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RESEARCH ARTICLE

Multi-Objective Aircraft Robust Trajectory Optimization Considering Various Predictability Metrics Under Uncertain Wind

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ABSTRACT Trajectory uncertainty presents a major challenge for air traffic management. In order to improving the predictability of flights to facilitate air traffic management while also considering the economic benefits of airlines, in this paper, a trajectory multi-objective optimization method based on robust optimal control with flexible chosen horizontal route is proposed. Both the trajectory predictability and trajectory economic cost are taken as the optimization objectives. The method takes the ensemble weather forecast data as input to consider the impact of wind uncertainty on trajectory operation. To consider the accumulated predictability of arrival time at important sampling points throughout the entire trajectory and the impacts of increased predictability on the passed through sectors with different busyness levels, the concept of robustness coefficient based on sector busyness weighting is proposed and new trajectory predictability objective functions are established. A scenario from Hong Kong to Amsterdam is constructed to verify the effectiveness of the proposed model and algorithm. The results show that the proposed method can improve the predictability of the arrival time at the destination by 67.30% when compared with the initial flight plan, and can further improve the accumulated predictability for the entire trajectory predictability by 21.30% using the entire trajectory predictability metric as the objective. The robustness coefficient's effect on improving the arrival time predictability for busy sectors traversed by the trajectory without bringing unacceptable costs to the trajectory predictability in other sectors is also verified. By the experiments results, we can draw the conclusions that the proposed method can obtain wind optimized trajectory with both economy efficiency and improved predictability.

INDEX TERMS Air traffic management, robust optimal control, trajectory optimization, high-altitude wind, ensemble prediction system.

I. INTRODUCTION

Air traffic is currently in a rapid growth phase, and although there is a small decline in traffic in 2020 and 2021 due to COVID-19, there is already a rebound in 2022 [1], and air traffic will continue to grow in the long run. To accommodate the continued growth of air traffic in the future, the new generation of Air Traffic Control (ATC) will adopt the Trajectory-Based Operation (TBO) approach [2]. Under TBO, each flight is represented by a four-dimensional trajectory, which is expressed by a series of points from departure to arrival, each of which includes a three-dimensional position and time [3]. The basic feature of TBO is Controlled Time of Arrival (CTA) [4], which is a time constraint imposed on a series of points on the four-dimensional trajectory, i.e., the fourth dimension in the four-dimensional trajectory in addition to the three-dimensional position, and the use of feedback control to control the aircraft within the allowable control accuracy can effectively ensure the aircraft reaches the specified position within the CTA range [5].

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Airlines and ATC interact through mutual trajectory negotiation to enable safe and efficient flight operations. Airlines submit initial planned flight trajectories, called Business Development Trajectories (BDTs). Based on the BDT submitted by the airline, ATC detects the air traffic capacity imbalance, identifies the air traffic bottleneck areas over time, and releases them to the airline. The airlines can adjust the trajectory according to the bottleneck area and generate the shared business trajectory (SBT). And the ATC will generate the SBT based on. According to the newly generated trajectory, ATC manages the new capacity flow balance and generates reference business trajectory (RBT).

In the background of TBO, the traditional airspace operation mode is difficult to meet the operation requirements, so FRA was born. Free Route Airspace is a flexible use of airspace. In the FRA, users can plan their trajectory more flexibly between the given approach and exit points [6], which extends the operational space available for flights from the route to the whole airspace, helping to further improve flight operation efficiency, and save flight operation cost. The method of grid discretization of the airspace is usually used for route planning within the airspace of flexible flight paths to convert the continuous space into discrete space, thus facilitating route optimization. This paper focuses on the method of generating optimized trajectories considering the bottleneck areas issued by the ATC side, considering the demand of the flight division for economy and punctuality on the one hand, and the demand of the ATC for high safety and predictability on the other hand, and studying the method of multi-objective optimization of trajectories in the environment of FRA.

The trajectory optimization is different from the path planning problem, which needs to consider the performance limitations of the aircraft and give a reasonable arrival time at each point of the trajectory to ensure that the aircraft can fly according to the arrival time within the flight envelope. The currently used trajectory optimization methods include dynamic planning [7], heuristic methods (genetic algorithm [8], simulated annealing algorithm [9], particle swarm algorithm [10], and differential evolution algorithm [11]), and optimal control methods [12], etc. The optimization objectives consider the flight time and fuel consumption, etc., among which the optimal control algorithm differs from other algorithms in that the motion model is considered, and there may be large errors between the trajectories generated by other algorithms and the real flyable trajectories, the generated trajectory may be infeasible because the aircraft equations of motion are ignored, while the optimal control takes into account the aircraft equations of motion and can generate a very accurate trajectory with guaranteed flyability, which has become the main method used in recent years [13].

High-altitude winds have an important influence on trajectory optimization, and some studies assume that the high-altitude winds are deterministic, under which the trajectory optimization problem under deterministic wind forecasting conditions is studied [14], [15]. The actual high-altitude winds usually differ from the forecasted high-altitude winds, and the high-altitude wind forecast results usually have uncertainties. Among the mainstream meteorological forecast products, high-altitude wind forecast products such as the ensemble forecast system (EPS) [16] give ensemble forecasts of high-altitude winds, which are sets of possible high-altitude wind forecast results, consisting of about 10 to 100 ''ensemble members ''Under TBO, for a single flight trajectory, the uncertainty of high altitude winds can make it difficult for the flight to follow the predefined 4D trajectory and meet the planned CTA, thus affecting the flight's operational safety and operational cost. For air traffic as a whole, the impact of uncertainty in high-altitude wind forecasting is reflected in the reduced predictability of the overall trajectory, which makes it necessary to reserve more capacity to cope with the uncertainty in airspace usage and reduces the actual available capacity.

To quantify the effects of high-altitude wind uncertainty. EPS data is used and a deterministic trajectory predictor is constructed to obtain a trajectory set to quantify the impact of high-altitude wind uncertainty over the North Atlantic, and the results showed that high-altitude wind uncertainty reduced the predictability of flight time and fuel consumption and increased flight costs [17], [18]; And the effect of high-altitude wind uncertainty has also been investigated using a probabilistic trajectory prediction method, where the probability density functions of wind speed and direction are obtained from discrete EPS data by probability distribution fitting methods [19]; To quantify the additional economic cost of uncertainty, data from 810,227 American Airlines flights are collected to analyze the benefits of reducing forecast uncertainty, which showed that reducing the dispersion of flight time forecasts by one minute would save at least 120 million dollars [20]. Improving flight predictability can effectively increase airspace capacity and thus improve the efficiency of the entire air traffic network, and improving the overall predictability of trajectories by reducing the effect of wind uncertainty at high altitudes has an important impact on the safe and efficient operation of the entire airspace, especially the busy airspace where the capacity is close to saturation [21], [22].

To reduce the impact of high-altitude wind uncertainty, a series of studies have been conducted by researchers. Lindner et al.. use historical meteorological data to pre-optimize a flight corridor that includes all possible trajectories and re-optimize the trajectory within the corridor in real-time during the tactical traffic management phase based on updates of meteorological forecasts. the existence of flight corridors improves flight [23] Álvaro Rodríguez-Sanz et al. quantified the impact of uncertainty factors on flight times based on Monte Carlo methods and used Bayesian networks to identify flight parameters that are strongly associated with uncertainty and achieved uncertainty management by changing these parameters [24], [25]. Legrand et al. first used Bellman's algorithm to plan the optimal trajectory in a deterministic wind field, then combined it with EPS to obtain the optimal trajectory set, and proposed a new method to calculate the distance between trajectories, using a clustering method to obtain a robust trajectory over the North Atlantic [26].

