IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 24 August 2023, accepted 7 September 2023, date of publication 18 September 2023, date of current version 2 October 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3316879

RESEARCH ARTICLE

Decision-Making by Using TOPSIS Techniques in the Framework of Bipolar Complex Intuitionistic Fuzzy N-Soft Sets

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This work was supported by the Princess Nourah Bint Abdulrahman University, Riyadh, Saudi Arabia, under Project PNURSP2023R259.

ABSTRACT The major influence of this manuscript is to diagnose the well-recognized and achievable theory of bipolar complex intuitionistic fuzzy N-soft (BCIFN-S) information, which is the generalization of two different theories, bipolar complex intuitionistic fuzzy set (BCIF) and N-soft sets. The diagnosed theory of BCIFN-S set (BCIFN-SS) would cope with information that contains the 2nd dimension along with truth degree (containing both positive and negative aspects) and falsity degree (containing both positive and negative aspects) and parameterization along with grades. We also discuss various algebraic operations like union, intersection, compliments, and some of their other types for BCIFN-SS. More, in this manuscript, we interpret the TOPSIS (a technique for order preference by similarity to ideal solution) approach which is dominant and skillful for managing strategic decision-making (DM) dilemmas under the setting of BCIFN-SS. To reveal the applicability and practicality of the diagnosed approach, we interpret a numerical example. In the last of the manuscript, we compare the devised work with certain prevailing theories to reveal supremacy and dominance.

INDEX TERMS Bipolar complex fuzzy N-soft sets, TOPSIS techniques, aggregation operators, similarity measures, decision-making evaluations.

I. INTRODUCTION

Due to the enhancement of the vagueness and ambiguities in real-life circumstances, it became almost impossible for the crisp set theory to handle this vagueness and ambiguities. To handle these situations, Zadeh [1] devised the primary structure of the fuzzy set (FS). The domain in the structure of FS is $[0, 1]$ instead of $\{0, 1\}$. Zhang et al. [2] discussed the stability of solutions for FS optimization issues along with applications. Chen et al. [3] presented an FS qualitative comparative analysis technique. Youg [4] devised the TOPSIS approach in the setting of FS. Moreover, the notion of FS has got great success in numerous areas to manage vagueness and ambiguities. Numerous researchers investigated various modifications of FS because of the enhancement of vague and ambiguous data in genuine life. Atanassov and Stoeva [5] devised modified FS and devised the notion of intuitionistic FS (IFS), which is a great tool to cope with uncertainty. The structure of IFS is described by truth degree and falsity degree with the condition that the sum of both truth and falsity degree must belong to $[0, 1]$. Alcantud [6] investigated complemental FSs, a semantic justification of q-rung orthopiar FS. Li [7] devised multi-attribute decision-making (MADM) structures and techniques by employing IFS. Tian et al. [8] devised the anomaly detection of network traffic relying on IFS. Pandey et al. [9] devised an intuitionistic fuzzy (IF) entropy approach.

The associate editor coordinating the review of this manuscript and approving it for publication was Francisco J. Garcia-Penalvo.

Haktanır and Kahraman [10] studied IF risk-adjusted discount rates and approaches for risky projects. Boran et al. [11] and Rouyendegh et al. [12] devised the IF TOPSIS approach. Zhang [13] devised a bipolar fuzzy set (BFS) to cope with bipolarity information that is the information contains both positive and negative sides. The structure of BFS is described by the positive truth degree and negative truth degree placed in $[0, 1]$ and $[-1, 0]$ respectively. Akram [14] and Samanta and Pal [15] devised bipolar fuzzy (BF) graphs and irregular BF graphs respectively. Akram et al. [16], and Alghamdi et al. [17] invented the TOPSIS technique in the setting of BFS.

The 2nd dimension i.e., extra fuzzy information involved in numerous circumstances, thus, Ramot et al. [18] established the structure of complex FS (CFS). The structure of CFS is described by the truth degree placed in a unit circle of a complex plane. Tamir et al. [19] discussed the truth degree in cartesian form and placed it in the unit square of a complex plane. Barbat et al. [20] invented the TOPSIS approach for CFS. Alkouri and Salleh [21] devised the notion of complex IFS (CIFS). Azam et al. [22] interpreted the DM technique under CIFS. What would happen if the positive and negative sides of an object and extra fuzzy information related to the objects need to be handled simultaneously. To answer this question, Mahmood and Ur Rehman [23] devised the structure of bipolar CFS (BCFS), which is a great tool to cope with complicated and uncertain information. The structure of BCFS is described by the positive truth degree and negative truth degree placed in $[0, 1] + \iota\ [0, 1]$ and $[-1, 0] + \iota\ [-1, 0]$ respectively. MADM approaches in the setting of BCFS were diagnosed by Mahmood and Ur Rehman [24], Mahmood et al. [25], and Mahmood et al. [26]. The BCFS is utilized in pattern recognition and medical diagnosis by Ur Rehman and Mahmood [27]. Rehman et al. [28] investigated the AHP approach in the setting of BCFS. Al-Husban [29] discussed bipolar complex IFS (BCIFS) in the polar form of complex numbers.

Molodtsov [30] devised the notion of the soft set (SS) which is the modification of FS to cope with uncertainties and ambiguities in a parametric manner. The parameterized group of sets is termed a SS. SS attracts various researchers due to its applications in numerous areas such as data analysis, decision-making (DM), forecasting, etc. Ali et al. [31] devised a primary operation for SS. Maji et al. [32] discussed the application of SS in DM. Maji et al. [33] devised the notion of fuzzy SS (FSS) and Maji et al. [34] also invented the IF soft set (IFSS). Abdullah et al. [35] investigated BF soft set (BFSS). The notion of bipolar IFSS (BIFSS) was devised by Jana and Pal [36]. Thirunavukarasu et al. [37] devised the complex FSS (CFSS). Kumar and Bajaj [38] established complex IFSS (CIFSS) and Mahmood et al. [39] studied bipolar CFSS (BCFSS). Gwak et al. [40] discussed bipolar complex intuitionistic fuzzy soft relation. N-soft set (N-SS) is the modification of the soft set investigated by Fatimah et al. [41]. After that, Alcantud et al. [42] devised the N-SS approach to rough set. Akram et al. [43] presented Hesitant N-SS. Akram et al. [44] studied parameter reduction in N-SS. The semantics of N-SS was investigated by Alcantud [45]. Akram et al. [46] modified N-SS and devised fuzzy N-SS (FN-SS). Fatimah and Alcantud [47] invented multi-fuzzy N-SS. Akram et al. [48] invented IF N-SS (IFN-SS). The notion of bipolar fuzzy N-SS (BFN-SS) was propounded by Akram et al. [49]. Mahmood et al. [50] studied complex fuzzy N-SS (CFN-SS). Rehman and Mahmood [51] investigated complex IFN-SS (CIFN-SS).

There are numerous genuine-life circumstances, where the information is complicated and ambiguous that is the information contains the 2nd dimension along with truth degree (containing both positive and negative aspects) and falsity degree (containing both positive and negative aspects) and parameterization along with grades. For modeling such sort of information, we need a mathematical tool but the prevailing and above-mentioned mathematical structures in the literature can't model this information. There is no mathematical structure in the literature that can model such sort of information. This observation leads us that there is a research gap in the literature which needs to be addressed. Thus, in this script, we devise the notion of BCFIFN-SS which would easily tackle such sort of information. BCIFN-SS is important because they provide a flexible mathematical framework that unifies the ideas of BCFS, IFS, and N-SS. enabling the representation and unified management of uncertain, imprecise, and vague information. Because prevailing notions in the literature are unable to adequately represent ambiguity, they fall short when used to simulate real-world situations. This hybrid method fills this gap.

BCIFN-SS is the generalization of various notions such as:

- Bipolar complex IF soft set (BCIFSS): by letting $\aleph = 2$, BCIFN-SS would degenerate to BCIFSS.
- Bipolar complex IFS (BCIFS): by letting $|\mathbb{D}| = 1$, and $\aleph = 2$, BCIFN-SS would degenerate to BCIFS.
- Bipolar IF soft set (BIFSS): by letting $\aleph = 2$, and neglecting unreal parts in both truth and falsity degree, then BCIFN-SS would be disintegrated to BIFSS.
- Bipolar IFS (BIFS): by letting $\aleph = 2$, $|\mathbb{D}| = 1$ and neglecting unreal parts in both truth and falsity degree, then BCIFN-SS would be disintegrated to BIFS.
- Complex IF soft set (CIFSS): by letting $\aleph = 2$, and neglecting the negative aspects in both truth and falsity degrees, then BCIFN-SS would degenerate to CIFSS.
- Complex IFS (CIFS): by letting $|\mathbb{D}| = 1$, and $\aleph = 2$, and neglecting the negative aspects in both truth and falsity degrees, then BCIFN-SS would degenerate to CIFS.
- IF soft set (IFSS): by letting $\aleph = 2$, neglecting the negative aspects in both truth and falsity degrees, and ignoring the unreal parts in both positive aspects of the truth and falsity degree, then BCIFN-SS would degenerate to IFSS.
- IFS: by letting $\aleph = 2$, $|\mathbb{D}| = 1$ and neglecting the negative aspects in both truth and falsity degrees and ignoring

the unreal parts in both positive aspects of the truth and falsity degree, then BCIFN-SS would degenerate to IFS.

- BCFSS: by letting $\aleph = 2$, and neglecting the falsity degree, then BCIFN-SS would degenerate to BCFSS.
- BCFS: by letting $\aleph = 2$, $|\mathrm{D}| = 1$, and neglecting the falsity degree, then BCIFN-SS would be reduced to BCFS.
- BFSS: by letting $\aleph = 2$, and neglecting the falsity degree and unreal parts in both positive and negative aspects of the truth degree, then BCIFN-SS would be diminished to BFSS.
- BFS: by letting $\aleph = 2$, $|\mathrm{D}| = 1$, and neglecting the falsity degree and unreal parts in both positive and negative aspects of the truth degree, then BCIFN-SS would be diminished to BFS.
- CFSS: by letting $\aleph = 2$, and neglecting the falsity degree, and the negative aspect in the truth degree, then BCIFN-SS would be decreased to CFSS.
- CFS: by letting $\aleph = 2$, $|\mathrm{D}| = 1$, and neglecting the falsity degree, and the negative aspect in the truth degree, then BCIFN-SS would be decreased to CFS.
- FSS: by letting $\aleph = 2$, and neglecting the falsity degree, and the negative aspect and unreal part in the truth degree, then BCIFN-SS would be decreased to FSS.
- FS: by letting $\aleph = 2$, $|\mathrm{D}| = 1$, and neglecting the falsity degree, and the negative aspect and unreal part in the truth degree, then BCIFN-SS would be decreased to FS.
- N-SS: by neglecting the truth and falsity degrees, BCIFN-SS would be decreased to N-SS.

where, $\aleph = \{2, 3, 4, \ldots, \}$, and $\mathrm{H} = \{0, 1, 2, \ldots, \aleph - 1\}$ as a set of ordered grades. Similarly, the notion of BCIFN-SS can degenerate to the setting of BCFN-SS, BFN-SS, CFN-SS, IFN-SS, and FN-SS.

The rest of the script is handled as: In Section II, a few primary notions associated with prevailing notions are discussed. In Section III, we devise BCIFN-SS and explain it through an example. Further, we interpret associated operations of BCFIN-SS such as weak complement, BCIF complement, weak BCIF complement, extended and restricted unions, intersections, etc. In Section IV, we interpret the approach of TOPSIS in the setting of BCIFN-SS and provide a numerical example. Moreover, we compare the established work with certain prevailing work. In Section V, the conclusion of this script is given.

II. PRELIMINARIES

This Section contains a few primary notions associated with prevailing notions.

Definition 1: The structure of BCFS is devised as [23]:

$$\begin{aligned}&\mathbb{W}\\&=\left\{\left(\omega,\ \mathrm{F}^{+}_{\mathbb{W}}(\omega),\ \mathrm{F}^{-}_{\mathbb{W}}(\omega)\right)\mid \omega\in\mathfrak{C}\right\}\\&\left\{\left(\omega,\ \left(Z^{+}_{\mathbb{W}}(\omega)+\iota\,\mathrm{R}^{+}_{\mathbb{W}}(\omega),\ Z^{-}_{\mathbb{W}}(\omega)+\iota\,\mathrm{R}^{-}_{\mathbb{W}}(\omega)\right)\right)\mid\omega\in\mathfrak{C}\right\}\end{aligned}\tag{1}$$

where $Z^{+}_{\mathbb{W}}(\omega), \mathrm{R}^{+}_{\mathbb{W}}(\omega) \in [0, 1]$ and $Z^{-}_{\mathbb{W}}(\omega), \mathrm{R}^{-}_{\mathbb{W}}(\omega) \in [-1, 0]$. $\mathbb{W} = \left(\mathrm{F}^{+}_{\mathbb{W}}, \mathrm{F}^{-}_{\mathbb{W}}\right) = \left(Z^{+}_{\mathbb{W}} + \iota\,\mathrm{R}^{+}_{\mathbb{W}}, Z^{-}_{\mathbb{W}} + \iota\mathrm{R}^{-}_{\mathbb{W}}\right)$ signified the BCF number (BCFN).

Definition 2: Underneath is the score value of a BCFN [24]:

$$\begin{aligned}\mathbb{W} &= \left(\omega,\ \mathrm{F}^{+}_{\mathbb{W}}(\omega),\ \mathrm{F}^{-}_{\mathbb{W}}(\omega)\right)\\&=\left(\omega,\ Z^{+}_{\mathbb{W}}(\omega)+\iota\,\mathrm{R}^{+}_{\mathbb{W}}(\omega),\ Z^{-}_{\mathbb{W}}(\omega)+\iota\mathrm{R}^{-}_{\mathbb{W}}(\omega)\right)\\ \mathfrak{S}_{\mathfrak{B}}(\mathbb{W}) &= \frac{1}{4}\left(2+Z^{+}_{\mathbb{W}}(\omega)+\mathrm{R}^{+}_{\mathbb{W}}(\omega)+Z^{-}_{\mathbb{W}}(\omega)\right.\\&\quad\left.+\mathrm{R}^{-}_{\mathbb{W}}(\omega)\right),\ \mathfrak{S}_{\mathfrak{B}}\in[0,\ 1]\end{aligned}\tag{2}$$

Definition 3: Underneath is the accuracy value of a BCFN [24]:

$$\begin{aligned}\mathbb{W} &= \left(\omega,\ \mathrm{F}^{+}_{\mathbb{W}}(\omega),\ \mathrm{F}^{-}_{\mathbb{W}}(\omega)\right)\\&=\left(\omega,\ Z^{+}_{\mathbb{W}}(\omega)+\iota\,\mathrm{R}^{+}_{\mathbb{W}}(\omega),\ Z^{-}_{\mathbb{W}}(\omega)+\iota\mathrm{R}^{-}_{\mathbb{W}}(\omega)\right)\\ \mathcal{H}_{\mathfrak{B}}(\mathbb{W}) &= \frac{Z^{+}_{\mathbb{W}}(\omega)+\mathrm{R}^{+}_{\mathbb{W}}(\omega)-Z^{-}_{\mathbb{W}}(\omega)-\mathrm{R}^{-}_{\mathbb{W}}(\omega)}{4},\\&\mathcal{H}_{\mathfrak{B}}\in[0,\ 1]\end{aligned}\tag{3}$$

With the help of Eq. (2) and Eq. (3), we have:

1) If $\mathfrak{S}_{\mathfrak{B}}(\mathbb{W}) < \mathfrak{S}_{\mathfrak{B}}(\tilde{\mathrm{V}})$, then $\mathbb{W} < \tilde{\mathrm{V}}$;
2) If $\mathfrak{S}_{\mathfrak{B}}(\mathbb{W}) > \mathfrak{S}_{\mathfrak{B}}(\tilde{\mathrm{V}})$, then $\mathbb{W} > \tilde{\mathrm{V}}$;
3) If $\mathfrak{S}_{\mathfrak{B}}(\mathbb{W}) = \mathfrak{S}_{\mathfrak{B}}(\tilde{\mathrm{V}})$, then
   i) If $\mathcal{H}_{\mathfrak{B}}(\mathbb{W}) < \mathcal{H}_{\mathfrak{B}}(\tilde{\mathrm{V}})$, then $\mathbb{W} < \tilde{\mathrm{V}}$;
   ii) If $\mathcal{H}_{\mathfrak{B}}(\mathbb{W}) > \mathcal{H}_{\mathfrak{B}}(\tilde{\mathrm{V}})$, then $\mathbb{W} > \tilde{\mathrm{V}}$;
   iii) If $\mathcal{H}_{\mathfrak{B}}(\mathbb{W}) = \mathcal{H}_{\mathfrak{B}}(\tilde{\mathrm{V}})$, then $\mathbb{W} = \tilde{\mathrm{V}}$.

Definition 4: Utilizing two BCFNs [24] i.e. $\mathbb{W} = \left(\mathrm{F}^{+}_{\mathbb{W}}, \mathrm{F}^{-}_{\mathbb{W}}\right) = \left(Z^{+}_{\mathbb{W}} + \iota\,\mathrm{R}^{+}_{\mathbb{W}}, Z^{-}_{\mathbb{W}} + \iota\,\mathrm{R}^{-}_{\mathbb{W}}\right)$, and $\tilde{\mathrm{V}} = \left(\mathrm{F}^{+}_{\tilde{\mathrm{V}}}, \mathrm{F}^{-}_{\tilde{\mathrm{V}}}\right) = \left(Z^{+}_{\tilde{\mathrm{V}}} + \iota\,\mathrm{R}^{+}_{\tilde{\mathrm{V}}}, Z^{-}_{\tilde{\mathrm{V}}} + \iota\,\mathrm{R}^{-}_{\tilde{\mathrm{V}}}\right)$, with $\varrho > 0$ We have

$$\mathbb{W}\oplus\tilde{\mathrm{V}} = \begin{pmatrix} Z^{+}_{\mathbb{W}}+Z^{+}_{\tilde{\mathrm{V}}}-Z^{+}_{\mathbb{W}}Z^{+}_{\tilde{\mathrm{V}}}+\iota\left(\mathrm{R}^{+}_{\mathbb{W}}+\mathrm{R}^{+}_{\tilde{\mathrm{V}}}-\mathrm{R}^{+}_{\mathbb{W}}\mathrm{R}^{+}_{\tilde{\mathrm{V}}}\right),\\ -\left(Z^{-}_{\mathbb{W}}Z^{-}_{\tilde{\mathrm{V}}}\right)+\iota\left(-\left(\mathrm{R}^{-}_{\mathbb{W}}\mathrm{R}^{-}_{\tilde{\mathrm{V}}}\right)\right)\end{pmatrix}\tag{4}$$

$$\mathbb{W}\otimes\tilde{\mathrm{V}} = \begin{pmatrix} Z^{+}_{\mathbb{W}}Z^{+}_{\tilde{\mathrm{V}}}+\iota\ \mathrm{R}^{+}_{\mathbb{W}}\mathrm{R}^{+}_{\tilde{\mathrm{V}}},\\ Z^{-}_{\mathbb{W}}+Z^{-}_{\tilde{\mathrm{V}}}+Z^{-}_{\mathbb{W}}Z^{-}_{\tilde{\mathrm{V}}}+\iota\left(\mathrm{R}^{-}_{\mathbb{W}}+\mathrm{R}^{-}_{\tilde{\mathrm{V}}}+\mathrm{R}^{-}_{\mathbb{W}}\mathrm{R}^{-}_{\tilde{\mathrm{V}}}\right)\end{pmatrix}\tag{5}$$

$$\begin{aligned}\varrho\mathbb{W} &= \left(1-(1-Z^{+}_{\mathbb{W}})^{\varrho}+\iota\left(1-\left(1-\mathrm{R}^{+}_{\mathbb{W}}\right)^{\varrho}\right),-|Z^{-}_{\mathbb{W}}|^{\varrho}\right.\\&\quad\left.+\iota\left(-\left|\mathrm{R}^{-}_{\mathbb{W}}\right|^{\varrho}\right)\right)\end{aligned}\tag{6}$$

$$\begin{aligned}\mathbb{W}^{\varrho} &= \left(\left((Z^{+}_{\mathbb{W}})^{\varrho}+\iota\left(\mathrm{R}^{+}_{\mathbb{W}}\right)^{\varrho},\ -1+(1+Z^{-}_{\mathbb{W}})^{\varrho}\right.\right.\\&\quad\left.\left.+\iota\left(-1+(1+Z^{-}_{\mathbb{W}})^{\varrho}\right)\right)\right)\end{aligned}\tag{7}$$

Theorem 1: Utilizing two BCFNs [24] i.e. $\mathbb{W} = \left(\mathrm{F}^{+}_{\mathbb{W}}, \mathrm{F}^{-}_{\mathbb{W}}\right) = \left(Z^{+}_{\mathbb{W}} + \iota\,\mathrm{R}^{+}_{\mathbb{W}}, Z^{-}_{\mathbb{W}} + \iota\,\mathrm{R}^{-}_{\mathbb{W}}\right)$, and $\tilde{\mathrm{V}} = \left(\mathrm{F}^{+}_{\tilde{\mathrm{V}}}, \mathrm{F}^{-}_{\tilde{\mathrm{V}}}\right) = \left(Z^{+}_{\tilde{\mathrm{V}}} + \iota\,\mathrm{R}^{+}_{\tilde{\mathrm{V}}}, Z^{-}_{\tilde{\mathrm{V}}} + \iota\,\mathrm{R}^{-}_{\tilde{\mathrm{V}}}\right)$, with $\varrho, \varrho_1, \varrho_2 > 0$ we achieved

1) $\text{₩}\oplus\tilde{V} = \tilde{V}\oplus\text{₩}$
2) $\text{₩}\otimes\tilde{V} = \tilde{V}\otimes\text{₩}$
3) $\varrho\left(\text{₩}\oplus\tilde{V}\right) = \varrho\text{₩}\oplus\varrho\tilde{V}$
4) $\left(\text{₩}\otimes\tilde{V}\right)^{\varrho} = \text{₩}^{\varrho}\otimes\tilde{V}^{\varrho}$
5) $\varrho_1\text{₩}\oplus\varrho_2\text{₩} = (\varrho_1+\varrho_2)\text{₩}$
6) $\text{₩}^{\varrho_1}\otimes\text{₩}^{\varrho_2} = \text{₩}^{\varrho_1+\varrho_2}$
7) $(\text{₩}^{\varrho_1})^{\varrho_2} = \text{₩}^{\varrho_1\varrho_2}$.

Definition 5: A FN-SS [42] would be termed by $(\Xi, (\mathfrak{U}, D, \aleph))$ over $\mathfrak{C}$ where $\Xi : D \rightarrow \bigcup_{\underline{d}\in D}\mathcal{F}\left(\Xi\left(\underline{d}\right)\right), \underline{d} \in D\subseteq X$ specifies by $\mu'\left(\underline{d}\right)\in\mathcal{F}\left(\Xi\left(\underline{d}\right)\right)$ for each $\underline{d}\in D$ and $H = \{0, 1, 2, \ldots, \aleph-1\}$. if $\underline{d}\in D$, then $\mu'(\underline{d})\subseteq\mathcal{F}\left(\Xi\left(\underline{d}\right)\right)$ is termed as $\underline{d}-$ approximation elements of $(\Xi, (\mathfrak{U}, D, \aleph))$.

III. BIPOLAR COMPLEX INTUITIONISTIC FUZZY N-SOFT SETS

Here, firstly, we devise the conception of bipolar complex intuitionistic FS (BCIFS). After that, we merge BCIFS with N-SS to interpret BCIFN-SS. Secondly, we invent weak complement and other related complements for BCIFN-SS. Further, we investigate restricted and extended unions and intersections based on BCIFN-SS. We also invent primary operations for BCIFN-SS.

Definition 6: The model of BCIFS over a fixed set $\mathfrak{C}$ is devised as:

$$\text{₩} = \left\{\left(\omega,\ \Theta^{T}_{\text{₩}}(\omega),\ \Theta^{F}_{\text{₩}}(\omega)\right) \mid \omega\in\mathfrak{C}\right\} = \left\{\left(\omega,\ \text{₣}^{+}_{\text{₩}}(\omega),\ \text{₣}^{-}_{\text{₩}}(\omega),\ \text{₮}^{+}_{\text{₩}}(\omega),\ \text{₮}^{-}_{\text{₩}}(\omega)\right) \mid \omega\in\mathfrak{C}\right\} \quad (8)$$

where, $\text{₣}^{+}_{\text{₩}}(\omega) = Z^{+}_{\text{₩}}(\omega) + \iota\,\text{Ʀ}^{+}_{\text{₩}}(\omega)$, $\text{₣}^{-}_{\text{₩}}(\omega) = Z^{-}_{\text{₩}}(\omega) + \iota\,\text{Ʀ}^{-}_{\text{₩}}(\omega)$, $\text{₮}^{+}_{\text{₩}}(\omega) = \text{Ł}^{+}_{\text{₩}}(\omega) + \iota\,\text{₱}^{+}_{\text{₩}}(\omega)$ and $\text{₮}^{-}_{\text{₩}}(\omega) = \text{Ł}^{-}_{\text{₩}}(\omega) + \iota\,\text{₱}^{-}_{\text{₩}}(\omega)$, $0\leq Z^{+}_{\text{₩}}(\omega) + \text{Ł}^{+}_{\text{₩}}(\omega)\leq 1$, $0\leq\text{Ʀ}^{+}_{\text{₩}}(\omega) + \text{₱}^{+}_{\text{₩}}(\omega)\leq 1$, $-1\leq Z^{-}_{\text{₩}}(\omega) + \text{Ł}^{-}_{\text{₩}}(\omega)\leq 0$, $-1\leq\text{Ʀ}^{-}_{\text{₩}}(\omega) + \text{₱}^{-}_{\text{₩}}(\omega)\leq 0$, $Z^{+}_{\text{₩}}(\omega), \text{Ʀ}^{+}_{\text{₩}}(\omega), \text{Ł}^{+}_{\text{₩}}(\omega)\text{₱}^{+}_{\text{₩}}(\omega)\in[0,\ 1]$ and $Z^{-}_{\text{₩}}(\omega), \text{Ʀ}^{-}_{\text{₩}}(\omega), \text{Ł}^{-}_{\text{₩}}(\omega)\text{₱}^{-}_{\text{₩}}(\omega)\in[-1,\ 0]$. $\Theta^{T}_{\text{₩}}(\omega)$ would identify truth degree and $\Theta^{F}_{\text{₩}}(\omega)$ would identify falsity degree. The BCIF number would be devised as $\text{₩} = (\text{₣}^{+}_{\text{₩}}, \text{₣}^{-}_{\text{₩}}, \text{₮}^{+}_{\text{₩}}, \text{₮}^{-}_{\text{₩}}) = (Z^{+}_{\text{₩}} + \iota\,\text{Ʀ}^{+}_{\text{₩}}, Z^{-}_{\text{₩}} + \iota\text{Ʀ}^{-}_{\text{₩}}, \text{Ł}^{+}_{\text{₩}} + \iota\,\text{₱}^{+}_{\text{₩}}, \text{Ł}^{-}_{\text{₩}} + \iota\,\text{₱}^{-}_{\text{₩}})$.

