

Received 12 August 2023, accepted 31 August 2023, date of publication 8 September 2023, date of current version 13 September 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3313175

RESEARCH ARTICLE

Improved Cross Entropy Method for Well-Being Evaluation of Composite Generation and Transmission Systems

DONGLI XU¹, YUQI WANG^{1,2}, FANG WANG¹, AND FAN CHEN¹

¹School of Electric Power Engineering, Nanjing Institute of Technology, Nanjing 211167, China

²Nanjing Gaochun District Power Supply Branch, State Grid Jiangsu Electric Power Company Ltd., Nanjing 211300, China

Corresponding author: Fan Chen (chenfan@njit.edu.cn)

This work was supported by the Scientific Research Fund of the Nanjing Institute of Technology of China under Grant CKJC202102.

ABSTRACT Well-Being analysis is an approach that integrates deterministic criteria with probabilistic methods, and it plays a crucial role in the operational planning of power systems. However, assessing the Well-Being of composite generation and transmission systems presents a formidable challenge, characterized by significant computational burdens and sluggish processing speeds. To tackle this issue, we embarked on an effort to enhance the computational efficiency of Well-Being assessment by employing the cross-entropy method (CEM). Nonetheless, our experimental pursuits revealed that the conventional employment of CEM for Well-Being assessment can lead to protracted convergence of the marginal index. To overcome this limitation, we introduce an enhanced multi-objective cross-entropy method (MCEM) that integrates weight factors, thereby ensuring an accelerated convergence rate for both the risk and marginal indices. To validate the effectiveness and advancement of our proposed MCEM approach, we conduct a comprehensive comparative analysis using the IEEE RTS79 and MRTS79 test systems as case studies. We contrast our method with the conventional MCS and CEM approaches, conducting a thorough examination of the computational performance of MCEM. This comprehensive comparative study unequivocally confirms the efficacy and progressive nature of the MCEM framework presented in this paper.


INDEX TERMS Well-being evaluation, composite generation and transmission systems, convergence of risk index and marginal index, cross entropy, optimal multiplier.

I. INTRODUCTION

In the domain of system reliability assessment, the power system reliability criteria are typically classified into two primary categories: deterministic and probabilistic methods. The deterministic method offers simplicity and practicality, as it allows for easy comprehension and implementation. However, it fails to account for the uncertainties of system operations. On the other hand, the probabilistic method incorporates the influence of various stochastic factors but requires sophisticated analytical techniques, making it challenging for field personnel to grasp [1], [2], [3]. To bridge the gap between these criteria, Billiton proposed the Well-Being model in 1994 [4]. This model is founded upon the N-1

criterion, which holds paramount importance in power system planning, design, and operational dispatching.

Researchers have conducted many studies on Well-Being analysis of power systems. These studies focused on Well-Being evaluation of different subsystems, including generation system, distribution system, and composite generation and transmission systems. For the Well-Being analysis of generation system, researchers studied the effect of load shifting, electric vehicle (EV) on system Well-Being. Reference [5] studied the load shifting among different kinds of load sectors on Well-Being indices of generation system integrated with wind power. In [6], the electric vehicle (EV) charging was being treated as interruptible load and served as emergency units, and its contribution in Well-Being enhancement of generation system was investigated. Based on the work in [6], the uncertainties of EV charging were

The associate editor coordinating the review of this manuscript and approving it for publication was R. K. Saket .

formulated and being considered in the Well-Being evaluation in [7]. In [8], the effects of charging modes of EVs and load shifting on well-being of generation system were investigated. For the Well-Being analysis of distribution system, researchers studied the effect of distributed generators and EV integration on system Well-Being. In [9], a reliability model for distribution transformers considering the aging rate of the transformer under different levels of photovoltaic permeability on rooftops was proposed and then the distribution system Well-Being indices were evaluated. In [3], the models of distributed generators including solar, wind, and tidal energy sources are established based on Markov framework for Well-Being evaluation of distribution network with DGs. Besides considering the integration of DGs, the managed charging and discharging of the EVs on Well-Being indices of distribution system were studied in [10] and [11]. In [12], factors including cyber-attack, power uncertainties of distributed generators in the supply side and of plug-in hybrid electric vehicles in the demand side were investigated for Well-Being analysis of cyber-physical distribution system. For the Well-Being analysis of composite generation and transmission systems, research has been conducted on the characteristics of power system with renewable energy integration. In existing literature, the uncertainties including wind power [13], [14], [15] and ocean wave energy [16] were simulated by Monte Carlo method, and their effects on Well-Being indices were investigated. Besides the uncertainty of the wind power generation, the effects of control measurements including energy storage [14] and demand response [15] on Well-Being indices were further studied.

As can be concluded in the literature above, Monte Carlo simulation (MCS) was used for system state simulation in the Well-Being analysis. When MCS is used for Well-Being analysis of generation or distribution system, the calculation speed of system Well-Being is fast, as the system state analysis of generation or distribution system is simple [17]. However, it is pointed out that the Well-Being evaluation of composite generation and transmission systems based on MCS is extremely time-consuming [18]. There are two reasons for this. First, it is due to the judgment of the N-1 criterion, which requires a lot of additional state evaluation. Second, it is due to the slow convergence of the MCS method for high reliability systems [19]. As for the poor performance of MCS in evaluating high reliability systems, researchers have proposed variance reduction techniques such as the importance sampling method [20], Latin hypercube sampling method [21] and cross-entropy method (CEM) [22], [23], [24], [25], [26], which can effectively improve the sampling probability of rare events and overcome the problem of slow convergence of MCS. Among the variance reduction techniques, CEM has attracted much attention in recent years. This method can calculate an appropriate sampling probability density function according to the convergence characteristics of the target index and effectively reduce the sample variance. Based on this, we were inspired to use CEM to replace MCS in the Well-Being evaluation of

composite generation and transmission systems. However, it was observed that when applying CEM for Well-Being evaluation of composite generation and transmission systems, there could be large discrepancies in convergence speed between the marginal index and the risk index. The convergence of the risk index does not necessarily ensure the convergence of the marginal index, rendering the risk index unsuitable as a convergence condition. To solve this problem, we propose a multi-objective cross-entropy method (MCEM) in this paper. The specific contributions of this paper are as follows.

