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RESEARCH ARTICLE

# Adaptive Fuzzy Event-Triggered Control for Non-Strict Feedback Nonlinear Systems With Input Delay and Full State Constraints

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**ABSTRACT** An event-triggered-based adaptive fuzzy control strategy is proposed for full state constraints nonlinear systems against time-varying disturbance, non-strict feedback structure, and input delay in this paper. Fuzzy logic systems (FLSs) are implemented to estimate the unknown nonlinear functions in the systems. The influence of input delay can be compensated by the method of Pade approximation, and the barrier Lyapunov function (BLF) is exploited to handle the problem of full state constraints. The input-to-state stability (ISS) assumption regarding measurement errors can be removed through the co-design of the event-triggered mechanism and the controller. Based on Lyapunov stability theory, all the signals in the closed-loop system are proved semi-globally uniformly ultimately bounded (SGUUB), and only a tiny tracking error between the system output and the reference signals, moreover, the Zeno behavior is avoided. The effectiveness of the adaptive control scheme can be verified by two simulation examples.

**INDEX TERMS** Adaptive fuzzy control, event-triggered mechanism, full state constraints, input delay, non-strict feedback structure.

## I. INTRODUCTION

In the past few decades, the study of nonlinear control systems has attracted a lot of attention because of its great theoretical and practical significance. As a result, a variety of design approaches for nonlinear system controllers have emerged. For instance, the adaptive backstepping method is considered to be an effective tool for solving the control problem of nonlinear systems, but it is no longer applicable for nonlinear systems with completely unknown nonlinear terms. In addition, neural networks (NNs) or fuzzy logic systems (FLSs) are widely used to estimate unknown terms of nonlinear systems due to their universal approximation properties for unknown nonlinear functions. Combined with the adaptive backstepping technique, the iconic results of adaptive control strategy have been widely reported [1], [2], [3], [4], [5], [6], [7], [8], [9]. For example, to realize the fixed-time tracking control of switched nonlinear systems with unmodeled dynamics and actuator fault, an adaptive fuzzy fault-tolerant control strategy was designed in [3] and [9]. The structural characteristics of NNs were exploited to approximate the unknown functions in the system and an adaptive tracking control scheme was established for pure feedback stochastic systems [7].

It is worth noting that in a wide variety of physical systems, constraint problems are always inevitable. Such as robot manipulator [10], multi-agent system [11], vehicle active suspension system [12], etc. Once the system violates certain constraints, it may lead to deterioration of system performance and even induce security incidents. It is clear that the study of constraint problems has deep practical significance. In order to solve the constraint problem, Professor S. S. Ge proposed an adaptive control method based on the barrier Lyapunov function (BLF) for single-input single-output (SISO) nonlinear systems in 2009 [13]. Due to the mathematical properties of the BLF, the constraints of the system can be guaranteed not to be violated [14], [15].

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Recently, a new time-varying BLF that can be applied to fractional-order systems was proposed in [16], when the initial tracking conditions are completely unknown or the constraints are initially violated, the introduced shifting function and error transformation scheme can still ensure that the output of system satisfies the constraints within the specified time. In the above work, most of the consideration is the output constraints, that is only the BLF is introduced in the controller design in the first step, compared with it, the full-state constraints are difficult as well as more convincing in practice [17], [18], [19], [20], [21], [22], [23], [24]. For instance, an adaptive tracking control method with full-state constraints was proposed for multi-coupled nonlinear systems and applied to chemical continuous stirred reactors [20]. Meanwhile, the problem of time delay is a major hindrance to the control performance of systems, so eliminating the influence of time delay on nonlinear systems has been a hot issue over the past few decades. In [25], [26], and [27], the Lyapunov-Krasovskii functions were employed to address the time delay problem of the system. Note that the above approach focuses on state delay, but it is not effective when dealing with input delay. Therefore, several other control strategies have been proposed to compensate for the input delay of nonlinear systems. In [28], aiming at the active suspension system, the influence of input delay was compensated by introducing an integral control input signal. Recently, the Pade approximation method was introduced into different nonlinear systems to handle input delay effectively [29], [30], [31].

Obviously, most of the adaptive control schemes proposed above are for specific system forms such as strict feedback or pure feedback systems, while nonlinear systems with non-strict feedback structures are more general in actual systems. Unlike the above-mentioned systems, the nonlinear terms in non-strict feedback nonlinear systems contain whole state vectors of the system. In the process of designing a controller with the backstepping method, for the $i$th subsystem, the virtual controller $\alpha_i$ is designed to stabilize the subsystem, in order to ensure the feasibility of the virtual controller design, $\alpha_i$ is usually the equation of partial state vector $\underline{x}_i = [x_1, x_2, \ldots, x_i]^T$ $(i = 1, 2, \ldots, n-1)$. If the previously proposed methods are applied to non-strict feedback nonlinear systems, the virtual controller $\alpha_i$ will contain whole state vector $x = [x_1, x_2, \ldots, x_n]^T$, and the algebraic loop problem will be induced. To solve this problem, the idea of variable separation was first proposed in [32]. Subsequently, in [33] and [34], the algebraic loop problem was solved by using the structural characteristics of the neural network and the variable separation method, respectively, but these control schemes all have assumptions about the monotonous increase of the nonlinear function in the system. Therefore, this Methodological progress is severely hindered. To eliminate this deficiency, a new technique, which exploits the mathematical structure of fuzzy basis functions to eliminate the above assumptions and handle the algebraic loop problem, was proposed by Tong et al. [35]. Furthermore, adaptive fuzzy control schemes for non-strict feedback nonlinear systems were studied in [36], [37], and [38].

Additionally, compared with the traditional time-triggered control (TTC) method, which requires continuous periodic updates of control input, event-triggered control (ETC) is more intelligent and activates the controller through a trigger mechanism designed according to certain requirements. Therefore, ETC is a more feasible technique, especially when system communication resources are limited. Based on the framework of ETC, a large number of excellent research results can be obtained [39], [40], [41], [42]. Specifically, an event-triggered adaptive controller design scheme integrating static reliability information and dynamic online information was proposed in [41], and the problem of unknown actuator faults for nonlinear systems was solved. However, the above ETC mechanisms all rely on the assumption that measurement errors in systems are input-to-state stable (ISS). In practice, this assumption is difficult to verify in nonlinear uncertain systems. In [43], the authors proposed an ETC mechanism and adaptive controller co-design method, the ISS assumption was eliminated, and the switching threshold mechanism was proposed. On this basis, some significant achievements can be obtained [44], [45], [46], [47], [48], [49], [50], [51], [52], [53], [54]. For example, an observer-based event-triggered adaptive fuzzy backstepping synchronization control method was proposed in [52]. Dong et al. used the compensation tracking error signal to construct an intermediate control function, which more accurately reflected the actual measurement error and the impact of ETC on synchronization accuracy was reduced. Moreover, due to the discontinuity of the output signal and the strong coupling of state variables, it is more challenging to apply ETC to the sensor-to-actuator stage [55]. In [56], a new model-based adaptive ETC strategy for discrete-time nonlinear systems was studied. However, so far, to our knowledge, few achievements have investigated the full state constraints of nonlinear systems with input delay and non-strict feedback structure based on the ETC strategy.

Inspired by the discussion of the above research work, in this study, an adaptive ETC for constrained nonlinear systems against non-strict feedback structure and input delay is proposed. To approximate the unknown nonlinear functions, FLSs are employed. The BLF is introduced to deal with the full state constraints problem. The effect of input delay is removed by the method of Pade approximation. By co-designing the controller and the ETC mechanism, the ISS assumption about measurement errors is not required. ETC strategy based on relative threshold can effectively reduce the consumption of computing resources. By comparing with previous literature, the main contributions and innovations of this study are as follows:

(1) Unlike the time delay problem or constraint control of strict feedback and pure feedback system [22], [23], [24], [25], [26], [27], [28], a new fuzzy adaptive ETC scheme for non-strict feedback nonlinear system with full-state

constraints and input delay is proposed. Taking advantage of the properties of fuzzy basis functions, the algebraic loop problem is solved, and the monotonically increasing limit of unknown nonlinear functions is relaxed.

(2) Different from the traditional TTC strategy [20], [21], [31], the ETC is introduced into the design of the adaptive controller, and the proposed adaptive ETC strategy can greatly reduce the consumption of communication resources and bandwidth occupation and the Zeno behavior is avoided.

(3) We extend the improved relative threshold-based ETC mechanism to the constrained non-strict feedback nonlinear systems. From the simulation results, it not only can save communication resources but also has smaller tracking errors than [30], moreover, all states do not violate the constraint interval.

