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RESEARCH ARTICLE

A Method to Improve Parameter Estimation Success

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ABSTRACT This paper introduces a method for improving parameter estimation in statistical models. Parameter estimation is a popular area of study in statistics, and recent years have seen the introduction of new distributions with more parameters to enhance modelling success. While finding a suitable model for a dataset is crucial, accurately estimating parameters is equally important. In some cases, classical parameter estimation methods fail to provide a closed form of estimation for parameters. As a result, researchers commonly resort to numerical methods and software programs for parameter estimation in models. The success rates of models have gained significance with the rising popularity of novel techniques like machine learning algorithms and artificial neural networks. Robust and reliable models are built on the strong theoretical foundations of statistical distributions. Specific distributions are used in various research fields to model datasets, and the assumptions associated with these distributions provide valuable insights into observations. Additionally, parameter estimation results sometimes lead researchers to direct conclusions. This paper presents an improvement method that relies on the estimation of parameters from other statistical distributions. This novel approach aims to make parameter estimation easier and more successful in certain situations. In the applications in this paper, the proposed methodology improves the success rate by up to 10% which provides an additional 6% success in the models.

INDEX TERMS Parameter estimation, point estimation, statistics distribution, statistical theory.

I. INTRODUCTION

Parameter estimation is a crucial process in statistics, particularly in modelling. The quest for improved modelling capabilities has led to the development of numerous new distributions with multiple parameters [1], [2], [3]. While these distributions are capable of modelling datasets, their complex structures with multiple parameters present challenges, especially when it comes to parameter estimation. Researchers often rely on programs to address this issue. However, researchers need to obtain a closed form of parameter calculation to extract valuable insights from the data [3].

The utilization of statistical distributions in various disciplines greatly aids the decision-making process. However, there is a need for user-friendly structures. Researchers desire a reliable statistical distribution that can be easily understood by anyone, enabling them to convey their studies more effectively and enhancing clarity for users in any discipline [2].

Statistical distribution users find themselves at a crossroads. On one hand, complex distributions offer significant modelling capabilities but require powerful statistical programs for parameter estimation. On the other hand, simpler structures offer limited capabilities but are easier to use [3].

In experimental studies, researchers often encounter situations where a statistical probability density function is appropriate [51]. Various standard statistical distributions are commonly used in different disciplines for modelling purposes. For example, rainfall data have been modelled using numerous statistical models, such as the 2 or 3-parameter Log-Normal distribution, the Asymptotic Extreme Value Type I (also known as Gumbel) [4], [5], [6], [7], [8], [9], [10], the 2-parameter Gamma [11], the Generalized Extreme Value [12], [13], [14], [15], the Generalized Logistic [16], [17], [18], and the Generalized Pareto [17], [18], [19] distributions, to achieve successful models.

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Certain lifetime distributions are commonly used for modelling specific datasets, including the Pareto and Weibull distributions. The Pareto distribution is named after its inventor Vilfredo Pareto and was proposed for modelling the distribution of wages in society [20]. Weibull introduced a new distribution named after himself in 1951, which proved to be a valuable model for strength datasets [21]. Since then, the Weibull distribution has been widely used for strength data analysis and is considered the most appropriate and capable distribution for such datasets [22], [23].

This study aims to provide an improved method for parameter estimation in complex structures. Researchers working with parametric models may use the proposed method in this paper to gain a better modeling success rate with better parameter estimation calculations. Especially the researchers who have to work with a determined statistical distribution or parametric statistical model with sufficient statistics could directly improve their models' success rates with the new methodology in this paper.

The main theory this study was built on is the uniqueness theorem. By using this theorem two different models are matched and the proposed methodology in this paper tries to improve the model which is wanted to be used in the dataset modeling. In the matching procedure, the capability of sufficient statistics helps researchers to use the other model without any information loss.

In this article initially, related works are investigated in the literature review section. After this section, the theory of using statistical distributions is explained, with two planned sections: probability theorems that provide important principles for utilizing distributions, and statistical theorems that elucidate the logic behind the statistical structure of distributions. After theorems, the details of the proposed methodology are explained and in the suggested methodology section advantages, disadvantages and limitations of the proposed methodology are discussed. Following these explanations, two illustrations are presented to demonstrate the success of our improvement models. The application section provides two different examples to showcase the efficacy of our proposed method.

II. LITERATURE REVIEW

In literature, many different studies have proposed new methodologies for parameter estimation to improve success or improve speed in calculations, especially in difficult modeling structures. The methodologies commonly focused on some disciplines, especially in signal processes, electronic component modeling or some devices working procedures in these disciplines. In all these studies researchers proposed to improve the efficiency of parameter estimations.

In one of these studies, authors proposed a new lower bound in parameter estimation with which the efficiency of the parameter estimation was tried to improve. Each problem is constructed on the details of the research field and some models are indispensable for researchers because according to these details and some special assumptions the models are created. Generally in these models, parameters are the results of some measurements that directly show the efficiency of some important metrics [24].

In another study, authors try to propose an algorithmic approach to improve parameter estimation efficiency for a specific model [25]. This is another indicator that some statistical models are indispensable in some disciplines. Especially high sensitive events and activities such as the reconstruction of radar targets, parameter estimation has vital importance. The quality of the parameter estimation results in expensive outputs in both success and failure [25]. Another study in the same field is to improve the efficiency of parameter estimation in different radar tasks [26]. The methodology depends on maximum likelihood estimation and Bayes estimation usage in the estimation of parameters for targets of radar.