Although the above studies reduce the effect of high-altitude wind uncertainty, the consideration of aircraft motion models is not sufficient to ensure the flyability of the trajectory, therefore, a large number of studies have started to use optimal control methods for trajectory optimization in recent years. González-Arribas et al. considered the effect of high-altitude wind uncertainty on the North Atlantic route and used a robust optimal control method to improve the predictability of the flight arrival destination moment by changing the horizontal path in a static high-altitude wind forecast wind field [27]. Manuel et al., also based on static forecast data, optimized the horizontal path over the North Atlantic and increased the flight cost as an optimization objective to obtain a Pareto optimal trajectory with different preferences between time predictability and average cost [28]. The advantage of using static forecast data is to reduce the computational effort in the optimization process, but the second half of the optimized trajectory gradually decreases in flyability with flight time, making it difficult to ensure effective application on long-haul flights. In order to solve the flyability problem of the trajectory planned before takeoff in the second half of the operation, Ramon et al. used the meteorological observation data shared by ADS-B in the tactical phase with the optimization objective of minimizing the arrival time error at the airport, and updated the 4D trajectory of the aircraft in the terminal area during continuous descent in real-time, which effectively improved the flyability of the second half of the trajectory, but this method requires high continuity and timeliness of the ADS-B data of the nearby aircraft, which is difficult to be applied in the high-altitude flight path phase [15]. Shumpei Kamo et al. similarly used the optimal control method to achieve continuous descent trajectory optimization for flights within the terminal area 1-2 h before the flight arrives at the TOD point for re-optimization of the trajectory within the terminal area in case of significant weather changes, but it is also limited by the forecast data source and faces the same problem of difficulty in extending to the airway stage [29].

Although recent research has yielded notable achievements, it exhibits certain deficiencies. Primarily, current studies predominantly employ static weather data for forecasts, disregarding the temporal evolution of weather patterns. Additionally, existing optimization models prioritize enhancing flight arrival time predictability at the destination, neglecting the significance of predictability at intermediate waypoints within the route sector. This oversight lacks comprehensive investigation into trajectory optimization methods that account for robustness in waypoint arrival times.

Therefore, to address the above shortcomings, this paper focuses on the trajectory optimization problem of FRA under the influence of uncertain and time-varying high-altitude wind forecasts, and investigates the robust trajectory optimization method based on optimal control theme, considering the improvement of flight operation efficiency and trajectory predictability. The main contributions of this paper include the following three points: First, considering the impact of time-varying high-altitude wind forecasts with uncertainty on the trajectory and the aircraft motion constraints, a multi-objective robust trajectory optimization model for aircraft is established based on the optimal control; Then, because the optimization objective of the existing robust trajectory optimization model only considers the predictability of the arrival time of the flight to the destination, but not the predictability of the arrival time of the waypoints during the flight, the robustness of the entire crossing time of the optimized trajectory is proposed as one of the objective functions. Finally, considering the fact that the flight passes through different airspace with different levels of busyness during the actual operation, the concept of robustness weight factor is introduced, and the effectiveness of the weight factor to improve the predictability of the critical flight phases of the flight path passing through busy sectors is investigated. The effectiveness of the weight coefficients in improving the predictability of the critical flight phases of the trajectory through busy sectors is investigated.

The rest of the paper is organized as follows: Section II describes and analyzes the problem in detail and gives the assumptions of the model developed in this paper. Section III provides an introduction to the models developed in this paper. Section IV describes the non-dominated ranking genetic algorithm (NSGA-II) with an elite policy, which is used to solve the discretized optimal model in Section III-E. Section V analyzes the trajectory of VHHH-EHAM long-range domestic cruise flight as an example solution to explore the effect of the optimization model on improving the robustness of the trajectory under different scenarios. The conclusions and future research prospects are shown in Section VI.

II. PROBLEM DESCRIPTION AND ANALYSIS

In the concept of TBO, the 4D trajectory consists of a three-dimensional position with CTA. Influenced by the uncertainty of high-altitude wind forecast, the arrival time of each point on the trajectory also has uncertainty. The purpose of this paper is to establish a robust trajectory optimization model for aircraft considering the uncertainty of high-altitude wind forecast, and to enhance the predictability of the arrival time of the destination and the arrival time of waypoints along the optimized trajectory affected by the uncertainty of high-altitude wind forecast, while improving the operational efficiency of the trajectory. In this paper, the optimization model is proposed based on the robust optimal control approach, and the model is based on the following assumptions:

(1) Considering only the effect of uncertainty in high-altitude wind forecasts, without considering other uncertainties such as convective weather and human factors;

(2) Considering only the cruise phase of the flight without considering the limitations of the approach and departure procedures within the terminal area, helps us to focus on the cruise phase, which is more affected by the uncertainty of the wind forecast at high altitudes, and to ignore the effects of the other phases of the flight;

(3) The aircraft is in a glossy configuration, additional fuel consumption due to different configurations is not taken into account;

(4) The model uses the ensemble wind forecast data of ECMWF, and the forecast ensemble contains all possible time-varying high-altitude wind scenarios, but the model is also applicable to other sources of EPS data;

(5) The airspace environment is free route airspace (FRA), so that the flight route can be planned freely without considering the fixed route network;

(6) With the help of the existing CNS technology, the aircraft can be guaranteed to operate autonomously according to the set trajectory, which ensures that the aircraft meets the operational requirements under TBO;

(7) Excluding restricted areas and exclusion zones due to traffic control, military activities, etc.

The above assumptions can help us exclude the interference of other factors and focus on the analysis of the impact of high-altitude wind forecast uncertainty on trajectory optimization.

III. ROBUST TRAJECTORY OPTIMIZATION MODEL BASED ON OPTIMAL CONTROL

The construction of a robust trajectory optimization model for aircraft, taking into account the effect of high-altitude wind, involves several key components: the wind effect on the ground speed model, aircraft motion model, and trajectory ensemble model under the influence of high-altitude wind. The wind model captures the relationship between wind speed, aircraft ground speed, heading angle, track angle, and wind direction. It provides insights into how the wind affects the aircraft's motion and helps in understanding the impact of wind on the trajectory. The aircraft motion model describes the state equations that govern the aircraft's motion. It considers various factors such as aerodynamics, thrust, and control inputs to determine the aircraft's position, velocity, and other state variables as a function of time. The trajectory ensemble model, influenced by high-altitude wind, represents a set of motion equations associated with the aircraft's ground speed under different sets of high-altitude wind forecasts. This model accounts for the uncertainties in wind forecasts and allows for the generation of multiple trajectories corresponding to different wind scenarios. To optimize the trajectory, an optimal control model is developed, and it is discretized to convert it into a numerical optimization problem. The discretization process involves dividing the trajectory into a finite number of points, and the optimization problem is then solved using numerical optimization techniques.

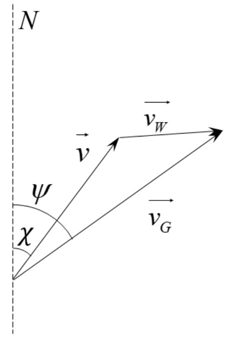


FIGURE 1. Diagram of the relationship between wind speed and ground speed, heading angle and track angle.

A. WIND EFFECT ON GROUND SPEED MODEL

Within the aircraft operation process, wind uncertainty stands as the primary source of uncertainty. Before constructing the aircraft dynamics model, it is crucial to establish the model for the influence of wind on ground speed. As depicted in Fig.1, the ground speed $\overrightarrow{v_G}$ is determined by calculating the vector sum of the true airspeed vector $\overrightarrow{v}$ and the wind velocity vector $\overrightarrow{v_w}$, resulting in the following equation:

$$\overrightarrow{v_G} = \overrightarrow{v} + \overrightarrow{v_W} \tag{1}$$

To ensure the aircraft stays on a predetermined course, specifically to maintain a constant trajectory angle, it is necessary to decompose the velocity vector into two components aligned with the east-west and north-south directions. This decomposition is achieved by altering the direction of the airspeed vector, known as the heading angle. Equation (1)can be expressed as follows:

$$v \cdot \cos(\chi) + w_{sn}(t, \phi, \lambda, h) = v_G \cdot \cos(\psi) \tag{2}$$

$$v \cdot \sin(\chi) + w_{ew}(t, \phi, \lambda, h) = v_G \cdot \sin(\psi) \tag{3}$$

where, v is the true air speed, v_G(m/s) is the ground speed, ψ(°) is the track angle, χ(°) is the heading angle, w_{sn}(m/s) is the wind component of wind velocity in the north-south direction, and w_{ew}(m/s) is the wind component of wind velocity in the east-west direction, ϕ(°) is the latitude, λ(°) is the longitude, h(m) is the flight altitude, t(s) is the time, w_{sn} and w_{ew} are both functions of t, ϕ, λ, and h, which can be obtained in a 4D wind field by the 3D position expressed by longitude, latitude, and altitude, and the time helps us to deal with the time variation in high altitude wind forecast.

