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RESEARCH ARTICLE

Design of Disturbance Rejection-Based Quantized Resilient Control for Fuzzy Chaotic Systems

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ABSTRACT In this paper, the design of a disturbance rejection-based fuzzy quantized resilient control for T-S fuzzy chaotic systems with randomly occurring parameter uncertainties and external disturbances is discussed. Specifically, a disturbance rejection technique is adopted in order to estimate the external disturbances with high precision that are impacting the system. Subsequently, by including a logarithmic quantizer into the feedback control, the control signals are quantized. From thereon, the estimated disturbance is fed into the quantized control path to counteract the impacts of disturbances on the system with an intent to achieve intended performance of the system. Moreover, through the utilisation of Lyapunov's stability theory, adequate criteria affirming the stabilization of the undertaken system are procured in the form of linear matrix inequalities. On the basis of specified criteria, the design procedure of controller and observer gain matrices are presented. In the end, numerical examples together with the results of simulations are laid out so as to corroborate the relevance and effectiveness of the developed techniques.

INDEX TERMS Fuzzy chaotic system, input quantization, resilient control, external disturbances, disturbance rejection approach.

I. INTRODUCTION

In recent years, chaotic systems (CSs) have received a great deal of interest from both industrial and research communities in light of the wide prospects of physics and engineering systems, including information processing, chemical reactions, mechanical systems, power converters, secure communications, and so on [1]. Even so, chaotic systems exhibit erratic and unexpected behaviours all through implementation, including sensitive dependency on initial conditions a lack of periodicity and a pseudo random feature [2]. In the spotlight of these factors, chaotic behaviour is highly detrimental to the performance of the system since it hampers the performance of the system. Therefore, it is desirable to minimise or eliminate chaos throughout the system to the greatest extent possible. As such, it is of the utmost importance to investigate CSs, and there have been an intriguing number of findings reported so far on the subject of stabilization of CSs [3], [4], [5]. On the other hand, in recent years, as an impact of the substantial nonlinear aspects of the contemporary industrial framework, the task of analysing and synthesising nonlinear controller for CSs has evolved into a prominent study area. And so, investigating how to control nonlinear CSs is an intriguing topic of study, and several approaches have been reported [6], [7], [8]. Amid them, Takagi-Sugeno (TS) fuzzy strategy has acquired broad adoption for nonlinear CSs as an outcome of its propensity to unify the concept of linear systems and the intrinsic value fuzzy technique [9]. To be precise, the nonlinear CSs are depicted as a weighted sum of linear-sub subsystems employing a set of fuzzy membership-based IF-THEN statements, which allows for the swift implementation of the extant linear system notions [10], [11]. Yet, despite its wider spectrum of benefits, only a limited amount of research

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works that deal with T-S fuzzy CSs have been published up to this point (see [12], [1] and references therein), which motivates the current investigation.

As is known, the fundamental intent of controller configuration is to offer efficient controllers that make sure the closed-loop system is stable or operates as anticipated. Nevertheless, in real-world settings, the controllers may typically accumulate considerable gain fluctuations due to parameter drift, actuator disintegration, round-off imperfections and so on [13]. In light of this, it is claimed that controllers are vulnerable to their own fluctuations and that minor modifications to their settings might potentially impair the system's performance and triggering instability [14]. Thereby, a controller ought to be built to be resilient or non-fragile, so that it sustains insensitivity to defects or disruptions in its coefficient. As a result, the control community as a whole has been quite interested in the issue of resilient control layout, and as a result, multiple findings have been produced [15], [16], [17]. As well, robust control strategies are of utmost importance to preserving stability across a wide range of applications, including but not limited to communication systems, power systems, automotive control, and aerospace systems. Specifically, fuzzy resilient control has been configured in [15] to tackle the gain fluctuations in fractional order nonlinear systems. Moreover, to deal with the gain perturbations inactive suspension system, the resilient frequency control has been frameworked in [17]. Thereby, devising a resilient control for T-S fuzzy CSs is of vast theoretical and practical importance, which serves as the impetus behind our research endeavour.

In real-world settings, it is worth noting that parameter uncertainties and different forms of disruptions occur in control systems and have a considerable effect on the effectiveness of controls and the stability of a system. Specifically, various factors, including changes in the surrounding environment, unexpected human input, or malfunctioning equipment, could all lead to the manifestation of disturbances [18], [19]. Therefore, the researchers have looked into designing unique disturbance attenuation and rejection strategies for diverse dynamical systems in an effort to address these concerns [20], [21], [22]. Regrettably, most of the approaches need precise knowledge of a disturbance, differentiability of the outputs, rank constraints on the control input and so on. But, owing to the complex behavior of real-world models, it may be challenging to precisely locate such preconditions, and doing so is not always attainable. To run over these issues, a technique known as equivalent input disturbance (EID) has been propounded by the researchers, which does not require any conditions that has been afore-discussed [23]. An EID is a disturbance on the control input path that mimics the impact of a real disturbance on the controlled output [24]. The EID method has the benefit of being easily constructed and rejecting both matched and mismatched disruptions [25]. Thereafter, the researchers developed an improved EID (IEID) technique in order to boost the disturbance rejection and allow for more freedom in system design, in which a gain factor has been incorporated in the standard EID estimator [26]. As a consequence of its profitable characterization, some promising results in relation to the IEID technique have been published (see [27], [28] and references cited therein). Moreover, the use of this sort of technology has been implemented across a diverse range of fields, notably aeronautical systems, chemical and process engineering, electrical power systems, and manufacturing systems. Therewithal, it is vital to pinpoint the fact that the stabilization problem for T-S fuzzy CSs in the presence of external disturbances is significant and complex owing to the trade-off among disturbance rejection and intended stabilization. In light of this, we must thus focus firmly on the development of a disturbance rejection-based controller to achieve an ideal performance of the T-S fuzzy CSs in spite of the existence of the external disturbances.

Aside from the above, it is important to note that the communication route over which signals are sent from the plant to the controller is normally designed to have a modest amount of data transfer capacity. Also, the controllers need to merit low levels of energy usage and affordability, both of which broaden the applicability of the system and have an immense effect on the results that it produces [29]. With this in mind, it is crucial to quantize the input signals before they are sent in order to reduce the range of traffic in the communication signals [30]. More specifically, the system's efficiency would increase if the data were converted to discrete signals before transmission, which can be achieved using quantizing mechanism. As a result, quantization control has emerged as a vital tool for achieving the desired system performance by virtue of its ability to simultaneously achieve high precision and low transmission costs [31], [32]. Thus, it has a major influence on system performance when analysing the dynamical aspects of T-S fuzzy CSs and thereby we are driven to undertake this study.

Inspired by the above arguments, this research delves into further in-depth explorations on the disturbance rejection approach for fuzzy chaotic systems. Precisely, we focus on designing the disturbance rejection (DR)-based quantized resilient control (QRC) protocol for T-S fuzzy CSs in the presence of randomly occurring uncertainties, gain fluctuations and external disturbances. Specifically, our research endeavour of this study can be summarized as follows:

- An unified stabilization and disturbance rejection problems for T-S fuzzy CSs that are susceptible to randomly occurring uncertainties, gain fluctuations and external disturbances is addressed through IEID strategy and DR-based QRC.
- Predominantly, the IEID approach is what we used to get an precise estimation on the external disturbances that are being acted upon by the system without any prior knowledge of the disturbances.
- Thereafter, a controller is proposed by incorporating IEID estimator state with feedback loop, which

proficiently tackle the external disturbances acting on the system under investigation.
- Moreover, the developed controller includes gain fluctuations in its synthesis, which solidifies the resilience of the proposed control technique. Subsequently, logarithmic quantizer is used to quantize the control input signals.
- Further, by the setup of a proper Lyapunov function, the prerequisites for stabilization and disturbance rejection are attained as linear matrix inequalities. After that, based on the gathered limitations, the precise design procedure of gain matrices of the proposed DR-based QRC and the configured observer are provided.

