IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received 12 August 2023, accepted 31 August 2023, date of publication 4 October 2023, date of current version 12 October 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3322032

RESEARCH ARTICLE

Selective Maintenance on a Multistate System Executing Multiple Consecutive Missions Under Sequential Maintenance

YAO SUN[1], JIE ZHOU[1], ZHILI SUN[2], AND ZHE WEI[1]
[1]School of Mechanical Engineering, Shenyang University of Technology, Shenyang 110870, China
[2]School of Mechanical Engineering and Automation, Northeastern University, Shenyang 110819, China

Corresponding author: Zhe Wei (weizhe@sut.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 51775097 and Grant 51975386; in part by the National Key Research and Development Program of China under Grant 2020YFB2007800; in part by the Key Science and Technology Program of Liaoning Province under Grant 2022020630-JH1/108; and in part by the Science and Technology Research and Development Program, China National Railway Group Company Ltd., under Grant N2022J014.

ABSTRACT A type of multistate system executes multiple consecutive missions over a period, and maintenance activities are also performed sequentially during this period, to improve system reliability during the consecutive missions. Sequential maintenance and dependence on the system state introduce new challenges for the calculation of system reliability during consecutive missions, which makes selective maintenance difficult to perform. In order to provide theoretical support for the selective maintenance of this type of multistate system and obtain a reasonable and effective selective maintenance strategy, a reliability calculation method is proposed for a multistate system that executes multiple consecutive missions under sequential maintenance. Based on this, a new selective maintenance model for a multistate system executing multiple consecutive missions is developed, and the ant colony optimization algorithm is customized to address the resulting maintenance strategy optimization problem. Finally, using an oil transportation system as an example, the accuracy and computational effectiveness of the proposed reliability calculation method are verified by comparison with the Monte–Carlo simulation method. Moreover, the selective maintenance strategy optimization is performed on the oil transportation system to illustrate the effectiveness of the proposed selective maintenance model and the customized optimization method, and the results can provide practical guidance for the reasonable adjustment of mission durations.

INDEX TERMS Maintenance strategy optimization, multistate system, selective maintenance, system reliability.

I. INTRODUCTION

Owing to the inevitable deterioration and failure of engineering systems during operation, maintenance becomes increasingly important for improving system reliability and prolonging system life [1]. Various maintenance strategies have been investigated to maximize system reliability or minimize resource consumption [2], [3]. In some industrial environments, not all desired maintenance activities can be completed because of limited resources. A subset of the desired maintenance activities is selected to be performed to satisfy specific requirements, and this is called selective maintenance [4].

Selective maintenance was first applied to binary–state (perfect function or complete failure) systems. It was initially applied to a parallel–series system comprising units with a constant failure rate, and only a replacement policy was considered [5]. Subsequently, selective maintenance was extended to a more general case, where the lifetime of units followed the Weibull distribution and three optional maintenance actions existed: minimal repair, corrective replacement, and preventive replacement [6]. Then, selective

The associate editor coordinating the review of this manuscript and approving it for publication was R. K. Saket.

maintenance was applied to manufacturing lines to minimize maintenance cost and production loss within limited maintenance time [7]. Additionally, the selective maintenance problem of systems subject to propagated failures with global effect and failure isolation phenomena was addressed [8]. However, engineering systems such as manufacturing systems [9], networked systems [10], generating systems [11], and transportation systems [12] can operate at intermediate states between perfect function and complete failure, exhibiting a multistate characteristic.

In recent years, increasing interest has been given to the selective maintenance of multistate systems (MSSs) [13]. A selective maintenance model for systems composed of binary–state units was proposed, in which imperfect maintenance was initially incorporated [14]. This model was further extended to a generalized case in which systems comprise multistate units [4]. Three selective maintenance models were developed to address economic dependence [15], structural dependence [16], and stochastic dependence [17], respectively, among units. In a load–varying environment, a joint optimization model for load distribution and selective maintenance was proposed [18], based on which load-dependent deterioration was introduced into the selective maintenance model [19]. With uncertainty being considered, a selective maintenance model under stochastic maintenance durations was developed [20], and uncertainties associated with the operation time of each unit and the durations of future missions were introduced into the selective maintenance model [21]. Additionally, for a simple selective maintenance model, the resulting optimization problem can be transformed into mathematical programming [22] or solved by exhausting all the maintenance strategies [23]. As selective maintenance models become increasingly complex, advanced computational intelligence technologies such as genetic algorithms [14] and differential evolution algorithms [4] have been widely adopted to seek a global optimal maintenance strategy in an efficient computational manner. However, in most previous studies on selective maintenance, only one mission was considered and maintenance was performed only during breaks.

Further research on the selective maintenance in the context of multiple missions is required. A selective maintenance strategy optimization approach was proposed for systems performing a sequence of identical missions with the same durations of breaks between missions, with the aim of integrating redundancy allocation and maintenance resource allocation decisions [24]. For situations where failed units can be minimally repaired immediately during a mission, the selective maintenance problem for systems executing multiple missions was addressed [25], [26]. Under the assumption that the lifetime of components complies with exponential distribution, the selective maintenance optimization for a system was formulated as a stochastic dynamic programming [27], and the approximate dynamic programming algorithm was used to address this problem when the number of components is extremely large [28]. The selective maintenance strategy for systems executing multiple missions over a finite horizon was dynamically optimized using a deep reinforcement learning method [29], and a multi–mission selective maintenance problem for MSSs was addressed to minimize the total cost [30]. Nevertheless, in these studies, a break exists between any two consecutive missions. Another scenario exists in engineering practice where MSSs must execute multiple consecutive missions over a predetermined period, during which maintenance is also performed to ensure that the system is reliable during consecutive missions. Because maintenance is performed during missions, all maintenance activities are performed sequentially to prevent a temporary system breakdown. Moreover, a dependence on the system state exists between any two consecutive missions. Sequential maintenance and dependence on the system state introduce new challenges for calculating the system reliability during consecutive missions, which makes the corresponding selective maintenance difficult to perform effectively. A recursive algorithm to calculate the system reliability during consecutive missions was proposed [31], [32], and performance sharing was introduced into the algorithm [33]; however, sequential maintenance was not considered. Additionally, the sequence of the selected maintenance activities to be performed needs to be arranged. The maintenance sequence arrangement for a predetermined period during which only one mission existed was addressed by customizing the ant colony optimization (ACO) algorithm [34]. Nevertheless, the maintenance sequence arrangement for the scenario in which multiple consecutive missions exist during a predetermined period has not yet been explored. The main contributions of this study are as follows:

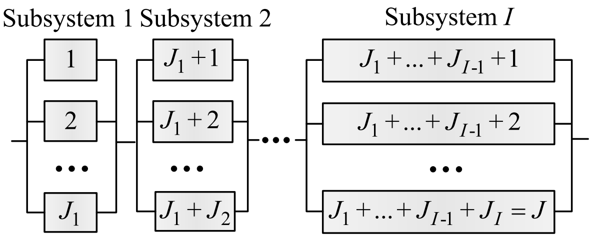

FIGURE 1. Series–parallel structure of the MSS studied in this paper.

(1) A reliability calculation method for a MSS executing multiple consecutive missions under sequential maintenance, considering the dependence on the system state across consecutive missions, is proposed. (2) A new selective maintenance model for a MSS executing multiple consecutive missions is developed to maximize system reliability during consecutive missions, in which the maintenance sequence arrangement is considered. Additionally, the ACO algorithm is customized to address the resulting optimization problem. These contributions would advance the state–of–the–art of selective maintenance optimization.

The remainder of this paper is organized as follows: Section II describes the MSS and selective maintenance

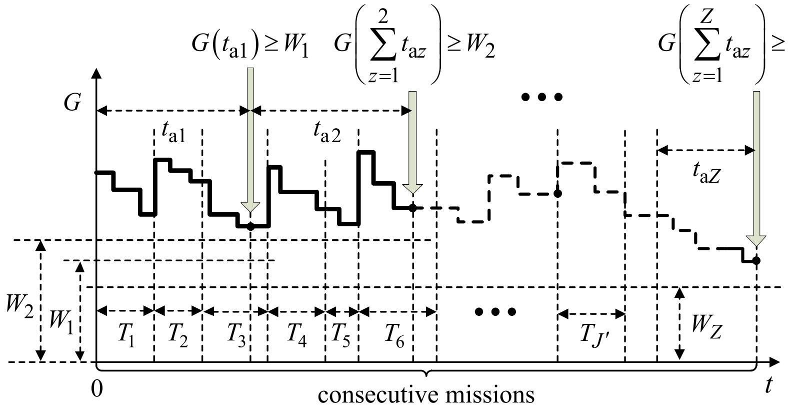

FIGURE 2. Predetermined period studied for the selective maintenance problem and the variation in the system efficiency in this period.

problem studied in this paper. The reliability calculation method for a MSS executing multiple consecutive missions under sequential maintenance is described in detail in Section III. Section IV elaborates the selective maintenance modeling and the customized ACO algorithm. In Section V, the accuracy and computational effectiveness of the proposed reliability calculation method are verified using an oil transportation system as an example, and the effectiveness of the proposed selective maintenance model and the customized ACO algorithm is illustrated by analyzing the influence of the mission duration on the maintenance strategy optimization results. Finally, Section VI provides the conclusions and outlines future research.

II. PROBLEM STATEMENTS

A. SYSTEM DESCRIPTION

As shown in Fig. 1, the MSS studied in this paper has a series–parallel structure.

I s-independent subsystems are connected in series, and subsystem $i\,(i \in \{1, \ldots, I\})$ has J_i s-independent units connected in parallel. The number of all units is denoted as $J = J_1 + \ldots + J_I$, and the units are numbered $1, 2, \ldots, J$ in sequence.