## II. PROBLEM FORMULATION

### A. PROBLEM STATEMENT

Consider the following non-strict feedback nonlinear state constrained system with input delay.

$$\begin{cases} \dot{x}_i = h_i(x) + x_{i+1} + \varepsilon_i(x,t), i = 1, 2, \ldots, n-1 \\ \dot{x}_n = h_n(x) + u(t - t_d) + \varepsilon_n(x,t), \\ y = x_1, \end{cases} \tag{1}$$

where $\underline{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$ $(x = \underline{x}_n, i = 1, 2, \ldots, n)$ stands for the state vector, $u \in R$ and $y \in R$ are input and output of the system. $h_i(x), i = 1, 2, \ldots, n$ denotes unknown nonlinear function. $\varepsilon_i(x,t), i = 1, 2, \ldots, n$ represents external disturbance, and $\varepsilon_i(x,t)$ have positive constant upper bound $\bar{\varepsilon}_i$. $t_d$ is defined as input delay.

The control aim of this article is with the new controller for system(1), which enables the tracking error and the system states can be adjusted to the prescribed constraint interval. Moreover, all variables are bounded and avoid Zeno behavior.

*Assumption 1 ([30]):* For $y_r(t)$ and $y_r^{(k)}(t)$, which are reference signals and their time derivatives, there are some positive constants $\omega_1$ and $Y_0, Y_1, \ldots, Y_n$ such that $|y_r(t)| \le Y_0 \le \omega_1$, $|y_r^{(k)}(t)| \le Y_k, k = 1, 2, \ldots, n$.

*Assumption 2 ([37]):* There exist known constants $l_i, i = 1, 2, \ldots, n$, for $\forall x_1, x_2$, the inequality holds:

$$\|h_i(x_1) - h_i(x_2)\| \le l_i \|x_1 - x_2\|,$$

where $\|x\|$ is the 2-norm of a vector $x$.

*Remark 1:* During the controller design, it can be obtained from Assumption 1 that all signals are bounded because $y_r(t)$ and $y_r^{(k)}(t)$ are bounded. Assumption 1 is often constructed in the study of adaptive control of nonlinear systems, like in [6], [14], [30], and [44].

### B. FUZZY LOGIC SYSTEMS

Among the components of an FLS, fuzzy rules are the core embodiment of its approximation ability, which can usually be described in the following form:

$\mathcal{R}^l$: If $x_1$ is $\mathcal{H}_1^l$ and …and $x_n$ is $\mathcal{H}_n^l$, then $y$ is $\mathcal{B}^l$, $l = 1, 2, \ldots, N$, where $y$ is the output of the FLS. $\mathcal{H}_i^l$ and $\mathcal{B}^l$ stand for the fuzzy sets, $i = 1, 2, \ldots, n$. From [39], $y$ can be described as follows:

$$y(x) = \frac{\sum_{l=1}^N \bar{y}_l \prod_{i=1}^n \mu_{\mathcal{H}_i^l}(x_i)}{\sum_{l=1}^N [\prod_{i=1}^n \mu_{\mathcal{H}_i^l}(x_i)]}, \tag{2}$$

where $\bar{y}_l = \max_{y \in R} \mu_{\mathcal{B}^l}(y)$.

$\psi(x) = [\psi_1(x), \psi_2(x), \ldots, \psi_N(x)]^T$ denotes the fuzzy basis function, and $\psi_l(x)$ is defined as

$$\psi_l(x) = \frac{\prod_{i=1}^n \mu_{\mathcal{H}_i^l}(x_i)}{\sum_{l=1}^N [\prod_{i=1}^n \mu_{\mathcal{H}_i^l}(x_i)]},$$

substituting $\Theta = [\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_N]^T = [\Theta_1, \Theta_2, \ldots, \Theta_N]^T$ and $\psi(x) = [\psi_1(x), \psi_2(x), \ldots, \psi_N(x)]^T$ into (2), we can obtain:

$$y(x) = \Theta^T \psi(x). \tag{3}$$

*Lemma 1 ([57]):* If $h_i(x)$ is a continuous function defined on a compact set $\Omega$, for any constant $\epsilon > 0$, there exists the above FLS such that:

$$\sup_{x \in \Omega} |h(x) - \Theta^T \psi(x)| \le \epsilon.$$

### C. BARRIER LYAPUNOV FUNCTION

Different from the general form of Lyapunov function, BLF has the ability to constrain related states due to its special mathematical properties, and BLF has three different types that can be flexibly selected according to requirements. The log-type BLF proposed in [13] will be adopted:

$$\bar{V}_i = \frac{1}{2} \log \frac{\omega_{ei}^2}{\omega_{ei}^2 - e_i^2}. \tag{4}$$

where $\log(\bullet)$ denotes the natural logarithm of $\bullet$, $e_i$ satisfies $|e_i| < \omega_{ei}$, and the BLF will soar to infinity if $|e_i| \to \omega_{ei}$. In addition, $\bar{V}_i$ is a positive definition function, and it is continuous in set $|e_i| < \omega_{ei}$.

*Lemma 2 ([37]):* Given that $|e_i| < \omega_{ei}$, for any positive constant $\omega_{ei}$, the inequality holds:

$$\log \frac{\omega_{ei}^2}{\omega_{ei}^2 - e_i^2} \le \frac{e_i^2}{\omega_{ei}^2 - e_i^2}.$$

*Remark 2:* In practical physical systems, the state constraint is necessary, and Once the constraint interval is violated by the system state, it will cause the system performance degradation and even equipment damage. In this paper, according to the mathematical properties of BLF, the tracking error is limited as well as the whole state is constrained by introducing BLF at each step of the backstepping method.

## III. ADAPTIVE CONTROLLER DESIGN

The input delay term $u(t - t_d)$ is treated by the method of Pade approximation in [31], which eliminates the effect of the input delay by the Laplace transform, it can be presented as follows:

$$\mathcal{L}\{u(t-t_d)\} = e^{(-t_d s)}\mathcal{L}\{u(t)\} = \frac{e^{(\frac{-t_d s}{2})}}{e^{(\frac{t_d s}{2})}}\mathcal{L}\{u(t)\}.$$

Since the time delay is small, one obtains

$$\frac{e^{(\frac{-t_d s}{2})}}{e^{(\frac{t_d s}{2})}}\mathcal{L}\{u(t)\} \approx \frac{1-\frac{t_d s}{2}}{1+\frac{t_d s}{2}}\mathcal{L}\{u(t)\},$$

Then, an intermediate variable $x_{n+1}$ is introduced. We can obtain:

$$\frac{1-\frac{t_d s}{2}}{1+\frac{t_d s}{2}}\mathcal{L}\{u(t)\} = \mathcal{L}\{x_{n+1}(t)\} - \mathcal{L}\{u(t)\}.$$

$$\dot{x}_{n+1} = \frac{4}{t_d}u - \frac{2}{t_d}x_{n+1},$$

let $\rho = \frac{2}{t_d}$, we have

$$\dot{x}_{n+1} = 2\rho u - \rho x_{n+1}.$$

The system (1) can be rewritten by the above transformation,

$$\begin{cases} \dot{x}_i = h_i(x) + x_{i+1} + \varepsilon_i(x,t),\ i = 1,2,\ldots,n-1 \\ \dot{x}_n = x_{n+1} - u + h_n(x) + \varepsilon_n(x,t), \\ \dot{x}_{n+1} = 2\rho u - \rho x_{n+1}, \\ y = x_1. \end{cases} \tag{5}$$

With the aid of the backstepping technique, The coordinate transformation and virtual controller can be constructed.

First, the change of coordinates is defined as follows:

$$\begin{cases} e_1 = x_1 - y_r, \\ e_i = x_i - \alpha_{i-1},\ i = 1,2,\ldots,n-1 \\ e_n = x_n - \alpha_{n-1} + x_{n+1}/\rho. \end{cases} \tag{6}$$

*Remark 3:* A special variable $x_{n+1}$, which unlike the real state variable $\underline{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$, is introduced in the $n$th subsystem, and $x_{n+1}$ is only a defined intermediate variable. The unknown time delay $t_d$ leads to the uncertainty of parameter $\rho$ and variable $x_{n+1}$, consider (6), the term $x_{n+1}/\rho$ is employed to eliminate the intermediate variable $x_{n+1}$ to ensure that the designed controller $u$ can be used to compensate the influence of intermediate variable and input delay. Moreover, the Pade approximation method can also handle time-varying input delay. This will be proved in simulation results.

Then, the virtual controller $\alpha_1$ and adaptive laws $\dot{\vartheta}_1$ can be design as

$$\alpha_1 = -\frac{\kappa_1 e_1 \vartheta_1}{2\delta_1^2(\omega_{e1}^2 - e_1^2)\psi_1^T(x_1,y_r)\psi_1(x_1,y_r)} - \frac{\kappa_1 e_1}{2\eta_1^2(\omega_{e1}^2 - e_1^2)} - \sigma_1 e_1, \tag{7}$$

$$\dot{\vartheta}_1 = \frac{\zeta_1\kappa_1 e_1^2}{2\delta_1^2(\omega_{e1}^2 - e_1^2)^2\psi_1^T(x_1,y_r)\psi_1(x_1,y_r)} - \gamma_1\vartheta_1, \tag{8}$$

where $\kappa_1, \delta_1, \eta_1, \sigma_1, \zeta_1$ and $\gamma_1$ are positive design parameters. $\tilde{\vartheta}_i = \vartheta_i^* - \vartheta_i, i = 1,2,\ldots,n-1$ represents the estimation error, $\vartheta_i$ is the estimation of $\vartheta_i^*$.

