

Received 12 September 2023, accepted 30 September 2023, date of publication 4 October 2023, date of current version 10 October 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3321702

RESEARCH ARTICLE

An Adaptive MP Algorithm for Underwater Acoustic Channel Estimation Based on Compressed Sensing

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This work was supported in part by the Open Research Fund of Anhui Key Laboratory of Detection Technology and Energy Saving Devices, Anhui Polytechnic University, under Grant JCKJ2021A02; and in part by the Anhui Polytechnic University Research Startup Foundation under Grant 2021YQQ039.

ABSTRACT In underwater acoustic (UWA) communication systems, inter-carrier interference (ICI) caused by the Doppler effect has significant negative impacts on system performance. To address this issue, this paper introduces a delay-Doppler spread function (DDSF) to account for the effect of ICI and proposes a new compressed sensing (CS) algorithm to estimate channels. Typically, proper termination of the iterative process is a major challenge when applying the orthogonal matching pursuit (OMP) algorithms in channel estimation, while other CS algorithms in the paper have high requirements in terms of complexity and system power consumption. To overcome these limitations, a sparse channel estimation algorithm with an adaptive sparse decision threshold is proposed. Given certain signal-to-noise ratio (SNR) conditions, the proposed algorithm achieves comparable estimation accuracy to OMP with much lower computational cost. Simulation results demonstrate that the proposed algorithm can achieve similar estimation accuracy to OMP at lower computational cost with high SNRs. In conclusion, this paper presents a novel approach to address ICI in UWA communication systems and offers a more efficient algorithm for channel estimation. The results are significant for improving the performance of underwater communication systems and have potential applications in various underwater communication scenarios.

INDEX TERMS Underwater acoustic communication, delayed-Doppler spreading function (DDSF), adaptive algorithm, iteration termination condition.

I. INTRODUCTION

The exploitation of marine resources has significantly increased, leading to a growing need for communication technologies in ocean development [1], [2], [3]. However, the penetration of electromagnetic waves into seawater is weak, making it difficult for mature electromagnetic wave communication technologies to be widely used in the ocean [4]. Nowadays, the exploitation of marine resources has increased dramatically in the development of the ocean, a large number

The associate editor coordinating the review of this manuscript and approving it for publication was Jiajia Jiang^(D).

of communication technologies are needed, which leads to the urgent need for high-speed UWA communication. Due to the conductivity of seawater, the penetration of electromagnetic wave to seawater is very weak, which makes it difficult to use electromagnetic wave communication technology in large scale in the ocean. Nowadays, UWA communication technology is widely used in marine communication, but the development of UWA technology is slow. Due to the maturity of electromagnetic wave communication technology, people usually transplant electromagnetic wave communication technology to UWA communication, which makes marine communication technology develop rapidly. Due to the different propagation characteristics of UWA and electromagnetic waves, UWA communication also faces many challenges [5].

Among the challenges of UWA communication systems, the main problems are the multipath effect and the Doppler effect. Due to the complex UWA environment, there are many reflection and refraction phenomena when the signal is transmitted to the surface, causing severe multipath effects in the UWA channel. Additionally, if objects on both sides of the communication are moving, even at slow speeds, the Doppler effect occurs due to the slow transmission speed of underwater sound, which has a significant impact on the transmission of channel data [6]. Therefore, DDSF is used to describe the effects of UWA channel delay and Doppler shift [7].

Using DDSF to track first-order channel dynamics does not require dynamic modeling of UWA channels and can balance the complexity and accuracy of sparse channels. Compared to the recursive least squares (RLS) algorithm for estimating channel state information (CIR), the complexity of using the search algorithm to estimate DDSF is relatively low [8]. This is mainly due to two factors: there are few important weight elements to predict in DDSF, and DDSF executes once for each data block, rather than once for each symbol [9].

When using the DDSF model, the signals received at the receiver are sparse due to the sparsity of the channel environment. Most of the atoms in the observation matrix have very small weights for the original signal, and only a few of them contribute more. These atoms with small contributions require a lot of calculation, which wastes both system overhead and calculation time. Therefore, it is essential to find a suitable estimation algorithm to select them.

Traditional channel estimation algorithms include the least-square (LS) [10], maximum likelihood estimation (ML) [11], minimum mean square error (MMSE) [12], Kalman filtering [5], [13], [14], [15], etc. These algorithms usually rely on the prior information of the channel. When the UWA channel changes rapidly, the outdated channel information is not conducive to the estimation of the existing channel, resulting in a significant reduction in the estimation performance [9]. Therefore, people usually add CS channel estimation algorithms to UWA channel estimation. Among them, the OMP algorithm is usually used on a large scale because of its good balance between the calculation cost and the estimation accuracy, and has been widely used at present [16], [17]. With the rapid development of UWA channel estimation, the problem of long estimation time and large computational cost of orthogonal match tracking OMP algorithm has become increasingly prominent, which is difficult to adapt to the operation of UWA channel estimation [18]. In complex underwater communication scenarios, due to the large sparsity of the channel matrix, the OMP algorithm has a high calculation cost and a long calculation time, resulting in a great impact on the communication delay. In order to fully tap the potential of the OMP algorithm, people began to use thresholds to optimize the number of iterations, leading to more and more OMP class algorithms [19], [20], [21]. However, most algorithms emphasize the estimation accuracy and ignore the system computational cost. To solve this problem, this paper adopts an improved MP algorithm, which can set a fixed value to limit the number of iterations according to the channel tapping value, thus balancing the estimation accuracy and computational effort. Meanwhile, this paper also employs Ordered Recursive Least Squares MP (ORLSMP) [9], Sparsity Adaptive MP (SaMP) [22] and Adaptive Step Size SaMP (AS-SaMP) [23] for sparse time-varying underwater channel estimation simulation.

