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RESEARCH ARTICLE

Near Feasibility Driven Adaptive Penalty Functions Embedded MOEA/D

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ABSTRACT This work extends a near feasibility threshold (NFT) based adaptive penalty function for constrained multiobjective optimization. The NFT zone adjoining the feasible region is considered as good one, where infeasible solutions are relatively less penalized. The modified penalty function, denoted by TAP with five different settings of NFT is embedded in a renowned multiobjective evolutionary algorithm based on decomposition, MOEA/D. This offers five constrained variants, namely CMOEA/D-TAP1 to CMOEA/D-TAP5, of the base algorithm. These variants are tested on well-known constrained multiobjective benchmark test suits, the CTP series and the CF series. The proposed variants are compared with the four best performing algorithms through HV (hyper volume) metric statistics for CTP series, and with seven state-of-the-art algorithms through Wilcoxon rank sum test employed to mean values of both HV and IGD (inverted generational distance) metrics for CF series. Simulation results reflect that overall performance of the newly introduced variants is better than the competitors for the taken benchmark test suits.

INDEX TERMS Constrained optimization, penalty function methods, near feasibility threshold, multiobjective optimization based on decomposition.

I. INTRODUCTION

An optimization method looks for the best possible solution (s) that optimizes a given function (s) under bound and/or functional constraints. Many researchers are drawn to optimization because it has so many common real-world applications that process or call for numerical data. It can be classified as deterministic or stochastic optimization, unconstrained and constrained optimization, and single, multi, or many objectives optimization. The majority of problems in daily life contain two or more, frequently conflicting, objectives. Such problems are known as multiobjective optimization problems (MOPs). Certain bound, inequality, and equality-type constraints frequently apply to MOP. The constrained multiobjective optimization problem (CMOP), which is subject to some functional and bound constraints, can be stated in generic form as follows [1]:

$$\begin{aligned} &\text{Minimize } \mathbf{Obj}(\mathbf{u}) = (Obj_1(\mathbf{u}), Obj_2(\mathbf{u}), \ldots, Obj_m(\mathbf{u}))^T; \\ &\text{Subject to } Con_j(\mathbf{u}) \geq 0, j = 1, 2, \ldots, q; \\ &\qquad u_k^l \leq u_k \leq u_k^u, k = 1, 2 \ldots, n; \end{aligned} \quad (1)$$

where $\mathbf{u} = (u_1, \ldots, u_n)^T \in \mathcal{R}^n$ is the vector of n decision variables, $Obj_i, i = 1, 2 \ldots, m$ represents the m objective functions, **Obj** contains m objectives of the problem, Con_j reflects q number of inequality constraints of "greater equal to type". Obj_i and Con_j are linear or non-linear functions of real variables, u_k^l and u_k^u are bound constraints for k^{th} variable u_k, where $k = 1, \ldots, n$. Bound constraints constitute the search space, $\mathcal{S}$ for a given problem. A solution belonging

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to the search space and satisfying all imposed constraints of the problem (1) is nominated as a feasible solution; while a solution that is not feasible is called as an infeasible solution. For a given solution $\mathbf{u} \in \mathcal{S}$, the overall violation of the constraints can be expressed as the sum of the individual violations [2], [3]:

$$V(\mathbf{u}) = \sum_{j=1}^{q} |\min(Con_j(\mathbf{u}), 0)|. \tag{2}$$

Clearly, $V(\mathbf{u}) = 0$ means $\mathbf{u}$ is feasible; otherwise, it is considered as infeasible.

The objectives of a CMOP may be competing or contradictory to each other. For example in buying a car if cost and comfort are taken as objectives then cost and comfort need to be minimized and maximized, respectively. Likewise, for an economist of certain sector/industry, the common objectives are maximizing profit/benefit while minimizing loss [2]. In the stated instances, the objectives are conflicting in nature. Hence, the objectives frequently end up being at odds in multiobective optimization. Therefore, it is difficult for a single solution to concurrently optimize all the objectives. In such cases, a group of tradeoff/compromise solutions are discovered. Normally, the concept of Pareto-optimality [1], [2], [3], [4] is employed to find the best tradeoffs among the competing objectives.

A feasible vector $\mathbf{u}$ Pareto-dominates (means ''be better than'') vector $\mathbf{v}$, denoted as $\mathbf{u} \preceq \mathbf{v}$, if $\mathbf{u}$ is as good as $\mathbf{v}$ in all objectives and is strictly better in at least one objective. More precisely, A feasible solution $\mathbf{u}^*$ is *Pareto-optimal* to problem (1) if there exists no feasible solution $\mathbf{v}$ such that $\mathbf{Obj}(\mathbf{v}) \preceq \mathbf{Obj}(\mathbf{u}^*)$. $\mathbf{Obj}(\mathbf{u}^*)$ is then called a Pareto-optimal (objective) vector. When neither $\mathbf{u}$ nor $\mathbf{v}$ dominates the other, $\mathbf{u}$ and $\mathbf{v}$ are referred to as nondominated decision vectors or solutions. If no other decision vector in the decision space outperforms $\mathbf{u}$, then $\mathbf{u}$ is called Pareto optimal vector. The Pareto Set (PS) is composed of all of these Pareto optimal vectors, and its related image set, the Pareto Front (PF), is made up of all of the Pareto optimal vectors in the objective space.

Practical optimization problems are constrained multi-objective in nature. For the solutions of these problems, CMOEAs are often employed, which are combinations of some constraint handling techniques and MOEAs. Penalty function methods are frequently used by the researcher as CHTs in constrained multiobjective optimization, since they push the solutions from both feasible and infeasible regions to the PS that consequently cover the PF. In this work, an NFT based adaptive penalty (AP) with five settings is proposed as CHTs. The suggested CHTs are integrated with MOEA/D-DE that results in five constrained variants of the base algorithm for solving CMOPs. The proposed constrained algorithms are evaluated and compared with the state-of-the-art algorithms for the two benchmark suits: the CTP series and the CF series with the remark that the proposed algorithms outperformed among the competing algorithms.

The following is the arrangement and composition of the remaining portions of this paper: Section II is devoted to a survey of the literature, which briefly discusses several methods with a focus on decomposition-based approaches and penalty function methods for solving CMOPs. The proposed algorithmic framework is in Section III. The system specifications, experimental set up and performance evaluation metrics are detailed in Section IV. The experimental results, comparison, and ranking of the proposed scheme with four well-known CMOEAs and seven state-of-the-art algorithms are shown in Section V for the CTP series and the CF series, respectively. Section VI discusses the simulation results of the competing algorithms. The paper is concluded in Section VII with a brief summary of the findings and suggestions for further research.

II. LITERATURE REVIEW

In many real-life applications, the provided budget is often fixed; therefore, its efficient utilization is always desirable. In such situations, the employment of optimization schemes is essential. Leading categories of the optimization methods are traditional optimization techniques and evolutionary algorithms (EAs) or evolutionary computation (EC) techniques. Major limitations for traditional optimization methods that necessarily to be held are continuity and differentiability of the objective function (s) and subjected constraint (s). However, for EAs the stated limitations are not compulsory. EAs evolve over the course of time and do not remain static [5]. Dynamic nature of EAs enables them to tackle complex problems of every-day life where the fitness function (s) composed of diverse search space like continuous, discrete, and even discontinuous.

Mathematical modeling of most of the real-world problems are closely connected with MOPs. To solve MOPs, multiobjective evolutionary algorithms (MOEAs) are adopted most often [6]. In an MOEA, the goal is to optimize more than one objectives simultaneously (which are usually conflicting in nature or competing with each other). An MOEA can generally provide the PS consisting of optimal solutions. In broader sense, MOEAs can be classified into three categories which are mainly based on different survival/selection schemes [7], [8]: The Pareto dominance based, Indicator based and Decomposition based MOEAs.

In Pareto dominance based MOEAs, objective vectors are checked component wise for domination and solutions are divided into ranks. Highest/first rank is assigned to individuals which are not dominated by any other member. Further, the crowding distance concept is used to preserve diversity. Through this approach, every and multiple Pareto optimal solutions can be obtained in single run. However, its performance is not too much significant on problems with more than three objectives (many objectives problems). Further, in this approach, the problem is tackled as vector optimization. Examples of such MOEAs include but not limited to NSGA [9], NSGA-II [10] and NSGA-III [11]. Some other very famous MOEAs of this class are SPEA, SPEA-II [12], [13], and PAES [14].

In indicator based MOEAs, such as HypE [15] and IBEA [16], performance indicator is used to evaluate and select members in the population. Performance indicators are set in such a way that they preserve Pareto dominance relation and individuals that contributing more to the indicator are selected. One issue with this approach is when PFs are degenerate (discontinuous) or irregular, then some individuals not closer to the PF and having more contribution to the performance indicator are selected. This type of behavior misleads such MOEAs to converge to the PF [8], [15].

Finally, in decomposition based MOEAs, an MOP is converted into a series of scalar optimization subproblems, where each subproblem is a weighted aggregate of all the objectives. In this class of MOEAs, uniformly distributed weights are supplied to each subproblem, and these MOEAs solve all the subproblems concurrently to get the desired PS/PF. The most widely used decomposition-based MOEAs are MOEA/D [4], MOEA/D-DE [17], and MOEA/D-DRA [18]. These MOEAs have high search ability for combinatorial optimization [19], [20], high compatibility with local search and less time complexity. Recently some advance versions of MOEA/D are proposed like MOEA/D-APP [21] which uses angle-based adaptive penalty scheme which dynamically adjusts penalty values for each vector during the evolutionary process. MOEA/D-SCS [22] utilizes self-organizing collaborative scheme to obtain best combination of DE operator, control parameters, and neighbourhood size. Currently in MOEA/D-WVA [23], adaptive adjustment weight vector scheme is employed in MOEA/D.

So far, many decomposition-based techniques have been proposed but the weighted sum approach and the weighted Tchebycheff approach are two commonly used weight-based decomposition techniques [1].

In the following, we detail the weighted Tchebycheff technique. It is not much sensitive to the shape of PF. Also, it can find the Pareto optimal solutions in both convex and nonconvex PFs. It can be defined as follows [1]:

$$\begin{aligned} &\text{Minimize} \quad g^{te}(\mathbf{u}|\mathbf{w}, \mathbf{r}^*) = \max_{i=1}^{m}\{w_i|obj_i(\mathbf{u}) - r_i^*|\}; \quad (3)\\ &\text{Subject to} \quad \mathbf{u} \in \mathcal{F} \subset \mathcal{R}^n; \end{aligned}$$

where $\mathbf{r}^* = (r_1^*, \ldots, r_m^*)^T$ is the reference point, i.e., r_i^* is the minimum value of obj_i, $\mathbf{w} = (w_1, \ldots, w_m)^T$ is a weight vector with positive components such that $\sum_{i=1}^{m} w_i = 1$, and $\mathcal{F}$ is the region of all feasible solutions.

To solve CMOPs, MOEAs are extended to constrained MOEAs (CMOEAs), where a CHT is combined with them to tackle constraints. To handle constraints in single objective optimization, interested readers are referred to [24]. Penalty function methods are simple and widely used techniques among other CHTs. The first penalty function method was proposed in 1940 by Courant and Carroll, which is further improved by Fiacco and McCormick [24]. In a penalty function method, a constrained optimization problem is converted to the corresponding unconstrained optimization problem by adding (in case of minimization) or subtracting (in case of maximization) a penalty term to/from the objective function of the given problem. The penalty term is normally equal to the scaled amount of the constraints' violations of a given solution vector. The scaling factor is generally called penalty factor/coefficient/parameter. The general form of the objective of a penalty function method is given below [24]:

$$Obj_p(\mathbf{u}) = Obj(\mathbf{u}) + \sum_{j=1}^{q} r_j \times |\min(Con_j(\mathbf{u}), 0)|, \quad (4)$$

where Obj_p is the new (expanded) objective function/fitness function to be optimized and $\sum_{j=1}^{p} r_j \times |\min(Con_j(\mathbf{u}), 0)|$ is the penalty term for $\mathbf{u}$, where r_j is the penalty parameter. There are three commonly used mechanisms of penalty function methods [24]: static, dynamic and adaptive. In a static penalty function method, constant values of r_js are employed. In a dynamic penalty function method, r_js are updated with generation/iteration counter. While, in an adaptive penalty (AP) function method, r_js are updated based on the search history. A major issue in penalty function methods is how to set the penalty parameters/coefficients, r_js, since they are problem depended. High values of r_js greatly penalize infeasible solutions. As a result, the search process converges to local optima and stuck there. On the other hand, small values of r_js less penalize infeasible solutions. Consequently, the search focuses on exploring more the infeasible region and might diverge. Hence, a common answer is to tune them as per problem at hand, which is not desirable. The AP techniques are promising, since they tune the penalty parameters on the go while learning from the search history. This work introduces an AP with the following details.

Smith and Tate [25] proposed a near feasibility threshold (NFT) based adaptive penalty function to address constrained single objective optimization problem. In this work, the usage of stated penalty is extended to solve CMOPs. Mathematical formulation for it is given as follows:

$$g^{te}_{ap}(\mathbf{u}) = g^{te}(\mathbf{u}) + (g^{te}_{feas} - g^{te}_{all})\left(\frac{V(\mathbf{u})}{NFT}\right)^k, \quad (5)$$

where $g^{te}_{ap}(\mathbf{u})$ is the penalized Tchebycheff value, $g^{te}(\mathbf{u})$ is the un-penalized Tchebycheff value, g^{te}_{all} is the overall found minimum Tchebycheff value irrespective of constraints and g^{te}_{feas} is the minimum Tchebycheff value of feasible solutions of the current population. However, if the current population does not contain any feasible solution, then the Tchebycheff value of the solution with minimum overall constraints's violation is assigned to g^{te}_{feas}, k is the severity constant (a value of $k = 2$ is adopted which has been previously suggested by Coit and Smith [26]), and NFT is the distance from the feasible region where infeasible solutions are considered reasonable. Infeasible solutions at NFT distance are normally allowed, along with other feasible solutions, to propagate in the next generation and produce offspring as compared to infeasible solutions beyond NFT distance.

In case of all the solutions of running population become feasible then $g^{te}_{feas} = g^{te}_{all}$ which reflects that $g^{te}_{feas} - g^{te}_{all} = 0$, and subsequently the penalty term becomes zeros. When infeasible solutions come into consideration then $g^{te}_{feas} - g^{te}_{all} > 0$ indicates that penalty term affects the penalized Tchebycheff value of infeasible solutions. If g^{te}_{all} and g^{te}_{feas} are much different from each other, then the difference $g^{te}_{feas} - g^{te}_{all}$ results in a large value. This results in severe penalties for the infeasible solutions. Since NFT determines the infeasible region to be explored well along with the feasible region, a suitable value for it is desired. This work fully investigates the sensitivity of the adaptive penalty function, Eq.(5) with different settings of NFT which are presented as follows:

$$NFT_1 = 3\% \times V(\mathbf{u}), \tag{6}$$

$$NFT_2 = 5\% \times V(\mathbf{u}), \tag{7}$$

$$NFT_3 = 7\% \times V(\mathbf{u}), \tag{8}$$

$$NFT_4 = \frac{NFT_0}{1+\mu t}, \tag{9}$$

$$NFT_5 = V_{min} + s(V_{max} - V_{min}), \tag{10}$$

where $V(\mathbf{u})$ is defined in Eq. (2), NFT_0 represents mean of the overall constraints's violations of the solutions in the current population, $V_{min} = Min\{V(\mathbf{u_i})\}$, $V_{max} = Max\{V(\mathbf{u_i})\}$, where $i = 1, 2, 3, \cdots, N$ (population size), $\mu = 0.2$, $s = 0.3$, and t is iteration counter. Main purpose of introducing various settings for NFT is to examine its impact in the proposed adaptive penalty platform.

III. PROPOSED ALGORITHMIC FRAMEWORK

In this work, the extended NFT based AP, Eq. (5) with five different settings as given in Eqs.(6-10) is employed in the framework of MOEA/D-DE for competing and selecting solutions for next generations. This results in five different constrained variants of MOEA/D-DE, namely CMOEA/D-TAP1 to CMOEA/D-TAP5 associated to the defined $NFTs$, respectively for solving CMOPs. In MOEA/D-DE new position for current solution is generated based on the following mathematical formulations:

$$v'_{ij} = \begin{cases} u^t_{ij} + F(u^t_{r_1 j} - u^t_{r_2 j}), & \text{if } r < pCR \text{ or } I_r = j, \\ u^t_{ij}, & \text{otherwise,} \end{cases} \tag{11}$$

where $r \in (0, 1)$ is a uniformly randomly generated number, $I_r \in \{1, 2, 3, \cdots, D\}$ is randomly drawn integer, r_1 and r_2 are stochastically taken solutions from the population, and $F = 0.5$ and $pCR = 0.95$ are two control parameters [27].

$$v_{ij} = \begin{cases} v'_{ij} + \sigma_i \times (u^u_{ij} - u^l_{ij}), & \text{if } rand < p_m \\ v'_{ij}, & \text{otherwise,} \end{cases} \tag{12}$$

$$\sigma_i = \begin{cases} (2 \times rand)^{\frac{1}{\eta+1}} - 1, & \text{if } rand < 0.5 \\ 1 - (2 - 2 \times rand)^{\frac{1}{\eta+1}}, & \text{otherwise,} \end{cases} \tag{13}$$

where $rand \in (0, 1)$ is a uniformly generated random number, u^u_{ij} and u^u_{ij} are lower and upper bounds for j^{th} variable of i^{th} solution, $p_m = 1/n$ (n = problem dimensionality) is the probability of mutation, and $\eta = 20$ is the distribution index. The pseudo code of newly designed variants is presented in Algorithm 1; while the schematic diagram of the proposed algorithm is displayed in Figure. 1.

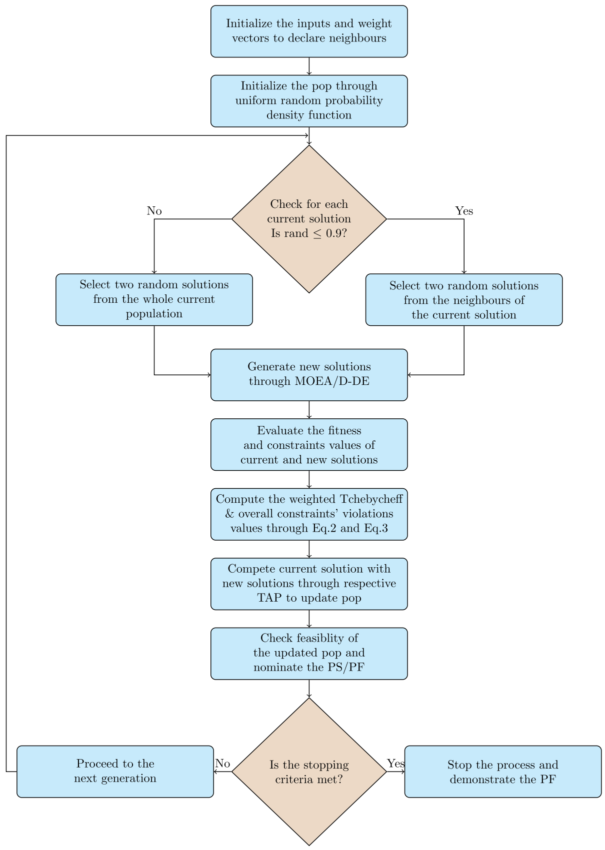

FIGURE 1. Schematic diagram of the proposed algorithm.

IV. SYSTEM SPECIFICATIONS, EXPERIMENTAL SET UP AND PERFORMANCE METRICS

A. SYSTEM SPECIFICATIONS

The computer system utilized for experiment execution has Windows 10 installed, 8 GB of RAM, and an Intel(R) Core(TM) i3-7100U CPU running at 2.40 GHz. Software used for programming is MATLAB R2022b.

B. EXPERIMENTAL SET UP

For performance evaluation, well-known CMOPs series of CTP and CF are selected. Since, the problems of CTP series were not available in PlatEMO. So the codes of proposed algorithms are developed separately in MATLAB to perform experiments on the stated instances and obtained simulation results are compared with four best performances. While for CF series the codes of introduced variants are embedded into the platform PlatEMO [28] and compared with seven state-of-the-art algorithms. Details of the parameters' settings adopted during simulations are as follows:

1) PARAMETERS' SETTINGS

- Population size for CTP series: $N = 200$;

Algorithm 1 Pseudocode of the Proposed Algorithm

Inputs: Population Size $= N$, Max Function Evaluations $= maxFE$, Neighbourhood Size $= T$, Maximal number of neighbours $= n_r$;
Outputs:
Pareto Set: $PS = \{\mathbf{u}_1, \ldots, \mathbf{u}_N\}$;
Pareto Front: $PF = \{\mathbf{Obj}(\mathbf{u}_1), \ldots, \mathbf{Obj}(\mathbf{u}_N)\}$;
Set Function Evaluation counter$= nFE \leftarrow 0$;
Generate set of N weight vectors $\mathbf{W} \leftarrow \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_N\}$;
Find $\{\mathbf{w}_{i1}, \mathbf{w}_{i2}, \ldots, \mathbf{w}_{iT}\}$ and $I(i) = \{i_1, i_2, \ldots, i_T\}$ according to the procedure defined in IV-B2 for each solution;
Generate initial population $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_N\}$ uniformly randomly;
Evaluate each solution $\mathbf{u}_i$ to find $\mathbf{Obj}(\mathbf{u}_i)$ and $V(\mathbf{u}_i)$;
Increase nFE by N;
Calculate reference vector $\mathbf{r}^* = min\{\mathbf{Obj}(\mathbf{u}_i), i = 1, 2, \cdots, N\}$;
while $nFE < maxFE$ **do**
  for $i = 1 : N$ **do**
    if $rand < \delta$ **then**
      Extract the indices of neighboring parents NP from $I(i)$;
    else
      Extract the indices of neighboring parents NP from $\{1, 2, 3, \cdots, N\}$;
    end if
    Select randomly two indices r_1 and r_2 from respective NP;
    Employ Eq. (11) to get v_i' and clip it through boundary conditions;
    Use Eqs. (12-13) to get v_i and clip it through boundary conditions;
    Evaluate v_i to find $\mathbf{Obj}(v_i)$ and $V(v_i)$;
    Update $\mathbf{r}^* = min\{\mathbf{Obj}(v_i), \mathbf{r}^*\}$;
    Calculate $g^{te}(v_i|\mathbf{w}_l, \mathbf{r}^*)$ and $g^{te}(\mathbf{u}_{NP}|\mathbf{w}_l, \mathbf{r}^*)$
    Find $g^{te}_{ap}(v_i|\mathbf{w}_l, \mathbf{r}^*)$ and $g^{te}_{ap}(\mathbf{u}_{NP}|\mathbf{w}_l, \mathbf{r}^*)$ according to Eq. 5 with defined NFT rule of the proposed variant, for NFT_4 $t = \frac{nFE}{N}$;
    if $any(g^{te}_{ap}(v_i|\mathbf{w}_l, \mathbf{r}^*) < g^{te}_{ap}(\mathbf{u}_{NP}|\mathbf{w}_l, \mathbf{r}^*))$ **then**
      Find $Indices = g^{te}_{ap}(v_i|\mathbf{w}_l, \mathbf{r}^*) < g^{te}_{ap}(\mathbf{u}_{NP}|\mathbf{w}_l, \mathbf{r}^*)$;
      $u_r = NP(Indices)$;
      if $length(u_r) > n_r$ **then**
        $u_r = randsample(u_r, n_r)$;
      end if
      Update PS and PF as $\mathbf{u}(u_r) = v_i$ and $\mathbf{Obj}(\mathbf{u}(u_r)) = \mathbf{Obj}(v_i)$
    end if
  end for
  Set $nFE = nFE + N$;
end while
Get outputs PS and PF;

- Population size for CF series: $N = 600$ for problems with two objectives (CF1-CF7) and $N = 1000$ for problems with three objectives (CF8-CF10);
- Neighbourhood size for both CTP series and CF series: $T = 0.1N$;
- Update number: $n_r = 0.01N$;
- Number of decision variables or problem dimensionality: $n = 2$ for CTP series and $n = 10$ for CF series;
- Probability to update/replace solution in neighbourhood: $\delta = 0.9$;
- Probability to update/replace solution in whole population is $\delta = 0.1$;
- Maximal generations: $MaxIt = 200$ for CTP series, $MaxIt = 500$ for two objectives CF series and $MaxIt = 300$ for three objectives CF series;
- Stopping Criteria: The algorithm stops after function evaluation counter reaches to $MaxFeval$. $MaxFeval = 40,000$ for CTP series and $MaxFeval = 300,000$ for CF series;
- Number of Runs: $runs = 30$ for both CTP series and CF series;

2) WEIGHT VECTORS GENERATION

The following criteria is adopted to generate a set W of N weight vectors [3], [29] for CTP series:

1) Initialize the set W of weight vectors with set $\{(1, 0, \ldots, 0, 0), (0, 1, \ldots, 0, 0), \ldots, (0, 0, \ldots, 0, 1)\}$ and then uniformly and randomly generate a set W_1 of 5000 weight vectors, where every component belongs to the interval (0, 1);
2) Choose the vector in W_1 with the largest distance to W. Add it to W and delete it from W_1;
3) Repeat the Step 2 until the size of W equals to N.