*Step i* ($i = 1,2,\ldots,n-1$)*:* The virtual controller $\alpha_i$ is stated as

$$\alpha_i = -\frac{\kappa_i e_i \vartheta_i}{2\delta_i^2(\omega_{ei}^2 - e_i^2)\psi_i^T(\underline{x}_i,y_r)\psi_i(\underline{x}_i,y_r)} - \frac{\kappa_i e_i}{2\eta_i^2(\omega_{ei}^2 - e_i^2)} - \sigma_i e_i, \tag{9}$$

$\dot{\vartheta}_i$ is designed as follows

$$\dot{\vartheta}_i = \frac{\zeta_i\kappa_i e_i^2}{2\delta_i^2(\omega_{ei}^2 - e_i^2)^2\psi_i^T(\underline{x}_i,y_r)\psi_i(\underline{x}_i,y_r)} - \gamma_i\vartheta_i, \tag{10}$$

where $\kappa_i, \delta_i, \eta_i, \sigma_i, \zeta_i$ iand $\gamma_i$ are positive parameters to be designed.

### *A. ADAPTIVE FUZZY CONTROL DESIGN*

*Step 1:* $\dot{e}_1$ can be calculated as

$$\dot{e}_1 = \dot{x}_1 - \dot{y}_r = x_2 + h_1(x) + \varepsilon_1(x,t) - \dot{y}_r. \tag{11}$$

Consider the BLF as the following form:

$$V_1 = \frac{1}{2}\log\frac{\omega_{e1}^2}{\omega_{e1}^2 - e_1^2} + \frac{\tilde{\vartheta}_1^2}{2\zeta_1}, \tag{12}$$

The derivative of (12) can be clearly induced as

$$\dot{V}_1 = \frac{e_1\dot{e}_1}{\omega_{e1}^2 - e_1^2} - \frac{\tilde{\vartheta}_1\dot{\vartheta}_1}{\zeta_1} = \frac{e_1}{\omega_{e1}^2 - e_1^2}(e_2 + \alpha_1 + h_1(x) + \varepsilon_1(x,t) - \dot{y}_r) - \frac{\tilde{\vartheta}_1\dot{\vartheta}_1}{\zeta_1}. \tag{13}$$

Adopting Young's inequality, one obtains

$$\frac{e_1}{\omega_{e1}^2 - e_1^2}\varepsilon_1(x,t) \le \frac{e_1^2}{2(\omega_{e1}^2 - e_1^2)^2} + \frac{\bar{\varepsilon}_1^2}{2}. \tag{14}$$

Then, (13) is rewritten as

$$\dot{V}_1 \le \frac{e_1}{\omega_{e1}^2 - e_1^2}(\alpha_1 + h_1(x) + \frac{e_1}{2(\omega_{e1}^2 - e_1^2)} - \dot{y}_r) + \frac{e_1 e_2}{\omega_{e1}^2 - e_1^2} + \frac{\bar{\varepsilon}_1^2}{2} - \frac{\tilde{\vartheta}_1\dot{\vartheta}_1}{\zeta_1}. \tag{15}$$

According to Lemma 1, we can have

$$h_1(x) + \frac{e_1}{2(\omega_{e1}^2 - e_1^2)} - \dot{y}_r = \Theta_1^T\psi_1(x,y_r) + \epsilon_1(x,y_r), \tag{16}$$

where $\epsilon_1(x,y_r) \le \epsilon_1, \epsilon_1 > 0$ is a constant.

Since $0 < \psi_i^T(\cdot)\psi_i(\cdot) < 1$, the following inequalities hold:

$$\frac{e_1}{\omega_{e1}^2 - e_1^2}\Theta_1^T\psi_1(x,y_r) \le \frac{e_1^2[\Theta_1^T\psi_1(x,y_r)]^2}{2\delta_1^2(\omega_{e1}^2 - e_1^2)^2} + \frac{\delta_1^2}{2} \le \frac{\kappa_1 e_1^2\vartheta_1^*\psi_1^T(x,y_r)\psi_1(x,y_r)}{2\delta_1^2(\omega_{e1}^2 - e_1^2)^2} + \frac{\delta_1^2}{2}$$

$$\leq \frac{\kappa_1 e_1^2 \vartheta_1^* \psi_1^T(x,y_r)\psi_1(x,y_r)}{2\delta_1^2(\omega_{e1}^2-e_1^2)^2[\psi_1(x_1,y_r)]^2}+\frac{\delta_1^2}{2}$$
$$\leq \frac{\kappa_1 e_1^2 \vartheta_1^*}{2\delta_1^2(\omega_{e1}^2-e_1^2)^2[\psi_1(x_1,y_r)]^2}+\frac{\delta_1^2}{2}, \tag{17}$$

$$\frac{e_1}{\omega_{e1}^2-e_1^2}\epsilon_1(x,y_r) \leq \frac{\kappa_1 e_1^2}{2\eta_1^2(\omega_{e1}^2-e_1^2)^2}+\frac{\eta_1^2\epsilon_1^2}{2\kappa_1}, \tag{18}$$

where $\vartheta_1^* = \frac{||\Theta_1||^2}{\kappa_1}$.

Substituting (16), (17) and (18) into (15) leads to

$$\dot{V}_1 \leq \frac{e_1}{\omega_{e1}^2-e_1^2}(\alpha_1+\frac{\kappa_1 e_1}{2\eta_1^2(\omega_{e1}^2-e_1^2)} + \frac{\kappa_1 e_1 \vartheta_1^*}{2\delta_1^2(\omega_{e1}^2-e_1^2)\psi_1^T(x_1,y_r)\psi_1(x_1,y_r)}) + \frac{e_1e_2}{\omega_{e1}^2-e_1^2}+\frac{\bar{\varepsilon}_1^2}{2}+\frac{\delta_1^2}{2}+\frac{\eta_1^2\epsilon_1^2}{2\kappa_1}-\frac{\tilde{\vartheta}_1\dot{\vartheta}_1}{\zeta_1}. \tag{19}$$

According to $\alpha_1$ and adaptive laws $\dot{\vartheta}_1$, we have

$$\dot{V}_1 \leq \frac{-\sigma_1 e_1^2}{\omega_{e1}^2-e_1^2}+\frac{e_1e_2}{\omega_{e1}^2-e_1^2}+\frac{\bar{\varepsilon}_1^2}{2}+\frac{\delta_1^2}{2}+\frac{\eta_1^2\epsilon_1^2}{2\kappa_1}+\frac{\gamma_1\tilde{\vartheta}_1\vartheta_1}{\zeta_1}, \tag{20}$$

where

$$\frac{\gamma_1\tilde{\vartheta}_1\vartheta_1}{\zeta_1}=\frac{\gamma_1\tilde{\vartheta}_1(\vartheta_1^*-\tilde{\vartheta}_1)}{\zeta_1}\leq\frac{\gamma_1\vartheta_1^{*2}}{2\zeta_1}-\frac{\gamma_1\tilde{\vartheta}_1^2}{2\zeta_1}. \tag{21}$$

Then (20) can be rewritten as

$$\dot{V}_1 \leq \frac{-\sigma_1 e_1^2}{\omega_{e1}^2-e_1^2}+\frac{e_1e_2}{\omega_{e1}^2-e_1^2}+\varsigma_1-\frac{\gamma_1\tilde{\vartheta}_1^2}{2\zeta_1}, \tag{22}$$

where $\varsigma_1 = \frac{\bar{\varepsilon}_1^2}{2}+\frac{\delta_1^2}{2}+\frac{\eta_1^2\epsilon_1^2}{2\kappa_1}+\frac{\gamma_1\vartheta_1^{*2}}{2\zeta_1}$.

