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# **HII RESEARCH ARTICLE**

# Scale-Free Patterns by Synchronous Word Connectivity Sustainably for Chronological Variation

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**ABSTRACT** The evolution of natural language is characterized by the incorporation of a dependency structure into word sequences. Scale-free pattern regularity, in which the frequency of words conforms to a common pattern based on their rank in a list, is empirically well-known. As statistical regularity has been observed in natural and social phenomena, understanding its standard principle is of great significance. To yield a scale-free rank-size relation, procedures have been established for distributing resources such as words to elements based on the rich-get-richer mechanism. However, such procedures do not address the relevance of the dependency among elements. Using a scheme that considers the occurrence of a word as the task of assigning an energy particle to an element in a dissipative system, I implemented a computational system to select synchronously connected elements in conjunction with a model of the noiseinduced synchronization phenomenon. I empirically demonstrated that the energy particle distribution for spatio-chronological freedom could provide a scale-free rank-size relation to reach a steady state of dynamic equilibrium corresponding to the most probable case in the system. Furthermore, I observed an interference fringe-like pattern in the scale-free energy particle distribution that self-organized into a stationary wave through the superposition of a wave function for energy particle assignments. The results obtained by the computational system demonstrate that statistical regularity can be provided by dependency relations with chronological variation sustainability, paving the way for the development of a methodology for evolutionary language modeling.

**INDEX TERMS** Dynamic equilibrium, evolutionary systems, natural language processing, noise-induced synchronization phenomenon, scale-free pattern regularity, word dependency.

### **I. INTRODUCTION**

According to structural linguistics  $[1]$ , a set of words is constructed based on a balanced interdependence relationship among adjacent concepts that are socially used because words are defined by their correspondence to objects. Regarding the frequency of words, the scale-free ranksize relation exhibits an empirical regularity known as Zipf's law [\[2\]. B](#page-5-1)ecause statistical regularity has been observed in both natural and social phenomena, such as city size distributions  $[3]$ , firm size distributions  $[4]$ , password distributions  $[5]$ , gene expression  $[6]$ , and hyperlinks on the

<span id="page-0-5"></span><span id="page-0-4"></span><span id="page-0-2"></span><span id="page-0-1"></span>The associate editor coordinating the review of this manuscript and approving it for publication was Chun-Wei Tsai<sup>D</sup>[.](https://orcid.org/0000-0003-0128-4052)

<span id="page-0-17"></span><span id="page-0-16"></span><span id="page-0-15"></span><span id="page-0-14"></span><span id="page-0-13"></span><span id="page-0-12"></span><span id="page-0-11"></span><span id="page-0-10"></span><span id="page-0-9"></span><span id="page-0-8"></span><span id="page-0-7"></span><span id="page-0-6"></span><span id="page-0-3"></span><span id="page-0-0"></span>World Wide Web [\[7\], un](#page-5-6)derstanding its standard principle is of great interest [\[8\],](#page-5-7) [\[9\],](#page-5-8) [\[10\],](#page-5-9) [\[11\],](#page-5-10) [\[12\],](#page-5-11) [\[13\]. T](#page-5-12)o yield a scalefree rank-size relation, resource distribution procedures have been established, such as words to elements based on the richget-richer mechanism [\[14\]. U](#page-5-13)nder this mechanism, elements are additionally selected with their probabilities proportional to their selected frequencies  $[15]$ . In 2019, a procedure using a pseudorandom number generator for periodical resource distribution with phase modulation to form a scale-free pattern was introduced [\[16\]](#page-6-0) from the perspective of self-organized criticality in complex systems [\[17\]. A](#page-6-1)s the procedures were concerned with the derivation of the rank-size relation [\[18\],](#page-6-2) they failed to address the relevance of the dependency among elements. The origin of the dependency structure for

acquiring languages has been discussed [\[19\]; i](#page-6-3)t is viewed as driven by a human-specific innate capacity  $[20]$ ,  $[21]$ . Although physics-inspired approaches have been used to investigate the growth of words and phrases [\[22\],](#page-6-6) [\[23\], h](#page-6-7)erein the occurrence of a word as the task of assigning an energy particle to an element in a dissipative system [\[24\],](#page-6-8) [\[25\]](#page-6-9) comprising interacting elements was considered. In a dissipative system, the regularity of an element's behavior can be observed when the system reaches a steady state, which is a balance between the energy supplied from outside the system and the energy dissipated within the system. Noise-induced synchronization [\[26\],](#page-6-10) [\[27\]](#page-6-11) in a dissipative system has been studied to provide a quasi-periodic rhythm through synchronization entrained by noise-loaded nonlinear interactions among oscillators based on the interaction of the heartbeat and respiration [\[28\]. B](#page-6-12)ecause synchronous word connectivity can give rise to the appearance of scale-free patterns, this study implemented a synchronously connected element selection system in which the elements were located to link oscillators in a noise-induced synchronization model. Using the computational system, the relevance of the dependency among elements in yielding scale-free rank-size distributions was examined. Subsequently, the connectivity among the elements that were synchronously selected owing to dissipative system behavior was investigated.

