in L} \theta_l^p x_l \geq 1 \forall p \in P \tag{8}$$

$$\sum_{l \in L} \sigma_l^{p,p'} x_l + \sum_{l \in L} \rho_l^{p,p'} x_l \geq 1 \ \forall p, p' \in P \tag{9}$$

TABLE 3. Set, parameters, and decision variables.

Symbols	Descriptions	Metrics
Set		
L	Set of links	
P	Set of paths	
Ω	Set of scenarios	
Parameter		
c	The unit price of RSUs	\$
d_l	Length of link l	m
H	RSU communication range on the link	m
θ_l^p	0 or 1 parameter; if the link l belongs to the path p, the value is 1; otherwise 0	
$\sigma_l^{p,p'}$	0 or 1 parameter; For arbitrary two paths p and p', if there is exactly one path including link l, the value is 1; otherwise 0	
$\rho_l^{p,p'}$	0 or 1 parameter; For arbitrary two paths p and p', if link l belongs to both paths, but these two paths exist at least one non-shared link before link l, the value is 1; otherwise 0	
γ	Value of time	\$/h
v_0	The speed of the vehicle moving freely on the link	m/s
α , β	Undetermined parameters of the BPR function	
p	The proportion of intelligent connected vehicles in the traffic flow	
h_R	The critical time headway of regular vehicles following a intelligent connected vehicle or regular vehicle	s
h_C	The critical time headway of intelligent connected vehicles following a intelligent connected vehicle	s
h_A	Without the RSU coverage, the critical time headway of intelligent connected vehicles following a regular vehicle	s
h_U	Within the RSU coverage，the critical time headway of Intelligent connected vehicles following a regular vehicle	s
$u_p(\omega)$	In the scenario ω, the traffic flow of path p	pcu/s
Variables		
x_l	The number of RSU installed on the link l	
$\bar{t}_l(\omega)$	Without RSU, in scenario ω, travel time for the whole link l	s
$t_l(\omega)$	Considering RSU deployment,in scenario ω, travel time for the whole link l	s

$$\bar{t}_l = \frac{d_l}{v_0}(1+\alpha(\sum_{p\in P}\theta_l^p u_p((1-p)h_R + p^2 h_C + p(1-p)h_A))^{\beta})\ \forall l \in L \tag{10}$$

$$t_l = \frac{(d_l - x_l H)}{v_0}(1+\alpha(\sum_{p\in P}\theta_l^p u_p((1-p)h_R + p^2 h_C + p(1-p)h_A))^{\beta}) + \frac{x_l H}{v_0}(1+\alpha(\sum_{p\in P}\theta_l^p u_p((1-p)h_R + p^2 h_C + p(1-p)h_U))^{\beta}) \tag{11}$$

$$0 \le x_l \le \left\lfloor \frac{d_l}{H} \right\rfloor, int \quad \forall l \in L \tag{12}$$

$$\bar{t}_l, t_l \ge 0\ \forall l \in L \tag{13}$$

where Equation (7) is the objective function, which minimizes the total cost of RSU minus the value of reduced travel time; Equation (8) ensures that for each path, at least one link deploys a RSU; Equation (9) ensures that for arbitrary two paths p and p', at least one RSU is installed in the non-shared link l ($\sigma_l^{p,p'} = 1$) or on a non-shared link before link l ($\rho_l^{p,p'} = 1$); Equations (8) and (9) express the preconditions of fully path flow reconstruction (Salari et al [26]); To address the difficulty in understanding the parameters involved in equation (9), this paper introduces two methods for distinguishing different paths in Section II Description, specifically in A. Path Flow Reconstruction, through illustrative examples. The content of this section contributes to a better comprehension of equation (9). Equation (10) calculates the link travel time without RSU; Equation (11) calculates the link travel time considering the deployment of RSU; Equations (12) ensures that the communication range of RSUs is not redundant and (13) ensure that the link travel time is non-negative.

B. TWO-STAGE STOCHASTIC PROGRAMMING MODEL

For a given origin-destination (OD) pair, there are multiple paths available for selection and the uncertainty comes from the path flow volume, which may be variant for a deterministic path combination. Although we can utilize roadside units to identify different paths through vehicle data, drivers have the possibility to change their original path and opt for an alternative one during their journey because of the dynamic traffic environment. In such cases, the roadside units are still capable of distinguishing between different paths and there is no uncertainty with historical trajectories identification. However, the traffic volume for each path changes, thereby affecting the traffic flow on the links. Due to the variable nature of the path flow, it becomes challenging to predict the definite benefits derived from the installation of RSUs by equation (6). To better describe and address the path flow uncertainty, this paper further builds a two-stage stochastic programming model for RSU functions.

The two-stage mathematical optimization model is expressed as follows:

$$\min \sum_{l\in L} c x_l + E_\omega [Q(\mathrm{x}, \omega)] \tag{14}$$

Such that (7)-(10) and (12)

$$Q(x,\omega) = \min -\gamma(\sum_{l\in L} \bar{t}_l(\omega) - t_l(\omega)) \tag{15}$$

$$\bar{t}_l(\omega) = \frac{d_l}{v_0}(1+\alpha(\sum_{p\in P}\theta_l^p u_p(\omega)((1-p)h_R + p^2 h_C + p(1-p)h_A))^{\beta}) \quad \forall l\in L\ \forall\omega\in\Omega \tag{16}$$

$$t_l(\omega) = \frac{(d_l - x_l H)}{v_0}(1+\alpha(\sum_{p\in P}\theta_l^p u_p(\omega)((1-p)h_R + p^2 h_C + p(1-p)h_A))^{\beta}) + \frac{x_l H}{v_0}(1+\alpha(\sum_{p\in P}\theta_l^p u_p(\omega)((1-p)h_R + p^2 h_C + p(1-p)h_U))^{\beta}) \quad \forall l\in L \quad \forall\omega\in\Omega \tag{17}$$

$$\bar{t}_l(\omega), t_l(\omega) \geq 0 \quad \forall l\in L \quad \forall\omega\in\Omega \tag{18}$$