*Step i* ($i = 1, 2, \ldots, n-1$)*:* Calculating $\dot{e}_i$ as follows:

$$\dot{e}_i = \dot{x}_i - \dot{\alpha}_{i-1} = x_{i+1}+h_i(x)+\varepsilon_i(x,t)-\dot{\alpha}_{i-1}. \tag{23}$$

Furthermore, we get

$$\dot{\alpha}_{i-1} = \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j)}}y_r^{(j+1)}+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(x_{j+1} + h_j(x)+\varepsilon_j(x,t))+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\vartheta_j}\dot{\vartheta}_j. \tag{24}$$

The BLF is selected as

$$V_i = V_{i-1}+\frac{1}{2}\log\frac{\omega_{ei}^2}{\omega_{ei}^2-e_i^2}+\frac{\tilde{\vartheta}_i^2}{2\zeta_i}, \tag{25}$$

Then calculating $\dot{V}_i$ as

$$\dot{V}_i = \dot{V}_{i-1}+\frac{e_i\dot{e}_i}{\omega_{ei}^2-e_i^2}-\frac{\tilde{\vartheta}_i\dot{\vartheta}_i}{\zeta_i}$$
$$= \dot{V}_{i-1}+\frac{e_i}{\omega_{ei}^2-e_i^2}(e_{i+1}+\alpha_i+h_i(x)+\varepsilon_i(x,t) - \dot{\alpha}_{i-1})-\frac{\tilde{\vartheta}_i\dot{\vartheta}_i}{\zeta_i}. \tag{26}$$

Applying Young's inequality, we have

$$\frac{e_i}{\omega_{ei}^2-e_i^2}\varepsilon_i(x,t)\leq\frac{e_i^2}{2(\omega_{ei}^2-e_i^2)^2}+\frac{\bar{\varepsilon}_i^2}{2}. \tag{27}$$

$$\frac{-e_i}{\omega_{ei}^2-e_i^2}\frac{\partial\alpha_{i-1}}{\partial x_j}\varepsilon_j(x,t)\leq\frac{e_i^2}{2(\omega_{ei}^2-e_i^2)^2}(\frac{\partial\alpha_{i-1}}{\partial x_j})^2+\frac{\bar{\varepsilon}_j^2}{2}. \tag{28}$$

And the $\dot{V}_{i-1}$ is

$$\dot{V}_{i-1}\leq\sum_{j=1}^{i-1}\frac{-\sigma_je_j^2}{\omega_{ej}^2-e_j^2}+\frac{e_{i-1}e_i}{\omega_{ei-1}^2-e_{i-1}^2}+\sum_{j=1}^{i-1}\varsigma_j-\sum_{j=1}^{i-1}\frac{\gamma_j\tilde{\vartheta}_j^2}{2\zeta_j}, \tag{29}$$

where $\varsigma_j = \sum_{k=1}^{j}\frac{\bar{\varepsilon}_k^2}{2}+\frac{\delta_j^2}{2}+\frac{\eta_j^2\epsilon_j^2}{2\kappa_j}+\frac{\gamma_j\vartheta_j^{*2}}{2\zeta_j}$.

Substituting (27), (28) and (29) into (26), we have

$$\dot{V}_i\leq\sum_{j=1}^{i-1}\frac{-\sigma_je_j^2}{\omega_{ej}^2-e_j^2}+\frac{e_{i-1}e_i}{\omega_{ei-1}^2-e_{i-1}^2}-\sum_{j=1}^{i-1}\frac{\gamma_j\tilde{\vartheta}_j^2}{2\zeta_j} + \frac{e_i}{\omega_{ei}^2-e_i^2}(e_{i+1}+\alpha_i+h_i(x)+\frac{e_i}{2(\omega_{ei}^2-e_i^2)} - \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j)}}y_r^{(j+1)}-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(x_{j+1}+h_j(x)) - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\vartheta_j}\dot{\vartheta}_j+\frac{e_i}{2(\omega_{ei}^2-e_i^2)}\sum_{j=1}^{i-1}(\frac{\partial\alpha_{i-1}}{\partial x_j})^2) + \sum_{k=1}^{i}\frac{\bar{\varepsilon}_k^2}{2}+\sum_{j=1}^{i-1}\varsigma_j-\frac{\tilde{\vartheta}_i\dot{\vartheta}_i}{\zeta_i}. \tag{30}$$

Based on Lemma 1, one has

$$h_i(x)+\frac{e_i}{2(\omega_{ei}^2-e_i^2)}-\sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_r^{(j)}}y_r^{(j+1)} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(x_{j+1}+h_j(x))-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\vartheta_j}\dot{\vartheta}_j + \frac{e_i}{2(\omega_{ei}^2-e_i^2)}\sum_{j=1}^{i-1}(\frac{\partial\alpha_{i-1}}{\partial x_j})^2+\frac{\omega_{ei}^2-e_i^2}{\omega_{ei-1}^2-e_{i-1}^2}e_{i-1} = \Theta_i^T\psi_i(x,y_r)+\epsilon_i(x,y_r). \tag{31}$$

where $\epsilon_i(x,y_r)\leq\epsilon_i$, $\epsilon_i>0$ is a constant.

Similarly to (17), we can get

$$\frac{e_i}{\omega_{ei}^2-e_i^2}\Theta_i^T\psi_i(x,y_r)\leq\frac{e_i^2[\Theta_i^T\psi_i(x,y_r)]^2}{2\delta_i^2(\omega_{ei}^2-e_i^2)^2}+\frac{\delta_i^2}{2}$$
$$\leq\frac{\kappa_ie_i^2\vartheta_i^*\psi_i^T(x,y_r)\psi_i(x,y_r)}{2\delta_i^2(\omega_{ei}^2-e_i^2)^2[\psi_i(\underline{x}_i,y_r)]^2}+\frac{\delta_i^2}{2}$$
$$\leq\frac{\kappa_ie_i^2\vartheta_i^*}{2\delta_i^2(\omega_{ei}^2-e_i^2)^2[\psi_i(\underline{x}_i,y_r)]^2}+\frac{\delta_i^2}{2}, \tag{32}$$

$$\frac{e_i}{\omega_{ei}^2 - e_i^2}\epsilon_i(x, y_r) \le \frac{\kappa_i e_i^2}{2\eta_i^2(\omega_{ei}^2 - e_i^2)^2} + \frac{\eta_i^2 \epsilon_i^2}{2\kappa_i}, \tag{33}$$

where $\vartheta_i^* = \frac{||\Theta_i||^2}{\kappa_i}$.

Substituting (31) – (33) into (30) leads to

$$\begin{aligned}\dot{V}_i \le & \sum_{j=1}^{i-1} \frac{-\sigma_j e_j^2}{\omega_{ej}^2 - e_j^2} + \frac{e_{i-1}e_i}{\omega_{ei-1}^2 - e_{i-1}^2} - \sum_{j=1}^{i-1} \frac{\gamma_j \tilde{\vartheta}_j^2}{2\zeta_j} \\ & + \frac{e_i}{\omega_{ei}^2 - e_i^2}(e_{i+1} + \alpha_i - \frac{\omega_{ei}^2 - e_i^2}{\omega_{ei-1}^2 - e_{i-1}^2} e_{i-1} \\ & + \frac{\kappa_i e_i \vartheta_i^*}{2\delta_i^2(\omega_{ei}^2 - e_i^2)[\psi_i(\underline{x}_i, y_r)]^2} + \frac{\kappa_i e_i}{2\eta_i^2(\omega_{ei}^2 - e_i^2)}) \\ & + \sum_{k=1}^{i} \frac{\bar{\varepsilon}_k^2}{2} + \sum_{j=1}^{i-1} \varsigma_j + \frac{\delta_i^2}{2} + \frac{\eta_i^2 \epsilon_i^2}{2\kappa_i} - \frac{\tilde{\vartheta}_i \dot{\vartheta}_i}{\zeta_i}. \end{aligned} \tag{34}$$

According to (9) and (10), we have

$$\begin{aligned}\dot{V}_i \le & \sum_{j=1}^{i} \frac{-\sigma_j e_j^2}{\omega_{ej}^2 - e_j^2} + \sum_{j=1}^{i-1} \varsigma_j - \sum_{j=1}^{i-1} \frac{\gamma_j \tilde{\vartheta}_j^2}{2\zeta_j} \\ & + \frac{e_i e_{i+1}}{\omega_{ei}^2 - e_i^2} + \sum_{k=1}^{i} \frac{\bar{\varepsilon}_k^2}{2} + \frac{\delta_i^2}{2} + \frac{\eta_i^2 \epsilon_i^2}{2\kappa_i} + \frac{\gamma_i \tilde{\vartheta}_i \vartheta_i}{\zeta_i}, \end{aligned} \tag{35}$$

where

$$\frac{\gamma_i \tilde{\vartheta}_i \vartheta_i}{\zeta_i} = \frac{\gamma_i \tilde{\vartheta}_i(\vartheta_i^* - \tilde{\vartheta}_i)}{\zeta_i} \le \frac{\gamma_i \vartheta_i^{*2}}{2\zeta_i} - \frac{\gamma_i \tilde{\vartheta}_i^2}{2\zeta_i}.$$

Then one obtains

$$\dot{V}_i \le \sum_{j=1}^{i} \frac{-\sigma_j e_j^2}{\omega_{ej}^2 - e_j^2} + \frac{e_i e_{i+1}}{\omega_{ei}^2 - e_i^2} + \sum_{j=1}^{i} \varsigma_j - \sum_{j=1}^{i} \frac{\gamma_j \tilde{\vartheta}_j^2}{2\zeta_j}, \tag{36}$$

where $\varsigma_i = \sum_{k=1}^{i} \frac{\bar{\varepsilon}_k^2}{2} + \frac{\delta_i^2}{2} + \frac{\eta_i^2\epsilon_i^2}{2\kappa_i} + \frac{\gamma_i \vartheta_i^{*2}}{2\zeta_i}$.

### B. EVENT-TRIGGERED CONTROLLER DESIGN

The ETC signal is defined as

$$u(t) = \nu(t_h), \forall t \in [t_h, t_{h+1}). \tag{37}$$

Then, the ETC strategy is expressed as

$$t_{h+1} = \inf\{t > t_h | \, |\underline{e}(t)| - \lambda |u(t)| - \mu \ge 0\} \tag{38}$$

$\underline{e}(t) = \nu(t) - u(t)$ denotes the measured error, $\nu(t)$ is a control law which will be defined later, $0 < \lambda < 1$ and $\mu > 0$ are known positive parameters. $t_h, h \in Z^+$ defines updating time. If the triggering condition (38) is achieved, the control input will be updated. On the contrary, $\nu(t_h)$ is always unchanging.