(1) To the best of the authors' knowledge, this paper is the first to highlight the issue that when using CEM for Well-Being evaluation of generation and transmission systems, the convergence of the risk index may not guarantee the convergence of the marginal index.

(2) MCEM is developed which takes the simultaneous convergence of the variance coefficients of the marginal index and the risk index as the convergence. Specifically, the optimal distribution parameters of component failure for both indices are calculated separately, and the comprehensive optimal distribution parameters are obtained by adjusting the weight coefficient. The optimal outage rates of components for risk index are obtained by CEM, while the optimal outage rates of components for marginal index are obtained by using the optimal multiplier method.

(3) A comparative analysis has been conducted between the proposed MCEM method and the traditional MCS and CME methods using the IEEE RTS79 and MRTS 79 test systems.

The rest of the paper is organized as follow. Section II introduces the theory of CEM and its limitation in application for Well-Being evaluation of power systems. Section III presents the MCEM for Well-Being evaluation of composite power systems. Case studies have been illustrated on RTS79 and MRTS79 systems to demonstrate the effectiveness of the proposed MCEM in Section IV. Finally, conclusions are made in Section V.

II. CROSS ENTROPY METHOD

A. BASIC PRINCIPLE OF CEM

Suppose that $X=[X_1, X_2, \dots, X_N]$ represents the state variable of system components, and N is the number of components; $f(X; \mathbf{a})$ is the joint probability density function of X (where \mathbf{a} is the parameter of the probability density function of X); $H(X)$ is the indicator function of system performance, and the reliability index h is given by [27]:

$$h = \int H(X)f(X; \mathbf{a})dX \quad (1)$$

To accelerate the reliability assessment, the cross-entropy method replaces the original probability density function $f(X; \mathbf{a})$ with an importance sampling probability density function $g(X; \mathbf{b})$. Then h is calculated by

$$h = \int H(X)g(X; \mathbf{b})\frac{f(X; \mathbf{a})}{g(X; \mathbf{b})}dX$$

$$\begin{aligned}
 &= E_g(H(X) \frac{f(X; \mathbf{a})}{g(X; \mathbf{b})}) \\
 &= E_g(H(X)W(X)) \tag{2}
 \end{aligned}
 \qquad
 \begin{aligned}
 &= \frac{\prod_{j=1}^N (1 - u_j)^{X_{k,j}} (u_j)^{1-X_{k,j}}}{\prod_{j=1}^N (1 - v_j)^{X_{k,j}} (v_j)^{1-X_{k,j}}} \tag{7}
 \end{aligned}$$

where \mathbf{b} is the parameter of $g(X; \mathbf{b})$; $E_g(\cdot)$ is the expected value of the index calculated when the system state variable X is sampled according to $g(X; \mathbf{b})$, and $W(X)$ is the likelihood ratio.

The unbiased estimation of h can be obtained by

$$\hat{h} = \frac{1}{n} \sum_{k=1}^n H(X_k)W(X_k) \tag{3}$$

where, n represents the total number of samples; X_k represents the system state variable obtained from the k -th sampling; $H(X_k)$ represents the system performance indicator function obtained from the k -th sampling; and $W(X_k)$ represents the likelihood ratio obtained from the k -th sampling.

There exists a theoretical optimal solution $g^*(X)$ such that the variance of \hat{h} is zero and $g^*(X)$ is given by

$$g^*(X) = \frac{H(X)f(X; \mathbf{a})}{h} \tag{4}$$

However, the value of h is unknown before evaluation, and $g^*(X)$ cannot be obtained directly. Therefore, the goal of the cross-entropy method is to approximately solve the important probability density function that is close to $g^*(X)$ by minimizing the KL distance $D(g^*(X), g(X; \mathbf{b}))$ between $g(X; \mathbf{b})$ and $g^*(X)$ as given by

$$\begin{aligned}
 D(g^*(X), g(X; \mathbf{b})) &= \int g^*(X) \ln g^*(X) dX \\
 &\quad - \int g^*(X) \ln g(X; \mathbf{b}) dX \tag{5}
 \end{aligned}$$

B. LIKELIHOOD RATIO AND PARAMETER OPTIMIZATION OF CEM

The distribution parameter of random variables is denoted by $\mathbf{u} = [u_1, \dots, u_j, \dots, u_N]$, where N is the total number of random variables, j is the j -th random variable, and the optimal distribution parameter corresponding to \mathbf{u} is denoted by \mathbf{v} . The analytical expression of the optimal distribution parameter of random variables is given by

$$v_j = 1 - \frac{\sum_{k=1}^n H(X_k)W(X_k)X_{k,j}}{\sum_{k=1}^n H(X_k)W(X_k)} \tag{6}$$

where v_j represents the optimal distribution parameter of the j -th random variable, $X_{k,j}$ represents the state of the j -th random variable obtained from the k -th sampling, the state "1" represents available, and "0" represents unavailable.

The likelihood ratio of random variables is calculated by

$$W(X_k) = \frac{f(X_k; \mathbf{u})}{f(X_k; \mathbf{v})}$$

where $f(X_k; \mathbf{u})$ is the probability of obtaining the state of X_k by the k -th sampling with \mathbf{u} as the parameter; and $f(X_k; \mathbf{v})$ is the probability of obtaining the state of X_k by the k -th sampling with \mathbf{v} as the parameter.

C. ISSUES IN CALCULATING WELL-BEING INDEX USING CEM

When CEM is applied to evaluate the Well-Being of power systems, it is observed that the convergence speed of the marginal index may be slower than that of risk index, and the gap may be large after using cross entropy optimization technology. To illustrate this problem, we use the traditional CEM method for Well-being evaluation of RTS79 system under peak load. Considering that the sample size is approximately proportional to the reciprocal of the square of variance coefficient [28], we depict the relation curve of these two variables in Figure 1.