Definition 7: Take X as an attribute set, $D\subseteq X$, $H = \{0, 1, 2, .., \aleph-1\}$ as a set of ordered grades where $\aleph = \{2, 3, 4, .., \}$, then a set $(\text{Ꝁ}, \mathfrak{B}) = (\text{Ꝁ}, (\mathfrak{U}, D, \aleph))$ is interpreted as BCIFN-SS, where $\mathfrak{B} = (\mathfrak{U}, D, \aleph)$ connotes N-SS and Ꝁ is a function from D to $2^{X\times H}\times F - BCIFN$ i.e.

$$\begin{aligned}(\text{Ꝁ}, \mathfrak{B}) &= (\text{Ꝁ}, (\mathfrak{U}, D, \aleph)) \\ &= \left\{\left(\underline{d},\ \left(\mathfrak{G}\left(\underline{d}\right), \mathfrak{I}\left(\underline{d}\right)\right)\right) | \underline{d}\in D,\ \left(\mathfrak{G}\left(\underline{d}\right), \mathfrak{I}\left(\underline{d}\right)\right)\in 2^{X\times H}\times BCIFN\right\} \\ &= \left\{\left(\underline{d},\ \left(\left(\omega,\ \mathfrak{h}^{\omega}_{\underline{d}}\right), \text{₣}^{+}_{\underline{d}}, \text{₣}^{-}_{\underline{d}}, \text{₮}^{+}_{\underline{d}}, \text{₮}^{-}_{\underline{d}}\right)\right) | \underline{d}\in D,\ \omega\in\mathfrak{C},\ \mathfrak{h}^{\omega}_{\underline{d}}\in H\right\} \\ &= \left\{\left(\underline{d},\ \left(\left(\omega,\ \mathfrak{h}^{\omega}_{\underline{d}}\right), Z^{+}_{\underline{d}} + \iota\,\text{Ʀ}^{+}_{\underline{d}}, Z^{-}_{\underline{d}} + \iota\,\text{Ʀ}^{-}_{\underline{d}}, \text{Ł}^{+}_{\underline{d}} + \iota\,\text{₱}^{+}_{\underline{d}}, \text{Ł}^{-}_{\underline{d}} + \iota\,\text{₱}^{-}_{\underline{d}}\right)\right) | \underline{d}\in D,\ \omega\in\mathfrak{C},\ \mathfrak{h}^{\omega}_{\underline{d}}\in H\right\}\end{aligned} \quad (9)$$

TABLE 1. The smash products are given by the expert to the alternatives based on parameters.

$S_{\mathfrak{A}\mathfrak{I}}$/D	$\underline{d}_1$	$\underline{d}_2$	$\underline{d}_3$	$\underline{d}_4$
$S_{\mathfrak{A}\mathfrak{I}-1}$	⋇⋇⋇⋇⋇	⋇⋇	⋇⋇	⋇⋇
$S_{\mathfrak{A}\mathfrak{I}-2}$	⋇⋇	⋇	⋇⋇⋇	⋇⋇⋇⋇
$S_{\mathfrak{A}\mathfrak{I}-3}$	⋇⋇⋇⋇	o	⋇⋇⋇⋇⋇	o
$S_{\mathfrak{A}\mathfrak{I}-4}$	⋇⋇⋇	⋇⋇⋇	⋇⋇	⋇⋇

TABLE 2. The associated grades with smash products of Table 1.

$S_{\mathfrak{A}\mathfrak{I}}$/D	$\underline{d}_1$	$\underline{d}_2$	$\underline{d}_3$	$\underline{d}_4$
$S_{\mathfrak{A}\mathfrak{I}-1}$	5	2	2	2
$S_{\mathfrak{A}\mathfrak{I}-2}$	2	1	3	4
$S_{\mathfrak{A}\mathfrak{I}-3}$	4	0	5	0
$S_{\mathfrak{A}\mathfrak{I}-4}$	3	3	2	2

where the gathering of BCFINs would be identified by $F - BCIFN$, $\mathfrak{G} : D \rightarrow 2^{X\times H}$, and $\mathfrak{I} : D \rightarrow F - BCIFN$. The bipolar complex intuitionistic fuzzy N-soft number (BCIFN-SN) would be interpreted as $Z_{m\mathfrak{l}} = (\mathfrak{h}^{m}_{\mathfrak{l}}, (Z^{+}_{m\mathfrak{l}} + \iota\,\text{Ʀ}^{+}_{m\mathfrak{l}}, Z^{-}_{m\mathfrak{l}} + \iota\,\text{Ʀ}^{-}_{m\mathfrak{l}}, \text{Ł}^{+}_{m\mathfrak{l}} + \iota\,\text{₱}^{+}_{m\mathfrak{l}}, \text{Ł}^{-}_{m\mathfrak{l}} + \iota\,\text{₱}^{-}_{m\mathfrak{l}}))$ in the BCFN-SS $\text{Ꝁ}(\underline{d}_{\mathfrak{l}}) = ((\omega_m, \mathfrak{h}^{m}_{\mathfrak{l}}), \text{₣}^{+}_{m\mathfrak{l}}, \text{₣}^{-}_{m\mathfrak{l}}, \text{₮}^{+}_{m\mathfrak{l}}, \text{₮}^{-}_{m\mathfrak{l}}) = ((\omega_m, \mathfrak{h}^{m}_{\mathfrak{l}}), Z^{+}_{m\mathfrak{l}} + \iota\,\text{Ʀ}^{+}_{m\mathfrak{l}}, Z^{-}_{m\mathfrak{l}} + \iota\,\text{Ʀ}^{-}_{m\mathfrak{l}}, \text{Ł}^{+}_{m\mathfrak{l}} + \iota\,\text{₱}^{+}_{m\mathfrak{l}}, \text{Ł}^{-}_{m\mathfrak{l}} + \iota\,\text{₱}^{-}_{m\mathfrak{l}})$.

Example 1: A company requires artificial intelligence (AI) software for enhancing the performance of the company. The IT experts team of the company would select the finest AI software in the described 4 AI software that is $S_{\mathfrak{A}\mathfrak{I}-1} = Cortana$, $S_{\mathfrak{A}\mathfrak{I}-2} = Google\ assistant$, $S_{\mathfrak{A}\mathfrak{I}-3} = IBM\ watson$, and $S_{\mathfrak{A}\mathfrak{I}-4} = H20.AI$. The IT experts team would assess this AI software by taking into account 4 various parameters which is $\underline{d}_1 = Deep\ learning$, $\underline{d}_2 = Automate\ tasks$, $\underline{d}_3 = Quantum\ computing$, $\underline{d}_4 = Data\ Ingestion$. The team of experts interpreted their evaluation in the model of grades to each AI software relying on the parameters. Table 1 would signify the 6-SS.

In Table 1, five smash products interpret "Excellent" four smash products interpret "very good", three smash products interpret "good" two smash products interpret "fair", two smash products interpret "poor" and the circle interprets "very poor". The grades would be associated with smash products as follows

0 would describe "o"

1 would describe "⋇"2 would describe "⋇⋇"

3 would describe "⋇⋇⋇

4 would describe "⋇⋇⋇⋇

5 would describe "⋇⋇⋇⋇⋇

Consequently, Table 2 would interpret the tabular interpretation of 6-SS.

In this example, we employ specific grading criteria. (one can employ any other grading criteria).

For 0 grade, $0.0\leq\text{₣}'^{+}_{\underline{d}} < 0.15$, and $-1.0\leq\text{₣}'^{-}_{\underline{d}} < -0.75$,

For 1 grade , $0.15\leq\text{₣}'^{+}_{\underline{d}} < 0.3$, and $-0.75\leq\text{₣}'^{-}_{\underline{d}} < -0.6$,

For 2 grade , $0.3\leq\text{₣}'^{+}_{\underline{d}} < 0.45$, and $-0.6\leq\text{₣}'^{-}_{\underline{d}} < -0.45$,

For 3 grade , $0.45\leq \text{₣}_{\underline{d}}^{'+} < 0.6$, and $-0.45\leq \text{₣}_{\underline{d}}^{'-} < -0.3$,
For 4 grade , $0.6\leq \text{₣}_{\underline{d}}^{'+} < 0.75$, and $-0.3\leq \text{₣}_{\underline{d}}^{'-} \leq -0.15$,
For 5 grade , $0.75\leq \text{₣}_{\underline{d}}^{'+} \leq 1.0$, and $-0.15\leq \text{₣}_{\underline{d}}^{'-} \leq -0.0$

where, $\text{₣}_{\underline{d}}^{'+} = \frac{Z_{\underline{d}}^{+}+ \text{Ʀ}_{\underline{d}}^{+}}{2}$, and $\text{₣}_{\underline{d}}^{'-} = \frac{-Z_{\underline{d}}^{+}- \text{Ʀ}_{\underline{d}}^{-}}{2}$, $0\leq Z_{\underline{d}}^{+} + \text{Ƚ}_{m\mathfrak{l}}^{+}\leq 1$, $0\leq \text{Ʀ}_{\underline{d}}^{+} + \text{Ᵽ}_{m\mathfrak{l}}^{+}\leq 1$, $-1\leq Z_{\underline{d}}^{-} + \text{Ƚ}_{m\mathfrak{l}}^{-}\leq 0$ and $-1\leq \text{Ʀ}_{\underline{d}}^{-} + \text{Ᵽ}_{m\mathfrak{l}}^{-}\leq 0$. The BCIF6-SS $(\text{Ꝁ}, (\mathfrak{U}, D, \aleph))$ would be exhibited as shown at the bottom of the next page.

Table 3 exhibits the tabular display of BCIF6-SS.

The assessment grades in the genuine-life dilemmas can be any, here in example 1, we are taking 6 grades. Further, any BCIFN-SS can be called BCIF(N+1)-SS and by letting $\aleph = 2$, the BCIFN-SS would degenerate to BCIFSS.

Definition 8: Underneath is the score and accuracy values of a BCIFN-SN $\mathfrak{V}_{m\mathfrak{l}} = (\mathfrak{h}_{\mathfrak{l}}^{m}, (Z_{m\mathfrak{l}}^{+} + \iota\, \text{Ʀ}_{m\mathfrak{l}}^{+}, Z_{m\mathfrak{l}}^{-} + \iota\, \text{Ʀ}_{m\mathfrak{l}}^{-}))$.

$$\mathfrak{S}(\mathfrak{V}_{m\mathfrak{l}}) = \frac{\mathfrak{h}_{\mathfrak{l}}^{m}}{\aleph-1} + \frac{1}{8}\Big(2 + Z_{m\mathfrak{l}}^{+} + \text{Ʀ}_{m\mathfrak{l}}^{+} + Z_{m\mathfrak{l}}^{-} + \text{Ʀ}_{m\mathfrak{l}}^{-} + \text{Ƚ}_{m\mathfrak{l}}^{+} + \text{Ᵽ}_{m\mathfrak{l}}^{+}, \text{Ƚ}_{m\mathfrak{l}}^{-} + \text{Ᵽ}_{m\mathfrak{l}}^{-}\Big)\ \mathfrak{S}(\mathfrak{V}_{m\mathfrak{l}}) \in [0,\ 2] \tag{10}$$

$$\mathcal{H}(\mathfrak{V}_{m\mathfrak{l}}) = \frac{\mathfrak{h}_{\mathfrak{l}}^{m}}{\aleph-1} + \frac{Z_{m\mathfrak{l}}^{+} + \text{Ʀ}_{m\mathfrak{l}}^{+} + Z_{m\mathfrak{l}}^{-} + \text{Ʀ}_{m\mathfrak{l}}^{-} + \text{Ƚ}_{m\mathfrak{l}}^{+} + \text{Ᵽ}_{m\mathfrak{l}}^{+}, \text{Ƚ}_{m\mathfrak{l}}^{-} + \text{Ᵽ}_{m\mathfrak{l}}^{-}}{8}\quad \mathcal{H}(\mathfrak{V}_{m\mathfrak{l}}) \in [0,\ 3] \tag{11}$$

Definition 9: Utilizing two BCIFN-SSs $\mathfrak{V}_{m\mathfrak{l}} = (\mathfrak{h}_{\mathfrak{l}}^{m}, Z_{m\mathfrak{l}}^{+} + \iota\, \text{Ʀ}_{m\mathfrak{l}}^{+}, Z_{m\mathfrak{l}}^{-} + \iota\, \text{Ʀ}_{m\mathfrak{l}}^{-} \text{Ƚ}_{m\mathfrak{l}}^{+} + \iota\, \text{Ᵽ}_{m\mathfrak{l}}^{+}, \text{Ƚ}_{m\mathfrak{l}}^{-} + \iota\, \text{Ᵽ}_{m\mathfrak{l}}^{-})$ and $\mathfrak{V}_{s\mathfrak{l}} = (\mathfrak{h}_{\mathfrak{l}}^{s}, Z_{s\mathfrak{l}}^{+} + \iota\, \text{Ʀ}_{s\mathfrak{l}}^{+}, Z_{s\mathfrak{l}}^{-} + \iota\, \text{Ʀ}_{s\mathfrak{l}}^{-}, \text{Ƚ}_{s\mathfrak{l}}^{+} + \iota\, \text{Ᵽ}_{s\mathfrak{l}}^{+}, \text{Ƚ}_{s\mathfrak{l}}^{-} + \iota\, \text{Ᵽ}_{s\mathfrak{l}}^{-})$, we have

1) If $\mathfrak{S}(\mathfrak{V}_{m\mathfrak{l}}) < \mathfrak{S}(\mathfrak{V}_{s\mathfrak{l}})$, then $\mathfrak{V}_{m\mathfrak{l}} < \mathfrak{V}_{s\mathfrak{l}}$
2) If $\mathfrak{S}(\mathfrak{V}_{m\mathfrak{l}}) > \mathfrak{S}(\mathfrak{V}_{s\mathfrak{l}})$, then $\mathfrak{V}_{m\mathfrak{l}} > \mathfrak{V}_{s\mathfrak{l}}$
3) If $\mathfrak{S}(\mathfrak{V}_{m\mathfrak{l}}) = \mathfrak{S}(\mathfrak{V}_{s\mathfrak{l}})$, then
   i) If $\mathcal{H}(\mathfrak{V}_{m\mathfrak{l}}) < \mathcal{H}(\mathfrak{V}_{s\mathfrak{l}})$, then $\mathfrak{V}_{m\mathfrak{l}} < \mathfrak{V}_{s\mathfrak{l}}$
   ii) If $\mathcal{H}(\mathfrak{V}_{m\mathfrak{l}}) > \mathcal{H}(\mathfrak{V}_{s\mathfrak{l}})$, then $\mathfrak{V}_{m\mathfrak{l}} > \mathfrak{V}_{s\mathfrak{l}}$
   iii) If $\mathcal{H}(\mathfrak{V}_{m\mathfrak{l}}) = \mathcal{H}(\mathfrak{V}_{s\mathfrak{l}})$, then $\mathfrak{V}_{m\mathfrak{l}} = \mathfrak{V}_{s\mathfrak{l}}$

Definition 10: For a BCIFN-SS $(\text{Ꝁ}, \mathfrak{B})$, the weak complement would be signified by $(\text{Ꝁ}, \mathfrak{B}^{c})$, where $\mathfrak{B}^{c} = (\mathfrak{U}^{c}, D, \aleph)$ symbolize the weak complement of $(\mathfrak{U}, D, \aleph)$ that is $\mathfrak{U}^{c}(\underline{d}_{\mathfrak{l}}) \cap \mathfrak{U}(\underline{d}_{\mathfrak{l}}) = \emptyset\ \forall\ \underline{d}_{\mathfrak{l}} \in D$.

Example 2: For a BCIF6-SS $(\text{Ꝁ}, \mathfrak{B}) = (\text{Ꝁ}, (\mathfrak{U}, D, 6))$ of example 1, the weak complement $(\text{Ꝁ}, \mathfrak{B}^{c}) = (\text{Ꝁ}, (\mathfrak{U}^{c}, D, 6))$ is revealed in Table 4.

Definition 11: For a BCIFN-SS $(\text{Ꝁ}, \mathfrak{B})$, the (bipolar complex intuitionistic fuzzy) BCIF complement would be signified by $(\text{Ꝁ}^{c}, \mathfrak{B})$, where $\text{Ꝁ}^{c} : D \to F - BCIFN^{(\mathfrak{X}\times H)}$ and

$$\text{Ꝁ}^{c}(\underline{d}_{\mathfrak{l}}) = \Big(\big(\omega_m, \mathfrak{h}_{\mathfrak{l}}^{m}\big), \big(\text{Ƚ}_{m\mathfrak{l}}^{+} + \iota\, \text{Ᵽ}_{m\mathfrak{l}}^{+}, \text{Ƚ}_{m\mathfrak{l}}^{-} + \iota\, \text{Ᵽ}_{m\mathfrak{l}}^{-}, Z_{m\mathfrak{l}}^{+} + \iota\, \text{Ʀ}_{m\mathfrak{l}}^{+}, Z_{m\mathfrak{l}}^{-} + \iota\, \text{Ʀ}_{m\mathfrak{l}}^{-}\big)\Big) \tag{12}$$

Example 3: For a BCIF6-SS $(\text{Ꝁ}, \mathfrak{B}) = (\text{Ꝁ}, (\mathfrak{U}, D, 6))$ of example 1, the BCIF complement $(\text{Ꝁ}^{c}, \mathfrak{B})$ is revealed in Table 5.

In the BCIF complement the associated grades would not change.

Definition 12: For a BCIFN-SS $(\text{Ꝁ}, \mathfrak{B})$, the weak BCIF complement would be signified by $(\text{Ꝁ}^{c}, \mathfrak{B}^{c}) = (\text{Ꝁ}^{c}, (\mathfrak{U}^{c}, D, \aleph))$, where $(\text{Ꝁ}, \mathfrak{B}^{c})$ would be a weak complement and $(\text{Ꝁ}^{c}, \mathfrak{B})$ would be BCIF complement.

Example 4: For a BCIF6-SS $(\text{Ꝁ}, \mathfrak{B}) = (\text{Ꝁ}, (\mathfrak{U}, D, 6))$ of example 1, the weak BCIF complement $(\text{Ꝁ}^{c}, \mathfrak{B}^{c})$ is revealed in Table 6.

Definition 13: For a BCIFN-SS $(\text{Ꝁ}, \mathfrak{B})$, the top weak complement would be implied as (13), shown at the bottom of the next page.

Example 5: For a BCIF6-SS $(\text{Ꝁ}, \mathfrak{B}) = (\text{Ꝁ}, (\mathfrak{U}, D, 6))$ of example 1, the top weak complement $(\text{Ꝁ}, \mathfrak{B}^{\tau}) = (\text{Ꝁ}, (\mathfrak{U}^{\tau}, D, 6))$ is revealed in Table 7.

Definition 14: For a BCIFN-SS $(\text{Ꝁ}, \mathfrak{B})$, the top weak BCIF complement would be implied as (14), shown at the bottom of the next page.

Example 6: For a BCIF6-SS $(\text{Ꝁ}, \mathfrak{B}) = (\text{Ꝁ}, (\mathfrak{U}, D, 6))$ of example 1, the top weak BCIF complement $(\text{Ꝁ}^{c}, \mathfrak{B}^{\tau}) = (\text{Ꝁ}^{c}, (\mathfrak{U}^{\tau}, D, 6))$ is revealed in Table 8.

Definition 15: For a BCIFN-SS $(\text{Ꝁ}, \mathfrak{B})$, the bottom weak complement would be implied as (15), shown at the bottom of page 7.

Example 7: For a BCIF6-SS $(\text{Ꝁ}, \mathfrak{B}) = (\text{Ꝁ}, (\mathfrak{U}, D, 6))$ of example 1, the bottom weak complement $(\text{Ꝁ}, \mathfrak{B}^{\beta}) = (\text{Ꝁ}, (\mathfrak{U}^{\beta}, D, 6))$ is revealed in Table 9.

Definition 16: For a BCIFN-SS $(\text{Ꝁ}, \mathfrak{B})$, the bottom weak BCIF complement would be implied as (16), shown at the bottom of page 7.

Example 8: For a BCIF6-SS $(\text{Ꝁ}, \mathfrak{B}) = (\text{Ꝁ}, (\mathfrak{U}, D, 6))$ of example 1, the bottom weak BCIF complement $(\text{Ꝁ}^{c}, \mathfrak{B}^{\beta}) = (\text{Ꝁ}^{c}, (\mathfrak{U}^{\beta}, D, 6))$ is revealed in Table 10.

Definition 17: For two BCIFN-SSs $(\text{Ꝁ}_1, \mathfrak{B}_1) = (\text{Ꝁ}_1, (\mathfrak{U}_1, D_1, \aleph_1))$ and $(\text{Ꝁ}_2, \mathfrak{B}_2) = (\text{Ꝁ}_2, (\mathfrak{U}_2, D_2, \aleph_2))$, their restricted union would be implied as

$$(\text{Ꝁ}_1, \mathfrak{B}_1) \cup_{\mathbb{R}} (\text{Ꝁ}_2, \mathfrak{B}_2) = (\text{Ꝁ}_1, (\mathfrak{U}_1, D_1, \aleph_1)) \cup_{\mathbb{R}} (\text{Ꝁ}_2, (\mathfrak{U}_2, D_2, \aleph_2)) = (\zeta, \mathfrak{B}_1 \cup_{\mathcal{R}} \mathfrak{B}_2, \max(\aleph_1, \aleph_2)) \tag{17}$$

where $\mathfrak{B}_1 \cup_{\mathcal{R}} \mathfrak{B}_2 = (\psi, D_1 \cap D_2, \max(\aleph_1, \aleph_2))$, that is $\forall\ \underline{d}_{\mathfrak{l}} \in D_1 \cap D_2$, $\omega_m \in \mathfrak{C}$, $((\omega_m, \mathfrak{h}_{\mathfrak{l}}^{m}), Z^{+} + \iota\, \text{Ʀ}^{+}, Z^{-} + \iota\, \text{Ʀ}^{-}, \text{Ƚ}^{+} + \iota\, \text{Ᵽ}^{+}, \text{Ƚ}^{-} + \iota\, \text{Ᵽ}^{-}) \in \zeta(\underline{d}_{\mathfrak{l}}) \iff \mathfrak{h}_{\mathfrak{l}}^{m} = \max(\mathfrak{h}_{\mathfrak{l}}^{m^1}, \mathfrak{h}_{\mathfrak{l}}^{m^2})$, $Z^{+} = \max(Z_{\mathbb{C}}^{+}, Z_{\mathbb{D}}^{+})$, $\text{Ʀ}^{+} = \max(\text{Ʀ}_{\mathbb{C}}^{+}, \text{Ʀ}_{\mathbb{D}}^{+})$, $Z^{-} = \min(Z_{\mathbb{C}}^{-}, Z_{\mathbb{D}}^{-})$, $\text{Ʀ}^{-} = \min(\text{Ʀ}_{\mathbb{C}}^{-}, \text{Ʀ}_{\mathbb{D}}^{-})$, $\text{Ƚ}^{+} = \min(\text{Ƚ}^{+}, \text{Ƚ}_{\mathbb{D}}^{+})$, $\text{Ᵽ}^{+} = \min(\text{Ᵽ}^{+}, \text{Ᵽ}_{\mathbb{D}}^{+})$, $\text{Ƚ}^{-} = \max(\text{Ƚ}^{-}, \text{Ƚ}_{\mathbb{D}}^{-})$, $\text{Ᵽ}^{-} = \max(\text{Ᵽ}^{-}, \text{Ᵽ}_{\mathbb{D}}^{-})$ if $((\omega_m, \mathfrak{h}_{\mathfrak{l}}^{m^1}), Z_{\mathbb{C}}^{+} + \iota\, \text{Ʀ}_{\mathbb{C}}^{+}, Z_{\mathbb{C}}^{+} + \iota\, \text{Ʀ}_{\mathbb{C}}^{+}, \text{Ƚ}_{\mathbb{C}}^{+} + \iota\, \text{Ᵽ}^{+}, \text{Ƚ}_{\mathbb{C}}^{-} + \iota\, \text{Ᵽ}_{\mathbb{C}}^{-}) \in D_1(\underline{d}_{\mathfrak{l}})$ and $((\omega_m, \mathfrak{h}_{\mathfrak{l}}^{m^2}), Z_{\mathbb{D}}^{+} + \iota\, \text{Ʀ}_{\mathbb{D}}^{+}, Z_{\mathbb{D}}^{+} + \iota\, \text{Ʀ}_{\mathbb{D}}^{+}, \text{Ƚ}_{\mathbb{D}}^{+} + \iota\, \text{Ᵽ}_{\mathbb{D}}^{+}, \text{Ƚ}_{\mathbb{D}}^{-} + \iota\, \text{Ᵽ}_{\mathbb{D}}^{-}) \in D_2(\underline{d}_{\mathfrak{l}})$, $\mathbb{C}$, $\mathbb{D}$ are BCIFSs on $\mathfrak{U}_1(\underline{d}_{\mathfrak{l}})$ and $\mathfrak{U}_2(\underline{d}_{\mathfrak{l}})$ respectively.

TABLE 3. The tabular exhibition of BCIF6-SS.

$\left(\begin{smallmatrix}\mathcal{K},\\ (\mathfrak{U},\mathrm{D},\aleph)\end{smallmatrix}\right)$	đ_1	đ_2	đ_3	đ_4
$\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-1}$	$\left(5,\begin{pmatrix}0.75+\iota 0.87,\\ -0.1-\iota 0.12,\\ 0.24+\iota 0.1\\ -0.8-\iota 0.75\end{pmatrix}\right)$	$\left(2,\begin{pmatrix}0.5+\iota 0.3,\\ -0.48-\iota 0.52,\\ 0.28+\iota 0.47,\\ -0.27-\iota 0.15\end{pmatrix}\right)$	$\left(2,\begin{pmatrix}0.33+\iota 0.36,\\ -0.47-\iota 0.67,\\ 0.46+\iota 0.53,\\ -0.26-\iota 0.12\end{pmatrix}\right)$	$\left(2,\begin{pmatrix}0.38+\iota 0.43,\\ -0.53-\iota 0.59,\\ 0.42+\iota 0.56,\\ -0.27-\iota 0.16\end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-2}$	$\left(2,\begin{pmatrix}0.34+\iota 0.39,\\ -0.55-\iota 0.49,\\ 0.5+\iota 0.61\\ -0.45-\iota 0.33\end{pmatrix}\right)$	$\left(1,\begin{pmatrix}0.23+\iota 0.18,\\ -0.7-\iota 0.61,\\ 0.71+\iota 0.68,\\ -0.11-\iota 0.21\end{pmatrix}\right)$	$\left(3,\begin{pmatrix}0.48+\iota 0.52,\\ -0.43-\iota 0.34,\\ 0.42+\iota 0.32,\\ -0.59-\iota 0.57\end{pmatrix}\right)$	$\left(4,\begin{pmatrix}0.67+\iota 0.74,\\ -0.23-\iota 0.19,\\ 0.2+\iota 0.1,\\ -0.53-\iota 0.72\end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-3}$	$\left(4,\begin{pmatrix}0.73+\iota 0.68,\\ -0.21-\iota 0.18,\\ 0.15+\iota 0.39\\ -0.39-\iota 0.63\end{pmatrix}\right)$	$\left(0,\begin{pmatrix}0.1+\iota 0.05,\\ -0.9-\iota 0.8,\\ 0.83+\iota 0.69,\\ -0.08-\iota 0.15\end{pmatrix}\right)$	$\left(5,\begin{pmatrix}0.8+\iota 0.87,\\ -0.1-\iota 0.14,\\ 0.17+\iota 0.12,\\ -0.82-\iota 0.76\end{pmatrix}\right)$	$\left(0,\begin{pmatrix}0.1+\iota 0.09,\\ -0.77-\iota 0.83,\\ 0.68+\iota 0.82,\\ -0.16-\iota 0.05\end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-4}$	$\left(3,\begin{pmatrix}0.37+\iota 0.44,\\ -0.41-\iota 0.31,\\ 0.49+\iota 0.5,\\ -0.27-\iota 0.62\end{pmatrix}\right)$	$\left(3,\begin{pmatrix}0.45+\iota 0.5,\\ -0.4-\iota 0.3,\\ 0.45+\iota 0.37,\\ -0.36-\iota 0.23\end{pmatrix}\right)$	$\left(2,\begin{pmatrix}0.3+\iota 0.4,\\ -0.6-\iota 0.48,\\ 0.15+\iota 0.35,\\ -0.33-\iota 0.51\end{pmatrix}\right)$	$\left(2,\begin{pmatrix}0.33+\iota 0.44,\\ -0.49-\iota 0.59,\\ 0.44+\iota 0.33,\\ -0.31-\iota 0.12\end{pmatrix}\right)$

Example 9: Take two BCIFN-SSs that is $(\mathcal{K}_1, \mathfrak{B}_1) = (\mathcal{K}_1, (\mathfrak{U}_1, \mathrm{D}_1, 6))$ interpreted in Table 11 and $(\mathcal{K}_2, \mathfrak{B}_2) = (\mathcal{K}_2, (\mathfrak{U}_2, \mathrm{D}_2, 5))$ interpreted in Table 12. Then Table 13 revealed their restricted union.