B. AIRCRAFT MOTION MODEL

The impact of high-altitude wind uncertainty on the trajectory has a cumulative effect, and the impact of wind forecast uncertainty on long-haul flights is more obvious. In the case of long-haul flights, the cruise phase comprises a significant portion of the journey, and the distribution of high-altitude wind forecast uncertainty is not significant in the vertical direction. Therefore, our focus lies on the cruise phase, assuming that the aircraft maintains a consistent altitude throughout the flight. When we take the aircraft's motion model into account, we treat the Earth as a uniform sphere and do not consider variations in gravitational acceleration across different positions. In the model, the wind force is considered as a parameter and is obtained through the ensemble forecasting system (EPS).

Our initial focus is on the aircraft dynamics model in the presence of a deterministic wind field. To simplify the analysis of forces acting on the aircraft, we adopt the point mass model. This assumes that all forces act on the aircraft's center of gravity, while neglecting the influence of aircraft slope on operations. We consider the aircraft as a controlled dynamic system, with the aircraft operation time serving as the independent variable for the system. The motion model of the aircraft system can be represented by the following set of differential equations:

$$\frac{d\phi}{dt} = \frac{v_G \cdot \cos(\psi)}{R+h} \tag{4}$$

$$\frac{d\lambda}{dt} = \frac{v_G \cdot \sin(\psi)}{(R+h)\cdot\cos(\phi)} \tag{5}$$

$$\frac{dv}{dt} = \dot{v} = \frac{1}{m}(Thr - D) \tag{6}$$

$$\frac{dm}{dt} = \dot{m} = -FF \tag{7}$$

where $\phi(^\circ)$ is the latitude, $\lambda(^\circ)$ is the longitude, R(m) is the earth radius, h(m) is the flight altitude, m is the aircraft weight, Thr(N) is the aircraft engine thrust, and FF(kg/s) is the fuel flow rate.

In the aircraft motion model described above, time is used as the independent variable and all state variables are described as functions concerning time. The robust optimal control framework in this paper aims to plan a robust trajectory for the aircraft that adapts to all high-altitude wind forecast scenarios, where the aircraft follows the planned unique trajectory in all high-altitude wind scenarios, and the uncertainty of the arrival time due to the uncertainty of the high-altitude wind forecast scenarios cannot be represented if time is used as the independent variable.

To address this, the independent variable of the system in this paper is taken as the distance 's' traveled by the aircraft along the route. The time 't' is then considered as a state variable, and the relationship between time 't' and distance 's' is expressed as follows:

$$\frac{ds}{dt} = v_G \tag{8}$$

Correspondingly, the expression conversion of the motion model of the above aircraft according to (8) is expressed as the following equation:

$$\frac{d\phi}{ds} = \frac{\cos\psi}{R+h} \tag{9}$$

$$\frac{d\lambda}{ds} = \frac{\sin\psi}{(R+h)\cdot\cos\phi} \tag{10}$$

$$\frac{dv}{ds} = \frac{thr - D}{m}\cdot\frac{1}{v_G} \tag{11}$$

$$\frac{dm}{ds} = \frac{-FF}{v_G} \tag{12}$$

$$\frac{dt}{ds} = \frac{1}{v_G} \tag{13}$$

C. TRAJECTORY ENSEMBLE MODEL UNDER THE INFLUENCE OF HIGH-ALTITUDE WIND

EPS contains several high-altitude wind forecast members forming an ensemble of high-altitude wind forecasts, and the high-altitude wind forecast uncertainty is represented in the form of discrete high-altitude wind scenarios through the en semble forecast members. The optimization model in this paper aims to plan a 4D trajectory for the aircraft consisting of a 3D position and a TAS, with the trajectory remaining unique in all high-altitude wind forecast scenarios.

The model assumes that the aircraft remains at the same altitude level at all times, so it is also required that the state variables, ϕ, λ and v are consistent across different high-altitude wind forecast members. To satisfy the consistency of the above state variables for the planned 4D trajectory under different members, the aircraft needs to have different heading angles and thrusts for each forecast member. In the state equation, the variable directly affected by the high-altitude wind is the ground speed of the aircraft, and the equation of the state equation of the aircraft system regarding the ground speed needs to be expressed as a set of equations corresponding to the high-altitude wind scenario, i.e., equations (11) to (13) need to be written as

$$\frac{dv_i}{ds} = \frac{thr_i - D_i}{m_i}\cdot\frac{1}{v_{G,i}}, \quad i \in [1, \ldots N] \tag{14}$$

$$\frac{dm_i}{ds} = \frac{-FF_i}{v_{G,i}}, \quad i \in [1, \ldots N] \tag{15}$$

$$\frac{dt_i}{ds} = \frac{1}{v_{G,i}}, \quad i \in [1, \ldots N] \tag{16}$$

where N denotes the number of members in the set forecast.

D. OPTIMAL CONTROL MODEL FOR AIRCRAFT TRAJECTORY OPTIMIZATION

Considering the flight process of an aircraft as a controlled dynamic system, the motion model described in Section III-B is the state equation of this system, and this set of differential equations can model the nonlinear motion equations well, in which the control vector is $u = [Thr_1, \ldots, Thr_N, \chi_1, \ldots, \chi_N]$ and the state vector is $x = [\phi, \lambda, \psi, v, m_1, \ldots, m_N, t_1, \ldots, t_N]$

Before defining the objective function of the optimization problem, first define "time window of arrival" (TWA):

$$t_w(s) = \max(t(s)) - \min(t(s)) \tag{17}$$

where, t_w is a function of the flight distance s and represents the difference between the time of the earliest and the latest two trajectories in the set of trajectories at s. It is used to measure the predictability of the aircraft's arrival time at this location, i.e., to measure the robustness of the trajectory under different sets of high-altitude winds, and the smaller the value of TWA, the higher the robustness of the trajectory at this location.

The optimization objectives in optimal control are employed to assess the effectiveness of the control strategy. In the robust trajectory optimization model for aircraft presented in this paper, the following three optimization objectives are proposed, the first of these optimization objectives is related to the efficiency of the trajectory operation, while the second and third optimization objectives are related to the robustness of the trajectory under the influence of high-altitude wind uncertainty:

1) MINIMIZE FUEL CONSUMPTION

The fuel consumption objective is used to evaluate the flight cost of the trajectory, and the fuel consumption is obtained by calculating the difference between the initial mass of the aircraft and the mass at the destination, and the average of the fuel consumption of all trajectories in the set of minimized trajectories is used as the optimization objective, i.e.:

$$\min J_1 = m_0 - \frac{1}{N}\sum_{i=1}^{N} m_i(s_f) \tag{18}$$

2) MINIMIZE TWA OF THE DESTINATION

This objective is used to evaluate the robustness of the aircraft arrival time at the destination by calculating the TWA created by all trajectories at the destination, and minimizing the TWA at the destination is taken as the optimization objective, i.e.:

$$\min J_2 = t_w(s_f) \tag{19}$$

3) MINIMIZE TWA OF WHOLE ROUTE POINT

In response to the fact that most of the existing studies focus on the robustness of destination arrival time and lack of studies on the robustness of the arrival time of the whole route point of the trajectory, the robustness of the arrival time of the whole flight is defined as the optimization objective.