Finally, the formulated theoretical results are verified via the chaotic Lorenz system. It is shown that the IEID-based estimator rejects the external disturbances in a satisfactory manner and the proposed DR-based QRC aids in achieving the stability of the T-S fuzzy CSs.

II. PROBLEM FORMULATION AND PRELIMINARIES

The section starts with an introductory overview of the chaotic system that is represented by the T-S fuzzy framework. It subsequently proceeds with the establishment of the fuzzy-based observer system. The IEID estimator and filter system is then set up to offer accurate estimate of the disturbances, and a DR-based QRC is developed from thereon.

A. DESCRIPTION OF T-S FUZZY CHAOTIC SYSTEMS

To get things off, we provide a model of T-S fuzzy CSs that are vulnerable to parameter uncertainty and external disturbances, which is outlined below adopting IF-THEN rule:

Plant rule i: **IF** $\phi_1(t)$ is Φ_1^i and ... and $\phi_h(t)$ is Φ_h^i, **THEN**

$$\begin{cases} \dot{x}(t) = (A_i + \nu(t)\Delta A_i(t))x(t) + B_i u_q(t) \\ \qquad +E_i W(t), \\ y(t) = C_i x(t), \end{cases} \tag{1}$$

where $\phi_{\mathfrak{b}}(t)$ indicates premise variable; $\Phi_{\mathfrak{b}}^i$ ($i = 1, 2, \ldots, l$; $\mathfrak{b} = 1, 2, \ldots, h$) symbolises the fuzzy set, here l and $\mathfrak{b}$, respectively, mean the number of fuzzy rules and premise variables; $x(t) \in \mathbb{R}^n$, $u_q(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^q$ denote, respectively, the state vector, control input, and measured output vector of the system; $W(t) \in \mathbb{R}^w$ is an unknown external disturbance; A_i, B_i, E_i and C_i are known system matrices with suitable dimensions; $\Delta A_i(t)$ is perturbation matrix which is of the form $\Delta A_i(t) = \mathfrak{A}_{1i}\mathfrak{F}_i(t)\mathfrak{C}_i$ in which $\mathfrak{A}_{1i}$, $\mathfrak{C}_i$ are known apt dimensioned real matrices and $\mathfrak{F}_i(t)$ is an unknown continuous time-varying function that met the criteria $\mathfrak{F}_i(t)^T\mathfrak{F}_i(t) \leq I$; $\nu(t)$ is a stochastic parameter signifying an incident of uncertainty at random, conforms to the Bernoulli-distributed white sequences and probability rules underneath:

$$\begin{cases} Pr\{\nu(t) = 1\} = \mathbb{E}\{\nu(t)\} = \bar{\nu}, \\ Pr\{\nu(t) = 0\} = 1 - \mathbb{E}\{\nu(t)\} = 1 - \bar{\nu}, \end{cases} \tag{2}$$

where $\bar{\nu} \in [0, 1]$ is a known scalar.

In the subsequent step, by utilizing the fuzzy blending techniques, we can write the defuzzified structure of T-S fuzzy CSs (1), whose dynamics are presented below:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))\big((A_i + \nu(t)\Delta A_i(t))x(t) \\ \qquad +B_i u_q(t) + E_i W(t)\big) \\ y(t) = \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))C_i x(t), \end{cases} \tag{3}$$

where $\phi(t) = [\phi_1(t), \phi_2(t), \ldots, \phi_h(t)]$, $\mathfrak{q}_i(\phi(t))$ symbolises the membership function that met the ensuing criterion:

$$\mathfrak{q}_i(\phi(t)) = \frac{\prod_{\mathfrak{b}=1}^{h} \Phi_{\mathfrak{b}}^i(\phi_{\mathfrak{b}}(t))}{\sum_{i=1}^{l}\prod_{\mathfrak{b}=1}^{h} \Phi_{\mathfrak{b}}^k(\phi_{\mathfrak{b}}(t))} \geq 0, \quad \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t)) = 1,$$

here $\Phi_{\mathfrak{b}}^i(\phi_{\mathfrak{b}}(t))$ denotes the grade of membership of $\phi_{\mathfrak{b}}(t)$ in $\Phi_{\mathfrak{b}}^i$.

B. CONFIGURATION OF T-S FUZZY OBSERVER

Typically, in diverse practical systems, it might be difficult or impossible to measure reliably the system states. Given this, the T-S fuzzy observer is configured and the following articulates the dynamics of the observer system:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))\big(A_i\hat{x}(t) + B_i u_f(t) \\ \qquad +L_i\big(y(t) - \hat{y}(t)\big)\big) \\ \hat{y}(t) = \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))C_i\hat{x}(t), \end{cases} \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^n$ and $\hat{y}(t) \in \mathbb{R}^q$ signify the estimated states of $x(t)$ and output of $y(t)$, respectively; $u_f(t)$ delineates the feedback control vector; $L_i \in \mathbb{R}^{n\times q}$ delineates the gain matrix of the observer, that will be calculated in the upcoming segment.

C. DESIGN OF IEID ESTIMATOR

Our prime intent of this work lies on estimating the external disturbances using the IEID approach. To do so, the T-S fuzzy CSs (1) can equitably be portrayed as follows without the loss of generality:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))\big(A_i x(t) + B_i u_q(t) + B_i W_e(t)\big), \\ y(t) = \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))\big(C_i x(t)\big). \end{cases} \tag{5}$$

where $W_e(t)$ stands for the lumped disturbances.

To advance ahead, we define the error dynamics between system (5) and observer (4) as $x_\delta(t) = x(t) - \hat{x}(t)$. With this settings, the underneath expression can thus be easily procured:

$$\begin{aligned}\dot{x}_\delta(t) + \dot{\hat{x}}(t) &= \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))\big(A_i[x_\delta(t) + \hat{x}(t)] + B_i u_q(t) \\ &\quad + B_i W(t)\big) \\ \dot{\hat{x}}(t) &= \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))\big(A_i\hat{x}(t) + A_i x_\delta(t) + B_i[u_q(t) \\ &\quad + W_e(t)] - \dot{x}_\delta(t)\big). \end{aligned} \tag{6}$$

Subsequently, we present an tunable gain matrix K_{wi} that allow us to alter the value of $x_\delta(t)$ for enhanced EID estimation and overall system performance. Thereafter, we let $B_i\delta W_d(t) = -\dot{x}_\delta(t) + A_i x_\delta(t) + (K_{wi} - I)L_i C_i x_\delta(t)$ and $\hat{W}_c(t) = W_e(t) + \delta W_d(t)$. With this combination, equation (6) can be rephrased in the form described below:

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))\big(A_i\hat{x}(t) + B_i\big(u_q(t) \\ &\quad + \hat{W}_c(t)\big) - (K_{wi} - I)L_i C_i x_\delta(t)\big). \end{aligned} \tag{7}$$

According to settings (4) and (7), the estimated disturbance $\hat{W}_c(t)$ can be generated by straightforward calculation and it is described as

$$\begin{aligned} B_i\hat{W}_c(t) &= \dot{\hat{x}}(t) - A\hat{x}(t) - B_i u_q(t) \\ \hat{W}_c(t) &= \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))B_i^{+}K_{wi}L_i C_i x_\delta(t) + u_f(t) - u_q(t), \end{aligned} \tag{8}$$

where B_i^{+} is a Moore-Penrose inverse of B_i.