Unit $j\,(j \in \{1, \ldots, J\})$ has $K_j + 1$ states defined as $0, 1, \ldots, K_j$. The states improve from 0 to K_j, and states 0 and K_j indicate complete failure and perfect function, respectively. The respective working efficiencies are termed $g_{j0}, g_{j1}, \ldots, g_{jK_j}$. The state and working efficiency of unit j are denoted as Y_j and g_j, respectively; $Y_j \in \{0, 1, \ldots, K_j\}$ and $g_j \in \{g_{j0}, g_{j1}, \ldots, g_{jK_j}\}$. For the MSS, with its working efficiency denoted as G, its structure function is expressed as follows:

$$G = (g_1, \ldots, g_J) = \min\left(\sum_{j=1}^{J_1} g_j, \sum_{j=J_1+1}^{J_1+J_2} g_j, \ldots, \sum_{j=J_1+\ldots+J_{I-1}+1}^{J} g_j\right). \quad (1)$$

Based on the structure function, all the possible working efficiencies of the MSS can be calculated, and they are denoted as $G_0, G_1, \ldots, G_Q$, respectively, which improve from G_0 to G_Q.

B. SELECTIVE MAINTENANCE PROBLEM

As shown in Fig. 2, the predetermined period consists of Z consecutive missions, and the mission durations are successively denoted as $t_{a1}, \ldots, t_{aZ}$. Owing to the continuity and non–overlap of missions, the duration of the predetermined period is $\sum_{z=1}^{Z} t_{az}$, and the time from the beginning of the predetermined period is denoted as $t\left(t \in \left[0, \sum_{z=1}^{Z} t_{az}\right]\right)$. The selected units are maintained sequentially, and sequential maintenance starts at $t = 0$. Immediately after its maintenance, each selected unit is connected to the system and begins working. The number of selected units is J', with the maintenance time of unit $j'\,(j' \in \{1, \ldots, J'\})$ denoted as $T_{j'}$; thus, $\sum_{j'=1}^{J'} T_{j'} \leq \sum_{z=1}^{Z} t_{az}$. The unselected units are always connected to the system and operating. The details of sequential maintenance can be found in [34], where the maintenance sequence arrangement for a MSS executing one mission during a predetermined period was studied.

To slow the system aging, G is generally required to be not excessively low at the end of each mission, which is the objective of maintenance. The minimum values of G corresponding to each mission are denoted sequentially as $W_1, \ldots, W_Z$ obtained empirically. Therefore, denoted as A_z, the event that the system is reliable in the zth mission can be formulated as

$$A_z = G\left(\sum_{z'=1}^{z} t_{az'}\right) \geq W_z, \quad z = 1, \ldots, Z. \quad (2)$$

Meanwhile, denoted as A, the event in which the system is reliable during consecutive missions means that the system is reliable in all Z missions, which can be expressed as

$$A = \bigcap_{z=1}^{Z} A_z. \tag{3}$$

In addition, the system completes a certain amount of work per mission and incurs operating cost during consecutive missions.

Therefore, the optimization problem is described as follows: First, maintenance must be completed within multiple consecutive missions. Second, the work completed during each mission must satisfy the workload requirement corresponding to the associated mission. Third, the total cost (the sum of the maintenance and operating costs [35]) should be the budget at most. Under these three constraints, the maintenance sequence to be performed is determined to maximize the reliability of the MSS executing multiple consecutive missions under sequential maintenance.

III. RELIABILITY CALCULATION METHOD

When a unit selected for maintenance has not yet been connected to the system, its state is equivalent to a complete failure (state 0) from a system perspective. Based on this consideration, any unit has two types of state: its own state and its equivalent state. When a unit is not connected to the system, its equivalent state is 0; otherwise, its equivalent state is its own state.

For unit j, its state at the end of its maintenance is denoted as Y_j^{A}. In particular, if unit j is not selected to be maintained, Y_j^{A} is equal to $Y_j(0)$. Thus, if unit j is selected to be maintained, $Y_j^{\mathrm{A}} > Y_j(0)$; otherwise, $Y_j^{\mathrm{A}} = Y_j(0)$. If unit j is not selected for maintenance, its equivalent state is always its own state. At time $t = 0$, its equivalent state is $Y_j(0)$, and during the period $\left(0, \sum_{z=1}^{Z} t_{\mathrm{a}z}\right]$, the sample space of its equivalent state is $\{0, \ldots, Y_j(0)\}$. If unit j is selected to be maintained, with its order in the maintenance sequence denoted as w_j, during the period $\left[0, \sum_{j'=1}^{w_j} T_{j'}\right)$, its equivalent state is 0; at time $t = \sum_{j'=1}^{w_j} T_{j'}$, its equivalent state is Y_j^{A}; and during the period $\left(\sum_{j'=1}^{w_j} T_{j'}, \sum_{z=1}^{Z} t_{\mathrm{a}z}\right]$, the sample space of its equivalent state is $\{0, \ldots, Y_j^{\mathrm{A}}\}$.

The above describes the changes in the sample space of the equivalent state for one unit. Based on this, the sample space of the equivalent–state combination of all units also varies during the consecutive missions, and it is represented as a vector $\boldsymbol{Y}_{\mathrm{D}} = (Y_{\mathrm{D}1}, \ldots, Y_{\mathrm{D}J})$, where $Y_{\mathrm{D}j}\ (j = 1, \ldots, J)$ is the equivalent state of unit j. Meanwhile, G corresponding to $\boldsymbol{Y}_{\mathrm{D}}$ is denoted as $G_{\boldsymbol{Y}_{\mathrm{D}}}$. Additionally, the numbers of units selected to be maintained are recorded as $j_1^{\mathrm{m}}, \ldots, j_{J'}^{\mathrm{m}}$ according to the maintenance sequence, and the numbers of units not selected are recorded as $j_1^{\mathrm{nm}}, \ldots, j_{J-J'}^{\mathrm{nm}}$, respectively.

Therefore, the sample space and number of samples can be expressed in the following form at different times, as in (4) and (5), shown at the bottom of the next page, where $Y_{\mathrm{D}j_c^{\mathrm{nm}}} \in \left\{0, \ldots, Y_{j_c^{\mathrm{nm}}}^{\mathrm{A}}\right\}$ $c = 1, \ldots, J - J'$ during the period $t \in (0, T_1+, \ldots, +T_{J'}]$.

Equations (4) and (5) show that whenever a unit is connected to the system after its maintenance, the sample space of the equivalent–state combination increases. Until the last unit for maintenance is connected, the sample space increases to its maximum and remains unchanged.

Because the missions are performed sequentially, based on conditional probability theory, the reliability of a MSS executing multiple consecutive missions under sequential maintenance is as follows:

$$\begin{aligned} R_{\mathrm{MS}} &= P(A) = P\left(\bigcap_{z=1}^{Z} A_z\right) \\ &= P(A_1)\, P(A_2|A_1) \cdots P(A_Z|A_1 \cap \ldots \cap A_{Z-1}). \end{aligned} \tag{6}$$

According to the changes in the sample space of the equivalent–state combination, (6) can be calculated in a recursion manner, which is illustrated as in Fig. 3 and detailed as follows.

Step 1: According to $Y_j(0)\ (j = 1, \ldots, J)$ and maintenance sequence, all the possible values and the corresponding occurrence probabilities of $\boldsymbol{Y}_{\mathrm{D}}$ at the end of the first mission are obtained, based on which the system reliability during the first mission is calculated.

All equivalent–state combinations of the units at the end of the first mission, i.e., all elements in $YH_{\mathrm{D}}(t_{\mathrm{a}1})$, are denoted as $\boldsymbol{Y}_{\mathrm{D}}^{1\,(1)}, \ldots, \boldsymbol{Y}_{\mathrm{D}}^{1(H_{\mathrm{D}}(t_{\mathrm{a}1}))}$, where $\boldsymbol{Y}_{\mathrm{D}}^{1(h)} = \left(Y_{\mathrm{D}1}^{1(h)}, \ldots, Y_{\mathrm{D}J}^{1(h)}\right)$ $(h \in \{1, \ldots, H_{\mathrm{D}}(t_{\mathrm{a}1})\})$. Thus, the probabilities of $\boldsymbol{Y}_{\mathrm{D}}$ being equal to $\boldsymbol{Y}_{\mathrm{D}}^{1(h)}$ $(h = 1, \ldots, H_{\mathrm{D}}(t_{\mathrm{a}1}))$ at the end of the first mission are calculated first as follows:

$$p\left(\boldsymbol{Y}_{\mathrm{D}}(t_{\mathrm{a}1}) = \boldsymbol{Y}_{\mathrm{D}}^{1\,(h)}\right) = \prod_{j=1}^{J} p_{j_{Y_{\mathrm{D}j}^{1(h)}}(1)}. \tag{7}$$

where $p_{j_{Y_{\mathrm{D}j}^{1(h)}}(1)}$ is the probability that the equivalent state of unit j is $Y_{\mathrm{D}j}^{1\,(h)}$ at the end of the first mission and can be calculated as follows:

$$p_{j_{Y_{\mathrm{D}j}^{1(h)}}(1)} = \begin{cases} p_j^{Y_j^{\mathrm{A}}}(t_{\mathrm{a}1})_{Y_{\mathrm{D}j}^{1(h)}}, & Y_j^{\mathrm{A}} = Y_j(0) \\ p_j^{Y_j^{\mathrm{A}}}\left(t_{\mathrm{a}1} - \sum_{j'=1}^{w_j} T_{j'}\right)_{Y_{\mathrm{D}j}^{1(h)}}, & Y_j^{\mathrm{A}} > Y_j(0),\ t_{\mathrm{a}1} > \sum_{j'=1}^{w_j} T_{j'} \\ 1, & Y_j^{\mathrm{A}} > Y_j(0),\ t_{\mathrm{a}1} \le \sum_{j'=1}^{w_j} T_{j'}, \end{cases} \tag{8}$$

where $p_j^{Y_j^{\mathrm{A}}}(t_{\mathrm{a}1})_{Y_{\mathrm{D}j}^{1\,(h)}}$ and $p_j^{Y_j^{\mathrm{A}}}\left(t_{\mathrm{a}1}-\sum_{j'=1}^{w_j}T_{j'}\right)_{Y_{\mathrm{D}j}^{1\,(h)}}$ are the probabilities of unit j being in state $Y_{\mathrm{D}j}^{1\,(h)}$ after the operating time of $t_{\mathrm{a}1}$ and $t_{\mathrm{a}1}-\sum_{j'=1}^{w_j}T_{j'}$, respectively, and Y_j^{A} is the initial state. Because the degenerative process of each unit is frequently modeled as a homogeneous Markov process with continuous time and discrete states [4], the two items can be solved via the Chapman–Kolmogorov equation [36], which is detailed in the Appendix. Meanwhile, for $Y_j^{\mathrm{A}}>Y_j(0)$, $t_{\mathrm{a}1}\leq\sum_{j'=1}^{w_j}T_{j'}$, the equivalent state of unit j in $YH_{\mathrm{D}}(t_{\mathrm{a}1})$ is specific values 0 and Y_j^{A}, respectively, when $t_{\mathrm{a}1}<\sum_{j'=1}^{w_j}T_{j'}$ and $t_{\mathrm{a}1}=\sum_{j'=1}^{w_j}T_{j'}$. Therefore, in this case, $p_{j_{Y_{\mathrm{D}j}^{1(h)}}(1)}$ is equal to 1.