In a study, authors try to propose a parameter estimation approach for process control systems. In that study, the dynamic systems which are vital in control systems, are evaluated and the requirements of these systems in modeling are assessed. The authors showed the importance of the parametric approach in their research field [27].

Jiang et al. proposed a special signal parameter estimation algorithm that depends on a correlation [28]. The authors proposed this model for highly sensitive signal processing events such as military communications [28]. In another study, authors compared methodologies to find the best method to improve parameter estimations in lines [29].

Moreover, parameter estimations in highly sensitive calculation fields in which systems composed of solar, atomic or nuclear components, need more attention. The successes of the models must be calculated sensitively and the model success rates must be higher. In these situations, some complex structures are proposed by researchers to improve parameter estimations in the models [30], [31].

Last but not least, there are many applications and proposals to improve parameter estimations in the applications for some specific research fields. Some studies are; improving parameter estimations in network services, quality of services and digital signal processing via machine learning classification [32], improving parameter estimations in Photovoltaic models via dynamic switch probability [33], improving parameter estimation for lithium batteries models via neural network [34], improving parameter estimation for lithium batteries models with comparing methods to gain better method [35], improving generalized and group-generalized parameter estimations for multi-criteria decision-making via Pythagorean fuzzy set [36], improving electrical power system stability via power system stabilizer parameter estimation [37], improving parameter estimation for the robot manipulators via a separation technique [38], improving parameter estimation for the 3D scanner models via plane fitting [39], improving parameter estimation for inductive power transfer system via load voltage estimation [40], improving parameter estimation for power system parameters via a rapid estimation algorithm [41] and improving parameter estimation for epidemic models via likelihood functions and Kalman filtering [42].

Besides these studies, researchers have tried to improve the general quality of the parameter estimations for each discipline. In these studies, the methodologies have been handled to create a new method to improve parameter estimation qualities in each parametric model in each research field in each situation under some assumptions. Some of these studies are; improving Bayes estimation via sparse sum of squares relaxations [43], improving maximum likelihood estimation via sparse sum of squares relaxations [44], improving parameter estimation of change point models via using Poisson distribution as discrete in the exponential changes as continuous sampler [45] and improving parameter estimation quality via an Experimental Design methodology [46].

III. MATERIAL AND METHODS

A. SOME NOTES FROM PROBABILITY THEORY

To construct an improvement method, at first, the theory behind the methodology was examined. The main theory behind the proposed method in this paper depends on the uniqueness theory in probability measurement space. Some important definitions that will be used in later theorems are below.

Definition 1: $\Omega \neq \emptyset$, and τ is a class (for definition [47]) in Ω.

1) $\emptyset \in \tau, \Omega \in \tau$
2) For $\forall A, B \in \tau, A \cap B \in \tau$
3) For every I index set (finite or infinite) under $A_i \in \tau$, $i \in I$, $\bigcup_{i \in I} A_i \in \tau$, τ class which provide all these three conditions are called a topology in Ω and (Ω, τ) binary is called as topological space.

The components of τ are called an open set, and complements of components of τ are named a closed set.

Definition 2: $\Omega \neq \emptyset$, and τ is a topology (for definition [def.1]) in Ω. In this situation with the definition of $B(\Omega) = \sigma(\tau)$, $B(\Omega)$ which is σ-algebra (for definition [47]) calls Borel algebra in Ω. Components of $B(\Omega)$ are called Borel sets.

Definition 3: Assume $\Omega = \mathbb{R}$,

$$\mathcal{U} = \{A : \text{For } \forall x \in A, \text{ there is at least one } (a, b) \in \mathbb{R} \text{ which provides } x \in (a, b) \subset A\}$$

this class is a topology in $\mathbb{R}$. This is named the usual topology of $\mathbb{R}$.

Definition 4: Assume $\Omega = \mathbb{R}$,

$B(\mathbb{R}) = \sigma(\{A \subset \mathbb{R}: A \text{ is open}\})$, algebra of $B(\mathbb{R})$ calls Borel-algebra in $\mathbb{R}$.

With the definitions 1-4, an equation in measurement field can be written as follows.

Theorem 1: $B(\mathbb{R}) = \sigma(\{(a, b) : a < b, \ a, b \in \mathbb{R}\})$.

proof: For simplicity, assume $B_1 = \{(a, b) : a < b, a, b \in \mathbb{R}\}$. $(a, b) \in B_1$. In this situation $(a, b) \in \{A : A \text{ is open}\}$, which means that $B_1 \subset \{A : A \text{ is open in } \mathbb{R}\}$. Thus $\sigma(B_1) \subset \sigma(\mathbb{R})$.

Now trying to show $\sigma(\mathbb{R}) \subset \sigma(B_1)$.

Assume A is open in $\mathbb{R}$ then there is a sequence of separate sets as (B_n) in class B_1 which is $A = \bigcup_{n=1}^{\infty} B_n$.

Thus $B_n \in \sigma(B_1)$ and $A = \bigcup_{n=1}^{\infty} B_n \in \sigma(B_1)$, meaning that $\{A : A \text{ is open in } \mathbb{R}\} \subset \sigma(B_1)$

and, $B(\mathbb{R}) \subset \sigma(B_1)$. Then $B(\mathbb{R}) = \sigma(B_1)$.