For the proposed ETC strategy (38), we consider that the system normally operates, and it can be obtained:

$$\nu(t) = (1 + k_1(t)\lambda)u(t) + k_2(t)\mu, \tag{39}$$

where $|k_1(t)| \le 1$ and $|k_2(t)| \le 1$ are time-varying parameters.

Then one obtains

$$u(t) = \frac{\nu(t)}{1 + k_1(t)\lambda} - \frac{k_2(t)\mu}{1 + k_1(t)\lambda}. \tag{40}$$

Define

$$\nu(t) = -(1 + \lambda)\left(\alpha_n \tanh\left(\frac{e_n \alpha_n}{\varpi}\right) + \bar{\mu} \tanh\left(\frac{e_n \bar{\mu}}{\varpi}\right)\right), \tag{41}$$

$$\begin{aligned}\alpha_n = & \frac{\kappa_n e_n \vartheta_n}{2\delta_n^2(\omega_{en}^2 - e_n^2)\psi_n^T(x, y_r)\psi_n(x, y_r)} \\ & + \frac{\kappa_n e_n}{2\eta_n^2(\omega_{en}^2 - e_n^2)} + \sigma_n e_n, \end{aligned} \tag{42}$$

$$\dot{\vartheta}_n = \frac{\zeta_n \kappa_n e_n^2}{2\delta_n^2(\omega_{en}^2 - e_n^2)^2 \psi_n^T(x, y_r)\psi_n(x, y_r)} - \gamma_n \vartheta_n, \tag{43}$$

where $\kappa_n, \delta_n, \eta_n, \sigma_n, \zeta_n, \gamma_n, \varpi$ and $\bar{\mu} > \mu/(1 - \lambda)$ are positive design parameters.

*Remark 4:* Event-triggered technology is considered to be an effective way to reduce the controller calculation cost and communication burden, observing (38) and (41), when the control input approaches zero, the threshold becomes smaller, and $\lambda$ is a parameter that can promote better system performance through proper selection. In addition, the larger the value of $\bar{\mu}$, the larger the update interval of the control signal, which will reduce the number of triggers, but the system performance may deteriorate. Therefore, we must comprehensively consider the selection of parameters.

## IV. STABILITY ANALYSIS

By the proof of the following theorem, we can give the stability analysis of the system (1).

*Theorem 1:* For the constraints system (1) with input delay and non-strict feedback structure, under Assumptions 1 and 2, the ETC signal $\nu$, the virtual control functions (7) and (9), the adaptive laws (8), (10) and (43) can be designed to ensure that the tracking errors and the system states can be adjusted to the prescribed constraint interval, all variables of the system are SGUUB, Besides, the Zeno behavior can be averted.

*Proof: step n:* Differentiating $e_n$, it follows

$$\begin{aligned}\dot{e}_n &= \dot{x}_n - \dot{\alpha}_{n-1} + \dot{x}_{n+1}/\rho \\ &= x_{n+1} - u + h_n(x) + \varepsilon_n(x, t) - \dot{\alpha}_{n-1} + \dot{x}_{n+1}/\rho. \end{aligned} \tag{44}$$

the derivative of $\alpha_{n-1}$ can be obtained

$$\begin{aligned}\dot{\alpha}_{n-1} = & \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(j)}} y_r^{(j+1)} + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}(x_{j+1} \\ & + h_j(x) + \varepsilon_j(x, t)) + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \vartheta_j} \dot{\vartheta}_j. \end{aligned} \tag{45}$$

Similar to (25), the BLF is selected as

$$V_n = V_{n-1} + \frac{1}{2} \log \frac{\omega_{en}^2}{\omega_{en}^2 - e_n^2} + \frac{\tilde{\vartheta}_n^2}{2\zeta_n}, \tag{46}$$

Calculating $\dot{V}_n$, we have

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + \frac{e_n \dot{e}_n}{\omega_{en}^2 - e_n^2} - \frac{\tilde{\vartheta}_n \dot{\vartheta}_n}{\zeta_n} \\ &= \dot{V}_{n-1} + \frac{e_n}{\omega_{en}^2 - e_n^2}(x_{n+1} - u + h_n(x) + \varepsilon_n(x,t) \\ &\quad - \dot{\alpha}_{n-1} + \dot{x}_{n+1}/\rho) - \frac{\tilde{\vartheta}_n \dot{\vartheta}_n}{\zeta_n}. \end{aligned} \tag{47}$$

Employing Young's inequality follows

$$\frac{e_n}{\omega_{en}^2 - e_n^2}\varepsilon_n(x,t) \le \frac{e_n^2}{2(\omega_{en}^2 - e_n^2)^2} + \frac{\bar{\varepsilon}_n^2}{2}. \tag{48}$$

$$\frac{-e_n}{\omega_{en}^2 - e_n^2}\frac{\partial \alpha_{n-1}}{\partial x_j}\varepsilon_j(x,t) \le \frac{e_n^2}{2(\omega_{en}^2 - e_n^2)^2}(\frac{\partial \alpha_{n-1}}{\partial x_j})^2 + \frac{\bar{\varepsilon}_j^2}{2}. \tag{49}$$

Similarly, differentiating $V_{n-1}$ yields

$$\begin{aligned}\dot{V}_{n-1} \le& \sum_{j=1}^{n-1}\frac{-\sigma_j e_j^2}{\omega_{ej}^2 - e_j^2} + \frac{e_{n-1}e_n}{\omega_{en-1}^2 - e_{n-1}^2} + \sum_{j=1}^{n-1}\varsigma_j \\ & - \sum_{j=1}^{n-1}\frac{\gamma_j \tilde{\vartheta}_j^2}{2\zeta_j}, \end{aligned} \tag{50}$$

where $\varsigma_j = \sum_{k=1}^{j}\frac{\bar{\varepsilon}_k^2}{2} + \frac{\delta_j^2}{2} + \frac{\eta_j^2\epsilon_j^2}{2\kappa_j} + \frac{\gamma_j \vartheta_j^{*2}}{2\zeta_j}$.

Substituting (48) – (50) into (47) leads to

$$\begin{aligned}\dot{V}_n \le& \sum_{j=1}^{n-1}\frac{-\sigma_j e_j^2}{\omega_{ej}^2 - e_j^2} + \frac{e_{n-1}e_n}{\omega_{en-1}^2 - e_{n-1}^2} - \sum_{j=1}^{n-1}\frac{\gamma_j \tilde{\vartheta}_j^2}{2\zeta_j} \\ & + \frac{e_n}{\omega_{en}^2 - e_n^2}(x_{n+1} - u + h_n(x) + \frac{e_n}{2(\omega_{en}^2 - e_n^2)} \\ & - \sum_{j=0}^{n-1}\frac{\partial \alpha_{n-1}}{\partial y_r^{(j)}}y_r^{(j+1)} - \sum_{j=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_j}(x_{j+1} + h_j(x)) \\ & - \sum_{j=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial \vartheta_j}\dot{\vartheta}_j + \frac{e_n}{2(\omega_{en}^2 - e_n^2)}\sum_{j=1}^{n-1}(\frac{\partial \alpha_{n-1}}{\partial x_j})^2 \\ & + (-\rho x_{n+1} + 2\rho u)/\rho) + \sum_{k=1}^{n}\frac{\bar{\varepsilon}_k^2}{2} + \sum_{j=1}^{n-1}\varsigma_j - \frac{\tilde{\vartheta}_n \dot{\vartheta}_n}{\zeta_n}. \end{aligned} \tag{51}$$

Considering that $\tanh(\cdot)$ has the following properties [58]:

$$0 \le |\varrho| - \varrho \tanh\left(\frac{\varrho}{d}\right) \le 0.2785d, \tag{52}$$

where $d > 0$ and $\varrho \in R$. In light of $|k_1(t)| \le 1$, $|k_2(t)| \le 1$ and (40), we have $|(e_n k_1(t)\mu)/(1+k_1(t)\lambda)| < (e_n\mu)/(1-\lambda)$, then, one obtains:

$$\begin{aligned}\frac{e_n u}{\omega_{en}^2 - e_n^2} &\le \frac{1}{\omega_{en}^2 - e_n^2}\left(\frac{e_n \nu(t)}{1 + k_1(t)\lambda} - \frac{e_n k_2(t)\mu}{1 + k_1(t)\lambda}\right) \\ &\le \frac{1}{\omega_{en}^2 - e_n^2}\left(\frac{e_n \nu(t)}{1+\lambda} + |e_n\bar{\mu}|\right) \\ &\le \frac{1}{\omega_{en}^2 - e_n^2}\Big(- e_n\alpha_n \tanh\left(\frac{e_n\alpha_n}{\varpi}\right) \\ &\quad - e_n\bar{\mu}\tanh\left(\frac{e_n\bar{\mu}}{\varpi}\right) + |e_n\bar{\mu}|\Big) \\ &\le \frac{1}{\omega_{en}^2 - e_n^2}\Big(- e_n\alpha_n \tanh\left(\frac{e_n\alpha_n}{\varpi}\right) \\ &\quad + e_n\alpha_n - e_n\alpha_n\Big) + 0.2785d \\ &\le -\frac{e_n\alpha_n}{\omega_{en}^2 - e_n^2} + 0.557d. \end{aligned} \tag{53}$$