#### **II. RELATED WORK**

<span id="page-1-11"></span><span id="page-1-10"></span>An investigation into the scale-free rank-size regularity of word frequency has been deemed an essential avenue to understanding the usage and evolution of natural language [\[13\]. T](#page-5-12)he emergence of statistical regularity within the context of a communication framework involving speakers and listeners has been attributed to the principle of least effort [\[29\]. T](#page-6-13)his principle posits that speakers minimize effort by favoring commonly used words, while listeners reduce effort by expecting predictable words. In 2021, a study employing a statistical index to elucidate the variety within sets of words [\[30\]](#page-6-14) observed that statistical regularity could coincide with maximal diversity, as gauged by the informational content conveyed through word sequences. Diverging from this previous research, the present study adopts a system dynamics approach [\[31\], f](#page-6-15)ocusing on a framework in which words are progressively exchanged within a social communication system. Therefore, assuming that synchronous word connectivity is linked to the noiseinduced synchronization observed in dissipative systems, this study implements a computational system that consecutively selects synchronously connected elements to investigate the evolutionary dynamics of the system leading to the appearance of scale-free patterns.

#### **III. METHODS**

# A. SYNCHRONOUSLY CONNECTED ELEMENT SELECTION SYSTEM

In a synchronously connected element selection system, synchronous word occurrences are associated with energy

<span id="page-1-6"></span><span id="page-1-5"></span><span id="page-1-4"></span><span id="page-1-3"></span><span id="page-1-2"></span><span id="page-1-1"></span><span id="page-1-0"></span>particle assignments to elements using a noise-induced synchronization model [\[27\]. T](#page-6-11)he model is based on a circle map that is used to describe the quasi-periodic behavior of the phase difference between oscillators and was developed based on a study of the synchrony between human heartbeats and pedaling motion with music modulation [\[27\]. R](#page-6-11)andom dynamics with two stochastic terms regarding internal and external noise was modeled to reproduce a time series of experimental physiological data. The distribution procedure for the synchronously connected element selection system using the model is as follows:

- <span id="page-1-8"></span><span id="page-1-7"></span>1: *N* energy particles are initially assigned to elements  $m_{(i)}$  ( $i = 1, \dots, M$ )
- 2: A random angle  $\theta_{(t=0)}$  is initially set
- <span id="page-1-9"></span>3:  $t = 1$ 4: **loop**
- 5: Angle  $\theta(t)$  is calculated as  $\theta(t) = \theta(t-1) \phi$
- 6: An element  $m_{(i^c)}$  corresponding to angle  $\theta_{(t)}$  is selected
- 7: **while** the number of energy particles assigned to  $m_{(i^c)} > 0$  do
- 8: Angle  $\theta_{(t)}^p$  is calculated by the following synchronous dynamical model
- 9: θ  $\frac{p}{(t)} = \theta_{(t)} + \varphi + \xi_{(t)} - \eta_{(t)} \frac{K}{2\pi} \cos(4\pi \theta_{(t)})$
- 10: A randomly chosen energy particle assigned to  $m_{(i^c)}$  is reassigned to  $m_{(i^p)}$  selected by the corresponding angle  $\theta_{(t)}^p$
- 11: **end while**
- $12 \cdot \quad t = t + 1$