Based on the total probability formula, the system reliability during the first mission is then calculated as follows:

$$P(A_1)=\sum_{h=1}^{H_{\mathrm{D}}(t_{\mathrm{a}1})}p\left(\boldsymbol{Y}_{\mathrm{D}}(t_{\mathrm{a}1})=\boldsymbol{Y}_{\mathrm{D}}^{1(h)}\right)I\left(G_{\boldsymbol{Y}_{\mathrm{D}}^{1(h)}},W_1\right),\tag{9}$$

where $I(x,c)$ is an indicative function of x defined as

$$I(x,c)=\begin{cases}1, & x\geq c\\ 0, & x<c.\end{cases}\tag{10}$$

Even if the system is determined to be reliable during the first mission, $\boldsymbol{Y}_{\mathrm{D}}$ at the beginning of the second mission cannot be determined. However, only the occurrence probabilities of the equivalent–state combinations of all units can be determined and calculated as in (11), shown at the bottom of page 7.

Equation (11) represents the condition that the system is reliable during the first mission. Additionally,

$$YH_{\mathrm{D}}(t)=\begin{cases}\left\{Y_{\mathrm{D}}|Y_{\mathrm{D}j_c^{\mathrm{nm}}}=Y_{j_c^{\mathrm{nm}}}^{\mathrm{A}}c=1,\ldots,J-J';\,Y_{\mathrm{D}j_c^{\mathrm{m}}}=0c=1,\ldots,J'\right\}, & t=0\\
\left\{Y_{\mathrm{D}}|Y_{\mathrm{D}j_1^{\mathrm{m}}}=Y_{j_1^{\mathrm{m}}}^{\mathrm{A}};\,Y_{\mathrm{D}j_2^{\mathrm{m}}},\ldots,Y_{\mathrm{D}j_{J'}^{\mathrm{m}}}=0\right\}, & t=T_1\\
\left\{Y_{\mathrm{D}}|Y_{\mathrm{D}j_c^{\mathrm{m}}}\in\left\{0,\ldots,Y_{j_c^{\mathrm{m}}}^{\mathrm{A}}\right\}c=1,\ldots,C-1;\,Y_{\mathrm{D}j_C^{\mathrm{m}}}=Y_{j_C^{\mathrm{m}}}^{\mathrm{A}};\,Y_{\mathrm{D}j_{C+1}^{\mathrm{m}}},\ldots,Y_{\mathrm{D}j_{J'}^{\mathrm{m}}}=0\right\}, & t=\sum_{j'=1}^{C}T_{j'},\ C=2,\ldots,J'-1\\
\left\{Y_{\mathrm{D}}|Y_{\mathrm{D}j_c^{\mathrm{m}}}\in\left\{0,\ldots,Y_{j_c^{\mathrm{m}}}^{\mathrm{A}}\right\}c=1,\ldots,J'-1;\,Y_{\mathrm{D}j_{J'}^{\mathrm{m}}}=Y_{j_{J'}^{\mathrm{m}}}^{\mathrm{A}}\right\}, & t=\sum_{j'=1}^{J'}T_{j'}\\
\left\{Y_{\mathrm{D}}|Y_{\mathrm{D}j_1^{\mathrm{m}}},\ldots,Y_{\mathrm{D}j_{J'}^{\mathrm{m}}}=0\right\}, & t\in(0,T_1)\\
\left\{Y_{\mathrm{D}}|Y_{\mathrm{D}j_c^{\mathrm{m}}}\in\left\{0,\ldots,Y_{j_c^{\mathrm{m}}}^{\mathrm{A}}\right\}c=1,\ldots,C;\,Y_{\mathrm{D}j_{C+1}^{\mathrm{m}}},\ldots,Y_{\mathrm{D}j_{J'}^{\mathrm{m}}}=0\right\}, & t\in\left(\sum_{j'=1}^{C}T_{j'},\sum_{j'=1}^{C+1}T_{j'}\right),\ C=1,\ldots,J'-1\\
\left\{Y_{\mathrm{D}}|Y_{\mathrm{D}j}\in\left\{0,\ldots,Y_j^{\mathrm{A}}\right\}\right\}=YH_{\mathrm{D}\max}, & t\in\left(\sum_{j'=1}^{J'}T_{j'},\sum_{z=1}^{Z}t_{\mathrm{a}z}\right],
\end{cases}\tag{4}$$

$$H_{\mathrm{D}}(t)=\begin{cases}1, & t=0\\
\prod_{c=1}^{J-J'}\left(Y_{j_c^{\mathrm{nm}}}^{\mathrm{A}}+1\right)\times\underbrace{1\times\ldots\times1}_{J'}, & t=T_1\\
\prod_{c=1}^{J-J'}\left(Y_{j_c^{\mathrm{nm}}}^{\mathrm{A}}+1\right)\times\prod_{c=1}^{C-1}\left(Y_{j_c^{\mathrm{m}}}^{\mathrm{A}}+1\right)\times\underbrace{1\times\ldots\times1}_{J'-C+1}, & t=\sum_{j'=1}^{C}T_{j'},\ C=2,\ldots,J'\\
\prod_{c=1}^{J-J'}\left(Y_{j_c^{\mathrm{nm}}}^{\mathrm{A}}+1\right)\times\underbrace{1\times\ldots\times1}_{J'}, & t\in(0,T_1)\\
\prod_{c=1}^{J-J'}\left(Y_{j_c^{\mathrm{nm}}}^{\mathrm{A}}+1\right)\times\prod_{c=1}^{C}\left(Y_{j_c^{\mathrm{m}}}^{\mathrm{A}}+1\right)\times\underbrace{1\times\ldots\times1}_{J'-C}, & t\in\left(\sum_{j'=1}^{C}T_{j'},\sum_{j'=1}^{C+1}T_{j'}\right),\ C=1,\ldots,J'-1\\
\prod_{j=1}^{J}\left(Y_j^{\mathrm{A}}+1\right)=H_{\mathrm{D}\max}, & t\in\left(\sum_{j'=1}^{J'}T_{j'},\sum_{z=1}^{Z}t_{\mathrm{a}z}\right],
\end{cases}\tag{5}$$

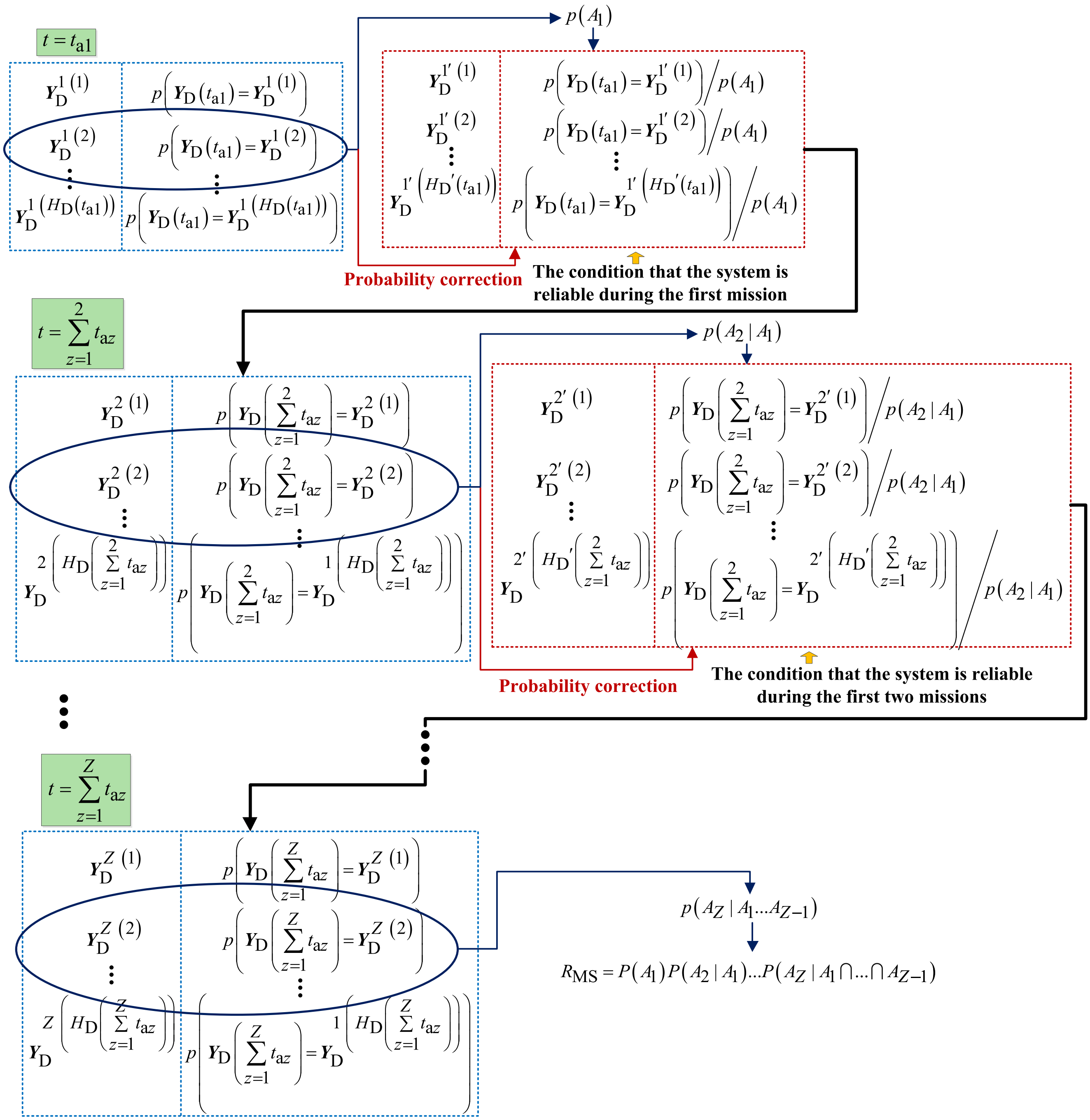

FIGURE 3. Recursion mechanism of the reliability calculation for a MSS executing multiple consecutive missions under sequential maintenance.

the equivalent–state combinations of all units with an occurrence probability greater than zero calculated according to (11) are denoted as $\boldsymbol{Y}_{\mathrm{D}}^{1'(1)}, \ldots, \boldsymbol{Y}_{\mathrm{D}}^{1'(H_{\mathrm{D}}'(t_{\mathrm{a}1}))}$ in sequence.