Considering that the busier airspace is affected by the uncertainty of the trajectory during busy hours, the more predictable the trajectory is required, so the trajectory robustness weight factor is defined as W, which is a function of longitude λ,latitude ϕ,and time t and is used to distinguish the importance of the robustness of the trajectory at a specific spatial location and at a specific time, and the larger the value of the factor, the higher the requirement of the robustness of the trajectory arrival time at that location, the optimization objective can be written as:

$$\min J_3 = \frac{1}{s_f}\int_{s_0}^{s_f} t_w(s)\cdot W(\lambda, \phi, t)ds \tag{20}$$

This equation expresses the integration of the TWA along the trajectory over the flight distance and normalizes the objective function by dividing it according to the total flight distance.

Constraints are categorized into terminal constraints and control constraints. Terminal constraints are used to represent constraints on the initial and end states of the aircraft, including the following constraints:

$$(\phi, \lambda)(0) = (\phi_0, \lambda_0) \tag{21}$$

$$(\phi, \lambda)(s_f) = (\phi_f, \lambda_f) \tag{22}$$

$$t_i(0) = 0, \quad \forall i \in \{1, \ldots, N\} \tag{23}$$

$$m_i(0) = m_{TOC}, \quad \forall i \in \{1, \ldots, N\} \tag{24}$$

Equation (21) and (22) indicates that the position and velocity of the starting and ending points of the trajectory are equal to the given starting and ending points of the flight optimization. Equation (23) indicates that the moment of the starting position of the optimization process is 0. Equation (24) indicates that the starting mass of all trajectories in the set is equal to the mass at the beginning of the cruise phase of the flight, i.e., when the flight reaches the top of the climb (TOC).

Control constraints are the aircraft performance and aircraft actual operating rules and other constraints on the range of values of aircraft control vectors, including:

$$Thr < Thr_{\max}(h) \tag{25}$$

$$0^\circ \leq \chi < 360^\circ \tag{26}$$

Equation(25) indicates that the thrust of the aircraft should not exceed the maximum thrust corresponding to the current altitude during the flight, and equation(26) indicates that the heading angle of the aircraft takes a value range between 0° and 360°.

E. MODEL DISCRETIZATION

Optimal control problems are typically solved using either indirect or direct methods. The indirect method involves deriving the solution through the variational method and the principle of minimum value, which requires an initial assumption about the solution structure, generally, it can only handle linear models and cannot contain complex constraints. However, for the complex aircraft robust trajectory optimization problem addressed in this paper, with intricate coupling relationships among the system's state variables, the indirect method is not suitable. Instead, this paper adopts the trapezoidal collocation method, which is a direct method, to discretize the optimal control problem into a Nonlinear Programming (NLP) problem for subsequent solutions.

To apply the trapezoidal collocation method in the direct solution approach, the first step is to determine the number of configuration points K. Then, the trajectory is discretized to obtain a finite set of decision variables at each configuration point. The trapezoidal matching point method is employed to discretize the continuous system dynamics equation, also known as the state equation, into a set of nonlinear constraints. These constraints ensure that the change of state between any two adjacent configuration points is equivalent to the integral of the continuous dynamics. As an example, consider Eq.(12), which represents one of these nonlinear constraints resulting from the application of the trapezoidal matching point method.

$$\int_{s_k}^{s_{k+1}} \frac{dm}{ds} ds = \int_{s_k}^{s_{k+1}} \frac{-FF}{v_G} ds \tag{27}$$

$$m_{k+1} - m_k = \frac{\Delta s_k}{2}\left[\left(-\frac{FF_{k+1}}{v_{Gk+1}}\right) + \left(-\frac{FF_k}{v_{Gk}}\right)\right] \tag{28}$$

Equation(28) is a representation of (27) after integrating both sides of the equation. In (28), the right side represents the integral approximation using the trapezoidal rule for numerical integration. The distance $\Delta s_k = s_{k+1} - s_k$ between the two points in the equation is calculated using the formula for the distance between two points on a rhumb line. The formula is as follows:

$$\begin{cases} a_k = \sin(\dfrac{\lambda_{k+1} - \lambda_k)}{2}^2 + \sin(\dfrac{\phi_{k+1} - \phi_k}{2})^2 \\ \qquad \cdot \cos(\lambda_k) \cdot \cos(\lambda_{k+1}) \\ \Delta s_k = 2 \cdot \mathrm{atan2}(\sqrt{a_k}, \sqrt{1 - a_k}) \end{cases} \tag{29}$$

In the equation, atan2 refers to the four-quadrant arctangent function, compared with the great circle route, the equiangular route is not the shortest path, but it is easier to navigate and calculate, and can effectively reduce the amount of calculation in the process of track optimization.

The objective in the optimal control model is discretized and expressed as:

$$\min J_1 = m_0 - \frac{1}{N}\sum_{i=1}^{N} m_{iK} \tag{30}$$

$$\min J_2 = t_{wK} \tag{31}$$

$$\min J_3 = \frac{1}{s_f}\sum_{k=1}^{K-1} t_{wk} \cdot W \cdot \Delta s_k \tag{32}$$

The equation of state after discretization can be represented as follows:

$$\phi_{k+1} - \phi_k = \frac{\Delta s_k}{2}\frac{(\cos\psi_{k+1} + \cos\psi_k)}{R + h} \tag{33}$$

$$\lambda_{k+1} - \lambda_k = \frac{\Delta s_k}{2}\left(\frac{\sin\psi_{k+1}}{(R+h)\cdot\cos\phi_{k+1}} + \frac{\sin\psi_k}{(R+h)\cdot\cos\phi_k}\right) \tag{34}$$

$$v_{k+1} - v_k = \frac{\Delta s_k}{2}(\frac{thr_{k+1} - D_{k+1}}{m_{k+1}} \cdot \frac{1}{v_{Gk+1}} + \frac{thr_k - D_k}{m_k} \cdot \frac{1}{v_{Gk}}) \tag{35}$$

$$t_{k+1} - t_k = \frac{\Delta s_k}{2}\cdot\left(\frac{1}{v_{Gk+1}} + \frac{1}{v_{Gk}}\right) \tag{36}$$

$$m_{k+1} - m_k = \frac{\Delta s_k}{2}\cdot\left(-\frac{FF_{k+1}}{v_{Gk+1}} - \frac{FF_k}{v_{Gk}}\right) \tag{37}$$

To ensure the accuracy and validity of the trajectory optimization model, the following additional constraints are added:

$$s(t_{k+1}) > s(t_k) \tag{38}$$

$$m(t_{k+1}) < m(t_k) \tag{39}$$

$$t_{k+1} > t_k \tag{40}$$

where constraint (38) ensures that the horizontal distance covered by the aircraft during the flight always increases, indicating forward progress. Constraint (39) ensures that the fuel consumption is always positive during the flight, implying that the aircraft's mass decreases over time. Lastly, constraint (40) indicates that the arrival time of discrete points is kept incremental. These additional constraints contribute to the accuracy and validity of the trajectory optimization model.

IV. MULTI-OBJECTIVE MODEL SOLVING ALGORITHM AND PARETO FRONTIER SOLUTION EVALUATION

The aircraft trajectory optimization problem exhibits a high level of complexity, which escalates exponentially with the number of decision variables. Consequently, obtaining and verifying the solution in polynomial time becomes challenging, rendering it an NP-Hard problem. Moreover, due to the strong coupling between variables, a metaheuristic algorithm is employed for its solution. The genetic algorithm, a well-established stochastic search algorithm inspired by natural selection, genetics, and variation, offers a versatile framework for tackling complex optimization problems. It has been successfully applied in various fields. The research findings demonstrate that the Non-dominated Sorting Genetic Algorithm II (NSGA-II), which incorporates an elite retention strategy, exhibits comprehensive advantages in terms of solution quality and convergence efficiency. Hence, this paper utilizes NSGA-II to solve the proposed trajectory optimization model. The algorithm's flow is depicted in Fig.2, outlining the step-by-step procedure for finding optimal or near-optimal solutions.