For next theorems, additional definitions are given below.

Definition 5: Assume $\Omega \neq \emptyset$, and U is a class in Ω. In this situation a function μ which goes from U to $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, \infty\}$ extended reel numbers set named as set function.

Definition 6: Assume μ is a set function defined in class U.

1) For $A \in U$, $B \in U$, A and B are separate, $A \cup B \in U$, If $\mu(A \cup B) = \mu(A) + \mu(B)$, μ is defined as a finite additive.
2) Assume (A_n) is a sequence of separate sets in U,

Under $\bigcup_{n=1}^{\infty} A_n \in U$, If $\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n)$, μ is defined as countable additive.

Definition 7: $\Omega \neq \emptyset$, and U is an algebra in Ω. For set function $\mu : U \to \overline{\mathbb{R}}$,

1) For $\forall A \in U$, $\mu(A) \in [0, \infty]$
2) $\mu(\emptyset) = 0$
3) Assume (A_n) is a sequence of disjoint sets in U,

Under $\bigcup_{n=1}^{\infty} A_n \in U$, If $\mu\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} \mu(A_n)$, μ is defined as a measure.

Definition 8: Assume μ is a measure. Under $\mu(\Omega) < \infty$, μ is defined as finite measure, otherwise if $\mu(\Omega) = \infty$, μ is defined as an infinite measure. The measure of $\mu(\Omega) = 1$ defined as a probability measure.

When μ is a measure on a σ-algebra U, if Ω can be written as an additive of sets whose measures are finite in U, μ is defined as σ-finite, and τ is a topology (for definition [def.1]) in Ω. In this situation with the definition of $B(\Omega) = \sigma(\tau)$, $B(\Omega)$ which (for definition refer to [47]) is called Borel algebra in Ω. The components of $B(\Omega)$ are called Borel sets.

Example 1 (Discrete Probability Measures): Under $\Omega = \{w_1, w_2, \ldots\}$ and $P_i \in [0, 1]$ condition,

$$\sum_{i=1}^{\infty} P_i = 1.$$

Set function P with the definition of $P(A) = \sum_{i=1}^{\infty} P_i I_A(w_i)$, $A \in P(\Omega)$ is a probability measure on power set of Ω. Here,

$$I_A(w_i) = \begin{cases} 1, & w_i \in A \\ 0, & w_i \notin A \end{cases}$$
$$I_{A \cup B}(w) = I_A(w) + I_B(w) - I_{A \cap B}(w)$$
$$I_{A \cap B}(w) = I_A(w) I_B(w)$$
$$I_{\emptyset}(w) = 0, I_{\Omega}(w) = 1$$

If $A_1, A_2, \ldots$ are separate, $I_{\bigcup_{n=1}^{\infty} A_n}(w) = \sum_{n=1}^{\infty} I_{A_n}(w)$.

Example 2 (Lebesque-Stieltjes Measures in $\mathbb{R}$): A large class of measures on $B(\mathbb{R})$ Borel-algebra is Lebesque-Stieltjes measures which are continuous, non-decreasing and emerged from $F:\mathbb{R} \rightarrow \mathbb{R}$. μ_F measures corresponding to each F with these features provides for each $-\infty < a < b < \infty$, $\mu_F((a,b]) = F(b) - F(a)$.

$$\mathbb{R} = \bigcup_{n=1}^{\infty} (-n, n], \quad \mu_F((-n,n]) = F(n) - F(-n)$$

μ_F Lebesque-Stieltjes measure is σ-finite every time.

Definition 9, gives a finite measure specialty which is needed to use in theorem 2, to reach uniqueness theorem.

Definition 9: A measure which is finite in a limited interval on $B(\mathbb{R})$ is a Radon measure. Each Radon measure is σ-finite. Thus, if a Lebesque-Stieltjes measure returns a finite value in a limited interval it is a Radon measure.

Theorem 2: Assume U is a σ-algebra on Ω and μ_1 and μ_2 are two defined measure on U. $\Lambda \subset U$, $\sigma(\Lambda) = U$ is a π-system (for definition [47]) and for each $\forall A \in \Lambda$, $\mu_1(A) = \mu_2(A)$.

1) Under μ_1 and μ_2 are finite condition, on U $\mu_1(\Omega) = \mu_2(\Omega) \Rightarrow \mu_1 = \mu_2$.
2) Assume that $A_n \uparrow \Omega$ and there are $A_1, A_2, \ldots$ on Λ which provides $\mu_1(A_n) = \mu_2(A_n) < \infty$, $n = 1, 2, \ldots$ In this situation $\mu_1 = \mu_2$ on U.■

Now assuming that $(\mathbb{R}, B(\mathbb{R}, P))$ is a probability space, examine the function below;

$$F(x) = P((-\infty, x]), \quad x \in \mathbb{R}$$

In that;

1) F is non-decreasing,
2) F is right continuous,
3) $\lim_{x\to\infty} F(x) = 1$, $\lim_{x\to-\infty} F(x) = 0$
4) F has maximum jump point which is countable.

When $F:\mathbb{R} \rightarrow \mathbb{R}$ provides (i), (ii), (iii) it names as a distribution function on $\mathbb{R}$.