In this two-stage stochastic programming model, the first stage, in which integer optimization is performed, is designed to minimize the total RSU installation cost in path flow reconstruction, and the second stage is built for linear programming. Under any scenario ω, after the model determines the RSU locations in the first stage, the model calculates the reduced vehicle travel time and then converts it to benefits in the second stage. In particular, the compressed expression form $E_\omega[Q(x,\omega)]$ of Equation (14) is a recourse function, which addresses the path flow uncertainty by calculating expectations for different scenarios ω.

Equation (14) represents the minimized total RSU installation cost from the first stage of the model and the expectation of the objective function value from the second stage of the model; Equation (15) provides the shortened vehicle travel time after maximizing the RSU location decision from the first stage of the model, and to be formally consistent with the first stage, the objective function is multiplied by (-1) to transform the maximization problem into a minimization problem; Equation (16) represents the vehicle travel time in the original road segment under scenario ω; Equation (17) represents the vehicle travel time under scenario ω after installing the RSUs; and Equation (18) ensures that the vehicle travel time is nonnegative.

IV. SOLUTION ALGORITHM

This paper employs the L-shaped algorithm [32] to solve the two-stage stochastic programming model presented above. During the solving process, SP denotes the subproblem, and MP refers to the master problem. For simplicity, this paper uses a simplified mathematical version to represent SP:

$$Q(x^v,\omega) = \min \mathrm{q}^T \mathrm{y}(\omega) \tag{19}$$

$$s.t. \mathrm{Wy}(\omega) = \mathrm{h}(\omega) - \mathrm{T}(\omega)\mathrm{x}^v \tag{20}$$

$$\mathrm{y}(\omega) \geq 0 \tag{21}$$

Theorem 1: For any feasible solution x of the master problem, at least one feasible solution y can be found in every subproblem. Thus, it is unnecessary to add a feasibility cut to the master problem.

Proof 1: $\bar{t}_l(\omega)$ in the subproblem is independent of x. The road length is $d_l > 0$; therefore, $\bar{t}_l(\omega) > 0$. If any feasible solution x of the master problem satisfies constraints (7) and (10), then $d_l - x_l H \geq 0$. In the subproblem, $d_l - x_l H \geq 0$; therefore, for any feasible solution x, feasible $\bar{t}_l(\omega)$ and $t_l(\omega)$ can always be found in the subproblem. It is unnecessary to add a feasibility cut.

$Q(x^v,\omega) = \max \pi^T(\omega)(\mathrm{h}(\omega) - \mathrm{T}(\omega)\mathrm{x}^v)$ In the classic L-shaped algorithm, the dual form of the subproblem represented by DP is required. According to strong duality from duality theory, the optimal value of the dual problem is the same as the optimal value of the original problem. Hence, the objective function is still represented by $Q(x,\omega)$. DP is expressed as follows:

$$\pi^T(\omega)\mathrm{W} \leq \mathrm{q} \tag{22}$$

$$s.t. \forall\omega\in\Omega \tag{23}$$

$$\pi(\omega) free \tag{24}$$

The form of MP is

$$\min \mathrm{c}^T\mathrm{x} + \theta \tag{25}$$

$$s.t.\ \mathrm{Ax} = \mathrm{b} \tag{26}$$

$$\mathrm{E}_\ell \mathrm{x} + \theta \geq e_\ell, \ell = 1, \ldots\ldots, s \tag{27}$$

$$x_l \geq 0, int \tag{28}$$

where constraint (27) is called an optimality cut, which can be generated from the solutions of SP and DP. The generation method is as follows:

$$E_\ell = \sum_{\omega\in\Omega} p(\omega)\cdot\pi^T(\omega)\mathrm{T}(\omega) \tag{29}$$

$$e_l = \sum_{\omega\in\Omega} p(\omega)\pi^T(\omega)\mathrm{h}(\omega) \tag{30}$$

$\pi(\omega)$ in constraints (22)-(24) and constraints (29)-(30) is the optimal solution of DP. Thus, to generate an optimality cut, x^v must be specified so that corresponding optimal solution can be found for any subproblem. According to Theorem 1, an optimality cut will be definitely generated.

In MP, an artificial variable θ is used to replace the recourse function $E_\omega[Q(x,\omega)]$, and the constraint on the artificial variable θ is strengthened by adding an optimality cut. The pseudocode of the L-shaped algorithm is presented in Algorithm 1.

V. NUMERICAL EXPERIMENT

To analyze the performance of the model, this paper uses the parallel highway network and Nguyen-Dupuis network, which are frequently used in studies to conduct experiments.

In numerical experiments, we analyze the following items:

1) Effects of the value of time and the unit price of RSU on objective value and total number of RSU.
2) Effect of the unit price of RSU on the deployment scheme.
3) Effect of the penetration rate of intelligent connected vehicles on objective value.
4) Effects of the number of scenarios on objective value, the total number of RSU, and the deployment scheme.

Algorithm 1 L-Shaped Algorithm

1: We define $r = s = v = 0$.

2: We set $v = v + 1$. The master problem (MP) is solved to obtain the optimal solution (x^v, θ^v).

3: If there is no constraint (27), then θ^v is set to $-\infty$, and it is not used to calculate x^v.

4: For $\forall \omega \in \Omega$, the dual problem (DP) is solved to obtain the optimal solution under scenario ω, namely, $\pi^v(\omega)$.