For CF series the weights vectors are generated as per the internal settings of the used platform PlatEMO.

C. PERFORMANCE METRICS: HYPERVOLUME AND INVERTED GENERATIONAL DISTANCE

This work uses the HV values statistics for comparison of CTP series; while for CF series, both HV and IGD values metric statistics are employed for performance evaluation of the proposed algorithms. Details of the utilized matrices are as follows:

1) HYPERVOLUME METRIC (HV)

The HV metric is used to evaluate the convergence and diversity of the obtained approximate PF to the real PF. It is defined as follows [30], [31]: Let P be the set of non-inferior/non-dominated objective vectors which approximates the real PF and $R \in \mathbb{R}^m$ is a reference point in the objective space, then $HV(R, P)$ is the Lebesgue measure of the region in the objective space weakly dominated by P and bounded above by point R, i.e.,

$$HV(R, P) = Volume(\bigcup_{i=1}^{|P|} v_i), \tag{14}$$

where v_i is the hypercube with i^{th} element of P and R as diagonal corners. The HV is calculated by taking the union of the volumes of all these hypercubes. For two different approximating data sets to the real PF and with same reference point R, the one with greater HV value determines better approximation to real PF. In this work, $R = (2, 20)$ for CTP6 and CTP8, and $R = (2, 2)$ for other CTP series test instances, CTP1-CTP5 and CTP7, are used.

2) INVERTED GENERATIONAL DISTANCE METRIC (IGD)

The IGD metric value of the two sets P^* and P is defined as follows [4]:

$$D(P^*, P) = \frac{\sum_{p \in P^*} d(p, P)}{|P^*|},$$

where P^* is the set of points sampled uniformly from the real PF, while P is the set of approximated PF containing non-dominated objective vectors that are obtained through the proposed algorithm. Further, $d(p, P)$ is the minimum Euclidean distance between p and P. $D(P^*, P)$ could determine the convergence and diversity of the algorithm provided that P^* is large enough to cover the real PF very well.

V. SIMULATION RESULTS, COMPARISON AND RANKING OF ALGORITHMS

In this part, the simulation results of the suggested algorithms, CMOEA/D-TAP1-CMOEA/D-TAP5, are compared and ranked with the best-performances that are designed for CTP and CF series.

A. COMPARISON FOR CTP SERIES

For performance evaluation, NSGA-II, IDEA, MOEA/D-SR [29], where stochastic ranking [32], [33] as a CHT is embedded into MOEA/D, MOEA/D-CDP [29], and CMOEA/D-TAP1 to CMOEA/D-TAP5 are compared. Proposed algorithms are run in MATLAB environment 30 times independently for each of the CTP test instance; however, the results of remaining algorithms are taken directly from their respective papers. TABLE 1 displays the HV metric values statistics: Best, Mean, and Standard Deviation (SD) of the competitors. Based on these statistics, a Rank Point (RP), which is equal to the relative position of an algorithm in the competition is assigned for each CTP test instance and is presented besides it in the parentheses. Figures 2-3 show the convergence graphs of mean feasible ratio and mean HV values, respectively of the contestants. Figures 4-5 and 6-7 demonstrate the best run and all runs PFs of each algorithm.

B. COMPARISON FOR CF SERIES

For CF series, following seven peer algorithms from the literature are selected to evaluate the performances of the proposed variants.

1) ANSGAIII

Adaptive Non-dominated Sorting Genetic Algorithm known as ANSGAIII [34] is the extended constrained version of NSGAIII [11]. While NSGAIII is the modified form of NSGAII [10] which was designed to solve unconstrained Many (more than three) Objectives Optimization Problems (MaOPs). Crowding distance selection scheme in NSGAII is replaced in NSGAIII by adaptively updated reference points mechanism to improve the diversity among the obtained solutions. To solve CMOPs as well as CMaOPs, ANSGAIII was proposed which uses constraint domination principle (CDP) [35] as CHT and modified tournament selection in the framework of NSGAIII.

2) BiCo

Bidirectional Coevolution CMOEA (BiCo) [36], a novel CHT is used to solve CMOPs through evolving two populations, namely main population and archive population. Each of them pushes the solutions towards PF of CMOP from feasible and infeasible sides, respectively. Main population is updated using NSGAII and a variant of CDP; while angle based selection scheme is designed to update archive population.

3) CMOEAD

The algorithm CMOEAD [37] is the constrained version of MOEAD. In MOEAD, selection operator compares parents and offspring based on an aggregation function. However in CMOEAD, two solutions are compared as: if both solutions have overall constraint violation less than or equal to a violation threshold, then comparison is made based on objective values (aggregation function values); otherwise, comparison is made based on scaled constraint violations. The violation threshold is obtained through adaptive setting of the feasibility ratio.

4) CAEAD

To balance convergence, diversity, feasibility and avoid trapping in local optimal solutions, dual-population algorithm based on alternative evolution and degeneration (CAEAD) [38] uses dual populations (main and secondary) in two stages. The first stage is the evolution stage and the second stage is the degenerate stage. Initially, in the evolution stage, secondary population is forced towards unconstrained PF with the aim to get better convergence while avoiding to trap in local optimal fronts. The main population, on the other hand, focuses on feasibility. While in the second stage, both populations converge towards constrained PF with maintaining better diversity and coverage.

5) CCMO

Coevolutionary Constrained Multi-Objective Optimizer (CCMO) framework [39] is mainly concerned to solve CMOPs while using two populations which evolve separately. The main population is associated with the original CMOP and the other population is associated with a helper problem which is comparatively easier to tackle. The second population provides assistance to the first one to guide it toward the PF of CMOP. Unlike other algorithms evolving

TABLE 1. Comparison of algorithms for CTP series.

P.No	Statistics	NSGA-II	IDEA	SR	CDP	CMOEA/D-TAP1	CMOEA/D-TAP2	CMOEA/D-TAP3	CMOEA/D-TAP4	CMOEA/D-TAP5
CTP1	Best	-	-	2.7609(7)	2.7647(1)	2.7647(1)	2.7647(1)	2.7647(1)	2.7646(6)	2.7647(1)
	Mean	-	-	2.7564(7)	2.7646(1)	2.7643(5)	2.7641(6)	2.7645(2)	2.7644(3)	2.7644(3)
	SD	-	-	0.0033(7)	0.0001(1)	0.0007(5)	0.0009(6)	0.0005(2)	0.0006(3)	0.0006(3)
CTP2	Best	3.0593(3)	3.0592(7)	3.0431(9)	3.0595(1)	3.0592(7)	3.0594(2)	3.0593(3)	3.0593(3)	3.0593(3)
	Mean	2.8707(9)	3.0114(8)	3.0369(7)	3.0594(1)	3.0577(5)	3.0576(6)	3.0579(3)	3.0581(2)	3.0579(3)
	SD	0.2701(9)	0.1771(8)	0.0037(7)	0.0001(1)	0.0015(5)	0.0016(6)	0.0013(2)	0.0014(3)	0.0014(3)
CTP3	Best	3.0104(8)	3.0160(7)	2.9802(9)	3.0282(1)	3.0247(5)	3.0252(3)	3.0254(2)	3.0241(6)	3.0251(4)
	Mean	2.8281(9)	2.9608(7)	2.9551(8)	3.0238(1)	3.0158(6)	3.0176(3)	3.0173(4)	3.0162(5)	3.0185(2)
	SD	0.2547(9)	0.1638(8)	0.0093(7)	0.0024(1)	0.0038(3)	0.0048(5)	0.0035(2)	0.0049(6)	0.0041(4)
CTP4	Best	2.8485(8)	2.9190(3)	2.7170(9)	2.9761(1)	2.9154(4)	2.8917(7)	2.9284(2)	2.9114(5)	2.9034(6)
	Mean	2.4381(9)	2.7447(2)	2.6024(8)	2.9147(1)	2.6364(7)	2.6445(6)	2.6480(4)	2.6460(5)	2.6567(3)
	SD	0.3527(9)	0.1393(3)	0.0512(2)	0.0342(1)	0.1860(8)	0.1569(4)	0.1799(7)	0.1780(6)	0.1666(5)
CTP5	Best	3.0209(8)	3.0247(7)	2.9891(9)	3.0382(1)	3.0343(2)	3.0334(4)	3.0329(5)	3.0327(6)	3.0342(3)
	Mean	2.7235(9)	2.9529(7)	2.9498(8)	3.0302(1)	3.0196(4)	3.0194(6)	3.0251(2)	3.0196(4)	3.0235(3)
	SD	0.2926(9)	0.1621(8)	0.0203(7)	0.0062(1)	0.0136(6)	0.0122(5)	0.0071(2)	0.0108(4)	0.0103(3)
CTP6	Best	36.8227(1)	36.8191(2)	29.8372(8)	19.4142(9)	36.8170(6)	36.8169(7)	36.8176(5)	36.8190(3)	36.8184(4)
	Mean	36.1829(7)	36.7878(6)	24.8577(8)	19.4142(9)	36.8083(2)	36.8020(4)	36.8096(1)	36.8002(5)	36.8027(3)
	SD	2.1873(9)	0.0758(7)	1.9411(8)	0.0000(1)	0.0097(3)	0.0280(5)	0.0091(2)	0.0438(6)	0.0279(4)
CTP7	Best	3.6177(1)	3.6177(1)	3.6154(8)	3.6125(9)	3.6174(3)	3.6174(3)	3.6174(3)	3.6174(3)	3.6174(3)
	Mean	3.2402(9)	3.4359(8)	3.6129(6)	3.6124(7)	3.6171(3)	3.6171(3)	3.6172(1)	3.6170(5)	3.6172(1)
	SD	0.5941(8)	0.5945(9)	0.0020(7)	0.0000(1)	0.0004(5)	0.0003(4)	0.0002(3)	0.0005(6)	0.0001(2)
CTP8	Best	36.1708(8)	36.1804(6)	39.5436(1)	19.5498(9)	36.1815(3)	36.1809(4)	36.1817(2)	36.1804(6)	36.1807(5)
	Mean	32.0859(8)	35.9706(7)	39.0566(1)	19.5498(9)	36.1598(4)	36.1581(5)	36.1601(3)	36.1375(6)	36.1690(2)
	SD	5.1763(9)	0.4345(8)	0.4327(7)	0.0000(1)	0.0360(4)	0.0494(5)	0.0326(3)	0.0701(6)	0.0230(2)

TABLE 2. Algorithms ranking using their total rank points.

Statistics	NSGA-II	IDEA	SR	CDP	CMOEA/D-TAP1	CMOEA/D-TAP2	CMOEA/D-TAP3	CMOEA/D-TAP4	CMOEA/D-TAP5
Best	37	33	53	31	30	30	22	32	28
Mean	60	45	46	29	31	33	18	32	17
SD	62	51	45	7	34	34	21	37	23
TRPs	159	129	144	67	95	97	61	101	68
Rank	9	7	8	2	4	5	1	6	3

multi populations, the interaction among the populations in CCMO is very weak. Further, CCMO uses NSGAII together with CDP as an optimizer.

6) CMOEA-MS

CMOEA-MS [40] uses multi stage strategy to get better approximation of the constrained PF of CMOPs. This uses two stages A and B during evolution. A predefined parameter λ is used to decide which stage to be used. Each stage uses different evaluation strategy. If the feasibility ratio of the combined population is less than λ, then the population undergos through stage A, where objective values and constraints' violations are given same priority, else Stage B is utilized which gives more priority to feasibility and this stage is used to maintain a good diversity among the obtained solutions.

7) ToP

Two-Phase (ToP) framework is proposed in [41]. Which simultaneously addressing constraints in decision and

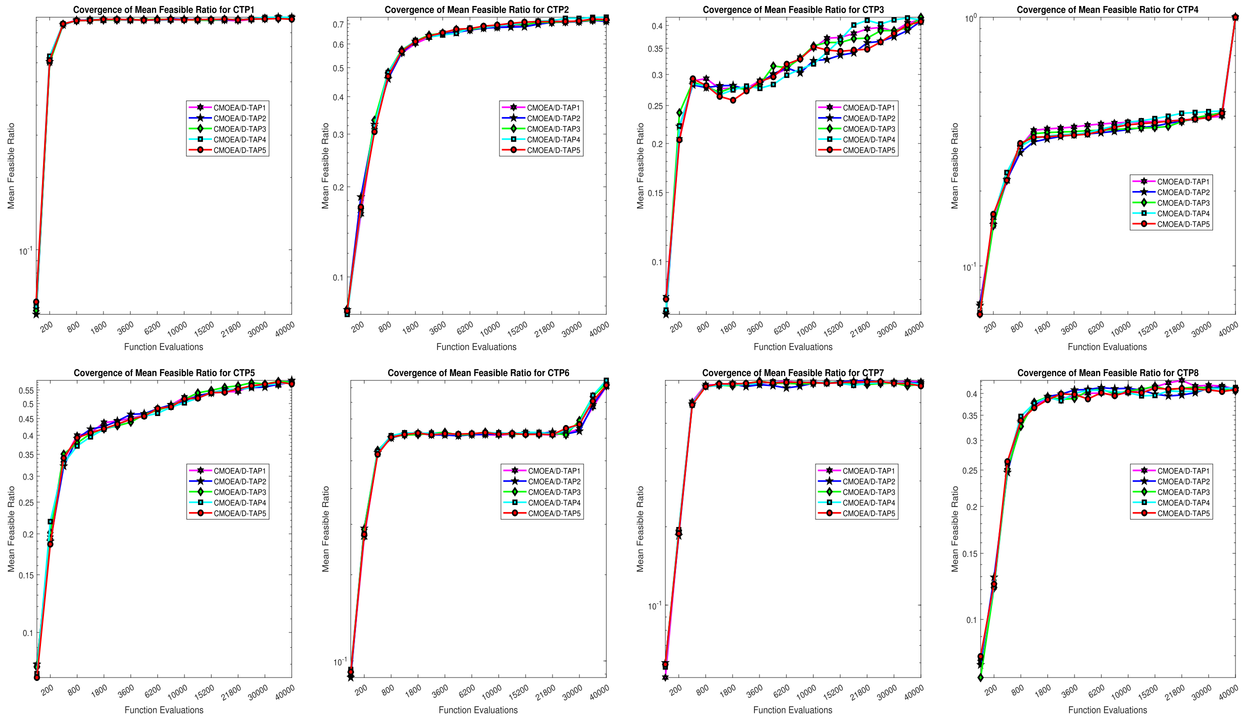

FIGURE 2. Convergence plots of mean feasibility ratio of the proposed algorithms for CTP series.

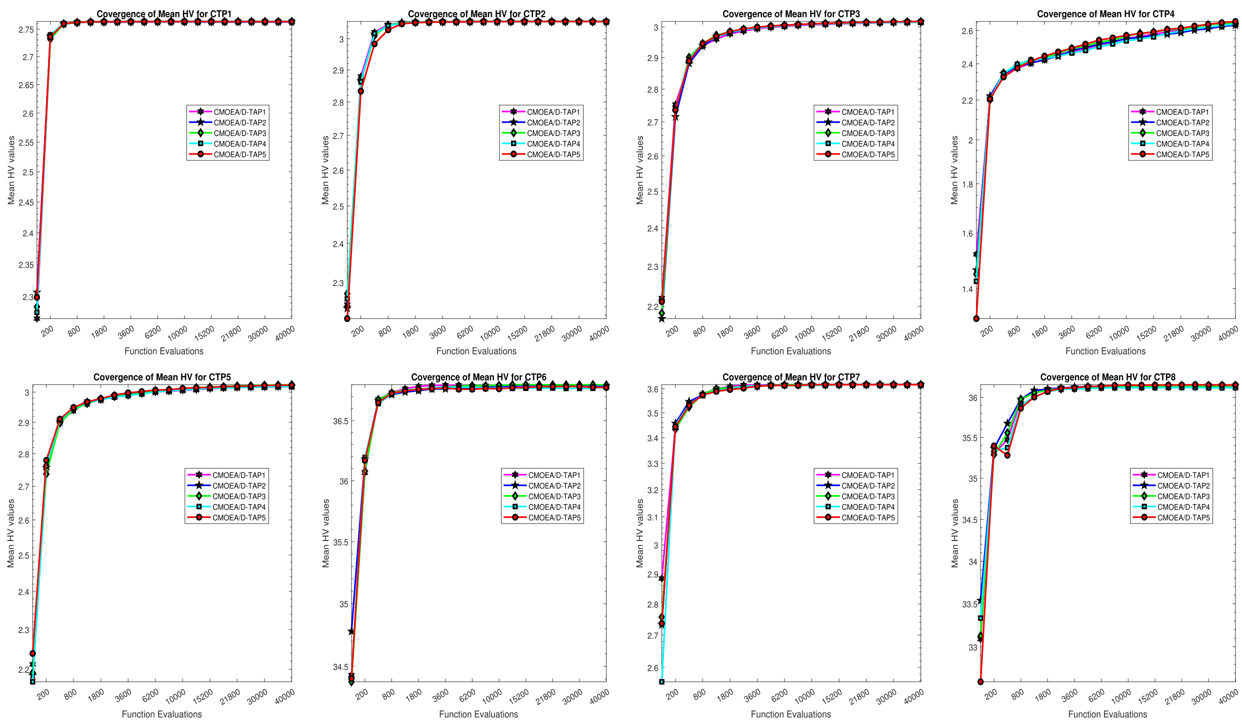

FIGURE 3. Convergence plots of mean HV of the proposed algorithms for CTP series.

objective spaces of CMOPs. In the first phase, a CMOP is converted into constrained single-objective problem to find promising feasible region. Then, a specific CMOEA is used to obtain final solutions.

The simulations for CF series are executed using MATLAB platform, namely, PlatEMO [28]. The MATLAB codes of aforementioned seven algorithms used for comparisons are available in PlatEMO; while codes of the proposed variants

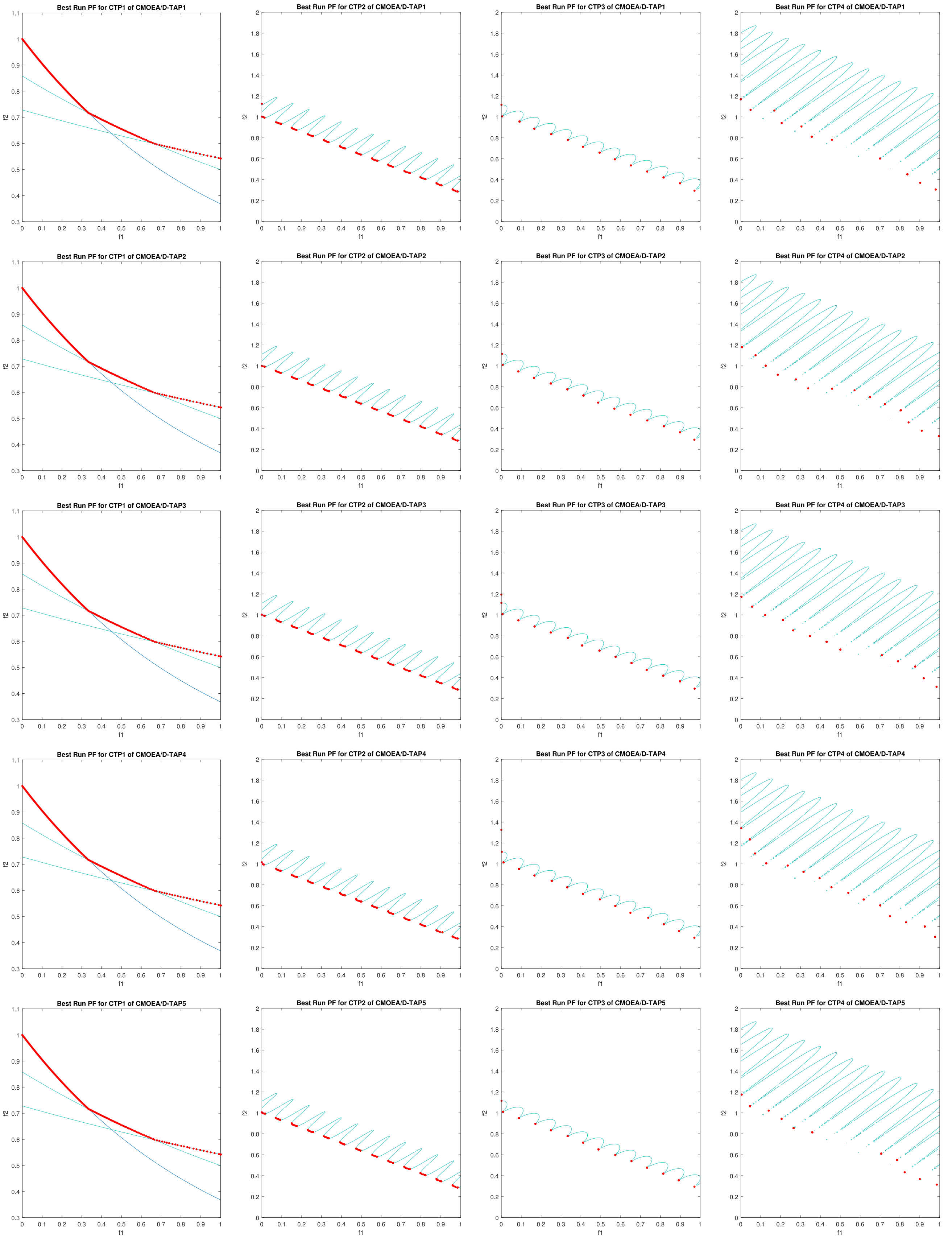

FIGURE 4. Best Run PF of the proposed algorithms for CTP1-CTP4.