After this, trying to answer whether F distribution function with the definition of$F(x) = P((-\infty, x])$, $x \in \mathbb{R}$, defines P probability measure uniquely.

When $(\mathbb{R}, B(\mathbb{R}), P_1)$ and $(\mathbb{R}, B(\mathbb{R}), P_2)$ are two probability measure with the same F distribution function, if $P_1 = P_2$ on $\mathbb{R}$, F distribution function defines P probability measure uniquely.

$$F(x) = P_1((-\infty, x]) = P_2((-\infty, x]), \ x \in \mathbb{R}$$

Now consider class $\Lambda = \{(-\infty, x] : x \in \mathbb{R}\}$ on $\mathbb{R}$. Λ is a π-system on $\mathbb{R}$. P_1, P_2 are both measures and on Λ $P_1((-\infty, x]) = P_2((-\infty, x])$, $\forall x \in \mathbb{R}$ and because $P_1(\mathbb{R}) = P_2(\mathbb{R}) = 1$ for each $A \in \sigma(\Lambda) = B(\mathbb{R})$, $P_1(\Lambda) = P_2(\Lambda)$. That means F defines P probability measure uniquely.

Another conclusion for the last statement is, the results of these two different probability functions are the same in each point in the probability space. The backbone of the theory in this paper depends on this inference.

B. SOME NOTES FROM STATISTICAL THEORY

For an unknown parameter θ, sufficient statistics are statistics which summarize the information of parameter in dataset. While sufficient statistics summarize the information for the parameter, it gives every information for the parameter without subtracting. Thus, researchers may use these statistics to gain every information from the dataset.

To identify sufficient statistics in theorem 3, additional definitions which are given in definition 10-12, are needed.

Definition 10: A set of probability measures P_θ on $(\Omega,\mathcal{F})$ indexed by a parameter $\theta \in \Theta$ is said to be a parametric family if and only if $\Theta \subset \mathbb{R}^d$ for some fixed positive integer d each P_θ is a known probability measure when θ is known. The set Θ is called the parameter space and d is called its dimension [48].■

Definition 11 (Exponential Families): A parametric family $\{P_\theta : \theta \in \Theta\}$ dominated by a σ-finite measure v on $(\Omega,\mathcal{F})$ is called an exponential family if and only if

$$\frac{dP_\theta}{dv}(w) = exp\left\{[\eta(\theta)]^\tau T(w) - \xi(\theta)\right\} h(w), \quad w \in \Omega,$$

where T is a random p-vector with a fixed positive integer p, η is a function from Θ to $\mathbb{R}^p$, h is a nonnegative Borel function on $(\Omega,\mathcal{F})$, and

$$\xi(\theta) = \log\left\{\int exp\left\{[\eta(\theta)]^\tau T(w)\right\} h(w) dv(w)\right\}.$$

[48].■

On def.11 η is called natural parameter.

Definition 12: Assume $X_1, X_2, \ldots, X_n$ is a sample with probability density function $f(., \theta) \in \mathcal{F}$ and $T(X_1, X_2, \ldots, X_m)$ is a statistics, when $T = t \in \mathbb{R}^m$ is known. If the conditional distribution of $X_1, X_2, \ldots, X_n$ is not depend on θ for t then the statistic T is named as a sufficient statistic for $\mathcal{F}$ family or θ parameter. ■

Assume $X_1, X_2, \ldots, X_n$ is a sample with probability density function $f(., \theta) \in \mathcal{F}$ and $T(X_1, X_2, \ldots, X_m)$ is a statistics. When $T(X_1, X_2, \ldots, X_m)$ is sufficient, for $\forall B \in B(\mathbb{R}^n)$, the probability of, $P((X_1, X_2, \ldots, X_n) \in B \mid T = t)$ is independent from θ. If there is a sufficient statistic, every function of this statistic is sufficient either.

Theorem 3: Assume that X is a sample from $P \in \mathcal{P}$ and $\mathcal{P}$ is a family of probability measures on $(\mathcal{R}^n, \mathcal{B}^n)$ dominated by σ-finite measure v. Then $T(X)$ is sufficient for $P \in \mathcal{P}$ if and only if there are nonnegative Borel function h which does not depend on P on $(\mathcal{R}^n, \mathcal{B}^n)$ and g_P which depends on P on the range of T such that

$$\frac{dP_\theta}{dv}(x) = g_P(T(X)) h(X).$$

(for proof please refer to [Chapter 2 in [48]])

After these definitions first conclusion is; if a parameter estimation depends on a sufficient statistic, every information for this parameter is taken from data and estimation includes this information completely. The second conclusion is that every probability distribution function, measures probability value uniquely and this specialty is very strong in probability theory. In addition to second conclusion, when two probability measure returns the same value on the same probability space, these functions are same on that space.

In the proposed method in this paper, a distribution increases its modeling capability on a data set in another distributions' capable probability distribution zone. Thus the modelling ability for datasets is improved.

While operating this process, for each probability point an equation is created and the equation number will be equal to observation number. In this process mean of these point estimations will be taken to reach new estimation value.