The contributions of this article are as follows:

- By cleverly designing a dynamic termination condition, the MP algorithm has been improved, making it more applicable. In special sparse channels, for difficult to achieve termination conditions, set a threshold for the iteration coefficient to terminate the operation, making the algorithm more applicable.
- This dynamic termination condition can greatly balance the calculation cost and estimation accuracy. Due to the reduced complexity of the algorithm, the power consumption and computational time of the system are reduced.
- Due to the dynamic termination condition of this algorithm, it has better estimation performance compared to other algorithms at low SNR.

In Section II, we present the DDSF model for UWA communication systems. In Section III, we introduce the proposed the threshold algorithm (T-MP), analyze the advantages and disadvantages of the OMP algorithm and the T-MP algorithm, and explain how the T-MP algorithm operates within the DDSF framework. Section IV, we perform simulations to compare the mean square error (MSE) and CPU computation time of the OMP algorithm, AS-SaMP algorithm, SaMP algorithm, ORLSMP algorithm, and T-MP algorithm. To ensure realistic simulations, the various algorithms are simulated in the environment of the Pacific Storm Experiment 2010 (PS'10) [24]. Finally, in Section V, we discuss the advantages and disadvantages of the T-MP algorithm.

II. SYSTEM MODEL

In a UWA communication system represented by a discrete time-varying signal x(n), n = 1, 2, ..., the impulse response is denoted by h(n). For a system with K channel delay taps, the impulse response of the kth channel delay tap is expressed as h(k, n), where k is the number of channel delay taps sampled by the system. The sampled discrete DDSF of the kth channel delay tap is denoted by u(l, k), where l represents the number of Doppler sampling points such that $0 \le l \le L$. Therefore, the expression for the impulse response of the kth channel delay tap [25] is given by:

$$h(k,n) = \sum_{l=1}^{L} u(l,k) e^{j2\pi f_l n \Delta t}$$
⁽¹⁾

where $f_l = f_{min} + (l - 1)\Delta f$ is the *l*th Doppler sampling frequency, f_{min} is minimum Doppler sampling frequency, Δf is Doppler sampling interval, Δt is time domain sampling interval. For the *k*th channel delay tap, The unit impulse response of the *k*th channel delay tap of time *n* is expressed as h(n - k). The output symbol can be expressed as

$$y(n) = \sum_{k=1}^{K} h(k, n) x(n-k+1) + w(n)$$
(2)

where x(n-k+1) indicates the *k*th channel delay tap symbol with transmission symbol x(n). And w(n) represents the background noise of the marine environment. By substituting equation (1) into equation (2), we can get the reception symbol of the *k*th channel delay tap as

$$y(n) = \sum_{k=1}^{K} \sum_{l=1}^{L} u(l,k) e^{j2\pi f_l n \Delta t} x(n-k+1) + w(n)$$
(3)

for the convenience of calculation, we can use the matrix form of the Kronecker product as

$$\mathbf{y} = [\mathbf{\Omega} \otimes \mathbf{X}]^T \mathbf{u} + \mathbf{w} \tag{4}$$

where $\mathbf{y} = [y(n) \ y(n + 1), \dots, \ y(n + M - 1)]^T$ is the $M \times 1$ receive symbol vector and M is the length of each received symbol, $\mathbf{w} = [w(n) \ w(n + 1), \dots, \ w(n + M - 1)]^T$ is the corresponding $M \times 1$ noise vector, $\Omega = [e^{j2\pi f_1 n\Delta t} \ e^{j2\pi f_2 n\Delta t}, \dots, \ e^{j2\pi f_L n\Delta t}]^T$ is an $L \times 1$ vector and $e^{j2\pi f_1 n\Delta t}$ is phase factor of Doppler sampling frequency, $\mathbf{X} = [x(n) \ x(n - 1), \dots, \ x(n - K + 1)]^T$ is the $K \times 1$ transmitted symbol vector, and $\mathbf{u} = [u(1, 1) \ u(1, 2), \dots, u(1, K), \dots, \ u(L, K)]^T$ is the $KL \times 1$ DDSF vector. It is assumed that the system operates under the assumption that the parameters controlling the relationship between transmitted and received symbols remain relatively constant for a given duration. To facilitate the analysis of the algorithm, equation (4) is written in the form of equation (5), as follows