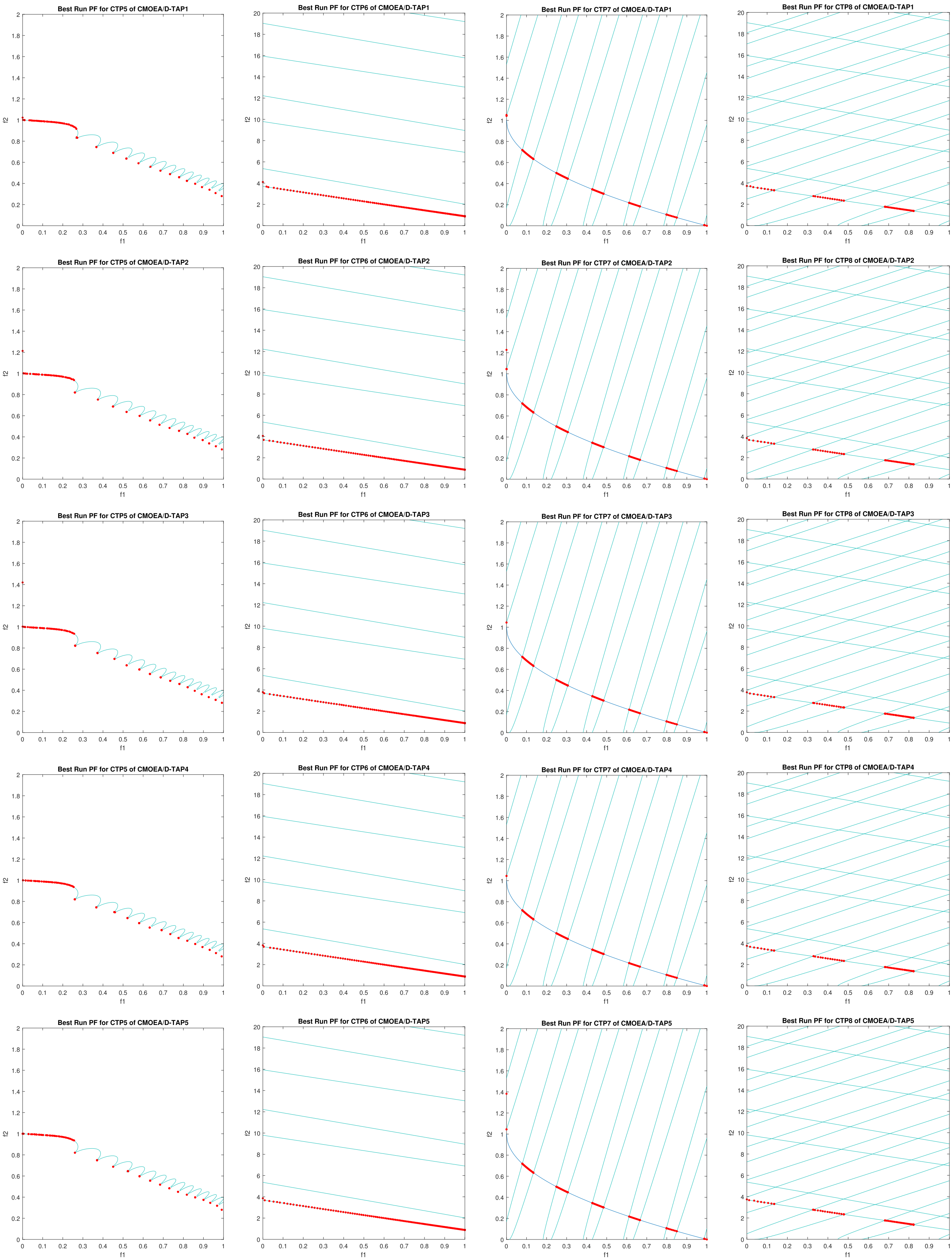

FIGURE 5. Best Run PF of the proposed algorithms for CTP5-CTP8.

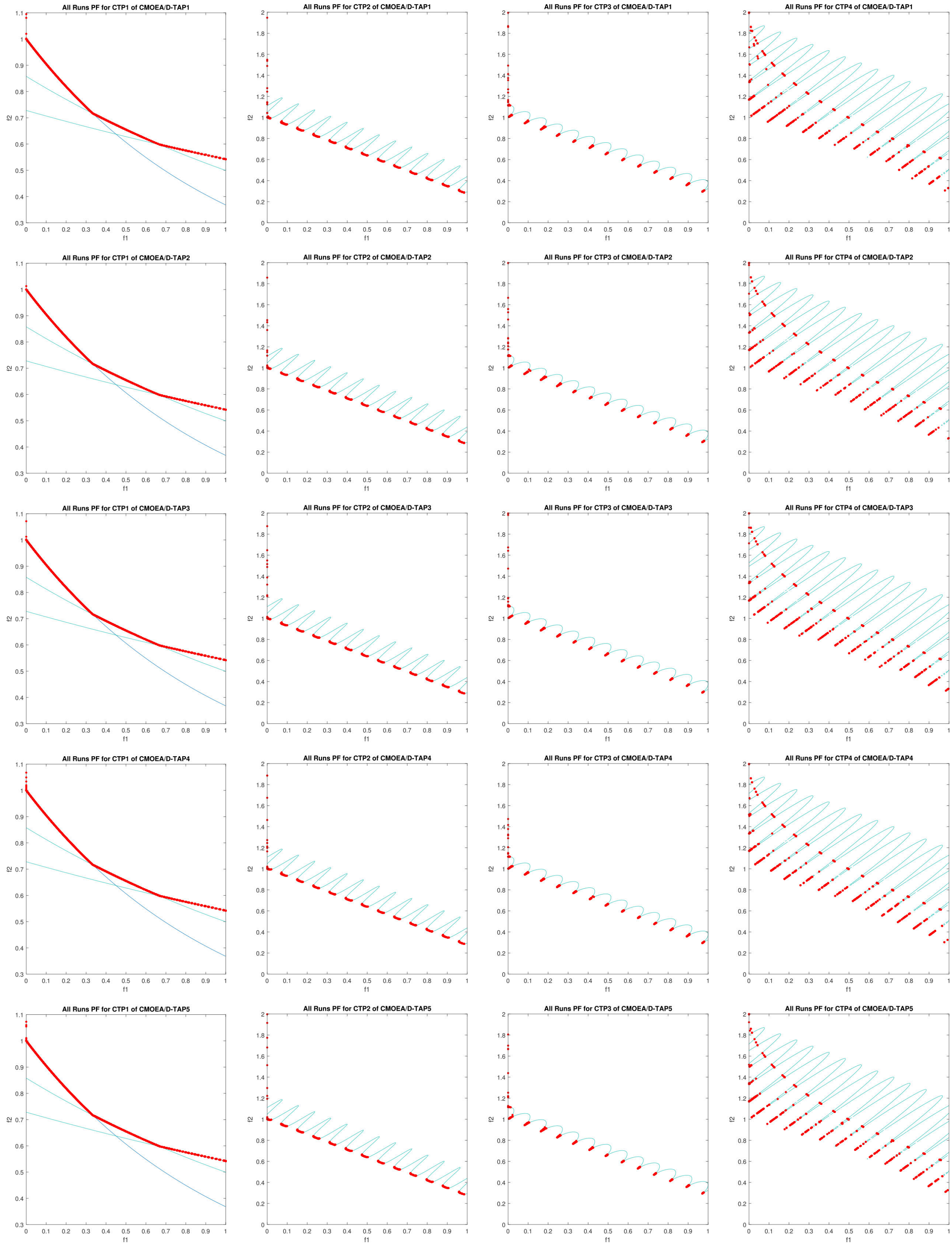

FIGURE 6. All Runs PF of the proposed algorithms for CTP1-CTP4.

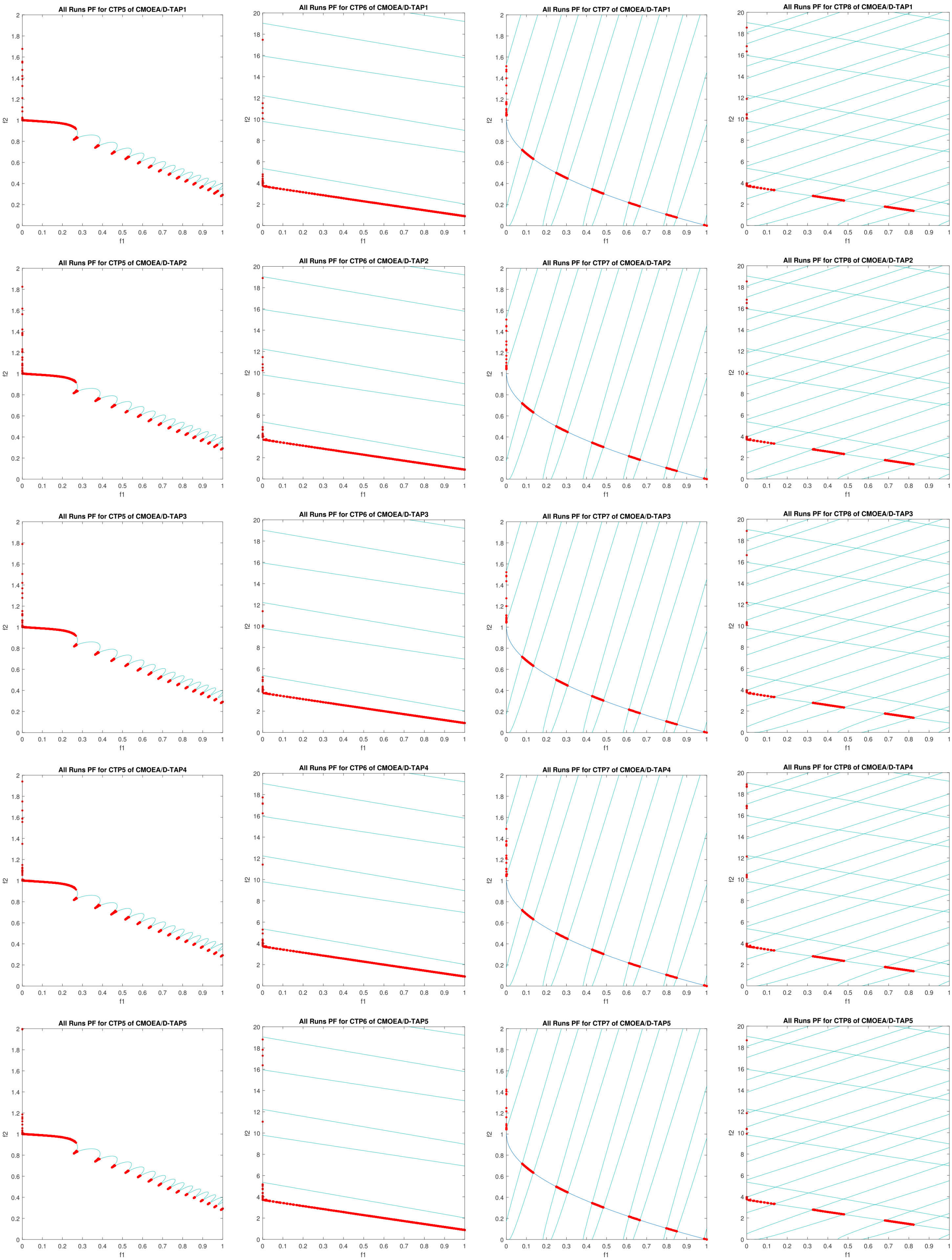

FIGURE 7. All Runs PF of the proposed algorithms for CTP5-CTP8.

TABLE 3. Wilcoxon rank sum test results of proposed vs state-of-the-arts algorithms using IGD values.

Problem	M	D	ANSGAIII	BiCo	CMOEAD	CAEAD	CCMO	CMOEA_MS	ToP	CMOEA/D-TAP1
CF1	2	10	1.4903e-2 (1.86e-3) −	1.0775e-2 (9.26e-4) +	9.5536e-3 (2.43e-3) +	1.2709e-3 (4.42e-3) +	2.7767e-3 (2.68e-4) ≈	1.3136e-2 (4.52e-3) +	8.0644e-3 (1.25e-3) +	1.3848e-2 (4.34e-2)
CF2	2	10	2.1753e-2 (8.67e-3) −	2.1238e-2 (9.13e-3) −	1.1475e-1 (5.13e-2) −	6.5803e-3 (1.79e-3) ≈	1.7322e-2 (8.07e-3) −	2.0850e-2 (8.56e-3) −	4.6186e-3 (7.08e-3) +	9.5087e-3 (1.04e-2)
CF3	2	10	2.2954e-1 (1.17e-1) ≈	2.0203e-1 (1.00e-1) ≈	5.7348e-1 (2.21e-1) −	5.4079e-1 (3.57e-2) −	2.3393e-1 (1.06e-1) ≈	2.4653e-1 (8.85e-2) ≈	5.2020e-1 (8.12e-2) −	2.1887e-1 (1.12e-1)
CF4	2	10	5.8099e-2 (1.10e-2) −	5.8816e-2 (1.14e-2) −	2.5711e-1 (1.07e-1) −	5.6385e-2 (3.85e-3) −	5.3180e-2 (1.25e-2) −	7.5274e-2 (3.12e-2) −	3.7019e-2 (4.84e-3) −	2.0687e-2 (2.22e-2)
CF5	2	10	2.0669e-1 (9.37e-2) ≈	1.8538e-1 (1.03e-1) +	4.0101e-1 (9.74e-2) −	1.5602e+0 (2.49e-1) −	1.9068e-1 (8.66e-2) +	2.6704e-1 (1.36e-1) ≈	4.6932e-1 (1.19e-1) −	2.1990e-1 (7.33e-2)
CF6	2	10	5.5901e-2 (3.16e-2) ≈	4.7785e-2 (2.04e-2) ≈	1.6813e-1 (5.83e-2) −	3.3979e-2 (2.84e-3) +	2.9657e-2 (7.61e-3) +	6.2408e-2 (3.27e-2) −	4.7269e-2 (2.68e-2) ≈	3.9852e-2 (2.09e-2)
CF7	2	10	1.6714e-1 (8.17e-2) +	1.6320e-1 (3.77e-2) +	4.4969e-1 (1.12e-1) −	1.2688e+0 (2.77e-1) −	1.5998e-1 (3.64e-2) +	1.8362e-1 (6.96e-2) +	2.9831e-1 (1.41e-1) ≈	2.1947e-1 (7.91e-2)
CF8	3	10	1.7030e-1 (8.53e-2) −	1.0310e+0 (1.10e+0) −	1.2052e-1 (2.97e-2) −	2.2910e-1 (9.73e-2) −	8.3529e-2 (2.84e-3) −	1.3848e-1 (2.08e-2) −	3.4011e-1 (8.47e-2) −	7.2318e-2 (6.79e-3)
CF9	3	10	3.9905e-2 (3.04e-3) −	3.8180e-2 (1.76e-3) −	5.5410e-2 (1.72e-2) −	6.3857e-2 (3.98e-3) −	3.6183e-2 (2.56e-3) −	3.6832e-2 (2.42e-3) −	5.9408e-2 (4.40e-3) −	2.3488e-2 (5.95e-4)
CF10	3	10	5.9293e-1 (1.05e-1) −	1.6792e+1 (8.92e+0) −	4.5022e-1 (2.98e-2) −	7.1372e+0 (7.22e+0) −	1.6063e-1 (5.61e-2) ≈	1.4304e-1 (9.29e-2) ≈	6.3285e+0 (8.58e+0) −	1.7349e-1 (1.42e-1)
+/ − / ≈			1/6/3	3/5/2	1/9/0	2/7/1	3/4/3	2/5/3	2/6/2	

Problem	M	D	ANSGAIII	BiCo	CMOEAD	CAEAD	CCMO	CMOEA_MS	ToP	CMOEA/D-TAP2
CF1	2	10	1.4903e-2 (1.86e-3) −	1.0775e-2 (9.26e-4) −	9.5536e-3 (2.43e-3) −	1.2709e-3 (4.42e-3) +	2.7767e-3 (2.68e-4) ≈	1.3136e-2 (4.52e-3) −	8.0644e-3 (1.25e-3) −	2.8479e-3 (1.06e-3)
CF2	2	10	2.1753e-2 (8.67e-3) −	2.1238e-2 (9.13e-3) −	1.1475e-1 (5.13e-2) −	6.5803e-3 (1.79e-3) ≈	1.7322e-2 (8.07e-3) −	2.0850e-2 (8.56e-3) −	4.6186e-3 (7.08e-3) +	7.1212e-3 (4.56e-3)
CF3	2	10	2.2954e-1 (1.17e-1) ≈	2.0203e-1 (1.00e-1) ≈	5.7348e-1 (2.21e-1) −	5.4079e-1 (3.57e-2) −	2.3393e-1 (1.06e-1) ≈	2.4653e-1 (8.85e-2) ≈	5.2020e-1 (8.12e-2) −	2.3337e-1 (1.08e-1)
CF4	2	10	5.8099e-2 (1.10e-2) −	5.8816e-2 (1.14e-2) −	2.5711e-1 (1.07e-1) −	5.6385e-2 (3.85e-3) −	5.3180e-2 (1.25e-2) −	7.5274e-2 (3.12e-2) −	3.7019e-2 (4.84e-3) −	2.7313e-2 (3.29e-2)
CF5	2	10	2.0669e-1 (9.37e-2) +	1.8538e-1 (1.03e-1) +	4.0101e-1 (9.74e-2) −	1.5602e+0 (2.49e-1) −	1.9068e-1 (8.66e-2) +	2.6704e-1 (1.36e-1) ≈	4.6932e-1 (1.19e-1) −	2.2762e-1 (7.19e-2)
CF6	2	10	5.5901e-2 (3.16e-2) ≈	4.7785e-2 (2.04e-2) ≈	1.6813e-1 (5.83e-2) −	3.3979e-2 (2.84e-3) +	2.9657e-2 (7.61e-3) +	6.2408e-2 (3.27e-2) −	4.7269e-2 (2.68e-2) ≈	4.0437e-2 (1.43e-2)
CF7	2	10	1.6714e-1 (8.17e-2) ≈	1.6320e-1 (3.77e-2) +	4.4969e-1 (1.12e-1) −	1.2688e+0 (2.77e-1) −	1.5998e-1 (3.64e-2) +	1.8362e-1 (6.96e-2) ≈	2.9831e-1 (1.41e-1) −	1.9668e-1 (8.40e-2)
CF8	3	10	1.7030e-1 (8.53e-2) −	1.0310e+0 (1.10e+0) −	1.2052e-1 (2.97e-2) −	2.2910e-1 (9.73e-2) −	8.3529e-2 (2.84e-3) −	1.3848e-1 (2.08e-2) −	3.4011e-1 (8.47e-2) −	7.1942e-2 (7.26e-3)
CF9	3	10	3.9905e-2 (3.04e-3) −	3.8180e-2 (1.76e-3) −	5.5410e-2 (1.72e-2) −	6.3857e-2 (3.98e-3) −	3.6183e-2 (2.56e-3) −	3.6832e-2 (2.42e-3) −	5.9408e-2 (4.40e-3) −	2.3214e-2 (4.18e-4)
CF10	3	10	5.9293e-1 (1.05e-1) −	1.6792e+1 (8.92e+0) −	4.5022e-1 (2.98e-2) ≈	7.1372e+0 (7.22e+0) −	1.6063e-1 (5.61e-2) ≈	1.4304e-1 (9.29e-2) ≈	6.3285e+0 (8.58e+0) −	2.3099e-1 (1.98e-1)
+/ − / ≈			1/6/3	2/6/2	0/9/1	2/7/1	3/4/3	0/6/4	1/8/1	

Problem	M	D	ANSGAIII	BiCo	CMOEAD	CAEAD	CCMO	CMOEA_MS	ToP	CMOEA/D-TAP3
CF1	2	10	1.4903e-2 (1.86e-3) −	1.0775e-2 (9.26e-4) −	9.5536e-3 (2.43e-3) −	1.2709e-3 (4.42e-3) +	2.7767e-3 (2.68e-4) ≈	1.3136e-2 (4.52e-3) −	8.0644e-3 (1.25e-3) −	2.8474e-3 (7.22e-4)
CF2	2	10	2.1753e-2 (8.67e-3) −	2.1238e-2 (9.13e-3) −	1.1475e-1 (5.13e-2) −	6.5803e-3 (1.79e-3) +	1.7322e-2 (8.07e-3) −	2.0850e-2 (8.56e-3) −	4.6186e-3 (7.08e-3) +	1.4744e-2 (1.47e-2)
CF3	2	10	2.2954e-1 (1.17e-1) ≈	2.0203e-1 (1.00e-1) ≈	5.7348e-1 (2.21e-1) −	5.4079e-1 (3.57e-2) −	2.3393e-1 (1.06e-1) ≈	2.4653e-1 (8.85e-2) ≈	5.2020e-1 (8.12e-2) −	2.1334e-1 (1.15e-1)
CF4	2	10	5.8099e-2 (1.10e-2) −	5.8816e-2 (1.14e-2) −	2.5711e-1 (1.07e-1) −	5.6385e-2 (3.85e-3) −	5.3180e-2 (1.25e-2) −	7.5274e-2 (3.12e-2) −	3.7019e-2 (4.84e-3) ≈	3.5022e-2 (3.89e-2)
CF5	2	10	2.0669e-1 (9.37e-2) ≈	1.8538e-1 (1.03e-1) +	4.0101e-1 (9.74e-2) −	1.5602e+0 (2.49e-1) −	1.9068e-1 (8.66e-2) ≈	2.6704e-1 (1.36e-1) ≈	4.6932e-1 (1.19e-1) −	2.2163e-1 (8.99e-2)
CF6	2	10	5.5901e-2 (3.16e-2) ≈	4.7785e-2 (2.04e-2) ≈	1.6813e-1 (5.83e-2) −	3.3979e-2 (2.84e-3) +	2.9657e-2 (7.61e-3) +	6.2408e-2 (3.27e-2) ≈	4.7269e-2 (2.68e-2) ≈	4.4950e-2 (1.56e-2)
CF7	2	10	1.6714e-1 (8.17e-2) ≈	1.6320e-1 (3.77e-2) +	4.4969e-1 (1.12e-1) −	1.2688e+0 (2.77e-1) −	1.5998e-1 (3.64e-2) +	1.8362e-1 (6.96e-2) ≈	2.9831e-1 (1.41e-1) −	2.0976e-1 (1.02e-1)
CF8	3	10	1.7030e-1 (8.53e-2) −	1.0310e+0 (1.10e+0) −	1.2052e-1 (2.97e-2) −	2.2910e-1 (9.73e-2) −	8.3529e-2 (2.84e-3) −	1.3848e-1 (2.08e-2) −	3.4011e-1 (8.47e-2) −	7.1908e-2 (6.15e-3)
CF9	3	10	3.9905e-2 (3.04e-3) −	3.8180e-2 (1.76e-3) −	5.5410e-2 (1.72e-2) −	6.3857e-2 (3.98e-3) −	3.6183e-2 (2.56e-3) −	3.6832e-2 (2.42e-3) −	5.9408e-2 (4.40e-3) −	2.3343e-2 (5.60e-4)
CF10	3	10	5.9293e-1 (1.05e-1) −	1.6792e+1 (8.92e+0) −	4.5022e-1 (2.98e-2) ≈	7.1372e+0 (7.22e+0) −	1.6063e-1 (5.61e-2) ≈	1.4304e-1 (9.29e-2) +	6.3285e+0 (8.58e+0) −	2.1310e-1 (1.68e-1)
+/ − / ≈			0/6/4	2/6/2	0/9/1	3/7/0	2/4/4	1/5/4	1/7/2	