Let $F_1\left(x, \underline{\mu}, \underline{\varphi}\right)$ be a candidate distribution for a defined data set and parameter vector $\underline{\varphi}$ contains sufficient statistics, and $F_2\left(x, \underline{\sigma}, \underline{\omega}\right)$ is another candidate with sufficient statistics for parameter vector $\underline{\omega}$. Then for each observation;

$$F_1\left(x_i, \underline{\mu}, \underline{\varphi}\right) = F_2\left(x_i, \underline{\sigma}, \underline{\omega}\right) \tag{1}$$

Thus, the conclusion in this process with the mean of estimations may be gained as below.

$$\widehat{\underline{\varphi}} = \frac{1}{n}\sum_{i=1}^{n}\widehat{\underline{\varphi_i}} \tag{2}$$

$$\widehat{\underline{\omega}} = \frac{1}{n}\sum_{i=1}^{n}\widehat{\underline{\omega_i}} \tag{3}$$

The main principle for this improvement method is increasing the modelling capability of candidate statistical distribution which has a lower modelling ability. As mentioned earlier, researchers may decide to use distribution with lower modelling ability. After using the improvement method, modelling capability converges to another candidate distribution which has the higher ability.

IV. SUGGESTED METHODOLOGY

A. DETAILS OF THE PROPOSED METHODOLOGY

With the proposed methodology in this article, users may increase their proposed model success rates by using another candidate model proposal under the theory of probability and statistics. With the capability of the uniqueness theorem and the power of sufficiency, the probability measurements in the same probability space can be equalized and new equations for parameters may be gained.

The steps in the proposed methodology are;

1) Create a model with the statistical distribution you want to use which has sufficient statistics.
2) Check if the model is appropriate according to a goodness of fit test.
3) Check if the plausibility rate of your model is high enough and if not find a different model proposal with another distribution which has sufficient statistics again.
4) Check if the last model is appropriate according to a goodness of fit test and evaluate the plausibility.
5) If the plausibility is high enough, match the model you want and the model with higher plausibility by using sufficient statistics and calculate your parameter estimation in the model you want to use.

The flow diagram of the proposed is in Figure 1.

In Figure 1, it is clear that the aim is to increase the plausibility rate of the proposed model with better parameter estimation quality. In each step, the methodology searches for better options to calculate parameter estimation and with this methodology, the plausibility rate of the proposed model which is detected at the beginning of the researcher's study is expected to converge the plausibility rate of the model which has a bigger success rate.

One of the most important issues which researchers have to pay attention to is using sufficient statistics. While using this methodology, to be successful and reliable, researchers have to be sure that there is not any information loss during the parameter estimation process. Because of this reason, sufficient statistics usage has to be validated.

B. ADVANTAGES OF THE PROPOSED METHODOLOGY

To evaluate the proposed methodology clearly, itemising advantages, disadvantages and limitations may be helpful. The advantages of this proposed methodology are;

1) Using robust probability theory and statistics theory behind the methodology, provides reliability.
2) Using sufficient statistics protects methodology from information loss.
3) Improve success rate in the same parametric model, therefore researchers do not have to change the proposed model structure.
4) The calculation of parameter estimation is easy after matching models. The closed form of parameter estimation equations can be gained easily.
5) A wide range of using areas. If your model includes sufficient statistics, you can use this method to improve parameter estimation.
6) Many new studies may be constructed on this methodology such as creating software, overcoming other obstacles by using theory, etc.

C. DISADVANTAGES OF THE PROPOSED METHODOLOGY

The disadvantages of this proposed methodology are;

1) To use this proposed methodology, the researcher has to find a better model which has sufficient statistics.
2) The proposed methodology converges the success rate to the other candidate model which means with using this methodology, the success rate is expected to be close to the other candidate, not more than the candidate's success rate.

D. LIMITATIONS OF THE PROPOSED METHODOLOGY

The limitations of this proposed methodology are;

1) The parametric model has to have sufficient statistics.
2) The researcher has to find a better model which has sufficient statistics and the model has to be successful enough.