Step 2: For missions 2 to Z, the recursive calculation is performed as follows.

The equivalent–state combinations of all units at the end of the zth mission, i.e., all elements in $YH_{\mathrm{D}}\,(t_{\mathrm{a}1}+\ldots+t_{\mathrm{a}z})$, are sequentially denoted as $\boldsymbol{Y}_{\mathrm{D}}^{z\,(1)}, \ldots, \boldsymbol{Y}_{\mathrm{D}}^{z(H_{\mathrm{D}}(t_{\mathrm{a}1}+\ldots+t_{\mathrm{a}z}))}$, where $\boldsymbol{Y}_{\mathrm{D}}^{z(h)} = \left(Y_{\mathrm{D}1}^{z(h)}, \ldots, Y_{\mathrm{D}J}^{z(h)}\right)$ $(h \in \{1, \ldots, H_{\mathrm{D}}\,(t_{\mathrm{a}1} + \ldots + t_{\mathrm{a}z})\})$. Thus, under the condition that the system is reliable during the first z-1 missions, the probabilities of $\boldsymbol{Y}_{\mathrm{D}}$ being equal to $\boldsymbol{Y}_{\mathrm{D}}^{z(h)}$ $(h = 1, \ldots, H_{\mathrm{D}}\,(t_{\mathrm{a}1} + \ldots + t_{\mathrm{a}z}))$ at the end of the zth mission is calculated as in (12), shown at the bottom of the next page, where $\boldsymbol{Y}_{\mathrm{D}}^{z-1'(1)}, \ldots, \boldsymbol{Y}_{\mathrm{D}}^{z-1'(H_{\mathrm{D}}'(t_{\mathrm{a}1}+,\ldots,+t_{\mathrm{a}z-1}))}$

are the equivalent–state combinations of all units with an occurrence probability greater than zero at the end of the (z-1)th mission under the condition that the system is reliable during the first z-1 missions, $p_{j_{Y_{Dj}^{z(h)}}}^{Y_{Dj}^{z\text{-}1'(\mathrm{hh})}}(z)$ is the probability that the equivalent states of unit j are $Y_{Dj}^{z-1\,'(\mathrm{hh})}$ and $Y_{Dj}^{z(\mathrm{hh})}$ at the end of the (z-1)th and zth missions, respectively, and are calculated as in (13), shown at the bottom of the page, where $p_j^{Y_{Dj}^{z\text{-}1'(\mathrm{hh})}}(t_{\mathrm{a}z})_{Y_{Dj}^{z(h)}}$ and $p_j^{Y_j^{\mathrm{A}}}\left(\sum_{z'=1}^{z} t_{\mathrm{a}z'}-\sum_{j'=1}^{w_j} T_{j'}\right)_{Y_{Dj}^{z(h)}}$ are the probabilities of unit j being in state $Y_{Dj}^{z\,(h)}$ after the operating time of $t_{\mathrm{a}z}$ and $\sum_{z'=1}^{z} t_{\mathrm{a}z'}-\sum_{j'=1}^{w_j} T_{j'}$ with $Y_{Dj}^{2'(\mathrm{hh})}$ and Y_j^{A} as the initial states, respectively, and they are solved using the Chapman–Kolmogorov equation.

On this basis, the system reliability during the zth mission under the condition that the system is reliable during the first z-1 missions is calculated as in (14), shown at the bottom of the page.

Except for the Zth mission, to calculate the conditional reliability of the system during the (z+1)th mission, the probabilities of $\boldsymbol{Y}_{\mathrm{D}}$ being $\boldsymbol{Y}_{\mathrm{D}}^{z(h)}$ ($h=1,\ldots,H_{\mathrm{D}}\left(t_{\mathrm{a}1}+\ldots+t_{\mathrm{a}z}\right)$) at the beginning of the ($z+1$)th mission under the condition that the system is reliable during missions 1 to Z-1 are calculated as in (15), shown at the bottom of the page.

Equation (15) represents the condition that the system is reliable during the first z missions. It can be seen that $p\left(\boldsymbol{Y}_{\mathrm{D}}\left(\sum_{z'=1}^{z} t_{\mathrm{a}z'}\right)=\boldsymbol{Y}_{\mathrm{D}}^{z(h)}|A_1\ldots A_z\right)$ is corrected with respect to $p\left(\boldsymbol{Y}_{\mathrm{D}}\left(\sum_{z'=1}^{z} t_{\mathrm{a}z'}\right)=\boldsymbol{Y}_{\mathrm{D}}^{z(h)}|A_1\ldots A_{z-1}\right)$, i.e., times the correction coefficient $\frac{I\left(G_{Y_{\mathrm{D}}^{z(h)}},W_z\right)}{P(A_z|A_1\ldots A_{z-1})}$.

$$p\left(\boldsymbol{Y}_{\mathrm{D}}\left(t_{\mathrm{a}1}\right)=\boldsymbol{Y}_{\mathrm{D}}^{1\,(h)}|A_1\right)=\begin{cases}\dfrac{p\left(\boldsymbol{Y}_{\mathrm{D}}\left(t_{\mathrm{a}1}\right)=\boldsymbol{Y}_{\mathrm{D}}^{1(h)}\right)}{P\left(A_1\right)}, & I\left(G_{\boldsymbol{Y}_{\mathrm{D}}^{1(h)}},W_1\right)=1\\ 0, & I\left(G_{\boldsymbol{Y}_{\mathrm{D}}^{1(h)}},W_1\right)=0\end{cases}\quad h=1,\ldots,H_{\mathrm{D}}\left(t_{\mathrm{a}1}\right), \tag{11}$$

$$p\left(\boldsymbol{Y}_{\mathrm{D}}\left(\sum_{z'=1}^{z} t_{\mathrm{a}z'}\right)=\boldsymbol{Y}_{\mathrm{D}}^{z(h)}|A_1\ldots A_{z-1}\right)=\sum_{\mathrm{hh}=1}^{H'_{\mathrm{D}}\left(t_{\mathrm{a}1}+\ldots,+t_{\mathrm{a}z-1}\right)}\left(p\left(\boldsymbol{Y}_{\mathrm{D}}\left(\sum_{z'=1}^{z-1} t_{\mathrm{a}z'}\right)=\boldsymbol{Y}_{\mathrm{D}}^{z\text{-}1'(\mathrm{hh})}|A_1\ldots A_{z-1}\right)\prod_{j=1}^{J} p_{j_{Y_{\mathrm{D}j}^{z(h)}}}^{Y_{\mathrm{D}j}^{z\text{-}1'(\mathrm{hh})}}(z)\right), \tag{12}$$

$$p_{j_{Y_{\mathrm{D}j}^{z(h)}}}^{Y_{\mathrm{D}j}^{z\text{-}1'(\mathrm{hh})}}(z)=\begin{cases}p_j^{Y_{Dj}^{z\text{-}1'(\mathrm{hh})}}\left(t_{\mathrm{a}z}\right)_{Y_{\mathrm{D}j}^{z(h)}}, & Y_j^{\mathrm{A}}=Y_j(0)\text{ or }Y_j^{\mathrm{A}}>Y_j(0),\ \sum_{z'=1}^{z-1} t_{\mathrm{a}z'}\geq\sum_{j'=1}^{w_j} T_{j'}\\ p_j^{Y_j^{\mathrm{A}}}\left(\sum_{z'=1}^{z} t_{\mathrm{a}z'}-\sum_{j'=1}^{w_j} T_{j'}\right)_{Y_{\mathrm{D}j}^{z\,(h)}}, & Y_j^{\mathrm{A}}>Y_j(0),\ \sum_{z'=1}^{z} t_{\mathrm{a}z'}>\sum_{j'=1}^{w_j} T_{j'}>\sum_{z'=1}^{z-1} t_{\mathrm{a}z'}\\ 1, & Y_j^{\mathrm{A}}>Y_j(0),\ \sum_{z'=1}^{z} t_{\mathrm{a}z'}\leq\sum_{j'=1}^{w_j} T_{j'},\end{cases} \tag{13}$$

$$P\left(A_z|A_1\ldots A_{z-1}\right)=\sum_{h=1}^{H_{\mathrm{D}}\left(t_{\mathrm{a}1}+\ldots+t_{\mathrm{a}z}\right)} p\left(Y_{\mathrm{D}}\left(\sum_{z'=1}^{z} t_{\mathrm{a}z'}\right)=Y_{\mathrm{D}}^{z(h)}|A_1\ldots A_{z-1}\right)I\left(G_{Y_{\mathrm{D}}^{z(h)}},W_z\right). \tag{14}$$

$$\begin{aligned}&p\left(\boldsymbol{Y}_{\mathrm{D}}\left(\sum_{z'=1}^{z} t_{\mathrm{a}z'}\right)=\boldsymbol{Y}_{\mathrm{D}}^{z(h)}|A_1\ldots A_z\right)\\ &\quad=\begin{cases}\dfrac{p\left(\boldsymbol{Y}_{\mathrm{D}}\left(\sum_{z'=1}^{z} t_{\mathrm{a}z'}\right)=\boldsymbol{Y}_{\mathrm{D}}^{z(h)}|A_1\ldots A_{z-1}\right)}{P\left(A_z|A_1\ldots A_{z-1}\right)}, & I\left(G_{\boldsymbol{Y}_{\mathrm{D}}^{z\,(h)}},W_z\right)=1\\ 0, & I\left(G_{\boldsymbol{Y}_{\mathrm{D}}^{z(h)}},W_z\right)=0\end{cases}\quad h=1,\ldots,H_{\mathrm{D}}\left(t_{\mathrm{a}1}+\ldots+t_{\mathrm{a}z}\right).\end{aligned} \tag{15}$$

Step 3: $P(A_1), P(A_2|A_1), \ldots, P(A_Z|A_1 \cap \ldots \cap A_{Z-1})$ are multiplied to obtain R_{MS} according to (6).