5: E_{s+1} and e_{s+1} are calculated via Equations (29) and (30).

6: We set $w^v = e_{s+1} - E_{s+1}x^v$. If $\theta^v \geq \omega^v$, the algorithm is terminated, and the optimal solution x^v is obtained; otherwise, we set s = s + 1, impose constraint (27) and return to step 2.

TABLE 4. The setting of parameters.

Parameters	Values	Metrics
H	500	m
v_0	30	m/s
α	0.5	
β	4	
h_R	1.5	s
h_A	1.5	s
h_U	1.1	s
h_C	0.6	s
p	0.5	

Experiments are conducted on both networks for Problems 1 and 2. The Problem 3, the effect of penetration rate of intelligent connected vehicles, is relatively easy to analyze, so we just conduct the experiment on parallel highway network. As for Problem 4, due to the small size of the parallel highway network, the experiment is only conducted on the larger network, namely, the Nguyen-Dupuis network. In experiments, we refer to the settings of Li et al [22] for the parameters associated with vehicle travel time, as shown in Table 4. In the BPR (Bureau of Public Roads) function, the values of α and β are often set as $\alpha = 0.5$ and $\beta = 4$. These values are commonly used and have been found to provide a reasonable approximation of traffic flow behavior in many transportation studies. However, it's important to note that the specific values of α and β can vary depending on the specific characteristics of the road network and the study context. Adjustments to these values may be made based on empirical observations, calibration, or the specific requirements of the analysis being conducted.

A. EXPERIMENTAL DESIGN FOR THE PARALLEL HIGHWAY NETWORK

The Parallel Highway network contains a total of 9 nodes and 14 links, as shown in Figure 4. The length of links in the network is presented in Table 5.

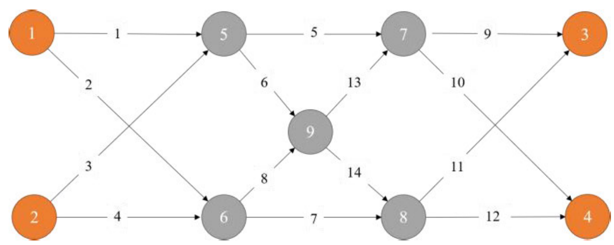


FIGURE 3. Parallel highway network.

TABLE 5. Length of links in the parallel highway network.

Link ID	Length (m)
1	2000
2	3000
3	3000
4	2000
5	1900
6	1200
7	1900
8	1200
9	2000
10	3000
11	3000
12	2000
13	1200
14	1200

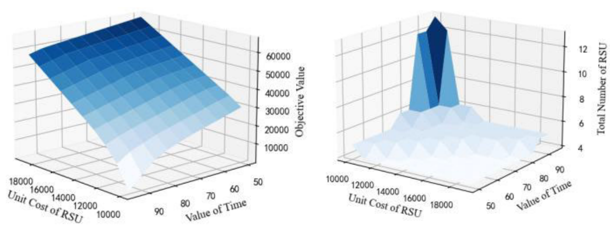


FIGURE 4. Results for parallel highway network: (a) Optimal objective value; (b) Optimal number of RSU.

The information of OD pairs and paths for the Parallel Highway network is shown in Table 6 below (Fu [24] et al.). It is noteworthy that the paths presented in Table 5 are generated by k-shortest path algorithm with k=4, so there exists some other path, e.g. 2-8-23-10, not appearing in corresponding OD flow.

1) SENSITIVITY ANALYSIS OF THE VALUE OF TIME AND THE UNIT PRICE OF RSU IN PARALLEL HIGHWAY NETWORK

This section examines the effect of value of time (γ) and unit price of RSU(c) on the value of the objective function, as shown in Figure 4(a). Numerically, the step in both experiments is 10% of initial value, respectively 10000$ for unit price of RSU and 50$/h for value of time. As value of time increases, the objective function value decreases; as unit price of RSU decreases, the objective function value also

TABLE 6. Paths in the parallel highway network.

OD flow	Path ID	Link
1-3	1	1-5-9
	2	1-6-13-9
	3	2-8-13-9
	4	1-6-14-11
1-4	5	1-6-14-12
	6	1-5-10
	7	2-7-12
	8	1-6-13-10
2-3	9	4-8-13-9
	10	3-5-9
	11	4-7-11
	12	3-6-13-9
2-4	13	4-7-12
	14	4-8-14-12
	15	4-8-13-10
	16	3-6-14-12

decreases. When unit price of RSU is high or value of time is low, the relationships between these two factor and objective value are both approximately linear. However, as unit price of RSU decreases and value of time increases, the objective value decreases substantially, which implies the deployment scheme of RSU changes greatly.

We further analyze the effect of value of time (γ) and unit price of RSU (c) on the total number of RSU, as shown in Figure 4(b). The trend is the same as the effect of value of time and unit price of RSU on the objective value. When value of time is high and unit price of RSU is low, the total number of RSU goes up drastically. In that cases, the deployment scheme of RSU changes which also reflects on the objective value in Figure 4(a).

2) NUMBER OF RSU ON EACH LINK IN THE PARALLEL HIGHWAY NETWORK

Unit price of RSU affects the number of RSU installed on different links, so we present the deployment scheme of RSU in Figure 5. Overall, as unit price of RSU increases from 10000$ to 20000$, the number of RSU on most links decreases. But it can be seen that RSUs are always installed on link 1, 9, 12, 13 whatever the unit price of RSU.

Recalling the intention of installing RSU, we would like to not only achieve the fully path flow reconstruction but also keep balance between deployment cost and value of reduced travel time. With high unit price of RSU, benefits of time cannot cover the expensive deployment cost, which implies the remained RSUs are used for fully path flow reconstruction.