$$y = Au + w \tag{5}$$

where $\mathbf{A} = [\Omega \otimes \mathbf{X}]^T$ is an $M \times KL$ matrix, its *i*th row equals to $[\Omega(n + i - 1) \otimes X(n + i - 1)]^T$. In CS theory, \mathbf{A} is the sensing matrix composed of the weighted sum of the Doppler effect and delay, and \mathbf{y} is the receiving symbol when it is sent as symbol \mathbf{X} . Our goal is to estimate \mathbf{u} given the known \mathbf{y} and \mathbf{A} , and reconstruct \mathbf{u} . Before calculation, we need to define the parameters in matrix \mathbf{A} . Let *KL* denote the length of matrix \mathbf{u} and *I* represent the number of dominant components in \mathbf{u} , which is referred to as the sparsity level of the target signal in CS theory. Through the CS algorithm, the process of compressing *KL* into *I* completes the reconstruction estimation of \mathbf{u} .

Due to the introduction of Doppler sampling, we need to adjust the parameters, including data block length M, delay number K, and Doppler sampling L. For data block length M, the estimation of an excessively long block will

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introduce significant error [26]. It takes a lot of time for the data block to process, and the dynamic change of the environment is too fast, which makes the estimated channel coefficients quickly outdated, resulting in the reconstructed **u** being unable to adapt to the estimation of the following parts of the data block. However, a longer block M is beneficial for reducing the correlation of the columns in matrix **A**, which is very convenient for the reconstruction of **u**. Therefore, the selection of M should balance the relationship between these factors. For the number of delayed samples (channel length), K should be greater than the maximum delay in multipath channels to aid subsequent equalization.

For Doppler sampling, *L* should be greater than the most important part of the frequency shift of the Doppler effect, and sampling the remaining part will cause waste in the system and result in little improvement in system performance. According to the derivation in [26], $\Delta f > \frac{1.4}{\pi M \Delta t}$. For M =600 and a sample frequency $f_s = 24$, 000 Hz, this corresponds to $\Delta f > 17.83$ Hz. The purpose is to make the correlation of columns in **A** low enough. To avoid aliasing in the time domain and Doppler domain when DDSF represents CIR, it is necessary to ensure that $\Delta f \Delta t \leq \frac{1}{N}$ and $\Delta t \leq \frac{1}{2f_{\text{max}}}$. Therefore, when selecting parameters, we must ensure that the above parameter conditions are met to achieve the best estimation effect.

III. CHANNEL ESTIMATION OF OMP ALGORITHM AND T-MP ALGORITHM

This section discusses the performance limitations of the conventional OMP algorithm, referred to as Algorithm 1 and introduces an T-MP algorithm, referred to as Algorithm 2, which can significantly enhance the algorithm's performance. The section also analyzes the reasons for the performance improvement achieved by Algorithm 2. T-MP algorithm given in Fig.1.

A. CHANNEL ESTIMATION OF OMP ALGORITHM

The OMP algorithm1 sequentially identifies the dominant channel tap coefficients and selects them in descending order of magnitude. In each iteration, the algorithm selects the column vector most correlated with the residual of the previous iteration from the matrix A [27]. The initial residual vector is set to $\mathbf{r}_0 = \mathbf{y}$, and the initial index set is $\Omega_0 = \emptyset$. The index for the *i*th iteration is expressed as

$$s_{i} = \arg \max_{j=1,2,\dots,KL} \frac{\left| \mathbf{a}_{j}^{H} \mathbf{r}_{i-1} \right|^{2}}{\left\| \mathbf{a}_{j} \right\|^{2}}$$
(6)

where \mathbf{a}_j is the *j*th column in matrix \mathbf{A} , and \mathbf{r}_{i-1} is the residual after the *i* – 1th iteration. After finding the index s_i , the index set is updated and used in the subsequent channel coefficient calculation. The *i*th index set, Ω_i , is updated as follows:

$$\Omega_i = \Omega_{i-1} \cup s_i \tag{7}$$

Algorithm 1 OMP Algorithm

- 1: Input
 - Dictionary matrix A
 - Received symbol **y**
 - Physical sparsity I
- 2: Output
 - Reconstructed signal $\hat{\mathbf{u}}$
- 3: Initialization

•Residual vector $\mathbf{r}_0 = \mathbf{y}$

- •Index set $\Omega_0 = \emptyset$
- 4: for i = 1, 2, ..., I do

Find the index: Select the atom with the maximum inner product with the residual

 $s_i = \arg\max_{j=1,2,\dots,KL} \frac{|\mathbf{a}_j^H \mathbf{r}_{i-1}|^2}{\|\mathbf{a}_j\|^2}$

Update the index set: Add the index obtained from the *i*th iteration to the index set constituted by the previous i - 1.

 $\Omega_i = \Omega_{i-1} \cup \{s_i\}$

Compute the coefficient vector (A_{Ω} denotes the starvation matrix of the composition of atoms selected from A).