Additionally, the pseudocode of the reliability calculation method is provided in the Appendix.

IV. SELECTIVE MAINTENANCE STRATEGY OPTIMIZATION PROBLEM

In this paper, the total cost C_{S} consists of the maintenance cost C_{MS} and operating cost C_{LS}. Its expected value is calculated as follows:

$$E(C_{\mathrm{S}}) = C_{\mathrm{MS}} + E(C_{\mathrm{LS}}) = \sum_{w=1}^{J}\sum_{j=1}^{J}\sum_{Y_j(0)+1}^{K_j} c^{j}_{Y_j(0)l} H_{w,j}\left(Y_j(0), l\right) + \sum_{i=1}^{J} V_j, \tag{16}$$

where $c^{j}_{Y_j(0)l}$ is the maintenance cost of unit j from state $Y_j(0)$ to l, and $H_{w,j}\left(Y_j(0), l\right)$ is a binary decision variable defined as follows:

$$H_{w,j}\left(Y_j(0), j\right) = \begin{cases} 1, & \text{the } w\text{th maintenance activity is restoring unit } j \text{ from state } Y_j(0) \text{ to } l \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

V_j is the expected operating cost of unit j during consecutive missions and is expressed as follows:

$$V_j = \begin{cases} v_j^{Y_j^{\mathrm{A}}}\left(\sum_{z=1}^{Z} t_{\mathrm{a}z}\right), & Y_j^{\mathrm{A}} = Y_j(0) \\ v_j^{Y_j^{\mathrm{A}}}\left(\sum_{z=1}^{Z} t_{\mathrm{a}z} - \sum_{j'=1}^{w_j} T_{j'}\right), & Y_j^{\mathrm{A}} > Y_j(0), \end{cases} \tag{18}$$

where $v_j^{Y_j^{\mathrm{A}}}\left(\sum_{z=1}^{Z} t_{\mathrm{a}z}\right)$ and $v_j^{Y_j^{\mathrm{A}}}\left(\sum_{z=1}^{Z} t_{\mathrm{a}z} - \sum_{j'=1}^{w_j} T_{j'}\right)$ are the expected operating cost for unit j after the operating time of $\sum_{z=1}^{Z} t_{\mathrm{a}z}$ and $\sum_{z=1}^{Z} t_{\mathrm{a}z} - \sum_{j'=1}^{w_j} T_{j'}$, respectively, and Y_j^{A} is the initial state. They can be calculated by modeling the operating cost of unit j as a Markov reward process, as detailed in the Appendix.

For multiple consecutive missions, the system can complete a certain amount of work during each mission. Typically, the amount of work completed within a period is the accumulation of system efficiency during this period [34]. Thus, the expected amount of work completed during the zth mission ($z = 1, \ldots, Z$) is expressed as

$$E(O_{\mathrm{S}z}) = \begin{cases} \int_0^{t_{\mathrm{a}1}} \sum_{q=0}^{Q} P(t)_q G_q dt, & z = 1 \\ \int_{\sum_{z'=1}^{z-1} t_{\mathrm{a}z'}}^{\sum_{z'=1}^{z} t_{\mathrm{a}z'}} \sum_{q=0}^{Q} P(t)_q G_q dt, & z = 2, \ldots, Z, \end{cases} \tag{19}$$

where $\sum_{q=0}^{Q} P(t)_q G_q$ is the expectation function of the system working efficiency.

Additionally, considering the sequential maintenance, the maintenance time is calculated as follows:

$$T_{\mathrm{MS}} = \sum_{j'=1}^{J'} T_{j'} = \sum_{w=1}^{J}\sum_{j=1}^{J}\sum_{Y_j(0)+1}^{K_j} H_{w,j}\left(Y_j(0), l\right) t^{j}_{Y_j(0)l}, \tag{20}$$

where $t^{j}_{Y_j(0)l}$ is the maintenance cost of unit j from state $Y_j(0)$ to l.

Based on this, the selective maintenance model investigated in this paper can be expressed as follows:

$$\text{Max } R_{\mathrm{MS}}, \tag{21}$$

$$s.t. \begin{cases} E(C_{\mathrm{S}}) \le C' & \text{(a)} \\ T_{\mathrm{MS}} \le \sum_{z=1}^{Z} t_{\mathrm{a}z} & \text{(b)} \\ E(O_{\mathrm{S}z}) \ge O'_z \; (z = 1, \ldots, Z) & \text{(c)} \\ \sum_{w=1}^{J}\sum_{j=Y_j(0)+1}^{K_j} H_{w,j}\left(Y_j(0), l\right) \le 1 \; (j = 1, \ldots, J) & \text{(d)} \\ \sum_{j=1}^{J}\sum_{j=Y_j(0)+1}^{K_j} H_{w,j}\left(Y_j(0), l\right) \le 1 \; (w = 1, \ldots, J) & \text{(e)} \\ H_{w,j}\left(Y_j(0), l\right) = 1 \text{or } 0 \; (w, j = 1, \ldots, J) & \text{(f)}, \end{cases} \tag{22}$$

where C' is the budget, and O'_z is the minimum workload requirement for the expected amount of work completed during the zth mission. This is similar to the case of the single mission studied in [34]. However, the difference is that the reliability calculation method is an innovative recursive operation based on the changes in the sample space of the equivalent–state combination and conditional probability, rather than using UGF method which is only suitable for the case of the single mission. Additionally, the maintenance strategy optimization problem can be addressed by extending the customized ACO algorithm proposed in [34]. The extensions are described as follows:

Because the objective is to maximize R_{MS}, the effectiveness of the next node is quantified by the increment in R_{MS}, which is expressed as:

$$R_{\mathrm{MS}(\varepsilon_a,\varepsilon_b)} - R_{\mathrm{MS}(\varepsilon_a)}, \tag{23}$$

The 11th row is changed as follows:

$C' \leftarrow C' - c(\varepsilon_{c1})$ and $T' \leftarrow \sum_{z=1}^{Z} t_{az} - t(\varepsilon_{c1})$;

The 25~34th rows are changed as follows:

For np=1 to NP **do**

  The $E(C_S)$ and $E(O_{Sz})(z=1,...,Z)$ corresponding to the path passed by the npth ant;

  If $E(C_S) \geq C' \cap E(O_{S1}) \geq O_1' \cap ... \cap E(O_{SZ}) \geq O_Z'$ **Then**

    Calculate the R_{MS} corresponding to the path passed by the npth ant using the proposed reliability calculation method;

  Else

    Set the R_{MS} corresponding to the path passed by the npth ant as 0;

  End if

  Calculate $\Delta\tau^{np}(\varepsilon_a, \varepsilon_b)_{ite}(\varepsilon_a, \varepsilon_b = 1,..., BY; \varepsilon_a \neq \varepsilon_b)$;

End for

Calculate $\tau(\varepsilon_a, \varepsilon_b)_{ite+1}(\varepsilon_a, \varepsilon_b = 1,..., BY; \varepsilon_a \neq \varepsilon_b)$;

$R_{MSite} \leftarrow$ the maximum value of R_{MS} corresponding to the paths passed by all the NP ants;

$R_{MSmax} \leftarrow \max\{R_{MS1},..., R_{MSite}\}$;

FIGURE 4. Differences of the pseudocode of the customized ACO algorithm for multiple consecutive missions under sequential maintenance relative to the single mission. R_{MSite} is the maximum value of the R_{MS} in the iteth generation.

where $R_{MS(\varepsilon_a)}$ is the R_{MS} corresponding to a maintenance sequence ending with ε_a, and $R_{MS(\varepsilon_a,\varepsilon_b)}$ is the R_{MS} corresponding to the above maintenance sequence in which ε_b is added after ε_a. Thus, an ant is assumed to be currently at node ε_a. If a node ε_b satisfies $R_{MS(\varepsilon_a,\varepsilon_b)} - R_{MS(\varepsilon_a)} > 0$, then $\forall \varepsilon \notin$ **Tabu**,

$$\eta(\varepsilon_a, \varepsilon) = \begin{cases} R_{MS(\varepsilon_a,\varepsilon)} - R_{MS(\varepsilon_a)}, & R_{MS(\varepsilon_a,\varepsilon)} - R_{MS(\varepsilon_a)} > 0 \\ 0, & \text{otherwise}, \end{cases} \tag{24}$$

otherwise, $\eta(\varepsilon_a, \varepsilon)$ are set as identical constants. Further, the quantity of pheromones left between ε_a and ε_b by the npth ant in the iteth iteration is defined as

$$\Delta\tau^{np}(\varepsilon_a, \varepsilon_b)_{ite} = \begin{cases} \omega R_{MS}^{(np,ite)}, & \text{the path from } \varepsilon_a \text{ to } \varepsilon_b \text{ is included in the path passed by the npth ant in the iteth iteration} \\ 0, & \text{otherwise}, \end{cases} \tag{25}$$

where ω is a positive constant, and $R_{MS}^{(np,ite)}$ is the R_{MS} corresponding to the maintenance sequence represented by the path passed by the npth ant in the iteth iteration.