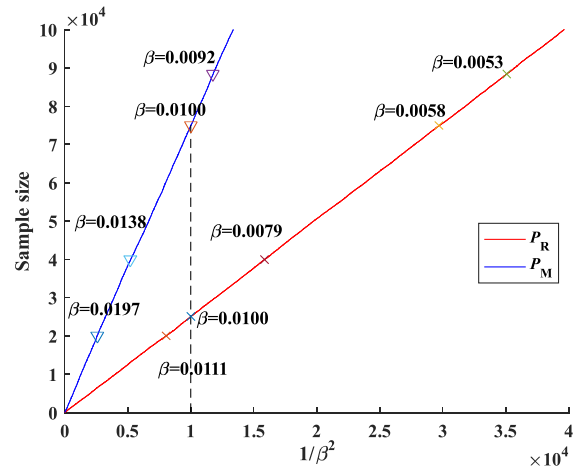


FIGURE 1. Convergence chart of Well-Being index based on CEM.

As shown in Figure 1, the abscissa is the reciprocal of the square of variance coefficient, and the ordinate is the sample size. It can be observed that the convergence speed of marginal index P_M is significantly slower than that of risk index P_R and EENS. This phenomenon arises from the fact that the indicator function of CEM for parameter optimization is based on risk indicators. Therefore, although the optimal distribution parameters obtained by CEM can significantly increase the probability of risk state, it may also lead to the phenomenon that the convergence rate of marginal index is significantly lower than that of risk index, resulting in the marginal index dragging down the overall convergence rate.

III. MCEM FOR WELL-BEING EVALUATION

A. WELL-BEING ANALYSIS

The Well-Being assessment model classifies system states into three categories: healthy, marginal, and at risk, based on

the N-1 criterion. The criteria for determining these states are as follows. (1) Healthy State: The system's operating state is designated as healthy if it adheres to all constraints and fulfills the N-1 criterion. (2) Marginal State: The system's operating state is designated as marginal if it satisfies all constraints but falls short of meeting the N-1 criterion. (3) At-Risk State: The system's operating state is designated as at risk if it breaches any constraints. Once the system states are determined, the Well-Being indices can be obtained by [5] and [11]:

$$P_R = \frac{\sum_{i=1}^n H_R(X_i)}{n} \quad (8)$$

$$P_M = \frac{\sum_{i=1}^n H_M(X_i)}{n} \quad (9)$$

$$P_H = \frac{\sum_{i=1}^n H_H(X_i)}{n} = 1 - P_M - P_R \quad (10)$$

$$EENS = \frac{\sum_{i=1}^n H_{EENS}(X_i)}{n} \quad (11)$$

where, P_H , P_M , and P_R represent the probabilities of the system being in a healthy, marginal, and risky state, respectively. i denotes the i -th sampling. $H_R(X_i)$, $H_M(X_i)$, and $H_H(X_i)$ are indicator functions for the at risk, marginal, and healthy states, respectively. Since the sum of the three probabilities is 1, only two independent indicators are needed. Generally, the probabilities of the risky and marginal states are relatively small, requiring more evaluations for convergence. Therefore, the indicator function $H_H(X_i)$ for the healthy state is usually not recorded. EENS (expected energy not supplied) is the expected amount of energy that is not supplied, measured in MWh/year. EENS is also considered a risk indicator, and $H_{EENS}(X_i)$ is the indicator function for the EENS indicator.

B. COMPONENT PARAMETER OPTIMIZATION FOR P_M

CEM is limited to optimizing distribution parameters for risk index and cannot consider optimizing distribution parameters for marginal index. Hence, this study introduces a multi-objective cross-entropy method (MCEM) based on the concept of identifying comprehensive optimal distribution parameters that promote the convergence of both marginal and risky indices.

For a single marginal state, there are several generation capacity margin functions based on the N-1 criterion. Hence, the traditional CEM cannot generate additional threshold parameters. To address this limitation, we employ the optimal multiplier method [29], [30] for parameter optimization of marginal index. The optimal multiplier method constructs the optimal probability distribution of components $f(X_j)$ by

$$f(X_j) = \begin{cases} \varepsilon u_j, & X_j = 0 \\ 1 - \varepsilon u_j, & X_j = 1 \end{cases} \quad (12)$$

where ε is the optimal multiplier, and it can be calculated by

$$\varepsilon = -\frac{B + \sqrt{B^2 - AC}}{A} \quad (13)$$

$$\begin{cases} A = \frac{n_1}{n_0 + n_1} \bar{u} - (1 - \frac{n_1}{n_0 + n_1}) \bar{u} (1 - \bar{u}) \\ B = -(\frac{n_1}{n_0 + n_1}) \bar{u} \\ C = \frac{n_1}{n_0 + n_1} \\ u^- = \frac{1}{N} \sum_{j=1}^N u_j \end{cases} \quad (14)$$

where n_1 and n_0 represent the number of normal and fault components when the system state is in the target event set, respectively.

C. PARAMETER OPTIMIZATION OF MCEM

To determine the optimal outage rate of components for both the marginal and risk state indices, MCEM calculates the optimal outage rates, denoted as v_M and v_R , respectively, for component failures associated with the marginal and risk state indices. The specific calculation formulas for are as follows:

$$v_M = \varepsilon u \quad (15)$$

$$v_{R,j} = 1 - \frac{\sum_{i=1}^n H_R(X_i) W(X_i) X_{ij}}{\sum_{k=1}^n H_R(X_i) W(X_i)} \quad (16)$$

After calculating v_M and v_R , the weight α is introduced to modify and calculate the comprehensive optimal outage rate v by

$$v = \alpha \times v_M + (1 - \alpha) \times v_R \quad (17)$$

where the value range of α is [0, 1].