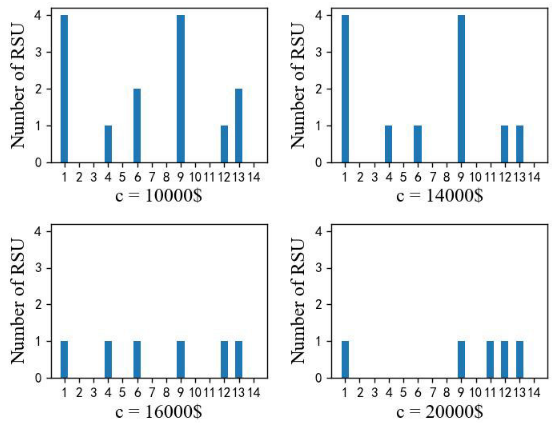


FIGURE 5. Number of RSU on different links in parallel highway network with $\gamma = 50\$/h$.

3) EFFECT OF PENETRATION RATE OF INTELLIGENT CONNECTED VEHICLES IN THE PARALLEL HIGHWAY NETWORK

For deterministic model, the penetration rate of intelligent connected vehicles influence the objective value, rather than path flow reconstruction. Thus, with equation (6), we can obtain the derivative of objective function with respective to p on the basis of fixed x_l.

$$\begin{aligned} f(p) = \lambda\beta[&(2(h_C - h_A)p + h_A - h_R)((1-p)h_R + p^2 h_C \\ &+ p(1-p)h_A)^{\beta-1} \\ &- (2(h_C - h_U)p + h_U - h_R)((1-p)h_R + p^2 h_C \\ &+ p(1-p)h_U)^{\beta-1}] \end{aligned} \tag{31}$$

where λ is negative and denotes the other parameters not corresponding to p.

As p increase from 0 to 1, if $f(p) < 0$ the objective value decrease and if $f(p) > 0$ the objective value increase. Figure 6 has proven that to some extent.

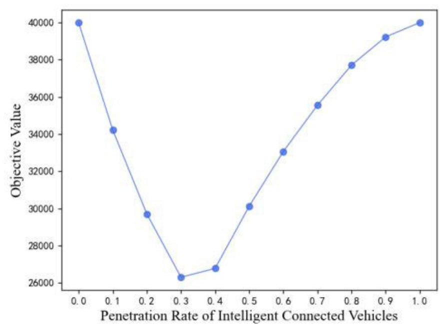


FIGURE 6. Objective Values For Different Penetration Rate of Intelligent Connected Vehicles in Parallel Highway Network with c = 10000$ and $\gamma = 50\$/h$.

However the deployment scheme of RSU is not necessarily fixed, which means x_l maybe not remains. As the penetration rate increase, the total number of RSU is shown in Figure 7.

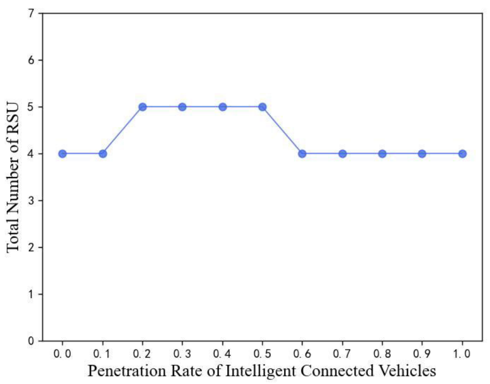


FIGURE 7. Total Number of RSU For Different Penetration Rate of Intelligent Connected Vehicles in Parallel Highway Network with $c = 10000\$$ and $\gamma = 50\$/h$.

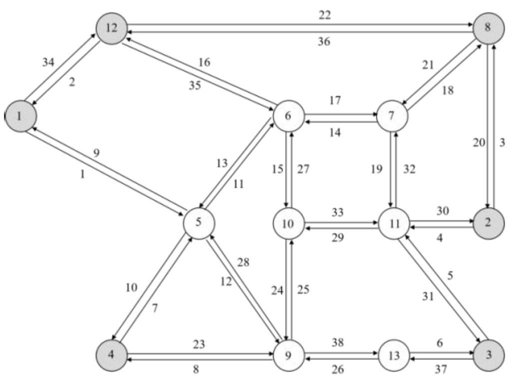
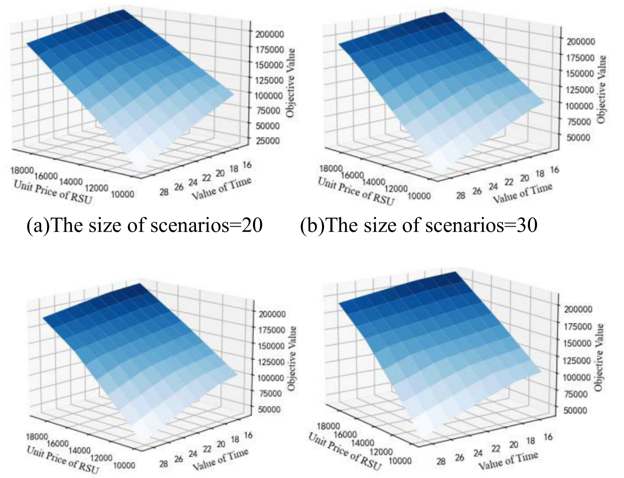


FIGURE 8. Nguyen-Dupuis network.

TABLE 7. Length of links in the nguyen-dupuis network.