Furthermore, it is vital to pinpoint that the estimated disturbance $\hat{W}_c(t)$ encompasses some measurement noise. To counteract this noise, the estimated disturbance $\hat{W}_c(t)$ is fed into a low pass filter $\mathcal{G}_w(s)$, wherein $\mathcal{G}_w(s)$ fulfill the setting $\left|\mathcal{G}_w(j\theta)\right| \approx 1, \ \forall\, \theta \in [0, \theta_r]$, where θ_r is the cutoff angular frequency. The state-space form of low pass filter $\mathcal{G}_w(s)$ is described as

$$\begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f \hat{W}_c(t), \\ y_f(t) = C_f x_f(t), \end{cases} \tag{9}$$

where x_f denotes the filter state of $\mathcal{G}_w(s)$; $y_f(t)$ is noiseless IEID estimation; A_f, B_f and C_f are constant diagonal matrices.

D. DISTURBANCE REJECTION-BASED QUANTIZED RESILIENT CONTROL DESIGN

In this phase, DR-based QRC is built to offset the effects of external disturbances and make the system to perform robust. Firstly, the control signal $u_q(t)$ has to be quantized prior to being sent to the actuator for the purpose of mitigating the amount of data sent in the network. To do so, the logarithmic quantizer has been employed, that is, $\mathfrak{S}(u_f(t)) = \left[\mathfrak{S}(u_{f1}(t)), \mathfrak{S}(u_{f2}(t)), \ldots, \mathfrak{S}(u_{fm}(t))\right]^T$ and it is assumed to be time-invariant, static and symmetric such that $\mathfrak{S}(-u_f(t)) = -\mathfrak{S}(u_f(t))$. The level of quantization with quantization density $0 < \mathfrak{h} < 1$ and scaling parameter $\mathfrak{g}_0 > 0$ can be depicted in the following context:

$$\begin{aligned} \mathfrak{H} = \{\pm\mathfrak{g}^{\mathfrak{z}} : \mathfrak{g}^{\mathfrak{z}} = \mathfrak{h}^{\mathfrak{z}}\mathfrak{g}_0^{\mathfrak{z}}, \mathfrak{z} = 0, \pm1, \pm2, \ldots\} \\ \cup \{\pm\mathfrak{g}_0^{\mathfrak{z}}\} \cup \{0\}, \mathfrak{g}_0^{\mathfrak{z}} > 0. \end{aligned} \tag{10}$$

Whilst, the associated quantizer $\mathfrak{S}(\cdot)$ can be structured in the ensuing context:

$$\mathfrak{S}(u_f(t)) = \begin{cases} \mathfrak{g}^{\mathfrak{z}}, & \text{if } \dfrac{1}{1+\wp}\mathfrak{g}^{\mathfrak{z}} < u_f(t) \\ & \quad < \dfrac{1}{1-\wp}\mathfrak{g}^{\mathfrak{z}}, u_f(t) > 0, \\ 0, & \text{if } u_f(t) = 0, \\ -\mathfrak{S}(-u_f(t)), & \text{if } u_f(t) < 0, \end{cases} \tag{11}$$

where $\wp < 1$ specifies the quantized parameter having the form $\wp = \frac{1-\mathfrak{h}}{1+\mathfrak{h}}$. Subsequently, with the aid of sector-bounded condition, we thus arrive at the following relation:

$$\mathfrak{S}(u_f(t)) = (I_m + \mathfrak{G}(t))u_f(t), \tag{12}$$

where $\mathfrak{G}(t) = \text{diag}\left\{\mathfrak{G}^1(t), \mathfrak{G}^2(t), \ldots, \mathfrak{G}^m(t)\right\}$ with $\mathfrak{G}(t) \in \left[-\wp^2, \wp^2\right]$ and satisfies $|\mathfrak{G}(t)| < \wp^2$.

Besides, as discussed formerly in the previous section, even small changes to the control design might have a catastrophic effect on the control performance. So, it is imperative to build a controller that is able to endure some fluctuations in gain value. For the foregoing reason, the following fuzzy-based resilient controller has been designed:

Control rule j: **IF** $\phi_1(t)$ is Φ_1^j and $\ldots$ and $\phi_h(t)$ is Φ_h^j, **THEN**

$$u_f(t) = (I_m + \mathfrak{G}(t))(K_j + \Delta K_j(t))\hat{x}(t), \tag{13}$$

where K_j delineates the controller gain matrices, which will be reckoned in the successive sections; $\Delta K_j(t)$ is the gain perturbation satisfying $\Delta K_j(t) = \mathfrak{A}_{2j}\mathfrak{F}_j(t)\mathfrak{N}_j$, where $\mathfrak{F}_j^T(t)\mathfrak{F}_j(t) \leq I$ and $\mathfrak{A}_{2j}$, $\mathfrak{N}_j$ are known real matrices. In a further, the defuzzified version of (13) is laid out below:

$$u_f(t) = \sum_{j=1}^{l} \mathfrak{q}_j(\phi(t))\Big((I_m + \mathfrak{G}(t))(K_j + \Delta K_j(t))\hat{x}(t)\Big). \tag{14}$$

In the next stage, the estimated noiseless disturbance is incorporated in the control law so that the footprints of the external disturbances can be compensated form the system. With these discussion, the overall DR-based QRC can be formulated as follows:

$$\begin{aligned} u_q(t) &= \sum_{j=1}^{l} \mathfrak{q}_j(\phi(t))\Big((I_m + \mathfrak{G}(t))(K_j + \Delta K_j(t))\hat{x}(t)\Big) \\ &\quad - y_f(t). \end{aligned} \tag{15}$$

Moreover, as the stability of the assayed T-S fuzzy CSs does not rely on the external disturbances, it is presumed that $W(t) = 0$. With this in mind and based on the above interpretations, we can obtain the following equations:

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{l}\sum_{j=1}^{l} \mathfrak{q}_j(\phi(t))\Big(A_i\hat{x}(t) + B_i\big(I_m + \mathfrak{G}(t)\big)\\ &\quad \times \big(K_j + \Delta K_j(t)\big)\hat{x}(t) + L_iC_ix_\delta(t)\Big), \qquad (16)\\ \dot{x}_\delta(t) &= \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))\Big(\big(A_i + \nu(t)\Delta A_i(t) - L_iC_i\big)x_\delta(t)\\ &\quad + \nu(t)\Delta A_i(t)\hat{x}(t) - B_iC_f x_f(t)\Big), \qquad (17)\\ \dot{x}_f(t) &= \sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))\Big(\big(A_f + B_fC_f\big)x_f(t) + B_fB_i^{+}\\ &\quad \times K_{wi}L_iC_ix_\delta(t)\Big). \qquad (18)\end{aligned}$$

Remark 1: The exploration of the dynamic behaviour of chaotic systems by executing fuzzy protocols has shown a substantial rise in recent years [1], [8], [9], [10], [11], [12]. In particular, a significant portion of the existing works use the disturbance attenuation approaches, which only deliver a specific amount of attenuation for disturbances. However, in specific cases of chaotic systems, the system's performance can be poor if the disturbances are not completely rejected from the system. Further, the impact of the gain fluctuations and quantization effects has not been considered in all of the prior studies in spite of their key impact on the stabilization of chaotic systems. Consequently, the presented investigation focuses on devising disturbance rejection-based control for T-S fuzzy chaotic systems using the IEID technique. Furthermore, the control design takes into account the effects of quantization and gain variations, thereby capturing the practical aspects of synthesizing the controller. To put it more precisely, the primary objective of this work is to close that gap by embarking on a pioneering effort to come up with a disturbance rejection-based quantized robust controller for chaotic systems.