Definition 18: For two BCIFN-SSs $(\mathcal{K}_1, \mathfrak{B}_1) = (\mathcal{K}_1, (\mathfrak{U}_1, \mathrm{D}_1, \aleph_1))$ and $(\mathcal{K}_2, \mathfrak{B}_2) = (\mathcal{K}_2, (\mathfrak{U}_2, \mathrm{D}_2, \aleph_2))$, their extended union would be implied as (18), shown at the bottom of page 8.

$$
\begin{aligned}
\mathcal{K}(\text{đ}_1) &= \left(\begin{matrix}(\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-1}, 5), \begin{pmatrix}0.75+\iota 0.87, & -0.1-\iota 0.12,\\ 0.24+\iota 0.1, & -0.8-\iota 0.75\end{pmatrix}, (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-2}, 2), \begin{pmatrix}0.34+\iota 0.39, & -0.55-\iota 0.49,\\ 0.5+\iota 0.61, & -0.45-\iota 0.33\end{pmatrix},\\ (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-3}, 4), \begin{pmatrix}0.73+\iota 0.68, & -0.21-\iota 0.18,\\ 0.15+\iota 0.3, & -0.39-\iota 0.63\end{pmatrix}, (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-4}, 3), \begin{pmatrix}0.37+\iota 0.44, & -0.41-\iota 0.31,\\ 0.49+\iota 0.5, & -0.27-\iota 0.62\end{pmatrix}\end{matrix}\right)\\
\mathcal{K}(\text{đ}_2) &= \left(\begin{matrix}(\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-1}, 2), \begin{pmatrix}0.5+\iota 0.3, & -0.48-\iota 0.52,\\ 0.28+\iota 0.47, & -0.27-\iota 0.15\end{pmatrix}, (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-2}, 1), \begin{pmatrix}0.23+\iota 0.18, & -0.7-\iota 0.61,\\ 0.71+\iota 0.68, & -0.11-\iota 0.21\end{pmatrix},\\ (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-3}, 0), \begin{pmatrix}0.1+\iota 0.05, & -0.9-\iota 0.8,\\ 0.83+\iota 0.69, & -0.08-\iota 0.15\end{pmatrix}, (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-4}, 3), \begin{pmatrix}0.45+\iota 0.5, & -0.4-\iota 0.3,\\ 0.45+\iota 0.37, & -0.36-\iota 0.23\end{pmatrix}\end{matrix}\right)\\
\mathcal{K}(\text{đ}_3) &= \left(\begin{matrix}(\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-1}, 2), \begin{pmatrix}0.33+\iota 0.36, & -0.47-\iota 0.67,\\ 0.46+\iota 0.53, & -0.26-\iota 0.12\end{pmatrix}, (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-2}, 3), \begin{pmatrix}0.48+\iota 0.52, & -0.43-\iota 0.34,\\ 0.42+\iota 0.32, & -0.59-\iota 0.57\end{pmatrix},\\ (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-3}, 5), \begin{pmatrix}0.8+\iota 0.87, & -0.1-\iota 0.14,\\ 0.17+\iota 0.12, & -0.82-\iota 0.76\end{pmatrix}, (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-4}, 2), \begin{pmatrix}0.3+\iota 0.4, & -0.6-\iota 0.48,\\ 0.15+\iota 0.35, & -0.33-\iota 0.51\end{pmatrix}\end{matrix}\right)\\
\mathcal{K}(\text{đ}_4) &= \left(\begin{matrix}(\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-1}, 2), \begin{pmatrix}0.38+\iota 0.43, & -0.53-\iota 0.59,\\ 0.42+\iota 0.56, & -0.27-\iota 0.16\end{pmatrix}, (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-2}, 4), \begin{pmatrix}0.67+\iota 0.74, & -0.23-\iota 0.19,\\ 0.2+\iota 0.1, & -0.53-\iota 0.72\end{pmatrix},\\ (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-3}, 0), \begin{pmatrix}0.1+\iota 0.09, & -0.77-\iota 0.83,\\ 0.68+\iota 0.82, & -0.16-\iota 0.05\end{pmatrix}, (\mathcal{S}_{\mathfrak{A}\mathfrak{Z}-4}, 2), \begin{pmatrix}0.33+\iota 0.44, & -0.49-\iota 0.59,\\ 0.44+\iota 0.33, & -0.31-\iota 0.12\end{pmatrix}\end{matrix}\right)
\end{aligned}
$$

$$
(\mathcal{K}, \mathfrak{B}^{\tau}) = (\mathcal{K}, (\mathfrak{U}^{\tau}, \mathrm{D}, \aleph)) = \begin{cases} \mathcal{K}(\text{đ}_{\mathfrak{l}}) = \left((\omega_{\mathrm{m}}, \aleph-1), \begin{pmatrix} Z^{+}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ꞧ}^{+}_{\mathrm{m}\mathfrak{l}}, Z^{-}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ꞧ}^{-}_{\mathrm{m}\mathfrak{l}}, \\ \text{Ƚ}^{+}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ᵽ}^{+}_{\mathrm{m}\mathfrak{l}}, \text{Ƚ}^{-}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ᵽ}^{-}_{\mathrm{m}\mathfrak{l}} \end{pmatrix}\right), & \text{if } \mathfrak{h}^{\mathrm{m}}_{\mathfrak{z}\mathfrak{l}} < \aleph - 1 \\ \mathcal{K}(\text{đ}_{\mathfrak{l}}) = \left((\omega_{\mathrm{m}}, 0), \begin{pmatrix} Z^{+}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ꞧ}^{+}_{\mathrm{m}\mathfrak{l}}, Z^{-}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ꞧ}^{-}_{\mathrm{m}\mathfrak{l}}, \\ \text{Ƚ}^{+}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ᵽ}^{+}_{\mathrm{m}\mathfrak{l}}, \text{Ƚ}^{-}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ᵽ}^{-}_{\mathrm{m}\mathfrak{l}} \end{pmatrix}\right), & \text{if } \mathfrak{h}^{\mathrm{m}}_{\mathfrak{z}\mathfrak{l}} = \aleph - 1 \end{cases} \tag{13}
$$

$$
(\mathcal{K}, \mathfrak{B}^{\tau}) = (\mathcal{K}, (\mathfrak{U}^{c}, \mathrm{D}, \aleph)) = \begin{cases} \mathcal{K}(\text{đ}_{\mathfrak{l}}) = \left((\omega_{\mathrm{m}}, \aleph-1), \begin{pmatrix} \text{Ƚ}^{+}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ᵽ}^{+}_{\mathrm{m}\mathfrak{l}}, \text{Ƚ}^{-}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ᵽ}^{-}_{\mathrm{m}\mathfrak{l}}, \\ Z^{+}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ꞧ}^{+}_{\mathrm{m}\mathfrak{l}}, Z^{-}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ꞧ}^{-}_{\mathrm{m}\mathfrak{l}} \end{pmatrix}\right), & \text{if } \mathfrak{h}^{\mathrm{m}}_{\mathfrak{z}\mathfrak{l}} < \aleph - 1 \\ \mathcal{K}(\text{đ}_{\mathfrak{l}}) = \left((\omega_{\mathrm{m}}, 0), \begin{pmatrix} \text{Ƚ}^{+}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ᵽ}^{+}_{\mathrm{m}\mathfrak{l}}, \text{Ƚ}^{-}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ᵽ}^{-}_{\mathrm{m}\mathfrak{l}}, \\ Z^{+}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ꞧ}^{+}_{\mathrm{m}\mathfrak{l}}, Z^{-}_{\mathrm{m}\mathfrak{l}} + \iota \text{Ꞧ}^{-}_{\mathrm{m}\mathfrak{l}} \end{pmatrix}\right), & \text{if } \mathfrak{h}^{\mathrm{m}}_{\mathfrak{z}\mathfrak{l}} = \aleph - 1 \end{cases} \tag{14}
$$

TABLE 4. The weak complement of BCIF6-SS (interpreted in Table 3) is interpreted in Table 3.

$(\mathrm{K}, \mathfrak{B}^c)$	$\underline{d}_1$	$\underline{d}_2$	$\underline{d}_3$	$\underline{d}_4$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-1}$	$\left(1, \begin{pmatrix} 0.75 + \iota\, 0.87, \\ -0.1 - \iota\, 0.12, \\ 0.24 + \iota\, 0.1 \\ -0.8 - \iota\, 0.75 \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.5 + \iota\, 0.3, \\ -0.48 - \iota\, 0.52, \\ 0.28 + \iota\, 0.47, \\ -0.27 - \iota\, 0.15 \end{pmatrix}\right)$	$\left(1, \begin{pmatrix} 0.33 + \iota\, 0.36, \\ -0.47 - \iota\, 0.67, \\ 0.46 + \iota\, 0.53, \\ -0.26 - \iota\, 0.12 \end{pmatrix}\right)$	$\left(1, \begin{pmatrix} 0.38 + \iota\, 0.43, \\ -0.53 - \iota\, 0.59, \\ 0.42 + \iota\, 0.56, \\ -0.27 - \iota\, 0.16 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-2}$	$\left(0, \begin{pmatrix} 0.34 + \iota\, 0.39, \\ -0.55 - \iota\, 0.49, \\ 0.5 + \iota\, 0.61 \\ -0.45 - \iota\, 0.33 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.23 + \iota\, 0.18, \\ -0.7 - \iota\, 0.61, \\ 0.71 + \iota\, 0.68, \\ -0.11 - \iota\, 0.21 \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.48 + \iota\, 0.52, \\ -0.43 - \iota\, 0.34, \\ 0.42 + \iota\, 0.32, \\ -0.59 - \iota\, 0.57 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.67 + \iota\, 0.74, \\ -0.23 - \iota\, 0.19, \\ 0.2 + \iota\, 0.1, \\ -0.53 - \iota\, 0.72 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-3}$	$\left(1, \begin{pmatrix} 0.73 + \iota\, 0.68, \\ -0.21 - \iota\, 0.18, \\ 0.15 + \iota\, 0.39 \\ -0.39 - \iota\, 0.63 \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.1 + \iota\, 0.05, \\ -0.9 - \iota\, 0.8, \\ 0.83 + \iota\, 0.69, \\ -0.08 - \iota\, 0.15 \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.8 + \iota\, 0.87, \\ -0.1 - \iota\, 0.14, \\ 0.17 + \iota\, 0.12, \\ -0.82 - \iota\, 0.76 \end{pmatrix}\right)$	$\left(1, \begin{pmatrix} 0.1 + \iota\, 0.09, \\ -0.77 - \iota\, 0.83, \\ 0.68 + \iota\, 0.82, \\ -0.16 - \iota\, 0.05 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-4}$	$\left(0, \begin{pmatrix} 0.37 + \iota\, 0.44, \\ -0.41 - \iota\, 0.31, \\ 0.49 + \iota\, 0.5, \\ -0.27 - \iota\, 0.62 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.45 + \iota\, 0.5, \\ -0.4 - \iota\, 0.3, \\ 0.45 + \iota\, 0.37, \\ -0.36 - \iota\, 0.23 \end{pmatrix}\right)$	$\left(0, \begin{pmatrix} 0.3 + \iota\, 0.4, \\ -0.6 - \iota\, 0.48, \\ 0.15 + \iota\, 0.35, \\ -0.33 - \iota\, 0.51 \end{pmatrix}\right)$	$\left(3, \begin{pmatrix} 0.33 + \iota\, 0.44, \\ -0.49 - \iota\, 0.59, \\ 0.44 + \iota\, 0.33, \\ -0.31 - \iota\, 0.12 \end{pmatrix}\right)$

TABLE 5. The BCIF complement of BCIF6-SS of example 1.

$(\mathrm{K}^c, \mathfrak{B})$	$\underline{d}_1$	$\underline{d}_2$	$\underline{d}_3$	$\underline{d}_4$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-1}$	$\left(5, \begin{pmatrix} 0.24 + \iota\, 0.1, \\ -0.8 - \iota\, 0.75, \\ 0.75 + \iota\, 0.87, \\ -0.1 - \iota\, 0.12 \end{pmatrix}\right)$	$\left(2, \begin{pmatrix} 0.28 + \iota\, 0.47, \\ -0.27 - \iota\, 0.15, \\ 0.5 + \iota\, 0.3, \\ -0.48 - \iota\, 0.52 \end{pmatrix}\right)$	$\left(2, \begin{pmatrix} 0.46 + \iota\, 0.53, \\ -0.26 - \iota\, 0.12, \\ 0.33 + \iota\, 0.36, \\ -0.47 - \iota\, 0.67 \end{pmatrix}\right)$	$\left(2, \begin{pmatrix} 0.42 + \iota\, 0.56, \\ -0.27 - \iota\, 0.16, \\ 0.38 + \iota\, 0.43, \\ -0.53 - \iota\, 0.59 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-2}$	$\left(2, \begin{pmatrix} 0.5 + \iota\, 0.61, \\ -0.45 - \iota\, 0.33, \\ 0.34 + \iota\, 0.39, \\ -0.55 - \iota\, 0.49 \end{pmatrix}\right)$	$\left(1, \begin{pmatrix} 0.71 + \iota\, 0.68, \\ -0.11 - \iota\, 0.21, \\ 0.23 + \iota\, 0.18, \\ -0.7 - \iota\, 0.61 \end{pmatrix}\right)$	$\left(3, \begin{pmatrix} 0.42 + \iota\, 0.32, \\ -0.59 - \iota\, 0.57, \\ 0.48 + \iota\, 0.52, \\ -0.43 - \iota\, 0.34 \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.2 + \iota\, 0.1, \\ -0.53 - \iota\, 0.72, \\ 0.67 + \iota\, 0.74, \\ -0.23 - \iota\, 0.19 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-3}$	$\left(4, \begin{pmatrix} 0.15 + \iota\, 0.39, \\ -0.39 - \iota\, 0.63, \\ 0.73 + \iota\, 0.68, \\ -0.21 - \iota\, 0.18 \end{pmatrix}\right)$	$\left(0, \begin{pmatrix} 0.83 + \iota\, 0.69, \\ -0.08 - \iota\, 0.15, \\ 0.1 + \iota\, 0.05, \\ -0.9 - \iota\, 0.8 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.17 + \iota\, 0.12, \\ -0.82 - \iota\, 0.76, \\ 0.8 + \iota\, 0.87, \\ -0.1 - \iota\, 0.14 \end{pmatrix}\right)$	$\left(0, \begin{pmatrix} 0.68 + \iota\, 0.82, \\ -0.16 - \iota\, 0.05, \\ 0.1 + \iota\, 0.09, \\ -0.77 - \iota\, 0.83 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-4}$	$\left(3, \begin{pmatrix} 0.49 + \iota\, 0.5, \\ -0.27 - \iota\, 0.62, \\ 0.37 + \iota\, 0.44, \\ -0.41 - \iota\, 0.31 \end{pmatrix}\right)$	$\left(3, \begin{pmatrix} 0.45 + \iota\, 0.37, \\ -0.36 - \iota\, 0.23, \\ 0.45 + \iota\, 0.5, \\ -0.4 - \iota\, 0.3 \end{pmatrix}\right)$	$\left(2, \begin{pmatrix} 0.15 + \iota\, 0.35, \\ -0.33 - \iota\, 0.51, \\ 0.3 + \iota\, 0.4, \\ -0.6 - \iota\, 0.48 \end{pmatrix}\right)$	$\left(2, \begin{pmatrix} 0.44 + \iota\, 0.33, \\ -0.31 - \iota\, 0.12, \\ 0.33 + \iota\, 0.44, \\ -0.49 - \iota\, 0.59 \end{pmatrix}\right)$

$$\begin{aligned}(\mathrm{K}_1,\ \mathfrak{B}_1) \cup_{\mathbb{E}} (\mathrm{K}_2,\ \mathfrak{B}_2) &= \left(\mathrm{K}_1, (\mathfrak{U}_1,\ \mathrm{D}_1,\ \aleph_1)\right) \cup_{\mathbb{E}} \left(\mathrm{K}_2, (\mathfrak{U}_2,\ \mathrm{D}_2,\ \aleph_2)\right) \\ &= (\varsigma,\ \mathfrak{B}_1 \cup_{\mathcal{E}} \mathfrak{B}_2, \max(\aleph_1,\ \aleph_2))\end{aligned}$$

where $\mathfrak{B}_1 \cup_{\mathcal{E}} \mathfrak{B}_2 = (\varphi,\ \mathrm{D}_1 \cup \mathrm{D}_2,\ \max(\aleph_1,\ \aleph_2))$, and

Example 10: Take two BCIFN-SSs that is $(\mathrm{K}_1,\ \mathfrak{B}_1) = (\mathrm{K}_1, (\mathfrak{U}_1,\ \mathrm{D}_1,\ 6))$ interpreted in Table 11 and $(\mathrm{K}_2,\ \mathfrak{B}_2) =$

$$(\mathrm{K},\ \mathfrak{B}^{\beta}) = (\mathrm{K},\ (\mathfrak{U}^{\beta}, \mathrm{D}, \aleph)) = \begin{cases} \mathrm{K}(\underline{d}_{\mathfrak{l}}) = \left((\omega_{\mathrm{m}},\ 0),\ \begin{pmatrix} Z^{+}_{\mathrm{m}\mathfrak{l}} + \iota\, \mathrm{R}^{+}_{\mathrm{m}\mathfrak{l}},\ Z^{-}_{\mathrm{m}\mathfrak{l}} + \iota\, \mathrm{R}^{-}_{\mathrm{m}\mathfrak{l}}, \\ Ⱡ^{+}_{\mathrm{m}\mathfrak{l}} + \iota\, Ᵽ^{+}_{\mathrm{m}\mathfrak{l}},\ Ⱡ^{-}_{\mathrm{m}\mathfrak{l}} + \iota\, Ᵽ^{-}_{\mathrm{m}\mathfrak{l}} \end{pmatrix}\right), & \text{if } \mathfrak{h}^{\mathrm{m}}_{\mathfrak{l}} > 0 \\ \mathrm{K}(\underline{d}_{\mathfrak{l}}) = \left((\omega_{\mathrm{m}},\ \aleph - 1),\ \begin{pmatrix} Z^{+}_{\mathrm{m}\mathfrak{l}} + \iota\, \mathrm{R}^{+}_{\mathrm{m}\mathfrak{l}},\ Z^{-}_{\mathrm{m}\mathfrak{l}} + \iota\, \mathrm{R}^{-}_{\mathrm{m}\mathfrak{l}}, \\ Ⱡ^{+}_{\mathrm{m}\mathfrak{l}} + \iota\, Ᵽ^{+}_{\mathrm{m}\mathfrak{l}},\ Ⱡ^{-}_{\mathrm{m}\mathfrak{l}} + \iota\, Ᵽ^{-}_{\mathrm{m}\mathfrak{l}} \end{pmatrix}\right), & \text{if } \mathfrak{h}^{\mathrm{m}}_{\mathfrak{l}} = 0 \end{cases} \tag{15}$$

$$(\mathrm{K},\ \mathfrak{B}^{\beta}) = (\mathrm{K},\ (\mathfrak{U}^{\beta}, \mathrm{D}, \aleph)) = \begin{cases} \mathrm{K}(\underline{d}_{\mathfrak{l}}) = \left((\omega_{\mathrm{m}},\ 0),\ \begin{pmatrix} Ⱡ^{+}_{\mathrm{m}\mathfrak{l}} + \iota\, Ᵽ^{+}_{\mathrm{m}\mathfrak{l}},\ Ⱡ^{-}_{\mathrm{m}\mathfrak{l}} + \iota\, Ᵽ^{-}_{\mathrm{m}\mathfrak{l}}, \\ Z^{+}_{\mathrm{m}\mathfrak{l}} + \iota\, \mathrm{R}^{+}_{\mathrm{m}\mathfrak{l}},\ Z^{-}_{\mathrm{m}\mathfrak{l}} + \iota\, \mathrm{R}^{-}_{\mathrm{m}\mathfrak{l}} \end{pmatrix}\right), & \text{if } \mathfrak{h}^{\mathrm{m}}_{\mathfrak{l}} > 0 \\ \mathrm{K}(\underline{d}_{\mathfrak{l}}) = \left((\omega_{\mathrm{m}},\ \aleph - 1),\ \begin{pmatrix} Ⱡ^{+}_{\mathrm{m}\mathfrak{l}} + \iota\, Ᵽ^{+}_{\mathrm{m}\mathfrak{l}},\ Ⱡ^{-}_{\mathrm{m}\mathfrak{l}} + \iota\, Ᵽ^{-}_{\mathrm{m}\mathfrak{l}}, \\ Z^{+}_{\mathrm{m}\mathfrak{l}} + \iota\, \mathrm{R}^{+}_{\mathrm{m}\mathfrak{l}},\ Z^{-}_{\mathrm{m}\mathfrak{l}} + \iota\, \mathrm{R}^{-}_{\mathrm{m}\mathfrak{l}} \end{pmatrix}\right), & \text{if } \mathfrak{h}^{\mathrm{m}}_{\mathfrak{l}} = 0 \end{cases} \tag{16}$$

TABLE 6. The weak BCIF complement of BCIF6-SS of example 1.

$(Ꝁ^c, \mathfrak{B}^c)$	$đ_1$	$đ_2$	$đ_3$	$đ_4$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-1}$	$\left(1, \begin{pmatrix} 0.24 + \iota 0.1, \\ -0.8 - \iota 0.75, \\ 0.75 + \iota 0.87, \\ -0.1 - \iota 0.12 \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.28 + \iota 0.47, \\ -0.27 - \iota 0.15, \\ 0.5 + \iota 0.3, \\ -0.48 - \iota 0.52 \end{pmatrix}\right)$	$\left(1, \begin{pmatrix} 0.46 + \iota 0.53, \\ -0.26 - \iota 0.12, \\ 0.33 + \iota 0.36, \\ -0.47 - \iota 0.67 \end{pmatrix}\right)$	$\left(1, \begin{pmatrix} 0.42 + \iota 0.56, \\ -0.27 - \iota 0.16, \\ 0.38 + \iota 0.43, \\ -0.53 - \iota 0.59 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-2}$	$\left(0, \begin{pmatrix} 0.5 + \iota 0.61, \\ -0.45 - \iota 0.33, \\ 0.34 + \iota 0.39, \\ -0.55 - \iota 0.49 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.71 + \iota 0.68, \\ -0.11 - \iota 0.21, \\ 0.23 + \iota 0.18, \\ -0.7 - \iota 0.61 \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.42 + \iota 0.32, \\ -0.59 - \iota 0.57, \\ 0.48 + \iota 0.52, \\ -0.43 - \iota 0.34 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.2 + \iota 0.1, \\ -0.53 - \iota 0.72, \\ 0.67 + \iota 0.74, \\ -0.23 - \iota 0.19 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-3}$	$\left(1, \begin{pmatrix} 0.15 + \iota 0.39, \\ -0.39 - \iota 0.63, \\ 0.73 + \iota 0.68, \\ -0.21 - \iota 0.18 \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.83 + \iota 0.69, \\ -0.08 - \iota 0.15, \\ 0.1 + \iota 0.05, \\ -0.9 - \iota 0.8 \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.17 + \iota 0.12, \\ -0.82 - \iota 0.76, \\ 0.8 + \iota 0.87, \\ -0.1 - \iota 0.14 \end{pmatrix}\right)$	$\left(1, \begin{pmatrix} 0.68 + \iota 0.82, \\ -0.16 - \iota 0.05, \\ 0.1 + \iota 0.09, \\ -0.77 - \iota 0.83 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-4}$	$\left(0, \begin{pmatrix} 0.49 + \iota 0.5, \\ -0.27 - \iota 0.62, \\ 0.37 + \iota 0.44, \\ -0.41 - \iota 0.31 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.45 + \iota 0.37, \\ -0.36 - \iota 0.23, \\ 0.45 + \iota 0.5, \\ -0.4 - \iota 0.3 \end{pmatrix}\right)$	$\left(0, \begin{pmatrix} 0.15 + \iota 0.35, \\ -0.33 - \iota 0.51, \\ 0.3 + \iota 0.4, \\ -0.6 - \iota 0.48 \end{pmatrix}\right)$	$\left(3, \begin{pmatrix} 0.44 + \iota 0.33, \\ -0.31 - \iota 0.12, \\ 0.33 + \iota 0.44, \\ -0.49 - \iota 0.59 \end{pmatrix}\right)$

TABLE 7. The top weak complement of BCIF6-SS of example 1.