The pseudocode of the customized ACO algorithm for the selective maintenance problem studied in this paper is similar to that of a single mission shown in Fig. 5 of [34], and its differences are shown in Fig. 4.

V. EXAMPLE ANALYSES

With its structure shown in Fig. 5, an oil transportation system is used as an example to verify the accuracy and computational effectiveness of the proposed reliability calculation method and perform the selective maintenance strategy optimization analysis. The oil transportation system is designed to transport oil from an oil tanker to carriages. Its working process is identical to that of the oil transportation system in [34], except that it performs three consecutive oil transportation missions during a predetermined period selected for sequential maintenance. The requirements for oil transportation volumes for different missions vary. The pipeline parameters are listed in Tables 1-3. The cost unit is USD and the time unit is one week.

Additionally, the initial states of the pipelines and the number of optional maintenance activities are listed in Table 4.

A. VERIFICATION OF THE RELIABILITY CALCULATION METHOD

It is assumed that the durations of the oil transportation missions, i.e., t_{a1}, t_{a2}, t_{a3}, are 1.2, 0.9, and 3 weeks, respectively. The requirements for oil transportation efficiency at the end of the missions, i.e., W_1, W_2, W_3, are 45, 60, and 55 kilotons per week, respectively. Maintenance activities 4, 2, and 7 are performed sequentially. Based on this, the reliability calculation method is verified by comparison with the Monte–Carlo simulation method.

With N representing the number of simulations, ten Monte–Carlo experiments are conducted for the cases of $N = 100$, $N = 1000$, $N = 10000$, and $N = 20000$, respectively. Taking the case of $N = 100$ as an example, one experiment, that is, 100 Monte–Carlo simulations, is firstly conducted. The mean values of 100 simulation results of $P(A_1)$, $P(A_2|A_1)$, $P(A_3|A_1A_2)$, and R_{MS} are calculated, respectively. Subsequently, the same experiment was

TABLE 1. Basic pipeline parameters in the oil transportation system.

j	K_j	g_{j0}	g_{j1}	g_{j2}	g_{j3}	λ_{10}^j	λ_{20}^j	λ_{21}^j	λ_{30}^j	λ_{31}^j	λ_{32}^j
1	3	0	20	40	60	0.008	0.006	0.008	0.002	0.004	0.006
2	3	0	30	50	65	0.012	0.008	0.01	0.004	0.007	0.013
3	2	0	25	40	-	0.008	0.005	0.007	-	-	-
4	2	0	20	40	-	0.007	0.004	0.01	-	-	-
5	2	0	15	45	-	0.012	0.004	0.009	-	-	-
6	3	0	10	35	60	0.007	0.004	0.009	0.001	0.006	0.01
7	3	0	20	45	60	0.013	0.01	0.012	0.005	0.008	0.011

λ_{ab}^j represents the transition rate of pipeline j from state a to b.

TABLE 2. Maintenance parameters of pipelines in the oil transportation system.

j	c_{01}^j	c_{02}^j	c_{03}^j	c_{12}^j	c_{13}^j	c_{23}^j	t_{01}^j	t_{02}^j	t_{03}^j	t_{12}^j	t_{13}^j	t_{23}^j
1	250	350	500	300	500	500	0.5	0.75	0.9	0.4	0.9	0.9
2	200	350	450	150	450	450	0.4	0.6	1	0.3	1	1
3	350	400	-	400	-	-	1	1.25	-	1.25	-	-
4	200	350	-	350	-	-	0.75	1.15	-	1.15	-	-
5	200	450	-	450	-	-	0.9	1	-	1	-	-
6	200	300	400	250	400	400	0.4	0.9	1	0.75	1	1
7	250	400	600	350	600	600	0.4	0.7	0.85	0.6	0.85	0.85

TABLE 3. Operating cost parameters of pipelines in the oil transportation system.

j	r_{10}^j	r_1^j	r_{20}^j	r_{21}^j	r_2^j	r_{30}^j	r_{31}^j	r_{32}^j	r_3^j
1	50	50	80	60	80	120	80	40	100
2	60	70	90	50	90	120	100	30	110
3	20	35	35	25	45	-	-	-	-
4	30	40	75	40	80	-	-	-	-
5	40	55	70	40	90	-	-	-	-
6	30	80	65	40	100	85	60	45	120
7	30	30	40	20	40	50	45	30	50

r_{ab}^j represents the depreciation cost of pipeline j from state a to b, and r_a^j represents the cost per unit time required to drive oil transportation when pipeline j is in state a.

TABLE 4. Operating cost parameters of pipelines in the oil transportation system.

j	$Y_j(0)$	Optional maintenance activities——Number
1	1	$1 \to 2$——1 $1 \to 3$——2
2	3	-
3	0	$0 \to 1$——3 $0 \to 2$——4
4	2	-
5	1	$1 \to 2$——5
6	3	-
7	1	$1 \to 2$——6 $1 \to 3$——7

$a \to b$ represents the maintenance activity restoring the pipeline from state a to b.

repeated nine more times. Additionally, the mean value and standard deviation of the run time of the ten simulations are calculated.

$P(A_1)$, $P(A_2|A_1)$, $P(A_3|A_1A_2)$, and R_{MS} are also calculated using the proposed calculation method, which is also executed ten times to obtain the mean value and standard deviation of the run time of calculation method. The comparison results are shown in Figs. 6 and 7.

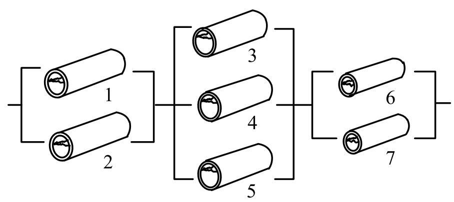

FIGURE 5. Schematic diagram of the series–parallel structure for the oil transportation system.

As shown in Fig. 6, with an increase in N, the simulation results gradually stabilize to the calculation results, indicating that the proposed reliability calculation method is accurate. It can be observed from Figs. 6 and 7 that when N is 100, although the simulation time is shorter than the calculation time, the simulation results have significant volatility, indicating that the current number of simulations is not enough. When N reaches 1000, the volatility of the simulation results is still obvious, that is, the simulation results have not yet reached an acceptable agreement with the calculation results; however, the simulation time is longer than the calculation time. When N is 10000, there is still some volatility in the simulation results. When N reaches 20000, the simulation results are almost consistent with the calculation results and relatively stable, but the simulation time is significantly longer than the calculation time. Additionally, the standard deviation of the calculation method is significantly smaller than that of the simulation method. The above analysis results indicate that the calculation method is more computationally efficient than the Monte–Carlo simulation method. In particular, the maintenance strategy optimization process requires

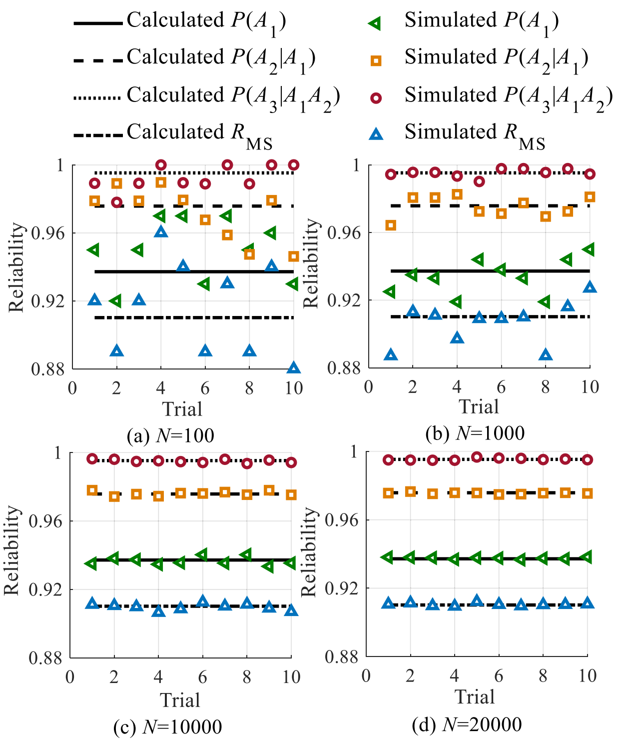

FIGURE 6. Results of solving the reliability using the calculation and Monte–Carlo simulation methods.

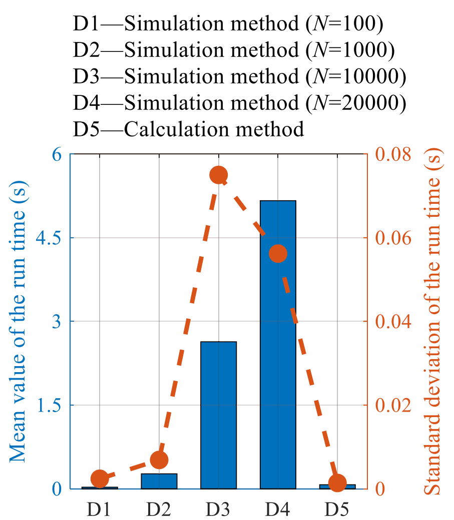

FIGURE 7. Mean value and standard deviation of the run time for solving the reliability by the calculation and Monte–Carlo methods.

numerous reliability calculations. Therefore, for maintenance strategy optimization, the advantage of the proposed reliability calculation method is more significant.

These analysis results indicate that the proposed reliability calculation method is accurate and computationally efficient.