 $\hat{\mathbf{u}}_{i} = (\mathbf{A}_{\Omega}^{H}\mathbf{A}_{\Omega})^{-1}\mathbf{A}_{\Omega}^{H}\mathbf{r}_{i-1}$ Update the residual vector $\mathbf{r}_{i} = \mathbf{r}_{i-1} - \mathbf{A}_{\Omega}\hat{\mathbf{u}}_{i}$ 5: end for

when calculating channel coefficients, the least squares of matrix A should be computed. The formula is given by:

$$\hat{\mathbf{u}}_i = (\mathbf{A}_{\Omega}^H \mathbf{A}_{\Omega})^{-1} \mathbf{A}_{\Omega}^H \mathbf{r}_{i-1}$$
(8)

where $\mathbf{A} = [\mathbf{a}_{1_i} \dots \mathbf{a}_{s_i}]$, in selecting the column vectors of \mathbf{A} , each column vector selected must be orthogonalized with all column vectors of the index set in the previous step. Here, $\hat{\mathbf{u}}_i$ denotes the channel coefficients generated by the *i*th iteration. After computing the channel coefficients, the residuals need to be updated as follows:

$$\mathbf{r}_i = \mathbf{r}_{i-1} - \mathbf{A}_{\Omega} \hat{\mathbf{u}}_i \tag{9}$$

repeat steps (6)-(9) until the number of repetitions equals the channel sparsity. The algorithm terminates when the number of repetitions reaches the specified value.

Although the OMP algorithm is widely used, it still exhibits several shortcomings in complex environments. For instance, implementing the OMP algorithm is challenging due to the difficulty of obtaining the physical sparsity of the UWA environment, which is a requirement for the algorithm. In the execution process of the OMP algorithm, coefficient calculation requires the least squares operation on the selected atoms, which represents the most computationally intensive part of the algorithm and requires a large amount of computing resources. However, the underwater environment is characterized by a large Doppler effect, and the number of unimportant elements in **u** increases, leading to greater sparsity. As sparsity increases, the computational power





FIGURE 1. The flowchart of the T-MP algorithm in channel estimation.

needed for least-squares operations on selected atoms in the matrix \mathbf{A} also grows substantially. The increase in sparsity results in the OMP algorithm requiring a large amount of computing resources, resulting in a longer calculation time for the system. Since underwater environments are complex and rapidly changing, extended computation times can limit the estimation accuracy of the OMP algorithm. Therefore, developing a suitable algorithm for such environments is essential.

B. CHANNEL ESTIMATION OF T-MP ALGORITHM

The original OMP algorithm suffers from the cumbersome balance between the estimation accuracy and the convergence rates [9]. Generally, the fixed threshold of original OMP algorithm may bring cumulative estimation errors and large computational cost due to the amounted iterations. Compared to the original OMP algorithm, the proposed T-MP algorithm adopts the adaptive tradeoff between the estimation accuracy and the convergence via utilizing the adaptive threshold $T = \alpha e^{-\gamma * SNR} + \beta$. Intuitively, the SNR [28] will impose intensive impact over the threshold *T*, which leads to low threshold with high SNR and high threshold with low SNR. Thus, the proposed T-MP algorithm can reduce the computational cost in low SNR and still keep accurate estimation in high SNR, which is validated in the simulation results in Fig.4.

For the T-MP algorithm2, it is essential to configure the algorithm parameters appropriately. Typically, the weight coefficient difference denoted as η , is set to 0.0015, while the parameter β plays a crucial role in adjusting the algorithm iteration threshold, which is determined based on the specific situation. In scenarios with high SNRs, β should be set to a value less than 0.02, which corresponds to an iteration threshold *T* less than 0.02. In the case of low SNRs, the bit error ratio(BER) of the system is high, and it is necessary to use a lower number of iterations to prevent the excessive BER from affecting the estimation accuracy. In such cases, the exponential part of the *T* expression plays a more significant role in adjusting *T*, which is typically set to a value greater than 0.13. By utilizing a lower number of

Algorithm 2 T-MP Algorithm

- 1: Input
 - Dictionary matrix A
 - Received symbol y
 - •Iterations parameter α , β , γ , η
- 2: Output
 - Reconstructed signal $\hat{\mathbf{u}}$
- 3: Initialization
 - •Residual vector $\mathbf{r}_0 = \mathbf{y}$

• Threshold value $T = \alpha e^{-\gamma * SNR} + \beta$

4: while $\hat{u}_i > T$ do

Find index:Select the atom with the maximum inner product with the residual

 $s_i = arg \max_{j=1,2,\dots,KL} |(\mathbf{a}_j^H \mathbf{r}_{i-1})|$

Compute coefficient: the inner product of the remaining vectors and the selected atoms

 $u_i = \langle \mathbf{r}_{i-1}, \mathbf{a}_{s_i} \rangle$ Update residual vector $\mathbf{r}_i = \mathbf{r}_{i-1} - u_i \mathbf{a}_{s_i}$ Optimize decision threshold if $1.2T \ge \hat{u}_i > T$ then if $\eta \ge \hat{u}_{i-1} - \hat{u}_i$ then $\hat{u}_{i+1} = 0$ end if update Iterations i = i + 15: end while

iterations, the error resulting from low SNRs is reduced, and the MSE performance in low SNR environments is improved. Simulation results show that the T-MP algorithm is superior to other algorithms in mean squared error when the SNR is low. This is because the T-MP algorithm has a lower threshold, fewer system iterations, and a smaller transmission BER. The number of iterations of other algorithms remains unchanged, and to obtain atoms with smaller weights, they absorb more errors during the iteration process, which has a significant impact on the estimation results.