D. DETERMINATION OF WEIGHT COEFFICIENT

In this paper, we introduce the concept of the equilibrium degree of variance coefficient, denoted as $\beta\%$, which quantifies the disparity between the variance coefficient of the marginal index and the variance coefficient of the risk index. The calculation formula for $\beta\%$ is given by

$$\beta\% = \frac{\beta_{P_M} - \min(\beta_{P_R}, \beta_{EENS})}{\max(\beta_{P_M}, \beta_{P_R}, \beta_{EENS})} \times 100\% \quad (18)$$

where β_{P_M} represents the variance coefficient of marginal probability P_M ; β_{P_R} represents the variance coefficient of risk probability P_R ; β_{EENS} is the variance coefficient of EENS; $\min(\beta_{P_R}, \beta_{EENS})$ refers to the risk index with faster convergence (i.e., the one with smaller variance coefficient). The risk index with faster convergence is selected for comparison to highlight the convergence speed gap between the two indices. We use $\max(\beta_{P_M}, \beta_{P_R}, \beta_{EENS})$ to limit the value range of $\beta\%$ within [0, 1].

Eqs. (17) and (18) indicate that when the convergence speed of marginal index is slower than that of risk index, $\beta\%$ is greater than 0, and the larger the value of $\beta\%$, the larger the weight coefficient α is expected to be. Based on this idea, this paper proposes a practical calculation method for the weight coefficient α , and the specific calculation steps are as follows.

Step 1: Parameter initialization. Set the variance coefficient equalization threshold γ , the initial value of iteration times $i = 0$, and the modified step size of weight coefficient ϑ .

Step 2: Calculate the initial value of variance coefficient equilibrium $\beta\%_{(0)}$ by

$$\beta\%_{(0)} = \frac{\beta_{P_M(0)} - \min(\beta_{P_R(0)}, \beta_{EENS(0)})}{\beta_{P_M(0)}} \quad (19)$$

where $\beta_{P_M(0)}$ and $\beta_{P_R(0)}$ represent the variance coefficients of P_M and P_R , respectively, and $\beta_{EENS(0)}$ denotes the variance coefficient of EENS.

Step 3: Check if $\beta\%_{(0)} \geq \gamma$. If so, proceed to Step 4; If not, CEM is directly used for evaluation.

Step 4: Set the initial value of the weight coefficient as $\alpha_1 = \beta\%_{(0)}$.

Step 5: Update the number of iterations by $i = i + 1$.

Step 6: Calculate the comprehensive distribution parameter ν according to the weight coefficient α_i and Eq. (17), and extract the system state according to the comprehensive distribution parameter for the system Well-Being evaluation.

Step 7: Calculate $\beta\%_{(i)}$ by

$$\beta\%_{(i)} = \frac{\beta_{P_M(i)} - \min(\beta_{P_R(i)}, \beta_{EENS(i)})}{\max(\beta_{P_M(i)}, \beta_{P_R(i)}, \beta_{EENS(i)})} \times 100\% \quad (20)$$

where $\beta_{P_M(i)}$, $\beta_{P_R(i)}$ and $\beta_{EENS(i)}$ are the variance coefficients of P_M , P_R and EENS, respectively, which are calculated by sampling with the comprehensive distribution parameters obtained in step 6.

Step 8: Check if the weight coefficient needs to be further updated according to the $\beta\%_{(i)}$ obtained in Step 7.

If $\beta\%_{(i)} \geq \gamma$, let $\alpha_{i+1} = \alpha_i + \vartheta$, and then go to Step 5 to continue the iteration. This is because $\beta\%_{(i)} \geq \gamma$ means that the weight coefficient correction is insufficient, that is, the modified weight coefficient α_i is still small, and the convergence speed of the marginal index is still significantly slower than that of the risk index, so it is necessary to further increase the weight coefficient.

If $\beta\%_{(i)} \leq -\gamma$, let $\alpha_{i+1} = \alpha_i - \vartheta$, and then go to Step 5 to continue the iteration. This is because $\beta\%_{(i)} \leq -\gamma$ means that the weight coefficient is over modified, which leads to the problem that the convergence of marginal index is significantly faster than that of risk index, which will increase the number of systematic samplings. Therefore, at this time, it is necessary to reduce α_i to shift the comprehensive distribution coefficient to the direction conducive to the convergence of risk index.

If $\beta\%_{(i)} \in (-\gamma, \gamma)$, it means that the modified weight coefficient can make the convergence of both the risk and marginal index reach equilibrium, so the iterative calculation

process of weight coefficient ends, and the weight coefficient α_i is output.

It is worth pointing that the calculation of weight coefficient in composite generation and transmission systems requires iterative calculations, which leads to a time-consuming process. To address this challenge, this paper proposes an improved adaptive calculation method for weight coefficient. This method involves performing iterative calculations at the generation level (i.e., only the reliability of generation system is considered), where the obtained results are used as the initial values for further iteration at the composite generation and transmission level (i.e., the reliability of composite generation and transmission is considered) until the weight coefficient satisfies the requirements of MCEM. Consequently, this method requires only a few iterations at the composite generation and transmission level, which significantly alleviates the time-consuming problem.

E. WELL-BEING EVALUATION OF COMPOSITE GENERATION AND TRANSMISSION SYSTEMS BASED ON MCEM

The specific steps for Well-Being evaluation of composite generation and transmission systems based on MCEM are presented as follows.

(1) Steps of component parameter optimization of P_R .

Step 1: Parameter initialization. Set the pre-sampling times of each iteration n_{pre} , quantile ρ , the optimal outage rate of the system components in two states to $\nu_R = \mathbf{u}$, where \mathbf{u} is the original outage rate of the components;

Step 2: Set the number of iterations $i_R = 0$, with the upper limit of the number of iterations being I ;

Step 3: $i_R = i_R + 1$;

Step 4: Conduct random sampling based on ν_R , generate system state samples $\{\mathbf{X}_k; k = 1, 2, \dots, n_{pre}\}$, and conduct load reduction analysis on these samples to calculate the corresponding likelihood ratio $W(\mathbf{X}_k)$;

Step 5: Calculate the generation capacity margin sequence $G(\mathbf{X}_k)$ corresponding to each system state by

$$G(\mathbf{X}_k) = \begin{cases} P_{G(k)} - L_{D(k)}, & \mathbf{X}_k \text{ is not load shedding} \\ -L_{C(k)}, & \mathbf{X}_k \text{ is load shedding} \end{cases} \quad (21)$$

where $P_{G(k)}$ is the total generating capacity of system state \mathbf{X}_k , $L_{D(k)}$ is the corresponding total load, and $L_{C(k)}$ is the corresponding total load shedding.