$(Ꝁ, \mathfrak{B}^{\tau})$	$đ_1$	$đ_2$	$đ_3$	$đ_4$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-1}$	$\left(0, \begin{pmatrix} 0.75 + \iota 0.87, \\ -0.1 - \iota 0.12, \\ 0.24 + \iota 0.1 \\ -0.8 - \iota 0.75 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.5 + \iota 0.3, \\ -0.48 - \iota 0.52, \\ 0.28 + \iota 0.47, \\ -0.27 - \iota 0.15 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.33 + \iota 0.36, \\ -0.47 - \iota 0.67, \\ 0.46 + \iota 0.53, \\ -0.26 - \iota 0.12 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.38 + \iota 0.43, \\ -0.53 - \iota 0.59, \\ 0.42 + \iota 0.56, \\ -0.27 - \iota 0.16 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-2}$	$\left(5, \begin{pmatrix} 0.34 + \iota 0.39, \\ -0.55 - \iota 0.49, \\ 0.5 + \iota 0.61 \\ -0.45 - \iota 0.33 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.23 + \iota 0.18, \\ -0.7 - \iota 0.61, \\ 0.71 + \iota 0.68, \\ -0.11 - \iota 0.21 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.48 + \iota 0.52, \\ -0.43 - \iota 0.34, \\ 0.42 + \iota 0.32, \\ -0.59 - \iota 0.57 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.67 + \iota 0.74, \\ -0.23 - \iota 0.19, \\ 0.2 + \iota 0.1, \\ -0.53 - \iota 0.72 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-3}$	$\left(5, \begin{pmatrix} 0.73 + \iota 0.68, \\ -0.21 - \iota 0.18, \\ 0.15 + \iota 0.39 \\ -0.39 - \iota 0.63 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.1 + \iota 0.05, \\ -0.9 - \iota 0.8, \\ 0.83 + \iota 0.69, \\ -0.08 - \iota 0.15 \end{pmatrix}\right)$	$\left(0, \begin{pmatrix} 0.8 + \iota 0.87, \\ -0.1 - \iota 0.14, \\ 0.17 + \iota 0.12, \\ -0.82 - \iota 0.76 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.1 + \iota 0.09, \\ -0.77 - \iota 0.83, \\ 0.68 + \iota 0.82, \\ -0.16 - \iota 0.05 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-4}$	$\left(5, \begin{pmatrix} 0.37 + \iota 0.44, \\ -0.41 - \iota 0.31, \\ 0.49 + \iota 0.5, \\ -0.27 - \iota 0.62 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.45 + \iota 0.5, \\ -0.4 - \iota 0.3, \\ 0.45 + \iota 0.37, \\ -0.36 - \iota 0.23 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.3 + \iota 0.4, \\ -0.6 - \iota 0.48, \\ 0.15 + \iota 0.35, \\ -0.33 - \iota 0.51 \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.33 + \iota 0.44, \\ -0.49 - \iota 0.59, \\ 0.44 + \iota 0.33, \\ -0.31 - \iota 0.12 \end{pmatrix}\right)$

$$\varphi\left(\underline{đ}_{\mathfrak{l}}\right)=\begin{cases} Ꝁ_1\left(\underline{đ}_{\mathfrak{l}}\right), & \text{if } \underline{đ}_{\mathfrak{l}} \in D_1 - D_2 \\ Ꝁ_2\left(\underline{đ}_{\mathfrak{l}}\right), & \text{if } \underline{đ}_{\mathfrak{l}} \in D_1 - D_2 \\ \left(\begin{array}{c} \left(\left(\omega_m, \mathfrak{h}_{\mathfrak{l}}^{m}\right), \begin{pmatrix} Z^{+} + \iota Ʀ^{+}, Z^{-} + \iota Ʀ^{-}, \\ Ƚ^{+} + \iota Ᵽ^{+}, Ƚ^{-} + \iota Ᵽ^{-} \end{pmatrix}\right) \\ \left(\begin{array}{c} \text{such that } \mathfrak{h}_{\mathfrak{l}}^{m} = \max\left(\mathfrak{h}_{\mathfrak{l}}^{m^1}, \mathfrak{h}_{\mathfrak{l}}^{m^2}\right), Z^{+} = \max\left(Z_{\mathbb{C}}^{+}, Z_{\mathbb{D}}^{+}\right) \\ Ʀ^{+} = \max\left(Ʀ_{\mathbb{C}}^{+}, Ʀ_{\mathbb{D}}^{+}\right), Z^{-} = \min\left(Z_{\mathbb{C}}^{-}, Z_{\mathbb{D}}^{-}\right), Ʀ^{-} = \min\left(Ʀ_{\mathbb{C}}^{-}, Ʀ_{\mathbb{D}}^{-}\right), \\ Ƚ^{+} = \min\left(Ƚ^{+}, Ƚ_{\mathbb{D}}^{+}\right), Ᵽ^{+} = \min\left(Ᵽ^{+}, Ᵽ_{\mathbb{D}}^{+}\right), Ƚ^{-} = \max\left(Ƚ^{-}, Ƚ_{\mathbb{D}}^{-}\right), \\ Ᵽ^{-} = \max\left(Ᵽ^{-}, Ᵽ_{\mathbb{D}}^{-}\right), \\ \text{if } \left(\left(\omega_m, \mathfrak{h}_{\mathfrak{l}}^{m^1}\right), \begin{pmatrix} Z_{\mathbb{C}}^{+} + \iota Ʀ_{\mathbb{C}}^{+}, Z_{\mathbb{C}}^{+} + \iota Ʀ_{\mathbb{C}}^{+}, \\ Ƚ_{\mathbb{C}}^{+} + \iota Ᵽ^{+}, Ƚ_{\mathbb{C}}^{-} + \iota Ᵽ_{\mathbb{C}}^{-} \end{pmatrix}\right) \in D_1\left(\underline{đ}_{\mathfrak{l}}\right) \text{ and} \\ \left(\left(\omega_m, \mathfrak{h}_{\mathfrak{l}}^{m^2}\right), \begin{pmatrix} Z_{\mathbb{D}}^{+} + \iota Ʀ_{\mathbb{D}}^{+}, Z_{\mathbb{D}}^{+} + \iota Ʀ_{\mathbb{D}}^{+}, \\ Ƚ_{\mathbb{D}}^{+} + \iota Ᵽ_{\mathbb{D}}^{+}, Ƚ_{\mathbb{D}}^{-} + \iota Ᵽ_{\mathbb{D}}^{-} \end{pmatrix}\right) \in D_2\left(\underline{đ}_{\mathfrak{l}}\right), \\ \mathbb{C}, \mathbb{D} \text{ are BCIFSs on } \mathfrak{U}_1\left(\underline{đ}_{\mathfrak{l}}\right) \text{ and } \mathfrak{U}_2\left(\underline{đ}_{\mathfrak{l}}\right) \text{ respectively.} \end{array}\right) \end{array}\right) & \text{if } \underline{đ}_{\mathfrak{l}} \in D_1 \cup D_2 \end{cases} \tag{18}$$

TABLE 8. The top weak BCIF complement of BCIF6-SS of example 1.

$(\text{Ƙ}^c, \mathfrak{B}^\tau)$	$đ_1$	$đ_2$	$đ_3$	$đ_4$
$S_{\mathfrak{uz}-1}$	$(0, (0.24+\iota 0.1, -0.8-\iota 0.75, 0.75+\iota 0.87, -0.1-\iota 0.12))$	$(5, (0.28+\iota 0.47, -0.27-\iota 0.15, 0.5+\iota 0.3, -0.48-\iota 0.52))$	$(5, (0.46+\iota 0.53, -0.26-\iota 0.12, 0.33+\iota 0.36, -0.47-\iota 0.67))$	$(5, (0.42+\iota 0.56, -0.27-\iota 0.16, 0.38+\iota 0.43, -0.53-\iota 0.59))$
$S_{\mathfrak{uz}-2}$	$(5, (0.5+\iota 0.61, -0.45-\iota 0.33, 0.34+\iota 0.39, -0.55-\iota 0.49))$	$(5, (0.71+\iota 0.68, -0.11-\iota 0.21, 0.23+\iota 0.18, -0.7-\iota 0.61))$	$(5, (0.42+\iota 0.32, -0.59-\iota 0.57, 0.48+\iota 0.52, -0.43-\iota 0.34))$	$(5, (0.2+\iota 0.1, -0.53-\iota 0.72, 0.67+\iota 0.74, -0.23-\iota 0.19))$
$S_{\mathfrak{uz}-3}$	$(5, (0.15+\iota 0.39, -0.39-\iota 0.63, 0.73+\iota 0.68, -0.21-\iota 0.18))$	$(5, (0.83+\iota 0.69, -0.08-\iota 0.15, 0.1+\iota 0.05, -0.9-\iota 0.8))$	$(0, (0.17+\iota 0.12, -0.82-\iota 0.76, 0.8+\iota 0.87, -0.1-\iota 0.14))$	$(5, (0.68+\iota 0.82, -0.16-\iota 0.05, 0.1+\iota 0.09, -0.77-\iota 0.83))$
$S_{\mathfrak{uz}-4}$	$(5, (0.49+\iota 0.5, -0.27-\iota 0.62, 0.37+\iota 0.44, -0.41-\iota 0.31))$	$(5, (0.45+\iota 0.37, -0.36-\iota 0.23, 0.45+\iota 0.5, -0.4-\iota 0.3))$	$(5, (0.15+\iota 0.35, -0.33-\iota 0.51, 0.3+\iota 0.4, -0.6-\iota 0.48))$	$(5, (0.44+\iota 0.33, -0.31-\iota 0.12, 0.33+\iota 0.44, -0.49-\iota 0.59))$

TABLE 9. The bottom weak complement of BCIF6-SS of example 1.

$(\text{Ƙ}, \mathfrak{B}^\beta)$	$đ_1$	$đ_2$	$đ_3$	$đ_4$
$S_{\mathfrak{uz}-1}$	$(0, (0.75+\iota 0.87, -0.1-\iota 0.12, 0.24+\iota 0.1, -0.8-\iota 0.75))$	$(0, (0.5+\iota 0.3, -0.48-\iota 0.52, 0.28+\iota 0.47, -0.27-\iota 0.15))$	$(0, (0.33+\iota 0.36, -0.47-\iota 0.67, 0.46+\iota 0.53, -0.26-\iota 0.12))$	$(0, (0.38+\iota 0.43, -0.53-\iota 0.59, 0.42+\iota 0.56, -0.27-\iota 0.16))$
$S_{\mathfrak{uz}-2}$	$(0, (0.34+\iota 0.39, -0.55-\iota 0.49, 0.5+\iota 0.61, -0.45-\iota 0.33))$	$(0, (0.23+\iota 0.18, -0.7-\iota 0.61, 0.71+\iota 0.68, -0.11-\iota 0.21))$	$(0, (0.48+\iota 0.52, -0.43-\iota 0.34, 0.42+\iota 0.32, -0.59-\iota 0.57))$	$(0, (0.67+\iota 0.74, -0.23-\iota 0.19, 0.2+\iota 0.1, -0.53-\iota 0.72))$
$S_{\mathfrak{uz}-3}$	$(0, (0.73+\iota 0.68, -0.21-\iota 0.18, 0.15+\iota 0.39, -0.39-\iota 0.63))$	$(5, (0.1+\iota 0.05, -0.9-\iota 0.8, 0.83+\iota 0.69, -0.08-\iota 0.15))$	$(0, (0.8+\iota 0.87, -0.1-\iota 0.14, 0.17+\iota 0.12, -0.82-\iota 0.76))$	$(5, (0.1+\iota 0.09, -0.77-\iota 0.83, 0.68+\iota 0.82, -0.16-\iota 0.05))$
$S_{\mathfrak{uz}-4}$	$(0, (0.37+\iota 0.44, -0.41-\iota 0.31, 0.49+\iota 0.5, -0.27-\iota 0.62))$	$(0, (0.45+\iota 0.5, -0.4-\iota 0.3, 0.45+\iota 0.37, -0.36-\iota 0.23))$	$(0, (0.3+\iota 0.4, -0.6-\iota 0.48, 0.15+\iota 0.35, -0.33-\iota 0.51))$	$(0, (0.33+\iota 0.44, -0.49-\iota 0.59, 0.44+\iota 0.33, -0.31-\iota 0.12))$

$(\text{Ƙ}_2, (\mathfrak{U}_2, D_2, 5))$ interpreted in Table 12. Then Table 14 revealed their extended union.

Definition 19: For two BCIFN-SSs $(\text{Ƙ}_1, \mathfrak{B}_1) = (\text{Ƙ}_1, (\mathfrak{U}_1, D_1, \aleph_1))$ and $(\text{Ƙ}_2, \mathfrak{B}_2) = (\text{Ƙ}_2, (\mathfrak{U}_2, D_2, \aleph_2))$, their restricted intersection would be implied as

$$(\text{Ƙ}_1, \mathfrak{B}_1) \cap_{\mathbb{R}} (\text{Ƙ}_2, \mathfrak{B}_2) = (\text{Ƙ}_1, (\mathfrak{U}_1, D_1, \aleph_1)) \cap_{\mathbb{R}} (\text{Ƙ}_2, (\mathfrak{U}_2, D_2, \aleph_2)) = (\eta, \mathfrak{B}_1 \cap_{\mathcal{R}} \mathfrak{B}_2, \min(\aleph_1, \aleph_2)) \quad (19)$$

where $\mathfrak{B}_1 \cap_{\mathcal{R}} \mathfrak{B}_2 = (\xi, D_1 \cap D_2, \min(\aleph_1, \aleph_2))$, that is $\forall\, đ_ſ \in D_1 \cap D_2$, $\omega_m \in \mathfrak{C}$, $((\omega_m, \mathfrak{h}_ſ^{m}), Z^+ + \iota\, \text{Ʀ}^+, Z^- + \iota\, \text{Ʀ}^-, \text{Ł}^+ + \iota\, \text{Ᵽ}^+, \text{Ł}^- + \iota\, \text{Ᵽ}^-) \in \zeta(đ_ſ) \iff \mathfrak{h}_ſ^{m} = \min(\mathfrak{h}_ſ^{m^1}, \mathfrak{h}_ſ^{m^2})$, $Z^+ = \min(Z_{\mathbb{C}}^+, Z_{\mathbb{D}}^+)$, $\text{Ʀ}^+ = \min(\text{Ʀ}_{\mathbb{C}}^+, \text{Ʀ}_{\mathbb{D}}^+)$, $Z^- = \max(Z_{\mathbb{C}}^-, Z_{\mathbb{D}}^-)$, $\text{Ʀ}^- = \max(\text{Ʀ}_{\mathbb{C}}^-, \text{Ʀ}_{\mathbb{D}}^-)$, $\text{Ł}^+ = \max(\text{Ł}^+, \text{Ł}_{\mathbb{D}}^+)$, $\text{Ᵽ}^+ = \max(\text{Ᵽ}^+, \text{Ᵽ}_{\mathbb{D}}^+)$, $\text{Ł}^- = \min(\text{Ł}^-, \text{Ł}_{\mathbb{D}}^-)$, $\text{Ᵽ}^- = \min(\text{Ᵽ}^-, \text{Ᵽ}_{\mathbb{D}}^-)$ if $((\omega_m, \mathfrak{h}_ſ^{m^1}), Z_{\mathbb{C}}^+ + \iota\, \text{Ʀ}_{\mathbb{C}}^+, Z_{\mathbb{C}}^+ + \iota\, \text{Ʀ}_{\mathbb{C}}^+, \text{Ł}_{\mathbb{C}}^+ + \iota\, \text{Ᵽ}^+, \text{Ł}_{\mathbb{C}}^- + \iota\, \text{Ᵽ}_{\mathbb{C}}^-) \in D_1(đ_ſ)$ and $((\omega_m, \mathfrak{h}_ſ^{m^2}), Z_{\mathbb{D}}^+ + \iota\, \text{Ʀ}_{\mathbb{D}}^+, Z_{\mathbb{D}}^+ + \iota\, \text{Ʀ}_{\mathbb{D}}^+, \text{Ł}_{\mathbb{D}}^+ + \iota\, \text{Ᵽ}_{\mathbb{D}}^+, \text{Ł}_{\mathbb{D}}^- + \iota\, \text{Ᵽ}_{\mathbb{D}}^-) \in D_2(đ_ſ)$, $\mathbb{C}$, $\mathbb{D}$ are BCIFSs on $\mathfrak{U}_1(đ_ſ)$ and $\mathfrak{U}_2(đ_ſ)$ respectively.

Example 11: Take two BCIFN-SSs that is $(\text{Ƙ}_1, \mathfrak{B}_1) = (\text{Ƙ}_1, (\mathfrak{U}_1, D_1, 6))$ interpreted in Table 11 and $(\text{Ƙ}_2, \mathfrak{B}_2) = (\text{Ƙ}_2, (\mathfrak{U}_2, D_2, 5))$ interpreted in Table 12. Then Table 15 revealed their restricted intersection.

Definition 20: For two BCIFN-SSs $(\text{Ƙ}_1, \mathfrak{B}_1) = (\text{Ƙ}_1, (\mathfrak{U}_1, D_1, \aleph_1))$ and $(\text{Ƙ}_2, \mathfrak{B}_2) = (\text{Ƙ}_2, (\mathfrak{U}_2, D_2, \aleph_2))$, their extended intersection would be implied as (20), shown at the bottom of page 12.

$$(\text{Ƙ}_1, \mathfrak{B}_1) \cap_{\mathbb{E}} (\text{Ƙ}_2, \mathfrak{B}_2) = (\text{Ƙ}_1, (\mathfrak{U}_1, D_1, \aleph_1)) \cap_{\mathbb{E}} (\text{Ƙ}_2, (\mathfrak{U}_2, D_2, \aleph_2)) = (\theta, \mathfrak{B}_1 \cap_{\mathcal{E}} \mathfrak{B}_2, \max(\aleph_1, \aleph_2))$$

where $\mathfrak{B}_1 \cap_{\mathcal{E}} \mathfrak{B}_2 = (\vartheta, D_1 \cup D_2, \max(\aleph_1, \aleph_2))$, and

TABLE 10. The bottom weak BCIF complement of BCIF6-SS of example 1.

$(\mathfrak{K}^c, \mathfrak{B}^{\beta})$	$\mathfrak{d}_1$	$\mathfrak{d}_2$	$\mathfrak{d}_3$	$\mathfrak{d}_4$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-1}$	$(0, (0.24+\iota 0.1, -0.8-\iota 0.75, 0.75+\iota 0.87, -0.1-\iota 0.12))$	$(0, (0.28+\iota 0.47, -0.27-\iota 0.15, 0.5+\iota 0.3, -0.48-\iota 0.52))$	$(0, (0.46+\iota 0.53, -0.26-\iota 0.12, 0.33+\iota 0.36, -0.47-\iota 0.67))$	$(0, (0.42+\iota 0.56, -0.27-\iota 0.16, 0.38+\iota 0.43, -0.53-\iota 0.59))$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-2}$	$(0, (0.5+\iota 0.61, -0.45-\iota 0.33, 0.34+\iota 0.39, -0.55-\iota 0.49))$	$(0, (0.71+\iota 0.68, -0.11-\iota 0.21, 0.23+\iota 0.18, -0.7-\iota 0.61))$	$(0, (0.42+\iota 0.32, -0.59-\iota 0.57, 0.48+\iota 0.52, -0.43-\iota 0.34))$	$(0, (0.2+\iota 0.1, -0.53-\iota 0.72, 0.67+\iota 0.74, -0.23-\iota 0.19))$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-3}$	$(0, (0.15+\iota 0.39, -0.39-\iota 0.63, 0.73+\iota 0.68, -0.21-\iota 0.18))$	$(5, 0.83+\iota 0.69, -0.08-\iota 0.15, 0.1+\iota 0.05, -0.9-\iota 0.8)$	$(0, (0.17+\iota 0.12, -0.82-\iota 0.76, 0.8+\iota 0.87, -0.1-\iota 0.14))$	$(5, (0.68+\iota 0.82, -0.16-\iota 0.05, 0.1+\iota 0.09, -0.77-\iota 0.83))$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-4}$	$(0, (0.49+\iota 0.5, -0.27-\iota 0.62, 0.37+\iota 0.44, -0.41-\iota 0.31))$	$(0, (0.45+\iota 0.37, -0.36-\iota 0.23, 0.45+\iota 0.5, -0.4-\iota 0.3))$	$(0, (0.15+\iota 0.35, -0.33-\iota 0.51, 0.3+\iota 0.4, -0.6-\iota 0.48))$	$(0, (0.44+\iota 0.33, -0.31-\iota 0.12, 0.33+\iota 0.44, -0.49-\iota 0.59))$

TABLE 11. The tabular exhibition of BCIF6-SS.

$(\mathfrak{K}_1, (\mathfrak{U}_1, D_1, 6))$	$\mathfrak{d}_1$	$\mathfrak{d}_2$	$\mathfrak{d}_3$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-1}$	$(5, (0.78+\iota 0.81, -0.15-\iota 0.11, 0.2+\iota 0.14, -0.55-\iota 0.75))$	$(1, (0.27+\iota 0.22, -0.73-\iota 0.66, 0.4+\iota 0.61, -0.14-\iota 0.19,))$	$(0, (0.08+\iota 0.11, -0.89-\iota 0.82, 0.75+\iota 0.62, -0.05-\iota 0.1,))$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-2}$	$(2, (0.33+\iota 0.39, -0.49-\iota 0.49, 0.53+\iota 0.43, -0.15-\iota 0.32,))$	$(3, (0.47+\iota 0.58, -0.41-\iota 0.35, 0.31+\iota 0.27, -0.39-\iota 0.18,))$	$(4, (0.73+\iota 0.69, -0.19-\iota 0.21, 0.16+\iota 0.25, -0.35-\iota 0.61,))$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-3}$	$(3, (0.53+\iota 0.54, -0.33-\iota 0.34, 0.24+\iota 0.24, -0.33-\iota 0.34,))$	$(2, (0.41+\iota 0.43, -0.53-\iota 0.49, 0.18+\iota 0.19, -0.27-\iota 0.23,))$	$(5, (0.8+\iota 0.81, -0.1-\iota 0.09, 0.1+\iota 0.09, -0.54-\iota 0.83,))$

TABLE 12. The tabular exhibition of BCIF6-SS.

$(\mathfrak{K}_2, (\mathfrak{U}_2, D_2, 5))$	$\mathfrak{d}_1$	$\mathfrak{d}_2$	$\mathfrak{d}_4$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-1}$	$(3, (0.79+\iota 0.65, -0.36-\iota 0.29, 0.15+\iota 0.22, -0.43-\iota 0.63))$	$(2, (0.57+\iota 0.47, -0.47-\iota 0.53, 0.35+\iota 0.18, -0.13-\iota 0.21))$	$(0, (0.15+\iota 0.17, -0.81-\iota 0.85, 0.71+\iota 0.64, -0.13-\iota 0.11))$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-2}$	$(4, (0.86+\iota 0.83, -0.19-\iota 0.14, 0.11+\iota 0.14, -0.81-\iota 0.83))$	$(3, (0.77+\iota 0.67, -0.32-\iota 0.23, 0.21+\iota 0.31, -0.53-\iota 0.52))$	$(2, (0.41+\iota 0.51, -0.42-\iota 0.52, 0.31+\iota 0.32, -0.34-\iota 0.33))$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-3}$	$(1, (0.25+\iota 0.29, -0.7-\iota 0.63, 0.59+\iota 0.48, -0.27-\iota 0.17))$	$(3, (0.76+\iota 0.77, -0.31-\iota 0.21, 0.16+\iota 0.17, -0.41-\iota 0.61))$	$(4, (0.9+\iota 0.87, -0.16-\iota 0.11, 0.05+\iota 0.09, -0.43-\iota 0.67))$

Example 12: Take two BCIFN-SSs that is $(\mathfrak{K}_1, \mathfrak{B}_1) = (\mathfrak{K}_1, (\mathfrak{U}_1, D_1, 6))$ interpreted in Table 11 and $(\mathfrak{K}_2, \mathfrak{B}_2) = (\mathfrak{K}_2, (\mathfrak{U}_2, D_2, 5))$ interpreted in Table 12. Then Table 16 revealed their extended intersection.

Definition 21: Assume a BCIFN-SS $(\mathfrak{K}, \mathfrak{B}) = (\mathfrak{K}, (\mathfrak{U}, D, \aleph))$ and $0 < \varpi < \aleph$ as a threshold, then a BCIF soft set (BCIFSS) associated with $(\mathfrak{K}, \mathfrak{B})$ and ϖ, implied by $(\mathfrak{K}^{\varpi}, D)$ and as (21), shown at the bottom of page 12.

Example 13: Take a BCIF6-SS $(\mathfrak{K}, \mathfrak{B}) = (\mathfrak{K}, (\mathfrak{U}, D, 6))$ interpreted in example 1 and $0 < \varpi < 5$ as a threshold, then Tables from 17 to 21 would exhibit the associated BCIFSSs with BCIF6-SS.

TABLE 13. The restricted union of BCIF6-SS (revealed in Table 11) and BCIF5-SS (revealed in Table 12).

$(\zeta, \mathfrak{B}_1 \cup_{\mathcal{R}} \mathfrak{B}_2, 6)$	$\underline{d}_1$	$\underline{d}_2$
$\mathcal{S}_{\mathfrak{AZ}-1}$	$\left(5, \begin{pmatrix} 0.79 + \iota\, 0.81, \\ -0.36 - \iota\, 0.29, \\ 0.15 + \iota\, 0.14, \\ -0.43 - \iota\, 0.63 \end{pmatrix}\right)$	$\left(2, \begin{pmatrix} 0.57 + \iota\, 0.47, \\ -0.73 - \iota\, 0.66, \\ 0.35 + \iota\, 0.18, \\ -0.13 - \iota\, 0.19, \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{AZ}-2}$	$\left(4, \begin{pmatrix} 0.86 + \iota\, 0.83, \\ -0.49 - \iota\, 0.49, \\ 0.11 + \iota\, 0.14, \\ -0.15 - \iota\, 0.32 \end{pmatrix}\right)$	$\left(3, \begin{pmatrix} 0.77 + \iota\, 0.67, \\ -0.41 - \iota\, 0.35, \\ 0.21 + \iota\, 0.27, \\ -0.39 - \iota\, 0.18, \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{AZ}-3}$	$\left(3, \begin{pmatrix} 0.53 + \iota\, 0.54, \\ -0.7 - \iota\, 0.63, \\ 0.24 + \iota\, 0.24, \\ -0.27 - \iota\, 0.17, \end{pmatrix}\right)$	$\left(3, \begin{pmatrix} 0.76 + \iota\, 0.77, \\ -0.53 - \iota\, 0.49, \\ 0.16 + \iota\, 0.17, \\ -0.27 - \iota\, 0.23, \end{pmatrix}\right)$

TABLE 14. The extended union of BCIF6-SS (revealed in Table 11) and BCIF5-SS (revealed in Table 12).

$\begin{pmatrix} \varsigma, \\ \mathfrak{B}_1 \cup_{\mathcal{E}} \mathfrak{B}_2, \\ 6 \end{pmatrix}$	$\underline{d}_1$	$\underline{d}_2$	$\underline{d}_3$	$\underline{d}_4$
$\mathcal{S}_{\mathfrak{AZ}-1}$	$\left(5, \begin{pmatrix} 0.79 + \iota\, 0.81, \\ -0.36 - \iota\, 0.29, \\ 0.15 + \iota\, 0.14, \\ -0.43 - \iota\, 0.63 \end{pmatrix}\right)$	$\left(2, \begin{pmatrix} 0.57 + \iota\, 0.47, \\ -0.73 - \iota\, 0.66, \\ 0.35 + \iota\, 0.18, \\ -0.13 - \iota\, 0.19, \end{pmatrix}\right)$	$\left(0, \begin{pmatrix} 0.08 + \iota\, 0.11, \\ -0.89 - \iota\, 0.82, \\ 0.75 + \iota\, 0.62, \\ -0.05 - \iota\, 0.1, \end{pmatrix}\right)$	$\left(0, \begin{pmatrix} 0.15 + \iota\, 0.17, \\ -0.81 - \iota\, 0.85, \\ 0.71 + \iota\, 0.64, \\ -0.13 - \iota\, 0.11 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{AZ}-2}$	$\left(4, \begin{pmatrix} 0.86 + \iota\, 0.83, \\ -0.49 - \iota\, 0.49, \\ 0.11 + \iota\, 0.14, \\ -0.15 - \iota\, 0.32 \end{pmatrix}\right)$	$\left(3, \begin{pmatrix} 0.77 + \iota\, 0.67, \\ -0.41 - \iota\, 0.35, \\ 0.21 + \iota\, 0.27, \\ -0.39 - \iota\, 0.18, \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.73 + \iota\, 0.69, \\ -0.19 - \iota\, 0.21, \\ 0.16 + \iota\, 0.25, \\ -0.35 - \iota\, 0.61, \end{pmatrix}\right)$	$\left(2, \begin{pmatrix} 0.41 + \iota\, 0.51, \\ -0.42 - \iota\, 0.52, \\ 0.31 + \iota\, 0.32, \\ -0.34 - \iota\, 0.33 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{AZ}-3}$	$\left(3, \begin{pmatrix} 0.53 + \iota\, 0.54, \\ -0.7 - \iota\, 0.63, \\ 0.24 + \iota\, 0.24, \\ -0.27 - \iota\, 0.17, \end{pmatrix}\right)$	$\left(3, \begin{pmatrix} 0.76 + \iota\, 0.77, \\ -0.53 - \iota\, 0.49, \\ 0.16 + \iota\, 0.17, \\ -0.27 - \iota\, 0.23, \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.8 + \iota\, 0.81, \\ -0.1 - \iota\, 0.09, \\ 0.1 + \iota\, 0.09, \\ -0.54 - \iota\, 0.83, \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.9 + \iota\, 0.87, \\ -0.16 - \iota\, 0.11, \\ 0.05 + \iota\, 0.09, \\ -0.43 - \iota\, 0.67 \end{pmatrix}\right)$

TABLE 15. The restricted intersection of BCIF6-SS (revealed in Table 11) and BCIF5-SS (revealed in Table 12).