Aircraft trajectory optimization problems, whose size grows geometrically with the number of decision variables, are NP-Hard problems and are solved using metaheuristic algorithms. Genetic algorithm is a stochastic search algorithm that draws on the laws of evolution in nature and provides a general framework for solving complex optimization problems, which has been successfully used in many disciplines. The research results show that the non-dominated ranking genetic algorithm (NSGA-II) based on the elite retention strategy has comprehensive advantages in terms of solution quality and convergence efficiency, so this paper uses this algorithm to solve the proposed model, and the algorithm flow is shown in Fig.2.

A. CHROMOSOME CODE

The chromosomes use a two-dimensional coding structure as shown in Fig.3, with a real number coding, and the chromosome length is the number of discrete points, and each gene indicates the longitude and latitude of the flight at that discrete point as well as the true airspeed. The advantage of designing the chromosomes in this way is that the variables that are consistent in each member of the trajectory set are

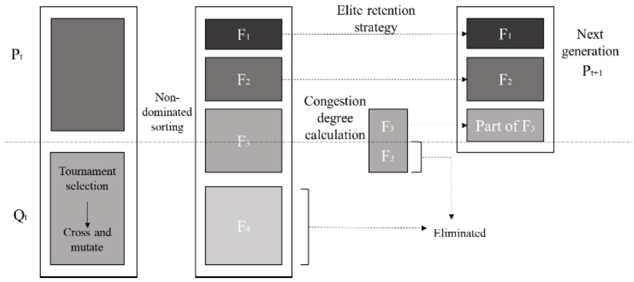


FIGURE 2. Flow of NSGA II algorithm.

designed as chromosomes to maintain consistency among the trajectories, and the adaptation function can be calculated by simply extrapolating the flight time and flight cost of each trajectory member separately according to the members of the different high-altitude wind sets.

λ_1	λ_2	$\cdots$	λ_k	$\cdots$	λ_{K-1}	λ_K
ϕ_1	ϕ_2	$\cdots$	ϕ_k	$\cdots$	ϕ_{K-1}	ϕ_K
v_1	v_2	$\cdots$	v_k	$\cdots$	v_{K-1}	v_K

FIGURE 3. Chromosome coding method and structure design.

B. INITIAL POPULATION

The initial population, as the starting point of the iteration of the heuristic algorithm, is an important factor affecting the population evolution results and the efficiency of the algorithm. In trajectory optimization, the flight history trajectory data is used as the initial population to perform the trajectory optimization.

C. CHROMOSOME CROSSOVER AND MUTATION

For the two-dimensional chromosome designed by the algorithm, a single-point crossover is used, and different crossover nodes are randomly selected for different decision variables, and the crossover operation is shown in Fig.4.

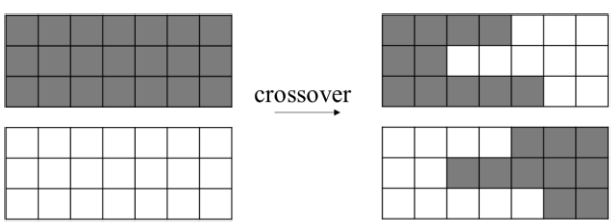


FIGURE 4. Chromosome crossover operation.

After the crossover operation on the parent chromosomes to obtain the offspring chromosomes, the mutation operation on the offspring chromosomes is performed by using multi-point mutation and randomly selecting the mutation nodes, and the number of mutation nodes is determined by the mutation probability. Here, if the mutation probability is large, then the whole search process degenerates into a random search process. To address this, the paper introduces an adaptive variation probability strategy. At the beginning of the evolutionary process, the mutation probability is set to a relatively large value, allowing for more exploration and diversification of the population. As the optimization progresses and the solutions begin to converge, the mutation probability gradually decreases. This adaptive approach helps strike a balance between exploration and exploitation, promoting a more focused search for refined solutions while avoiding premature convergence.

D. NON-DOMINATED SORTING

The non-dominated sorting method performs a fast stratification of all individuals within the population to form multiple Pareto fronts of different ranks. Crowding is used to measure the distance from one solution to two adjacent solutions on the same rank of non-dominated solution frontier.

E. EVALUATION METRICS FOR THE PARETO FRONTIER

After obtaining the Pareto frontier, the quality of the solution set needs to be evaluated, and in this paper, the solution set is evaluated by the following three metrics:

1) C-METRIC (CM), WHICH MEASURES THE DOMINANCE RELATIONSHIP BETWEEN SOLUTION SETS

$$CM(A, B) = \frac{|\{u \in B | v \in A : v \text{ dominate } u\}|}{|B|} \tag{41}$$

where, A, B denote the two solution sets, the numerator denotes the number of solutions governed by solution set A in solution set B, and the denominator denotes the number of solutions in solution set B.

2) HYPER VOLUME (HV), A MEASURE OF CONVERGENCE AND DIVERSITY OF SOLUTION SETS

$$HV(A) = \delta\left(\bigcup_{i=1}^{|A|} V_i\right) \tag{42}$$

where, $\delta(\cdot)$ denotes the Leberger measure, which is used to calculate the volume, and V_i denotes the hypervolume formed by the i point of solution and the reference point. Here, the solution set is normalized to the reference point $(1, 1)$. The larger the HV value, the more desirable the solution set is.

3) MEAN IDEAL DISTANCE (MID), WHICH MEASURES THE DISTANCE OF THE SOLUTION SET FROM THE IDEAL POINT

$$MID(A) = \frac{\sum_{i=1}^{|A|} \sqrt{\sum_{m=1}^{2} \left(\frac{f_i^m}{\max(f^m) - \min(f^m)}\right)^2}}{|A|} \tag{43}$$

where, denotes the m objective value of the i solution. The smaller the value of MID, the more ideal the solution set is.

V. CASE STUDY

A. HIGH-ALTITUDE WIND DATA AND UNCERTAINTY QUANTIFICATION

In the optimization model, the high-altitude wind data is obtained from the EPS of the European Centre for Medium-Range Weather Forecasts (ECMWF). The EPS is a numerical weather prediction method that generates forecast ensembles by running a weather model multiple times with different initial conditions and perturbations. The ECMWF data used in this paper consists of 51 ensemble members, which include 1 control forecast and 50 perturbation forecasts. These ensemble members provide a range of possible weather scenarios. The data have a temporal resolution of 1 h and a spatial resolution of 0.2° in the horizontal direction and are divided into 10 altitude layers represented by isobars at different pressure levels (100, 200, 250, 300, 400, 500, 700, 850, 925, and 1000 hPa). To incorporate the high-altitude wind data into the optimization model, a 3D linear interpolation method is used. This involves interpolating the wind speed and direction data based on the longitude, latitude, and altitude of the aircraft. By performing interpolation, the model can obtain wind information at specific locations and altitudes, allowing for the calculation of trajectory dynamics and the optimization of the aircraft's flight path in different wind conditions.

The study focuses on the domestic cruise segment of the VHHH-EHAM route. Fig.5 shows the planned route and route sectors for this segment and illustrates the horizontal distribution of wind uncertainty at an altitude of 9200m. Considering the high-altitude wind forecast is time-varying, we sliced and recombined the high-altitude wind forecast data corresponding to different hours according to the arrival time of the flight plan, and labeled them in the figure. The uncertainty in high-altitude wind forecasts is measured by the polar deviation of the maximum and minimum values in the forecast ensemble, which can be judged by the color depth in the figure, with darker regions indicating higher uncertainty

It can be seen that both the ZLLLAR06 sector and the ZLLLAR08 sector, through which the planned route passes, are areas with higher uncertainty of high-altitude wind forecast data. This particular route is analyzed in the context of KLM Flight 888, which is operated by a B747-400 aircraft. The flight departs at 4:20 (UTC) on June 8, 2019. For the analysis, the flight performance database used is BADA 3.11, which provides the necessary aircraft performance data for trajectory optimization. The initial KLM flight plan is used as a reference for comparison with the optimized trajectory obtained from the study. By comparing the optimized trajectory with the original KLM flight plan, the effectiveness and benefits of the trajectory optimization model can be evaluated.