13: **end loop**



<span id="page-1-14"></span><span id="page-1-13"></span><span id="page-1-12"></span>In the equation describing the synchronous dynamical model (line #9), the phase parameter  $\varphi$  provides one rotation per iteration and  $\xi$  denotes the internal noise in the form of independent white Gaussian noise,  $N(0, \sigma^2)$ . The multiplicative term is the external noise, which is smaller than the internal noise. The random variable  $\eta$  is independent noise whose amplitude is 0 and 1 with probabilities of *p* and  $1 - p$ , respectively, and  $K > 0$  is a control parameter for the amplitude. By locating elements linked to oscillators arranged at equal intervals along the circle, the state transition of energy particle reassignment to a selected element was performed by an iteration. Using a random number generator [\[32\], t](#page-6-16)he parameter values were set as  $\sigma = 0.05$ ,  $p = 0.28$ , and  $K = 0.8$  [\[27\]. F](#page-6-11)or reassigning an energy particle from a randomly chosen element, mapping by consecutive rotations was applied using the golden angle (line #5) to achieve chronologically unbiased choices of elements [\[33\]](#page-6-17) in the state transitions in the system. While rotations through a rational number-valued angle should be uneven with repeated returns to limited intrinsic angles, mapping by rotations through the golden angle was considered, which is the most difficult irrational number to approximate using rational numbers, resulting in an optimal circular spread arrangement.

# B. ANALYSIS OF SCALE-FREE PATTERN REGULARITY

For *k* frequency values observed for occurrence,  $X = \{x_1, x_2,$  $\cdots$ , $x_k$ } ( $x_1 \ge x_2 \ge \cdots \ge x_k > 0$ ), the relation between *X* and rank *n* is described using a power function of

$$
X = Cn^{-\lambda}, \quad (C: constant)
$$

which provides the regularity of scale-free patterns. Here,  $\lambda$  ( $\lambda$ >0) is a scaling exponent, and Zipf's law holds when  $\lambda = 1$  [\[8\]. S](#page-5-7)cale-free patterns are identified when the rank-ordered distribution forms a straight line on a graph with logarithmic axes. The scaling exponent is obtained by the maximum likelihood method [\[34\]](#page-6-18) to estimate the parameters of the data generation function that maximize the probability of data generation. For the probability function  $P(x|\Psi)$ , the parameter  $\Psi$  can be estimated as

<span id="page-2-1"></span>
$$
\hat{\Psi} = \arg \max_{\Psi} L ,
$$

$$
L = \prod_{i=1}^{n} P(x_i | \Psi) ,
$$

where likelihood *L* shows the degree of fitting to the probability function. The negative log-likelihood  $LL$  (= $-\log L$ ) is stated as an appropriate measure for the presence of the scale-free pattern, where smaller values indicate better regression [\[13\].](#page-5-12) For distribution changes with system state transitions, parameters estimated by the leastsquares method were employed to minimize the residual  $\varepsilon$ , where

$$
\varepsilon = \sqrt{\frac{1}{k} \sum_{i=1}^{k} (\log x_i - \log f_i)^2},
$$
  
\n
$$
f_i = Ci^{-\lambda},
$$

and the R-squared value ( $R^2$ ,  $0 \le R^2 \le 1$ ) was calculated by utilizing

$$
R^{2} = 1 - \frac{\sum_{i=1}^{k} (\log x_{i} - \log f_{i})^{2}}{\sum_{i=1}^{k} (\log x_{i} - \log \bar{f}_{i})^{2}}
$$

<span id="page-2-2"></span>.

This value was then used to evaluate the approximate degree to which the targeted distribution followed regularity.  $R<sup>2</sup>$  approaches 1 for a well-approximated distribution.

# C. ANALYSIS OF SYNTACTIC DEPENDENCY IN A **SENTENCE**

An analysis was performed using the Voice of America (VOA) Special English Word Book [\[35\], w](#page-6-19)hich provides 2,613 English sentences for learning 1,471 essential words.

<span id="page-2-0"></span>

**FIGURE 1.** Rank-ordered word frequency distribution for 3,601 words in the VOA English sentences. The apparent linearity shows the scale-free pattern regularity in a log-log plot ( $\lambda = 0.89$ , LL = 3.52, see Methods).

<span id="page-2-3"></span>Using the Natural Language Toolkit [\[36\],](#page-6-20) the number of words and total word frequency were obtained as 3,601 and 25,326, respectively. The words were counted along with isolated punctuation characters, including commas, full stops, and quotation marks. A syntactic dependency in a sentence is represented by a set of word pairs of head and dependent words, and the head is linked to its dependents. Using a dependency parser tool, i.e., DiaParser [\[37\],](#page-6-21) [\[38\], 2](#page-6-22)2,713 head-dependent pairs with 17,420 types were extracted from the sentences. The numbers of heads and dependents in the 3,601 words were 2,890 and 3,033 (79.7% and 84.2%), respectively.