Based on Lemma 1, we have

$$\begin{aligned}& h_n(x) + \frac{e_n}{2(\omega_{en}^2 - e_n^2)} - \sum_{j=0}^{n-1}\frac{\partial \alpha_{n-1}}{\partial y_r^{(j)}}y_r^{(j+1)} \\ & - \sum_{j=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_j}(x_{j+1} + h_j(x)) - \sum_{j=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial \vartheta_j}\dot{\vartheta}_j \\ & + \frac{e_n}{2(\omega_{en}^2 - e_n^2)}\sum_{j=1}^{n-1}(\frac{\partial \alpha_{n-1}}{\partial x_j})^2 + \frac{\omega_{en}^2 - e_n^2}{\omega_{en-1}^2 - e_{n-1}^2}e_{n-1} \\ &= \Theta_n^T\psi_n(x, y_r) + \epsilon_n(x, y_r). \end{aligned} \tag{54}$$

where $\epsilon_n(x, y_r) \le \epsilon_n$, $\epsilon_n > 0$ is a constant.

Adopting Young's inequality, one obtains

$$\begin{aligned}\frac{e_n}{\omega_{en}^2 - e_n^2}\Theta_n^T\psi_n(x, y_r) &\le \frac{e_n^2[\Theta_n^T\psi_n(x,y_r)]^2}{2\delta_n^2(\omega_{en}^2 - e_n^2)^2} + \frac{\delta_n^2}{2} \\ &\le \frac{\kappa_n e_n^2\vartheta_n^*\psi_n^T(x,y_r)\psi_n(x,y_r)}{2\delta_n^2(\omega_{en}^2 - e_n^2)^2[\psi_n(x,y_r)]^2} \\ &\quad + \frac{\delta_n^2}{2} \\ &\le \frac{\kappa_n e_n^2\vartheta_n^*}{2\delta_n^2(\omega_{en}^2 - e_n^2)^2[\psi_n(x,y_r)]^2} \\ &\quad + \frac{\delta_n^2}{2}, \end{aligned} \tag{55}$$

$$\frac{e_n}{\omega_{en}^2 - e_n^2}\epsilon_n(x, y_r) \le \frac{\kappa_n e_n^2}{2\eta_n^2(\omega_{en}^2 - e_n^2)^2} + \frac{\eta_n^2\epsilon_n^2}{2\kappa_n}, \tag{56}$$

where $\vartheta_n^* = \frac{||\Theta_n||^2}{\kappa_n}$.

Substituting (54), (55), and(56) into (51) produces

$$\begin{aligned}\dot{V}_n \le& \sum_{j=1}^{n-1}\frac{-\sigma_j e_j^2}{\omega_{ej}^2 - e_j^2} + \frac{e_{n-1}e_n}{\omega_{en-1}^2 - e_{n-1}^2} - \sum_{j=1}^{n-1}\frac{\gamma_j \tilde{\vartheta}_j^2}{2\zeta_j} \\ & + \frac{e_n}{\omega_{en}^2 - e_n^2}(u - \frac{\omega_{en}^2 - e_n^2}{\omega_{en-1}^2 - e_{n-1}^2}e_{n-1} \\ & + \frac{\kappa_n e_n \vartheta_n^*}{2\delta_n^2(\omega_{en}^2 - e_n^2)[\psi_n(x,y_r)]^2} + \frac{\kappa_n e_n}{2\eta_n^2(\omega_{en}^2 - e_n^2)}) \\ & + \sum_{k=1}^{n}\frac{\bar{\varepsilon}_k^2}{2} + \sum_{j=1}^{n-1}\varsigma_j + \frac{\delta_n^2}{2} + \frac{\eta_n^2\epsilon_n^2}{2\kappa_n} - \frac{\tilde{\vartheta}_n\dot{\vartheta}_n}{\zeta_n}. \end{aligned} \tag{57}$$

Along with (42), (43) and (53), (57) can be rewritten as

$$\dot{V}_n \le \sum_{j=1}^{n} \frac{-\sigma_j e_j^2}{\omega_{ej}^2 - e_j^2} + \sum_{j=1}^{n-1} \varsigma_j - \sum_{j=1}^{n-1} \frac{\gamma_j \tilde{\vartheta}_j^2}{2\zeta_j} + \sum_{k=1}^{n} \frac{\bar{\varepsilon}_k^2}{2} + \frac{\delta_n^2}{2} + \frac{\eta_n^2 \epsilon_n^2}{2\kappa_n} + \frac{\gamma_n \tilde{\vartheta}_n \vartheta_n}{\zeta_n} + 0.557d, \quad (58)$$

where

$$\frac{\gamma_n \tilde{\vartheta}_n \vartheta_n}{\zeta_n} = \frac{\gamma_n \tilde{\vartheta}_n (\vartheta_n^* - \tilde{\vartheta}_n)}{\zeta_n} \le \frac{\gamma_n \vartheta_n^{*2}}{2\zeta_n} - \frac{\gamma_n \tilde{\vartheta}_n^2}{2\zeta_n}.$$

Then one can have

$$\dot{V}_n \le \sum_{j=1}^{n} \frac{-\sigma_j e_j^2}{\omega_{ej}^2 - e_j^2} + \sum_{j=1}^{n} \varsigma_j - \sum_{j=1}^{n} \frac{\gamma_j \tilde{\vartheta}_j^2}{2\zeta_j}, \quad (59)$$

where $\varsigma_n = \sum_{k=1}^{n} \frac{\bar{\varepsilon}_k^2}{2} + \frac{\delta_n^2}{2} + \frac{\eta_n^2 \epsilon_n^2}{2\kappa_n} + \frac{\gamma_n \vartheta_n^{*2}}{2\zeta_n} + 0.557d$.

Based on lemma 2, it can be obtained that

$$\frac{-\sigma_i e_i^2}{\omega_{ei}^2 - e_i^2} \le -\sigma_i \log \frac{\omega_{ei}^2}{\omega_{ei}^2 - e_i^2}, i = 1, 2, \ldots, n. \quad (60)$$

Hence, one obtains

$$\dot{V}_n \le -\xi V_n + \bar{\varsigma}. \quad (61)$$

where $\xi = \min\{2\sigma_i, \gamma_i, i = 1, 2, \ldots, n\}$, $\bar{\varsigma} = \sum_{i=1}^{n} \varsigma_i$.

For $t > 0$, (61) is integrated as

$$V(t) \le (V(0) - \frac{\bar{\varsigma}}{\xi})e^{-\xi t} + \frac{\bar{\varsigma}}{\xi} \le V(0)e^{-\xi t} + \frac{\bar{\varsigma}}{\xi}. \quad (62)$$

With the definition of $V(t)$ and (62), the variables $x_i$, $\vartheta_1$, $e_i$ and $u$ are bounded.

From (62), it follows that:

$$|e_1| \le \omega_{e1} \sqrt{1 - e^{-2V(0)e^{-\xi t} - 2\frac{\bar{\varsigma}}{\xi}}}. \quad (63)$$

According to Assumption 1, (6) and (62), we can obtain $|x_1| \le |e_1| + |y_r| < \omega_{e1} + Y_0 \le \omega_1$. From (7), because $\alpha_1$ is a function which consisting of $y_r$, $\dot{y}_r$, $\vartheta_1$ and $x_1$, the boundedness of $\alpha_1$ is ensured. Moreover, the supremum $\bar{\alpha}_1$ of $\alpha_1$ exists. From $x_2 = e_2 + \alpha_1$, we obtain $|x_2| < \omega_{e2} + \bar{\alpha}_1 \le \omega_2$. Similarly, by $x_i = e_i + \alpha_{i-1}, i = 1, 2, \ldots, n-1, x_n = e_n + \alpha_{n-1} - x_{n+1}/\rho$, we get $|x_i| < \omega_i, i = 1, 2, \ldots, n$.

Next, the proposed adaptive fuzzy ETC will be illustrated can exclude the Zeno behavior, we suppose that $t^* > 0$ can be obtained, which is a time constant and satisfying $\forall h \in Z^+, \{t_{h+1} - t_h \ge t^*\}$.

Noting the $\underline{e}(t) = v(t) - u(t)$, then

$$\frac{d}{dt}|\underline{e}| = \frac{d}{dt}(\underline{e} \times \underline{e})^{\frac{1}{2}} = \text{sign}(\underline{e})\dot{\underline{e}} \le |\dot{v}|. \quad (64)$$

Observing (41), $v$ is differentiable, and $\dot{v}$ represents a function of the bounded variables. Furthermore, it must exist a constant $\ell > 0$ satisfying $|\dot{v}| \le \ell$. For $\underline{e}(t_h) = 0$ and $\lim_{t \to t_{h+1}} \underline{e}(t) = \mu$, the lower bound of interexecution intervals $t^* \ge (\mu/\ell)$ can be obtained, namely, the Zeno behavior is exempted.