III. THEORETICAL PROOF

The intent of this segment is to offer adequate requirements in a setting of matrix inequalities for certifying the stability of T-S fuzzy CSs in the face of randomly occurring uncertainties, gain fluctuations and external disturbances. In particular, the required sufficient criteria are derived with the help of Lyapunov stability theory.

A. STABILITY ANALYSIS

Under the pretence that the controller's and disturbance observer's gain values are already known, the requirements for stability criteria are laid out in this section.

Theorem 1: Let the scalars $\wp \in (0, 1)$, $\bar{\nu} \in [0, 1]$ and gain matrices K_j, L_i and K_{wi} be known. Then, the closed-loop systems (16)-(18) are asymptotically stable, if there exist scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, $\varepsilon_4 > 0$, $\varepsilon_5 > 0$, $\varepsilon_6 > 0$ and matrices $\mathcal{P}_1 < 0$, $\mathcal{P}_2 < 0$, $\mathcal{P}_3 < 0$, such that the following condition met:

$$\begin{cases} \Upsilon^{ii} < 0, & (i = j),\\ \Upsilon^{ij} + \Upsilon^{ji} < 0, & (i \neq j), \end{cases} \qquad (19)$$

where $\tilde{\Upsilon}^{ij}_{1,1} = sym(\mathcal{P}_1A_i + \mathcal{P}_1B_iK_j)$, $\tilde{\Upsilon}^{ij}_{1,2} = \mathcal{P}_1L_iC_i$, $\tilde{\Upsilon}^{ij}_{1,4} = \varepsilon_1\mathcal{P}_1B_i\mathfrak{A}_{2i}$, $\tilde{\Upsilon}^{ij}_{1,5} = \mathfrak{N}_j^T$, $\tilde{\Upsilon}^{ij}_{1,6} = \varepsilon_2\mathcal{P}_1B_i$, $\tilde{\Upsilon}^{ij}_{1,7} = \wp K_j^T$, $\tilde{\Upsilon}^{ij}_{1,8} = \varepsilon_3\mathcal{P}_1B_i$, $\tilde{\Upsilon}^{ij}_{1,10} = \bar{\nu}\mathfrak{C}_i^T$, $\tilde{\Upsilon}^{ij}_{1,14} = \wp\mathfrak{A}_{2i}$, $\tilde{\Upsilon}^{ij}_{2,2} = sym\{\mathcal{P}_2A_i - \mathcal{P}_2L_iC_i\}$, $\tilde{\Upsilon}_{2,3} = -\mathcal{P}_2B_iC_f + (\mathcal{P}_3B_fB_i^{+}K_{wi}L_iC_i)^T$, $\tilde{\Upsilon}^{ij}_{2,11} = \varepsilon_4\mathcal{P}_2B_i$, $\tilde{\Upsilon}^{ij}_{2,12} = \varepsilon_5\mathcal{P}_2\mathfrak{A}_{1i}$, $\tilde{\Upsilon}^{ij}_{2,13} = \mathfrak{C}_i^T$, $\tilde{\Upsilon}^{ij}_{3,3} = sym(\mathcal{P}_3A_f + \mathcal{P}_3B_fC_f)$ $\tilde{\Upsilon}^{ij}_{4,4} = -\varepsilon_1I$, $\tilde{\Upsilon}^{ij}_{5,5} = -\varepsilon_1I$, $\tilde{\Upsilon}^{ij}_{6,6} = -\varepsilon_2I$, $\tilde{\Upsilon}^{ij}_{7,7} = -\varepsilon_2I$, $\tilde{\Upsilon}^{ij}_{8,8} = -\varepsilon_3I$, $\tilde{\Upsilon}^{ij}_{9,9} = -\varepsilon_3I$, $\tilde{\Upsilon}^{ij}_{9,15} = \varepsilon_6\mathfrak{N}_j^T$, $\tilde{\Upsilon}^{ij}_{10,10} = -\varepsilon_4I$, $\tilde{\Upsilon}^{ij}_{11,11} = -\varepsilon_4I$, $\tilde{\Upsilon}^{ij}_{12,12} = -\varepsilon_5I$ $\tilde{\Upsilon}^{ij}_{13,13} = -\varepsilon_5I$, $\tilde{\Upsilon}^{ij}_{14,14} = -\varepsilon_6I$ and $\tilde{\Upsilon}^{ij}_{15,15} = -\varepsilon_6I$.

Proof: With the objective to acquire the stability criteria, the Lyapunov function is framed as

$$\begin{aligned}\mathcal{V}(t) &= \hat{x}^T(t)\mathcal{P}_1\hat{x}(t) + x_\delta^T(t)\mathcal{P}_2x_\delta(t)\\ &\quad + x_f^T(t)\mathcal{P}_3x_f(t). \qquad (20)\end{aligned}$$

By means of the the infinitesimal operator £ and taking mathematical expectation $\mathbb{E}$ in relation (20), we arrive the following equation

$$\begin{aligned}\mathbb{E}\{\pounds\mathcal{V}(t)\} &= \mathbb{E}\{\dot{\hat{x}}^T(t)\mathcal{P}_1\hat{x}(t) + \hat{x}^T(t)\mathcal{P}_1\dot{\hat{x}}(t) + \dot{x}_\delta^T(t)\mathcal{P}_2x_\delta(t)\\ &\quad + x_\delta^T(t)\mathcal{P}_2\dot{x}_\delta(t) + \dot{x}_f^T(t)\mathcal{P}_3x_f(t)\\ &\quad + x_f^T(t)\mathcal{P}_3\dot{x}_f(t)\}\\ &= \mathbb{E}\Bigg\{\Bigg[\sum_{i=1}^{l}\sum_{j=1}^{l}\mathfrak{q}_j(\phi(t))\Big(A_i\hat{x}(t) + B_i\big(I_m + \mathfrak{G}(t)\big)\\ &\quad \times \big(K_j + \Delta K_j(t)\big)\hat{x}(t) + L_iC_ix_\delta(t)\Big)\Bigg]^T\mathcal{P}_1\hat{x}(t)\\ &\quad + \hat{x}^T(t)\mathcal{P}_1\Bigg[\sum_{i=1}^{l}\sum_{j=1}^{l}\mathfrak{q}_j(\phi(t))\Big(A_i\hat{x}(t) + B_i\big(I_m\\ &\quad + \mathfrak{G}(t)\big)\big(K_j + \Delta K_j(t)\big)\hat{x}(t) + L_iC_ix_\delta(t)\Big)\Bigg]\\ &\quad + \Bigg[\sum_{i=1}^{l}\mathfrak{q}_i(\phi(t))\Big(\big(A_i + \nu(t)\Delta A_i(t) - L_iC_i\big)\\ &\quad \times x_\delta(t) + \nu(t)\Delta A_i(t)\hat{x}(t) - B_iC_fx_f(t)\Big)\Bigg]^T\\ &\quad \times \mathcal{P}_2x_\delta(t) + x_\delta^T(t)\mathcal{P}_2\Bigg[\sum_{i=1}^{l}\mathfrak{q}_i(\phi(t))\Big(\big(A_i\\ &\quad + \nu(t)\Delta A_i(t) - L_iC_i\big)x_\delta(t) + \nu(t)\Delta A_i(t)\hat{x}(t)\\ &\quad - B_iC_fx_f(t)\Big)\Bigg] + \Bigg[\sum_{i=1}^{l}\mathfrak{q}_i(\phi(t))\Big(\big(A_f + B_fC_f\big)\end{aligned}$$

$$\times x_f(t) + B_f B_i^+ K_{wi} L_i C_i x_\delta(t)\Big)\Big]^T \mathcal{P}_3 x_f(t) + x_f^T(t)\mathcal{P}_3\Big[\sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))\Big(\big(A_f + B_f C_f\big)x_f(t) + B_f B_i^+ K_{wi} L_i C_i x_\delta(t)\Big)\Big]\Big\}. \tag{21}$$