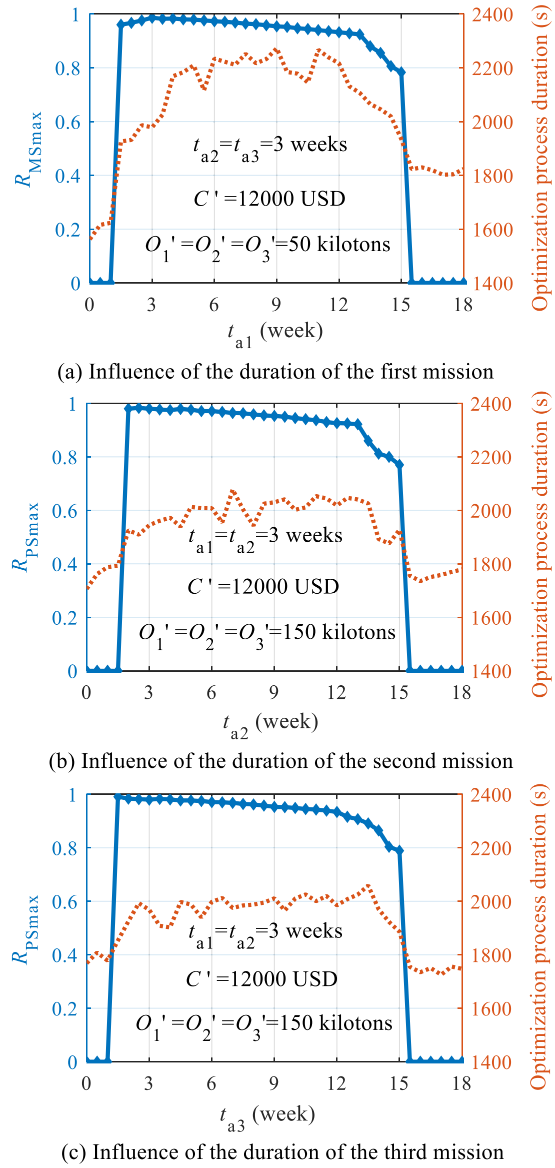

FIGURE 8. Influence of the mission duration on the selective maintenance strategy optimization results.

B. OPTIMIZATION ANALYSIS OF THE SELECTIVE MAINTENANCE STRATEGY

It is assumed that the requirements for oil transportation efficiency at the end of the missions, i.e., W_1, W_2, W_3, are 45, 50, and 45 kilotons per week, respectively. Based on this, the influence of the mission duration on the optimization results of the selective maintenance strategy are analyzed, and the results are shown in Fig. 8. As shown in Fig. 8, when the duration of a mission is relatively short, R_{MSmax} is 0, and the optimization process duration is also significantly short. This is because a short mission duration results in no maintenance sequence satisfying the requirements of the oil transportation volume. As the mission duration increases, R_{MSmax} suddenly increases to an extremely large value when the mission duration reaches a particular value, and the optimization process duration also significantly increases, indicating that feasible maintenance sequences begin to appear. Subsequently, R_{MSmax} fluctuates as follows: In Fig. 8a, R_{MSmax} first increases significantly, then decreases slightly, and then increases slightly; in Fig. 8b, R_{MSmax} first increases slightly, then decreases slightly, and then increases slightly; in Fig. 8c, R_{MSmax} first decreases slightly and then increases slightly. Thus, the duration of a mission has a dual impact on R_{MSmax}: the increase in mission duration provides maintenance opportunities for more pipelines, thereby increasing R_{MSmax}; it also increases the working time of the system, thereby causing more severe system degradation, with a reduced impact on R_{MSmax}. As the mission duration increases further, R_{MSmax} gradually decreases, indicating that the second impact is gradually dominant. When the mission duration increases to a certain extent and exceeds a particular value, R_{MSmax} becomes 0 and no longer changes, and the optimization process duration significantly decreases, indicating no feasible maintenance sequence. This is because the mission duration is relatively long, resulting in no maintenance sequence satisfying the budget requirement.

In summary, the proposed selective maintenance model and customized ACO algorithm are successfully applied to the oil transportation system, which indicates the effectiveness of them. Moreover, mission duration should be set within a reasonable range; otherwise, the workload requirements or the cost budget cannot be satisfied. Under the premise that it is reasonable, when it is in the first 70% of the reasonable range, extending it to a certain extent will not result in a significant decrease in R_{MSmax}, and occasionally even an increase in R_{MSmax}. Therefore, this extension is feasible. If it is within the back 30% of the reasonable range, extending it to the same extent would result in a significant decrease in R_{MSmax}, which is not feasible.

VI. CONCLUSION AND FUTURE RESEARCH

A reliability calculation method is developed for a MSS executing multiple consecutive missions under sequential maintenance. Based on this, the corresponding selective maintenance model is proposed, and the ACO algorithm is customized to address the corresponding maintenance strategy optimization problem. As demonstrated in the illustrative example, the proposed reliability calculation method is accurate and computationally effective, and the selective maintenance model and customized optimization method have engineering effectiveness. The optimization results show that mission duration should be set within a reasonable range, and under the premise that the mission duration is reasonably set, whether extending it is reasonable depends on its current value. These results can provide guidance for the reasonable adjustment of mission durations in practical engineering.

The proposed selective maintenance model and the customized optimization method can provide theoretical support for the selective maintenance of a MSS executing multiple consecutive missions under sequential maintenance. This research is helpful for enterprises to allocate resources scientifically and improve the quality of mission completion. It provides engineers and researchers with insights for effectively managing industrial systems. This research can be applied to large-scale complex equipment in industrial production, transportation and other fields. Typically, the equipment in these fields needs to work continuously for a long time. To ensure the smooth progress of the missions, maintenance had to be carried out during this period.

Nevertheless, there are limitations in this work as follows:

(1) The proposed selective maintenance model and the customized optimization method are illustrated to have a theoretical support role. In the future, the proposed model and optimization method will be applied to other oil transportation systems and engineering systems in other areas to further illustrate their engineering effectiveness.

(2) The proposed reliability calculation method and corresponding selective maintenance apply only to one MSS. A type of MSS combination characterized by performance sharing is practical [11], [37], [38]. Therefore, in the future, the proposed reliability calculation method for a MSS executing multiple consecutive missions under sequential maintenance and the corresponding selective maintenance will be developed for MSSs in which performance sharing exists.

APPENDIX

A. SOLUTION OF THE STATE TRANSITION PROBABILITY FOR A UNIT

It is assumed that the stochastic degenerative behavior of unit j $(j \in \{1, \ldots, J\})$ follows a homogeneous Markov process with continuous time and discrete states. Its transition intensity matrix is expressed as follows:

$$\boldsymbol{E}_j = \begin{bmatrix} \lambda_{00}^j & \lambda_{01}^j & \cdots & \lambda_{0K_j}^j \\ \lambda_{10}^j & \lambda_{11}^j & \cdots & \lambda_{1K_j}^j \\ \cdots & \cdots & \cdots & \cdots \\ \lambda_{K_j0}^j & \lambda_{K_j1}^j & \cdots & \lambda_{K_jK_j}^j \end{bmatrix}, \tag{26}$$

where $\lambda_{ab}^j \left(a, b = 0, 1, \ldots, K_j\right)$ represents the transition rate of unit j from state a to b. The state of a unit does not increase during degradation; thus, $\lambda_{ab}^j = 0\,(a < b)$. Additionally, based on the properties of the homogeneous Markov process, it can be known that $\lambda_{aa}^j = -\left(\lambda_{a0}^j + \ldots + \lambda_{aa-1}^j\right) (a = 1, \ldots, K_j)$.

With the initial state d $\left(d \in \{0, 1, \ldots, K_j\}\right)$ and operating time t', the state probabilities of unit j are denoted as the vector $\boldsymbol{p}_j^d(t') = \left(p_j^d(t')_0, p_j^d(t')_1, \ldots, p_j^d(t')_{K_j}\right)^{\mathrm{T}}$, which can be obtained by solving the following equations:

$$\begin{cases} \dfrac{d\boldsymbol{p}_j^d(t')}{dt'} = \boldsymbol{E}^T \times \boldsymbol{p}_j^d(t') \\ p^d(0)_d = 1, p^d(0)_u = 0 \ (u \neq d), \end{cases} \tag{27}$$