$(\eta, \mathfrak{B}_1 \cap_{\mathcal{R}} \mathfrak{B}_2, 5)$	$\underline{d}_1$	$\underline{d}_2$
$\mathcal{S}_{\mathfrak{AZ}-1}$	$\left(3, \begin{pmatrix} 0.78 + \iota\, 0.65, \\ -0.15 - \iota\, 0.11, \\ 0.2 + \iota\, 0.22, \\ -0.55 - \iota\, 0.75 \end{pmatrix}\right)$	$\left(1, \begin{pmatrix} 0.27 + \iota\, 0.22, \\ -0.47 - \iota\, 0.53, \\ 0.4 + \iota\, 0.61, \\ -0.14 - \iota\, 0.21, \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{AZ}-2}$	$\left(2, \begin{pmatrix} 0.33 + \iota\, 0.39, \\ -0.19 - \iota\, 0.14, \\ 0.53 + \iota\, 0.43, \\ -0.81 - \iota\, 0.83, \end{pmatrix}\right)$	$\left(3, \begin{pmatrix} 0.47 + \iota\, 0.58, \\ -0.32 - \iota\, 0.23, \\ 0.31 + \iota\, 0.31, \\ -0.53 - \iota\, 0.52, \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{AZ}-3}$	$\left(1, \begin{pmatrix} 0.25 + \iota\, 0.29, \\ -0.33 - \iota\, 0.34, \\ 0.59 + \iota\, 0.48, \\ -0.33 - \iota\, 0.34, \end{pmatrix}\right)$	$\left(2, \begin{pmatrix} 0.41 + \iota\, 0.43, \\ -0.31 - \iota\, 0.21, \\ 0.18 + \iota\, 0.19, \\ -0.41 - \iota\, 0.61, \end{pmatrix}\right)$

Some elementary operational laws for BCIFN-SNs are devised as:

Definition 22: Take two BCIFN-SNs that is $\mathfrak{V}_{m\mathfrak{l}} = (\mathfrak{h}_{\mathfrak{z}\mathfrak{l}}^{m}, Z_{m\mathfrak{l}}^{+} + \iota\, R_{m\mathfrak{l}}^{+}, Z_{m\mathfrak{l}}^{-} + \iota\, R_{m\mathfrak{l}}^{-}, Ł_{m\mathfrak{l}}^{+} + \iota\, P_{m\mathfrak{l}}^{+}, Ł_{m\mathfrak{l}}^{-} + \iota\, P_{m\mathfrak{l}}^{-})$ and $\mathfrak{V}_{s\mathfrak{l}} = (\mathfrak{h}_{\mathfrak{z}\mathfrak{l}}^{s}, Z_{s\mathfrak{l}}^{+} + \iota\, R_{s\mathfrak{l}}^{+}, Z_{s\mathfrak{l}}^{-} + \iota\, R_{s\mathfrak{l}}^{-}, Ł_{s\mathfrak{l}}^{+} + \iota\, P_{s\mathfrak{l}}^{+}, Ł_{s\mathfrak{l}}^{-} + \iota\, P_{s\mathfrak{l}}^{-})$ and $\nu > 0$, we have as shown at the bottom of the next page.

Theorem 2: Assume two BCFNs that is $\mathfrak{V}_{m\mathfrak{l}} = (\mathfrak{h}_{\mathfrak{z}\mathfrak{l}}^{m}, Z_{m\mathfrak{l}}^{+} + \iota\, R_{m\mathfrak{l}}^{+}, Z_{m\mathfrak{l}}^{-} + \iota\, R_{m\mathfrak{l}}^{-}, Ł_{m\mathfrak{l}}^{+} + \iota\, P_{m\mathfrak{l}}^{+}, Ł_{m\mathfrak{l}}^{-} + \iota\, P_{m\mathfrak{l}}^{-})$ and $\mathfrak{V}_{s\mathfrak{l}} = (\mathfrak{h}_{\mathfrak{z}\mathfrak{l}}^{s}, Z_{s\mathfrak{l}}^{+} + \iota\, R_{s\mathfrak{l}}^{+}, Z_{s\mathfrak{l}}^{-} + \iota\, R_{s\mathfrak{l}}^{-}, Ł_{s\mathfrak{l}}^{+} + \iota\, P_{s\mathfrak{l}}^{+}, Ł_{s\mathfrak{l}}^{-} + \iota\, P_{s\mathfrak{l}}^{-})$ and $\nu,\ \nu_1,\ \nu_2 > 0$ then,

5) $\mathfrak{V}_{m\mathfrak{l}} \oplus \mathfrak{V}_{s\mathfrak{l}} = \mathfrak{V}_{s\mathfrak{l}} \oplus \mathfrak{V}_{m\mathfrak{l}}$
6) $\mathfrak{V}_{m\mathfrak{l}} \otimes \mathfrak{V}_{s\mathfrak{l}} = \mathfrak{V}_{s\mathfrak{l}} \otimes \mathfrak{V}_{m\mathfrak{l}}$
7) $\nu \mathfrak{V}_{m\mathfrak{l}} \oplus \nu \mathfrak{V}_{s\mathfrak{l}} = \nu \left(\mathfrak{V}_{m\mathfrak{l}} \oplus \mathfrak{V}_{s\mathfrak{l}}\right)$
8) $\mathfrak{V}_{m\mathfrak{l}}^{\nu} \otimes \mathfrak{V}_{s\mathfrak{l}}^{\nu} = \left(\mathfrak{V}_{m\mathfrak{l}} \otimes \mathfrak{V}_{s\mathfrak{l}}\right)^{\nu}$
9) $\nu_1 \mathfrak{V}_{m\mathfrak{l}} \oplus \nu_2 \mathfrak{V}_{m\mathfrak{l}} = (\nu_1 + \nu_2)\, \mathfrak{V}_{m\mathfrak{l}}$
10) $\mathfrak{V}_{m\mathfrak{l}}^{\nu_1} \otimes \mathfrak{V}_{m\mathfrak{l}}^{\nu_2} = \mathfrak{V}_{m\mathfrak{l}}^{\nu_1 + \nu_2}$.

TABLE 16. The extended intersection of BCIF6-SS (revealed in Table 11) and BCIF5-SS (revealed in Table 12).

$\left(\mathfrak{B}_1 \cap_{\varepsilon} \mathfrak{B}_2, \eta, 6\right)$	$ꝱ_1$	$ꝱ_2$	$ꝱ_3$	$ꝱ_4$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-1}$	$\left(3, \begin{pmatrix} 0.78+\iota 0.65, \\ -0.15-\iota 0.11, \\ 0.2+\iota 0.22, \\ -0.55-\iota 0.75 \end{pmatrix}\right)$	$\left(1, \begin{pmatrix} 0.27+\iota 0.22, \\ -0.47-\iota 0.53, \\ 0.4+\iota 0.61, \\ -0.14-\iota 0.21, \end{pmatrix}\right)$	$\left(0, \begin{pmatrix} 0.08+\iota 0.11, \\ -0.89-\iota 0.82, \\ 0.75+\iota 0.62, \\ -0.05-\iota 0.1, \end{pmatrix}\right)$	$\left(0, \begin{pmatrix} 0.15+\iota 0.17, \\ -0.81-\iota 0.85, \\ 0.71+\iota 0.64, \\ -0.13-\iota 0.11 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-2}$	$\left(2, \begin{pmatrix} 0.33+\iota 0.39, \\ -0.19-\iota 0.14, \\ 0.53+\iota 0.43, \\ -0.81-\iota 0.83, \end{pmatrix}\right)$	$\left(3, \begin{pmatrix} 0.47+\iota 0.58, \\ -0.32-\iota 0.23, \\ 0.31+\iota 0.31, \\ -0.53-\iota 0.52, \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.73+\iota 0.69, \\ -0.19-\iota 0.21, \\ 0.16+\iota 0.25, \\ -0.35-\iota 0.61, \end{pmatrix}\right)$	$\left(2, \begin{pmatrix} 0.41+\iota 0.51, \\ -0.42-\iota 0.52, \\ 0.31+\iota 0.32, \\ -0.34-\iota 0.33 \end{pmatrix}\right)$
$\mathcal{S}_{\mathfrak{U}\mathfrak{Z}-3}$	$\left(1, \begin{pmatrix} 0.25+\iota 0.29, \\ -0.33-\iota 0.34, \\ 0.59+\iota 0.48, \\ -0.33-\iota 0.34, \end{pmatrix}\right)$	$\left(2, \begin{pmatrix} 0.41+\iota 0.43, \\ -0.31-\iota 0.21, \\ 0.18+\iota 0.19, \\ -0.41-\iota 0.61, \end{pmatrix}\right)$	$\left(5, \begin{pmatrix} 0.8+\iota 0.81, \\ -0.1-\iota 0.09, \\ 0.1+\iota 0.09, \\ -0.54-\iota 0.83, \end{pmatrix}\right)$	$\left(4, \begin{pmatrix} 0.9+\iota 0.87, \\ -0.16-\iota 0.11, \\ 0.05+\iota 0.09, \\ -0.43-\iota 0.67 \end{pmatrix}\right)$

$$\vartheta\left(\underline{ꝱ}_{\mathfrak{l}}\right)=\begin{cases} Ꝃ_1\left(\underline{ꝱ}_{\mathfrak{l}}\right), & \text{if } \underline{ꝱ}_{\mathfrak{l}} \in D_1 - D_2 \\ Ꝃ_2\left(\underline{ꝱ}_{\mathfrak{l}}\right), & \text{if } \underline{ꝱ}_{\mathfrak{l}} \in D_1 - D_2 \\ \left(\begin{array}{c} \left(\left(\omega_m, \mathfrak{h}_{\mathfrak{l}}^{m}\right), \begin{pmatrix} Z^{+}+\iota Ꝛ^{+}, Z^{-}+\iota Ꝛ^{-}, \\ Ꜳ^{+}+\iota Ᵽ^{+}, Ꜳ^{-}+\iota Ᵽ^{-} \end{pmatrix}\right) \\ \left(\begin{array}{c} \text{such that } \mathfrak{h}_{\mathfrak{l}}^{m}=\min\left(\mathfrak{h}_{\mathfrak{l}}^{m^1}, \mathfrak{h}_{\mathfrak{l}}^{m^2}\right), Z^{+}=\min\left(Z_{\mathbb{C}}^{+}, Z_{\mathbb{D}}^{+}\right) \\ Ꝛ^{+}=\min\left(Ꝛ_{\mathbb{C}}^{+}, Ꝛ_{\mathbb{D}}^{+}\right), Z^{-}=\max\left(Z_{\mathbb{C}}^{-}, Z_{\mathbb{D}}^{-}\right), Ꝛ^{-}=\max\left(Ꝛ_{\mathbb{C}}^{-}, Ꝛ_{\mathbb{D}}^{-}\right), \\ Ꜳ^{+}=\max\left(Ꜳ^{+}, Ꜳ_{\mathbb{D}}^{+}\right), Ᵽ^{+}=\max\left(Ᵽ^{+}, Ᵽ_{\mathbb{D}}^{+}\right), Ꜳ^{-}=\min\left(Ꜳ^{-}, Ꜳ_{\mathbb{D}}^{-}\right), \\ Ᵽ^{-}=\min\left(Ᵽ^{-}, Ᵽ_{\mathbb{D}}^{-}\right), \\ \text{if } \left(\left(\omega_m, \mathfrak{h}_{\mathfrak{l}}^{m^1}\right), \begin{pmatrix} Z_{\mathbb{C}}^{+}+\iota Ꝛ_{\mathbb{C}}^{+}, Z_{\mathbb{C}}^{+}+\iota Ꝛ_{\mathbb{C}}^{+}, \\ Ꜳ_{\mathbb{C}}^{+}+\iota Ᵽ^{+}, Ꜳ_{\mathbb{C}}^{-}+\iota Ᵽ_{\mathbb{C}}^{-} \end{pmatrix}\right) \in D_1\left(\underline{ꝱ}_{\mathfrak{l}}\right) \text{ and} \\ \left(\left(\omega_m, \mathfrak{h}_{\mathfrak{l}}^{m^2}\right), \begin{pmatrix} Z_{\mathbb{D}}^{+}+\iota Ꝛ_{\mathbb{D}}^{+}, Z_{\mathbb{D}}^{+}+\iota Ꝛ_{\mathbb{D}}^{+}, \\ Ꜳ_{\mathbb{D}}^{+}+\iota Ᵽ_{\mathbb{D}}^{+}, Ꜳ_{\mathbb{D}}^{-}+\iota Ᵽ_{\mathbb{D}}^{-} \end{pmatrix}\right) \in D_2\left(\underline{ꝱ}_{\mathfrak{l}}\right), \\ \mathbb{C}, \mathbb{D} \text{ are BCIFSs on } \mathfrak{U}_1\left(\underline{ꝱ}_{\mathfrak{l}}\right) \text{ and } \mathfrak{U}_2\left(\underline{ꝱ}_{\mathfrak{l}}\right) \text{ respectively.} \end{array}\right) \end{array}\right) & \text{if } \underline{ꝱ}_{\mathfrak{l}} \in D_1 \cup D_2 \end{cases} \tag{20}$$

$$Ꝃ^{\varpi}\left(\underline{ꝱ}_{\mathfrak{l}}\right)=\begin{cases} \begin{pmatrix} Z_{m\mathfrak{l}}^{+}+\iota Ꝛ_{m\mathfrak{l}}^{+}, Z_{m\mathfrak{l}}^{-}+\iota Ꝛ_{m\mathfrak{l}}^{-}, \\ Ꜳ_{m\mathfrak{l}}^{+}+\iota Ᵽ_{m\mathfrak{l}}^{+}, Ꜳ_{m\mathfrak{l}}^{-}+\iota Ᵽ_{m\mathfrak{l}}^{-} \end{pmatrix}, & \text{if } Ꝃ^{\varpi}\left(\underline{ꝱ}_{\mathfrak{l}}\right)=\left(\left(\omega_m, \mathfrak{h}_{\mathfrak{l}}^{m}\right), \begin{pmatrix} Z_{m\mathfrak{l}}^{+}+\iota Ꝛ_{m\mathfrak{l}}^{+}, Z_{m\mathfrak{l}}^{-}+\iota Ꝛ_{m\mathfrak{l}}^{-}, \\ Ꜳ_{m\mathfrak{l}}^{+}+\iota Ᵽ_{m\mathfrak{l}}^{+}, Ꜳ_{m\mathfrak{l}}^{-}+\iota Ᵽ_{m\mathfrak{l}}^{-} \end{pmatrix}\right), \text{ and } \mathfrak{h}_{\mathfrak{l}}^{m} \geq \varpi \\ \begin{pmatrix} 0.0+\iota 0.0, & -1.0-\iota 1.0, \\ 1.0+\iota 1.0, & -0.0-\iota 0.0 \end{pmatrix}, & \text{otherwise} \end{cases} \tag{21}$$

1) $\mathfrak{V}_{m\mathfrak{l}} \oplus \mathfrak{V}_{\mathcal{s}\mathfrak{l}}=\left(\max\left(\mathfrak{h}_{\mathfrak{l}}^{m}, \mathfrak{h}_{\mathfrak{l}}^{\mathcal{s}}\right), \begin{pmatrix} Z_{m\mathfrak{l}}^{+}+Z_{\mathcal{s}\mathfrak{l}}^{+}-Z_{m\mathfrak{l}}^{+}Z_{\mathcal{s}\mathfrak{l}}^{+}+\iota\left(Ꝛ_{m\mathfrak{l}}^{+}+Ꝛ_{\mathcal{s}\mathfrak{l}}^{+}-Ꝛ_{m\mathfrak{l}}^{+}Ꝛ_{\mathcal{s}\mathfrak{l}}^{+}\right), -\left(Z_{m\mathfrak{l}}^{-}Z_{\mathcal{s}\mathfrak{l}}^{-}\right)+\iota\left(-\left(Ꝛ_{m\mathfrak{l}}^{-}Ꝛ_{\mathcal{s}\mathfrak{l}}^{-}\right)\right), \\ Ꜳ_{m\mathfrak{l}}^{+}Ꜳ_{\mathcal{s}\mathfrak{l}}^{+}+\iota Ᵽ_{m\mathfrak{l}}^{+}Ᵽ_{\mathcal{s}\mathfrak{l}}^{+}, Ꜳ_{m\mathfrak{l}}^{-}+Ꜳ_{\mathcal{s}\mathfrak{l}}^{-}+Ꜳ_{m\mathfrak{l}}^{-}Ꜳ_{\mathcal{s}\mathfrak{l}}^{-}+\iota\left(Ᵽ_{m\mathfrak{l}}^{-}+Ᵽ_{\mathcal{s}\mathfrak{l}}^{-}+Ᵽ_{m\mathfrak{l}}^{-}Ᵽ_{\mathcal{s}\mathfrak{l}}^{-}\right) \end{pmatrix}\right)$

2) $\mathfrak{V}_{m\mathfrak{l}} \otimes \mathfrak{V}_{\mathcal{s}\mathfrak{l}}=\left(\min\left(\mathfrak{h}_{\mathfrak{l}}^{m}, \mathfrak{h}_{\mathfrak{l}}^{\mathcal{s}}\right), \begin{pmatrix} Z_{m\mathfrak{l}}^{+}Z_{\mathcal{s}\mathfrak{l}}^{+}+\iota Ꝛ_{m\mathfrak{l}}^{+}Ꝛ_{\mathcal{s}\mathfrak{l}}^{+}, Z_{m\mathfrak{l}}^{-}+Z_{\mathcal{s}\mathfrak{l}}^{-}+Z_{m\mathfrak{l}}^{-}Z_{\mathcal{s}\mathfrak{l}}^{-}+\iota\left(Ꝛ_{m\mathfrak{l}}^{-}+Ꝛ_{\mathcal{s}\mathfrak{l}}^{-}+Ꝛ_{m\mathfrak{l}}^{-}Ꝛ_{\mathcal{s}\mathfrak{l}}^{-}\right) \\ Ꜳ_{m\mathfrak{l}}^{+}+Ꜳ_{\mathcal{s}\mathfrak{l}}^{+}-Ꜳ_{m\mathfrak{l}}^{+}Ꜳ_{\mathcal{s}\mathfrak{l}}^{+}+\iota\left(Ᵽ_{m\mathfrak{l}}^{+}+Ᵽ_{\mathcal{s}\mathfrak{l}}^{+}-Ᵽ_{m\mathfrak{l}}^{+}Ᵽ_{\mathcal{s}\mathfrak{l}}^{+}\right), -\left(Ꜳ_{m\mathfrak{l}}^{-}Ꜳ_{\mathcal{s}\mathfrak{l}}^{-}\right)+\iota\left(-\left(Ᵽ_{m\mathfrak{l}}^{-}Ᵽ_{\mathcal{s}\mathfrak{l}}^{-}\right)\right) \end{pmatrix}\right)$

3) $\nu\mathfrak{V}_{m\mathfrak{l}}=\left(\mathfrak{h}_{\mathfrak{l}}^{m}, \begin{pmatrix} 1-\left(1-Z_{m\mathfrak{l}}^{+}\right)^{\nu}+\iota\left(1-\left(1-Ꝛ_{m\mathfrak{l}}^{+}\right)^{\nu}\right), -\left|Z_{m\mathfrak{l}}^{-}\right|^{\nu}+\iota\left(-\left|Ꝛ_{m\mathfrak{l}}^{-}\right|^{\nu}\right), \\ \left(Ꜳ_{m\mathfrak{l}}^{+}\right)^{\nu}+\iota\left(Ᵽ_{m\mathfrak{l}}^{+}\right)^{\nu}, -1+\left(1+Ꜳ_{m\mathfrak{l}}^{-}\right)^{\nu}+\iota\left(-1+\left(1+Ᵽ_{m\mathfrak{l}}^{-}\right)^{\nu}\right) \end{pmatrix}\right)$

4) $\mathfrak{V}_{m\mathfrak{l}}^{\nu}=\left(\mathfrak{h}_{\mathfrak{l}}^{m}, \begin{pmatrix} \left(Z_{m\mathfrak{l}}^{+}\right)^{\nu}+\iota\left(Ꝛ_{m\mathfrak{l}}^{+}\right)^{\nu}, -1+\left(1+Z_{m\mathfrak{l}}^{-}\right)^{\nu}+\iota\left(-1+\left(1+Ꝛ_{m\mathfrak{l}}^{-}\right)^{\nu}\right) \\ 1-\left(1-Ꜳ_{m\mathfrak{l}}^{+}\right)^{\nu}+\iota\left(1-\left(1-Ᵽ_{m\mathfrak{l}}^{+}\right)^{\nu}\right), -\left|Ꜳ_{m\mathfrak{l}}^{-}\right|^{\nu}+\iota\left(-\left|Ᵽ_{m\mathfrak{l}}^{-}\right|^{\nu}\right) \end{pmatrix}\right)$

TABLE 17. BCIFSS is linked with BCIFN-SS and the threshold is 1.

$(\mathfrak{K}^1, \mathrm{D})$	$\mathfrak{d}_1$	$\mathfrak{d}_2$	$\mathfrak{d}_3$	$\mathfrak{d}_4$
$\mathcal{S}_{\mathfrak{az}-1}$	$\begin{pmatrix} 0.75+\iota 0.87, \\ -0.1-\iota 0.12, \\ 0.24+\iota 0.1 \\ -0.8-\iota 0.75 \end{pmatrix}$	$\begin{pmatrix} 0.5+\iota 0.3, \\ -0.48-\iota 0.52, \\ 0.28+\iota 0.47, \\ -0.27-\iota 0.15 \end{pmatrix}$	$\begin{pmatrix} 0.33+\iota 0.36, \\ -0.47-\iota 0.67, \\ 0.46+\iota 0.53, \\ -0.26-\iota 0.12 \end{pmatrix}$	$\begin{pmatrix} 0.38+\iota 0.43, \\ -0.53-\iota 0.59, \\ 0.42+\iota 0.56, \\ -0.27-\iota 0.16 \end{pmatrix}$
$\mathcal{S}_{\mathfrak{az}-2}$	$\begin{pmatrix} 0.34+\iota 0.39, \\ -0.55-\iota 0.49, \\ 0.5+\iota 0.61 \\ -0.45-\iota 0.33 \end{pmatrix}$	$\begin{pmatrix} 0.23+\iota 0.18, \\ -0.7-\iota 0.61, \\ 0.71+\iota 0.68, \\ -0.11-\iota 0.21 \end{pmatrix}$	$\begin{pmatrix} 0.48+\iota 0.52, \\ -0.43-\iota 0.34, \\ 0.42+\iota 0.32, \\ -0.59-\iota 0.57 \end{pmatrix}$	$\begin{pmatrix} 0.67+\iota 0.74, \\ -0.23-\iota 0.19, \\ 0.2+\iota 0.1, \\ -0.53-\iota 0.72 \end{pmatrix}$
$\mathcal{S}_{\mathfrak{az}-3}$	$\begin{pmatrix} 0.73+\iota 0.68, \\ -0.21-\iota 0.18, \\ 0.15+\iota 0.39 \\ -0.39-\iota 0.63 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.8+\iota 0.87, \\ -0.1-\iota 0.14, \\ 0.17+\iota 0.12, \\ -0.82-\iota 0.76 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$
$\mathcal{S}_{\mathfrak{az}-4}$	$\begin{pmatrix} 0.37+\iota 0.44, \\ -0.41-\iota 0.31, \\ 0.49+\iota 0.5, \\ -0.27-\iota 0.62 \end{pmatrix}$	$\begin{pmatrix} 0.45+\iota 0.5, \\ -0.4-\iota 0.3, \\ 0.45+\iota 0.37, \\ -0.36-\iota 0.23 \end{pmatrix}$	$\begin{pmatrix} 0.3+\iota 0.4, \\ -0.6-\iota 0.48, \\ 0.15+\iota 0.35, \\ -0.33-\iota 0.51 \end{pmatrix}$	$\begin{pmatrix} 0.33+\iota 0.44, \\ -0.49-\iota 0.59, \\ 0.44+\iota 0.33, \\ -0.31-\iota 0.12 \end{pmatrix}$

TABLE 18. BCIFSS is linked with BCIFN-SS and the threshold is 2.

$(\mathfrak{K}^2, \mathrm{D})$	$\mathfrak{d}_1$	$\mathfrak{d}_2$	$\mathfrak{d}_3$	$\mathfrak{d}_4$
$\mathcal{S}_{\mathfrak{az}-1}$	$\begin{pmatrix} 0.75+\iota 0.87, \\ -0.1-\iota 0.12, \\ 0.24+\iota 0.1 \\ -0.8-\iota 0.75 \end{pmatrix}$	$\begin{pmatrix} 0.5+\iota 0.3, \\ -0.48-\iota 0.52, \\ 0.28+\iota 0.47, \\ -0.27-\iota 0.15 \end{pmatrix}$	$\begin{pmatrix} 0.33+\iota 0.36, \\ -0.47-\iota 0.67, \\ 0.46+\iota 0.53, \\ -0.26-\iota 0.12 \end{pmatrix}$	$\begin{pmatrix} 0.38+\iota 0.43, \\ -0.53-\iota 0.59, \\ 0.42+\iota 0.56, \\ -0.27-\iota 0.16 \end{pmatrix}$
$\mathcal{S}_{\mathfrak{az}-2}$	$\begin{pmatrix} 0.34+\iota 0.39, \\ -0.55-\iota 0.49, \\ 0.5+\iota 0.61 \\ -0.45-\iota 0.33 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.48+\iota 0.52, \\ -0.43-\iota 0.34, \\ 0.42+\iota 0.32, \\ -0.59-\iota 0.57 \end{pmatrix}$	$\begin{pmatrix} 0.67+\iota 0.74, \\ -0.23-\iota 0.19, \\ 0.2+\iota 0.1, \\ -0.53-\iota 0.72 \end{pmatrix}$
$\mathcal{S}_{\mathfrak{az}-3}$	$\begin{pmatrix} 0.73+\iota 0.68, \\ -0.21-\iota 0.18, \\ 0.15+\iota 0.39 \\ -0.39-\iota 0.63 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.8+\iota 0.87, \\ -0.1-\iota 0.14, \\ 0.17+\iota 0.12, \\ -0.82-\iota 0.76 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$
$\mathcal{S}_{\mathfrak{az}-4}$	$\begin{pmatrix} 0.37+\iota 0.44, \\ -0.41-\iota 0.31, \\ 0.49+\iota 0.5, \\ -0.27-\iota 0.62 \end{pmatrix}$	$\begin{pmatrix} 0.45+\iota 0.5, \\ -0.4-\iota 0.3, \\ 0.45+\iota 0.37, \\ -0.36-\iota 0.23 \end{pmatrix}$	$\begin{pmatrix} 0.3+\iota 0.4, \\ -0.6-\iota 0.48, \\ 0.15+\iota 0.35, \\ -0.33-\iota 0.51 \end{pmatrix}$	$\begin{pmatrix} 0.33+\iota 0.44, \\ -0.49-\iota 0.59, \\ 0.44+\iota 0.33, \\ -0.31-\iota 0.12 \end{pmatrix}$

TABLE 19. BCIFSS is linked with BCIFN-SS and the threshold is 3.