In the subsequent step, the above equation can be recast with the aid of the relation (2) in the following form:

$$\mathbb{E}\{\pounds\mathcal{V}(t)\} = \zeta^T(t)\Big[\sum_{i=1}^{l} \mathfrak{q}_i(\phi(t))\check{\Upsilon}\Big]\zeta(t), \tag{22}$$

where $\zeta^T(t) = \big[\hat{x}^T(t)\ x_\delta^T(t)\ x_f^T(t)\big]$, $\check{\Upsilon}_{1,1} = sym\{\mathcal{P}_1 A_i + \mathcal{P}_1 B_i\big(I_m + \mathfrak{G}(t)\big)\big(K_j + \Delta K_j(t)\big)\}$, $\check{\Upsilon}_{1,2} = \mathcal{P}_1 L_i C_i + (\mathcal{P}_2 \bar{\nu}\Delta A_i(t))^T$, $\check{\Upsilon}_{2,2} = sym\{\mathcal{P}_2 A_i + \mathcal{P}_2\bar{\nu}\Delta A_i(t) - \mathcal{P}_2 L_i C_i\}$, $\check{\Upsilon}_{2,3} = -\mathcal{P}_2 B_i C_f + (\mathcal{P}_3 B_f B_i^+ K_{wi} L_i C_i)^T$, $\check{\Upsilon}_{3,3} = \mathcal{P}_3 A_f + \mathcal{P}_3 B_f C_f$.

Thereafter, through the utilization of Schur complement lemma to the aforementioned matrix $\check{\Upsilon}$ in (22), we able to procure the matrix Υ defined in (19). Moreover, if the relation (19) met, then it is straightforward to achieve the relation $\mathbb{E}\{\pounds\mathcal{V}(t)\} < 0$. Then, in light of Lyapunov stability theory, the closed-loop system (16)-(18) are asymptotically stable, which brings an end to this argument.

B. STABILIZATION ANALYSIS

In the forthcoming theorem, suitable conditions to vouch for the anticipated outcome are set up by treating the gain matrices as unknown.

Theorem 2: For given scalars $\wp \in (0,1)$, $\alpha > 0$, $\beta > 0$, $\eta > 0$, $\bar{\nu} \in [0,1]$, the closed-loop system (16)-(18) are asymptotically stable, if there exist positive scalars κ, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, $\varepsilon_4 > 0$, $\varepsilon_5 > 0$, $\varepsilon_6 > 0$, symmetric matrices $\mathcal{Q} < 0$, $\mathcal{R} < 0$, $\mathcal{S} < 0$ and appropriate dimensioned matrices $\mathcal{Z}_i$, $\mathcal{Y}_j$, $\mathcal{X}_i$ such that the following conditions hereunder are satisfied:

$$\begin{cases} \beth^{ii} < 0, & (i = j), \\ \beth^{ij} + \beth^{ji} < 0, & (i \neq j), \end{cases} \tag{23}$$

$$\begin{bmatrix} -\kappa I & C_i\mathcal{R} - \bar{\mathcal{R}}C_i \\ * & -I \end{bmatrix} < 0, \tag{24}$$

where $\beth^{ij}_{1,1} = sym(\alpha A_i \mathcal{Q} + B_i \mathcal{Y}_j)$, $\beth^{ij}_{1,2} = \beta\mathcal{Z}_i C_i$, $\beth^{ij}_{1,4} = \varepsilon_1 B_i \mathfrak{A}_{2i}$, $\beth^{ij}_{1,5} = \alpha\mathcal{Q}\mathfrak{N}_j^T$, $\beth^{ij}_{1,6} = \varepsilon_2 B_i$, $\beth^{ij}_{1,7} = \wp\mathcal{Y}_j^T$, $\beth^{ij}_{1,8} = \varepsilon_3 B_i$, $\beth^{ij}_{1,10} = \bar{\nu}\alpha\mathcal{Q}\mathfrak{C}_i^T$, $\beth^{ij}_{1,14} = \wp\alpha\mathcal{Q}\mathfrak{A}_{2i}$, $\beth^{ij}_{2,2} = sym\{A_i\beta\mathcal{R} - \beta\mathcal{Z}_i C_i\}$, $\beth_{2,3} = -\eta B_i C_f \mathcal{S} + (\beta B_f B_i^+ \mathcal{X}_i C_i)^T$, $\beth^{ij}_{2,11} = \varepsilon_4 B_i$, $\beth^{ij}_{2,12} = \varepsilon_5 \mathfrak{A}_{1i}$, $\beth^{ij}_{2,13} = \beta\mathcal{R}\mathfrak{C}_i^T$, $\beth^{ij}_{3,3} = sym(\eta A_f \mathcal{S} + \eta B_f C_f \mathcal{S})$ $\beth^{ij}_{4,4} = -\varepsilon_1 I$, $\beth^{ij}_{5,5} = -\varepsilon_1 I$, $\beth^{ij}_{6,6} = -\varepsilon_2 I$, $\beth^{ij}_{7,7} = -\varepsilon_2 I$, $\beth^{ij}_{8,8} = -\varepsilon_3 I$, $\beth^{ij}_{9,9} = -\varepsilon_3 I$, $\beth^{ij}_{9,15} = \varepsilon_6\mathfrak{N}_j^T$, $\beth^{ij}_{10,10} = -\varepsilon_4 I$, $\beth^{ij}_{11,11} = -\varepsilon_4 I$, $\beth^{ij}_{12,12} = -\varepsilon_5 I$ $\beth^{ij}_{13,13} = -\varepsilon_5 I$, $\beth^{ij}_{14,14} = -\varepsilon_6 I$ and $\beth^{ij}_{15,15} = -\varepsilon_6 I$. Furthermore, if the constraints (23) and (24) have workable solutions, then the gain values can be computed by $L_i = \mathcal{Z}_i\mathcal{R}_i^{-1}$, $K_j = \mathcal{Y}_j\mathcal{Q}_i^{-1}$ and $K_{wi} = \mathcal{X}_i\bar{\mathcal{R}}_i^{-1}L_i^+$.

Proof: We have utilised the same Lyapunov function (20) from the prior theorem in an effort to establish this theorem as well. Beginning with the similar technique as in the Theorem 1, we are able to grab the matrix Υ. Nevertheless, in this theorem, we assume the unknown gain matrices, and as a direct consequence of this supposition, the nonlinearities appear in Υ. To convert into linear matrix inequality, let us assume $\mathcal{P}_1^{-1} = \alpha\mathcal{Q}$, $\mathcal{P}_2^{-1} = \beta\mathcal{R}$, $\mathcal{P}_3^{-1} = \eta\mathcal{S}$ and then pre- and post- multiply the matrix Υ by $\Big\{\mathcal{P}_1^{-1},\ \mathcal{P}_2^{-1}\ \mathcal{P}_3^{-1}, \underbrace{I,\ I, \ldots,\ I,\ I}_{12 times}\Big\}$ and adopt the following congruent transformations $C_i\mathcal{R} = \bar{\mathcal{R}}C_i$, $\mathcal{Z}_i = L_i\bar{\mathcal{R}}$, $\mathcal{Y}_j = K_j\mathcal{Q}$ and $\mathcal{X}_i = K_{wi}L_i\bar{\mathcal{R}}_i$. As a result of this, the matrix $\beth$ defined in (23) can be readily arrived. Then, if the condition in (23) met, we can say that the closed-loop systems (16)-(18) are asymptotically stable in accordance with Lyapunov stability theory.