1. Obtain $T_1,\ldots,T_{J'}$ and vector $\boldsymbol{Y}^{\mathrm{A}}=\left[Y_1^{\mathrm{A}},\ldots,Y_J^{\mathrm{A}}\right]$ according to the maintenance sequence;
2. $H_{\mathrm{D}\max}\leftarrow\prod_{j=1}^{J}\left(\boldsymbol{Y}^{\mathrm{A}}[j]+1\right)$, solve the matrix $\boldsymbol{YH}$ and calculate $G_{\boldsymbol{YH}(h)}\left(h=1,\ldots,H_{\mathrm{D}\max}\right)$;
3. In $\boldsymbol{YH}$, find the rows where the equivalent states of the units not connected to the system at the end of the zth mission are 0 and the equivalent states of the units that happen to be connected to the system at that moment are the post-maintenance states, and form these rows into a new matrix denoted as $\boldsymbol{YH}_z\left(z=1,\ldots,Z\right)$;
4. $\boldsymbol{pYH}_{z1}\leftarrow$ a $l(\boldsymbol{YH}_z)$-dimensional vector whose elements are all 1 $\left(z=1,\ldots,Z\right)$;
5. **For** $j\leftarrow 1$ to J **do**
6. **If** $\boldsymbol{Y}^{\mathrm{A}}[i]>Y_j(0)$ **then**
7. **If** $t_{\mathrm{a}1}>\sum_{j'=1}^{w_j}T_{j'}$ **then**
8. $\boldsymbol{pYH}_{11}[h]\leftarrow\boldsymbol{pYH}_{11}[h]\boldsymbol{p}_j\left(t_{\mathrm{a}1}-\sum_{j'=1}^{w_j}T_{j'}\right)\left[\boldsymbol{Y}^{\mathrm{A}}[j]+1,\boldsymbol{YH}_1[h,j]+1\right]$ $\left(h=1,\ldots,\mathrm{le}(\boldsymbol{YH}_1)\right)$;
9. **End if**
10. **Else**
11. $\boldsymbol{pYH}_{11}[h]\leftarrow\boldsymbol{pYH}_{11}[h]\boldsymbol{p}_j\left(t_{\mathrm{a}1}\right)\left[\boldsymbol{Y}^{\mathrm{A}}[j]+1,\boldsymbol{YH}_1[h,j]+1\right]$ $\left(h=1,\ldots,\mathrm{le}(\boldsymbol{YH}_1)\right)$;
12. **End if**
13. **End for**
14. $P\left(A_1\right)\leftarrow\sum_{h=1}^{\mathrm{le}(\boldsymbol{YH}_1)}\boldsymbol{pYH}_{11}[h]I\left(G_{\boldsymbol{YH}_1(h)},W_1\right)$;
15. Divide the items greater than 0 in $\boldsymbol{pYH}_{11}$ by $P\left(A_1\right)$ and list them as a new vector $\boldsymbol{pYH}_{12}$, and list the corresponding rows in $\boldsymbol{YH}_1$ as a new matrix $\boldsymbol{YH}_{12}$;
16. **For** $z\leftarrow 2$ to Z **do**
17. **For** $h\leftarrow 1$ to $\mathrm{le}(\boldsymbol{YH}_z)$ **do**
18. $\boldsymbol{L}\leftarrow\boldsymbol{pYH}_{z-1\,2}$;
19. **For** $j\leftarrow 1$ to J **do**
20. **If** $\boldsymbol{Y}^{\mathrm{A}}[j]==Y_j(0)$ or $\boldsymbol{Y}^{\mathrm{A}}[j]>Y_j(0)\cap\sum_{z'=1}^{z\text{-}1}t_{\mathrm{a}z'}\geq\sum_{j'=1}^{w_j}T_{j'}$ **then**
21. $\boldsymbol{L}[h']\leftarrow\boldsymbol{L}[h']\boldsymbol{p}_j\left(t_{\mathrm{a}2}\right)\left[\boldsymbol{YH}_{z-1\,2}[h',j]+1,\boldsymbol{YH}_z[h,j]+1\right]$ $\left(h'=1,\ldots,\mathrm{le}(\boldsymbol{L})\right)$;
22. **Else if** $\sum_{z'=1}^{z}t_{\mathrm{a}z'}>\sum_{j'=1}^{w_j}T_{j'}>\sum_{z'=1}^{z\text{-}1}t_{\mathrm{a}z'}$
23. $\boldsymbol{L}[h']\leftarrow\boldsymbol{L}[h']\boldsymbol{p}_j\left(\sum_{z'=1}^{z}t_{\mathrm{a}z'}-\sum_{j'=1}^{w_j}T_{j'}\right)\left[\boldsymbol{YH}_{z-1\,2}[h',j]+1,\boldsymbol{YH}_z[h,j]+1\right]$ $\left(h'=1,\ldots,\mathrm{le}(\boldsymbol{L})\right)$;
24. **End if**
25. **End for**
26. $\boldsymbol{pYH}_{z1}[h]\leftarrow\sum_{h'=1}^{\mathrm{le}(\boldsymbol{L})}\boldsymbol{L}(h')$;
27. **End for**
28. $P\left(A_z|A_1\ldots A_{z-1}\right)\leftarrow\sum_{h=1}^{\mathrm{le}(\boldsymbol{YH}_z)}\boldsymbol{pYH}_{z1}[h]I\left(G_{\boldsymbol{YH}_z(h)},W_z\right)$;
29. **If** $z<Z$ **then**
30. Divide the items greater than 0 in $\boldsymbol{pYH}_{z1}$ by $P\left(A_z|A_1\ldots A_{z-1}\right)$ and list them as a new vector $\boldsymbol{pYH}_{z2}$, and list the corresponding rows in $\boldsymbol{YH}_z$ as a new matrix $\boldsymbol{YH}_{z2}$;
31. **End if**
32. **End for**
33. $R_{\mathrm{MS}}\leftarrow P\left(A_1\right)\ldots P\left(A_Z|A_1\ldots A_{Z-1}\right)$.

FIGURE 9. Pseudocode of the reliability calculation method for a MSS executing multiple consecutive missions under sequential maintenance.

where $p_j^d(t')_u$ $(u = 0, 1, \ldots, K_j)$ represents the occurrence probabilities of state u with initial state d and operating duration t' for unit j. Equation (27) is the Chapman–Kolmogorov equation.

B. SOLUTION OF THE OPERATING COST FOR A UNIT

Generally, when a unit operates in a specific state, it requires a certain amount of cost consumption per unit of time to support its operation. Additionally, the unit would depreciate over time. The greater the degree of depreciation, the higher the depreciation cost. Considering the Markov nature of unit degradation in this paper, the unit operating cost can be established as a Markov reward model. For unit j $(j \in \{1, \ldots, J\})$, the reward value matrix is expressed as follows:

$$\boldsymbol{R}_j = \begin{bmatrix} r_0^j & r_{01}^j & \cdots & r_{0K}^j \\ r_{10}^j & r_1^j & \cdots & r_{1K}^j \\ \cdots & \cdots & \cdots & \cdots \\ r_{K0}^j & r_{K1}^j & \cdots & r_K^j \end{bmatrix}, \tag{28}$$

where r_a^j $(a = 0, \ldots, K_j)$ represents the cost per unit time required to support the operation of unit j in state a, and r_{ab}^j $(a, b = 0, 1, \ldots, K_j)$ represents the depreciation cost for transitioning unit j from state a to b. A unit does not incur any cost to ensure its operation when it is in a complete failure state; thus, $r_0^j = 0$. Additionally, no state improvement would occur during the degradation process of a unit; thus, r_{ab} $(a, b = 0, \ldots, K, a < b)$ is not involved in this paper and is recommended to be set to 0.

With the initial state d $\left(d \in \{0, 1, \ldots, K_j\}\right)$ and operating time t', the expected cumulative operating cost of unit j is denoted as $v_j^d(t')$, and can be obtained by solving the following equations:

$$\begin{cases} \dfrac{\mathrm{d}\boldsymbol{v}_j(t')}{\mathrm{d}t'} = \boldsymbol{u}_j + \boldsymbol{E}_j \times \boldsymbol{v}_j(t') \\ v_j^d(0) = 0 \left(d = 0, \ldots, K_j\right), \end{cases} \tag{29}$$

where vector $\boldsymbol{v}_j(t') = \left(v_j^0(t'), v_j^1(t'), \ldots, v_j^{K_j}(t')\right)^{\mathrm{T}}$ and $\boldsymbol{u}_j = \left(u_0^j, u_1^j, \ldots, u_{K_j}^j\right)^{\mathrm{T}}$, in which $u_a^j = r_a^j + \sum_{b=0, b\neq a}^{K_j} \lambda_{ab}^j r_{ab}^j$ $(a = 0, 1, \ldots, K_j)$.

C. PSEUDOCODE OF THE RELIABILITY CALCULATION METHOD FOR A MSS EXECUTING MULTIPLE CONSECUTIVE MISSIONS UNDER SEQUENTIAL MAINTENANCE

The pseudocode for the reliability calculation method for a MSS executing multiple consecutive missions under sequential maintenance is shown in Fig. 9.

In Fig. 9, $\boldsymbol{p}_j(a)$ is the transition rate matrix of unit j passing through the duration of a; $\boldsymbol{e}[u]$ is the uth element of vector $\boldsymbol{e}$; $\boldsymbol{e}[u_1, u_2]$ is the u_2th element in the u_1th row of matrix $\boldsymbol{e}$; $G_{\boldsymbol{e}(u)}$ represents the value of G corresponding to the uth row of matrix $\boldsymbol{e}$; $l\mathrm{e}(\boldsymbol{e})$ represents the dimension of vector $\boldsymbol{e}$; and matrix $\boldsymbol{YH}$ represents $YH_{\mathrm{D\,max}}$ with each row being a combination of elements taken from $\left\{0, \ldots, Y_j^{\mathrm{A}}\right\}$ $(j = 1, \ldots, J)$, respectively.

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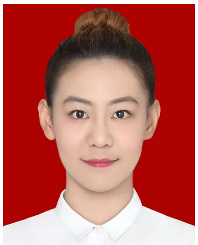

YAO SUN was born in Liaoning, China, in 1993. She received the M.S. degree in mechanical engineering and the Ph.D. degree in mechanical design and theory from Northeastern University, Shenyang, China, in 2018 and 2023, respectively.

She is currently a Postdoctoral Researcher with the School of Mechanical Engineering, Shenyang University of Technology, Shenyang. Her research interests include motion reliability analysis, system reliability evaluation, and maintenance decision.

JIE ZHOU received the B.Eng. degree in mechanical engineering from the Jiangxi University of Science and Technology, Jiangxi, China, in 2015, the M.S. degree in mechanical design and theory from Northeastern University, Shenyang, China, in 2018, and the Ph.D. degree in biomedical engineering from Sun Yat-sen University, Guangzhou, China, in 2022. He studied with the Laboratory of Space Automation Technology, Shenyang Institute of Automation, Chinese Academy of Sciences, from 2016 to 2017. He is currently a Postdoctoral Researcher with the School of Mechanical Engineering, Shenyang University of Technology, China. His research interests include physical human–robot interaction, musculoskeletal modeling, rehabilitation robots, and mechanism reliability.

ZHILI SUN was born in Shandong, China, in 1957. He received the B.S., M.S., and Ph.D. degrees in mechanical design and theory from Northeastern University, Shenyang, China, in 1982, 1985, and 1997, respectively.

He is currently a Professor with the School of Mechanical Engineering and Automation, Northeastern University. His research interests include mechanical reliability engineering, tribology design, structural design, and fault diagnosis.

ZHE WEI received the B.S. degree in industrial design from Northeastern University, Shenyang, China, in 2004, and the M.S. and Ph.D. degrees in mechanical design and theory from Zhejiang University, Hangzhou, China, in 2009.

From 2009 to 2014, he was the Dean of the Research Institute and a Chief Process Information Officer with Sany Heavy Equipment Company Ltd. From 2014 to 2018, he was an Associate Professor with the State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University. He is currently a Professor with the School of Mechanical Engineering, Shenyang University of Technology, Shenyang, and the Director of the Key Laboratory of Intelligent Manufacturing and Industrial Robotics in Liaoning Province. His current research interests include human–robot interaction, virtual reality, digital twin, and intelligent design and manufacturing.