Problem	M	D	ANSGAIII	BiCo	CMOEAD	CAEAD	CCMO	CMOEA_MS	ToP	CMOEA/D-TAP4
CF1	2	10	1.4903e-2 (1.86e-3) +	1.0775e-2 (9.26e-4) +	9.5536e-3 (2.43e-3) +	1.2709e-3 (4.42e-3) +	2.7767e-3 (2.68e-4) +	1.3136e-2 (4.52e-3) +	8.0644e-3 (1.25e-3) +	1.7116e-2 (3.52e-3)
CF2	2	10	2.1753e-2 (8.67e-3) −	2.1238e-2 (9.13e-3) −	1.1475e-1 (5.13e-2) −	6.5803e-3 (1.79e-3) ≈	1.7322e-2 (8.07e-3) −	2.0850e-2 (8.56e-3) −	4.6186e-3 (7.08e-3) +	1.3203e-2 (1.38e-2)
CF3	2	10	2.2954e-1 (1.17e-1) ≈	2.0203e-1 (1.00e-1) ≈	5.7348e-1 (2.21e-1) −	5.4079e-1 (3.57e-2) −	2.3393e-1 (1.06e-1) ≈	2.4653e-1 (8.85e-2) ≈	5.2020e-1 (8.12e-2) −	2.5361e-1 (1.08e-1)
CF4	2	10	5.8099e-2 (1.10e-2) −	5.8816e-2 (1.14e-2) −	2.5711e-1 (1.07e-1) −	5.6385e-2 (3.85e-3) −	5.3180e-2 (1.25e-2) −	7.5274e-2 (3.12e-2) −	3.7019e-2 (4.84e-3) −	2.3921e-2 (3.18e-2)
CF5	2	10	2.0669e-1 (9.37e-2) ≈	1.8538e-1 (1.03e-1) ≈	4.0101e-1 (9.74e-2) −	1.5602e+0 (2.49e-1) −	1.9068e-1 (8.66e-2) ≈	2.6704e-1 (1.36e-1) −	4.6932e-1 (1.19e-1) −	1.8945e-1 (9.13e-2)
CF6	2	10	5.5901e-2 (3.16e-2) ≈	4.7785e-2 (2.04e-2) ≈	1.6813e-1 (5.83e-2) −	3.3979e-2 (2.84e-3) +	2.9657e-2 (7.61e-3) +	6.2408e-2 (3.27e-2) −	4.7269e-2 (2.68e-2) ≈	3.9357e-2 (1.20e-2)
CF7	2	10	1.6714e-1 (8.17e-2) +	1.6320e-1 (3.77e-2) +	4.4969e-1 (1.12e-1) −	1.2688e+0 (2.77e-1) −	1.5998e-1 (3.64e-2) +	1.8362e-1 (6.96e-2) +	2.9831e-1 (1.41e-1) ≈	2.3446e-1 (8.17e-2)
CF8	3	10	1.7030e-1 (8.53e-2) −	1.0310e+0 (1.10e+0) −	1.2052e-1 (2.97e-2) ≈	2.2910e-1 (9.73e-2) −	8.3529e-2 (2.84e-3) +	1.3848e-1 (2.08e-2) −	3.4011e-1 (8.47e-2) −	1.0962e-1 (6.54e-3)
CF9	3	10	3.9905e-2 (3.04e-3) −	3.8180e-2 (1.76e-3) −	5.5410e-2 (1.72e-2) −	6.3857e-2 (3.98e-3) −	3.6183e-2 (2.56e-3) −	3.6832e-2 (2.42e-3) −	5.9408e-2 (4.40e-3) −	2.7401e-2 (5.58e-3)
CF10	3	10	5.9293e-1 (1.05e-1) −	1.6792e+1 (8.92e+0) −	4.5022e-1 (2.98e-2) −	7.1372e+0 (7.22e+0) −	1.6063e-1 (5.61e-2) −	1.4304e-1 (9.29e-2) ≈	6.3285e+0 (8.58e+0) −	1.2489e-1 (6.94e-2)
+/ − / ≈			2/5/3	2/5/3	1/8/1	2/7/1	4/4/2	2/6/2	2/6/2	

Problem	M	D	ANSGAIII	BiCo	CMOEAD	CAEAD	CCMO	CMOEA_MS	ToP	CMOEA/D-TAP5
CF1	2	10	1.4903e-2 (1.86e-3) +	1.0775e-2 (9.26e-4) +	9.5536e-3 (2.43e-3) +	1.2709e-3 (4.42e-3) +	2.7767e-3 (2.68e-4) +	1.3136e-2 (4.52e-3) +	8.0644e-3 (1.25e-3) +	2.8803e-1 (1.48e-1)
CF2	2	10	2.1753e-2 (8.67e-3) −	2.1238e-2 (9.13e-3) −	1.1475e-1 (5.13e-2) −	6.5803e-3 (1.79e-3) ≈	1.7322e-2 (8.07e-3) −	2.0850e-2 (8.56e-3) −	4.6186e-3 (7.08e-3) +	6.8104e-3 (5.21e-3)
CF3	2	10	2.2954e-1 (1.17e-1) ≈	2.0203e-1 (1.00e-1) ≈	5.7348e-1 (2.21e-1) −	5.4079e-1 (3.57e-2) −	2.3393e-1 (1.06e-1) ≈	2.4653e-1 (8.85e-2) ≈	5.2020e-1 (8.12e-2) −	2.4294e-1 (1.27e-1)
CF4	2	10	5.8099e-2 (1.10e-2) ≈	5.8816e-2 (1.14e-2) ≈	2.5711e-1 (1.07e-1) −	5.6385e-2 (3.85e-3) ≈	5.3180e-2 (1.25e-2) +	7.5274e-2 (3.12e-2) ≈	3.7019e-2 (4.84e-3) +	1.0429e-1 (7.44e-2)
CF5	2	10	2.0669e-1 (9.37e-2) ≈	1.8538e-1 (1.03e-1) ≈	4.0101e-1 (9.74e-2) −	1.5602e+0 (2.49e-1) −	1.9068e-1 (8.66e-2) ≈	2.6704e-1 (1.36e-1) ≈	4.6932e-1 (1.19e-1) −	1.9828e-1 (7.25e-2)
CF6	2	10	5.5901e-2 (3.16e-2) +	4.7785e-2 (2.04e-2) +	1.6813e-1 (5.83e-2) −	3.3979e-2 (2.84e-3) +	2.9657e-2 (7.61e-3) +	6.2408e-2 (3.27e-2) +	4.7269e-2 (2.68e-2) +	9.4487e-2 (4.84e-2)
CF7	2	10	1.6714e-1 (8.17e-2) +	1.6320e-1 (3.77e-2) +	4.4969e-1 (1.12e-1) −	1.2688e+0 (2.77e-1) −	1.5998e-1 (3.64e-2) +	1.8362e-1 (6.96e-2) +	2.9831e-1 (1.41e-1) ≈	2.8004e-1 (2.41e-1)
CF8	3	10	1.7030e-1 (8.53e-2) −	1.0310e+0 (1.10e+0) −	1.2052e-1 (2.97e-2) −	2.2910e-1 (9.73e-2) −	8.3529e-2 (2.84e-3) +	1.3848e-1 (2.08e-2) −	3.4011e-1 (8.47e-2) −	1.0014e-1 (1.27e-2)
CF9	3	10	3.9905e-2 (3.04e-3) −	3.8180e-2 (1.76e-3) −	5.5410e-2 (1.72e-2) −	6.3857e-2 (3.98e-3) −	3.6183e-2 (2.56e-3) −	3.6832e-2 (2.42e-3) −	5.9408e-2 (4.40e-3) −	2.5463e-2 (8.84e-4)
CF10	3	10	5.9293e-1 (1.05e-1) −	1.6792e+1 (8.92e+0) −	4.5022e-1 (2.98e-2) −	7.1372e+0 (7.22e+0) −	1.6063e-1 (5.61e-2) −	1.4304e-1 (9.29e-2) ≈	6.3285e+0 (8.58e+0) −	1.0391e-1 (5.95e-2)
+/ − / ≈			3/4/3	3/4/3	1/9/0	2/6/2	5/3/2	3/3/4	4/5/1	

are embedded in the platform so that all are dealt uniformly. Common parameters, like number of population, function evaluations, and runs were kept same for all algorithms; however, the parameters associated with each algorithm uses its default values as described in their original papers and available in PlatEMO. Tables 3-4, display the comparison of

TABLE 4. Wilcoxon rank sum test results of proposed vs state-of-the-arts algorithms using HV values.

Problem	M	D	ANSGAIII	BiCo	CMOEAD	CAEAD	CCMO	CMOEA_MS	ToP	CMOEA/D-TAP1
CF1	2	10	5.4750e-1 (2.36e-3) −	5.5261e-1 (1.20e-3) +	5.5348e-1 (3.12e-3) +	5.6460e-1 (5.23e-3) +	5.6263e-1 (3.33e-4) +	5.5204e-1 (4.23e-3) +	5.5634e-1 (1.47e-3) +	5.4899e-1 (5.11e-2)
CF2	2	10	6.5275e-1 (1.18e-2) −	6.5322e-1 (1.25e-2) −	5.4565e-1 (3.43e-2) −	6.7074e-1 (2.76e-3) ≈	6.5671e-1 (1.45e-2) −	6.4923e-1 (1.58e-2) −	6.7364e-1 (5.57e-3) +	6.6917e-1 (6.64e-3)
CF3	2	10	2.1798e-1 (4.30e-2) ≈	2.2522e-1 (4.56e-2) ≈	1.0356e-1 (5.12e-2) −	1.2919e-3 (2.42e-3) −	2.1035e-1 (4.41e-2) ≈	1.9959e-1 (3.64e-2) ≈	5.5298e-3 (6.10e-3) −	2.0457e-1 (4.72e-2)
CF4	2	10	4.6007e-1 (1.29e-2) −	4.5911e-1 (1.47e-2) −	3.1608e-1 (6.23e-2) −	4.5732e-1 (6.62e-3) −	4.6767e-1 (1.46e-2) −	4.4405e-1 (2.99e-2) −	4.8541e-1 (8.09e-3) −	5.0808e-1 (2.79e-2)
CF5	2	10	3.2168e-1 (6.18e-2) ≈	3.3128e-1 (6.30e-2) ≈	2.4108e-1 (5.59e-2) −	0.0000e+0 (0.00e+0) −	3.3271e-1 (5.79e-2) ≈	2.8709e-1 (7.52e-2) −	8.8013e-2 (7.30e-2) −	3.4075e-1 (4.51e-2)
CF6	2	10	6.5047e-1 (1.65e-2) ≈	6.5534e-1 (1.30e-2) ≈	6.0099e-1 (5.30e-2) −	6.6532e-1 (1.91e-3) +	6.6993e-1 (8.54e-3) +	6.5070e-1 (1.79e-2) ≈	6.6156e-1 (1.70e-2) ≈	6.5968e-1 (1.57e-2)
CF7	2	10	5.3257e-1 (4.62e-2) +	5.3723e-1 (2.15e-2) +	3.4192e-1 (9.09e-2) −	3.4490e-3 (1.23e-2) −	5.2484e-1 (4.21e-2) +	5.1595e-1 (4.50e-2) +	3.4195e-1 (1.25e-1) −	4.9249e-1 (6.51e-2)
CF8	3	10	3.9037e-1 (3.52e-2) −	1.3691e-1 (1.37e-1) −	4.0265e-1 (3.18e-2) −	3.1377e-1 (6.17e-2) −	4.6054e-1 (6.04e-3) −	4.4573e-1 (1.86e-2) −	2.6927e-1 (5.09e-2) −	4.8046e-1 (1.01e-2)
CF9	3	10	4.8428e-1 (6.28e-3) −	4.8322e-1 (6.59e-3) −	4.6137e-1 (2.99e-2) −	4.6295e-1 (1.20e-2) −	4.9614e-1 (1.01e-2) −	4.8926e-1 (9.48e-3) −	4.7301e-1 (1.53e-2) −	5.5075e-1 (2.28e-3)
CF10	3	10	1.0549e-1 (1.21e-2) −	0.0000e+0 (0.00e+0) −	1.1131e-1 (2.48e-3) −	4.0223e-4 (1.56e-3) −	3.0038e-1 (5.42e-2) −	3.0328e-1 (8.02e-2) ≈	1.9041e-3 (8.30e-3) −	3.2181e-1 (1.17e-1)
+/ − / ≈			1/6/3	2/5/3	1/9/0	2/7/1	3/5/2	2/5/3	2/7/1	

Problem	M	D	ANSGAIII	BiCo	CMOEAD	CAEAD	CCMO	CMOEA_MS	ToP	CMOEA/D-TAP2
CF1	2	10	5.4750e-1 (2.36e-3) −	5.5261e-1 (1.20e-3) −	5.5348e-1 (3.12e-3) −	5.6460e-1 (5.23e-3) +	5.6263e-1 (3.33e-4) ≈	5.5204e-1 (4.23e-3) −	5.5634e-1 (1.47e-3) −	5.6213e-1 (1.43e-3)
CF2	2	10	6.5275e-1 (1.18e-2) −	6.5322e-1 (1.25e-2) −	5.4565e-1 (3.43e-2) −	6.7074e-1 (2.76e-3) ≈	6.5671e-1 (1.45e-2) −	6.4923e-1 (1.58e-2) −	6.7364e-1 (5.57e-3) +	6.7077e-1 (3.16e-3)
CF3	2	10	2.1798e-1 (4.30e-2) ≈	2.2522e-1 (4.56e-2) ≈	1.0356e-1 (5.12e-2) −	1.2919e-3 (2.42e-3) −	2.1035e-1 (4.41e-2) ≈	1.9959e-1 (3.64e-2) ≈	5.5298e-3 (6.10e-3) −	2.0145e-1 (4.31e-2)
CF4	2	10	4.6007e-1 (1.29e-2) −	4.5911e-1 (1.47e-2) −	3.1608e-1 (6.23e-2) −	4.5732e-1 (6.62e-3) −	4.6767e-1 (1.46e-2) −	4.4405e-1 (2.99e-2) −	4.8541e-1 (8.09e-3) −	5.0222e-1 (3.59e-2)
CF5	2	10	3.2168e-1 (6.18e-2) ≈	3.3128e-1 (6.30e-2) ≈	2.4108e-1 (5.59e-2) −	0.0000e+0 (0.00e+0) −	3.3271e-1 (5.79e-2) ≈	2.8709e-1 (7.52e-2) −	8.8013e-2 (7.30e-2) −	3.3559e-1 (3.98e-2)
CF6	2	10	6.5047e-1 (1.65e-2) ≈	6.5534e-1 (1.30e-2) ≈	6.0099e-1 (5.30e-2) −	6.6532e-1 (1.91e-3) +	6.6993e-1 (8.54e-3) +	6.5070e-1 (1.79e-2) ≈	6.6156e-1 (1.70e-2) ≈	6.5692e-1 (1.17e-2)
CF7	2	10	5.3257e-1 (4.62e-2) ≈	5.3723e-1 (2.15e-2) +	3.4192e-1 (9.09e-2) −	3.4490e-3 (1.23e-2) −	5.2484e-1 (4.21e-2) ≈	5.1595e-1 (4.50e-2) ≈	3.4195e-1 (1.25e-1) −	5.1233e-1 (6.40e-2)
CF8	3	10	3.9037e-1 (3.52e-2) −	1.3691e-1 (1.37e-1) −	4.0265e-1 (3.18e-2) −	3.1377e-1 (6.17e-2) −	4.6054e-1 (6.04e-3) −	4.4573e-1 (1.86e-2) −	2.6927e-1 (5.09e-2) −	4.8052e-1 (1.10e-2)
CF9	3	10	4.8428e-1 (6.28e-3) −	4.8322e-1 (6.59e-3) −	4.6137e-1 (2.99e-2) −	4.6295e-1 (1.20e-2) −	4.9614e-1 (1.01e-2) −	4.8926e-1 (9.48e-3) −	4.7301e-1 (1.53e-2) −	5.5157e-1 (2.00e-3)
CF10	3	10	1.0549e-1 (1.21e-2) −	0.0000e+0 (0.00e+0) −	1.1131e-1 (2.48e-3) ≈	4.0223e-4 (1.56e-3) −	3.0038e-1 (5.42e-2) ≈	3.0328e-1 (8.02e-2) ≈	1.9041e-3 (8.30e-3) −	3.0106e-1 (1.21e-1)
+/ − / ≈			0/6/4	1/6/3	0/9/1	2/7/1	1/4/5	0/6/4	1/8/1	

Problem	M	D	ANSGAIII	BiCo	CMOEAD	CAEAD	CCMO	CMOEA_MS	ToP	CMOEA/D-TAP3
CF1	2	10	5.4750e-1 (2.36e-3) −	5.5261e-1 (1.20e-3) −	5.5348e-1 (3.12e-3) −	5.6460e-1 (5.23e-3) +	5.6263e-1 (3.33e-4) ≈	5.5204e-1 (4.23e-3) −	5.5634e-1 (1.47e-3) −	5.6218e-1 (9.70e-4)
CF2	2	10	6.5275e-1 (1.18e-2) −	6.5322e-1 (1.25e-2) −	5.4565e-1 (3.43e-2) −	6.7074e-1 (2.76e-3) +	6.5671e-1 (1.45e-2) −	6.4923e-1 (1.58e-2) −	6.7364e-1 (5.57e-3) +	6.6632e-1 (7.98e-3)
CF3	2	10	2.1798e-1 (4.30e-2) ≈	2.2522e-1 (4.56e-2) ≈	1.0356e-1 (5.12e-2) −	1.2919e-3 (2.42e-3) −	2.1035e-1 (4.41e-2) ≈	1.9959e-1 (3.64e-2) ≈	5.5298e-3 (6.10e-3) −	2.0334e-1 (4.78e-2)
CF4	2	10	4.6007e-1 (1.29e-2) −	4.5911e-1 (1.47e-2) −	3.1608e-1 (6.23e-2) −	4.5732e-1 (6.62e-3) −	4.6767e-1 (1.46e-2) −	4.4405e-1 (2.99e-2) −	4.8541e-1 (8.09e-3) ≈	4.9292e-1 (3.85e-2)
CF5	2	10	3.2168e-1 (6.18e-2) ≈	3.3128e-1 (6.30e-2) ≈	2.4108e-1 (5.59e-2) −	0.0000e+0 (0.00e+0) −	3.3271e-1 (5.79e-2) ≈	2.8709e-1 (7.52e-2) −	8.8013e-2 (7.30e-2) −	3.4955e-1 (5.59e-2)
CF6	2	10	6.5047e-1 (1.65e-2) ≈	6.5534e-1 (1.30e-2) ≈	6.0099e-1 (5.30e-2) −	6.6532e-1 (1.91e-3) +	6.6993e-1 (8.54e-3) +	6.5070e-1 (1.79e-2) ≈	6.6156e-1 (1.70e-2) ≈	6.5357e-1 (1.11e-2)
CF7	2	10	5.3257e-1 (4.62e-2) ≈	5.3723e-1 (2.15e-2) +	3.4192e-1 (9.09e-2) −	3.4490e-3 (1.23e-2) −	5.2484e-1 (4.21e-2) ≈	5.1595e-1 (4.50e-2) ≈	3.4195e-1 (1.25e-1) −	4.9490e-1 (8.30e-2)
CF8	3	10	3.9037e-1 (3.52e-2) −	1.3691e-1 (1.37e-1) −	4.0265e-1 (3.18e-2) −	3.1377e-1 (6.17e-2) −	4.6054e-1 (6.04e-3) −	4.4573e-1 (1.86e-2) −	2.6927e-1 (5.09e-2) −	4.8132e-1 (9.48e-3)
CF9	3	10	4.8428e-1 (6.28e-3) −	4.8322e-1 (6.59e-3) −	4.6137e-1 (2.99e-2) −	4.6295e-1 (1.20e-2) −	4.9614e-1 (1.01e-2) −	4.8926e-1 (9.48e-3) −	4.7301e-1 (1.53e-2) −	5.5103e-1 (1.93e-3)
CF10	3	10	1.0549e-1 (1.21e-2) −	0.0000e+0 (0.00e+0) −	1.1131e-1 (2.48e-3) −	4.0223e-4 (1.56e-3) −	3.0038e-1 (5.42e-2) +	3.0328e-1 (8.02e-2) ≈	1.9041e-3 (8.30e-3) −	2.9694e-1 (1.03e-1)
+/ − / ≈			0/6/4	1/6/3	0/10/0	3/7/0	2/4/4	0/6/4	1/7/2	

Problem	M	D	ANSGAIII	BiCo	CMOEAD	CAEAD	CCMO	CMOEA_MS	ToP	CMOEA/D-TAP4
CF1	2	10	5.4750e-1 (2.36e-3) ≈	5.5261e-1 (1.20e-3) +	5.5348e-1 (3.12e-3) +	5.6460e-1 (5.23e-3) +	5.6263e-1 (3.33e-4) +	5.5204e-1 (4.23e-3) +	5.5634e-1 (1.47e-3) +	5.4644e-1 (3.94e-3)
CF2	2	10	6.5275e-1 (1.18e-2) −	6.5322e-1 (1.25e-2) −	5.4565e-1 (3.43e-2) −	6.7074e-1 (2.76e-3) +	6.5671e-1 (1.45e-2) −	6.4923e-1 (1.58e-2) −	6.7364e-1 (5.57e-3) +	6.6656e-1 (7.77e-3)
CF3	2	10	2.1798e-1 (4.30e-2) +	2.2522e-1 (4.56e-2) +	1.0356e-1 (5.12e-2) −	1.2919e-3 (2.42e-3) −	2.1035e-1 (4.41e-2) ≈	1.9959e-1 (3.64e-2) ≈	5.5298e-3 (6.10e-3) −	1.8931e-1 (3.83e-2)
CF4	2	10	4.6007e-1 (1.29e-2) −	4.5911e-1 (1.47e-2) −	3.1608e-1 (6.23e-2) −	4.5732e-1 (6.62e-3) −	4.6767e-1 (1.46e-2) −	4.4405e-1 (2.99e-2) −	4.8541e-1 (8.09e-3) −	5.1005e-1 (2.81e-2)
CF5	2	10	3.2168e-1 (6.18e-2) −	3.3128e-1 (6.30e-2) −	2.4108e-1 (5.59e-2) −	0.0000e+0 (0.00e+0) −	3.3271e-1 (5.79e-2) −	2.8709e-1 (7.52e-2) −	8.8013e-2 (7.30e-2) −	3.7085e-1 (5.98e-2)
CF6	2	10	6.5047e-1 (1.65e-2) ≈	6.5534e-1 (1.30e-2) ≈	6.0099e-1 (5.30e-2) −	6.6532e-1 (1.91e-3) +	6.6993e-1 (8.54e-3) +	6.5070e-1 (1.79e-2) ≈	6.6156e-1 (1.70e-2) ≈	6.5990e-1 (1.37e-2)
CF7	2	10	5.3257e-1 (4.62e-2) +	5.3723e-1 (2.15e-2) +	3.4192e-1 (9.09e-2) −	3.4490e-3 (1.23e-2) −	5.2484e-1 (4.21e-2) +	5.1595e-1 (4.50e-2) ≈	3.4195e-1 (1.25e-1) −	4.8830e-1 (6.54e-2)
CF8	3	10	3.9037e-1 (3.52e-2) −	1.3691e-1 (1.37e-1) −	4.0265e-1 (3.18e-2) −	3.1377e-1 (6.17e-2) −	4.6054e-1 (6.04e-3) +	4.4573e-1 (1.86e-2) +	2.6927e-1 (5.09e-2) −	4.3511e-1 (1.21e-2)
CF9	3	10	4.8428e-1 (6.28e-3) −	4.8322e-1 (6.59e-3) −	4.6137e-1 (2.99e-2) −	4.6295e-1 (1.20e-2) −	4.9614e-1 (1.01e-2) −	4.8926e-1 (9.48e-3) −	4.7301e-1 (1.53e-2) −	5.3775e-1 (1.61e-2)
CF10	3	10	1.0549e-1 (1.21e-2) −	0.0000e+0 (0.00e+0) −	1.1131e-1 (2.48e-3) −	4.0223e-4 (1.56e-3) −	3.0038e-1 (5.42e-2) −	3.0328e-1 (8.02e-2) ≈	1.9041e-3 (8.30e-3) −	3.4337e-1 (6.17e-2)
+/ − / ≈			2/6/2	3/6/1	1/9/0	3/7/0	4/5/1	2/4/4	2/7/1	