*Remark 5:* By using the Pade approximation method, the input delay can be compensated, and the BLF is introduced at each step of the derivation process to meet the full state constraint requirements. According to the proposed adaptive fuzzy control approach, the restriction of monotonous increase of the nonlinear function in [32], [33], and [34] is removed, which promotes the control strategy more widely used. In addition, under the premise of ensuring excellent tracking performance, the ETC strategy is considered to achieve the purpose of saving communication resources.

*Remark 6:* The selection of design parameters can refer to the following points: (1) Better tracking effect can be obtained by reducing $\omega_{e_i}$, but the peak value of the system control input will become larger. (2) Increasing $\mu$ will reduce the update times of the control signal and save more resources, but if its value is too large, the system performance may deteriorate. (3) Reducing $\lambda$ will make the control effect of the system more precise, but the control signal of the system may fluctuate significantly.

*Remark 7:* The relative threshold ETC strategy will obtain a larger threshold when the amplitude of the control signal is large, therefore, the update time of the input signal of the system becomes longer, but if the amplitude of the control signal of the system is extremely large, it will inevitably large measurement errors of the control signal are generated. When the control input is updated, the system may be impacted by a large pulse signal, which will have a serious impact on the performance of the system. Additionally, in order to ensure that the approximation error of the Pade approximation method tends to zero, the input time delay $t_d$ of the system must be small. Therefore, the proposed control scheme is still insufficient in dealing with system control problems with ultra-large value control signals and long delays.

## V. SIMULATION

To illustrate the effectiveness of the proposed adaptive ETC scheme, two simulation examples will be conducted in this section, and The conventional TTC adaptive fuzzy control strategy proposed in [30] will be used as a comparison method.

### A. NUMERICAL EXAMPLE

Consider the non-strict feedback nonlinear system as follows.

$$\begin{cases} \dot{x}_1 = \sin(x_1 x_3) + x_3 + 2x_2 + \varepsilon_1(x, t), \\ \dot{x}_2 = x_1^2 \exp(x_2) x_3 + x_3 + \varepsilon_2(x, t), \\ \dot{x}_3 = x_1 x_2 \exp(x_3) + x_3 \sin(x_1 x_2) \\ \qquad + u(t - t_d) + \varepsilon_3(x, t), \\ y = x_1, \end{cases} \quad (65)$$

where $x_1$, $x_2$, $x_3$ stand for the state vectors. $u$ and $y$ are the system's input and output, respectively. $\varepsilon_1(x, t) = 0.1 \sin(x_1 x_2)$, $\varepsilon_2(x, t) = 0.1 x_3^2$, and $\varepsilon_3(x, t) = 0.2 \cos(t + 0.3)$

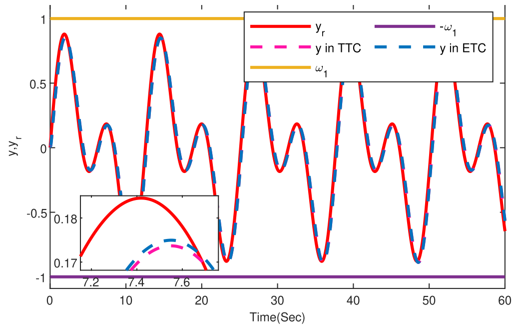


**FIGURE 1.** Trajectories of $y$, $y_r$ and constraints interval.

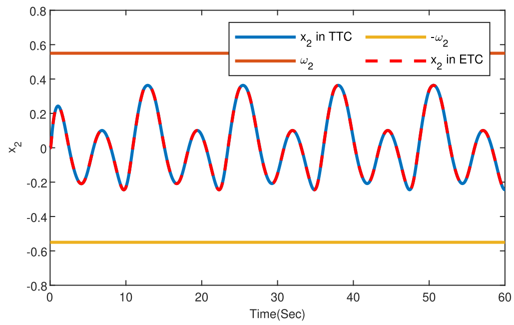


**FIGURE 2.** Trajectories of state $x_2$ and constraints interval.

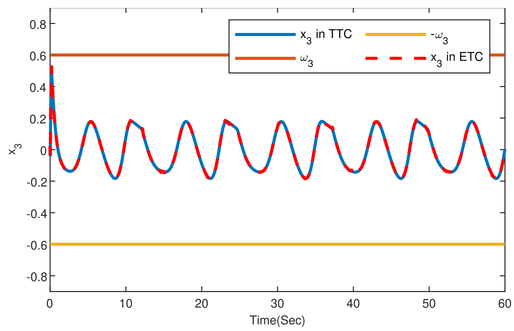


**FIGURE 3.** Trajectories of state $x_3$ and constraints interval.

are external disturbance. Input delay $t_d = 0.02\ s$, reference signal $y_r = \frac{1}{2}(\sin(t) + \sin(\frac{t}{2}))$.

The fuzzy membership functions are selected as follows:

$$\mu_{\mathcal{H}_i^j}(x_i) = \exp\left[\frac{-(x_i + 12 - 3j)}{2}\right], i = 1, 2, 3,$$
$$j = 1, 2, \ldots, 7.$$

The design parameters are chosen as $\lambda = 0.1$, $\mu = 0.1$, $\varpi = 0.3$, $\delta_1 = 10$, $\delta_2 = 9$, $\delta_3 = 15$, $\sigma_1 = 2.9$, $\sigma_2 = 2.7$, $\sigma_3 = 3$, $\eta_1 = 2$, $\eta_2 = 1$, $\eta_3 = 0.12$, $\zeta_1 = 0.1$, $\zeta_2 = 0.2$, $\zeta_3 = 0.2$, $\gamma_1 = 0.3$, $\gamma_2 = 0.5$, $\gamma_3 = 0.2$, $\omega_{e1} = 0.12$, $\omega_1 = 1$, $\omega_2 = 0.55$, $\omega_3 = 0.6$, the initial conditions are $x_1(0) = 0.01$, $x_2(0) = 0$, $x_3(0) = 0$.

From Figs. 1–6, it is clear to know that the tracking performance satisfies tracking error $|e_1| \leq \omega_{e1} = 0.12$, in addition, $|y(t)| \leq \omega_1 = 1$, $x_2$ and $x_3$ are constrained within intervals $|x_2| \leq \omega_2 = 0.55$ and $|x_3| \leq \omega_3 = 0.6$. The trajectories of adaptation laws and the actual control input

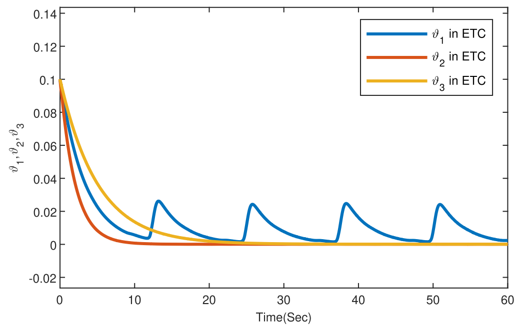


**FIGURE 4.** Adaptive laws $\vartheta_1$, $\vartheta_2$ and $\vartheta_3$.

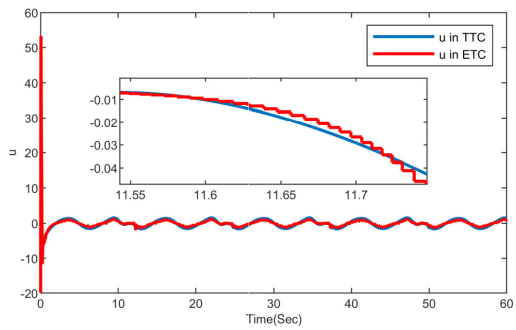


**FIGURE 5.** Trajectories of the system input.

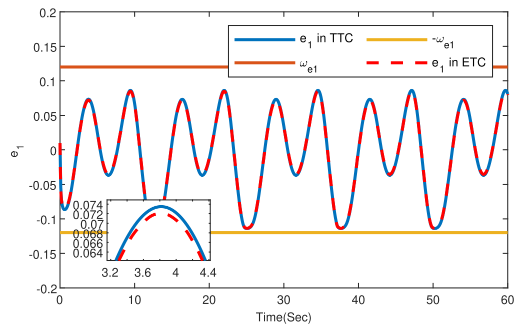


**FIGURE 6.** Trajectories of tracking error $e_1$ and constraints interval.

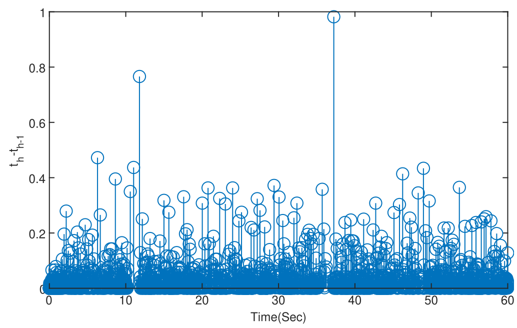


**FIGURE 7.** Time interval of triggering event.

are shown in Figs. 4 and 5, respectively. As can be seen from Fig. 6, the ETC strategy proposed in this paper produces a smaller tracking error. In Fig. 7, the event-triggered numbers are 2736 in 60 seconds. Therefore, the consumption of communication resources is reduced, and all variables of the system are bounded.