For the generation capacity margin sequence $G(\mathbf{X}_k)$, arrange it from small to large to obtain $M = [M_{[1]}, M_{[2]}, \dots, M_{[n_{pre}]}]$. If $M_{[\rho n_{pre}]} > 0$, then the threshold parameter $\gamma = M_{[\rho n_{pre}]}$; otherwise, $\gamma = 0$.

Step 6: Use the threshold parameter γ to correct $G(\mathbf{X}_k)$ by $G'(\mathbf{X}_k) = G(\mathbf{X}_k) - \gamma$, and the corresponding indication function $H_R(\mathbf{X}_k)$ is expressed as

$$H_R(\mathbf{X}_k) = \begin{cases} 0, & G'(\mathbf{X}_k) > 0 \\ 1, & G'(\mathbf{X}_k) \leq 0 \end{cases} \quad (22)$$

Step 7: Update the optimal outage rate of system components ν_R according to Eq. (16).

Step 8: If the threshold parameter $\gamma = 0$ or $i_R = I$, the pre-sampling process ends and output ν_R ; Otherwise, return to Step 3;

(2) Optimization steps of component parameters for P_M .

Step 1: Parameter initialization: Set the pre-sampling times of each iteration as n_{pre} , the initial value of the optimal multiplier ε_0 , and the optimal outage rate of system components as $\nu_M = \varepsilon_0 \mathbf{u}$;

Step 2: set the number of iterations $i_M = 0$, with the upper limit of the number of iterations being I ;

Step 3: $i_M = i_M + 1$;

Step 4: Conduct random sampling based on ν_M , generate system state samples $\{X_k; k = 1, 2, \dots, n_{pre}\}$, and conduct Well-Being evaluation to determine the system state type.

Step 5: Calculate the values of n_1 and n_0 when the system is in marginal state.

Step 6: Update the optimal outage rate ν_M and optimal multiplier ε_{i_M} of system components.

Step 7: If $|\varepsilon_{i_M} - \varepsilon_{i_M-1}| < 0.01$, the pre-sampling process ends and ν_M is the output; Otherwise, return to step 3.

(3) Comprehensive optimal parameter acquisition.

Calculate the weight coefficient α according to the improved weight coefficient calculation method in Section III, and update the comprehensive distribution parameter ν of the two-state variables using Eq. (17).

(4) Optimal sampling flow based on comprehensive optimal parameter ν .

Step 1: Parameter initialization. Input ν obtained in pre-sampling process of MCEM. Set the system state set vector as an empty set, and the limit of variance coefficient of convergence condition β_{max} ;

Step 2: Set the sampling times $n = 0$;

Step 3: $n = n + 1$;

Step 4: Calculate the likelihood ratio $W(X_n)$ based on the extracted system state sample X_n .

Step 5: Compare and store. The extracted system state sample is compared to the state combinations in the vector. If it has already been stored, the corresponding indicating function is called. Otherwise, the process moves to Step 6.

Step 6: Evaluate the status X_n and record the indication function $H(X_n)$ as follows.

If X_n is a risk state, then $H_R(X_n) = W(X_n)$, $H_M(X_n) = 0$, $H_{EENS}(X_n) = 8760 \times LC(n) \times W(X_n)$; If X_n is a marginal state, then $H_R(X_n) = 0$, $H_M(X_n) = W(X_n)$, $H_{EENS}(X_n) = 0$; If X_n is a healthy state, then $H_R(X_n) = 0$, $H_M(X_n) = 0$, $H_{EENS}(X_n) = 0$.

Step 7: Calculation the Well-Being index by

$$h = \frac{\sum_{k=1}^n H(X_k)W(X_k)}{n} \quad (23)$$

When $H(X)$ is $H_R(X)$, h represents P_R ; When $H(X)$ is $H_M(X)$, h represents P_M ; When $H(X)$ is $H_{EENS}(X)$, h represents $EENS$.

Step 8: Calculate the variance coefficient of Well-Being index including P_R , P_M and EENS, and compare the variance coefficient of these index to β_{max} . If they are all less than β_{max} , stop the iteration and output the Well-Being index; Otherwise, return to Step 3.

IV. CASE STUDIES

This section verifies the effectiveness and accuracy of the proposed MCEM by taking the IEEE RTS79 [31] system as an example. As mentioned in Section II, the convergence rate of the marginal index may be lower than the risk index. Therefore, this paper considers the variance coefficient of both risk index and marginal index. Only when the variance coefficients of both risk index and marginal index are less than β_{max} , stop the iterative process. The β_{max} is set to 1% in this paper. The evaluation of this example considers random faults of generators and transmission lines, and the state analysis adopts the minimum load shedding model based on DC power flow [24]. The example analysis is implemented in MATLAB 2018 on a microcomputer with an Intel Core i7-10700 CPU and 16GB memory.

A. RTS79 SYSTEM

This example verifies the effectiveness of MCEM using the RTS79 system under peak load level. The parameters in the pre-sampling stage are set as follows: $n_{pre} = 5000$, $\rho = 0.1$, $\varepsilon_0 = 1$, $\gamma = 5\%$, $\vartheta = 0.025$, and the weight is 0.6172. The Well-Being index using different methods including MCS, CEM and MCEM are shown in Table 1, where the percentage following Well-Being index indicates its corresponding variance coefficient.

TABLE 1. Well-Being index using different methods for RTS79 system.