# <span id="page-2-5"></span><span id="page-2-4"></span>**IV. RESULTS AND DISCUSSION**

# A. WORD DEPENDENCY IN SENTENCES

<span id="page-2-7"></span><span id="page-2-6"></span>First, this section describes an example of the dependency relations using the VOA English sentences for learning essential words. The dependency relations are defined for head-dependent word pairs to generate hierarchically structured word arrangements [\[39\]. T](#page-6-23)he rank-ordered frequency distribution of 3,601 words that comprise the sentences is shown in Fig. [1](#page-2-0) to confirm the scale-free rank-size relation by the apparent linearity on a log-log plot ( $\lambda = 0.89$ ,  $LL = 3.52$  $LL = 3.52$ . Table [1](#page-3-0) lists the 36 most frequent words. Table 2 lists the 20 most frequent dependencies of head-dependent word pairs. The list shows that a head accompanies its dependents, as a head having a lower rank in terms of word frequency specifies a dependent having a higher rank. While head-dependent pairs are expressed using arrows from the head to dependent, the paired words "for"  $\leftarrow$ "years" (10th) and "many"  $\leftarrow$  "years" (17th) can recall synchronous occurrence as a merge operation [\[40\]](#page-6-24) by the common head of ''years'' to compose the phrase ''for many years.'' In this manner, synchronous behavior can be observed in occurrences of words with dependency relations. Then, occurrences of words with synchronous connectivity can be referred to as consecutive selections of synchronously connected elements in a dissipative system.

#### <span id="page-3-0"></span>**TABLE 1.** Thirty-six most frequent words in the VOA English sentences.

Rank	Frequency	Word	Head	Dependent
T	2450			2450
	1319	the	$\frac{2}{0}$	1319
	763	The	$\overline{0}$	763
	584	to	$\boldsymbol{0}$	584
	529	a	$\mathbf{0}$	529
	496	of	$\boldsymbol{0}$	496
	387	in	$\overline{0}$	387
	359	is	33	349
23456789	306	He	$\mathbf{1}$	306
10	251	for	$\bar{0}$	251
11	240	She	$\mathbf{1}$	240
	200	was	17	196
$\frac{12}{13}$	191	you	9	191
14	186	his	$\overline{0}$	186
15	185	I	$\overline{\mathcal{L}}$	185
16	172	and	$\boldsymbol{0}$	172
17	166	$\overline{\mathcal{L}}$	$\overline{0}$	166
18	162	her	16	162
19	156	,	$\boldsymbol{0}$	156
20	139	on	$\boldsymbol{0}$	139
	134	will		134
	122	that	$\begin{matrix} 0 \\ 3 \\ 0 \end{matrix}$	122
	118	not		118
	115	he	$\bar{0}$	115
$21$ $22$ $23$ $24$ $25$	110	from	$\mathbf{0}$	110
$\frac{26}{27}$ 28	108	at	$\overline{0}$	108
	106	with	$\bf{0}$	106
	96	new	$\mathbf{1}$	96
29	90	are	$\overline{6}$	89
30	87	him	10	87
31	86	We	$\mathbf{0}$	86
32	81	by	$\boldsymbol{0}$	81
33	80	$\cdot$ <sub>S</sub>	$\frac{0}{177}$	80
34	79	have		42
35	78	it	10	78
36	76	has	124	38

The numbers of heads and dependents in the dependency word pairs (out of all  $22,713$ ) are shown for each word. The number of dependents is the same as the word frequency, except for verbs such as "is" (8th) and "have" (34th), whereas verbs dominate several heads.

<span id="page-3-1"></span>**TABLE 2.** Twenty most frequent dependencies in the VOA English sentences for learning essential words (the numbers in parentheses denote the word frequency ranks).

Rank	Frequency	$Head \rightarrow Dependent$
1	62	said $(44) \rightarrow . (1)$
2	45	made $(52) \rightarrow . (1)$
$\overline{\mathbf{3}}$	37	has $(36) \rightarrow$ . (1)
$\overline{\mathcal{L}}$	33	States (88) $\rightarrow$ United (74)
5	30	have $(34) \rightarrow . (1)$
6	28	had $(63) \rightarrow . (1)$
7	24	President (62) $\rightarrow$ The (3)
8	20	States $(88) \rightarrow$ the $(2)$
9	19	found $(128) \rightarrow . (1)$
10	18	years $(54) \rightarrow$ for $(10)$
11	17	boy $(123) \rightarrow$ The $(3)$
12	17	world $(144) \rightarrow$ the $(2)$
13	16	said (44) $\rightarrow$ She (11)
14	16	man $(66) \rightarrow$ The $(3)$
15	16	saw $(119) \rightarrow$ . $(1)$
16	15	house $(51) \rightarrow$ the $(2)$
17	15	years $(54) \rightarrow$ many $(53)$
18	15	time $(68) \rightarrow a(5)$
19	15	problem $(145) \rightarrow$ the $(2)$
20	15	says $(203) \rightarrow . (1)$

The ranks are given for descriptive purposes, even for the same frequency.