Link ID	Length (m)
(1,12)	1800
(1,5)	2800
(5,6)	2000
(5,9)	2000
(5,4)	1900
(12,8)	6000
(12,6)	3000
(6,7)	1200
(6,10)	1500
(9,4)	2400
(9,10)	1500
(9,13)	1200
(8,7)	2200
(8,2)	3600
(11,7)	1500
(11,10)	1200
(11,2)	1200
(11,3)	2100
(3,13)	1200

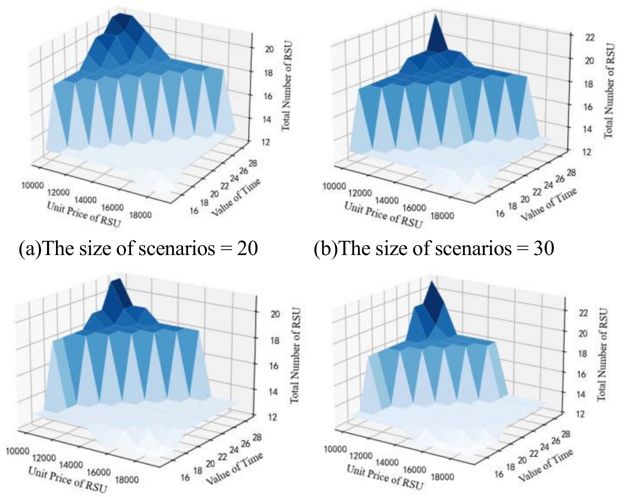
It might be confusing that when the penetration rate range from 0.2 to 0.5, the objective values, deployment cost minus value of reduced travel time, is relatively low. Recalling the



(a)The size of scenarios=20 (b)The size of scenarios=30

(c)The size of scenarios=40 (d)The size of scenarios=50

FIGURE 9. Objective value for different sizes of scenarios.



(a)The size of scenarios = 20 (b)The size of scenarios = 30

(c)The size of scenarios = 40 (d)The size of scenarios = 50

FIGURE 10. Number of RSU in different sizes of scenarios.

function of RSU discussed in Section II, only in the scenario where Intelligent connected vehicles follow regular vehicles, the critical headway can be reduced within communication range of RSU, which consequently brings reduced travel time. Therefore, a decrease in objective value depends on mixed traffic flow. For RSU, it is noteworthy that the function of reducing travel time is discussed in mixed traffic flow. When the penetration rate is close to 1, there exists other functions playing important roles, which otherwise is beyond the scope of this paper.

B. NUMERICAL EXPERIMENTAL FOR THE NGUYEN-DUPUIS NETWORK

The Nguyen-Dupuis network contains a total of 13 nodes and 38 links, as shown in Figure 8. The dark node is the start or end point of the OD flows, and the white node is the intermediate node of the OD flows. And for the generation of

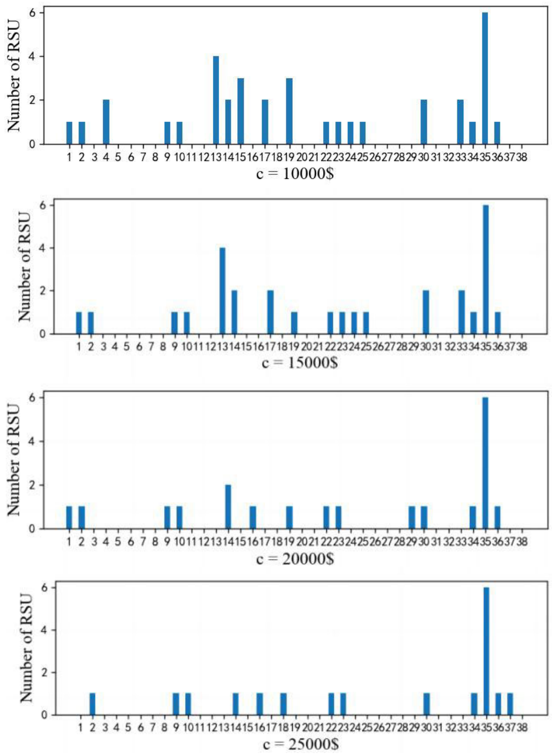


FIGURE 11. Number of RSU on different links under different unit price of RSU with $\gamma = 50\$/h$.

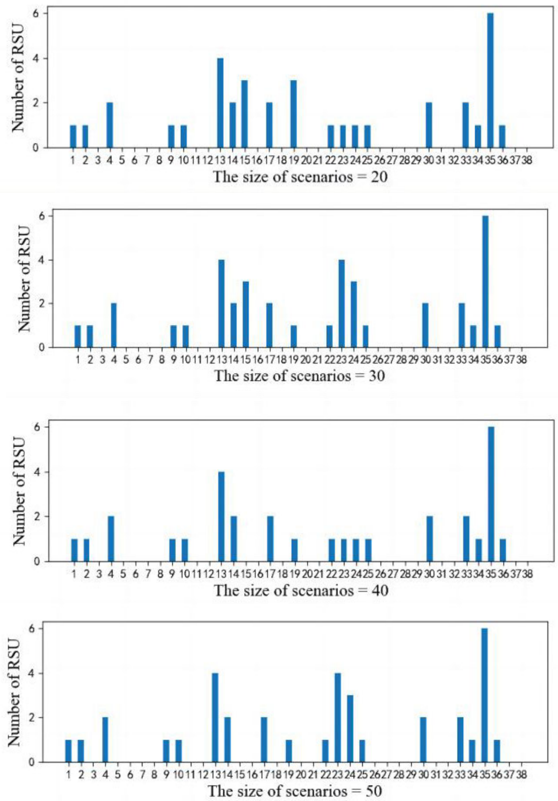


FIGURE 12. Number of RSU on different links under different sizes of scenarios.

paths in OD flows, the k-shortest path algorithm is employed. In this experiment, k is set to be 4, which means that each OD flow has at most four paths. The length of links in the network is presented in Table 7. Considering the scale of Nguyen-Dupuis network, writing all Path IDs will occupy too much space, so that content is omitted.