In intricate UWA communication systems, the T-MP algorithm swiftly computes elements with greater contributions in the dictionary matrix **A**, based on the magnitude of the atomic contributions to the linear approximation. This effectively balances estimation accuracy and speed. Unlike the OMP algorithm, the T-MP algorithm does not necessitate the atomic matrix generated in the prior iteration to be orthogonal to the atoms of the current iteration, i.e., the matrix's Least Squares operation. In complex UWA systems, such an approach would demand numerous calculations, consuming considerable computation time and restricting estimation accuracy in rapidly changing underwater environments. As such, the T-MP algorithm is better suited for quickly evolving UWA systems.

Nevertheless, when signal sparsity is excessively high, the size of the matrix \hat{u} to be reconstructed may become



FIGURE 2. Comparison of the NMSE versus iteration number in different SNRs. From the figure, it can be obtained that there exists an optimal iteration number under which the NMSE of the OMP algorithm is the smallest.

substantially large. In the dictionary matrix, the system selects column vectors with higher weights during early iterations, allowing the received signal to converge rapidly within the iterative process. Following multiple iterations, the weights of column vectors diminish and their magnitudes become similar. At this stage, satisfying the algorithm's termination condition may be challenging, and the remaining column vectors contribute minimally to the estimation. Therefore, it would be a waste of system resources to continue the calculation. To rectify this issue, it is essential to discard these column vectors and force the algorithm to halt. The termination decision is made by comparing the weight coefficients \hat{u}_i and \hat{u}_{i-1} of the current and previous iterations, respectively. When the magnitudes of \hat{u}_i and \hat{u}_{i-1} are similar, the weight coefficient \hat{u}_{i+1} is set to 0, fulfilling the termination condition and concluding the iteration. As a result, when the weights of the dictionary matrix become excessively small after several iterations, the system can maintain the normal operation.

$$MSE = \frac{1}{KL} \sum_{i=1}^{KL} (u_i - \hat{u}_i)^2, NMSE = \frac{\sum_{i=1}^{KL} |u_i - \hat{u}_i|^2}{\sum_{i=1}^{KL} |u_i - \bar{u}|^2}$$
(10)

Equation (10) is an expression for MSE and the normalized mean square error (NMSE), where K denotes the channel length and L represents the sampled DDSF of the channel, u_i , \hat{u}_i and \bar{u} correspond to the actual sparse weight, the estimated sparse weight and the average of actual sparse weight of UWA channels, respectively. The expression for SNR is shown in Equation (11). where P_S denotes the signal power and P_N denotes the noise power.

$$SNR = 10 \cdot \log_{10} \left(\frac{P_S}{P_N}\right) \tag{11}$$

Fig.2 shows the relationship between the number of iterations and the NMSE of the OMP algorithm for different



FIGURE 3. The influence of different types of dictionaries on the rate of convergence of T-MP algorithm.

SNRs. For SNR = 5 dB, the MSE converges to a minimum when the number of iterations is about 20. Thereafter, an increase in the number of iterations leads to a higher NMSE. At SNR = 10 dB, the MSE reaches a minimum at about 30 iterations. After that, the NMSE increases with the number of iterations. As seen in the figure, the number of iterations for the minimum value of NMSE varies as the SNR varies. For the OMP algorithm, although the residuals of the signal decrease with the number of iterations, the estimation error caused by the low SNR is introduced during the iteration process, which leads to an increase in the difference between the estimated signal and the original signal. Therefore, it is necessary to find a number of iterations that minimizes the NMSE of the signal and brings the estimated signal closest to the original signal.

Due to the inability of the OMP algorithm to adaptively determine the number of iterations based on SNR, the algorithm cannot always be at the optimal number of iterations in environments with varying SNRs. However, the adaptive iteration count of the T-MP algorithm effectively solves this problem*T* determines the number of iterations in the T-MP algorithm and corresponds to the relationship between the optimal number of iterations and SNR, providing better performance than the OMP algorithm in low SNR environments. Therefore, compared to the OMP algorithm, the adaptive T-MP algorithm provides greater flexibility and can meet the requirements of a wider range of scenarios.

For the T-MP algorithm, we use different dictionaries for simulation, and the results are shown in Fig.3. The Wavelet dictionary can better capture the signals of sparse channels, especially the DDSF underwater acoustic channel with obvious Doppler effect. The performance of Random dictionaries is the worst, which also indicates the need for targeted optimization of dictionaries for the special environment of underwater channels. The effect of Hadamard dictionary is average, possibly due to its simple structure and difficulty in handling complex underwater channels. Therefore, we use wavelet dictionaries for simulation in the following simulations.