Problem	M	D	ANSGAIII	BiCo	CMOEAD	CAEAD	CCMO	CMOEA_MS	ToP	CMOEA/D-TAP5
CF1	2	10	5.4750e-1 (2.36e-3) +	5.5261e-1 (1.20e-3) +	5.5348e-1 (3.12e-3) +	5.6460e-1 (5.23e-3) +	5.6263e-1 (3.33e-4) +	5.5204e-1 (4.23e-3) +	5.5634e-1 (1.47e-3) +	2.8655e-1 (1.12e-1)
CF2	2	10	6.5275e-1 (1.18e-2) −	6.5322e-1 (1.25e-2) −	5.4565e-1 (3.43e-2) −	6.7074e-1 (2.76e-3) ≈	6.5671e-1 (1.45e-2) −	6.4923e-1 (1.58e-2) −	6.7364e-1 (5.57e-3) +	6.7084e-1 (4.44e-3)
CF3	2	10	2.1798e-1 (4.30e-2) ≈	2.2522e-1 (4.56e-2) ≈	1.0356e-1 (5.12e-2) −	1.2919e-3 (2.42e-3) −	2.1035e-1 (4.41e-2) ≈	1.9959e-1 (3.64e-2) ≈	5.5298e-3 (6.10e-3) −	1.9419e-1 (4.87e-2)
CF4	2	10	4.6007e-1 (1.29e-2) ≈	4.5911e-1 (1.47e-2) ≈	3.1608e-1 (6.23e-2) −	4.5732e-1 (6.62e-3) ≈	4.6767e-1 (1.46e-2) +	4.4405e-1 (2.99e-2) ≈	4.8541e-1 (8.09e-3) +	4.4984e-1 (5.26e-2)
CF5	2	10	3.2168e-1 (6.18e-2) −	3.3128e-1 (6.30e-2) −	2.4108e-1 (5.59e-2) −	0.0000e+0 (0.00e+0) −	3.3271e-1 (5.79e-2) −	2.8709e-1 (7.52e-2) −	8.8013e-2 (7.30e-2) −	3.6610e-1 (4.67e-2)
CF6	2	10	6.5047e-1 (1.65e-2) ≈	6.5534e-1 (1.30e-2) +	6.0099e-1 (5.30e-2) −	6.6532e-1 (1.91e-3) +	6.6993e-1 (8.54e-3) +	6.5070e-1 (1.79e-2) +	6.6156e-1 (1.70e-2) +	6.4435e-1 (1.35e-2)
CF7	2	10	5.3257e-1 (4.62e-2) +	5.3723e-1 (2.15e-2) +	3.4192e-1 (9.09e-2) −	3.4490e-3 (1.23e-2) −	5.2484e-1 (4.21e-2) +	5.1595e-1 (4.50e-2) +	3.4195e-1 (1.25e-1) −	4.7474e-1 (1.06e-1)
CF8	3	10	3.9037e-1 (3.52e-2) −	1.3691e-1 (1.37e-1) −	4.0265e-1 (3.18e-2) −	3.1377e-1 (6.17e-2) −	4.6054e-1 (6.04e-3) +	4.4573e-1 (1.86e-2) +	2.6927e-1 (5.09e-2) −	4.3162e-1 (2.20e-2)
CF9	3	10	4.8428e-1 (6.28e-3) −	4.8322e-1 (6.59e-3) −	4.6137e-1 (2.99e-2) −	4.6295e-1 (1.20e-2) −	4.9614e-1 (1.01e-2) −	4.8926e-1 (9.48e-3) −	4.7301e-1 (1.53e-2) −	5.4224e-1 (3.17e-3)
CF10	3	10	1.0549e-1 (1.21e-2) −	0.0000e+0 (0.00e+0) −	1.1131e-1 (2.48e-3) −	4.0223e-4 (1.56e-3) −	3.0038e-1 (5.42e-2) −	3.0328e-1 (8.02e-2) −	1.9041e-3 (8.30e-3) −	3.7964e-1 (6.36e-2)
+/ − / ≈			2/5/3	3/4/3	1/9/0	2/6/2	5/4/1	4/4/2	4/6/0	

each proposed variant vs state-of-the-art algorithms through Wilcoxon rank sum test [42], [43] with the significance level of 0.05 using mean IGD and HV values besides their standard deviation of the executable runs. The test with stated statistics is applied with same level of significance for mutual comparison of the proposed algorithms and results are demonstrated in Tables 5-6. Last row of each table shows the overall ranking with "+/ − / ≈" notations, representing the performance

TABLE 5. Wilcoxon rank sum test results of the proposed algorithms using IGD values.

Problem	M	D	CMOEA/D-TAP2	CMOEA/D-TAP3	CMOEA/D-TAP4	CMOEA/D-TAP5	CMOEA/D-TAP1
CF1	2	10	2.8479e-3 (1.06e-3) ≈	2.8474e-3 (7.22e-4) ≈	1.7116e-2 (3.52e-3) −	2.8803e-1 (1.48e-1) −	1.3848e-2 (4.34e-2)
CF2	2	10	7.1212e-3 (4.56e-3) ≈	1.4744e-2 (1.47e-2) ≈	1.3203e-2 (1.38e-2) ≈	6.8104e-3 (5.21e-3) ≈	9.5087e-3 (1.04e-2)
CF3	2	10	2.3337e-1 (1.08e-1) ≈	2.1334e-1 (1.15e-1) ≈	2.5361e-1 (1.08e-1) ≈	2.4294e-1 (1.27e-1) ≈	2.1887e-1 (1.12e-1)
CF4	2	10	2.7313e-2 (3.29e-2) ≈	3.5022e-2 (3.89e-2) ≈	2.3921e-2 (3.18e-2) ≈	1.0429e-1 (7.44e-2) −	2.0687e-2 (2.22e-2)
CF5	2	10	2.2762e-1 (7.19e-2) ≈	2.2163e-1 (8.99e-2) ≈	1.8945e-1 (9.13e-2) ≈	1.9828e-1 (7.25e-2) ≈	2.1990e-1 (7.33e-2)
CF6	2	10	4.0437e-2 (1.43e-2) ≈	4.4950e-2 (1.56e-2) ≈	3.9357e-2 (1.20e-2) ≈	9.4487e-2 (4.84e-2) −	3.9852e-2 (2.09e-2)
CF7	2	10	1.9668e-1 (8.40e-2) ≈	2.0976e-1 (1.02e-1) ≈	2.3446e-1 (8.17e-2) ≈	2.8004e-1 (2.41e-1) ≈	2.1947e-1 (7.91e-2)
CF8	3	10	7.1942e-2 (7.26e-3) ≈	7.1908e-2 (6.15e-3) ≈	1.0962e-1 (6.54e-3) −	1.0014e-1 (1.27e-2) −	7.2318e-2 (6.79e-3)
CF9	3	10	2.3214e-2 (4.18e-4) ≈	2.3343e-2 (5.60e-4) ≈	2.7401e-2 (5.58e-3) −	2.5463e-2 (8.84e-4) −	2.3488e-2 (5.95e-4)
CF10	3	10	2.3099e-1 (1.98e-1) ≈	2.1310e-1 (1.68e-1) ≈	1.2489e-1 (6.94e-2) ≈	1.0391e-1 (5.95e-2) ≈	1.7349e-1 (1.42e-1)
$+/-/\approx$			0/0/10	0/0/10	0/3/7	0/5/5	

Problem	M	D	CMOEA/D-TAP1	CMOEA/D-TAP3	CMOEA/D-TAP4	CMOEA/D-TAP5	CMOEA/D-TAP2
CF1	2	10	1.3848e-2 (4.34e-2) ≈	2.8474e-3 (7.22e-4) ≈	1.7116e-2 (3.52e-3) −	2.8803e-1 (1.48e-1) −	2.8479e-3 (1.06e-3)
CF2	2	10	9.5087e-3 (1.04e-2) ≈	1.4744e-2 (1.47e-2) −	1.3203e-2 (1.38e-2) −	6.8104e-3 (5.21e-3) ≈	7.1212e-3 (4.56e-3)
CF3	2	10	2.1887e-1 (1.12e-1) ≈	2.1334e-1 (1.15e-1) ≈	2.5361e-1 (1.08e-1) ≈	2.4294e-1 (1.27e-1) ≈	2.3337e-1 (1.08e-1)
CF4	2	10	2.0687e-2 (2.22e-2) ≈	3.5022e-2 (3.89e-2) ≈	2.3921e-2 (3.18e-2) ≈	1.0429e-1 (7.44e-2) −	2.7313e-2 (3.29e-2)
CF5	2	10	2.1990e-1 (7.33e-2) ≈	2.2163e-1 (8.99e-2) ≈	1.8945e-1 (9.13e-2) ≈	1.9828e-1 (7.25e-2) ≈	2.2762e-1 (7.19e-2)
CF6	2	10	3.9852e-2 (2.09e-2) ≈	4.4950e-2 (1.56e-2) ≈	3.9357e-2 (1.20e-2) ≈	9.4487e-2 (4.84e-2) −	4.0437e-2 (1.43e-2)
CF7	2	10	2.1947e-1 (7.91e-2) ≈	2.0976e-1 (1.02e-1) ≈	2.3446e-1 (8.17e-2) ≈	2.8004e-1 (2.41e-1) ≈	1.9668e-1 (8.40e-2)
CF8	3	10	7.2318e-2 (6.79e-3) ≈	7.1908e-2 (6.15e-3) ≈	1.0962e-1 (6.54e-3) −	1.0014e-1 (1.27e-2) −	7.1942e-2 (7.26e-3)
CF9	3	10	2.3488e-2 (5.95e-4) ≈	2.3343e-2 (5.60e-4) ≈	2.7401e-2 (5.58e-3) −	2.5463e-2 (8.84e-4) −	2.3214e-2 (4.18e-4)
CF10	3	10	1.7349e-1 (1.42e-1) ≈	2.1310e-1 (1.68e-1) ≈	1.2489e-1 (6.94e-2) ≈	1.0391e-1 (5.95e-2) +	2.3099e-1 (1.98e-1)
$+/-/\approx$			0/0/10	0/1/9	0/4/6	1/5/4	

Problem	M	D	CMOEA/D-TAP1	CMOEA/D-TAP2	CMOEA/D-TAP4	CMOEA/D-TAP5	CMOEA/D-TAP3
CF1	2	10	1.3848e-2 (4.34e-2) ≈	2.8479e-3 (1.06e-3) ≈	1.7116e-2 (3.52e-3) −	2.8803e-1 (1.48e-1) −	2.8474e-3 (7.22e-4)
CF2	2	10	9.5087e-3 (1.04e-2) ≈	7.1212e-3 (4.56e-3) +	1.3203e-2 (1.38e-2) ≈	6.8104e-3 (5.21e-3) +	1.4744e-2 (1.47e-2)
CF3	2	10	2.1887e-1 (1.12e-1) ≈	2.3337e-1 (1.08e-1) ≈	2.5361e-1 (1.08e-1) ≈	2.4294e-1 (1.27e-1) ≈	2.1334e-1 (1.15e-1)
CF4	2	10	2.0687e-2 (2.22e-2) ≈	2.7313e-2 (3.29e-2) ≈	2.3921e-2 (3.18e-2) ≈	1.0429e-1 (7.44e-2) −	3.5022e-2 (3.89e-2)
CF5	2	10	2.1990e-1 (7.33e-2) ≈	2.2762e-1 (7.19e-2) ≈	1.8945e-1 (9.13e-2) ≈	1.9828e-1 (7.25e-2) ≈	2.2163e-1 (8.99e-2)
CF6	2	10	3.9852e-2 (2.09e-2) ≈	4.0437e-2 (1.43e-2) ≈	3.9357e-2 (1.20e-2) ≈	9.4487e-2 (4.84e-2) −	4.4950e-2 (1.56e-2)
CF7	2	10	2.1947e-1 (7.91e-2) ≈	1.9668e-1 (8.40e-2) ≈	2.3446e-1 (8.17e-2) ≈	2.8004e-1 (2.41e-1) ≈	2.0976e-1 (1.02e-1)
CF8	3	10	7.2318e-2 (6.79e-3) ≈	7.1942e-2 (7.26e-3) ≈	1.0962e-1 (6.54e-3) −	1.0014e-1 (1.27e-2) −	7.1908e-2 (6.15e-3)
CF9	3	10	2.3488e-2 (5.95e-4) ≈	2.3214e-2 (4.18e-4) ≈	2.7401e-2 (5.58e-3) −	2.5463e-2 (8.84e-4) −	2.3343e-2 (5.60e-4)
CF10	3	10	1.7349e-1 (1.42e-1) ≈	2.3099e-1 (1.98e-1) ≈	1.2489e-1 (6.94e-2) +	1.0391e-1 (5.95e-2) +	2.1310e-1 (1.68e-1)
$+/-/\approx$			0/0/10	1/0/9	1/3/6	2/5/3	

Problem	M	D	CMOEA/D-TAP1	CMOEA/D-TAP2	CMOEA/D-TAP3	CMOEA/D-TAP5	CMOEA/D-TAP4
CF1	2	10	1.3848e-2 (4.34e-2) +	2.8479e-3 (1.06e-3) +	2.8474e-3 (7.22e-4) +	2.8803e-1 (1.48e-1) −	1.7116e-2 (3.52e-3)
CF2	2	10	9.5087e-3 (1.04e-2) ≈	7.1212e-3 (4.56e-3) +	1.4744e-2 (1.47e-2) ≈	6.8104e-3 (5.21e-3) +	1.3203e-2 (1.38e-2)
CF3	2	10	2.1887e-1 (1.12e-1) ≈	2.3337e-1 (1.08e-1) ≈	2.1334e-1 (1.15e-1) ≈	2.4294e-1 (1.27e-1) ≈	2.5361e-1 (1.08e-1)
CF4	2	10	2.0687e-2 (2.22e-2) ≈	2.7313e-2 (3.29e-2) ≈	3.5022e-2 (3.89e-2) ≈	1.0429e-1 (7.44e-2) −	2.3921e-2 (3.18e-2)
CF5	2	10	2.1990e-1 (7.33e-2) ≈	2.2762e-1 (7.19e-2) ≈	2.2163e-1 (8.99e-2) ≈	1.9828e-1 (7.25e-2) ≈	1.8945e-1 (9.13e-2)
CF6	2	10	3.9852e-2 (2.09e-2) ≈	4.0437e-2 (1.43e-2) ≈	4.4950e-2 (1.56e-2) ≈	9.4487e-2 (4.84e-2) −	3.9357e-2 (1.20e-2)
CF7	2	10	2.1947e-1 (7.91e-2) ≈	1.9668e-1 (8.40e-2) ≈	2.0976e-1 (1.02e-1) ≈	2.8004e-1 (2.41e-1) ≈	2.3446e-1 (8.17e-2)
CF8	3	10	7.2318e-2 (6.79e-3) +	7.1942e-2 (7.26e-3) +	7.1908e-2 (6.15e-3) +	1.0014e-1 (1.27e-2) +	1.0962e-1 (6.54e-3)
CF9	3	10	2.3488e-2 (5.95e-4) +	2.3214e-2 (4.18e-4) +	2.3343e-2 (5.60e-4) +	2.5463e-2 (8.84e-4) +	2.7401e-2 (5.58e-3)
CF10	3	10	1.7349e-1 (1.42e-1) ≈	2.3099e-1 (1.98e-1) ≈	2.1310e-1 (1.68e-1) −	1.0391e-1 (5.95e-2) ≈	1.2489e-1 (6.94e-2)
$+/-/\approx$			3/0/7	4/0/6	3/1/6	3/3/4	

Problem	M	D	CMOEA/D-TAP1	CMOEA/D-TAP2	CMOEA/D-TAP3	CMOEA/D-TAP4	CMOEA/D-TAP5
CF1	2	10	1.3848e-2 (4.34e-2) +	2.8479e-3 (1.06e-3) +	2.8474e-3 (7.22e-4) +	1.7116e-2 (3.52e-3) +	2.8803e-1 (1.48e-1)
CF2	2	10	9.5087e-3 (1.04e-2) ≈	7.1212e-3 (4.56e-3) ≈	1.4744e-2 (1.47e-2) −	1.3203e-2 (1.38e-2) −	6.8104e-3 (5.21e-3)
CF3	2	10	2.1887e-1 (1.12e-1) ≈	2.3337e-1 (1.08e-1) ≈	2.1334e-1 (1.15e-1) ≈	2.5361e-1 (1.08e-1) ≈	2.4294e-1 (1.27e-1)
CF4	2	10	2.0687e-2 (2.22e-2) +	2.7313e-2 (3.29e-2) +	3.5022e-2 (3.89e-2) +	2.3921e-2 (3.18e-2) +	1.0429e-1 (7.44e-2)
CF5	2	10	2.1990e-1 (7.33e-2) ≈	2.2762e-1 (7.19e-2) ≈	2.2163e-1 (8.99e-2) ≈	1.8945e-1 (9.13e-2) ≈	1.9828e-1 (7.25e-2)
CF6	2	10	3.9852e-2 (2.09e-2) +	4.0437e-2 (1.43e-2) +	4.4950e-2 (1.56e-2) +	3.9357e-2 (1.20e-2) +	9.4487e-2 (4.84e-2)
CF7	2	10	2.1947e-1 (7.91e-2) ≈	1.9668e-1 (8.40e-2) ≈	2.0976e-1 (1.02e-1) ≈	2.3446e-1 (8.17e-2) ≈	2.8004e-1 (2.41e-1)
CF8	3	10	7.2318e-2 (6.79e-3) +	7.1942e-2 (7.26e-3) +	7.1908e-2 (6.15e-3) +	1.0962e-1 (6.54e-3) −	1.0014e-1 (1.27e-2)
CF9	3	10	2.3488e-2 (5.95e-4) +	2.3214e-2 (4.18e-4) +	2.3343e-2 (5.60e-4) +	2.7401e-2 (5.58e-3) −	2.5463e-2 (8.84e-4)
CF10	3	10	1.7349e-1 (1.42e-1) ≈	2.3099e-1 (1.98e-1) −	2.1310e-1 (1.68e-1) −	1.2489e-1 (6.94e-2) ≈	1.0391e-1 (5.95e-2)
$+/-/\approx$			5/0/5	5/1/4	5/2/3	3/3/4	

TABLE 6. Wilcoxon rank sum test results of the proposed algorithms using HV values.

Problem	M	D	CMOEA/D-TAP2	CMOEA/D-TAP3	CMOEA/D-TAP4	CMOEA/D-TAP5	CMOEA/D-TAP1
CF1	2	10	5.6213e-1 (1.43e-3) ≈	5.6218e-1 (9.70e-4) ≈	5.4644e-1 (3.94e-3) −	2.8655e-1 (1.12e-1) −	5.4899e-1 (5.11e-2)
CF2	2	10	6.7077e-1 (3.16e-3) ≈	6.6632e-1 (7.98e-3) ≈	6.6656e-1 (7.77e-3) ≈	6.7084e-1 (4.44e-3) ≈	6.6917e-1 (6.64e-3)
CF3	2	10	2.0145e-1 (4.31e-2) ≈	2.0334e-1 (4.78e-2) ≈	1.8931e-1 (3.83e-2) ≈	1.9419e-1 (4.87e-2) ≈	2.0457e-1 (4.72e-2)
CF4	2	10	5.0222e-1 (3.59e-2) ≈	4.9292e-1 (3.85e-2) ≈	5.1005e-1 (2.81e-2) ≈	4.4984e-1 (5.26e-2) −	5.0808e-1 (2.79e-2)
CF5	2	10	3.3559e-1 (3.98e-2) ≈	3.4955e-1 (5.59e-2) ≈	3.7085e-1 (5.98e-2) +	3.6610e-1 (4.67e-2) +	3.4075e-1 (4.51e-2)
CF6	2	10	6.5692e-1 (1.17e-2) ≈	6.5357e-1 (1.11e-2) ≈	6.5990e-1 (1.37e-2) ≈	6.4435e-1 (1.35e-2) −	6.5968e-1 (1.57e-2)
CF7	2	10	5.1233e-1 (6.40e-2) ≈	4.9490e-1 (8.30e-2) ≈	4.8830e-1 (6.54e-2) ≈	4.7474e-1 (1.06e-1) ≈	4.9249e-1 (6.51e-2)
CF8	3	10	4.8052e-1 (1.10e-2) ≈	4.8132e-1 (9.48e-3) ≈	4.3511e-1 (1.21e-2) −	4.3162e-1 (2.20e-2) −	4.8046e-1 (1.01e-2)
CF9	3	10	5.5157e-1 (2.00e-3) ≈	5.5103e-1 (1.93e-3) ≈	5.3775e-1 (1.61e-2) −	5.4224e-1 (3.17e-3) −	5.5075e-1 (2.28e-3)
CF10	3	10	3.0106e-1 (1.21e-1) ≈	2.9694e-1 (1.03e-1) ≈	3.4337e-1 (6.17e-2) ≈	3.7964e-1 (6.36e-2) +	3.2181e-1 (1.17e-1)
+/ − / ≈			0/0/10	0/0/10	1/3/6	2/5/3	