1) INFLUENCE ON THE OBJECTIVE VALUE IN THE NGUYEN-DUPUIS NETWORK

This section analyzes the influence of different parameters in the N-D network and the size of scenarios on objective value, presenting the trend of the objective function value under different sizes of scenarios, as shown in Figure 9. The effect of unit price of RSU and value of time on objective value on N-D network is similar to experiments on P-H network, so it is not repeated in this experiment. Considering the impact of the size of scenarios on the objective value, the value of time is set to take the range of 15 to 30 with a step of 1.5; the unit price of RSU was made to take the range of 10000 to 20000 with a step of 1000. From Figure 9, it shows that different sizes of scenarios have less impact on the trend of objective value.

2) INFLUENCE ON THE TOTAL NUMBER OF RSU IN THE NGUYEN-DUPUIS NETWORK

The influence of different parameters and size of scenarios on the total number of RSUs in the N-D network is shown in Figure 10. When value of time is small and the unit price of RSU is large, the total number of RSUs is not sensitive to them; As value of time increase or unit price of RSU decrease, a surge in total number of RSU appears. A reason for that is that there are several links with large traffic flow. When the benefits from reduced travel time of these links surpass the cost of an additional RSU, RSUs are recommended to be installed until the communication range of RSU on these links is large enough.

As for the size of scenarios, it has obvious influence on total number of RSU when the unit price of RSU is low and value of time is high. However, the size of scenarios doesn't change the trend of total number of RSU.

3) INFLUENCE ON DEPLOYMENT SCHEME OF RSU IN THE NGUYEN-DUPUIS NETWORK

The number of RSU on each link in the N-D network is shown in Figure 11. It is expected that as the unit price of RSU increases, the total number of RSU reduces. However, there exists some links always with at least one RSU whatever the unit price of RSU, e.g. link 2, which means these links play an important role in path flow reconstruction. Furthermore, when the cost of additional RSU far outweigh the benefit from reduced travel time, this problem degenerate to achieve fully path flow reconstruction with minimum number of RSU. The length of links also restrict the installation of RSU.

Considering the communication range of RSU, it is a waste if too many RSUs are put on the same link.

To explore the influence of the size of scenarios on each links, we set the unit price of RSU to 10000$, and the value of time to 50$/h, and the size of scenarios from 20 to 50 with step of 10, as shown in Figure 12. It shows that the number of RSU on each link is less variable. As the size of scenarios increases, the number of RSU on some links grows and tends to be stable. The reason would be that the traffic flow on some links increases in an average sense when considering more OD flows, which leads to more RSUs.

VI. CONCLUSION

The cooperative vehicle infrastructure system in the IoV environment provides a new solution to the problem of path flow reconstruction. Based on the ability of RSU to obtain historical trajectories of passing vehicles, this paper proposes an RSU deployment optimization model for fully path flow reconstruction. In addition, RSU can exchange information with intelligent connected vehicles to shorten the security distance from front cars, which consequently enlarge the road capacity and finally reduce the average travel time. On this basis, considering the uncertainty of path traffic flow, a two-stage stochastic model is built to balance the deployment cost of RSU and the value of reduced travel time in transportation system.

The achievements and contributions of this paper are as follows: (1) The role of RSU in path flow reconstruction and the balance between deployment cost and value of reduced travel time are fully considered. (2) The uncertainty in path flow are integrated into the calculation of average travel time. Consequently a two-stage stochastic model is proposed. (3) The importance of each links on path flow reconstruction can be identified. (4) Expanding the size of scenarios has little impact on experimental results, which indicating that this model has good applicability.

The following considerations should be taken into account when applying the findings of this paper to real-world transportation networks:

(1) Model Verification: To ensure the accuracy and reliability of our model, it is essential to establish strong cooperation with local governments or traffic departments in order to obtain the necessary data. The parameters required for the model include the length of different links, path combinations for specific OD pairs, free-flow speed of vehicles, the traffic condition parameter in the BPR function, the proportion of intelligent connected vehicles in the traffic flow, and the path flow during various time periods. These critical data will be utilized to run the model and conduct a comprehensive analysis of the deployment of roadside units. The major output of the model is to determine where to install the RSUs. The primary objective of this analysis is to verify whether the reduced travel time predicted by the model aligns with real-world observations. By comparing the model's outcomes with actual travel time measurements, we can assess the model's effectiveness and its potential for practical application.

(2) Promotion of Public Policies: As this research focuses on the installation of one or more roadside units on road sections, effective communication with the government is crucial to convey the research findings and the potential benefits it may offer. The stochastic model provides two key aspects in its outputs. Firstly, the model presents a well-defined deployment strategy for RSUs, highlighting the recommended locations and quantities for optimal installations across different links. This strategy aims to maximize the efficiency and effectiveness of RSU deployment. Secondly, the model also predicts the extent of reduced travel time in different links if the suggested RSUs are installed. This reduction in travel time signifies the positive impact that RSUs can have on enhancing transportation efficiency. By effectively conveying these two aspects, we can foster understanding and support from the government, paving the way for the adoption of public policies that promote the widespread implementation of roadside units, thereby improving overall transportation infrastructure and ensuring a more seamless travel experience for the public.

This paper can be enhanced by addressing the following aspects:

(1) Enhanced Path Randomness: To further enrich the study, it is essential to consider the incorporation of more sophisticated randomness in different paths within the OD flow. Analyzing the impact of this randomness on flow redistribution will provide a deeper understanding of traffic patterns and aid in optimizing traffic management strategies.