Method	P_M ($\beta_R\%$)	P_R ($\beta_M\%$)	EENS ($\beta_{EENS}\%$)	Number of samples	Computation time/s
MCS	0.371 (0.30%)	0.085 (0.74%)	129845 (1.00%)	194658	1948.47
CEM	0.368 (1.00%)	0.085 (0.57%)	128907 (0.33%)	83750	652.17
MCEM	0.372 (0.95%)	0.085 (1.00%)	128622 (0.85%)	22137	324.75

As shown in Table 1, the indicators obtained by CEM and MCEM are close to those calculated by MCS. However, the sampling times and calculation time of MCEM are significantly lower than those of CEM and MCS, indicating that MCEM is effective in accelerating the Well-Being evaluation of composite generation and transmission systems.

Figure 2 shows the efficiency of the three methods, where the computational efficiency represents the reciprocal of the calculation time and the sampling efficiency represents the reciprocal of the sampling number. Compared with MCS and CEM, the computational efficiency of MCEM has increased by 500% and 101%, and the sampling efficiency has increased by 779% and 278%. The results indicate that the

proposed MCEM has better performance than the traditional methods.

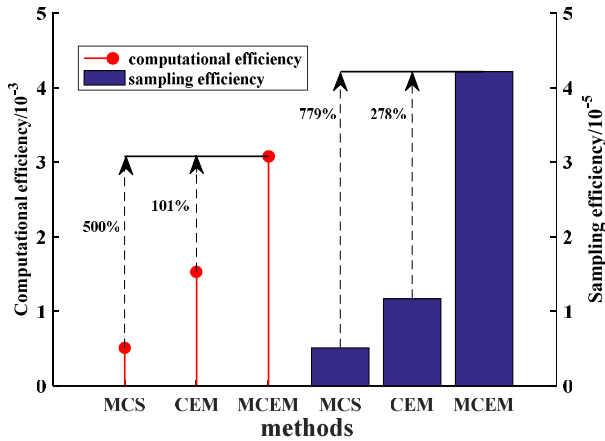


FIGURE 2. Efficiency comparison of different methods for RTS79.

To further elucidate the principle of how MCEM accelerates the convergence of well-being evaluation, a statistical analysis was performed on the marginal state, risk state, and healthy state extracted by three methods. Figure 3 displays the statistical distribution of the risk state and marginal state of the three methods for RTS79 system. The results show that when compared with MCS, the probability of risk states extracted by CEM is greater, but the probability of marginal states extracted by CEM is smaller. This indicates that when CEM is adopted, the convergence speed of the risk index improves, while the convergence speed of the marginal index worsens. However, when comparing MCEM with MCS, the probability of risk and marginal states extracted by MCEM are both greater than MCS, suggesting that MCEM can improve the convergence speeds of both risk and marginal index at the same time.

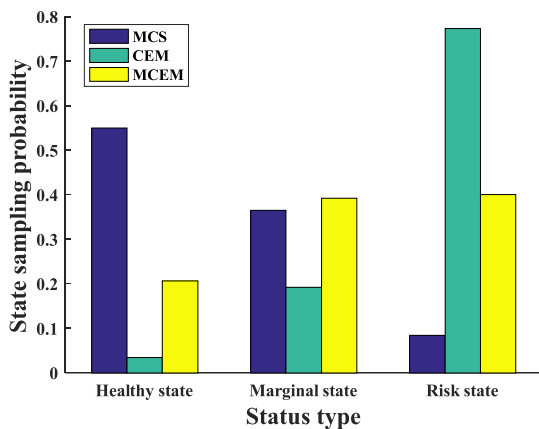


FIGURE 3. Statistical distribution of sampled system state for RTS79 system.

To further verify the efficiency and accuracy of the adaptive calculation method of weight coefficients proposed in this

TABLE 2. Results of weight coefficients for RTS79 system.

Cases	α	Iteration number at Generation level	Iteration number at generation and transmission level	time(s)
Case 1	0.6238	/	4	247.93
Case 2	0.6172	4	1	65.34

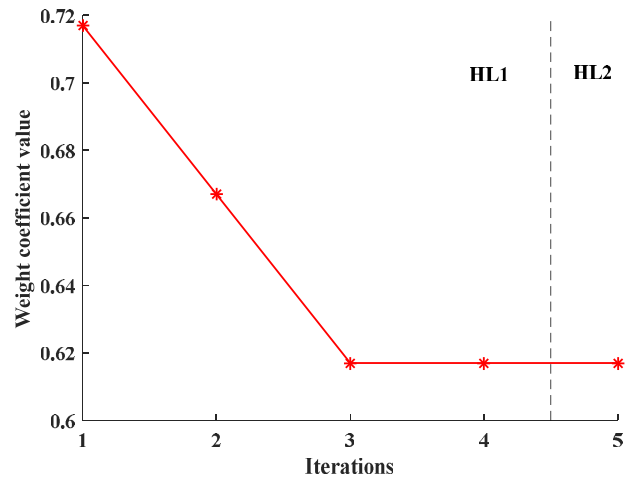


FIGURE 4. Iterative process of weight coefficient for RTS79 system.

paper, the weight coefficients were calculated based on the distribution parameters obtained by MCEM for two cases:

Case 1: Directly calculating the weight coefficient at the composite generation and transmission level.

Case 2: Calculating the weight coefficient according to the improved weight coefficient calculation method.

The Iteration number and computation time under the two cases based on MCEM are shown in Table 2. As shown in Table 2, it is observed that the time consumption of case 2 is only 26.3% of that of case 1, thereby confirming the efficiency of the improved adaptive calculation method of weight coefficient.

To further illustrate the advantages of the improved method, Figure 4 depicts the iterative process of the adaptive weight coefficient calculation method which is proposed in Section III. It can be observed that the first four iterations of the improved method are the iterative calculation at the generation level (also called the hierarchical level one, HL1), while the fifth iteration is the iterative calculation at the composite generation and transmission level (also called the hierarchical level two, HL2). After four iterations, the calculation of weight coefficients at the generation level yields a reasonable initial value for the composite generation and transmission level. Using this initial value, the calculation of weight coefficients at the composite generation and transmission level requires only one iteration to obtain an accurate value. This approach significantly reduces the number of iterations at the composite generation and transmission level and greatly reduces the overall calculation time.