$(\mathfrak{K}^3, \mathrm{D})$	$\mathfrak{d}_1$	$\mathfrak{d}_2$	$\mathfrak{d}_3$	$\mathfrak{d}_4$
$\mathcal{S}_{\mathfrak{az}-1}$	$\begin{pmatrix} 0.75+\iota 0.87, \\ -0.1-\iota 0.12, \\ 0.24+\iota 0.1 \\ -0.8-\iota 0.75 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$
$\mathcal{S}_{\mathfrak{az}-2}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.48+\iota 0.52, \\ -0.43-\iota 0.34, \\ 0.42+\iota 0.32, \\ -0.59-\iota 0.57 \end{pmatrix}$	$\begin{pmatrix} 0.67+\iota 0.74, \\ -0.23-\iota 0.19, \\ 0.2+\iota 0.1, \\ -0.53-\iota 0.72 \end{pmatrix}$
$\mathcal{S}_{\mathfrak{az}-3}$	$\begin{pmatrix} 0.73+\iota 0.68, \\ -0.21-\iota 0.18, \\ 0.15+\iota 0.39 \\ -0.39-\iota 0.63 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.8+\iota 0.87, \\ -0.1-\iota 0.14, \\ 0.17+\iota 0.12, \\ -0.82-\iota 0.76 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$
$\mathcal{S}_{\mathfrak{az}-4}$	$\begin{pmatrix} 0.37+\iota 0.44, \\ -0.41-\iota 0.31, \\ 0.49+\iota 0.5, \\ -0.27-\iota 0.62 \end{pmatrix}$	$\begin{pmatrix} 0.45+\iota 0.5, \\ -0.4-\iota 0.3, \\ 0.45+\iota 0.37, \\ -0.36-\iota 0.23 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$	$\begin{pmatrix} 0.0+\iota 0.0, \\ -1.0-\iota 1.0, \\ 1.0+\iota 1.0, \\ -0.0-\iota 0.0 \end{pmatrix}$

and then by Def (22), we achieve as shown at the bottom of page 15.

4) We have as shown at the bottom of page 15.

and then by Def (22), we achieve as shown at the bottom of page 16.

5) Next, we have as shown at the bottom of page 16.

then, as shown at the bottom of page 17.

By Def. (22), we have as shown at the bottom of page 17.

then, as shown at the bottom of page 18.

IV. TOPSIS APPROACH RELYING ON BCIFN-SS

Here, we would devise an approach of TOPSIS in the setting of BCIFN-SS. The primary goal of this notion is to achieve the most superb alternative in the described alternatives by employing both positive ideal solution (PIS) and negative ideal solution (NIS). Thus, we devise a BCIFN-S TOPSIS approach for tackling BCIFN-S information.

Take the gathering of þ alternatives $\mathcal{G}_{\mathfrak{at}-1}, \mathcal{G}_{\mathfrak{at}-2}, \ldots, \mathcal{G}_{\mathfrak{at}-\text{þ}}$ in which the most superb one would be selected. The

TABLE 20. BCIFSS is linked with BCIFN-SS and the threshold is 4.

$(\mathfrak{K}^{4}, \mathrm{D})$	$\mathfrak{d}_1$	$\mathfrak{d}_2$	$\mathfrak{d}_3$	$\mathfrak{d}_4$
$\mathcal{S}_{\mathfrak{AI}-1}$	$\begin{pmatrix}0.75+\iota\, 0.87,\\ -0.1-\iota\, 0.12,\\ 0.24+\iota\, 0.1\\ -0.8-\iota\, 0.75\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$
$\mathcal{S}_{\mathfrak{AI}-2}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.67+\iota\, 0.74,\\ -0.23-\iota\, 0.19,\\ 0.2+\iota\, 0.1,\\ -0.53-\iota\, 0.72\end{pmatrix}$
$\mathcal{S}_{\mathfrak{AI}-3}$	$\begin{pmatrix}0.73+\iota\, 0.68,\\ -0.21-\iota\, 0.18,\\ 0.15+\iota\, 0.39\\ -0.39-\iota\, 0.63\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.8+\iota\, 0.87,\\ -0.1-\iota\, 0.14,\\ 0.17+\iota\, 0.12,\\ -0.82-\iota\, 0.76\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$
$\mathcal{S}_{\mathfrak{AI}-4}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$

TABLE 21. BCIFSS is linked with BCIFN-SS and the threshold is 5.

$(\mathfrak{K}^{5}, \mathrm{D})$	$\mathfrak{d}_1$	$\mathfrak{d}_2$	$\mathfrak{d}_3$	$\mathfrak{d}_4$
$\mathcal{S}_{\mathfrak{AI}-1}$	$\begin{pmatrix}0.75+\iota\, 0.87,\\ -0.1-\iota\, 0.12,\\ 0.24+\iota\, 0.1\\ -0.8-\iota\, 0.75\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$
$\mathcal{S}_{\mathfrak{AI}-2}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.67+\iota\, 0.74,\\ -0.23-\iota\, 0.19,\\ 0.2+\iota\, 0.1,\\ -0.53-\iota\, 0.72\end{pmatrix}$
$\mathcal{S}_{\mathfrak{AI}-3}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.8+\iota\, 0.87,\\ -0.1-\iota\, 0.14,\\ 0.17+\iota\, 0.12,\\ -0.82-\iota\, 0.76\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$
$\mathcal{S}_{\mathfrak{AI}-4}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$	$\begin{pmatrix}0.0+\iota\, 0.0,\\ -1.0-\iota\, 1.0,\\ 1.0+\iota\, 1.0,\\ -0.0-\iota\, 0.0\end{pmatrix}$

expert would consider $\mathfrak{q}$ attributes that are $\mathcal{E}_{\mathfrak{av}-1}$, $\mathcal{E}_{\mathfrak{av}-2}$, …, $\mathcal{E}_{\mathfrak{av}-\mathfrak{q}}$ for the assessment of these alternatives. For the expert, the weight of the attributes may not be equal thus the expert can interpret the weight that is $\Omega_{\mathfrak{wv}-1}$, $\Omega_{\mathfrak{wv}-2}$, …, $\Omega_{\mathfrak{wv}-\mathfrak{q}}$ to each attribute such that $0\leq\Omega_{\mathfrak{wv}-\mathfrak{q}}\leq 1$ for each $\mathfrak{q}$ and $\sum_{\mathrm{Y}=1}^{\mathfrak{q}}\Omega_{\mathfrak{wv}-\mathfrak{q}}$. Underneath are the stages of the BCIFN-S TOPSIS approach.

Stage 1: The evaluation arguments described by the expert would be in the shape of BCIFN-SS and would construct a BCFIN-S decision matrix.

Stage 2: This stage contains the weighted BCIFN-S decision matrix. The weighted BCIFN-S decision matrix would be determined by employing Def (22).

Stage 3: In this stage, the BCIFN-S PIS (BCIFN-S-PIS) and BCIFN-S NIS (BCIFN-S-NIS) would be achieved by employing the underneath formulas as shown at the bottom of page 18.

Further, we can also utilize the BCIFN-S ideal PIS (BCIFN-S-IPIS) and BCIFN-S ideal NIS (BCIFN-S-INIS) which is in this stage.

$$\mathcal{P}^{+}=\left\{\left(\max_{\mathfrak{m}}\max_{\mathfrak{l}}\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}},\ \begin{pmatrix}1.0+\iota 1.0, & -0.0-\iota\, 0.0,\\ 0.0+\iota\, 0.0, & -1.0-\iota\, 1.0\end{pmatrix}\right)\right\}$$

$$\mathcal{P}^{-}=\left\{\left(\min_{\mathfrak{m}}\min_{\mathfrak{l}}\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}},\ \begin{pmatrix}0.0+\iota\, 0.0, & -1.0-\iota\, 1.0,\\ 1.0+\iota 1.0, & -0.0-\iota\, 0.0\end{pmatrix}\right)\right\}$$

Stage 4: Next, for the determination of the most superb alternative which is close to the BCIFN-S-PIS and far from the BCIFN-S-NIS. We assess the distance of every alternative from BCIFN-S-PIS and BCIFN-S-NIS by employing the underneath formulas shown at the bottom of page 18.

Stage 5: This stage contains the relative closeness corresponding to each alternative which would be determined as:

$$\mathfrak{C}=\frac{\left(\mathcal{G}_{\mathfrak{at}-\mathfrak{m}},\ \mathcal{P}^{+}\right)}{\left(\mathcal{G}_{\mathfrak{at}-\mathfrak{m}},\ \mathcal{P}^{+}\right)+\left(\mathcal{G}_{\mathfrak{at}-\mathfrak{m}},\ \mathcal{P}^{-}\right)}$$

Stage 6: Relying on the relative closeness, rank the alternative and achieve the most superb alternative.

A. ILLUSTRATED EXAMPLE

Reconsider example 1, in which a company requires AI software for enhancing the performance of the company. The under consideration 4 AI software is $\mathcal{S}_{\mathfrak{AI}-1}$ = *Cortana*, $\mathcal{S}_{\mathfrak{AI}-2}$ = *Google assistant*, $\mathcal{S}_{\mathfrak{AI}-3}$ = *IBM watson*, and $\mathcal{S}_{\mathfrak{AI}-4}$ = *H20.AI*. the grades and assessment value of this AI software by keeping in view 4 parameters that are $\mathfrak{d}_1$ = *Deep learning*, $\mathfrak{d}_2$ = *Automate tasks*, $\mathfrak{d}_3$ =

Quantum computing, $\underline{\mathfrak{d}}_4$ = *Data Ingestion* are displayed in Table 3.

Stage 1: Here, we consider Table 3 as a BCIFN-S decision matrix.

Stage 2: As every parameter has equal weight. So, no requirement for this stage.

Stage 3: The BCIFN-S-PIS and BCIFN-S-NIS are displayed as shown at the bottom of page 19.

Stage 4: The distance among alternatives and BCIFN-S-PIS, and BCIFN-S-NIS are displayed below

$$\begin{aligned}
&\mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-1},\ \mathcal{P}^{+}\right)=0324,\ \mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-2},\ \mathcal{P}^{+}\right)=0.319,\\
&\mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-e},\ \mathcal{P}^{+}\right)=0.381,\ \mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-4},\ \mathcal{P}^{+}\right)=0.329,\\
&\mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-1},\ \mathcal{P}^{-}\right)=0.297,\ \mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-2},\ \mathcal{P}^{-}\right)=0.328,\\
&\mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-3},\ \mathcal{P}^{-}\right)=0.295,\ \mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-4},\ \mathcal{P}^{-}\right)=0.281,
\end{aligned}$$

Stage 5: The relative closeness corresponding to each alternative is interpreted underneath

$$\begin{aligned}
&\mathfrak{C}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-1}\right)=0.522,\ \mathfrak{C}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-2}\right)=0.493,\\
&\mathfrak{C}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-3}\right)=0.563,\ \mathfrak{C}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-4}\right)=0.54,
\end{aligned}$$

Stage 6: Relying on the relative closeness, the ranking of the alternatives is

$$\mathcal{S}_{\mathfrak{A}\mathfrak{I}-3}>\mathcal{S}_{\mathfrak{A}\mathfrak{I}-4}>\mathcal{S}_{\mathfrak{A}\mathfrak{I}-1}>\mathcal{S}_{\mathfrak{A}\mathfrak{I}-2}$$

thus, $\mathcal{S}_{\mathfrak{A}\mathfrak{I}-3}$ is the most superb AI software.

Further, if we take the BCIFN-S-IPIS and BCIFN-S-INIS instead of BCIFN-S-PIS and BCIFN-S-NIS, in stage 3, that is as shown at the bottom of page 19.

Stage 7: The distance among alternatives and BCIFN-S-IPIS, and BCIFN-S-INIS are displayed below

$$\begin{aligned}
&\mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-1},\ \mathcal{P}^{+}\right)=0.551,\ \mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-2},\ \mathcal{P}^{+}\right)=0.564,\\
&\mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-e},\ \mathcal{P}^{+}\right)=0.64,\ \mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-4},\ \mathcal{P}^{+}\right)=0.543,\\
&\mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-1},\ \mathcal{P}^{-}\right)=0.538,\ \mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-2},\ \mathcal{P}^{-}\right)=0.523,\\
&\mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-3},\ \mathcal{P}^{-}\right)=0.57,\ \mathfrak{d}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-4},\ \mathcal{P}^{-}\right)=0.502,
\end{aligned}$$

Stage 8: The relative closeness corresponding to each alternative is interpreted underneath

$$\begin{aligned}
&\mathfrak{C}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-1}\right)=0.506,\ \mathfrak{C}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-2}\right)=0.519,\\
&\mathfrak{C}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-3}\right)=0.529,\ \mathfrak{C}\left(\mathcal{S}_{\mathfrak{A}\mathfrak{I}-4}\right)=0.52,
\end{aligned}$$

Stage 9: Relying on the relative closeness, the ranking of the alternatives is

$$\mathcal{S}_{\mathfrak{A}\mathfrak{I}-3}>\mathcal{S}_{\mathfrak{A}\mathfrak{I}-4}>\mathcal{S}_{\mathfrak{A}\mathfrak{I}-2}>\mathcal{S}_{\mathfrak{A}\mathfrak{I}-1}$$

thus, $\mathcal{S}_{\mathfrak{A}\mathfrak{I}-3}$ is the most superb AI software.

1)

$$\begin{aligned}
&\mathfrak{V}_{m\mathfrak{l}}\oplus\mathfrak{V}_{s\mathfrak{l}}\\
&=\left(\max\left(\mathfrak{h}_{\mathfrak{l}}^{m},\mathfrak{h}_{\mathfrak{l}}^{s}\right),\left(\begin{array}{c}Z_{m\mathfrak{l}}^{+}+Z_{s\mathfrak{l}}^{+}-Z_{m\mathfrak{l}}^{+}Z_{s\mathfrak{l}}^{+}+\iota\left(\text{Ʀ}_{m\mathfrak{l}}^{+}+\text{Ʀ}_{s\mathfrak{l}}^{+}-\text{Ʀ}_{m\mathfrak{l}}^{+}\text{Ʀ}_{s\mathfrak{l}}^{+}\right),-\left(Z_{m\mathfrak{l}}^{-}Z_{s\mathfrak{l}}^{-}\right)+\iota\left(-\left(\text{Ʀ}_{m\mathfrak{l}}^{-}\text{Ʀ}_{s\mathfrak{l}}^{-}\right)\right),\\ \text{Ƚ}_{m\mathfrak{l}}^{+}\text{Ƚ}_{s\mathfrak{l}}^{+}+\iota\text{Ᵽ}_{m\mathfrak{l}}^{+}\text{Ᵽ}_{s\mathfrak{l}}^{+},\text{Ƚ}_{m\mathfrak{l}}^{-}+\text{Ƚ}_{s\mathfrak{l}}^{-}+\text{Ƚ}_{m\mathfrak{l}}^{-}\text{Ƚ}_{s\mathfrak{l}}^{-}+\iota\left(\text{Ᵽ}_{m\mathfrak{l}}^{-}+\text{Ᵽ}_{s\mathfrak{l}}^{-}+\text{Ᵽ}_{m\mathfrak{l}}^{-}\text{Ᵽ}_{s\mathfrak{l}}^{-}\right)\end{array}\right)\right)\\
&=\left(\max\left(\mathfrak{h}_{\mathfrak{l}}^{s},\ \mathfrak{h}_{\mathfrak{l}}^{m}\right),\ \left(\begin{array}{c}Z_{s\mathfrak{l}}^{+}+Z_{m\mathfrak{l}}^{+}-Z_{s\mathfrak{l}}^{+}Z_{m\mathfrak{l}}^{+}+\iota\ \left(\text{Ʀ}_{s\mathfrak{l}}^{+}+\text{Ʀ}_{m\mathfrak{l}}^{+}-\text{Ʀ}_{s\mathfrak{l}}^{+}\text{Ʀ}_{m\mathfrak{l}}^{+}\right),\ -\left(Z_{s\mathfrak{l}}^{-}Z_{m\mathfrak{l}}^{-}\right)+\iota\ \left(-\left(\text{Ʀ}_{s\mathfrak{l}}^{-}\text{Ʀ}_{m\mathfrak{l}}^{-}\right)\right),\\ \text{Ƚ}_{s\mathfrak{l}}^{+}\text{Ƚ}_{m\mathfrak{l}}^{+}+\iota\ \text{Ᵽ}_{s\mathfrak{l}}^{+}\text{Ᵽ}_{m\mathfrak{l}}^{+}\ ,\text{Ƚ}_{s\mathfrak{l}}^{-}+\text{Ƚ}_{m\mathfrak{l}}^{-}+\text{Ƚ}_{s\mathfrak{l}}^{-}\text{Ƚ}_{m\mathfrak{l}}^{-}+\iota\ \left(\text{Ᵽ}_{s\mathfrak{l}}^{-}+\text{Ᵽ}_{m\mathfrak{l}}^{-}+\text{Ᵽ}_{s\mathfrak{l}}^{-}\text{Ᵽ}_{m\mathfrak{l}}^{-}\right)\end{array}\right)\right)\\
&=\mathfrak{V}_{s\mathfrak{l}}\oplus\mathfrak{V}_{m\mathfrak{l}}
\end{aligned}$$

2)

$$\begin{aligned}
&\mathfrak{V}_{s\mathfrak{l}}\otimes\mathfrak{V}_{m\mathfrak{l}}\\
&=\left(\min\left(\mathfrak{h}_{\mathfrak{l}}^{m},\ \mathfrak{h}_{\mathfrak{l}}^{s}\right),\ \left(\begin{array}{c}Z_{m\mathfrak{l}}^{+}Z_{s\mathfrak{l}}^{+}+\iota\,\text{Ʀ}_{m\mathfrak{l}}^{+}\text{Ʀ}_{s\mathfrak{l}}^{+},\ Z_{m\mathfrak{l}}^{-}+Z_{s\mathfrak{l}}^{-}+Z_{m\mathfrak{l}}^{-}Z_{s\mathfrak{l}}^{-}+\iota\ \left(\text{Ʀ}_{m\mathfrak{l}}^{-}+\text{Ʀ}_{s\mathfrak{l}}^{-}+\text{Ʀ}_{m\mathfrak{l}}^{-}\text{Ʀ}_{s\mathfrak{l}}^{-}\right)\\ \text{Ƚ}_{m\mathfrak{l}}^{+}+\text{Ƚ}_{s\mathfrak{l}}^{+}-\text{Ƚ}_{m\mathfrak{l}}^{+}\text{Ƚ}_{s\mathfrak{l}}^{+}+\iota\ \left(\text{Ᵽ}_{m\mathfrak{l}}^{+}+\text{Ᵽ}_{s\mathfrak{l}}^{+}-\text{Ᵽ}_{m\mathfrak{l}}^{+}\text{Ᵽ}_{s\mathfrak{l}}^{+}\right),\ -\left(\text{Ƚ}_{m\mathfrak{l}}^{-}\text{Ƚ}_{s\mathfrak{l}}^{-}\right)+\iota\ \left(-\left(\text{Ᵽ}_{m\mathfrak{l}}^{-}\text{Ᵽ}_{s\mathfrak{l}}^{-}\right)\right)\end{array}\right)\right)\\
&=\left(\min\left(\mathfrak{h}_{\mathfrak{l}}^{s},\ \mathfrak{h}_{\mathfrak{l}}^{m}\right),\ \left(\begin{array}{c}Z_{s\mathfrak{l}}^{+}Z_{m\mathfrak{l}}^{+}+\iota\,\text{Ʀ}_{s\mathfrak{l}}^{+}\text{Ʀ}_{m\mathfrak{l}}^{+},\ Z_{s\mathfrak{l}}^{-}+Z_{m\mathfrak{l}}^{-}+Z_{s\mathfrak{l}}^{-}Z_{m\mathfrak{l}}^{-}+\iota\ \left(\text{Ʀ}_{s\mathfrak{l}}^{-}+\text{Ʀ}_{m\mathfrak{l}}^{-}+\text{Ʀ}_{s\mathfrak{l}}^{-}\text{Ʀ}_{m\mathfrak{l}}^{-}\right)\\ \text{Ƚ}_{s\mathfrak{l}}^{+}+\text{Ƚ}_{m\mathfrak{l}}^{+}-\text{Ƚ}_{s\mathfrak{l}}^{+}\text{Ƚ}_{m\mathfrak{l}}^{+}+\iota\ \left(\text{Ᵽ}_{s\mathfrak{l}}^{+}+\text{Ᵽ}_{m\mathfrak{l}}^{+}-\text{Ᵽ}_{s\mathfrak{l}}^{+}\text{Ᵽ}_{m\mathfrak{l}}^{+}\right),\ -\left(\text{Ƚ}_{s\mathfrak{l}}^{-}\text{Ƚ}_{m\mathfrak{l}}^{-}\right)+\iota\ \left(-\left(\text{Ᵽ}_{s\mathfrak{l}}^{-}\text{Ᵽ}_{m\mathfrak{l}}^{-}\right)\right)\end{array}\right)\right)\\
&=\mathfrak{V}_{s\mathfrak{l}}\otimes\mathfrak{V}_{m\mathfrak{l}}
\end{aligned}$$

3) We have

$$\begin{aligned}
\nu\mathfrak{V}_{m\mathfrak{l}}&=\left(\mathfrak{h}_{\mathfrak{l}}^{m},\ \left(\begin{array}{c}1-\left(1-Z_{m\mathfrak{l}}^{+}\right)^{\nu}+\iota\ \left(1-\left(1-\text{Ʀ}_{m\mathfrak{l}}^{+}\right)^{\nu}\right),\ -\left|Z_{m\mathfrak{l}}^{-}\right|^{\nu}+\iota\ \left(-\left|\text{Ʀ}_{m\mathfrak{l}}^{-}\right|^{\nu}\right),\\ \left(\text{Ƚ}_{m\mathfrak{l}}^{+}\right)^{\nu}+\iota\ \left(\text{Ᵽ}_{m\mathfrak{l}}^{+}\right)^{\nu},-1+\left(1+\text{Ƚ}_{m\mathfrak{l}}^{-}\right)^{\nu}+\iota\ \left(-1+\left(1+\text{Ᵽ}_{m\mathfrak{l}}^{-}\right)^{\nu}\right)\end{array}\right)\right)\\
\nu\mathfrak{V}_{s\mathfrak{l}}&=\left(\mathfrak{h}_{\mathfrak{l}}^{s},\ \left(\begin{array}{c}1-\left(1-Z_{s\mathfrak{l}}^{+}\right)^{\nu}+\iota\ \left(1-\left(1-\text{Ʀ}_{s\mathfrak{l}}^{+}\right)^{\nu}\right),\ -\left|Z_{s\mathfrak{l}}^{-}\right|^{\nu}+\iota\ \left(-\left|\text{Ʀ}_{s\mathfrak{l}}^{-}\right|^{\nu}\right),\\ \left(\text{Ƚ}_{s\mathfrak{l}}^{+}\right)^{\nu}+\iota\ \left(\text{Ᵽ}_{s\mathfrak{l}}^{+}\right)^{\nu},-1+\left(1+\text{Ƚ}_{s\mathfrak{l}}^{-}\right)^{\nu}+\iota\ \left(-1+\left(1+\text{Ᵽ}_{s\mathfrak{l}}^{-}\right)^{\nu}\right)\end{array}\right)\right)
\end{aligned}$$

B. COMPARATIVE ANALYSIS

For revealing the supremacy and dominance of the devised work, it is necessary to compare it with a few prevailing works. Therefore, here we compare the devised work with the prevailing work investigated in [38], [43], [45], [46], [47], and [48].

- In [41], Fatimah et al. devised N-SS. The structure of N-SS can't handle information containing the 2nd dimension along with the truth degree (containing both positive and negative aspects) and falsity degree (containing both positive and negative aspects) and parameterization along with grades at the same time because the truth degree and falsity degree is missing in the structure of N-SS.
- In [46], Akram et al. invented the structure of FN-SS. The structure of FN-SS can't cope with the data containing 2nd dimension along with truth degree (containing both positive and negative aspects) and falsity degree (containing both positive and negative aspects) and parameterization along with grades at once because