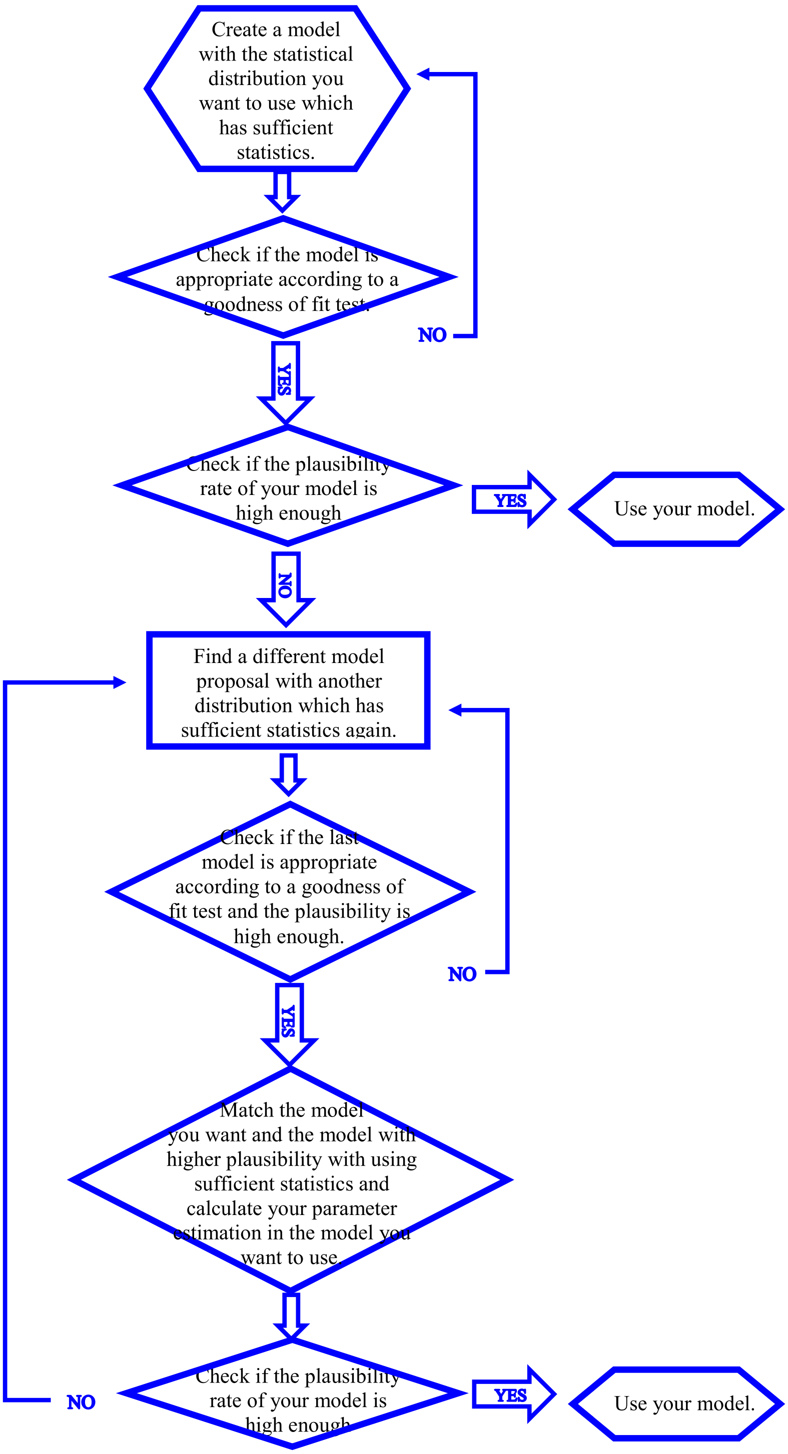

FIGURE 1. Flow diagram of the proposed method.

By examining the advantages, disadvantages and limitations of the proposed methodology, we can conclude that although there are some limitations and disadvantages, this methodology can support researchers in their studies efficiently. Nearly each method proposals have disadvantages and limitations, but the limitations of the proposed methodology in this article come from the statistics and probability theories which is the emerging point of measurement of all statistical science.

Moreover, in some studies, even a 1% improvement is very valuable for researchers. Especially in highly sensitive studies like space events, and military activities, some health science study areas researchers do not want to lose any opportunity to improve their model success rate. This study may help researchers to gain better parameter estimation performance.

V. APPLICATION

By using Theorem 3 the sufficient statistic for exponential distribution may be found. The structure of the probability density function of the exponential distribution is as follows.

$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$$

The vector form of exponential distributions' probability density function with n sample is as follows.

$$f\left(\underline{x}, \theta\right) = \frac{1}{\theta^n} e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i}$$

In this vector form $g\left(T\left(\underline{x}\right), \theta\right) = \frac{1}{\theta^n} e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i}$, and $h\left(\underline{x}\right) = 1$.

Thus $\sum_{i=1}^{n} x_i$ is a sufficient statistic for the exponential distribution parameter θ.

According to definition 12, every function of this statistic is sufficient, like $\frac{\sum_{i=1}^{n} x_i}{n} = \overline{x}$, where $\overline{x}$ is the sample mean.

Now the target is to gain maximum likelihood estimation for the parameter of the exponential distribution. For this, at first a likelihood function definition is given.

$$L\left(\underline{x} \mid \theta\right) = \frac{1}{\theta^n} e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i}$$

By using derivation, extreme points may be obtained.

$$\frac{\partial}{\partial \theta} L\left(\underline{x} \mid \theta\right) = e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i} \frac{1}{\theta^n}\left(\frac{1}{\theta^2}\sum_{i=1}^{n} x_i - \frac{n}{\theta}\right) = 0$$

Because $e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i}$ and $\frac{1}{\theta^n}$ can not be 0, other components in multiplication must be 0.

$$\frac{1}{\theta^2}\sum_{i=1}^{n} x_i = \frac{n}{\theta}$$
$$\widehat{\theta} = \overline{x} \tag{4}$$

Therefore a conclusion that $\overline{x}$ is maximum likelihood estimator for exponential distribution and this estimator is *sufficient.*

Once any researcher wants to look for sufficient statistics for Weibull parameters, it is not as easy as exponential distribution case. In a resource, by using Lehmann-Scheffe Theorem, $\overline{x}$ is sufficient statistics for shape parameter of Weibull [49]. When any researcher wants to look for sufficient statistics for Pareto parameters, there is a plain sufficient statistic for one parameter under a condition that the other parameter is known [50].

With the conclusion in section two, there is a question; whether any estimator like $\overline{x}$ in exponential distribution can improve parameter estimation success on alternative model for the same data set. Trying to find the answer to this question with examples 3 and 4.