(2) Mixed Deployment Strategy: To maximize the effectiveness of traffic sensing, a mixed deployment strategy involving different types of traffic sensors should be explored. This approach can provide comprehensive and diverse data inputs, enabling more accurate and robust traffic analysis.

(3) I2I Communication Technologies: The inclusion of new technologies like I2I communication (communication between RSU and RSU) is worth exploring. Analyzing and incorporating these advancements into the mathematical models will enhance the capabilities of the proposed system, leading to more efficient and reliable traffic management solutions.

(4) High-Efficiency Algorithms: For large-scale networks, the development of high-efficient algorithms is paramount. These algorithms will ensure that the proposed models can handle extensive data sets and complex traffic scenarios without compromising computational efficiency.

By addressing these aspects, the paper will offer a more comprehensive and sophisticated approach, ultimately contributing to the advancement of traffic management systems and fostering more efficient and sustainable transportation networks.

REFERENCES

[1] M. M. Hamdi, L. Audah, S. A. Rashid, A. H. Mohammed, S. Alani, and A. S. Mustafa, "A review of applications, characteristics and challenges in vehicular ad hoc networks (VANETs)," in *Proc. Int. Congr. Hum.-Comput. Interact., Optim. Robotic Appl. (HORA)*, Jun. 2020, pp. 1–7.

[2] R. Ali, R. Liu, A. Nayyar, I. Waris, L. Li, and M. A. Shah, "Intelligent driver model-based vehicular ad hoc network communication in real-time using 5G new radio wireless networks," *IEEE Access*, vol. 11, pp. 4956–4971, 2023.

[3] G. Yu, H. Li, Y. Wang, P. Chen, and B. Zhou, "A review on cooperative perception and control supported infrastructure-vehicle system," *Green Energy Intell. Transp.*, vol. 1, no. 3, Dec. 2022, Art. no. 100023.

[4] M. Wang, "Infrastructure assisted adaptive driving to stabilise heterogeneous vehicle strings," *Transp. Res. C, Emerg. Technol.*, vol. 91, pp. 276–295, Jun. 2018.

[5] A. Guerna, S. Bitam, and C. T. Calafate, "Roadside unit deployment in Internet of Vehicles systems: A survey," *Sensors*, vol. 22, no. 9, p. 3190, Apr. 2022.

[6] L. Xue, Y. Yang, and D. Dong, "Roadside infrastructure planning scheme for the urban vehicular networks," *Transp. Res. Proc.*, vol. 25, pp. 1380–1396, Jan. 2017.

[7] D. Kim, Y. Velasco, W. Wang, R. N. Uma, R. Hussain, and S. Lee, "A new comprehensive RSU installation strategy for cost-efficient VANET deployment," *IEEE Trans. Veh. Technol.*, vol. 66, no. 5, pp. 4200–4211, May 2017.

[8] M. Lehsaini, N. Gaouar, and T. Nebbou, "Efficient deployment of roadside units in vehicular networks using optimization methods," *Int. J. Commun. Syst.*, vol. 35, no. 14, pp. 1–12, Sep. 2022.

[9] S. Anbalagan, A. K. Bashir, G. Raja, P. Dhanasekaran, G. Vijayaraghavan, U. Tariq, and M. Guizani, "Machine-learning-based efficient and secure RSU placement mechanism for software-defined-IoV," *IEEE Internet Things J.*, vol. 8, no. 18, pp. 13950–13957, Sep. 2021.

[10] E. R. Magsino and I. W. Ho, "An enhanced information sharing roadside unit allocation scheme for vehicular networks," *IEEE Trans. Intell. Transp. Syst.*, vol. 23, no. 9, pp. 15462–15475, Sep. 2022.

[11] A. Guerna, S. Bitam, and C. T. Calafate, "AC-RDV: A novel ant colony system for roadside units deployment in vehicular ad hoc networks," *Peer-Peer Netw. Appl.*, vol. 14, no. 2, pp. 627–643, Mar. 2021.

[12] Y. Liang, S. Zhang, and Y. Wang, "Data-driven road side unit location optimization for connected-autonomous-vehicle-based intersection control," *Transp. Res. C, Emerg. Technol.*, vol. 128, Jul. 2021, Art. no. 103169.

[13] Y. Wang and J. Zheng, "Connectivity analysis of a highway with one entry/exit and multiple roadside units," *IEEE Trans. Veh. Technol.*, vol. 67, no. 12, pp. 11705–11718, Dec. 2018, doi: 10.1109/TVT.2018.2873706.

[14] M. Fogue, J. Sanguesa, F. Martinez, and J. Marquez-Barja, "Improving roadside unit deployment in vehicular networks by exploiting genetic algorithms," *Appl. Sci.*, vol. 8, no. 1, p. 86, Jan. 2018.

[15] Y. Ni, J. He, L. Cai, J. Pan, and Y. Bo, "Joint roadside unit deployment and service task assignment for Internet of Vehicles (IoV)," *IEEE Internet Things J.*, vol. 6, no. 2, pp. 3271–3283, Apr. 2019.

[16] Z. Ahmed, S. Naz, and J. Ahmed, "Minimizing transmission delays in vehicular ad hoc networks by optimized placement of road-side unit," *Wireless Netw.*, vol. 26, no. 4, pp. 2905–2914, May 2020.

[17] F. Yang, C. Zhao, X. Ding, and J. Han, "An analytical model for energy harvest road side units deployment with dynamic service radius in vehicular ad-hoc networks," *IEEE Access*, vol. 8, pp. 122589–122598, 2020.