TABLE 3. Well-Being index using different methods for MRTS79 system.

Method	P_M ($\beta_M\%$)	P_R ($\beta_R\%$)	EENS ($\beta_{EENS}\%$)	Number of samples	Time/s
MCS	0.398 (0.31%)	0.102 (0.75%)	153919 (1.00%)	178943	1830.94
CEM	0.396 (1.00%)	0.103 (0.53%)	155275 (0.29%)	96369	708.74
MCEM	0.395 (0.86%)	0.103 (1.00%)	155879 (0.93%)	21572	382.39

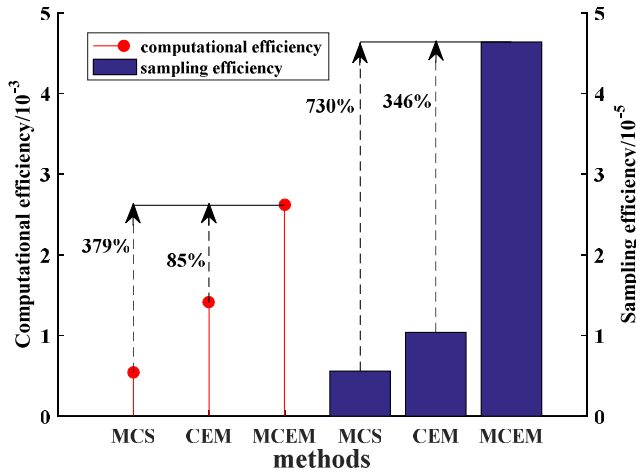


FIGURE 5. Efficiency comparison of different methods for MRTS79.

B. MRTS79 SYSTEM

To demonstrate the applicability of MCEM in evaluating systems with weak transmission, we designed MRTS79 system in this study by reducing the transmission capacity of all lines to 80% of the original. The Well-Being index of the MRTS79 system is calculated using MCS, CEM and MCEM, respectively. The parameters for MCEM are set as follows: $n_{pre} = 5000$, $\rho = 0.1$, $\epsilon_0 = 1$, $\gamma = 5\%$, $\vartheta = 0.025$, and the weight coefficient value $\alpha = 0.6574$ is obtained through iterative calculation.

The calculation results, as shown in Table 3, indicated that the calculation results of MCEM are very close to those of MCS, and has superior performance compared to MCS and CEM (with less sampling size and computation time).

Figure 5 illustrates the efficiency comparison of the three methods. Compared to MCS and CEM, MCEM has improved computational and sampling efficiency, demonstrating its excellent performance in the MRTS79 system.

Similarly, to further validate the efficiency and accuracy of the adaptive calculation method of weight coefficients for MRTS79 system, the weight coefficients were calculated for two cases. The Iteration number and computation time under the two cases for MRTS79 are shown in Table 4. As shown in Table 4, it is observed that the time consumption of case 2 is only 34.4% of that of case 1, thereby demonstrating the superiority of the improved calculation method of weight coefficient.

Figure 6 illustrates the iterative process of the improved weight coefficient for MRTS79 system. The first four

TABLE 4. Results of weight coefficients for MRTS79 system.

Cases	α	Iteration number at Generation level	Iteration number at generation and transmission level	time(s)
Case 1	0.6638	/	5	312.12
Case 2	0.6574	4	2	107.34

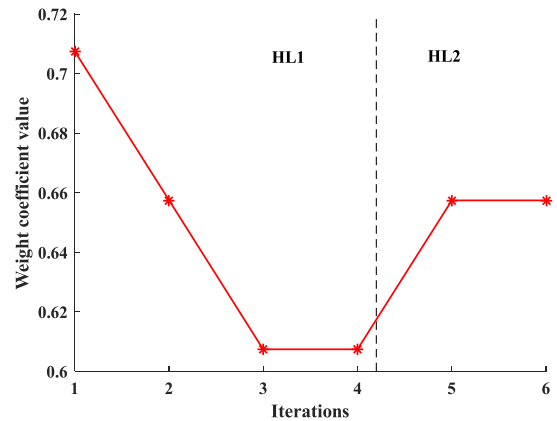


FIGURE 6. Iterative process of weight coefficient for MRTS79 system.

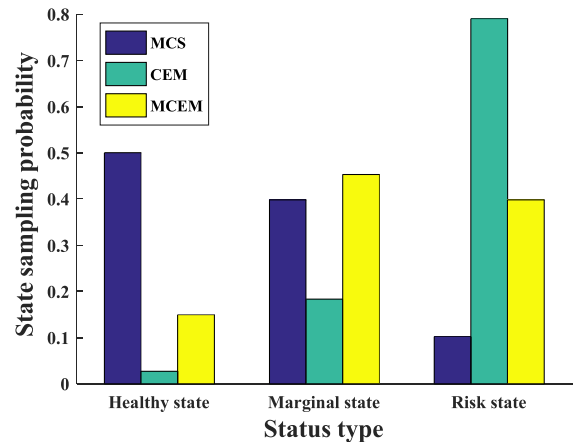


FIGURE 7. Statistical distribution of system state sampled for MRTS79 system.

iterations of the improved method involve the iterative calculation at the generation level (HL1), while the fifth and sixth iterations are the iterative calculation at the composite generation and transmission level (HL2). Only two iterations are implemented at the composite generation and transmission level, thus it greatly reduces the calculation time.

Figure 7 illustrates the sampling statistical distribution of the marginal, risk, and health states of the three methods for MRTS79 system. The sampling probability of marginal and risk states extracted by MCEM are higher than those extracted by MCS, which results in higher calculation efficiency of MCEM compared to MCS. On the other hand, the sampling probability of marginal states extracted by CEM is much smaller than that of marginal states extracted by MCS, leading to the convergence characteristics of CEM being dragged

down by marginal index. In contrast, the sampling probabilities for both marginal and risk states extracted by MCEM are relatively close, thereby enabling MCEM to ensure that the convergence characteristics of multiple indices are close, resulting in better comprehensive convergence than that of CEM.