$$
\begin{aligned}
&\nu\mathfrak{V}_{ml}\oplus\nu\mathfrak{V}_{\mathfrak{s}l}\\
&=\left(\mathfrak{h}^{m}_{\mathfrak{z}_l},\left(\begin{array}{c}1-(1-Z^{+}_{ml})^{\nu}+\iota\left(1-\left(1-\mathcal{R}^{+}_{ml}\right)^{\nu}\right),\ -\left|Z^{-}_{ml}\right|^{\nu}+\iota\left(-\left|\mathcal{R}^{-}_{ml}\right|^{\nu}\right),\\(\mathcal{L}^{+}_{ml})^{\nu}+\iota\,(\mathcal{P}^{+}_{ml})^{\nu},-1+(1+\mathcal{L}^{-}_{ml})^{\nu}+\iota\,\left(-1+(1+\mathcal{P}^{-}_{ml})^{\nu}\right)\end{array}\right)\right)\\
&\quad\oplus\left(\mathfrak{h}^{\mathfrak{s}}_{\mathfrak{z}_l},\left(\begin{array}{c}1-\left(1-Z^{+}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(1-\left(1-\mathcal{R}^{+}_{\mathfrak{s}l}\right)^{\nu}\right),\ -\left|Z^{-}_{\mathfrak{s}l}\right|^{\nu}+\iota\left(-\left|\mathcal{R}^{-}_{\mathfrak{s}l}\right|^{\nu}\right),\\\left(\mathcal{L}^{+}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(\mathcal{P}^{+}_{\mathfrak{s}l}\right)^{\nu},-1+\left(1+\mathcal{L}^{-}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(-1+\left(1+\mathcal{P}^{-}_{\mathfrak{s}l}\right)^{\nu}\right)\end{array}\right)\right)\\
&=\left(\max\left(\mathfrak{h}^{m}_{\mathfrak{z}_l},\mathfrak{h}^{\mathfrak{s}}_{\mathfrak{z}_l}\right),\left(\begin{array}{c}1-(1-Z^{+}_{ml})^{\nu}+1-\left(1-Z^{+}_{\mathfrak{s}l}\right)^{\nu}-\left(1-(1-Z^{+}_{ml})^{\nu}\times1-\left(1-Z^{+}_{\mathfrak{s}l}\right)^{\nu}\right)\\+\iota\left(1-\left(1-\mathcal{R}^{+}_{ml}\right)^{\nu}+1-\left(1-\mathcal{R}^{+}_{\mathfrak{s}l}\right)^{\nu}-\left(1-\left(1-\mathcal{R}^{+}_{ml}\right)^{\nu}\times1-\left(1-\mathcal{R}^{+}_{\mathfrak{s}l}\right)^{\nu}\right)\right),\\-\left((-|Z^{-}_{ml}|^{\nu})\left(-\left|Z^{-}_{\mathfrak{s}l}\right|^{\nu}\right)\right)+\iota\left(-\left(\left(-\left|\mathcal{R}^{-}_{ml}\right|^{\nu}\right)\left(-\left|\mathcal{R}^{-}_{\mathfrak{s}l}\right|^{\nu}\right)\right)\right)\\(\mathcal{L}^{+}_{ml})^{\nu}\left(\mathcal{L}^{+}_{\mathfrak{s}l}\right)^{\nu}+\iota\,(\mathcal{P}^{+}_{ml})^{\nu}\left(\mathcal{P}^{+}_{\mathfrak{s}l}\right)^{\nu},\\-1+(1+\mathcal{L}^{-}_{ml})^{\nu}+\left(-1+\left(1+\mathcal{L}^{-}_{\mathfrak{s}l}\right)^{\nu}\right)+\left(-1+(1+\mathcal{L}^{-}_{ml})^{\nu}\times\left(-1+\left(1+\mathcal{L}^{-}_{\mathfrak{s}l}\right)^{\nu}\right)\right)\\+\iota\left(-1+(1+\mathcal{P}^{-}_{ml})^{\nu}+\left(-1+\left(1+\mathcal{P}^{-}_{\mathfrak{s}l}\right)^{\nu}\right)+\left(-1+(1+\mathcal{P}^{-}_{ml})^{\nu}\times\left(-1+\left(1+\mathcal{P}^{-}_{\mathfrak{s}l}\right)^{\nu}\right)\right)\right)\end{array}\right)\right)\\
&=\left(\max\left(\mathfrak{h}^{m}_{\mathfrak{z}_l},\mathfrak{h}^{\mathfrak{s}}_{\mathfrak{z}_l}\right),\left(\begin{array}{c}1-\left(1-Z^{+}_{ml}-Z^{+}_{\mathfrak{s}l}+Z^{+}_{ml}Z^{+}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(1-\left(1-\mathcal{R}^{+}_{ml}-\mathcal{R}^{+}_{\mathfrak{s}l}+\mathcal{R}^{+}_{ml}\mathcal{R}^{+}_{\mathfrak{s}l}\right)^{\nu}\right)\\-\left(\left(|Z^{-}_{ml}|\left|Z^{-}_{\mathfrak{s}l}\right|\right)^{\nu}\right)+\iota\left(-\left(\left(\left|\mathcal{R}^{-}_{ml}\right|\left|\mathcal{R}^{-}_{\mathfrak{s}l}\right|\right)^{\nu}\right)\right)\\\left(\mathcal{L}^{+}_{ml}\mathcal{L}^{+}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(\mathcal{P}^{+}_{ml}\mathcal{P}^{+}_{\mathfrak{s}l}\right)^{\nu},\\-1+\left(1+\mathcal{L}^{-}_{\mathfrak{s}l}+\mathcal{L}^{-}_{ml}-\mathcal{L}^{-}_{ml}\mathcal{L}^{-}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(-1+\left(1+\mathcal{P}^{-}_{\mathfrak{s}l}+\mathcal{P}^{-}_{ml}-\mathcal{P}^{-}_{ml}\mathcal{P}^{-}_{\mathfrak{s}l}\right)^{\nu}\right)\end{array}\right)\right)\\
&=\left(\max\left(\mathfrak{h}^{m}_{\mathfrak{z}_l},\mathfrak{h}^{\mathfrak{s}}_{\mathfrak{z}_l}\right),\left(\begin{array}{c}1-\left(1-Z^{+}_{ml}-Z^{+}_{\mathfrak{s}l}+Z^{+}_{ml}Z^{+}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(1-\left(1-\mathcal{R}^{+}_{ml}-\mathcal{R}^{+}_{\mathfrak{s}l}+\mathcal{R}^{+}_{ml}\mathcal{R}^{+}_{\mathfrak{s}l}\right)^{\nu}\right)\\-\left(\left(Z^{-}_{ml}Z^{-}_{\mathfrak{s}l}\right)^{\nu}\right)+\iota\left(-\left(\left(\mathcal{R}^{-}_{ml}\mathcal{R}^{-}_{\mathfrak{s}l}\right)^{\nu}\right)\right),\\\left(\mathcal{L}^{+}_{ml}\mathcal{L}^{+}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(\mathcal{P}^{+}_{ml}\mathcal{P}^{+}_{\mathfrak{s}l}\right)^{\nu},\\-1+\left(1+\mathcal{L}^{-}_{\mathfrak{s}l}+\mathcal{L}^{-}_{ml}-\mathcal{L}^{-}_{ml}\mathcal{L}^{-}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(-1+\left(1+\mathcal{P}^{-}_{\mathfrak{s}l}+\mathcal{P}^{-}_{ml}-\mathcal{P}^{-}_{ml}\mathcal{P}^{-}_{\mathfrak{s}l}\right)^{\nu}\right)\end{array}\right)\right)\\
&=\nu\left(\max\left(\mathfrak{h}^{m}_{\mathfrak{z}_l},\mathfrak{h}^{\mathfrak{s}}_{\mathfrak{z}_l}\right),\left(\begin{array}{c}Z^{+}_{ml}-Z^{+}_{\mathfrak{s}l}+Z^{+}_{ml}Z^{+}_{\mathfrak{s}l}+\iota\left(\mathcal{R}^{+}_{ml}-\mathcal{R}^{+}_{\mathfrak{s}l}+\mathcal{R}^{+}_{ml}\mathcal{R}^{+}_{\mathfrak{s}l}\right),\ -\left(Z^{-}_{ml}Z^{-}_{\mathfrak{s}l}\right)+\iota\left(-\left(\mathcal{R}^{-}_{ml}\mathcal{R}^{-}_{\mathfrak{s}l}\right)\right),\\\mathcal{L}^{+}_{ml}\mathcal{L}^{+}_{\mathfrak{s}l}+\iota\,\mathcal{P}^{+}_{ml}\mathcal{P}^{+}_{\mathfrak{s}l},\mathcal{L}^{-}_{ml}+\mathcal{L}^{-}_{\mathfrak{s}l}+\mathcal{L}^{-}_{ml}\mathcal{L}^{-}_{\mathfrak{s}l}+\iota\left(\mathcal{P}^{-}_{ml}+\mathcal{P}^{-}_{\mathfrak{s}l}+\mathcal{P}^{-}_{ml}\mathcal{P}^{-}_{\mathfrak{s}l}\right)\end{array}\right)\right)\\
&=\nu\left(\mathfrak{V}_{ml}\oplus\mathfrak{V}_{\mathfrak{s}l}\right)
\end{aligned}
$$

$$
\mathfrak{V}^{\nu}_{ml}=\left(\mathfrak{h}^{m}_{\mathfrak{z}_l},\left(\begin{array}{c}(Z^{+}_{ml})^{\nu}+\iota\left(\mathcal{R}^{+}_{ml}\right)^{\nu},\ -1+(1+Z^{-}_{ml})^{\nu}+\iota\left(-1+\left(1+\mathcal{R}^{-}_{ml}\right)^{\nu}\right)\\1-(1-\mathcal{L}^{+}_{ml})^{\nu}+\iota\,\left(1-(1-\mathcal{P}^{+}_{ml})^{\nu}\right),-|\mathcal{L}^{-}_{ml}|^{\nu}+\iota\,\left(-|\mathcal{P}^{-}_{ml}|^{\nu}\right)\end{array}\right)\right)
$$

$$
\mathfrak{V}^{\nu}_{\mathfrak{s}l}=\left(\mathfrak{h}^{\mathfrak{s}}_{\mathfrak{z}_l},\left(\begin{array}{c}\left(Z^{+}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(\mathcal{R}^{+}_{\mathfrak{s}l}\right)^{\nu},\ -1+\left(1+Z^{-}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(-1+\left(1+\mathcal{R}^{-}_{\mathfrak{s}l}\right)^{\nu}\right)\\1-\left(1-\mathcal{L}^{+}_{\mathfrak{s}l}\right)^{\nu}+\iota\left(1-\left(1-\mathcal{P}^{+}_{\mathfrak{s}l}\right)^{\nu}\right),-\left|\mathcal{L}^{-}_{\mathfrak{s}l}\right|^{\nu}+\iota\left(-\left|\mathcal{P}^{-}_{\mathfrak{s}l}\right|^{\nu}\right)\end{array}\right)\right)
$$

$$\begin{aligned}
&\mathfrak{V}_{m\mathfrak{l}}^{\nu}\otimes\mathfrak{V}_{\mathfrak{s}\mathfrak{l}}^{\nu}\\
&=\left(\mathfrak{h}_{\mathfrak{l}}^{m},\left(\begin{array}{c}(Ƶ_{m\mathfrak{l}}^{+})^{\nu}+\iota\left(Ɽ_{m\mathfrak{l}}^{+}\right)^{\nu},\ -1+(1+Ƶ_{m\mathfrak{l}}^{-})^{\nu}+\iota\left(-1+\left(1+Ɽ_{m\mathfrak{l}}^{-}\right)^{\nu}\right)\\ 1-(1-Ƚ_{m\mathfrak{l}}^{+})^{\nu}+\iota\left(1-(1-Ᵽ_{m\mathfrak{l}}^{+})^{\nu}\right),-|Ƚ_{m\mathfrak{l}}^{-}|^{\nu}+\iota\left(-|Ᵽ_{m\mathfrak{l}}^{-}|^{\nu}\right)\end{array}\right)\right)\\
&\otimes\left(\mathfrak{h}_{\mathfrak{l}}^{\mathfrak{s}},\left(\begin{array}{c}\left(Ƶ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu}+\iota\left(Ɽ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu},\ -1+\left(1+Ƶ_{\mathfrak{s}\mathfrak{l}}^{-}\right)^{\nu}+\iota\left(-1+\left(1+Ɽ_{\mathfrak{s}\mathfrak{l}}^{-}\right)^{\nu}\right)\\ 1-\left(1-Ƚ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu}+\iota\left(1-\left(1-Ᵽ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu}\right),-\left|Ƚ_{\mathfrak{s}\mathfrak{l}}^{-}\right|^{\nu}+\iota\left(-\left|Ᵽ_{\mathfrak{s}\mathfrak{l}}^{-}\right|^{\nu}\right)\end{array}\right)\right)\\
&=\left(\min\left(\mathfrak{h}_{\mathfrak{l}}^{m},\mathfrak{h}_{\mathfrak{l}}^{\mathfrak{s}}\right),\left(\begin{array}{c}(Ƶ_{m\mathfrak{l}}^{+})^{\nu}\left(Ƶ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu}+\iota\left(Ɽ_{m\mathfrak{l}}^{+}\right)^{\nu}\left(Ɽ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu},\\ -1+(1+Ƶ_{m\mathfrak{l}}^{-})^{\nu}-1+\left(1+Ƶ_{\mathfrak{s}\mathfrak{l}}^{-}\right)^{\nu}+\left(-1+(1+Ƶ_{m\mathfrak{l}}^{-})^{\nu}\times\left(-1+\left(1+Ƶ_{\mathfrak{s}\mathfrak{l}}^{-}\right)^{\nu}\right)\right)\\ +\iota\left(-1+\left(1+Ɽ_{m\mathfrak{l}}^{-}\right)^{\nu}-1+\left(1+Ɽ_{\mathfrak{s}\mathfrak{l}}^{-}\right)^{\nu}+\left(-1+\left(1+Ɽ_{\mathfrak{s}\mathfrak{l}}^{-}\right)^{\nu}\times\left(-1+\left(1+Ɽ_{\mathfrak{s}\mathfrak{l}}^{-}\right)^{\nu}\right)\right)\right),\\ 1-(1-Ƚ_{m\mathfrak{l}}^{+})^{\nu}+1-\left(1-ZȽ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu}-\left(1-(1-Ƚ_{m\mathfrak{l}}^{+})^{\nu}\times1-\left(1-Ƚ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu}\right)\\ +\iota\left(1-(1-Ᵽ_{m\mathfrak{l}}^{+})^{\nu}+1-\left(1-Ᵽ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu}-\left(1-(1-Ᵽ_{m\mathfrak{l}}^{+})^{\nu}\times1-\left(1-Ᵽ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu}\right)\right),\\ -\left(\left(-|Ƚ_{m\mathfrak{l}}^{-}|^{\nu}\right)\left(-\left|Ƚ_{\mathfrak{s}\mathfrak{l}}^{-}\right|^{\nu}\right)\right)+\iota\left(-\left(\left(-|Ᵽ_{m\mathfrak{l}}^{-}|^{\nu}\right)\left(-\left|Ᵽ_{\mathfrak{s}\mathfrak{l}}^{-}\right|^{\nu}\right)\right)\right)\end{array}\right)\right)\\
&=\left(\min\left(\mathfrak{h}_{\mathfrak{l}}^{m},\mathfrak{h}_{\mathfrak{l}}^{\mathfrak{s}}\right),\left(\begin{array}{c}\left(Ƶ_{m\mathfrak{l}}^{+}Ƶ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu}+\iota\left(Ɽ_{m\mathfrak{l}}^{+}Ɽ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu},\\ -1+\left(1+Ƶ_{m\mathfrak{l}}^{-}+Ƶ_{\mathfrak{s}\mathfrak{l}}^{-}+Ƶ_{m\mathfrak{l}}^{-}Ƶ_{\mathfrak{s}\mathfrak{l}}^{-}\right)^{\nu}+\iota\left(-1+\left(1+Ƶ_{m\mathfrak{l}}^{-}+Ƶ_{\mathfrak{s}\mathfrak{l}}^{-}+Ƶ_{m\mathfrak{l}}^{-}Ƶ_{\mathfrak{s}\mathfrak{l}}^{-}\right)^{\nu}\right)\\ 1-\left(1-Ƚ_{m\mathfrak{l}}^{+}-Ƚ_{\mathfrak{s}\mathfrak{l}}^{+}+Ƚ_{m\mathfrak{l}}^{+}Ƚ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu}+\iota\left(1-\left(1-Ᵽ_{m\mathfrak{l}}^{+}-Ᵽ_{\mathfrak{s}\mathfrak{l}}^{+}+Ᵽ_{m\mathfrak{l}}^{+}Ᵽ_{\mathfrak{s}\mathfrak{l}}^{+}\right)^{\nu}\right)\\ -\left(\left(Ƚ_{m\mathfrak{l}}^{-}Ƚ_{\mathfrak{s}\mathfrak{l}}^{-}\right)^{\nu}\right)+\iota\left(-\left(\left(Ᵽ_{m\mathfrak{l}}^{-}Ᵽ_{\mathfrak{s}\mathfrak{l}}^{-}\right)^{\nu}\right)\right),\end{array}\right)\right)\\
&=\left(\mathfrak{V}_{m\mathfrak{l}}\otimes\mathfrak{V}_{\mathfrak{s}\mathfrak{l}}\right)^{\nu}.
\end{aligned}$$

$$\nu_1\mathfrak{V}_{m\mathfrak{l}}=\left(\mathfrak{h}_{\mathfrak{l}}^{m},\left(\begin{array}{c}1-(1-Ƶ_{m\mathfrak{l}}^{+})^{\nu_1}+\iota\left(1-\left(1-Ɽ_{m\mathfrak{l}}^{+}\right)^{\nu_1}\right),\ -|Ƶ_{m\mathfrak{l}}^{-}|^{\nu_1}+\iota\left(-\left|Ɽ_{m\mathfrak{l}}^{-}\right|^{\nu_1}\right),\\ (Ƚ_{m\mathfrak{l}}^{+})^{\nu_1}+\iota\,(Ᵽ_{m\mathfrak{l}}^{+})^{\nu_1},-1+(1+Ƚ_{m\mathfrak{l}}^{-})^{\nu_1}+\iota\left(-1+(1+Ᵽ_{m\mathfrak{l}}^{-})^{\nu_1}\right)\end{array}\right)\right)$$

$$\nu_2\mathfrak{V}_{m\mathfrak{l}}=\left(\mathfrak{h}_{\mathfrak{l}}^{m},\left(\begin{array}{c}1-(1-Ƶ_{m\mathfrak{l}}^{+})^{\nu_2}+\iota\left(1-\left(1-Ɽ_{m\mathfrak{l}}^{+}\right)^{\nu_2}\right),\ -|Ƶ_{m\mathfrak{l}}^{-}|^{\nu_2}+\iota\left(-\left|Ɽ_{m\mathfrak{l}}^{-}\right|^{\nu_2}\right),\\ (Ƚ_{m\mathfrak{l}}^{+})^{\nu_2}+\iota\,(Ᵽ_{m\mathfrak{l}}^{+})^{\nu_2},-1+(1+Ƚ_{m\mathfrak{l}}^{-})^{\nu_2}+\iota\left(-1+(1+Ᵽ_{m\mathfrak{l}}^{-})^{\nu_2}\right)\end{array}\right)\right)$$

$$\begin{aligned}
&\nu_1\mathfrak{V}_{m\mathfrak{l}}\oplus\nu_2\mathfrak{V}_{m\mathfrak{l}}\\
&=\left(\mathfrak{h}_{\mathfrak{l}}^{m},\left(\begin{array}{c}1-(1-Ƶ_{m\mathfrak{l}}^{+})^{\nu_1}+\iota\left(1-\left(1-Ɽ_{m\mathfrak{l}}^{+}\right)^{\nu_1}\right),\ -|Ƶ_{m\mathfrak{l}}^{-}|^{\nu_1}+\iota\left(-\left|Ɽ_{m\mathfrak{l}}^{-}\right|^{\nu_1}\right),\\ (Ƚ_{m\mathfrak{l}}^{+})^{\nu_1}+\iota\,(Ᵽ_{m\mathfrak{l}}^{+})^{\nu_1},-1+(1+Ƚ_{m\mathfrak{l}}^{-})^{\nu_1}+\iota\left(-1+(1+Ᵽ_{m\mathfrak{l}}^{-})^{\nu_1}\right)\end{array}\right)\right)\\
&\oplus\left(\mathfrak{h}_{\mathfrak{l}}^{m},\left(\begin{array}{c}1-(1-Ƶ_{m\mathfrak{l}}^{+})^{\nu_2}+\iota\left(1-\left(1-Ɽ_{m\mathfrak{l}}^{+}\right)^{\nu_2}\right),\ -|Ƶ_{m\mathfrak{l}}^{-}|^{\nu_2}+\iota\left(-\left|Ɽ_{m\mathfrak{l}}^{-}\right|^{\nu_2}\right),\\ (Ƚ_{m\mathfrak{l}}^{+})^{\nu_2}+\iota\,(Ᵽ_{m\mathfrak{l}}^{+})^{\nu_2},-1+(1+Ƚ_{m\mathfrak{l}}^{-})^{\nu_2}+\iota\left(-1+(1+Ᵽ_{m\mathfrak{l}}^{-})^{\nu_2}\right)\end{array}\right)\right)\\
&=\left(\max\left(\mathfrak{h}_{\mathfrak{l}}^{m},\mathfrak{h}_{\mathfrak{l}}^{m}\right),\left(\begin{array}{c}1-(1-Ƶ_{m\mathfrak{l}}^{+})^{\nu_1}+1-(1-Ƶ_{m\mathfrak{l}}^{+})^{\nu_2}-\left(1-(1-Ƶ_{m\mathfrak{l}}^{+})^{\nu_1}\times1-(1-Ƶ_{m\mathfrak{l}}^{+})^{\nu_2}\right)\\ +\iota\left(1-\left(1-Ɽ_{m\mathfrak{l}}^{+}\right)^{\nu_1}+1-\left(1-Ɽ_{m\mathfrak{l}}^{+}\right)^{\nu_2}-\left(1-\left(1-Ɽ_{m\mathfrak{l}}^{+}\right)^{\nu_1}\times1-\left(1-Ɽ_{m\mathfrak{l}}^{+}\right)^{\nu_2}\right)\right),\\ -\left(\left(-|Ƶ_{m\mathfrak{l}}^{-}|^{\nu_1}\right)\left(-|Ƶ_{m\mathfrak{l}}^{-}|^{\nu_2}\right)\right)+\iota\left(-\left(\left(-\left|Ɽ_{m\mathfrak{l}}^{-}\right|^{\nu_1}\right)\left(-\left|Ɽ_{m\mathfrak{l}}^{-}\right|^{\nu_1}\right)\right)\right)\\ (Ƚ_{m\mathfrak{l}}^{+})^{\nu_1}(Ƚ_{m\mathfrak{l}}^{+})^{\nu_2}+\iota\,(Ᵽ_{m\mathfrak{l}}^{+})^{\nu_1}(Ᵽ_{m\mathfrak{l}}^{+})^{\nu_2},\\ -1+(1+Ƚ_{m\mathfrak{l}}^{-})^{\nu_1}+\left(-1+(1+Ƚ_{m\mathfrak{l}}^{-})^{\nu_2}\right)+\left(-1+(1+Ƚ_{m\mathfrak{l}}^{-})^{\nu_1}\times\left(-1+(1+Ƚ_{m\mathfrak{l}}^{-})^{\nu_2}\right)\right)\\ +\iota\left(-1+(1+Ᵽ_{m\mathfrak{l}}^{-})^{\nu_1}+\left(-1+(1+Ᵽ_{m\mathfrak{l}}^{-})^{\nu_2}\right)+\left(-1+(1+Ᵽ_{m\mathfrak{l}}^{-})^{\nu_1}\times\left(-1+(1+Ᵽ_{m\mathfrak{l}}^{-})^{\nu_2}\right)\right)\right)\end{array}\right)\right)\\
&=\left(\max\left(\mathfrak{h}_{\mathfrak{l}}^{m},\mathfrak{h}_{\mathfrak{l}}^{\mathfrak{s}}\right),\left(\begin{array}{c}1-(1-Ƶ_{m\mathfrak{l}}^{+})^{\nu_1+\nu_2}+\iota\left(1-\left(1-Ɽ_{m\mathfrak{l}}^{+}\right)^{\nu_1+\nu_2}\right),\ -\left(|Ƶ_{m\mathfrak{l}}^{-}|^{\nu_1+\nu_2}\right)+\iota\left(-\left(\left|Ɽ_{m\mathfrak{l}}^{-}\right|^{\nu_1+\nu_2}\right)\right),\\ \left(Ƚ_{m\mathfrak{l}}^{+}\right)^{\nu_1+\nu_2}+\iota\,(Ᵽ_{m\mathfrak{l}}^{+})^{\nu_1+\nu_2},-1+(1+Ƚ_{m\mathfrak{l}}^{-})^{\nu_1+\nu_2}+\iota\left(-1+(1+Ᵽ_{m\mathfrak{l}}^{-})^{\nu_1+\nu_2}\right)\end{array}\right)\right)\\
&=(\nu_1+\nu_2)\,\mathfrak{V}_{m\mathfrak{l}}.
\end{aligned}$$

the FN-SS has merely the positive aspects of truth degree with N-SS but can't model the other perspectives.

- In [48], Akram et al. invented the structure of IFN-SS. The structure of IFN-SS can't cope with the data

$$\mathfrak{V}_{\mathfrak{ml}}^{\nu_1}=\left(\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}},\begin{pmatrix}\left(Z_{\mathfrak{ml}}^{+}\right)^{\nu_1}+\iota\left(\text{Ꝛ}_{\mathfrak{ml}}^{+}\right)^{\nu_1}, -1+\left(1+Z_{\mathfrak{ml}}^{-}\right)^{\nu_1}+\iota\left(-1+\left(1+\text{Ꝛ}_{\mathfrak{ml}}^{-}\right)^{\nu_1}\right)\\ 1-\left(1-\text{Ƚ}_{\mathfrak{ml}}^{+}\right)^{\nu_1}+\iota\left(1-\left(1-\text{Ᵽ}_{\mathfrak{ml}}^{+}\right)^{\nu_1}\right),-\left|\text{Ƚ}_{\mathfrak{ml}}^{-}\right|^{\nu_1}+\iota\left(-\left|\text{Ᵽ}_{\mathfrak{ml}}^{-}\right|^{\nu_1}\right)\end{pmatrix}\right)$$

$$\mathfrak{V}_{\mathfrak{ml}}^{\nu_2}=\left(\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}},\begin{pmatrix}\left(Z_{\mathfrak{ml}}^{+}\right)^{\nu_2}+\iota\left(\text{Ꝛ}_{\mathfrak{ml}}^{+}\right)^{\nu_2}, -1+\left(1+Z_{\mathfrak{ml}}^{-}\right)^{\nu_2}+\iota\left(-1+\left(1+\text{Ꝛ}_{\mathfrak{ml}}^{-}\right)^{\nu_2}\right)\\ 1-\left(1-\text{Ƚ}_{\mathfrak{ml}}^{+}\right)^{\nu_2}+\iota\left(1-\left(1-\text{Ᵽ}_{\mathfrak{ml}}^{+}\right)^{\nu_2}\right),-\left|\text{Ƚ}_{\mathfrak{ml}}^{-}\right|^{\nu_2}+\iota\left(-\left|\text{Ᵽ}_{\mathfrak{ml}}^{-}\right|^{\nu_2}\right)\end{pmatrix}\right)$$

$$\begin{aligned}
&\mathfrak{V}_{\mathfrak{ml}}^{\nu_1}\otimes\mathfrak{V}_{\mathfrak{ml}}^{\nu_2}\\
&=\left(\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}},\begin{pmatrix}\left(Z_{\mathfrak{ml}}^{+}\right)^{\nu_1}+\iota\left(\text{Ꝛ}_{\mathfrak{ml}}^{+}\right)^{\nu_1}, -1+\left(1+Z_{\mathfrak{ml}}^{-}\right)^{\nu_1}+\iota\left(-1+\left(1+\text{Ꝛ}_{\mathfrak{ml}}^{-}\right)^{\nu_1}\right)\\ 1-\left(1-\text{Ƚ}_{\mathfrak{ml}}^{+}\right)^{\nu_1}+\iota\left(1-\left(1-\text{Ᵽ}_{\mathfrak{ml}}^{+}\right)^{\nu_1}\right),-\left|\text{Ƚ}_{\mathfrak{ml}}^{-}\right|^{\nu_1}+\iota\left(-\left|\text{Ᵽ}_{\mathfrak{ml}}^{-}\right|^{\nu_1}\right)\end{pmatrix}\right)\\
&\quad\otimes\left(\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}},\begin{pmatrix}\left(Z_{\mathfrak{ml}}^{+}\right)^{\nu_2}+\iota\left(\text{Ꝛ}_{\mathfrak{ml}}^{+}\right)^{\nu_2}, -1+\left(1+Z_{\mathfrak{ml}}^{-}\right)^{\nu_2}+\iota\left(-1+\left(1+\text{Ꝛ}_{\mathfrak{ml}}^{-}\right)^{\nu_2}\right)\\ 1-\left(1-\text{Ƚ}_{\mathfrak{ml}}^{+}\right)^{\nu_2}+\iota\left(1-\left(1-\text{Ᵽ}_{\mathfrak{ml}}^{+}\right)^{\nu_2}\right),-\left|\text{Ƚ}_{\mathfrak{ml}}^{-}\right|^{\nu_2}+\iota\left(-\left|\text{Ᵽ}_{\mathfrak{ml}}^{-}\right|^{\nu_2}\right)\end{pmatrix}\right)\\
&=\left(\min\left(\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}},\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}}\right),\begin{pmatrix}\left(Z_{\mathfrak{ml}}^{+}\right)^{\nu_1}\left(Z_{\mathfrak{ml}}^{+}\right)^{\nu_2}+\iota\left(\text{Ꝛ}_{\mathfrak{ml}}^{+}\right)^{\nu_1}\left(\text{Ꝛ}_{\mathfrak{ml}}^{+}\right)^{\nu_2},\\ -1+\left(1+Z_{\mathfrak{ml}}^{-}\right)^{\nu_1}-1+\left(1+Z_{\mathfrak{ml}}^{-}\right)^{\nu_2}+\left(-1+\left(1+Z_{\mathfrak{ml}}^{-}\right)^{\nu_1}\times\left(-1+\left(1+Z_{\mathfrak{ml}}^{-}\right)^{\nu_2}\right)\right)\\ +\iota\left(-1+\left(1+\text{Ꝛ}_{\mathfrak{ml}}^{-}\right)^{\nu_1}-1+\left(1+\text{Ꝛ}_{\mathfrak{ml}}^{-}\right)^{\nu_2}+\left(-1+\left(1+\text{Ꝛ}_{\mathfrak{ml}}^{-}\right)^{\nu_1}\times\left(-1+\left(1+\text{Ꝛ}_{\mathfrak{ml}}^{-}\right)^{\nu_2}\right)\right)\right)\\ 1-\left(1-\text{Ƚ}_{\mathfrak{ml}}^{+}\right)^{\nu_1}+1-\left(1-\text{Ƚ}_{\mathfrak{ml}}^{+}\right)^{\nu_2}-\left(1-\left(1-\text{Ƚ}_{\mathfrak{ml}}^{+}\right)^{\nu_1}\times 1-\left(1-\text{Ƚ}_{\mathfrak{ml}}^{+}\right)^{\nu_2}\right)\\ +\iota\left(1-\left(1-\text{Ᵽ}_{\mathfrak{ml}}^{+}\right)^{\nu_1}+1-\left(1-\text{Ᵽ}_{\mathfrak{ml}}^{+}\right)^{\nu_2}-\left(1-\left(1-\text{Ᵽ}_{\mathfrak{ml}}^{+}\right)^{\nu_1}\times 1-\left(1-\text{Ᵽ}_{\mathfrak{ml}}^{+}\right)^{\nu_2}\right)\right),\\ -\left(\left(-\left|\text{Ƚ}_{\mathfrak{ml}}^{-}\right|^{\nu_1}\right)\left(-\left|\text{Ƚ}_{\mathfrak{ml}}^{-}\right|^{\nu_2}\right)\right)+\iota\left(-\left(\left(-\left|\text{Ᵽ}_{\mathfrak{ml}}^{-}\right|^{\nu_1}\right)\left(-\left|\text{Ᵽ}_{\mathfrak{ml}}^{-}\right|^{\nu_1}\right)\right)\right)\end{pmatrix}\right)\\
&=\left(\min\left(\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}},\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}}\right),\begin{pmatrix}\left(Z_{\mathfrak{ml}}^{+}\right)^{\nu_1+\nu_2}+\iota\left(\text{Ꝛ}_{\mathfrak{ml}}^{+}\right)^{\nu_1+\nu_2}, -1+\left(1+Z_{\mathfrak{ml}}^{-}\right)^{\nu_1+\nu_2}+\iota\left(-1+\left(1+\text{Ꝛ}_{\mathfrak{ml}}^{-}\right)^{\nu_1+\nu_2}\right)\\ 1-\left(1-\text{Ƚ}_{\mathfrak{ml}}^{+}\right)^{\nu_1+\nu_2}+\iota\left(1-\left(1-\text{Ᵽ}_{\mathfrak{ml}}^{+}\right)^{\nu_1+\nu_2}\right),-\left(\left|\text{Ƚ}_{\mathfrak{ml}}^{-}\right|^{\nu_1+\nu_2}\right)+\iota\left(-\left(\left|\text{Ᵽ}_{\mathfrak{ml}}^{-}\right|^{\nu_1+\nu_2}\right)\right)\end{pmatrix}\right)
\end{aligned}$$