Nevertheless, $C_i\mathcal{Q}_i = \bar{\mathcal{Q}}_i C_i$ is not a linear one and so, the condition can be equivalently viewed as $\big[C_i\mathcal{Q}_i - \bar{\mathcal{Q}}_i C_i\big]^T\big[C_i\mathcal{Q}_i - \bar{\mathcal{Q}}_i C_i\big] \prec \kappa I$, where $\kappa > 0$ is a known constant. Then, with the assistance of Schur complement we can write it as equivalent to (24). Thus, the proof is now completed.

Remark 2: It is important to point out that inside the primary findings portion, the trade-off between the formulated Lyapunov function and the produced LMI constraint plays an important role in the examination and development of control design and stability for the system under consideration. Moreover, the computational cost and time required are significantly influenced by the number of decision variables utilised in the LMI-based adequate constraints. In addition to the quantity of decision parameters, the computational complexity of the system is also directly linked to the dimensions of the matrices and the number of fuzzy rules. Furthermore, not including free-weighting matrices during the derivation of the primary results leads to the formulation of LMI-based constraints with a reduced number of choice variables, which significantly reduces the computational workload required for the analysis.

IV. SIMULATION VERIFICATION

In this section, simulation results are provided to ensure the asymptotic stability and disturbance rejection performance for the T-S fuzzy CSs under IEID-based QRC strategy. With this intent, we consider the following chaotic Lorenz system with input parameter as in [8]:

$$\begin{cases} \dot{x}_1(t) = -\mathfrak{a}x_1(t) + \mathfrak{a}x_2(t) + u_1(t), \\ \dot{x}_2(t) = \mathfrak{c}x_1(t) - x_2(t) - x_1(t)x_3(t) \\ \dot{x}_3(t) = x_1(t)x_2(t) - \mathfrak{b}x_3(t), \end{cases} \tag{25}$$

with $x_1(t) = [-\mathfrak{d}, \mathfrak{d}]$. Obviously, the chaotic Lorenz system (25) can be expressed as a fuzzy system with the

Algorithm 1 Brief Description of the Algorithm

Step 1: Design the mathematical model of T-S fuzzy CSs and configure the observer and low pass filter based on EID approach

Step 2: Devise the DR-based QRC utilising the information received from the constructed filter.

Step 3: Frame the Lyapunov function and derive the linear matrix inequality constraints.

Step 4: Input the parameters A_i, B_i, E_i, C_i, A_f,B_f and C_f in the established linear matrix inequality relations in Step 3.

Step 5: ***if*** the produced constraints holds:

- ▶ Compute the gain matrices K_j, $\mathcal{L}_i$ and K_{wi}
- ▶ The undertaken T-S fuzzy CSs is stable.

else

- ▶ Reload the parameters specified in Step 4 and repeat the procedure.

end

following parameters:

Rule 1:

$$A_1 = \begin{bmatrix} -a & a & 0 \\ c & -1 & -d \\ 0 & d & -b \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T,$$
$$C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \text{ and } D_1 = \begin{bmatrix} -0.1 & 1 & 0.5 \end{bmatrix}^T.$$

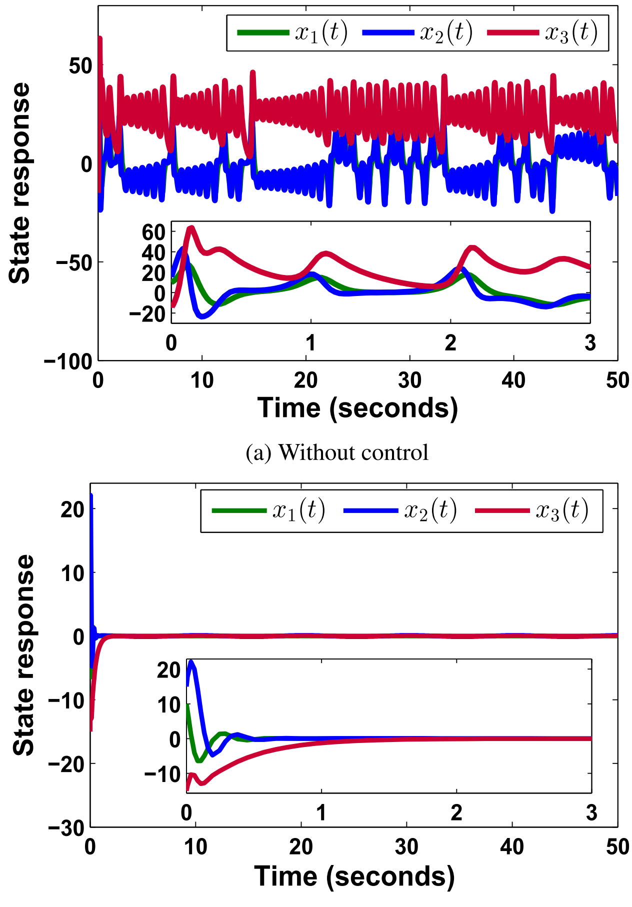
Rule 2:

$$A_2 = \begin{bmatrix} -a & a & 0 \\ c & -1 & d \\ 0 & -d & -b \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T,$$
$$C_2 = \begin{bmatrix} 6 & 0 & 0 \end{bmatrix} \text{ and } D_2 = \begin{bmatrix} -0.1 & 1 & 0.3 \end{bmatrix}^T.$$

Further, let us chose the parameters $\mathfrak{a} = 10$, $\mathfrak{b} = \frac{8}{3}$, $\mathfrak{c} = 28$ and $\mathfrak{d} = 25$ as in [8]. The uncertain parameter matrices are taken as $\mathfrak{A}_{11} = \mathfrak{A}_{12} = \begin{bmatrix} 0.5 & 0.01 & 0.05 \end{bmatrix}^T$, $\mathfrak{A}_{21} = \mathfrak{A}_{22} = \begin{bmatrix} 0.5 \end{bmatrix}$, and $\mathfrak{C} = \begin{bmatrix} 0.01 & 0.01 & 0.02 \end{bmatrix}$.

In addition, the parameters of filter system (9) are taken as $A_f = -81$, $B_f = 26$, and $C_f = 1$. Meanwhile, the membership function is picked as in [8] and displayed as $\mathfrak{q}_1(x_1(t)) = 0.5(1 + \frac{x_1(t)}{d})$ and $\mathfrak{q}_2(x_1(t)) = 1 - \mathfrak{q}_1(x_1(t))$. Further, we consider the remaining parameters which are involved in LMI constraints to test the feasibility as $\bar{\nu} = 0.052$, $\wp = 1.3$, $\alpha = 2.5, \beta = 1.0$ and $\mathfrak{y} = 1.3$. With the values of these parameters, the feasible solutions to the linear matrix inequalities (23) and (24) are obtained. Subsequently, the reckoned gain matrices are displayed hereunder:

$$L_1 = \begin{bmatrix} -20.2532 \\ 48.1910 \\ -24.7720 \end{bmatrix}, \quad K_1 = \begin{bmatrix} 13.7366 \\ -17.2002 \\ 3.6863 \end{bmatrix}^T,$$



(a) Without control

(b) With control

FIGURE 1. Time profile of state trajectories.