In order to quantify the impact of high-altitude wind forecast uncertainty on the flight, the paper first constructs a trajectory predictor. The predictor simulates and predicts the domestic cruise segment (TOC-SARIN segment) of the flight, which is affected by high-altitude wind forecasts.

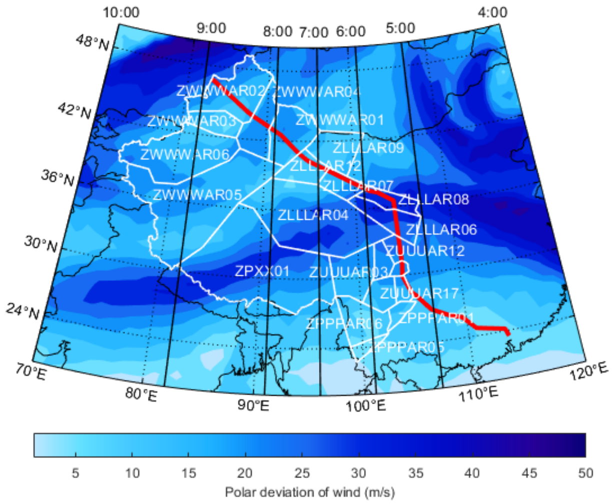


FIGURE 5. VHHH-EHAM route domestic cruise phase and route sector.

The weight of the aircraft at the TOC point is set to 370 tons. Using the 51 high-altitude wind forecast members released on June 8, 2019, the predictor generates a predicted trajectory set for the flight over the full segment. The average flight time for this predicted trajectory set is 275.5 minutes, and the average fuel consumption is 52.89 tons. At the SARIN point, the end point of the flight, the TWA is 152.9 seconds, indicating the uncertainty in the arrival time at that point.

The baseline trajectory is obtained using the ensemble forecast control forecast members. Fig.6 shows the trend of flight time deviation with flight distance for each trajectory relative to the baseline trajectory. Each curve represents the forecast trajectory corresponding to one high-altitude wind forecast member. In Fig. 6 the TWA formed at 2500km by the ensemble of trails is shown.

The difference between the uppermost and lowermost trajectory arrival time represents the TWA formed by the trajectory set at that point. The TWA formed by the trajectory set at other locations is also calculated using the above method. From the figure, it is evident that as the flight distance increases, the flight time deviation of each trajectory becomes larger. The TWA formed by the trajectory set also increases, indicating a higher level of flight time uncertainty. This decrease in predictability is a result of the influence of high-altitude wind forecast uncertainty on the trajectory. This analysis sets the stage for validating the optimization model by comparing the optimized trajectory with the predicted trajectory set obtained from the trajectory predictor.

B. RESULT ANALYSIS

Before conducting the case study, a comparison is made to validate the superiority of the NSGA-II algorithm in solving the aircraft robust trajectory optimization problem. Two other algorithms, the multi-objective particle swarm algorithm (MOPSO) and the linear weighting method based on Cost Index (CI), are selected for comparison. The linear weighting method based on Cost Index combines fuel consumption and

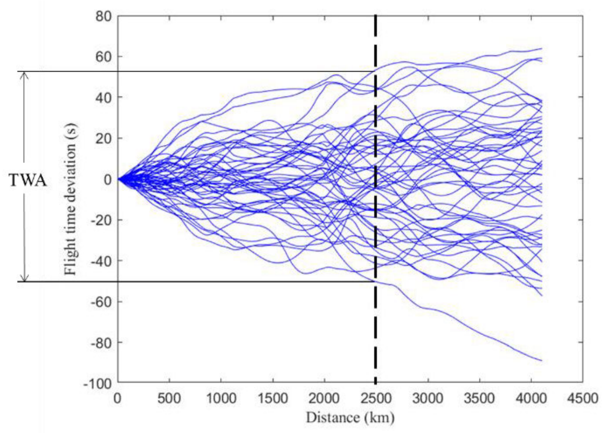


FIGURE 6. Flight time deviation of the predicted trajectory for the plan route.

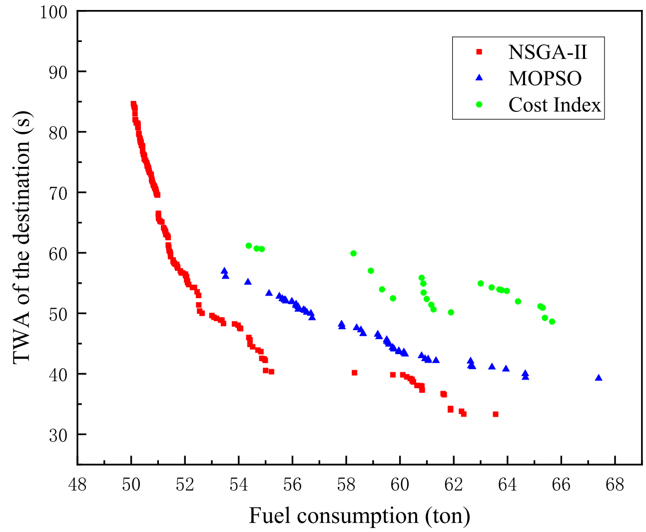


FIGURE 7. Comparison of Pareto front of three algorithms.

flight time robustness into a single objective by using CI. Multiple solutions are obtained by varying the CI values, resulting in a solution set. The optimization scenario for this comparison is the TOC-SARIN segment of flight KLM888 with an initial weight of 370t. The objectives are to minimize fuel consumption and minimize the TWA at the terminal. Each algorithm is applied to solve these objectives separately, resulting in respective solution sets.

The Pareto front solution sets obtained by the three algorithms are evaluated using three metrics: CM, HV, and MID, as mentioned in Section IV-E. Figure 7 shows the Pareto front solutions obtained by the optimization of the three algorithms. Each point represents a trajectory set, where the horizontal coordinate represents the average fuel consumption of all trajectory members in the set, and the vertical coordinate represents the time window of arrival formed by all trajectory members at the end position.

A comparison of the performance of the three types of solutions is presented in Table 1. It is observed that the NSGA-II algorithm outperforms both the MOPSO algorithm and the CI linear weighting method in all three metrics (CM, HV, and MID). For the same solution time, the NSGA-II algorithm obtains a higher quality solution set, indicating its capability of generating a better Pareto front solution set compared to the other two algorithms.

TABLE 1. Performance comparison of three algorithms.

	CM	HV	MID
NSGA2	1	0.1002	4.1085
PSO	0	0.0286	6.3518
CI	0	0.0154	6.9426

In order to analyze the effect of the optimization model proposed in this paper to reduce the impact of flights affected by the uncertainty of high-altitude wind forecast, in this section, the optimization and result analysis is carried out for the domestic cruise segment of flight KLM888 based on three optimization objectives setting approaches, which are:

Approach 1: The optimization model considers only the robustness of the flight when it reaches the end of the segment, i.e., the departure point, in terms of the robustness objective, i.e., minimizing the TWA of the destination, and verifying the ability of the model to improve the robustness of the trajectory at the moment it reaches the end based on the solution results.

Approach 2: Considering the robustness of the flight over the entire flight segment, minimizing the mean value of the TWA along the trajectory as the optimization objective, and verifying the ability of the optimization model to improve the overall robustness of the trajectory based on the solution results.

Approach 3: Determine the busy sectors through which the flights pass based on the historical operation data, consider the full-range robustness under the weighting of the predictability of the entry time of the busy sectors, narrow the TWA to enter the sector by increasing the weight of the robustness of the entry time of the busy sectors in the objective function, and verify the ability of the model to improve the robustness of the trajectory in the busy areas based on the solution results.