$$\mathcal{P}^{+}=\left\{\left(\max_{\mathfrak{m}}\max_{\mathfrak{l}}\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}},\begin{pmatrix}\max_{\mathfrak{m}}\max_{\mathfrak{l}}Z_{\mathfrak{ml}}^{+}+\iota\max_{\mathfrak{m}}\max_{\mathfrak{l}}\text{Ꝛ}_{\mathfrak{ml}}^{+},\max_{\mathfrak{m}}\max_{\mathfrak{l}}Z_{\mathfrak{ml}}^{-}+\iota\max_{\mathfrak{m}}\max_{\mathfrak{l}}\text{Ꝛ}_{\mathfrak{ml}}^{-}\\ \min_{\mathfrak{m}}\min_{\mathfrak{l}}\text{Ƚ}_{\mathfrak{ml}}^{+}+\iota\min_{\mathfrak{m}}\min_{\mathfrak{l}}\text{Ᵽ}_{\mathfrak{ml}}^{+},\min_{\mathfrak{m}}\min_{\mathfrak{l}}\text{Ƚ}_{\mathfrak{ml}}^{-}+\iota\min_{\mathfrak{m}}\min_{\mathfrak{l}}\text{Ᵽ}_{\mathfrak{ml}}^{-}\end{pmatrix}\right)\right\}$$

$$\mathcal{P}^{-}=\left\{\left(\min_{\mathfrak{m}}\min_{\mathfrak{l}}\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}},\begin{pmatrix}\min_{\mathfrak{m}}\min_{\mathfrak{l}}Z_{\mathfrak{ml}}^{+}+\iota\min_{\mathfrak{m}}\min_{\mathfrak{l}}\text{Ꝛ}_{\mathfrak{ml}}^{+},\min_{\mathfrak{m}}\min_{\mathfrak{l}}Z_{\mathfrak{ml}}^{-}+\iota\min_{\mathfrak{m}}\min_{\mathfrak{l}}\text{Ꝛ}_{\mathfrak{ml}}^{-}\\ \max_{\mathfrak{m}}\max_{\mathfrak{l}}\text{Ƚ}_{\mathfrak{ml}}^{+}+\iota\max_{\mathfrak{m}}\max_{\mathfrak{l}}\text{Ᵽ}_{\mathfrak{ml}}^{+},\max_{\mathfrak{m}}\max_{\mathfrak{l}}\text{Ƚ}_{\mathfrak{ml}}^{-}+\iota\max_{\mathfrak{m}}\max_{\mathfrak{l}}\text{Ᵽ}_{\mathfrak{ml}}^{-}\end{pmatrix}\right)\right\}$$

$$\eth\left(\mathcal{G}_{\mathfrak{at-m}},\mathcal{P}^{+}\right)=\sqrt{\frac{1}{9n}\sum_{\mathfrak{l}=1}^{n}\left\{\begin{matrix}\left(\frac{\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}}}{\aleph-1}-\frac{\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathcal{P}^{+}}}{\aleph-1}\right)^{2}+\left(Z_{\mathfrak{ml}}^{+}-Z_{\mathfrak{ml}}^{+\mathcal{P}^{+}}\right)^{2}+\left(\text{Ꝛ}_{\mathfrak{ml}}^{+}-\text{Ꝛ}_{\mathfrak{ml}}^{\mathcal{P}^{+}}\right)^{2}\\ +\left(Z_{\mathfrak{ml}}^{-}-Z_{\mathfrak{ml}}^{-\mathcal{P}^{+}}\right)^{2}+\left(\text{Ꝛ}_{\mathfrak{ml}}^{-}-\text{Ꝛ}_{\mathfrak{ml}}^{-\mathcal{P}^{+}}\right)^{2}+\left(\text{Ƚ}_{\mathfrak{ml}}^{+}-\text{Ƚ}_{\mathfrak{ml}}^{+\mathcal{P}^{+}}\right)^{2}+\left(\text{Ᵽ}_{\mathfrak{ml}}^{+}-\text{Ᵽ}_{\mathfrak{ml}}^{\mathcal{P}^{+}}\right)^{2}\\ \left(\text{Ƚ}_{\mathfrak{ml}}^{-}-\text{Ƚ}_{\mathfrak{ml}}^{-\mathcal{P}^{+}}\right)^{2}+\left(\text{Ᵽ}_{\mathfrak{ml}}^{-}-\text{Ᵽ}_{\mathfrak{ml}}^{-\mathcal{P}^{+}}\right)^{2}\end{matrix}\right\}}$$

$$\eth\left(\mathcal{G}_{\mathfrak{at-m}},\mathcal{P}^{-}\right)=\sqrt{\frac{1}{9n}\sum_{\mathfrak{l}=1}^{n}\left\{\begin{matrix}\left(\frac{\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathfrak{m}}}{\aleph-1}-\frac{\mathfrak{h}_{\mathfrak{z}_{\mathfrak{l}}}^{\mathcal{P}^{-}}}{\aleph-1}\right)^{2}+\left(Z_{\mathfrak{ml}}^{+}-Z_{\mathfrak{ml}}^{+\mathcal{P}^{-}}\right)^{2}+\left(\text{Ꝛ}_{\mathfrak{ml}}^{+}-\text{Ꝛ}_{\mathfrak{ml}}^{\mathcal{P}^{-}}\right)^{2}\\ +\left(Z_{\mathfrak{ml}}^{-}-Z_{\mathfrak{ml}}^{-\mathcal{P}^{-}}\right)^{2}+\left(\text{Ꝛ}_{\mathfrak{ml}}^{-}-\text{Ꝛ}_{\mathfrak{ml}}^{-\mathcal{P}^{-}}\right)^{2}+\left(\text{Ƚ}_{\mathfrak{ml}}^{+}-\text{Ƚ}_{\mathfrak{ml}}^{+\mathcal{P}^{-}}\right)^{2}+\left(\text{Ᵽ}_{\mathfrak{ml}}^{+}-\text{Ᵽ}_{\mathfrak{ml}}^{\mathcal{P}^{-}}\right)^{2}\\ \left(\text{Ƚ}_{\mathfrak{ml}}^{-}-\text{Ƚ}_{\mathfrak{ml}}^{-\mathcal{P}^{-}}\right)^{2}+\left(\text{Ᵽ}_{\mathfrak{ml}}^{-}-\text{Ᵽ}_{\mathfrak{ml}}^{-\mathcal{P}^{-}}\right)^{2}\end{matrix}\right\}}$$

containing 2^{nd} dimension along with the truth degree (containing both positive and negative aspects) and falsity degree (containing both positive and negative aspects) and parameterization along with grades at once, because in the model of IFN-SS, the negative aspects and unreal parts of the positive aspects in both truth and falsity degrees are missing.

- In [49], Akram et al. invented the structure of BFN-SS. The structure of BFN-SS can't cope with the data containing 2^{nd} dimension along with the truth degree (containing both positive and negative aspects) and falsity degree (containing both positive and negative aspects) and parameterization along with grades at once, because in the model of BFN-SS, the falsity degree is missing and also the unreal parts in the truth degree are missing.
- In [50], Mahmood et al. invented the model of CFN-SS. The structure of CFN-SS can't cope with the data containing 2^{nd} dimension along with the truth degree (containing both positive and negative aspects) and falsity degree (containing both positive and negative aspects) and parameterization along with grades at the same time because, in the model of CFN-SS, the negative aspects in truth degree and falsity degree is missing.
- In [51], Rehman and Mahmood invented the model of CIFN-SS. The model of CIFN-SS can't cope with the data containing the 2^{nd} dimension along with the truth degree (containing both positive and negative aspects) and falsity degree (containing both positive and negative aspects) and parameterization along with grades at the same time, because in the model of CIFN-SS, the negative aspects in both truth and falsity degree are missing.

Thus, the invented work is more advanced and dominant than [38], [43], [45], [46], [47], [48] and can be degenerated into these notions. Further, our investigated TOPSIS based on BCIFN-SS can also degenerate to the setting of N-SS, FN-SS, IFN-SS, BFN-SS, CFN-SS, CIFN-SS and tackle the information in the environment of these discussed notions. Consequently, the invented BCIFN-SS can also manage the MADM (multi-attribute DM) dilemmas existing in the prevailing notions.

V. CONCLUSION

In this script, we investigated the conception of BCIFN-SS which is the modification of numerous prevailing notions. The development of this notion aims to model the information which contains the 2^{nd} dimension along with the truth degree (containing both positive and negative aspects) and falsity degree (containing both positive and negative aspects) and parameterization along with grades at the same time. In this article, we also interpreted weak, top weak, bottom weak complement, BCIF complement, weak BCIF complement, top weak BCIF complement, and bottom weak BCIF complement. Further, we investigated the extended and restricted unions and intersections for the conception of BCIFN-SS. TOPSIS approach is a DM approach for various objectives and is appropriate for managing MADM dilemmas, this article contained the TOPSIS approach relying on the interpreted BCIFN-SS. After that, we solved a DM dilemma by employing the inverted TOPSIS approach to reveal the applicability of this approach. Moreover, in this article, we revealed the dominance and enhanced the worth of the proposed BCIFN-SS by comparing it with certain prevailing conceptions such as N-SS, FN-SS, IFN-SS, BFN-SS, CFN-SS, CIFN-SS.

In the future, we would wish to spread this work in numerous fields such as complex Pythagorean FSS [52], complex cubic picture fuzzy [53], etc.

$$\mathcal{P}^+ = \left\{ \left(4, \begin{pmatrix} 0.75 + \iota 0.87, \\ -0.1 - \iota 0.12, \\ 0.15 + \iota 0.1, \\ -0.8 - \iota 0.75 \end{pmatrix}\right), \left(2, \begin{pmatrix} 0.45 + \iota 0, .5 \\ -0.4 - \iota 0.3, \\ 0.28 + \iota 0.37, \\ -036. - \iota 0.23 \end{pmatrix}\right), \left(5, \begin{pmatrix} 0.8 + \iota 0.87, \\ -0.1 - \iota 0.14, \\ 0.15 + \iota 0.12, \\ -0.82 - \iota 0.76 \end{pmatrix}\right), \left(4, \begin{pmatrix} 0.67 + \iota 0.74, \\ -0.23 - \iota 0.19, \\ 0.2 + \iota 0.1, \\ -0.53 - \iota 0.72 \end{pmatrix}\right) \right\}$$

$$\mathcal{P}^- = \left\{ \left(2, \begin{pmatrix} 0.34 + \iota 0.39, \\ -0.55 - \iota 0.49, \\ 0.49 + \iota 0.61, \\ -0.27 - \iota 0.33 \end{pmatrix}\right), \left(0, \begin{pmatrix} 0.1 + \iota 0.05, \\ -0.9 - \iota 0.8, \\ 0.83 + \iota 0.69, \\ -0.08 - \iota 0.15 \end{pmatrix}\right), \left(2, \begin{pmatrix} 0.3 + \iota 0.36, \\ -0.47 - \iota 0.67, \\ 0.46 + \iota 0.53, \\ -0.26 - \iota 0.12 \end{pmatrix}\right), \left(0, \begin{pmatrix} 0.1 + \iota 0.09, \\ -0.77 - \iota 0.83, \\ 0.68 + \iota 0.82, \\ -0.16 - \iota 0.05 \end{pmatrix}\right) \right\}$$

$$\mathcal{P}^+ = \left\{ \left(5, \begin{pmatrix} 1.0 + \iota 1.0, \\ -0.0 - \iota 0.0, \\ 0.0 + \iota 0.0, \\ -1.0 - \iota 1.0 \end{pmatrix}\right), \left(5, \begin{pmatrix} 1.0 + \iota 1.0, \\ -0.0 - \iota 0.0, \\ 0.0 + \iota 0.0, \\ -1.0 - \iota 1.0 \end{pmatrix}\right), \left(5, \begin{pmatrix} 1.0 + \iota 1.0, \\ -0.0 - \iota 0.0, \\ 0.0 + \iota 0.0, \\ -1.0 - \iota 1.0 \end{pmatrix}\right), \left(5, \begin{pmatrix} 1.0 + \iota 1.0, \\ -0.0 - \iota 0.0, \\ 0.0 + \iota 0.0, \\ -1.0 - \iota 1.0 \end{pmatrix}\right) \right\}$$

$$\mathcal{P}^- = \left\{ \left(0, \begin{pmatrix} 0.0 + \iota 0.0, \\ -1.0 - \iota 1.0, \\ 1.0 + \iota 1.0, \\ -0.0 - \iota 0.0 \end{pmatrix}\right), \left(0, \begin{pmatrix} 0.0 + \iota 0.0, \\ -1.0 - \iota 1.0, \\ 1.0 + \iota 1.0, \\ -0.0 - \iota 0.0 \end{pmatrix}\right), \left(0, \begin{pmatrix} 0.0 + \iota 0.0, \\ -1.0 - \iota 1.0, \\ 1.0 + \iota 1.0, \\ -0.0 - \iota 0.0 \end{pmatrix}\right), \left(0, \begin{pmatrix} 0.0 + \iota 0.0, \\ -1.0 - \iota 1.0, \\ 1.0 + \iota 1.0, \\ -0.0 - \iota 0.0 \end{pmatrix}\right) \right\}$$

REFERENCES

[1] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965.

[2] C.-L. Zhang, N.-J. Huang, and D. O'Regan, "Stability of solutions for fuzzy set optimization problems with applications," *Fuzzy Sets Syst.*, vol. 466, Aug. 2023, Art. no. 108470.

[3] K. Chen, V. W. Lou, and C. Y. M. Cheng, "Intention to use robotic exoskeletons by older people: A fuzzy-set qualitative comparative analysis approach," *Comput. Hum. Behav.*, vol. 141, Apr. 2023, Art. no. 107610.

[4] D. Yong, "Plant location selection based on fuzzy TOPSIS," *Int. J. Adv. Manuf. Technol.*, vol. 28, nos. 7–8, pp. 839–844, Apr. 2006.

[5] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, no. 1, pp. 87–96, Aug. 1986.

[6] J. C. R. Alcantud, "Complemental fuzzy sets: A semantic justification of q-rung orthopair fuzzy sets," *IEEE Trans. Fuzzy Syst.*, early access, May 26, 2023, doi: 10.1109/TFUZZ.2023.3280221.

[7] D.-F. Li, "Multiattribute decision making models and methods using intuitionistic fuzzy sets," *J. Comput. Syst. Sci.*, vol. 70, no. 1, pp. 73–85, Feb. 2005.

[8] H. Tian, K. Guo, X. Guan, and Z. Wu, "Anomaly detection of network traffic based on intuitionistic fuzzy set ensemble," *IEICE Trans. Commun.*, vol. E106.B, no. 7, pp. 538–546, 2023.

[9] K. Pandey, A. Mishra, P. Rani, J. Ali, and R. Chakrabortty, "Selecting features by utilizing intuitionistic fuzzy Entropy method," *Decis. Mak. Appl. Manag. Eng.*, vol. 6, no. 1, pp. 111–133, 2023.

[10] E. Haktanır and C. Kahraman, "Intuitionistic fuzzy risk adjusted discount rate and certainty equivalent methods for risky projects," *Int. J. Prod. Econ.*, vol. 257, Mar. 2023, Art. no. 108757.

[11] F. E. Boran, K. Boran, and T. Menlik, "The evaluation of renewable energy technologies for electricity generation in Turkey using intuitionistic fuzzy TOPSIS," *Energy Sources, B, Econ., Planning, Policy*, vol. 7, no. 1, pp. 81–90, Jan. 2012.

[12] B. D. Rouyendegh, A. Yildizbasi, and P. Üstünyer, "Intuitionistic fuzzy TOPSIS method for green supplier selection problem," *Soft Comput.*, vol. 24, no. 3, pp. 2215–2228, Feb. 2020.

[13] W.-R. Zhang, "Bipolar fuzzy sets and relations: A computational framework for cognitive modeling and multiagent decision analysis," in *Proc. NAFIPS/IFIS/NASA 1st Int. Joint Conf. North Amer. Fuzzy Inf. Process. Soc. Biannual Conf. Ind. Fuzzy Control Intellige*, Dec. 1994, pp. 305–309.

[14] M. Akram, "Bipolar fuzzy graphs," *Inf. Sci.*, vol. 181, no. 24, pp. 5548–5564, Dec. 2011.

[15] S. Samanta and M. Pal, "Irregular bipolar fuzzy graphs," 2012, *arXiv:1209.1682*.

[16] M. Akram and M. Arshad, "Bipolar fuzzy TOPSIS and bipolar fuzzy ELECTRE-I methods to diagnosis," *Comput. Appl. Math.*, vol. 39, no. 1, pp. 1–21, Mar. 2020.

[17] M. A. Alghamdi, N. O. Alshehri, and M. Akram, "Multi-criteria decision-making methods in bipolar fuzzy environment," *Int. J. Fuzzy Syst.*, vol. 20, no. 6, pp. 2057–2064, Aug. 2018.

[18] D. Ramot, R. Milo, M. Friedman and A. Kandel, "Complex fuzzy sets," *IEEE Trans. Fuzzy Syst.*, vol. 10, no. 2, pp. 171–186, Aug. 2002.

[19] D. E. Tamir, L. Jin, and A. Kandel, "A new interpretation of complex membership grade," *Int. J. Intell. Syst.*, vol. 26, no. 4, pp. 285–312, Apr. 2011.

[20] S. Barbat, M. Barkhordariahmadi, and V. Kermani, "Extension of the TOPSIS method for decision making problems under complex fuzzy data based on the central point index," *Adv. Fuzzy Syst.*, vol. 2022, pp. 1–14, Oct. 2022.

[21] A. Moh'd Jumah, S. Alkouri, and A. R. Salleh, "Complex intuitionistic fuzzy sets," *AIP Conf. Proc.*, vol. 1482, no. 1, 2012, pp. 464–470.

[22] M. Azam, M. S. A. Khan, and S. Yang, "A decision-making approach for the evaluation of information security management under complex intuitionistic fuzzy set environment," *J. Math.*, vol. 2022, pp. 1–30, Feb. 2022.

[23] T. Mahmood and U. Rehman, "A novel approach towards bipolar complex fuzzy sets and their applications in generalized similarity measures," *Int. J. Intell. Syst.*, vol. 37, no. 1, pp. 535–567, Jan. 2022.

[24] T. Mahmood and U. U. Rehman, "A method to multi-attribute decision making technique based on dombi aggregation operators under bipolar complex fuzzy information," *Comput. Appl. Math.*, vol. 41, no. 1, p. 47, Feb. 2022.

[25] T. Mahmood, U. U. Rehman, J. Ahmmad, and G. Santos-García, "Bipolar complex fuzzy Hamacher aggregation operators and their applications in multi-attribute decision making," *Mathematics*, vol. 10, no. 1, p. 23, Dec. 2021.

[26] T. Mahmood, U. ur Rehman, and Z. Ali, "Analysis and application of Aczel–Alsina aggregation operators based on bipolar complex fuzzy information in multiple attribute decision making," *Inf. Sci.*, vol. 619, pp. 817–833, Jan. 2023.

[27] U. U. Rehman and T. Mahmood, "The generalized dice similarity measures for bipolar complex fuzzy set and its applications to pattern recognition and medical diagnosis," *Comput. Appl. Math.*, vol. 41, no. 6, p. 265, Sep. 2022.

[28] U. U. Rehman, T. Mahmood, M. Albaity, K. Hayat, and Z. Ali, "Identification and prioritization of DevOps success factors using bipolar complex fuzzy setting with Frank aggregation operators and analytical hierarchy process," *IEEE Access*, vol. 10, pp. 74702–74721, 2022.

[29] A. Al-Husban, "Bipolar complex intuitionistic fuzzy sets," *Earthline J. Math. Sci.*, vol. 8, no. 2, pp. 273–280, Jan. 2022.

[30] D. Molodtsov, "Soft set theory—First results," *Comput. Math. With Appl.*, vol. 37, nos. 4–5, pp. 19–31, Feb. 1999.

[31] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, "On some new operations in soft set theory," *Comput. Math. With Appl.*, vol. 57, no. 9, pp. 1547–1553, May 2009.

[32] P. K. Maji, A. R. Roy, and R. Biswas, "An application of soft sets in a decision making problem," *Comput. Math. With Appl.*, vol. 44, nos. 8–9, pp. 1077–1083, Oct. 2002.

[33] P. K. Maji, R. K. Biswas, and A. Roy, "Fuzzy soft sets," *J. Fuzzy Math.*, vol. 9, no. 3, pp. 589–602, 2001.

[34] P. K. Maji, R. Biswas, and A. R. Roy, "Intuitionistic fuzzy soft sets," *J. Fuzzy Math.*, vol. 9, no. 3, pp. 677–692, 2001.

[35] S. Abdullah, M. Aslam, and K. Ullah, "Bipolar fuzzy soft sets and its applications in decision making problem," *J. Intell. Fuzzy Syst.*, vol. 27, no. 2, pp. 729–742, 2014.

[36] C. Jana and M. Pal, "Application of bipolar intuitionistic fuzzy soft sets in decision making problem," *Int. J. Fuzzy Syst. Appl.*, vol. 7, no. 3, pp. 32–55, Jul. 2018.

[37] P. Thirunavukarasu, R. Suresh, and V. Ashokkumar, "Theory of complex fuzzy soft set and its applications," *Int. J. Innov. Res. Sci. Technol.*, vol. 3, no. 10, pp. 13–18, 2017.

[38] T. Kumar and R. K. Bajaj, "On complex intuitionistic fuzzy soft sets with distance measures and entropies," *J. Math.*, vol. 2014, pp. 1–12, Dec. 2014.

[39] T. Mahmood, U. U. Rehman, A. Jaleel, J. Ahmmad, and R. Chinram, "Bipolar complex fuzzy soft sets and their applications in decision-making," *Mathematics*, vol. 10, no. 7, p. 1048, Mar. 2022.

[40] J. Gwak, H. Garg, and N. Jan, "Investigation of robotics technology based on bipolar complex intuitionistic fuzzy soft relation," *Int. J. Fuzzy Syst.*, vol. 25, no. 5, pp. 1834–1852, Jul. 2023.

[41] F. Fatimah, D. Rosadi, R. B. F. Hakim, and J. C. R. Alcantud, "N-soft sets and their decision making algorithms," *Soft Comput.*, vol. 22, no. 12, pp. 3829–3842, Jun. 2018.

[42] J. C. R. Alcantud, F. Feng, and R. R. Yager, "An N-soft set approach to rough sets," *IEEE Trans. Fuzzy Syst.*, vol. 28, no. 11, pp. 2996–3007, Nov. 2020.

[43] M. Akram, A. Adeel, and J. C. R. Alcantud, "Group decision-making methods based on hesitant N-soft sets," *Expert Syst. Appl.*, vol. 115, pp. 95–105, Jan. 2019.

[44] M. Akram, G. Ali, J. C. R. Alcantud, and F. Fatimah, "Parameter reductions in N-soft sets and their applications in decision-making," *Expert Syst.*, vol. 38, no. 1, Jan. 2021, Art. no. e12601.

[45] J. C. R. Alcantud, "The semantics of N-soft sets, their applications, and a coda about three-way decision," *Inf. Sci.*, vol. 606, pp. 837–852, Aug. 2022.

[46] M. Akram, A. Adeel, and J. C. R. Alcantud, "Fuzzy N-soft sets: A novel model with applications," *J. Intell. Fuzzy Syst.*, vol. 35, no. 4, pp. 4757–4771, Oct. 2018.

[47] F. Fatimah and J. C. R. Alcantud, "The multi-fuzzy N-soft set and its applications to decision-making," *Neural Comput. Appl.*, vol. 33, no. 17, pp. 11437–11446, Sep. 2021.

[48] M. Akram, G. Ali, and J. C. R. Alcantud, "New decision-making hybrid model: Intuitionistic fuzzy N-soft rough sets," *Soft Comput.*, vol. 23, no. 20, pp. 9853–9868, Oct. 2019.

[49] M. Akram, U. Amjad, and B. Davvaz, "Decision-making analysis based on bipolar fuzzy N-soft information," *Comput. Appl. Math.*, vol. 40, no. 6, p. 182, Sep. 2021.

[50] T. Mahmood, U. ur Rehman, and Z. Ali, "A novel complex fuzzy N-soft sets and their decision-making algorithm," *Complex Intell. Syst.*, vol. 7, no. 5, pp. 2255–2280, Oct. 2021.

[51] U. U. Rehman and T. Mahmood, "Complex intuitionistic fuzzy N-Soft sets and their applications in decision making algorithm," *Tech. J.*, vol. 27, no. 1, pp. 95–117, 2022.
[52] N. Jan, J. Gwak, Y. Jeon, and B. Akram, "Investigation of blockchain technology by using the innovative concepts of complex Pythagorean fuzzy soft information," *Complexity*, vol. 2022, pp. 1–18, Nov. 2022, doi: 10.1155/2022/2274684.
[53] J. Gwak, N. Jan, R. Maqsood, and A. Nasir, "Analysis of risks and security of E-commerce by using the novel concepts of complex cubic picture fuzzy information," *J. Function Spaces*, vol. 2022, pp. 1–27, Jul. 2022, doi: 10.1155/2022/7254306.

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