C. CONVERGENCE ANALYSIS OF T-MP ALGORITHM

Let $\mathbf{y} \in \mathbb{R}^n$ be the input signal, and $\mathbf{A} = \{\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_e\}$ be an overcomplete dictionary with *e* atoms, where each atom $\mathbf{a}_i \in \mathbb{R}^n$ has unit norm ($||\mathbf{a}_i||_2 = 1$) [29]. The goal of the MP algorithm is to find a sparse representation of *y* using a linear combination of a few atoms from **A**:

$$\mathbf{y} \approx \sum_{i=1}^{I} u_i \mathbf{a}_{s_i},\tag{12}$$

where $I \ll KL$, u_i is the coefficients, and s_i is the indices of the selected atoms.

The MP algorithm iteratively selects the atom that has the maximum inner product with the current residual and updates the residual accordingly. The algorithm can be summarized as follows:

- 1) Initialize the residual $\mathbf{r}_0 = \mathbf{y}$.
- 2) For i = 1, 2, ..., I
 - a) Select the atom with the maximum inner product with the residual: $s_i = \arg \max_{j=1,2,...,KL} |\langle \mathbf{r}_{i-1}, \mathbf{a}_j \rangle|.$
 - b) Update the coefficient: $u_i = \langle \mathbf{r}_{i-1}, \mathbf{a}_{s_i} \rangle$.
 - c) Update the residual: $\mathbf{r}_i = \mathbf{r}_{i-1} u_i \mathbf{a}_{s_i}$.
- 3) Output the sparse representation $\mathbf{y} \approx \sum_{i=1}^{I} u_i \mathbf{a}_{s_i}$.

To analyze the convergence of the MP algorithm, we will examine the decrease in the residual's energy (squared norm) at each iteration:

$$E_i = \|\mathbf{r}_i\|_2^2 = \|\mathbf{y} - \sum_{i=1}^I u_i \mathbf{a}_{s_i}\|_2^2.$$
(13)

For any input signal \mathbf{y} and dictionary \mathbf{A} , the energy of the residual decreases monotonically at each iteration of the MP algorithm:

$$E_i \le E_{i-1} - u_i^2. \tag{14}$$

Let $\mathbf{r}_i = \mathbf{r}_{i-1} - u_i \mathbf{a}_{s_i}$. Then,

$$E_t = \|\mathbf{r}_i\|_2^2 \tag{15}$$

$$= \|\mathbf{r}_{i-1} - u_i \mathbf{a}_{s_i}\|_2^2 \tag{16}$$

$$= \|\mathbf{r}_{i-1}\|_{2}^{2} - 2u_{i}\langle \mathbf{r}_{i-1}\mathbf{a}_{s_{i}}\rangle + u_{i}^{2}\|\mathbf{a}_{s_{i}}\|_{2}^{2}.$$
 (17)

Since $\|\mathbf{a}_{s_i}\|_2 = 1$ and $u_i = \langle \mathbf{r}_{i-1}, \mathbf{a}_{s_i} \rangle$,

$$E_i = E_{i-1} - u_i^2. (18)$$

Thus, the energy of the residual decreases monotonically at each iteration. Since the T-MP algorithm only changes from MP in terms of the number of iterations, the T-MP algorithm and the MP algorithm have similar convergence properties.

D. COMPLEXITY ANALYSIS OF T-MP ALGORITHM

The main operations of the T-MP algorithm include the following steps:

1) Finding the index s_i .

TABLE 1. System parameters in experiments.

Parameters	Value
Training symbol length M	600
Frame length N	2800
Total number of block N_b	11000
The number of sampled deley points	126
Maximum Doppler shift [Hz]	30
Doppler frequency resolution [Hz]	4.3
The number of sampled Doppler points	4
The central carrier frequency [kHz]	11
Bandwith [kHz]	4
The depth of water [m]	100
Transmitter height [m]	8
Receiver height [m]	90
Distance between transmitter and receiver [m]	700
Relative velocity between transmitter and receiver [m/s]	0.5

2) Updating the index set Ω .

- 3) Computing the coefficient \hat{u}_i .
- 4) Updating the residual vector \mathbf{r}_i .
- 5) Optimizing the decision threshold.

First, let's consider step 1. For each iteration, the algorithm needs to calculate KL inner products and find the maximum. Calculating an inner product requires N multiplications and N - 1 additions, so the time complexity of calculating KL inner products is O(KLN). Finding the maximum requires KL - 1 comparisons, so the time complexity is O(KL). Therefore, the total time complexity of step 1 is O(KLN).

For steps 2 and 3, the time complexity is O(1) and O(N), respectively.

Step 4 involves vector-matrix multiplication and subtraction. Since a_{s_i} is an *N*-dimensional vector, the time complexity of this step is O(N).

Step 5 involves conditional judgment and threshold updating, with a time complexity of O(1).