Problem	M	D	CMOEA/D-TAP1	CMOEA/D-TAP3	CMOEA/D-TAP4	CMOEA/D-TAP5	CMOEA/D-TAP2
CF1	2	10	5.4899e-1 (5.11e-2) ≈	5.6218e-1 (9.70e-4) ≈	5.4644e-1 (3.94e-3) −	2.8655e-1 (1.12e-1) −	5.6213e-1 (1.43e-3)
CF2	2	10	6.6917e-1 (6.64e-3) ≈	6.6632e-1 (7.98e-3) −	6.6656e-1 (7.77e-3) −	6.7084e-1 (4.44e-3) ≈	6.7077e-1 (3.16e-3)
CF3	2	10	2.0457e-1 (4.72e-2) ≈	2.0334e-1 (4.78e-2) ≈	1.8931e-1 (3.83e-2) ≈	1.9419e-1 (4.87e-2) ≈	2.0145e-1 (4.31e-2)
CF4	2	10	5.0808e-1 (2.79e-2) ≈	4.9292e-1 (3.85e-2) ≈	5.1005e-1 (2.81e-2) ≈	4.4984e-1 (5.26e-2) −	5.0222e-1 (3.59e-2)
CF5	2	10	3.4075e-1 (4.51e-2) ≈	3.4955e-1 (5.59e-2) ≈	3.7085e-1 (5.98e-2) +	3.6610e-1 (4.67e-2) +	3.3559e-1 (3.98e-2)
CF6	2	10	6.5968e-1 (1.57e-2) ≈	6.5357e-1 (1.11e-2) ≈	6.5990e-1 (1.37e-2) ≈	6.4435e-1 (1.35e-2) −	6.5692e-1 (1.17e-2)
CF7	2	10	4.9249e-1 (6.51e-2) ≈	4.9490e-1 (8.30e-2) ≈	4.8830e-1 (6.54e-2) ≈	4.7474e-1 (1.06e-1) ≈	5.1233e-1 (6.40e-2)
CF8	3	10	4.8046e-1 (1.01e-2) ≈	4.8132e-1 (9.48e-3) ≈	4.3511e-1 (1.21e-2) −	4.3162e-1 (2.20e-2) −	4.8052e-1 (1.10e-2)
CF9	3	10	5.5075e-1 (2.28e-3) ≈	5.5103e-1 (1.93e-3) ≈	5.3775e-1 (1.61e-2) −	5.4224e-1 (3.17e-3) −	5.5157e-1 (2.00e-3)
CF10	3	10	3.2181e-1 (1.17e-1) ≈	2.9694e-1 (1.03e-1) ≈	3.4337e-1 (6.17e-2) ≈	3.7964e-1 (6.36e-2) +	3.0106e-1 (1.21e-1)
+/ − / ≈			0/0/10	0/1/9	1/4/5	2/5/3	

Problem	M	D	CMOEA/D-TAP1	CMOEA/D-TAP2	CMOEA/D-TAP4	CMOEA/D-TAP5	CMOEA/D-TAP3
CF1	2	10	5.4899e-1 (5.11e-2) ≈	5.6213e-1 (1.43e-3) ≈	5.4644e-1 (3.94e-3) −	2.8655e-1 (1.12e-1) −	5.6218e-1 (9.70e-4)
CF2	2	10	6.6917e-1 (6.64e-3) ≈	6.7077e-1 (3.16e-3) +	6.6656e-1 (7.77e-3) ≈	6.7084e-1 (4.44e-3) +	6.6632e-1 (7.98e-3)
CF3	2	10	2.0457e-1 (4.72e-2) ≈	2.0145e-1 (4.31e-2) ≈	1.8931e-1 (3.83e-2) ≈	1.9419e-1 (4.87e-2) ≈	2.0334e-1 (4.78e-2)
CF4	2	10	5.0808e-1 (2.79e-2) ≈	5.0222e-1 (3.59e-2) ≈	5.1005e-1 (2.81e-2) ≈	4.4984e-1 (5.26e-2) −	4.9292e-1 (3.85e-2)
CF5	2	10	3.4075e-1 (4.51e-2) ≈	3.3559e-1 (3.98e-2) ≈	3.7085e-1 (5.98e-2) ≈	3.6610e-1 (4.67e-2) ≈	3.4955e-1 (5.59e-2)
CF6	2	10	6.5968e-1 (1.57e-2) ≈	6.5692e-1 (1.17e-2) ≈	6.5990e-1 (1.37e-2) +	6.4435e-1 (1.35e-2) −	6.5357e-1 (1.11e-2)
CF7	2	10	4.9249e-1 (6.51e-2) ≈	5.1233e-1 (6.40e-2) ≈	4.8830e-1 (6.54e-2) ≈	4.7474e-1 (1.06e-1) ≈	4.9490e-1 (8.30e-2)
CF8	3	10	4.8046e-1 (1.01e-2) ≈	4.8052e-1 (1.10e-2) ≈	4.3511e-1 (1.21e-2) −	4.3162e-1 (2.20e-2) −	4.8132e-1 (9.48e-3)
CF9	3	10	5.5075e-1 (2.28e-3) ≈	5.5157e-1 (2.00e-3) ≈	5.3775e-1 (1.61e-2) −	5.4224e-1 (3.17e-3) −	5.5103e-1 (1.93e-3)
CF10	3	10	3.2181e-1 (1.17e-1) ≈	3.0106e-1 (1.21e-1) ≈	3.4337e-1 (6.17e-2) ≈	3.7964e-1 (6.36e-2) +	2.9694e-1 (1.03e-1)
+/ − / ≈			0/0/10	1/0/9	1/3/6	2/5/3	

Problem	M	D	CMOEA/D-TAP1	CMOEA/D-TAP2	CMOEA/D-TAP3	CMOEA/D-TAP5	CMOEA/D-TAP4
CF1	2	10	5.4899e-1 (5.11e-2) +	5.6213e-1 (1.43e-3) +	5.6218e-1 (9.70e-4) +	2.8655e-1 (1.12e-1) −	5.4644e-1 (3.94e-3)
CF2	2	10	6.6917e-1 (6.64e-3) ≈	6.7077e-1 (3.16e-3) +	6.6632e-1 (7.98e-3) ≈	6.7084e-1 (4.44e-3) +	6.6656e-1 (7.77e-3)
CF3	2	10	2.0457e-1 (4.72e-2) ≈	2.0145e-1 (4.31e-2) ≈	2.0334e-1 (4.78e-2) ≈	1.9419e-1 (4.87e-2) ≈	1.8931e-1 (3.83e-2)
CF4	2	10	5.0808e-1 (2.79e-2) ≈	5.0222e-1 (3.59e-2) ≈	4.9292e-1 (3.85e-2) ≈	4.4984e-1 (5.26e-2) −	5.1005e-1 (2.81e-2)
CF5	2	10	3.4075e-1 (4.51e-2) −	3.3559e-1 (3.98e-2) −	3.4955e-1 (5.59e-2) ≈	3.6610e-1 (4.67e-2) ≈	3.7085e-1 (5.98e-2)
CF6	2	10	6.5968e-1 (1.57e-2) ≈	6.5692e-1 (1.17e-2) ≈	6.5357e-1 (1.11e-2) −	6.4435e-1 (1.35e-2) −	6.5990e-1 (1.37e-2)
CF7	2	10	4.9249e-1 (6.51e-2) ≈	5.1233e-1 (6.40e-2) ≈	4.9490e-1 (8.30e-2) ≈	4.7474e-1 (1.06e-1) ≈	4.8830e-1 (6.54e-2)
CF8	3	10	4.8046e-1 (1.01e-2) +	4.8052e-1 (1.10e-2) +	4.8132e-1 (9.48e-3) +	4.3162e-1 (2.20e-2) ≈	4.3511e-1 (1.21e-2)
CF9	3	10	5.5075e-1 (2.28e-3) +	5.5157e-1 (2.00e-3) +	5.5103e-1 (1.93e-3) +	5.4224e-1 (3.17e-3) ≈	5.3775e-1 (1.61e-2)
CF10	3	10	3.2181e-1 (1.17e-1) ≈	3.0106e-1 (1.21e-1) ≈	2.9694e-1 (1.03e-1) ≈	3.7964e-1 (6.36e-2) +	3.4337e-1 (6.17e-2)
+/ − / ≈			3/1/6	4/1/5	3/1/6	2/3/5	

Problem	M	D	CMOEA/D-TAP1	CMOEA/D-TAP2	CMOEA/D-TAP3	CMOEA/D-TAP4	CMOEA/D-TAP5
CF1	2	10	5.4899e-1 (5.11e-2) +	5.6213e-1 (1.43e-3) +	5.6218e-1 (9.70e-4) +	5.4644e-1 (3.94e-3) +	2.8655e-1 (1.12e-1)
CF2	2	10	6.6917e-1 (6.64e-3) ≈	6.7077e-1 (3.16e-3) ≈	6.6632e-1 (7.98e-3) −	6.6656e-1 (7.77e-3) −	6.7084e-1 (4.44e-3)
CF3	2	10	2.0457e-1 (4.72e-2) ≈	2.0145e-1 (4.31e-2) ≈	2.0334e-1 (4.78e-2) ≈	1.8931e-1 (3.83e-2) ≈	1.9419e-1 (4.87e-2)
CF4	2	10	5.0808e-1 (2.79e-2) +	5.0222e-1 (3.59e-2) +	4.9292e-1 (3.85e-2) +	5.1005e-1 (2.81e-2) +	4.4984e-1 (5.26e-2)
CF5	2	10	3.4075e-1 (4.51e-2) −	3.3559e-1 (3.98e-2) −	3.4955e-1 (5.59e-2) ≈	3.7085e-1 (5.98e-2) ≈	3.6610e-1 (4.67e-2)
CF6	2	10	6.5968e-1 (1.57e-2) +	6.5692e-1 (1.17e-2) +	6.5357e-1 (1.11e-2) +	6.5990e-1 (1.37e-2) +	6.4435e-1 (1.35e-2)
CF7	2	10	4.9249e-1 (6.51e-2) ≈	5.1233e-1 (6.40e-2) ≈	4.9490e-1 (8.30e-2) ≈	4.8830e-1 (6.54e-2) ≈	4.7474e-1 (1.06e-1)
CF8	3	10	4.8046e-1 (1.01e-2) +	4.8052e-1 (1.10e-2) +	4.8132e-1 (9.48e-3) +	4.3511e-1 (1.21e-2) ≈	4.3162e-1 (2.20e-2)
CF9	3	10	5.5075e-1 (2.28e-3) +	5.5157e-1 (2.00e-3) +	5.5103e-1 (1.93e-3) +	5.3775e-1 (1.61e-2) ≈	5.4224e-1 (3.17e-3)
CF10	3	10	3.2181e-1 (1.17e-1) −	3.0106e-1 (1.21e-1) −	2.9694e-1 (1.03e-1) −	3.4337e-1 (6.17e-2) −	3.7964e-1 (6.36e-2)
+/ − / ≈			5/2/3	5/2/3	5/2/3	3/2/5	

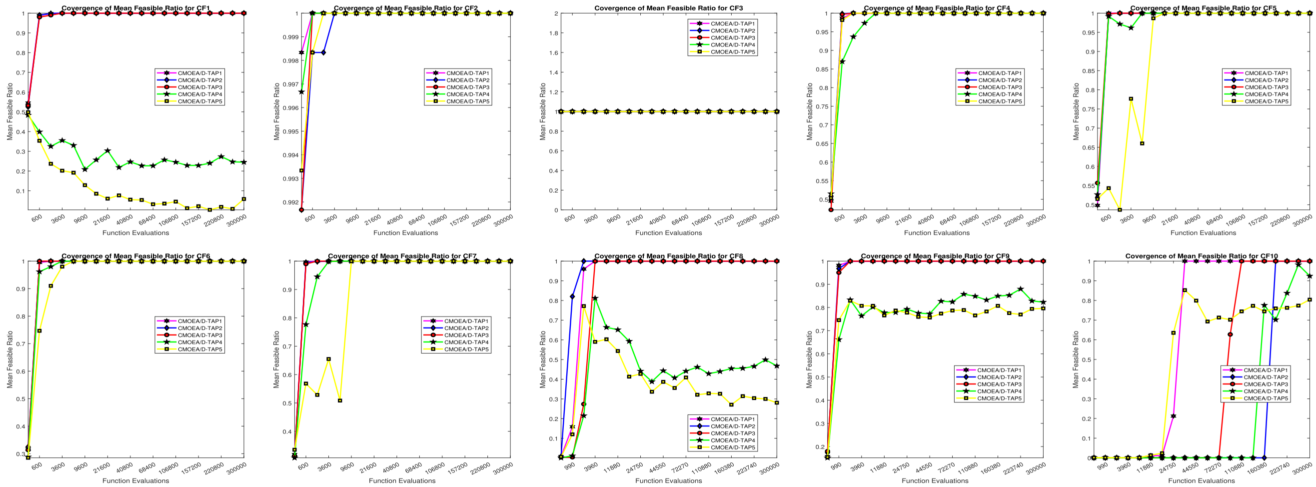

FIGURE 8. Convergence Plots of Mean Feasibility Ratio of the proposed algorithms for CF series.

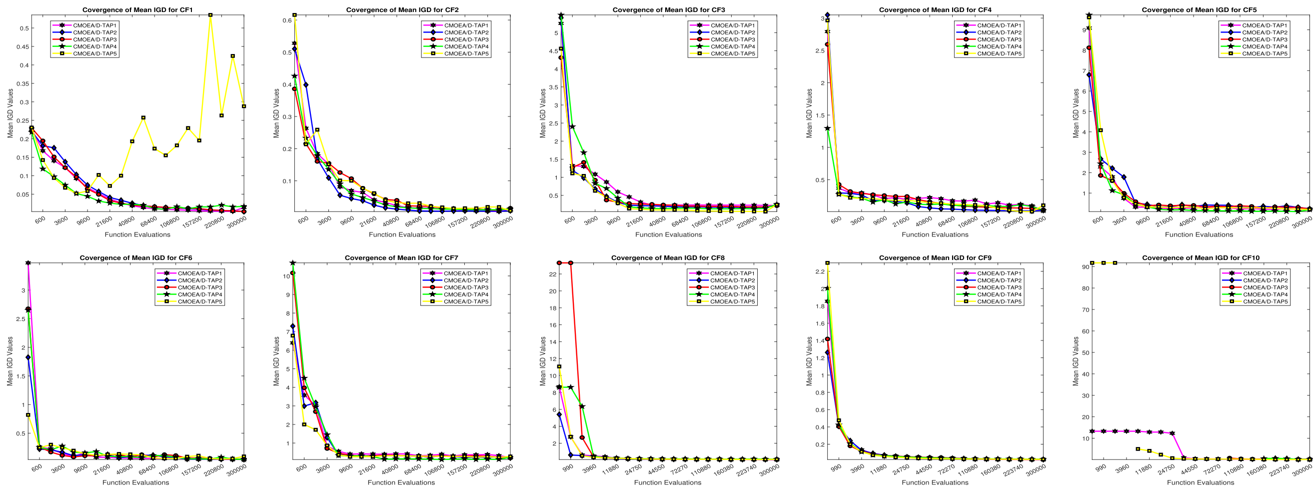

FIGURE 9. Convergence Plots of Mean IGD of the proposed algorithms for CF series.

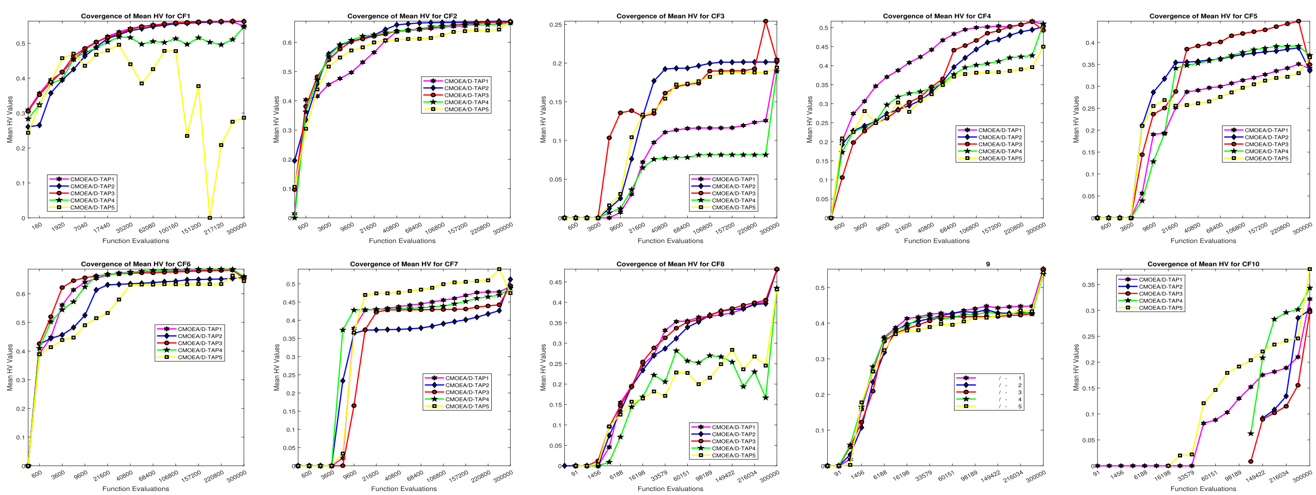

FIGURE 10. Convergence Plots of Mean HV of the proposed algorithms for CF series.

of an algorithm is better than, worse than and equivalent to the algorithm to which it is compared. Figures 8-10 show the convergence graphs of mean feasibility ratio and mean HV values, respectively of the contestants. Figures 11-15 and Figures 16-20 demonstrate the best run (based on IGD values) and all runs PF, respectively of the contenders.

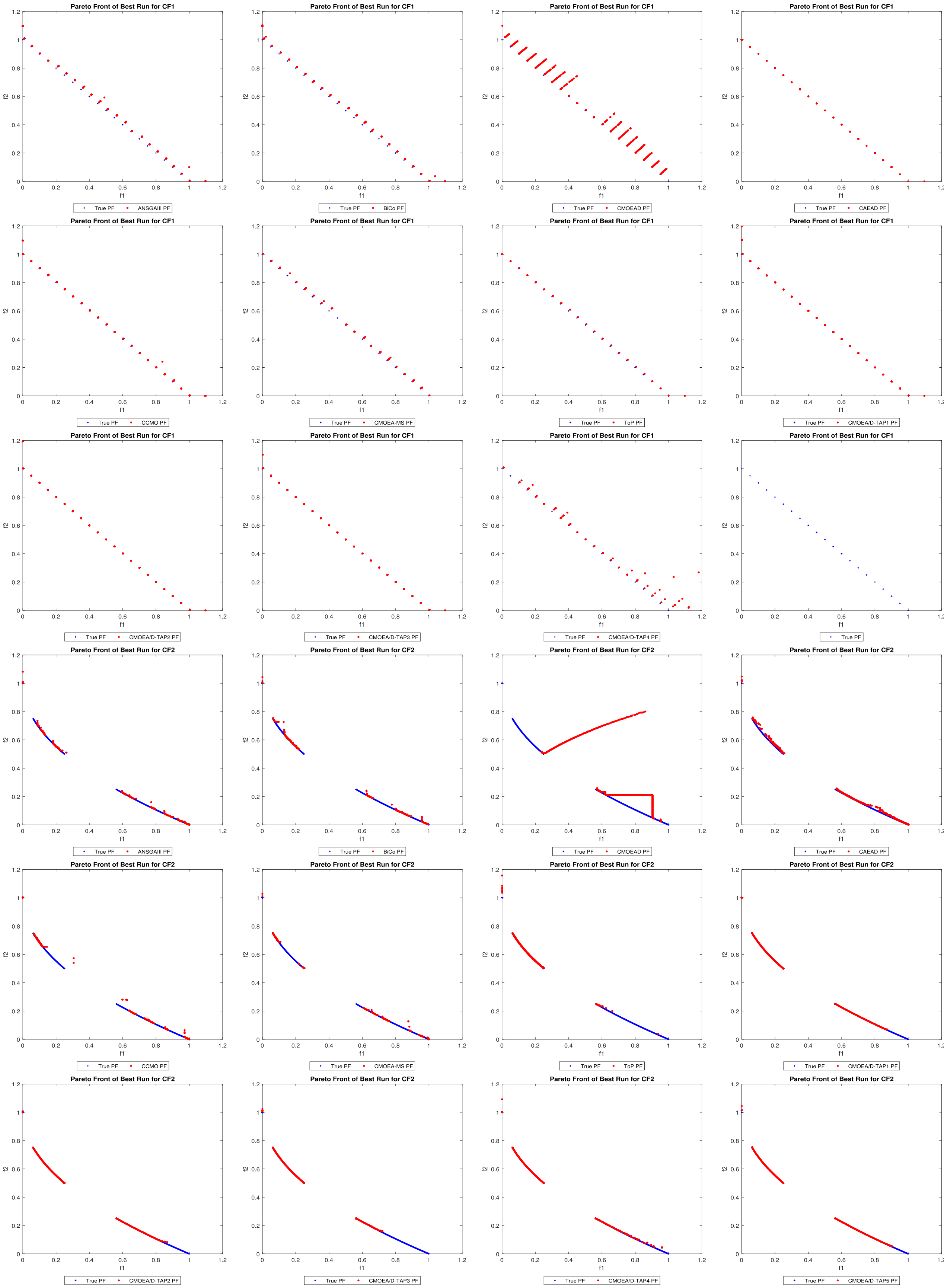

FIGURE 11. Best Run PF of the compared algorithms for CF1-CF2.

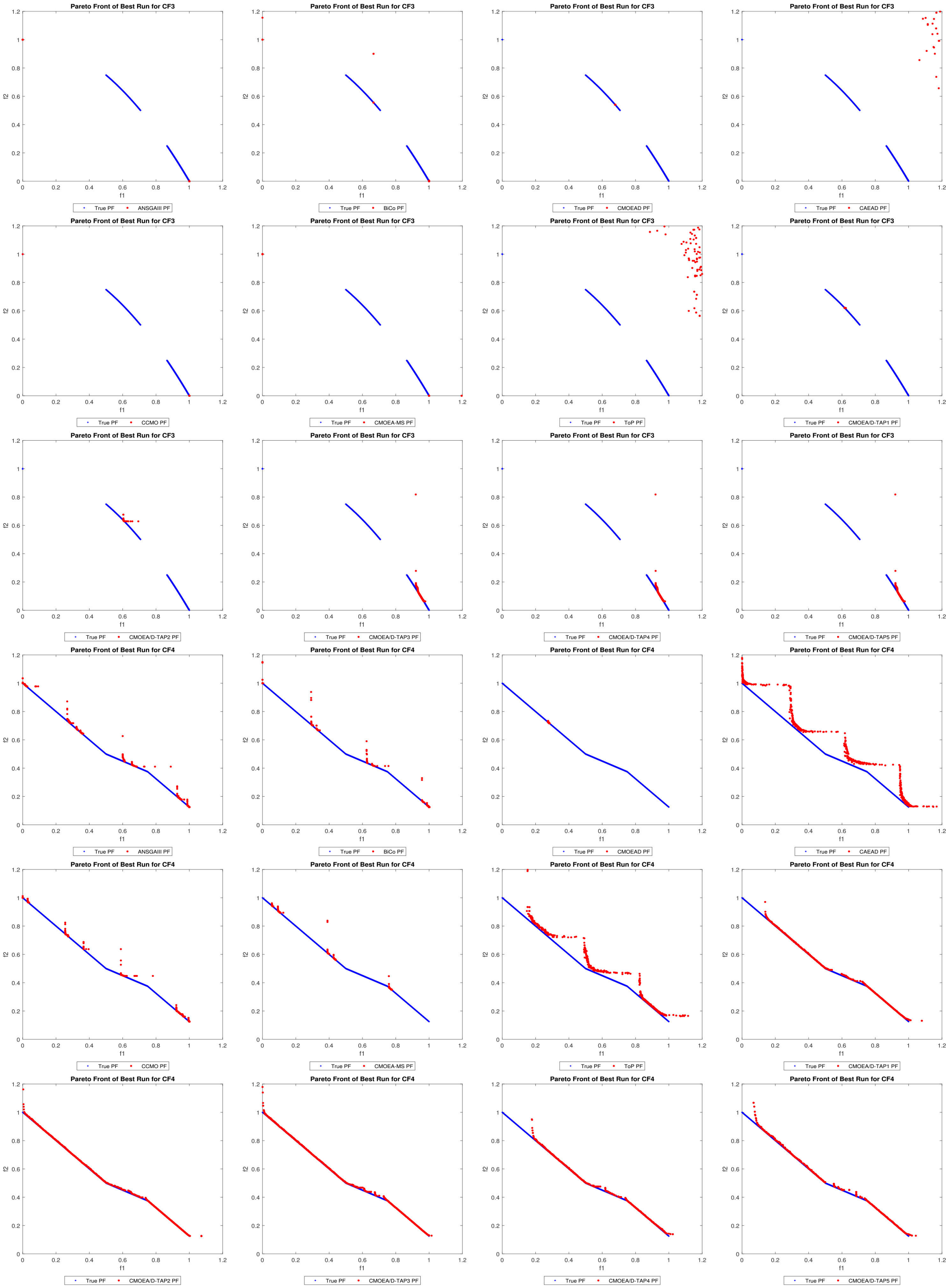

FIGURE 12. Best Run PF of the compared algorithms for CF3-CF4.