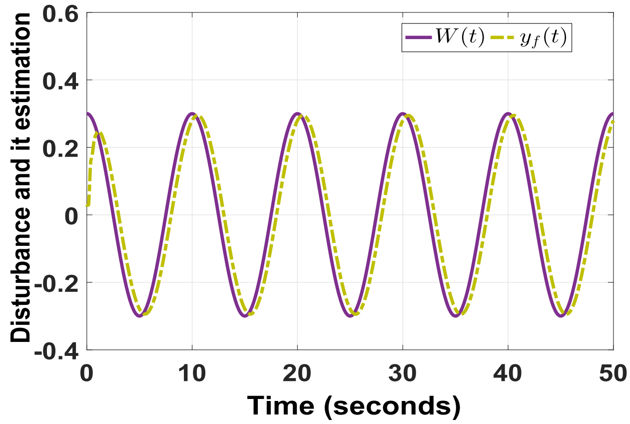
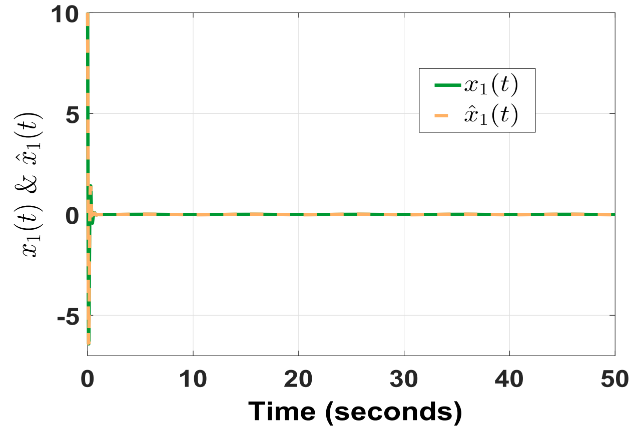


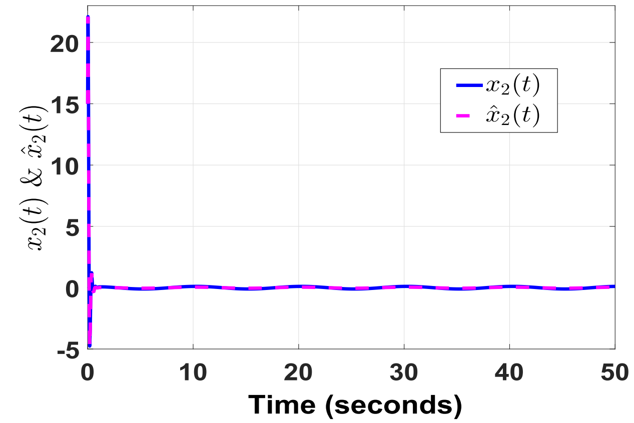
FIGURE 2. Disturbance and its estimation.

$$K_{w1} = \begin{bmatrix} -0.0083 & 0.0197 & -0.0101 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

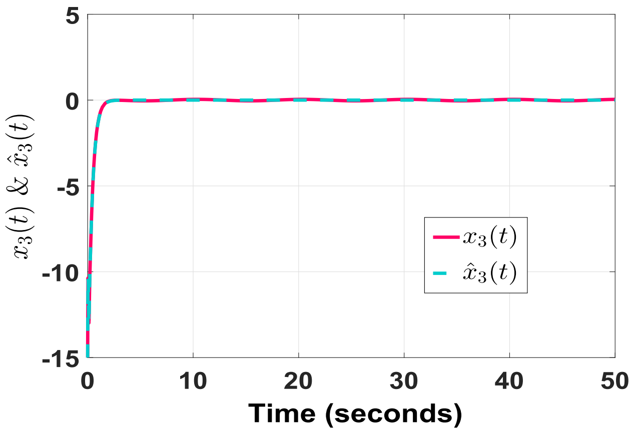
$$L_2 = \begin{bmatrix} 78.4498 \\ 62.0115 \\ 25.3952 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -20.0540 \\ -22.3022 \\ -3.2145 \end{bmatrix}^T,$$



(a) State $x_1(t)$ and its estimate $\hat{x}_1(t)$



(b) State $x_2(t)$ and its estimate $\hat{x}_2(t)$



(c) State $x_3(t)$ and its estimate $\hat{x}_3(t)$

FIGURE 3. Time profile of state trajectories and its corresponding observer trajectories.

$$\text{and } K_{w2} = \begin{bmatrix} 0.0101 & 0.0080 & 0.0033 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Moreover, for the simulation purposes, the following external disturbances acting on systems are taken as $W(t) = 0.3cos(0.2\pi t)$, respectively. Subsequently, the initial

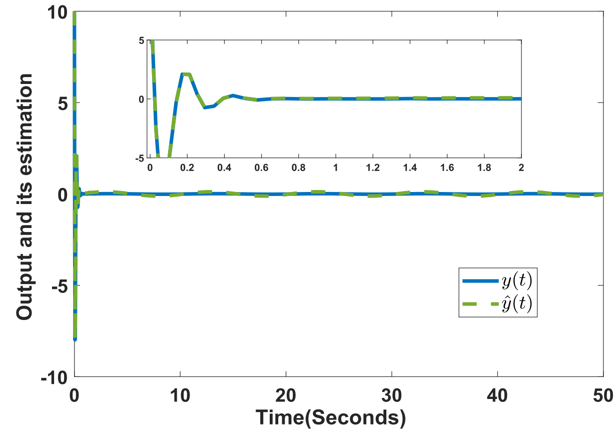


FIGURE 4. Output and its estimation.

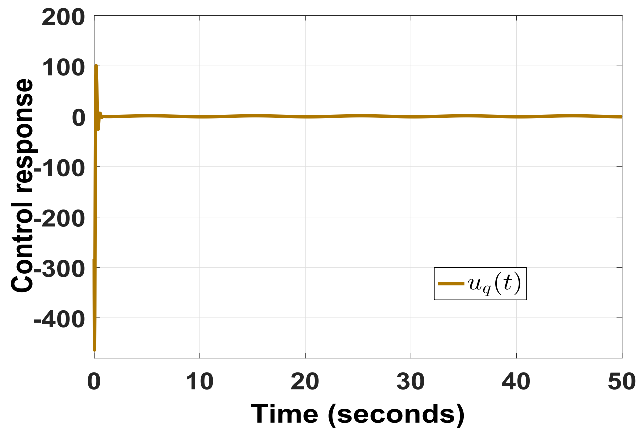
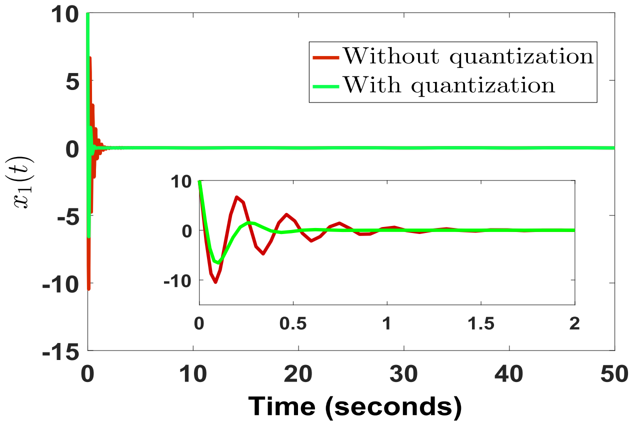


FIGURE 5. Time profile of control trajectory.