The experiments were conducted using a computer system equipped with an Intel Core i7-9750h 2.60GHz processor, 16GB of RAM, and running the Windows 11 operation system. The program was developed and solved using MATLAB R2019b.

C. ROBUST TRAJECTORY OPTIMIZATION RESULTS CONSIDERING ARRIVAL TIME PREDICTABILITY OF DESTINATION

Optimization objectives of minimizing the TWA of destination and minimizing fuel consumption for flights, i.e., the objective functions $\min J_1$ and $\min J_2$, and the Pareto front

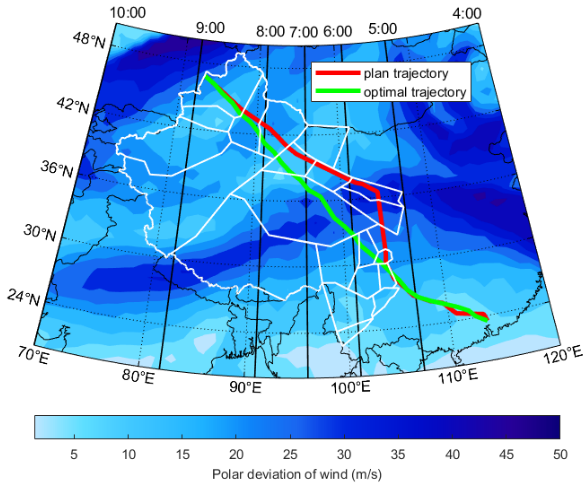


FIGURE 8. Figure 8 Comparison of optimized route and plan route.

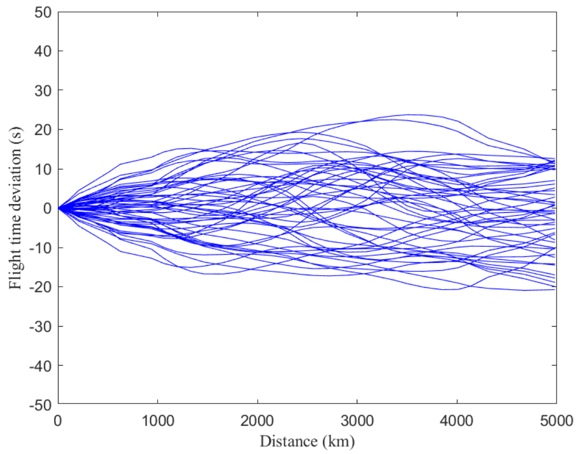


FIGURE 9. Flight time deviation of trajectory set 1.

surface is obtained using the NSGA-II algorithm as shown in red in Fig.7, where the top leftmost position in the Pareto front surface represents the set of trajectories with the lowest average fuel consumption, while the bottommost position represents the set of trajectories with the lowest average arrival time window. The lowest trajectory set, i.e., the set with the highest robustness of the trajectory, is called trajectory set 1, and the flight path corresponding to trajectory set 1 is compared with the initial planned path as shown in Fig.8. The red trajectory represents the planned route before optimization. The green trajectory represents the route corresponding to trajectory set 1 The TWA of the exit point SARIN of trajectory set 1 is 50.013s, which is 67.30% lower than the TWA of the planned route to reach the destination. It can be seen that the horizontal route corresponding to trajectory set 1 avoids the area of higher uncertainty by detouring, and thus improves the robustness of the trajectory, at the cost of extra flight cost and longer flight distance compared to the great circle trajectory.

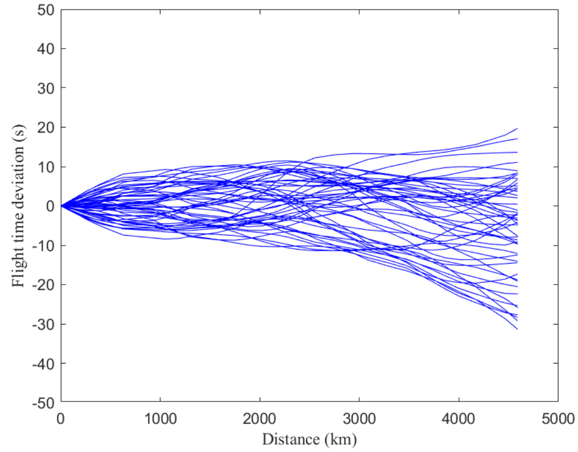


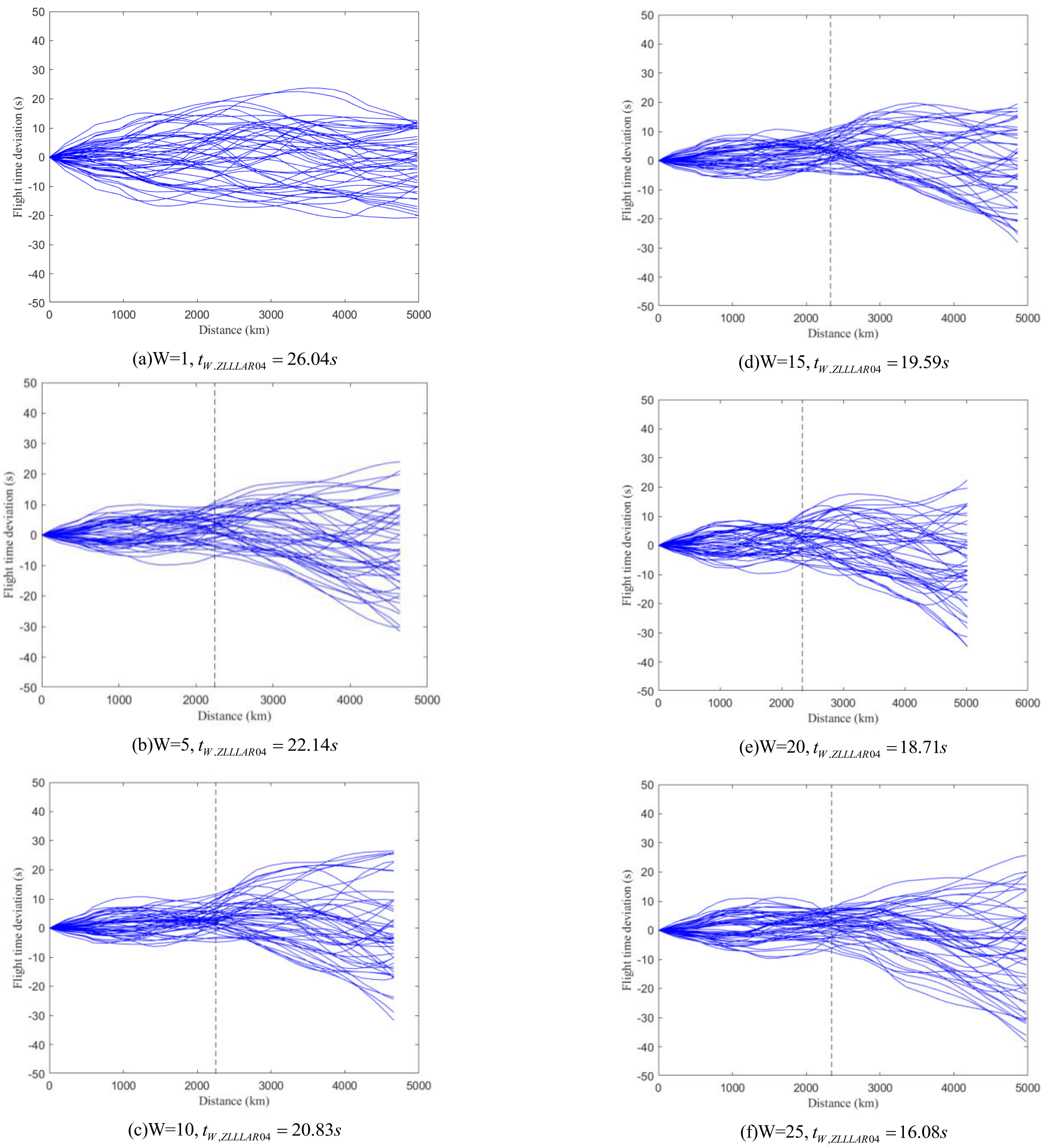
FIGURE 10. Flight time deviation of trajectory set 2.