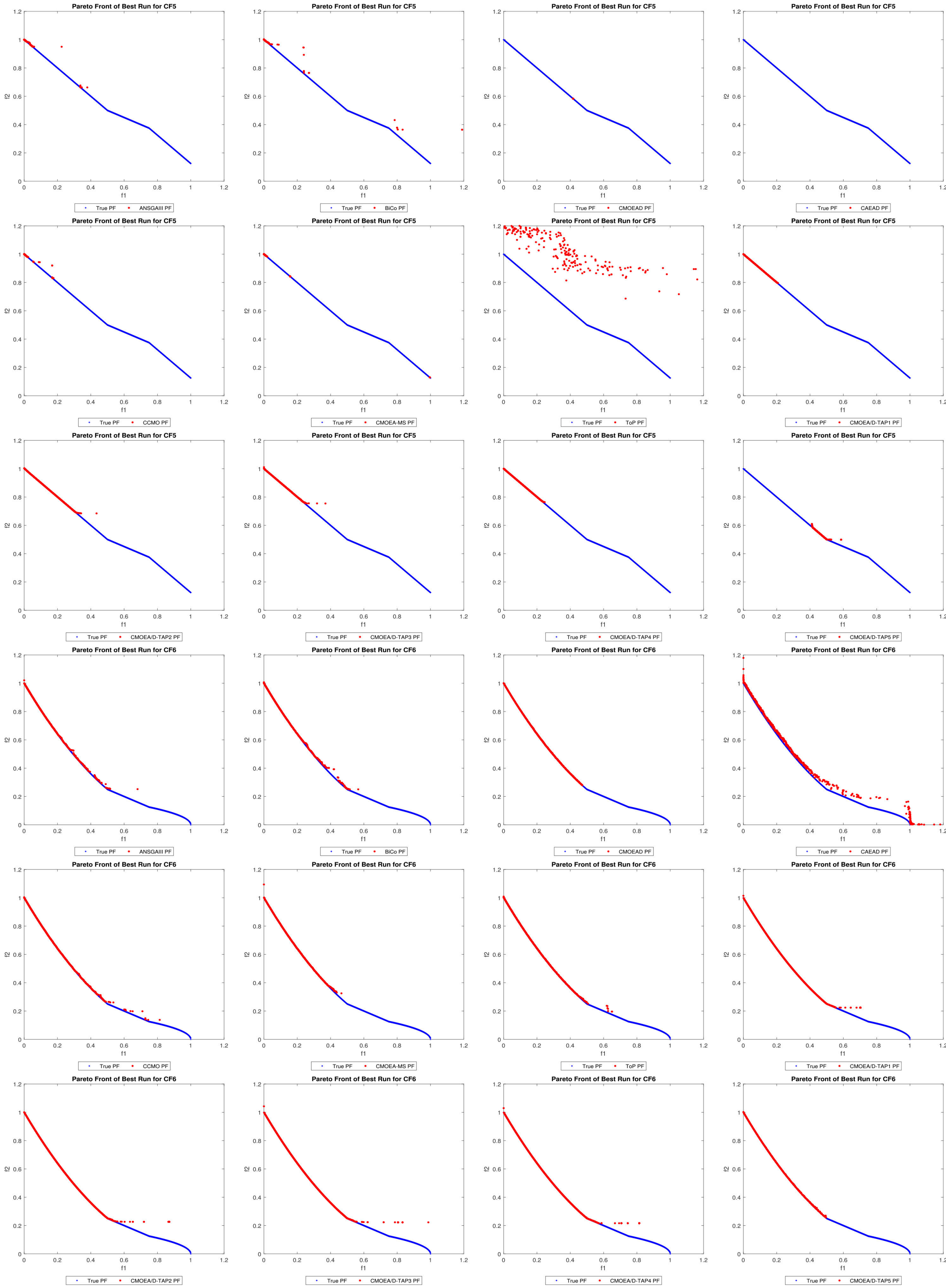

FIGURE 13. Best Run PF of the compared algorithms for CF5-CF6.

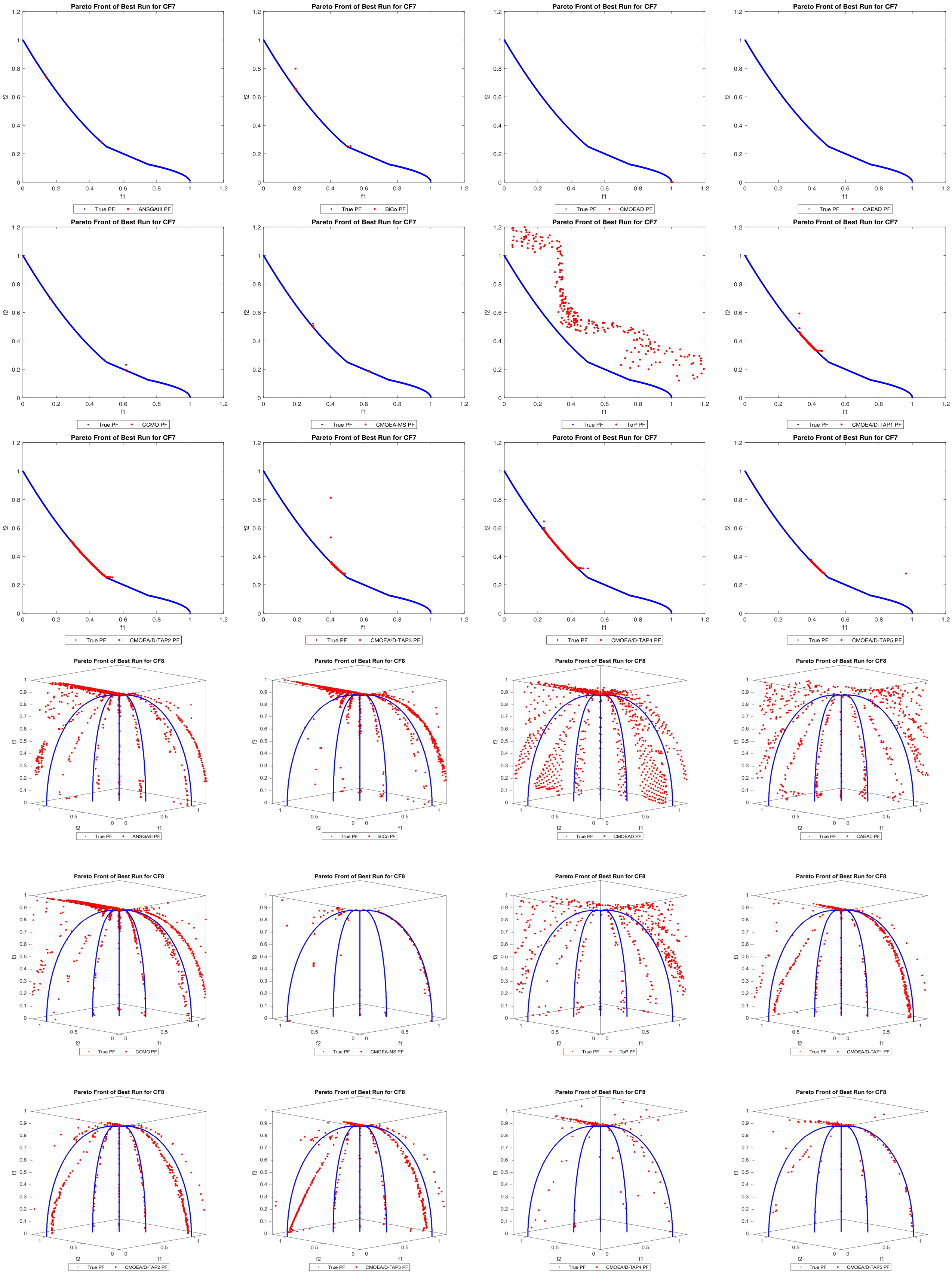

FIGURE 14. Best Run PF of the compared algorithms for CF7-CF8.

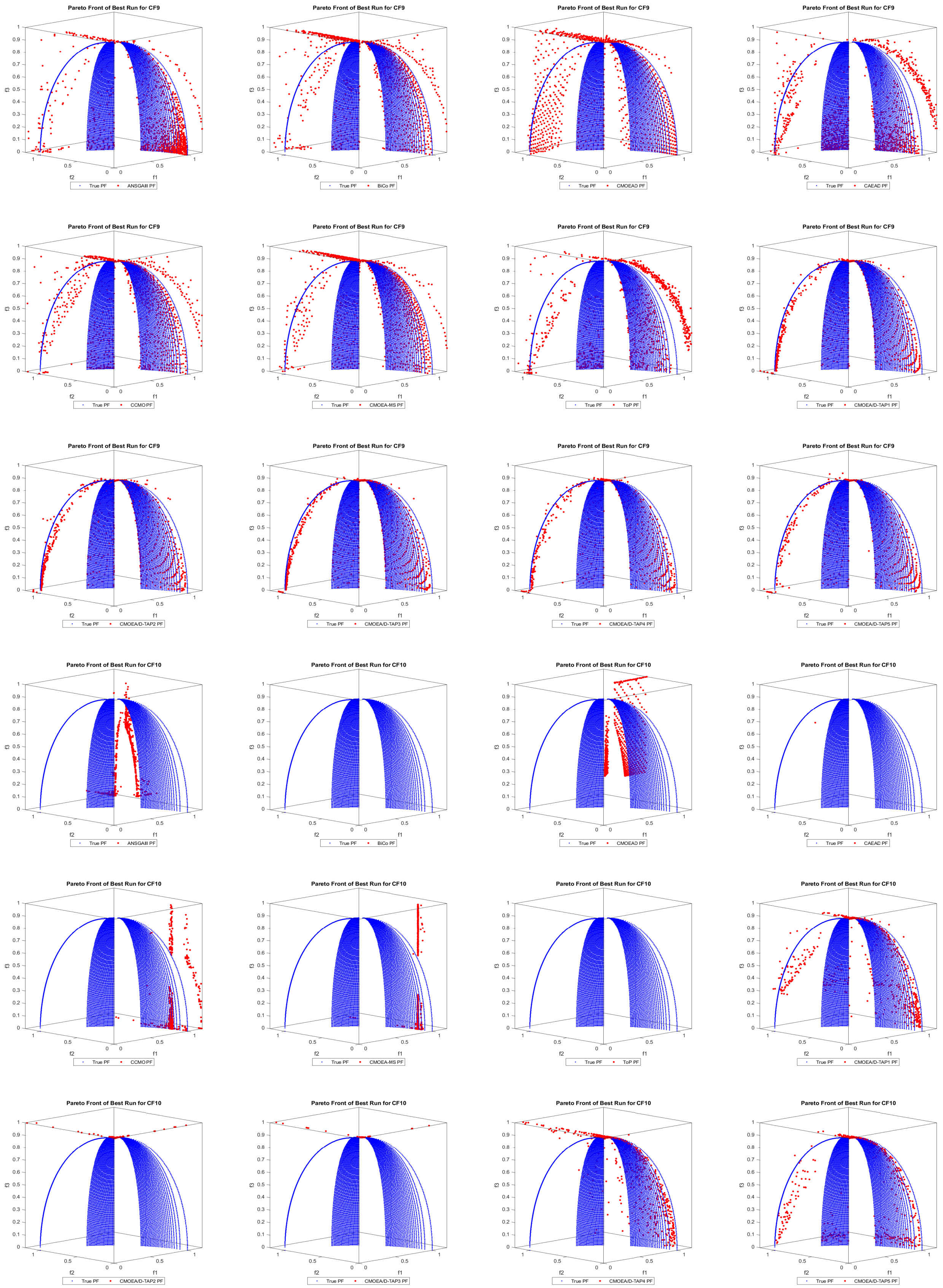

FIGURE 15. Best Run PF of the compared algorithms for CF9-CF10.

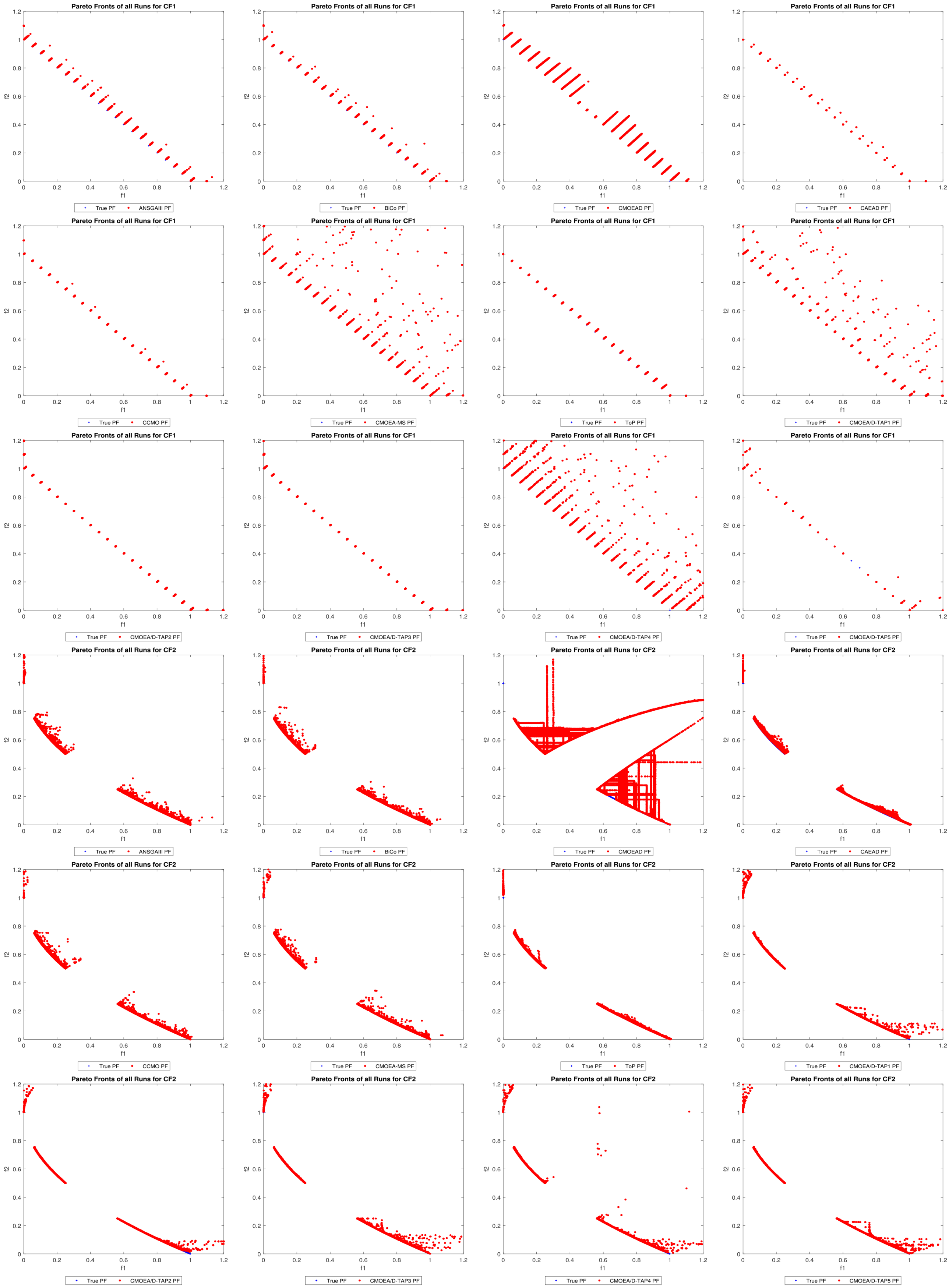

FIGURE 16. **All Runs PF of the compared algorithms for CF1-CF2.**

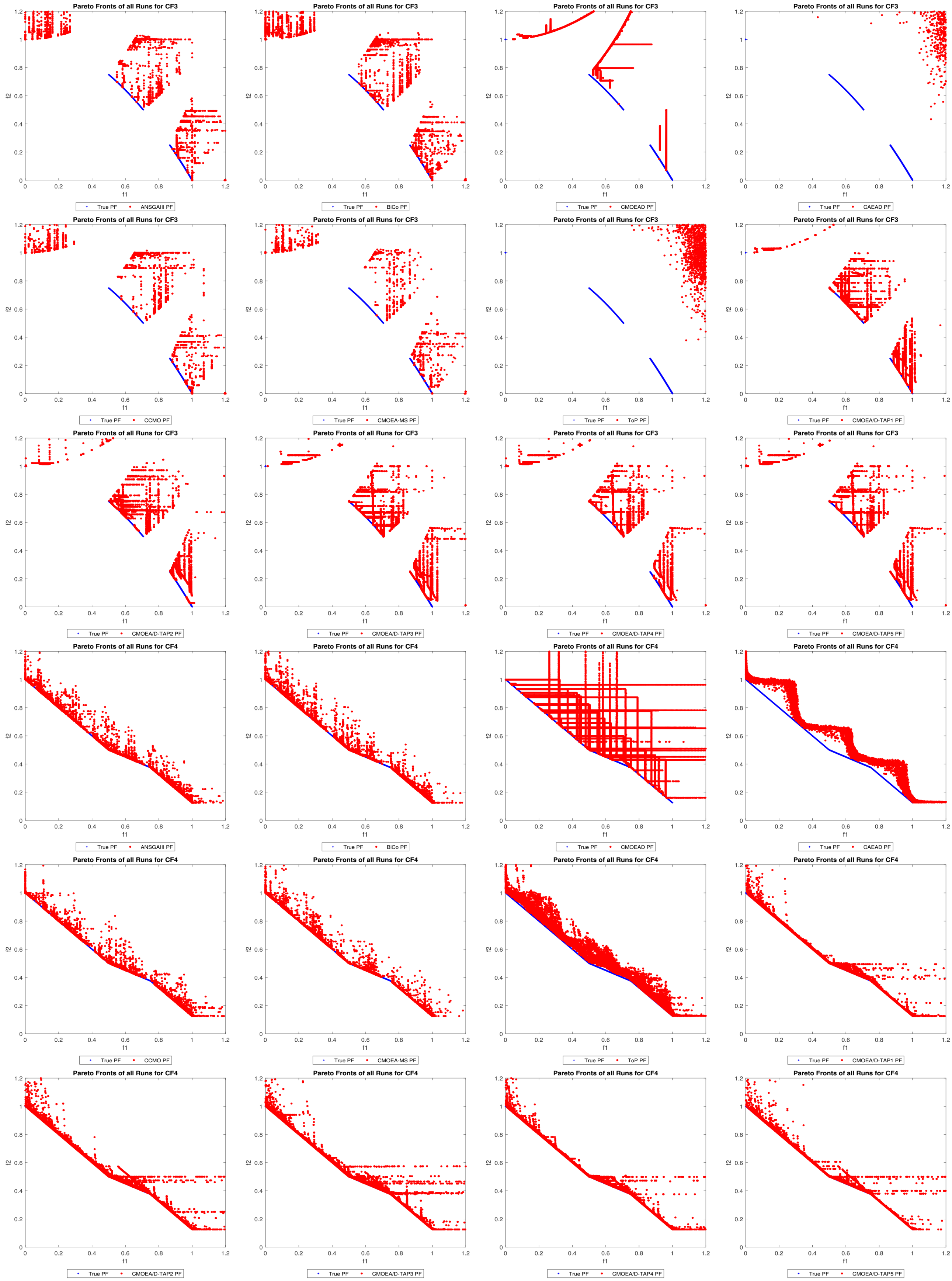

FIGURE 17. All Runs PF of the compared algorithms for CF3-CF4.

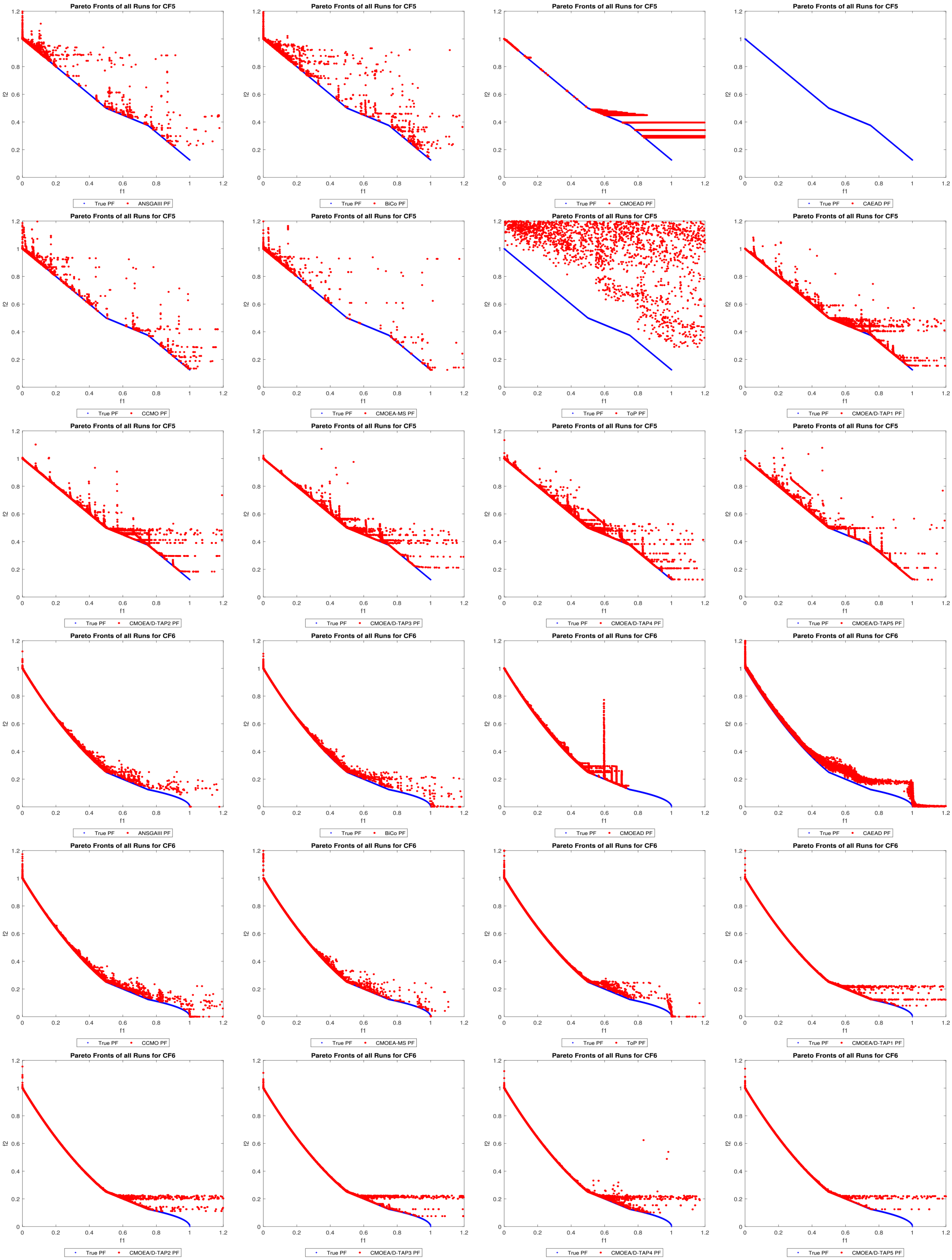

FIGURE 18. All Runs PF of the compared algorithms for CF5-CF6.