# B. RANK-SIZE RELATION IN SYNCHRONOUSLY CONNECTED ELEMENT SELECTION SYSTEM

A system of synchronously connected element selection was implemented by considering the occurrence of a word

<span id="page-3-2"></span>

**FIGURE 2.** Rank-size relations in the energy distribution of synchronously connected element selection systems with  $M = 12,000$ ,  $N = 24,000$  at the 500 millionth iteration when the phase parameter  $\varphi$  is 0.15 ( $\simeq$ 8.59°) (red), 1.0 (~57.3°) (green), and the golden angle (~137.5°) (blue). The blue points for the phase parameter of the golden angle exhibit linearity in a log-log plot ( $\lambda = 1.04$ ,  $LL = 3.57$ , see Methods), revealing a scale-free pattern.

<span id="page-3-3"></span>

**FIGURE 3.** Changes in the scaling exponent and R-squared value with state transitions in the system when the phase parameter is the golden angle. The values are stable after the 200 millionth iteration, indicating that the system reaches a steady state of dynamic equilibrium.

as the assignment of an energy particle to an element, assuming that the synchronous behavior observed in the word dependency would be related to the noise-induced synchronization phenomenon. The system performs a state transition using a circle map model [\[27\]](#page-6-11) by selecting an element, while elements are arranged at equal intervals along a circle. An energy particle from a randomly chosen element is reassigned to a selected element corresponding to the angle calculated based on the circle map through an iteration (see Methods for the system procedure). The occurrence frequencies of words correspond to the number of energy particles assigned to the elements. The energy distribution by counting the number of energy particles per element through state transition iterations was examined.

Fig. [2](#page-3-2) shows the rank-ordered energy distributions for 12,000 elements and 24,000 energy particles at the 500 millionth iteration when the phase parameter is 0.15  $(\simeq 8.59°)$  (red), 1.0 ( $\simeq 57.3°$ ) (green), and the golden angle (≃137.5◦ ) [\[33\]](#page-6-17) (blue). The phase parameter settings of

<span id="page-4-0"></span>

**FIGURE 4.** Characterization of sustainable state transition dynamics for chronological variation. (**A**) Energy distribution of the scale-free rank-size relation according to the angles of elements depicted in blue in Fig. [2.](#page-3-2) An interference fringe-like pattern with 13 peaks is visible when the horizontal axis is changed to the element angle. (**B**) Angle pairs between subsequently selected elements for the blue points in Fig. [2](#page-3-2) from the 200 millionth to the 500 millionth iteration (the angle pairs with frequencies greater than three are plotted in red). (**C**) Trajectories of selected element angles for two arbitrarily chosen energy particles. The lines show a common circulation form in which neighboring elements along a circle are contextually selected with a period of 13 state transitions. (**D**) Changes in correlation due to the lag between element sequences selected for two arbitrarily chosen energy particles. A mesh-like pattern with 13 wave-shaped curves due to the common fluctuating circulation form is displayed, suggesting that element selections have self-actively entangled adaptively to sustain chronological variation for subsequent transition states in a system.

<span id="page-4-1"></span>convex upward curves are related to the occurrence of crossover [\[13\]](#page-5-12) in distributions, as seen in the lower part of the rank for the phase parameter of 0.15 (red) and in the top part for 1.0 (green), demonstrating that they are due to spatio-chronologically biased circle map element selections. The blue points exhibit linearity, revealing a scale-free pattern ( $\lambda = 1.04$ ,  $LL = 3.57$ ) for element selection using the phase parameter of the golden angle for spatiochronological freedom [\[41\]](#page-6-25) in the subsequent energy particle assignments. The scaling exponent and R-squared value for the phase parameter of the golden angle are stable after the 200 millionth iteration and are about 1.04 and 0.97, respectively (Fig. [3\)](#page-3-3). These values suggest that the system reaches a steady state of dynamic equilibrium corresponding to the most probable case in the system. The results show that the appearance of the scale-free rank-size relation may be due to the adaptively sustaining chronological variation for subsequent transition states in a system.