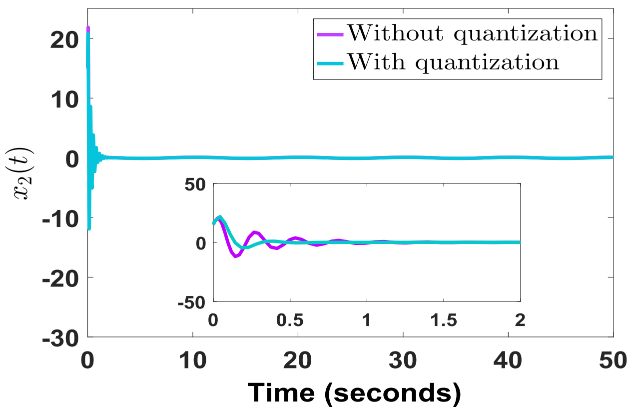
conditions for the state, observer and filter are considered as $x(0) = \hat{x}(0) = \begin{bmatrix}10 & 15 & -15\end{bmatrix}^T$ and $x_f = \begin{bmatrix}0.1 & 0.1 & 0.1\end{bmatrix}^T$.

After then, we perform the simulation through the use of the Simulink environment with the settings that were specified beforehand and as a follow-up, we provided Fig. 1-Fig. 5. To sum up, Fig. 1a reflects the result of state trajectories pertaining to the examined chaotic Lorenz system (25) in the devoid of the built controller (15). As perceived from this, the response of states fail to converge to zero, which implies that the open-loop chaotic Lorenz system being looked at is unstable. Moreover, the illustration in Fig. 1b exhibits the state responses of the system (25) for the closed-loop setting. Therein, it is clear from looking at Fig. 1b that the constructed DR-based QRC is able to get rid of disturbances in an efficient manner and swiftly drive the states of the chaotic Lorenz system to the convergence point. Therefore, these two figures exemplifies the significance of the intended DR-based QRC along with its potency.

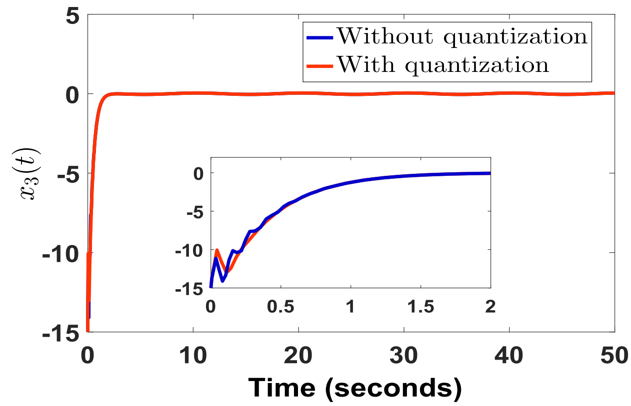
On the flip side, Fig. 2 is offered to showoff the relevance of the adopted IEID estimator approach. More precisely, this figure showcases the disturbance acting on the system and its estimate, where it ought to be noticed that the estimation of the disturbance is fairly close to the considered disturbance



(a) Response of first trajectory



(b) Response of second trajectory



(c) Response of third trajectory

FIGURE 6. Time profile of state trajectories with and without input quantization.

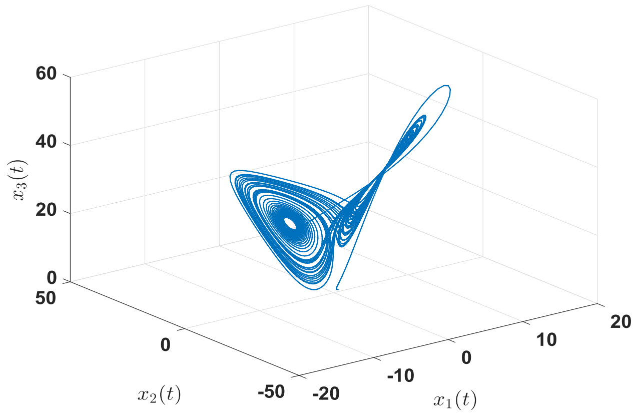


FIGURE 7. Chaotic behavior of Lorenz system without controller.

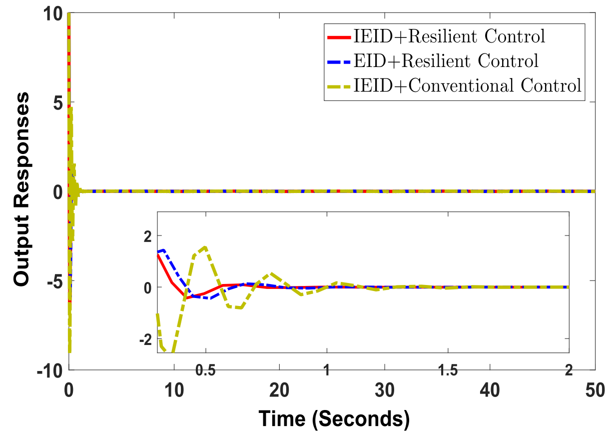


FIGURE 8. Time profile of output trajectories under different approach.

signal that was acting on the system. In a nutshell, the versatility of the IEID framework that was laid out in this investigation, as well as the advantages that are linked with adopting it, are well proven.

In a further, the simulation output of both the actual state of the chaotic Lorenz system and the state of the fuzzy observer are portrayed in Fig. 3a-Fig. 3c. In addition, the output estimation trajectories can be viewed on Fig. 4. These figures demonstrate that the estimated states consistently forecast the real states, reveal that the framed fuzzy observer functions seamlessly and delivers sufficient state estimation. In the end, the graphical illustration of the control trajectory is presented in Fig. 5. Afterwards, Fig. 6 illustrates the state trajectories in the presence and absence of quantizers, rendering the role of the deployed quantizers very evident. In addition, Fig. 7 depicts the chaotic behaviour associated with the relevant system (25) without the controller, wherein it can be seen that the system trajectories are wandering randomly.

On top of that, for the sake of showcasing the competencies of the controller and the approach deployed over existing ones, visual representation in the form of Fig. 8 have been included. To be precise, the output responses of the chaotic Lorentz system under IEID-based conventional controller, EID-based resilient controller and proposed approach are presented. Therein, it can be witnessed that performance of the system under proposed approach is better when compared to conventional techniques. To be precise, by comparing the convergence instant of output trajectories, we can infer that proposed approach improves 53.54% over IEID-based conventional controller and 25% over EID-based resilient controller. Upon examination at these results, it becomes evident that the theoretical outcomes developed in this article possess a considerable level of significance over existing techniques.

As a whole, the simulation studies indicate that the offered DR-based QRC design affords adequate performance even when parameter uncertainties, external disturbances and gain fluctuations are present.

V. CONCLUSION

In the work reported here, the issues of stabilization and disturbance rejection has been addressed for T-S fuzzy CSs through an DR-based QRC and IEID approach. At first, external disturbances occurring in the assayed system are precisely estimated using IEID approach. Secondly, perturbations have been added in the controller gain matrices to make the controller more resilient. Thirdly, the control input signal has been quantized by means of logarithmic quantizer with the intent to alleviate the traffic in the communication channels. Subsequently, by melding the estimated disturbance signal with the quantized resilient control, the disturbances in the system has been compensated and the intended stability is achieved. Moreover, by blending Lyapunov stability theory and linear matrix inequality technique, the sufficient condition assuring the stability of the assayed system are established and as a further step, we solve the linear matrix inequalities to get the gain matrices. Finally, numerical simulations demonstrate the potential of the proposed control scheme as well as the derived key conclusions. Specifically, from the simulation results, by comparing the convergence instant of output trajectories, we can see that the developed approach improves 53.54% over IEID-based conventional controller and 25% over EID-based resilient controller. As a prospective avenue for further investigation, the fuzzy-based actuator and sensor fault reconstruction approach will be developed for chaotic systems.

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