[18] S. Zhang, N. Zhang, X. Fang, P. Yang, and X. S. Shen, "Cost-effective vehicular network planning with cache-enabled green roadside units," in *Proc. IEEE Int. Conf. Commun. (ICC)*, May 2017, pp. 1–6.

[19] M. Al Shareeda, A. Khalil, and W. Fahs, "Towards the optimization of road side unit placement using genetic algorithm," in *Proc. Int. Arab Conf. Inf. Technol. (ACIT)*, Nov. 2018, pp. 1–5.

[20] J. Heo, B. Kang, J. M. Yang, J. Paek, and S. Bahk, "Performance-cost tradeoff of using mobile roadside units for V2X communication," *IEEE Trans. Veh. Technol.*, vol. 68, no. 9, pp. 9049–9059, Sep. 2019.

[21] J. Lee and S. Ahn, "Adaptive configuration of mobile roadside units for the cost-effective vehicular communication infrastructure," *Wireless Commun. Mobile Comput.*, vol. 2019, pp. 1–14, Jul. 2019.

[22] Y. Li, Z. Chen, Y. Yin, and S. Peeta, "Deployment of roadside units to overcome connectivity gap in transportation networks with mixed traffic," *Transp. Res. C, Emerg. Technol.*, vol. 111, pp. 496–512, Feb. 2020.

[23] C. Fu, N. Zhu, S. Ling, S. Ma, and Y. Huang, "Heterogeneous sensor location model for path reconstruction," *Transp. Res. B, Methodol.*, vol. 91, pp. 77–97, Sep. 2016.

[24] C. Fu, N. Zhu, and S. Ma, "A stochastic program approach for path reconstruction oriented sensor location model," *Transp. Res. B, Methodol.*, vol. 102, pp. 210–237, Aug. 2017.

[25] F. Álvarez-Bazo, R. Cerulli, S. Sánchez-Cambronero, M. Gentili, and A. Rivas, "An iterative multiparametric approach for determining the location of AVI sensors for robust route flow estimation," *Comput. Oper. Res.*, vol. 138, Feb. 2022, Art. no. 105596.

[26] M. Salari, L. Kattan, and M. Gentili, "Optimal roadside units location for path flow reconstruction in a connected vehicle environment," *Transp. Res. C, Emerg. Technol.*, vol. 138, May 2022, Art. no. 103625.

[27] T. S. Abraham and K. Narayanan, "Cooperative communication for vehicular networks," in *Proc. IEEE Int. Conf. Adv. Commun., Control Comput. Technol.*, May 2014, pp. 1163–1167.

[28] J. B. Kenney, "Dedicated short-range communications (DSRC) standards in the United States," *Proc. IEEE*, vol. 99, no. 7, pp. 1162–1182, Jul. 2011.

[29] D. Wu, Y. Ling, H. Zhu, and J. Liang, "The RSU access problem based on evolutionary game theory for VANET," *Int. J. Distrib. Sensor Netw.*, vol. 9, no. 7, Jul. 2013, Art. no. 143024.

[30] V. Milanés, S. E. Shladover, J. Spring, C. Nowakowski, H. Kawazoe, and M. Nakamura, "Cooperative adaptive cruise control in real traffic situations," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 1, pp. 296–305, Feb. 2014.

[31] D. Chen, S. Ahn, M. Chitturi, and D. A. Noyce, "Towards vehicle automation: Roadway capacity formulation for traffic mixed with regular and automated vehicles," *Transp. Res. B, Methodol.*, vol. 100, pp. 196–221, Jun. 2017.

[32] R. M. Van Slyke and R. Wets, "L-shaped linear programs with applications to optimal control and stochastic programming," *SIAM J. Appl. Math.*, vol. 17, no. 4, pp. 638–663, Jul. 1969.

ZIJIAN BAI received the B.S. and M.S. degrees from the School of Transportation, Jilin University, China, in 2003, and the Ph.D. degree from the Institute of Systems Engineering, Tianjin University, China, in 2007.

Since 2007, he has been with the Tianjin Municipal Engineering Design & Research Institute Company Ltd., where he is currently the Director and a Chief Engineer with the Urban and Traffic Planning and Design Institute with Management Science and Engineering. His research interests include ITS and urban traffic control and management.

ZIXUAN BAI received the B.S. degree in industrial engineering from Tianjin University, Tianjin, China, in 2023. He is currently pursuing the M.S. degree in management science and engineering with Shanghai Jiao Tong University, Shanghai, China.

From 2021 to 2022, he was a Research Assistant with the Department of Management and Economics, Tianjin University. His research interests include transportation system optimization, stochastic programming, and mixed integer programming.

HENGBO ZHU received the B.S. degree in information management and information systems from Tianjin University, Tianjin, China, in 2022, where he is currently pursuing the M.S. degree in management science and engineering.

From 2021 to 2022, he was a Research Assistant with the Department of Management and Economics, Tianjin University. His research interests include stochastic programming, integer programming, and robust optimization.

SHUIPING KE received the B.S. and M.S. degrees from the School of Transportation, Shanghai Maritime University, China, in 2009, and the Ph.D. degree from the Institute of Systems Engineering, Tianjin University, China, in 2015.

Since 2009, he has been with the Tianjin Municipal Engineering Design & Research Institute Company Ltd., where he is currently the Deputy Director of the Urban and Traffic Planning and Design Institute with Management Science and Engineering. His research interest includes urban traffic planning and design.

YAO SUN received the B.S. and M.S. degrees from the School of Transportation Management, Dalian Maritime University, China, in 2018, and the Ph.D. degree in vehicle engineering from the Dalian University of Technology, China, in 2022.

Since 2018, he has been with the Tianjin Municipal Engineering Design & Research institute Company Ltd., where he is currently studying on big data technology. His research interests include ITS and CAV.