In summary, in a single iteration, the main time complexity of the algorithm comes from step 1, which is O(KLN). Assuming the algorithm performs a maximum of I_{max} iterations, the total time complexity of the T-MP algorithm is $O(I_{max}KLN)$.

IV. SIMULATION RESULTS AND ANALYSIS

In this section, various algorithms are simulated for Doppler underwater channel modeling. The superiority of the T-MP algorithm in low SNR environments is derived by comparing the mean square error simulation results of the OMP algorithm, the AS-SaMP algorithm, the SaMP algorithm, the ORLSMP algorithm and the T-MP algorithm. Then, the CPU computation time of the above-mentioned algorithms is compared and the low power consumption of the T-MP algorithm is derived. After that, the MSE of T-MP algorithm and OMP algorithm with different lengths of lead frequency is simulated. Finally, the effect of sparsity on OMP algorithm, ORLSMP algorithm and T-MP algorithm is also compared.

A. MSE COMPARISON OF DIFFERENT ALGORITHMS

The parameter settings provided in Table 1 were employed to conduct simulations using the OMP algorithm, SaMP



FIGURE 4. MSE of different SNRs of OMP algorithm, AS-SaMP algorithm, SaMP algorithm, ORLSMP algorithm, and T-MP algorithm in the DDSF model.

algorithm, ORLSMP algorithm, as well as AS-SaMPand algorithm, respectively, and the results are shown in Fig.4.

For the OMP algorithm, due to its sensitivity to noise, it is easily affected by noise interference, and the MSE performance with low SNR is not as good as the T-MP algorithm [30]. Under high SNR, its MSE performance is inferior to other algorithms due to fewer iterations. However, compared to the T-MP algorithm, the vectors selected during the iteration process are orthogonal, reducing the impact of vector correlation during the iteration and enabling fast recovery of the original signal. Therefore, its MSE performance is better than the current T-MP algorithm under high SNR. The SaMP algorithm adapts to changes in the number of iterations through the sparsity of the channel, and has high stability. It has better estimation performance compared to the MP algorithm, and the adaptive iteration of the algorithm can more accurately select channel coefficients to prevent excessive non-zero coefficients from causing estimation errors to increase. For the AS-SaMP algorithm, it usually has better estimation performance than the SaMP algorithm [31]. By introducing the mechanism of dynamically adjusting the step size, the robustness and Rate of convergence of the estimation are improved, but it has high complexity. The ORLSMP algorithm is an iterative algorithm based on least squares, which uses the basis functions of the MP algorithm and then uses recursive least squares to update the coefficients of these basis functions. At each iteration, the channel parameters are updated recursively using previous historical data to improve the accuracy and stability of the estimation. Overall, none of the above algorithms have introduced the size of SNR into the estimation process of the algorithm, and SNR has a significant impact in the channel estimation process. When the SNR is low, the T-MP algorithm reduces the number of iterations through the effect of SNR, thereby reducing the introduction of estimation errors, resulting in better estimation performance than other algorithms. The T-MP algorithm is of great



FIGURE 5. Performance of the considered algorithms for M=600 with the PS'10 experiment.

significance in communication that requires low energy consumption and long cycles. For example, deep-sea oil well sensors, deep-sea seismic observation instruments, and ocean current meteorological observation instruments all have the characteristics of difficult deployment and long periods, and low-power communication plays a very important role.

TABLE 2. System parame	eters in e	xperiments.
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Parameters	Value
Training symbol length M	600
Frame length N	2600
Total number of block N_b	10000
The number of sampled deley points	126
Maximum Doppler shift [Hz]	30
Doppler frequency resolution [Hz]	4.3
The number of sampled Doppler points	4
The central carrier frequency [kHz]	10
Transmission symbol rate [kilo symbols per second]	4
Carrier frequency [kHz]	10
Bandwith [kHz]	4
The depth of water [m]	130
Transmitter height [m]	3
Receiver height [m]	127
Distance between transmitter and receiver [m]	500
Relative velocity between transmitter and receiver [m/s]	3

To accurately simulate real-world conditions, the experimental data from the PS'10 was employed [24]. The parameter settings for simulation are shown in Table2, and the simulation results of various algorithms are shown in Fig.5. The simulation results show that the T-MP algorithm has better performance than the OMP algorithm when the SNR is below 12. However, when the SNR is below 6, there is better performance than other algorithms. It is worth noting that compared with the above experiments, there is a significant difference in the performance of SaMP and ORLSMP. This may be due to the low sparsity during the experiment, which results in less information being transmitted during the iteration process of the SaMP algorithm and poorer estimation performance.



FIGURE 6. Comparison of CPU running time for various algorithms with M=600 based on DDSF.

B. CPU RUNNING TIME OF DIFFERENT ALGORITHMS

As shown in Fig.6, the CPU running time of the T-MP algorithm is significantly shorter compared to the other algorithms. The OMP algorithm requires picking atoms while orthogonal computations are being performed, which greatly increases the computational effort of the system. ORLSMP, on the other hand, introduces recursive updating, which has to be updated all the time during the computation, so it also has a considerable amount of computation. For both SaMP and AS-SaMP contain sparse approximation operations, which wastes the power consumption of the system for channel environments with little sparse variation. In summary, the T-MP algorithm has the characteristics of simplicity and high efficiency, which can greatly save the computation time and reduce the system power consumption.