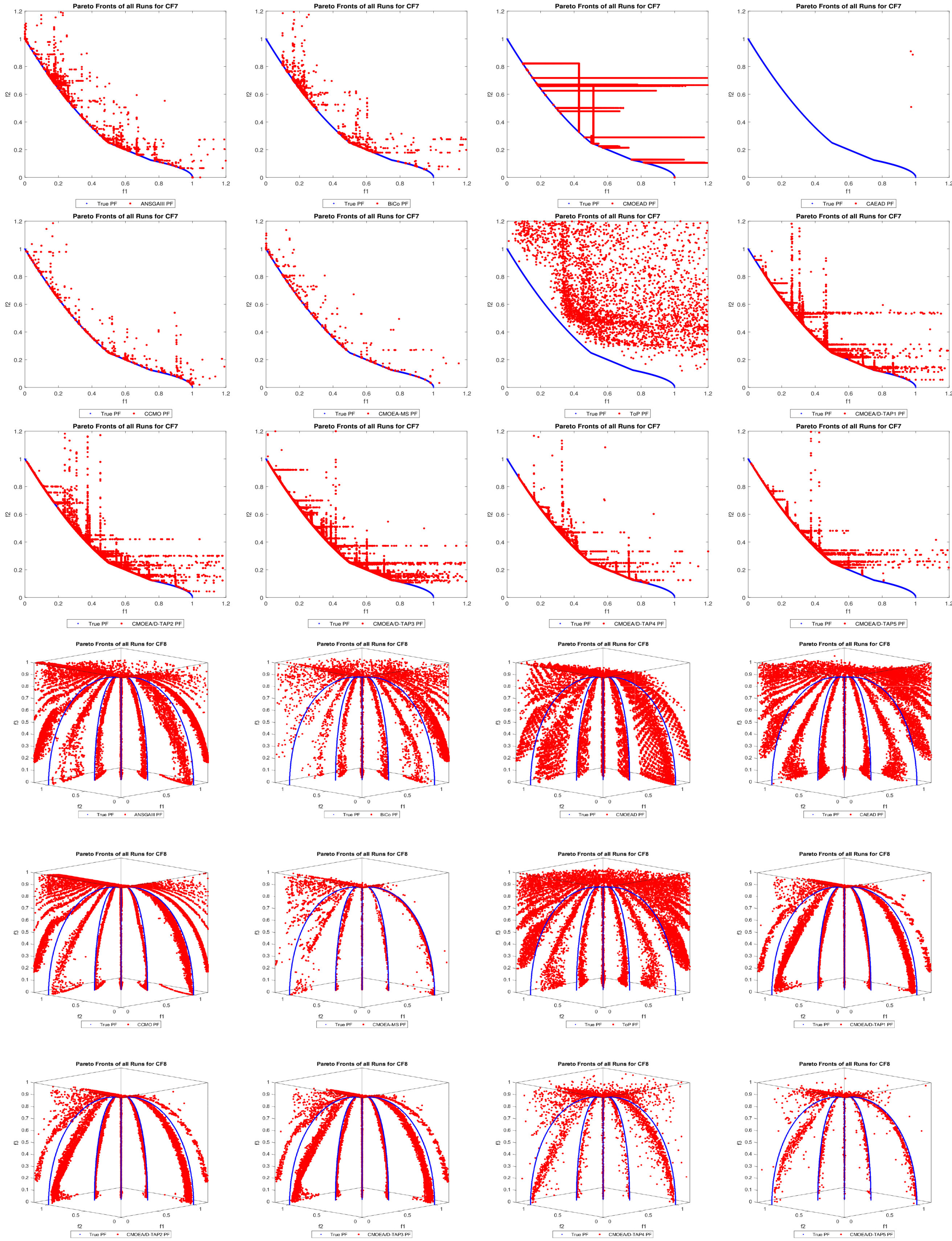

FIGURE 19. All Runs PF of the compared algorithms for CF7-CF8.

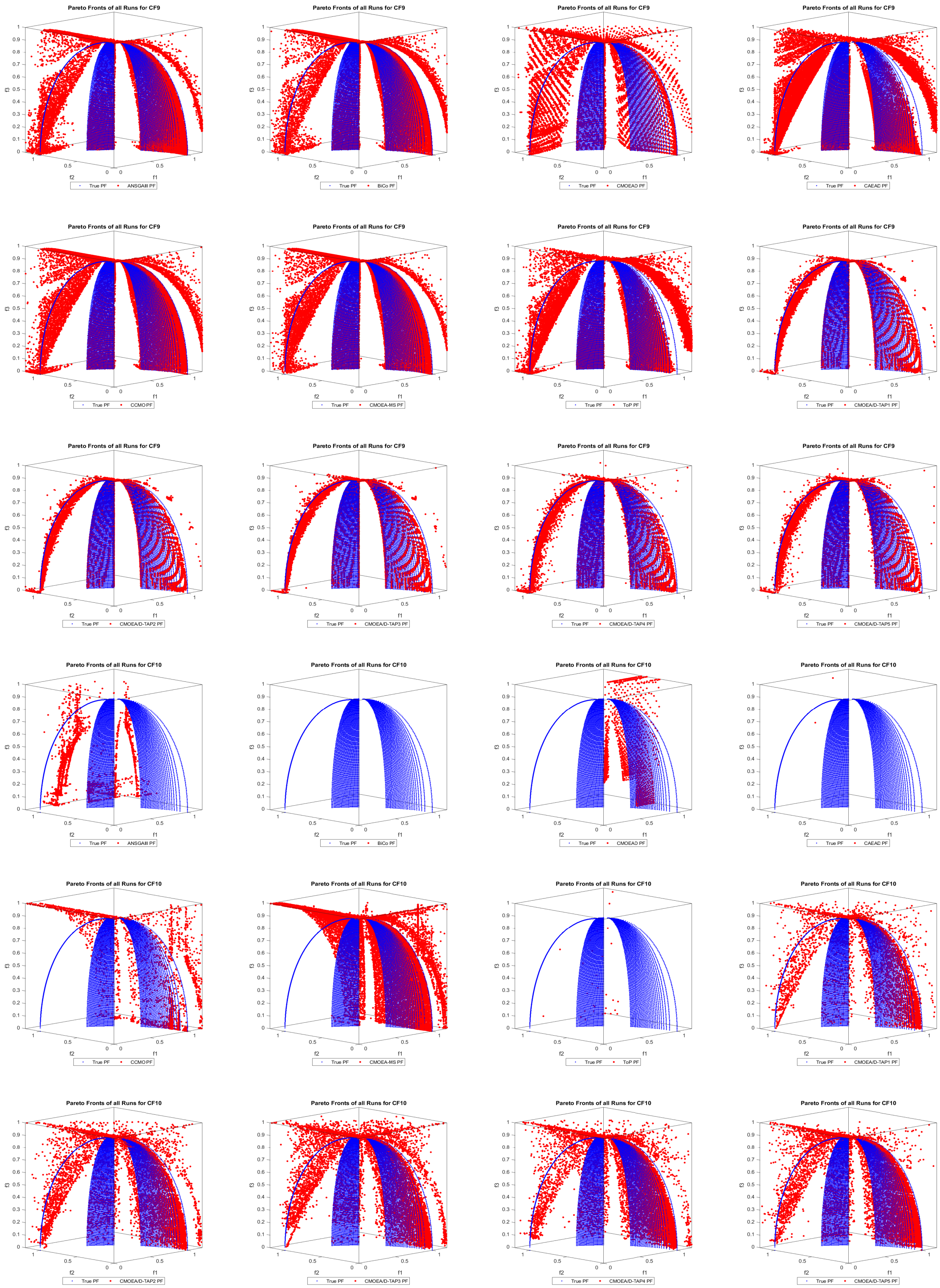

FIGURE 20. All Runs PF of the compared algorithms for CF8-CF10.

VI. DISCUSSION

This work extends the usage of a single objective constrained handling technique, threshold based adaptive penalty (TAP), to solve CMOPs. TAP is NFT depended CHT; therefore, various settings for it is also investigated during the current work. For tackling CMOPs, TAP with various NFT mechanisms are embedded into the framework of MOEA/D-DE to propose five constrained multiobjective optimization algorithms, namely CMOEA/D-TAP1 to CMOEA/D-TAP5. For performance evaluation of the introduced algorithms, two challenging suits, CTP series and CF series of CMOPs are taken. For CTP series, the comparison is made with four algorithms; while for CF series, seven recently developed state-of-the-art algorithms are picked. The comparison spotted the following points.

A. FOR CTP SERIES

- For CTP1, the best and the worst performances are shown by MOEA/D-DE-CDP and MOEA/D-DE-SR, respectively. Although, the proposed algorithms, CMOEA/D-TAP1-TAP5 attained the same best value as that of MOEA/D-DE-CDP, they remained in ranks 2-6 for mean and SD values, particularly CMOEA/D-TAP3 was ranked second for this problem. It is noted that NSGA-II and IDEA did not try to solve this problem.
- For CTP2, MOEA/D-DE-CDP outperforms all competing algorithms in terms of the best, mean and SD values. The second best value is achieved by CMOEA/D-TAP2. The worst value is obtained by MOEA/D-DE-SR. The second best mean value is obtained by CMOEA/D-TAP4. The second best SD value is achieved by CMOEA/D-TAP3. While the worst mean and SD values are achieved by NSGA-II. Thus, on CTP2, the MOEA/D-CDP performs better than all competing algorithms, while NSGA-II and MOEA/D-SR perform worse than other competitors.
- For CTP3, the best, mean and SD values are achieved by MOEA/D-CDP, the second best HV value is achieved by CMOEA/D-TAP3 and the worst value is obtained by MOEA/D-SR. The second best mean value is achieved by CMOEA/D-TAP5. The second best SD value is obtained by CMOEA/D-TAP3. The worst values of mean and SD are achieved by NSGA-II.
- For CTP4, the best, mean and SD values are achieved again by MOEA/D-CDP, the second best value is obtained by CMOEA/D-TAP3 and the worst value is attained by NSGA-II. The second best mean value is obtained by IDEA. The second best SD value is obtained by MOEA/D-SR. The worst mean and SD values are obtained by MOEA/D-SR.
- For CTP5, the best, mean and SD values are achieved yet again by MOEA/D-CDP. The second best value is achieved by CMOEA/D-TAP1 and the worst value is attained by NSGA-II. The second best mean value is achieved by CMOEA/D-TAP3 and the second best SD value is obtained by CMOEA/D-TAP3. The worst mean and SD values are obtained by MOEA/D-SR.
- For CTP6, the best value is obtained by NSGA-II. The second best value is achieved by IDEA. While the worst value is obtained by CMOEA/D-TAP2. The best mean and SD values are obtained by CMOEA/D-TAP3 and the second best mean and SD values are achieved by CMOEA/D-TAP1. The worst mean and SD values are attained again by NSGA-II.
- For CTP7, the best value is achieved by NSGA-II and IDEA. The second best value is attained by CMOEA/D-TAP1-CMOEA/D-TAP5. While the worst value is achieved by MOEA/D-CDP. The best mean value is achieved by CMOEA/D-TAP3 and CMOEA/D-TAP5 and the second best mean value is attained by CMOEA/D-TAP1-CMOEA/D-TAP2, while the worst mean value is achieved by NSGA-II. The best SD value is attained by MOEA/D-DE-CDP and the second best SD value by CMOEA/D-TAP5, while the worst SD value is obtained by IDEA.
- For CTP8, the best value is obtained by CMOEA/D-TAP3. The second best value is achieved by CMOEA/D-TAP2, while the worst value is attained by IDEA. The best mean and SD values are attained by CMOEA/D-TAP5. The second best mean and SD values are achieved CMOEA/D-TAP3. The worst mean value is attained by IDEA and the worst SD value is obtained by NSGA-II.

Total Rank Points (TRPs) based on HV metric statistics for each algorithm on seven CTP test instances: CTP2-CTP8 are displayed in TABLE 2. TRPs, which reflect an algorithm's overall effectiveness across all CTP test instances, are used in this case to determine ranking. The algorithm with the lowest value of TRPs is deemed to be the best performer, and the algorithm with the highest value of TRPs is deemed to be the worst performance. This table shows that CMOEA/D-TAP3 is rated first with the smallest TRPs of 61, followed by MOEA/D-DE-CDP with the second smallest TRPs of 67, and NSGA-II with the highest TRPs of 159, which is ranked ninth. As a result, the tested CTP test instances show that CMOEA/D-TAP3 surpasses all other algorithms, MOEA/D-DE-CDP performs second best, and NSGA-II performs worst.

Figure 2 shows the convergence plots of feasible ratio (FR) (the ratio of the number of feasible solutions to the total number of solutions in the whole population) of the proposed algorithm versus function evaluations for CTP series. It is reflected from the plots that the proposed algorithms attain an increase in FR values on most of the CTP series test instances, except CTP3 where an initial increase then some abrupt decrease and later on gradual increase with vibrations in the FR values is observed as the function evaluation increases.

Figure 3 depicts the convergence plots of HV of the proposed algorithm versus function evaluations for CTP series. It is observed that initially proposed algorithms attain rapid increase in HV values on all of the CTP series test instances, but later on the convergence rate become slow down.

For each CTP problem, the PFs obtained from the best runs of the suggested algorithm are shown in Figures 4-5. The introduced variants are successful in obtaining the majority of the pareto optimal solutions for all CTP problems with the exception of CTP4, where the proposed algorithms, CMOEA/D-TAP1, CMOEA/D-TAP3, and CMOEA/D-TAP5 are unable to get some optimal solutions.

Figures 6-7 shows the approximately calculated final PFs of 30 runs of the suggested algorithm on each CTP series test instance. With the exception of CTP4, where the proposed algorithms, using CMOEA/D-TAP1 and CMOEA/D-TAP5 consistently misses certain optimal solutions throughout all runs, it is clear from these figures that the proposed variants converged towards the same region over the course of the 30 runs while preserving better diversity.

B. FOR CF SERIES

Seven state-of-the-art algorithms are brought into the competition with each proposed variant through the Wilcoxon Rank Sum Test applied to their IGD and HV values. Results of the test reflects the following remarks:

- Overall performance of CMOEA/D-TAP1, CMOEA/D-TAP2, and CMOEA/D-TAP3 is better than seven state-of-the-art algorithms based on the applied test to their IGD values.
- Average performance of CMOEA/D-TAP4 is better than six state-of-the-art algorithms, however, a tie is observed with CCMO based on the applied test to their IGD values.
- CMOEA/D-TAP5 performs better than five state-of-the-art algorithms averagely based on the applied test to their IGD values. It is equivalent to CMOEA-MS while defeated by ToP.
- Overall performance of CMOEA/D-TAP1, CMOEA/D-TAP2, CMOEA/D-TAP3, and CMOEA/D-TAP4 is better than seven state-of-the-art algorithms based on the applied test to their HV values.
- Average performance of CMOEA/D-TAP5 is better than five state-of-the-art algorithms, a tie is notated with CMOEA-MS, and it is succeeded by CCMO based on the applied test to their HV values.
- It is also observed that in the seven state-of-the-arts CCMO showed prominent performance; while CMOEAD played badly in the competition.
- A comparison among the proposed algorithms in-lights that CMOEA/D-TAP1 and CMOEA/D-TAP3 are equivalent while CMOEA/D-TAP2 is slightly better than these based on the applied test to their IGD and HV values. So CMOEA/D-TAP2 is at first and CMOEA/D-TAP5 is at last rank in the competition; while remaining are in between these two.

For getting detailed information about the above discussion, consultation of Tables 3-6 is recommended.

Figure 8 reflects: (i) on CF1, CF8, CF9, and CF10 CMOEA/D-TAP4 and CMOEA/D-TAP5 show inconsistence convergence with zigzagging of FR (feasibility ratio) values (ii) On CF10, the suggested algorithms converge to feasible region after some time not abruptly like the remaining problems (ii) on CF3 the proposed algorithms get the maximum feasibility from the start of the evolution (iv) On the remaining instances, the algorithms converge to same FR values, finally.

Figure 9 demonstrates: (i) on CF1, CMOEA/D-TAP5 displays unstable convergence of IGD values with some fluctuations (ii) On CF10, CMOEA/D-TAP2-CMOEA/D-TAP4 are unable to achieve feasible solutions initially due to which their IGD values are not defined, but later on, they show some progress by gaining some convergence in IGD values (ii) for remaining problems proposed variants show nearly similar convergence of IGD values with minute variations.

Figure 10 exhibits: (i) on CF1 and CF8 CMOEA/D-TAP4 and CMOEA/D-TAP5 have uncertain and shaky HV progress (ii) on CF10, initially CMOEA/D-TAP2-CMOEA/D-TAP4 don't create feasible solutions, so their HV values are not visible in the graphs, however ultimately, the proposed algorithms gained feasibility and hence increase in HV values is observed (iii) for CF2, CF6, and CF8 the introduced variants present approximately uniform performance in the convergence of HV values (iv) for the rest of the problems the convergence of HV values are not uniform.

Figures 11-15 show the best approximation of the PFs that were obtained by the competitors for each of the CF series instances. Graphs reflect that (i) for CF1, CAEAD, CMOEA/D-TAP1, CMOEA/D-TAP2, and CMOEA/D-TAP3 attain all the discrete optimal solutions, although, remaining contestants converge but miss some portions or some of their solution are away from the true PF; while CMOEA/D-TAP5 showed huge deficiency in the coverage (ii) for CF2, CAEAD and all proposed variants achieve good approximations and spread of solutions to the true PF; however some parts are untaught (iii) for CF3 the possession of true PF of all contestants are not remarkable, but CMOEA/D-TAP1, CMOEA/D-TAP2, and CMOEA/D-TAP5 showed some success in the discussed perspective (iv) for CF4, CMOEA/D-TAP2 and CMOEA/D-TAP3 almost cover the true PF, CMOEA/D-TAP1 and CMOEA/D-TAP4 also displayed some performance while the rests do not reflect some thing valuable comparatively (v) for CF5, all the proposed algorithms demonstrate some performance while the progress of the rests are negligible (vi) for CF6, the compared algorithms show nearly equivalent role in covering the real PF; however, CMOEA/D-TAP4 has winning status among these (vii) for CF7, the performance of the proposed variants are approximately same but CMOEA/D-TAP4 is the leader while the rests are not worth considering (viii) for CF8, CMOEA/D-TAP1, CMOEA/D-TAP2, and CMOEA/D-TAP3 outperformed, while the rests are followed by these in covering the true PF (ix) for CF9, all the contestants show good performance but the proposed variants are prominent among these (x) for CF10, CMOEA/D-TAP1 and CMOEA/D-TAP5 showed class performance while the rests are not precious to discuss.

Figures 16-20 demonstrate the plotting of final PFs of all runs obtained by the contestants from simulations. Graphs spotted that (i) for CF1 approximated PF of CMOEA/D-TAP2 and CMOEA/D-TAP3 are prominent and nearly equivalent and followed by CCMO and ToP; while CMOEA/D-TAP5 played badly (ii) for CF2, ToP shows peak performance and remaining covered the true PF with same fashion; however, CMOEAD leaves some portion of the true PF untouched (iii) for CF3, CMOEA/D-TAP1, CMOEA/D-TAP3, CMOEA/D-TAP4, and CMOEA/D-TAP5 show similar performance in the covering of the true PF; while ToP and CAEAD are unable to cover the true PF (iv) for CF4, the proposed variants performed well, CMOEAD solution are far away from the true PF, and CAEAD doesn't cover the true PF in most places (v) for CF5, CMOEA/D-TAP1 and CMOEA/D-TAP4 cover the PF prominently, CMOEA/D-TAP2, CMOEA/D-TAP3, and CMOEA/D-TAP5 also show good performance; while remaining show nothing special, amongst these CAEAD doesn't bring even a single solution to the visible range of the PF (vi) for CF6. the proposed algorithms show nearly equivalent progress in covering the true PF where CMOEA/D-TAP1 and CMOEA/D-TAP3 are outstanding; while CAEAD leaves uncover some portions of the true PF (vii) for CF7, the performance of CMOEA/D-TAP1 to CMOEA/D-TAP4 are approximately same but CMOEA/D-TAP5 is behind these, while CAEAD performs worst (viii) for CF8, CMOEA/D-TAP1, CMOEA/D-TAP2, and CMOEA/D-TAP3 outperformed, the rest are followed by these in the coverage of the true PF, while CMOEA-MS plays worst among the contestants (ix) for CF9, all the contestants show good performance but the proposed variants are outstanding amongst these (x) for CF10, the proposed variants show stupendous performances while the rests are not worth considering comparatively.

VII. CONCLUSION AND FUTURE WORK

A. CONCLUSION

This study employed a modified adaptive penalty function method to the promising decomposition-based multiobjective optimization evolutionary algorithm, MOEA/D to tackle CMOPs. The modification is made based on extending the usage of a near feasibility threshold (NFT) based adaptive penalty function technique to constrained multiobjective optimization, which was initially employed for constrained single-objective optimization. For complete investigation of the suitable choice of NFT, five settings for it are tried, which result in five variants of the proposed algorithm. For performance evaluation, the constrained suits of CMOPs, the CTP series and the CF series are employed. Comparison of the introduced algorithms were made with four best performances for CTP series and seven state-of-the-art algorithms for CF series. Wilcoxon Rank Sum Test was applied to the IGD and HV values of the contesting algorithms for measuring the level of proximity. To determine the complete progress of the suggested variants, convergence graphs of FR, IGD, and HV values besides the best and all PFs are also displayed, which reflect the following pints.

- CMOEA/D-TAP3 was ranked first with the least TRPs of 61, followed by MOEA/D-DE-CDP with the second smallest TRPs of 67, while NSGA-II was at bottom with the highest TRPs of 159, for the instances of CTP series. The results of the examined CTP test instances demonstrate that the suggested algorithm CMOEA/D-TAP3 outperformed among the contesters, MOEA/D-DE-CDP comes at second; while NSGA-II stands at last in the CTP series competition.
- CMOEA/D-TAP1 and CMOEA/D-TAP2 achieved first position as both of them outperformed among the competitors, CMOEA/D-TAP3 attained second position as it performs better the rest of the algorithms for CF series, CMOEA/D-TAP4 and CCMO showed comparable performance but better than CMOEA/D-TAP5; therefore, CMOEA/D-TAP4 and CCMO both achieved third rank; while CMOEA/D-TAP5 attained fourth rank for the CF series problems comparison.
- Proposed variants have shown better performances on most of the CTP and CF series problems that reflect the effectiveness of the current research work.

B. FUTURE WORK

In future, this study will be extended in following directions:

- To evaluate the performance of proposed variants with real-world problems and other newly designed suits to measure their global impact.
- To employ the TAPs in the frameworks of other MOEAs to check its adaptability.
- To enhance the suggested schemes through other settings of penalty coefficient and NFT.

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