# C. ADAPTIVE SYSTEM BEHAVIOR SUSTAINABILITY FOR CHRONOLOGICAL VARIATION

<span id="page-4-2"></span>Fig [4A](#page-4-0) shows the energy distribution of the scale-free rank-size relation (blue points in Fig. [2\)](#page-3-2) according to the angles of elements arranged along the circle, presenting an interference fringe-like pattern. This pattern can be obtained as a stationary wave produced by the superposition of a wave function [\[42\]. T](#page-6-26)o explore a wave function for stochastic energy particle assignments, the trajectory obtained by successively selecting an element for each energy particle in a state transition after the 200 millionth iteration was investigated. Fig [4B](#page-4-0) shows the 1,018,532 angle pairs of the subsequently selected elements for each of the 24,000 energy particles from the 200 to 500 millionth iteration. The red points show 2,891 pairs with frequencies greater than 3, aligned with the 13 peaks shown in Fig [4A.](#page-4-0) The parallel band structure in the figure corresponds to the subsequent element selection performed all over the circle by a rotation about the

golden angle ( $\simeq$ 137.5°). Fig [4C](#page-4-0) depicts the trajectories for the selection of 40 elements from the 200 millionth iteration for two arbitrarily chosen energy particles. While the 4*th* , 17<sup>th</sup>, and 30<sup>th</sup> trajectory numbers in the solid line have angles near 360◦ , their last trajectory numbers (#3, 16, and 29) have angles of about 220◦ , where the differences are in terms of the golden angle. In the dashed line for the other particle, the subsequent trajectory numbers (#5, 18, and 31) have angles of approximately 300◦ . The lines in the figure reveal that energy particle assignments entrain a circulating element selection with a period of 13 state transitions. In these transitions, the neighboring elements along a circle are contextually selected by parallel rotation about the golden angle. Fig [4D](#page-4-0) shows the correlation of the 12,000 selected element sequences for two arbitrarily chosen particles as a function of the lag. Although the correlation for a significant lag of 5,000 is comparable to that for a lag of 1,000, the correlation increases and decreases in the successive pattern as the lag increases. This reveals an adaptive system behavior in which the element selection required to sustain the scale-free ranksize regularity has self-actively entangled [\[43\]](#page-6-27) to take a wave function under which the assignment of each energy particle should determine the assignments of other particles. Thus, the word dependency could be auto-regulated sustainably for chronological variation in successive social use.

#### **V. LIMITATIONS OF THE STUDY**

This study focused on addressing the dependency among interacting elements in a dissipative system aiming to derive a relevant principle for yielding scale-free pattern distributions. The main contribution of this study is the discovery that scale-free patterns can be created through a stationary wave resulting from the superposition of a wave function, generated from adaptively coordinated state transitions for chronological variation in a sustainable manner. However, some limitations should be noted. First, the wave function obtained in this study may be biased because of the adopted noise-induced synchronization model. Second, the results need to be verified through the development of other synchronization models [\[44\]](#page-6-28) to enhance the significance of this study. Additionally, an in-depth understanding of the synchronization phenomenon associated with word occurrences should be achieved in the future to clarify how synchronous connectivity can cause a wave function for a stationary wave to create a scale-free pattern.

# **VI. CONCLUSION**

To develop a method for assessing synchronous word connectivity from dependency relations, a computational system to select synchronously connected elements using a noise-induced synchronization model was implemented. The phenomenon of synchronous word occurrence was considered by assigning energy particles to interacting elements in a dissipative system. Thereafter, the connectivity among the elements that were synchronously selected owing to dissipative system behavior was investigated. The

findings demonstrated that the energy particle distribution for spatio-chronological freedom can render a scale-free pattern as a stationary wave that is self-organizationally formed by the superposition of a wave function for energy particle assignments. The obtained results suggest that word sequences in sentences can be adaptively constructed via self-actively generated dependency with chronological variation sustainability. In future studies, consistency of the system dynamics with real-world word occurrences will be investigated. The results from the synchronously connected element selection system based on the noise-induced synchronization phenomenon have demonstrated the relevance of dependency among elements in yielding scale-free pattern distributions, implying that the evolution of natural language can be studied by taking a physics-based approach, i.e., words occur owing to the energy supplied from outside the system, whereas word dependency is due to energy dissipated within the system. The dependency relations derived from the synchronization phenomenon in a dissipative system can provide perspectives on the development of an innovative approach to evolutionary language models.

#### <span id="page-5-15"></span>**ACKNOWLEDGMENT**

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