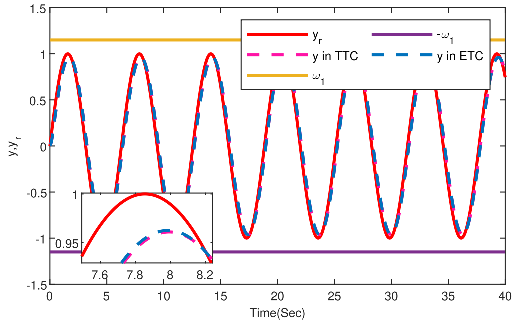


**FIGURE 8.** Trajectories of $y$, $y_r$ and constraints interval.

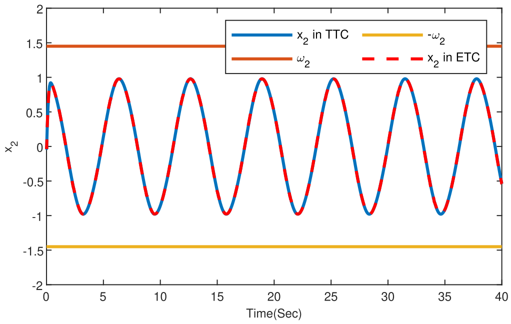


**FIGURE 9.** Trajectories of state $x_2$ and constraints interval.

## B. APPLICATION EXAMPLE

Take the electromechanical system in [34] into account, which is a permanent magnet brush dc motor, the dynamic is given as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -\dfrac{N\sin(x_1)}{H} - \dfrac{Bx_2}{H} + \dfrac{B\cos(x_2)\sin(x_3)}{H} + \dfrac{x_3}{H}, \\ \dot{x}_3 = -\dfrac{Kx_2}{L} - \dfrac{Rx_3}{L} + \dfrac{u}{L}, \\ y = x_1, \end{cases} \tag{66}$$

where $H = \frac{J}{K_T} + \frac{mL_0^2}{3K_T} + \frac{M_0L_0^2}{K_T} + \frac{2M_0R_0^2}{5K_T}$, $N = \frac{mL_0G}{2K_T} + \frac{M_0L_0G}{K_T}$, $B = \frac{B_0}{K_T}$, $m = 0.506$, $M_0 = 0.434$, $L = 0.025$, $L_0 = 0.305$, $R = 0.5$, $R_0 = 0.023$, $B_0 = 0.01625$, $K = 0.9$, $K_T = 0.9$, $J = 0.001625$, $G = 9.8$, reference signal $y_r = \sin(t)$, and $t_d = 0.06 + 0.03\sin(t)$. It is a time-varying input delay.

The fuzzy membership functions are given as

$$\mu_{\mathcal{H}_i^j}(x_i) = \exp\left[\frac{-(x_i + 3 - j)}{2}\right], i = 1, 2, 3, \quad j = 1, 2, \ldots, 5.$$

Design parameters as $\lambda = 0.2$, $\mu = 0.25$, $\varpi = 0.6$, $\delta_1 = 45$, $\delta_2 = 50$, $\delta_3 = 35$, $\sigma_1 = 6.5$, $\sigma_2 = 7.3$, $\sigma_3 = 7$, $\eta_1 = 3$, $\eta_2 = 3$, $\eta_3 = 5$, $\zeta_1 = 0.5$, $\zeta_2 = 0.7$, $\zeta_3 = 0.7$, $\gamma_1 = 0.7$, $\gamma_2 = 0.5$, $\gamma_3 = 0.1$, $\omega_{e1} = 0.15$, $\omega_1 = 1.15$, $\omega_2 = 1.45$, $\omega_3 = 2.6$, and the initial conditions are similar to example 1.

From Figs. 8–13, it is clearly to know that tracking error $|e_1| \leq \omega_{e1} = 0.15$ in ETC, and satisfies $|y(t)| \leq \omega_1 = 1.15$. System states $x_2$ and $x_3$ are constrained within intervals $|x_2| \leq \omega_2 = 1.45$ and $|x_3| \leq \omega_3 = 2.6$ respectively. In Fig. 14, the event-triggered numbers are 10699 in 40 seconds, but $u$ needs 87017 control updates to apply the classical time-triggered controller. Thus, the control scheme proposed in this paper can considerably reduce the communication

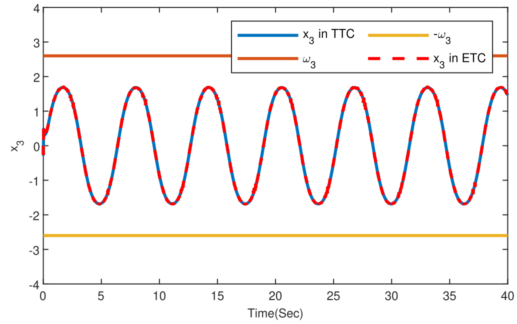


**FIGURE 10.** Trajectories of state $x_3$ and constraints interval.

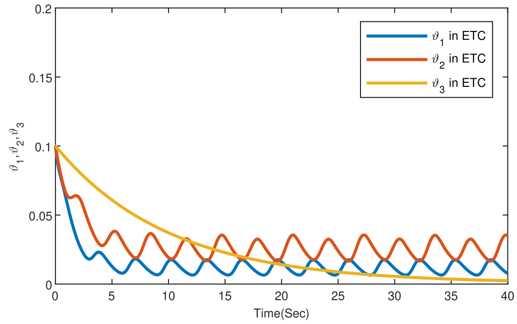


**FIGURE 11.** Adaptive laws $\vartheta_1$, $\vartheta_2$ and $\vartheta_3$.

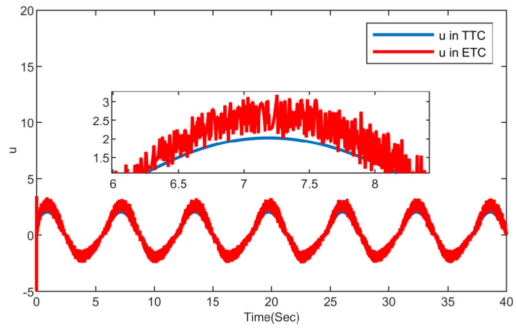


**FIGURE 12.** Trajectories of the system input.

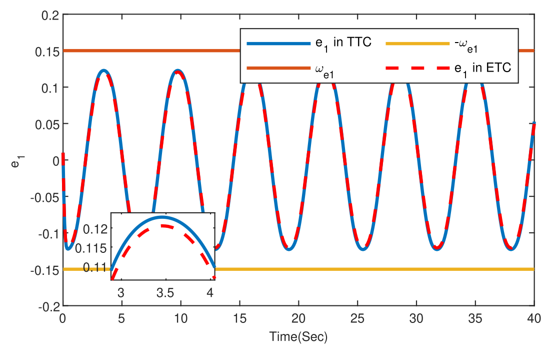


**FIGURE 13.** Trajectories of tracking error $e_1$ and constraints interval.

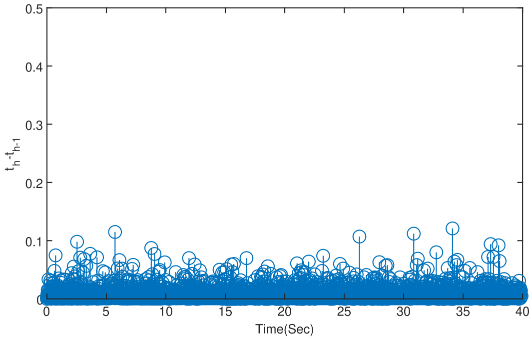


**FIGURE 14.** **Time interval of triggering event.**

burden, and the feasibility of the controller design in practice can be demonstrated by the above simulation results.

## VI. CONCLUSION

In this paper, combining the backstepping control with the relative threshold mechanism, a new adaptive event-triggered control approach is proposed for the constrained nonlinear system with input delay and non-strict feedback structure. With the aid of the Pade approximate method, the influence of the input delay is compensated. By adopting barrier Lyapunov function at every step of controller design, the whole state of the system is constrained to preset intervals, and the impact of completely unknown nonlinear items on the design is mitigated by fuzzy logic systems. Moreover, all variables of the system are semi-globally uniformly ultimately bounded, and the Zeno behavior does not exist in the developed approach. Finally, a constrained non-strict feedback system and an electromechanical system are provided to verify that the proposed approach has a smaller update frequency under the premise of ensuring excellent tracking accuracy. In future work, we will further consider the time-varying asymmetric state-constrained, and apply the proposed control strategy to the multi-agent systems.

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