Taking the trajectory corresponding to the control prediction member in the trajectory set as the baseline trajectory, the trend of flight time deviation of the trajectories corresponding to other high-altitude wind forecast members relative to the baseline trajectory with increasing flight distance is shown in Fig.9, it can be seen that the distribution of aircraft flight time gradually disperses with increasing flight distance and constitutes a gradually increasing TWA, but in the region near the end of the flight segment, the dispersion trend of flight time gradually tends to be concentrated and the TWA gradually decreases, which is because the objective function is set to minimize the TWA of destination, and this case shows that the model can effectively improve the robustness at the moment of reaching the destination.

D. ROBUST TRAJECTORY OPTIMIZATION RESULTS CONSIDERING ACCUMULATED ARRIVAL TIME PREDICTABILITY OF WHOLE TRAJECTORY

In this case, the focus is on considering the overall robustness of the flight along the entire flight segment. The optimization objectives are to minimize the average fuel consumption of the trajectory set and the average TWA along each trajectory in the trajectory set. In the objective function J_3, equal weight values W of 1 are assigned to all regions.

The NSGA-II algorithm is utilized to solve the optimization problem. The trajectory set corresponding to the point with the least uncertainty in the Pareto front (referred to as trajectory set 2) is selected. Fig.10 illustrates the trend of flight time deviation with increasing distance for the trajectories associated with other high-altitude wind forecast members, in comparison to Fig.9. It can be observed that compared to trajectory set 1, the flight time deviation of each member in trajectory set 2 is significantly reduced at various locations along the trajectory. Furthermore, the TWA is smaller, indicating a reduction in the flight time window at each point along the flight segment. As a result, the robustness of the flight is enhanced throughout the entire trajectory.



(a)W=1, $t_{W,ZLLLAR04} = 26.04s$
(b)W=5, $t_{W,ZLLLAR04} = 22.14s$
(c)W=10, $t_{W,ZLLLAR04} = 20.83s$
(d)W=15, $t_{W,ZLLLAR04} = 19.59s$
(e)W=20, $t_{W,ZLLLAR04} = 18.71s$
(f)W=25, $t_{W,ZLLLAR04} = 16.08s$

FIGURE 11. Comparison of flight time deviation with different weighting factors.

In this case, using the objective function for trajectory set 1, the average TWA along the trajectory for trajectory set 1 is 37.3726 seconds. On the other hand, for trajectory set 2, the average TWA is 29.4107 seconds over the full flight segment, which is 21.30% lower than trajectory set 1.

E. ROBUST TRAJECTORY OPTIMIZATION RESULTS CONSIDERING ARRIVAL TIME WEIGHTED PREDICTABILITY

To further extend the robust trajectory optimization model to a scenario that closely resembles actual operation, the optimization objective is focused on improving the robustness of the aircraft when entering the ZLLLAR04 sector.

The ZLLLAR04 sector was chosen because it avoids areas of high uncertainty in high-altitude wind forecasts and does not result in unacceptable additional fuel consumption for the flight.

By setting the weight of the ZLLLAR04 sector entry point in the objective function J_3, i.e., $(\lambda_{entry}, \phi_{entry})$ value, the effect of changing the weight value for a specific waypoint on reducing the TWA of the trajectory at that point is verified, and the corresponding optimized trajectories into the

ZLLLAR04 sector TWA for different values of weighting factors are shown in Fig.11.

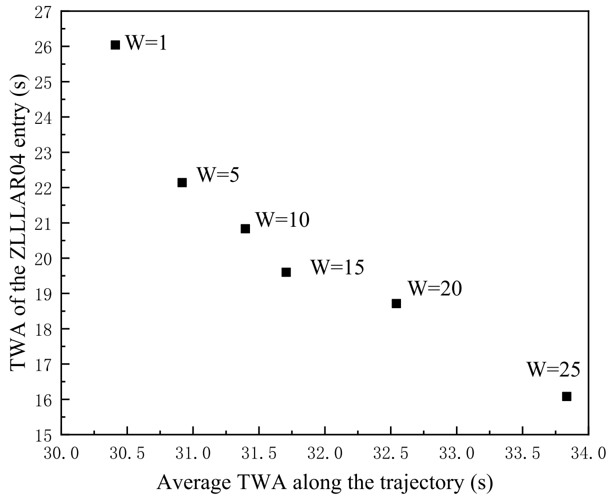


FIGURE 12. Comparison of TWA of the ZLLLAR04 entry and TWA along the trajectory with different sector predictability weighting factors.

Fig.12 shows the change of the TWA of each trajectory set at the entry point of ZLLLAR04 sector and the average TWA along the trajectory with the increase of the weight factor W. When W is 25, the time window for entering the ZLLLAR04 sector is reduced from 26.04s to 16.08s, a reduction of 38.25%, and the average TWA along the trajectory is increased from 30.41s to 33.83s, an increase of 11.25%.

This indicates that the robustness of the optimized trajectory at the entry time into the ZLLLAR04 sector increases with increasing values of the weighting factor, but at the same time, the average TWA along the trajectory gradually increases, but the overall robustness of the trajectory decreases by an acceptable amount compared to the increase in robustness in the high weighting factor region. This indicates that by increasing the weighting factor W in a region, the robustness of the trajectory through that region can be improved without overly affecting the robustness of the trajectory in other regions.

VI. CONCLUSION AND OUTLOOK

In this study, we focused on the problem of robust four-dimensional trajectory optimization for aircraft considering the uncertainty of high-altitude wind forecasts. Specifically, we conducted our research on the domestic cruise section of the Hongkong-Amsterdam route as our target scenario to validate and analyze the proposed model. The key findings of our research are summarized as follows:

(1) We have developed an aircraft control model that considers deterministic high-altitude winds. This model accounts for the time-varying nature and uncertainty of high-altitude wind forecasts. We address the trajectory consistency issue by solving for robust trajectories under various high-altitude wind forecast scenarios. The optimization objectives of our model are focused on enhancing the robustness of the trajectory destination as well as the overall trajectory. As a result, we have established an aircraft trajectory robustness optimization model based on optimal control principles.

(2) We have selected the domestic cruise segment of the Hong Kong-Amsterdam route as our research object. Through our experiments, we have successfully demonstrated that the proposed optimization model effectively mitigates the flight time dispersion caused by uncertain high-altitude wind forecasts. Our model focuses on enhancing the overall robustness of the trajectory by considering the entire flight segment. By adjusting the robustness target weights of different regions, we have observed significant improvements in trajectory robustness for flights passing through those specific regions, without imposing excessive costs on the robustness of the trajectory in other regions.

The optimization model proposed in this paper provides a method to optimize the trajectory in the future TBO framework, considering the efficiency objective of the trajectory optimization while also improving the predictability of the trajectory to the destination and during the flight, especially the predictability to the bottleneck airspace, which can help to ensure the safety and increase the airspace capacity. Although the paper has its shortcomings. Due to data limitations and other reasons, it was not possible to perform additional validation of the methodology and error propagation analysis of the trajectory using data from different sources. Moving forward, our future research will expand in two key directions. Firstly, we aim to incorporate additional sources of uncertainty, such as human factors uncertainties, equipment uncertainties and convective weather uncertainties into the optimization model. This will further enhance the model's ability to handle a broader range of uncertainties. Secondly, we plan to extend the research scope from the pre-tactical stage to the strategic stage, and consider the trajectory optimization problem for different time phases. This expansion will enable us to provide decision support for trajectory optimization and trajectory deconfliction management during the strategy phase.

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