Example 3: In section II it was shown that if a probability function is appropriate for a data set in a probability space, then this measure is unique. Many times in statistics, data sets can be modelled by different statistical distributions simultaneously. To illustrate this, the exponential and Weibull distributions are used in this example. Assume both distributions are capable of modelling a defined data set and assume both models have more than 0.5 in p-value in Kolmogorov-Smirnov test statistics (where p-value indicates explanation rate of offered model). Now try to match these two models with defined parameter estimation values. Survival functions of these distributions will be used as below.

$$S_{exponential}(x) = e^{-\frac{x}{\theta}} \tag{5}$$
$$S_{weibull}(x) = e^{-\left(\frac{x}{\lambda}\right)^k} \tag{6}$$

Let $X_1, X_2, \ldots, X_n$ be random variables. Under theorem 2 each probability measure defines each probability value in every point uniquely. When these two probability values are matched in each observation, the result is below.

$$e^{-\frac{x_i}{\theta}} = e^{-\left(\frac{x_i}{\lambda}\right)^k} \tag{7}$$

New estimation for shape parameter of Weibull is below.

$$\hat{k} = \frac{1}{n}\sum_{i=1}^{n} \frac{\log x_i - \log\theta}{\log x_i - \log\lambda} \tag{8}$$

Example 4: For another illustration Pareto distribution type 2 will be evaluated. Parameter equations of Pareto distribution and exponential distribution by the proposed method can be gained as below.

$$S_{exponential}(x) = e^{-\frac{x}{\theta}} \tag{9}$$
$$S_{pareto}(x) = \left(\frac{\beta}{x+\beta}\right)^{\alpha} \tag{10}$$

Let $X_1, X_2, \ldots, X_n$ be random variables. Under theorem 2 each probability measure defines each probability value in every point uniquely. When these two probability values are matched with condition of defined distributions' maximum likelihood estimation in each observation, the equation which is below is gained.

$$e^{-\frac{x_i}{\theta}} = \left(\frac{\beta}{x_i+\beta}\right)^{\alpha} \tag{11}$$

New estimations for parameters are below.

$$\widehat{\theta} = \frac{1}{n}\sum_{i=1}^{n} \frac{-x_i}{\alpha\left(\log\beta - \log\left(x_i+\beta\right)\right)} \tag{12}$$
$$\widehat{\alpha} = \frac{1}{n}\sum_{i=1}^{n} \frac{-x_i}{\theta\left(\log\beta - \log\left(x_i+\beta\right)\right)} \tag{13}$$

For parameter β numeric methods may be better to gain equation. To gain pure success with the new improvement method only α and θ improvements in Pareto and exponential distribution will be used.

TABLE 1. Vinyl chloride data.

5.1	1.2	1.3	0.6	0.5	2.4	0.5	1.1	8	0.8	0.4	0.1
0.4	2	0.5	5.3	3.2	2.7	2.9	2.5	2.3	1	0.2	
1.8	0.9	2	4	6.8	1.2	0.4	0.6	0.9	0.2	0.2	

TABLE 2. Vinyl chloride data test results.

Model	K-S	p	MLE Parameter estimations	
Exponential	0.0894	0.9257	$\widehat{\theta} = 1.8823$	
Weibull	0.0963	0.8377	$\widehat{\lambda} = 1.9$	$\hat{k} = 1.024$

TABLE 3. Vinyl chloride data test results after improvement (Improve Weibull by using Exponential.)

Model	K-S	p	MLE Parameter estimations	
Exponential	0.0894	0.9257	$\widehat{\theta} = 1.8823$	
Weibull	0.094	0.8971	$\widehat{\lambda} = 1.9$	$\hat{k} = 1.006$

TABLE 4. Strength of Kevlar epoxy material.

0.54	0.80	1.52	2.05	1.03	1.18	1.52	0.19	1.51	1.64
0.60	0.72	0.63	1.29	1.11	1.45	0.34	0.24	0.23	4.69
0.12	0.92	0.56	1.33	3.34	0.18	1.51	0.40	1.45	0.72
1.80	1.05	2.17	0.80	1.54	0.09	1.55	0.07	7.89	0.08
1.60	1.43	3.03	1.81	0.08	0.03	0.02	0.65	1.58	0.03

TABLE 5. Strength of Kevlar epoxy material test results.

Model	K-S	p	MLE Parameter estimations	
Exponential	0.1074	0.5739	$\widehat{\theta} = 1.2222$	
Weibull	0.1070	0.5786	$\widehat{\lambda} = 1.205$	$\hat{k} = 0.9681$
Pareto	0.1024	0.633	$\widehat{\alpha} = 18.95$	$\widehat{\beta} = 21.93$

With the applications made in Dataset 1 and Dataset 2, the use of the information given in examples 3 and 4 will be demonstrated. For illustration, two different data sets will be used. In these applications maximum likelihood estimations will be used and each distribution will have more than 0.5 p-value in Kolmogorov-Smirnov test statistics in modelling.

Dataset 1: This data set was used by [51], and later it was used in a study for indicating the efficiency of decreasing failure rate in lifetime distributions [52]. The data set carries the vinyl chloride level which was obtained from cleaned-up gradient monitoring wells in mg/l:

In Table 2 it is clear that the exponential distribution is a better modelling opportunity. As a lifetime distribution, Weibull is used commonly in data sets which are observed in nature. When a scientist decides on using Weibull in this kind of data sets, he/she does not want any decrease in modelling success. Moreover, he/she wants to improve modelling capability of their distribution with better point estimation.