Since the number of iterations of OMP is related to the sparsity of the system, it is generally believed that for a fixed sparsity, the number of iterations is fixed and the number of atoms selected in the operation is also fixed, so the computational cost of least squares is also fixed, and the change in SNR has no effect on the computational cost of OMP [32]. For the T-MP algorithm, its iteration stop threshold T is dynamic. At low SNR, the threshold T is large, the number of iterations is small, and the computation of the iterative process is reduced; At high SNR, the threshold T becomes small, and the corresponding computation increases.

The PS'10 experiment was conducted to compare the CPU running time of different algorithms, and the simulation results are shown in Fig.7. Among these algorithms, T-MP algorithm has the shortest computation time. The T-MP algorithm pursues atoms with significant weights, which greatly saves CPU computation time. In contrast, the other algorithms pursue as many atoms as possible, wasting a significant amount of system power consumption.



FIGURE 7. Comparison of CPU running time with M=600 in the PS'10 experiment.



FIGURE 8. Comparison of different pilot lengths of T-MP algorithm and OMP algorithm. Three pilot lengths M = 400, M = 600 and M = 800 are used to calculate the MSE.

C. INFLUENCE OF PILOTS WITH DIFFERENT LENGTHS ON ESTIMATION ALGORITHM

The T-MP algorithm and the OMP algorithm exhibit the same MSE points at various pilot lengths M, as demonstrated by the intersection points in Fig.8. The intersection point is near SNR = 5 dB for pilot length M = 800, near SNR = 8 dBfor M = 600, and near SNR = 14 dB for M = 400. This observation suggests that within a certain range, a longer pilot length M accelerates the convergence of OMP concerning T-MP algorithm. The reason behind this is that at low SNR, the over-iterations of the OMP algorithm generate many errors, resulting in a lower MSE for OMP than for T-MP algorithm. Increasing the pilot length decreases the error rate of the OMP algorithm in a low SNR environment, hence the longer the pilot length, the faster the OMP algorithm converges. For the T-MP algorithm, the smaller number of iterations reduces errors in the iterations, and atoms with larger weights are selected while those with smaller weights are discarded, leading to better tracking performance for



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FIGURE 9. Constellations with different M of T-MP algorithm. Figures (a) , (b) and (c) represent pilot length M=400, M=600 and M=800.

T-MP algorithm compared to OMP. We then simulate the constellation diagram of QPSK with different pilot lengths of the T-MP algorithm according to the experimental conditions in Fig.8, with results presented in Fig.9. For M = 400, the output constellation of the equalizer is disordered and exhibits a high BER. The output points of M = 600 have good phase differentiation and a low BER. With M = 800, only a few points are challenging for the system to



FIGURE 10. The impact of channel sparsity on different algorithms.

identify, meeting the requirements of systems with relatively high BER. This illustrates the significant impact of pilot length on channel estimation. Nonetheless, increasing the pilot length also introduces certain issues. First, a longer pilot length increases communication overhead, as more channel resources are required for pilot transmission. Second, when the pilot length surpasses the channel, the signal experiences multipath effects, causing increased errors in channel sparsity estimation. Moreover, excessively long pilot lengths may also contribute to a rise in the computational complexity of the algorithm, affecting real-time performance.

D. MSE OF DIFFERENT ALGORITHMS UNDER DIFFERENT SPARSITY

As shown in Fig.10, we simulated the sparsity of different algorithms at SNR = 10dB. The T-MP algorithm performs better when sparsity is low. The biggest reason is that under low sparsity, T-MP, OMP, and ORLSMP algorithms have similar performance in selecting atoms, but under low SNR conditions, T-MP has better performance. Under high sparsity conditions, the performance of T-MP algorithm in selecting matching atoms deteriorates, and the atoms selected in the later stage of iteration may have a significant correlation with the atoms selected in the earlier stage, leading to a decrease in recovery ability. The OMP algorithm selects atoms that are orthogonal, which has better recovery performance. The ORLSMP algorithm has better estimation performance than the OMP algorithm because it has the characteristic of recursive updating, reducing errors caused by noise, and also improving estimation stability.

V. CONCLUSION

In this study, the T-MP algorithm was employed to investigate underwater channel estimation using the DDSF. The performance of sparse reconstruction was compared between T-MP and OMP algorithms using the DDSF model. The dynamic iterative threshold of T-MP algorithm offers a superior balance between computational accuracy and cost. Simulation results demonstrate that T-MP algorithm outperforms OMP in low SNR scenarios, significantly reducing computational time. Under high SNR conditions, both algorithms exhibit comparable performance; However, T-MP algorithm requires less computational time. Consequently, T-MP algorithm emerges as a preferable choice for underwater communication applications necessitating lower power consumption. This study contributes to the development of more efficient UWA channel estimation algorithms and holds potential applications in a wide range of underwater communication scenarios.

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