Once the improvement method suggested in the current study is used, the last modelling results may be obtained as values in Table 3.

Table 3 shows that this method adds 6% success to the capability of the Weibull distribution in modelling dataset 1.

As stated in the previous section, the models that can be evaluated in the methodology must have sufficient statistics and be successful enough. Thus, only two known statistical distributions are used for this dataset.

Dataset 2: The strength of Kevlar epoxy material which was used in the NASA space shuttles was measured. The breaking strength was tested at the 90% pressure level. This data set represents time to failure (in hours) from 50 epoxy observations. [53].

In Table 5 it is clear that the modelling capabilities of Weibull and Exponential distributions are nearly equal. As a lifetime distribution, Weibull is commonly used in strength data sets [54], [55], [56]. When a scientist decides to use Weibull distribution in this kind of data set, the proposed method may be helpful.

From Table 6 it is concluded that this method adds 6% success to the capability of the Weibull distribution in modelling dataset 2.

After this improvement method, it was concluded that Weibull distribution offers a better modelling capability. In addition, the question of whether this increase is valid in the opposite case has been tried to be examined. When improvement method on exponential distribution under

TABLE 6. Strength of Kevlar epoxy material test results after improvement (Improve Weibull by using Pareto.)

Model	K-S	p	MLE Parameter estimations
Exponential	0.1074	0.5739	$\widehat{\theta} = 1.2222$
Weibull	0.1022	0.635	$\widehat{\lambda} = 1.205$ $\quad \hat{k} = 1.0029$
Pareto	0.1024	0.633	$\widehat{\alpha} = 18.95$ $\quad \widehat{\beta} = 21.93$

TABLE 7. Strength of Kevlar epoxy material test results after second improvement (Improve Exponential by using Weibull.)

Model	K-S	p	MLE Parameter estimations
Exponential	0.1032	0.6237	$\widehat{\theta} = 1.207$
Weibull	0.1022	0.635	$\widehat{\lambda} = 1.205$ $\quad \hat{k} = 1.0029$
Pareto	0.1024	0.633	$\widehat{\alpha} = 18.95$ $\quad \widehat{\beta} = 21.93$

TABLE 8. Strength of Kevlar epoxy material test results after third improvement (Improve Exponential by using Pareto.)

Model	K-S	p	MLE Parameter estimations
Exponential	0.1020	0.638	$\widehat{\theta} = 1.1892$
Weibull	0.1022	0.635	$\widehat{\lambda} = 1.205$ $\quad \hat{k} = 1.0029$
Pareto	0.1024	0.633	$\widehat{\alpha} = 18.95$ $\quad \widehat{\beta} = 21.93$

TABLE 9. Outputs of the proposed methodology.

Dataset	Model	Previous plausibility	Plausibility after using method	Improvement rate
Vinyl chloride	Weibull	0.8377	0.8971	7%
Kevlar epoxy	Weibull	0.5786	0.635	9.7%
Kevlar epoxy	Exponential	0.5739	0.6237	8.6%

Weibull values is carried out, the results shown in Table 7 are gained.

In Table 7 it is concluded that this method adds 5% success to the capability of the exponential distribution in modelling dataset 2.

When the Pareto distribution for increasing the capability of exponential distribution in this data set is used, the results shown in Table 8 are gained.

From the results presented in Table 8, it can be concluded that this improvement method increases a candidate distribution's capability with another candidate distribution. While doing this, the parameters of both distributions have to satisfy sufficiency.

VI. CONCLUSION

To improve the success of parameter estimation, a novel method is introduced in this study. Initially, the study defines the theoretical foundations of accurate parameter estimation. Subsequently, emphasis is placed on sufficiency and the specific theories behind point estimation methods. By leveraging these robust theories, a new method for enhancing parameter estimation is proposed.

The findings indicate that the newly developed improvement method enhances the modelling capability by achieving better point estimation for parameters. It is concluded that when two models possess sufficient statistics for parameter estimation, this method can lead to higher levels of success.

The application section presents two different examples showcasing the efficacy of this new method, particularly in statistical distributions with sufficient statistics. While implementing this methodology, we used exponential distribution, Weibull distribution which has a widespread usage area in strength datasets and Pareto distribution which has a widespread usage area in economy and finance related datasets. In the usage processes, we only used sufficient statistics in the estimations for parameters and only the closed form of equations were used. For instance, we used only one parameter of Weibull because the other parameter did not provide the assumptions. By using the proposed method, the model success rates increased by 6%, 6% and 5% in the three experiments in the application section. The outputs of the proposed methodology in the application section are summarized in Table 9.

From the summarized information presented in Table 9, it can be concluded that this improvement method provides important increase rates, especially in the fields in which researchers have to use determined parametric models. For instance, if any researcher has to use a parametric model which is declared in a project proposal and this usage is a must, the proposed methodology in this article may help the researcher to increase modeling capability.

These results demonstrate that the method can be valuable for researchers who rely on specific statistical distributions dictated by their theories and assumptions. By employing this innovative method, researchers in various disciplines can achieve higher levels of success in the modelling process.

COMPETING INTERESTS

No competing interest is declared.

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