V. CONCLUSION

To address the issue of computational time in the Well-Being assessment of composite generation and transmission systems, we investigated the potential of utilizing the cross-entropy method (CEM) to evaluate the Well-Being of these systems. However, during our experimentation, we observed that the conventional CEM approach employed for assessing the well-being of generation and transmission systems might result in sluggish convergence of the marginal index. To overcome this limitation, we present a solution in this paper: the multi-objective cross-entropy method (MCEM). Within the framework of the MCEM, we introduce a rapid approach for computing weight coefficients. This method entails an iterative process to determine the initial weight coefficient value at the generation level, followed by its application at the composite generation and transmission level, thus ultimately yielding dependable outcomes. The case studies conducted on the IEEE RTS79 and MRTS79 test systems confirm the validity and advancement of the proposed MCEM method.

Nevertheless, it's important to note that the MCEM framework we propose in this study does not currently accommodate correlations among random variables during system state simulation. As a future avenue of research, we suggest further exploration to enhance the MCEM by integrating correlations among random variables, including those related to the output of renewable energy plants. This refined method could then be applied to the well-being assessment of power systems characterized by a substantial proportion of renewable energy sources.

REFERENCES

- [1] W. Wangdee, "Deterministic-based power grid planning enhancement using system well-being analysis," *J. Mod. Power Syst. Clean Energy*, vol. 6, no. 3, pp. 438–448, Feb. 2018.
- [2] W. Wangdee, W. Li, and R. Billinton, "Locational transmission capacity reserve determination using system well-being analysis," *Electr. Power Syst. Res.*, vol. 119, pp. 329–336, Feb. 2015.
- [3] B. K. Talukdar, B. C. Deka, and A. K. Goswami, "Reliability analysis of an active distribution network integrated with solar, wind and tidal energy sources," *Int. Trans. Electr. Energy Syst.*, vol. 31, no. 12, Dec. 2021.
- [4] R. Billinton and M. Fotuhi-Firuzabad, "A basic framework for generating system operating health analysis," *IEEE Trans. Power Syst.*, vol. 9, no. 3, pp. 1610–1617, Aug. 1994.
- [5] M. Hassanzadeh, M. Fotuhi-Firuzabad, and A. Safdarian, "Wind energy penetration with load shifting from the system well-being viewpoint," *Int. J. Renew. Energy Res.*, vol. 7, no. 2, pp. 977–987, Feb. 2017.
- [6] N. Z. Xu and C. Y. Chung, "Well-being analysis of generating systems considering electric vehicle charging," *IEEE Trans. Power Syst.*, vol. 29, no. 5, pp. 2311–2320, Sep. 2014.
- [7] N. Z. Xu and C. Y. Chung, "Uncertainties of EV charging and effects on well-being analysis of generating systems," *IEEE Trans. Power Syst.*, vol. 30, no. 5, pp. 2547–2557, Sep. 2015.
- [8] A. Badakhsh and M. N. Hassanzadeh, "Evaluation of well-being criteria in presence of electric vehicles consumption increase and load shifting on different load sectors," *Int. J. Renew. Energy Res.*, vol. 7, no. 3, pp. 1484–1494, Jul. 2017.
- [9] M. Hamzeh and B. Vahidi, "Reliability evaluation of distribution transformers considering the negative and positive effects of rooftop photovoltaics," *IET Gener., Transmiss. Distrib.*, vol. 14, no. 15, pp. 3063–3069, Jun. 2020.
- [10] H. Hashemi-Dezaki, M. Hamzeh, H. Askarian-Abyaneh, and H. Haeri-Khiavi, "Risk management of smart grids based on managed charging of PHEVs and vehicle-to-grid strategy using Monte Carlo simulation," *Energy Convers. Manage.*, vol. 100, pp. 262–276, Aug. 2015.
- [11] H. Kheradmand-Khanekehdani and M. Gitizadeh, "Well-being analysis of distribution network in the presence of electric vehicles," *Energy*, vol. 155, pp. 610–619, Jul. 2018.
- [12] L. Chen, N. Zhao, Z. Cheng, and W. Gu, "Reliability evaluation of cyber-physical power systems considering supply- and demand-side uncertainties," *Energies*, vol. 15, no. 1, p. 118, Dec. 2021.
- [13] D. Huang and R. Billinton, "Effects of wind power on bulk system adequacy evaluation using the well-being analysis framework," *IEEE Trans. Power Syst.*, vol. 24, no. 3, pp. 1232–1240, Aug. 2009.
- [14] B. Sun, X. Pan, F. Wu, W. Li, and D. Xie, "Adequacy assessment for power systems with wind power and energy storage integration based on well-being theory," *Power Syst. Technol.*, vol. 40, no. 5, pp. 1363–1370, May 2016.
- [15] M. N. Hassanzadeh, M. Fotuhi-Firuzabad, A. Safdarian, and S. Soleymani, "Demand response as a complement for wind energy from the viewpoint of system well-being," *Sci. Iran.*, vol. 27, no. 3, pp. 1373–1383, Jun. 2020.
- [16] B. Johnson and E. Cotilla-Sanchez, "Estimating the impact of ocean wave energy on power system reliability with a well-being approach," *IET Renew. Power Gener.*, vol. 14, no. 4, pp. 608–615, Feb. 2020.
- [17] W. Li, *Risk Assessment of Power Systems: Models, Methods, and Applications*, 2nd ed. Hoboken, NJ, USA: Wiley, 2014, p. 560.
- [18] T. S. Amaral, C. L. T. Borges, and A. M. Rei, "Composite system well-being evaluation based on non-sequential Monte Carlo simulation," *Electr. Power Syst. Res.*, vol. 80, no. 1, pp. 37–45, Jan. 2010.
- [19] E. Tomasson and L. Soder, "Generation adequacy analysis of multi-area power systems with a high share of wind power," *IEEE Trans. Power Syst.*, vol. 33, no. 4, pp. 3854–3862, Jul. 2018.
- [20] C. Yan, T. Ding, Z. Bie, and X. Wang, "A geometric programming to importance sampling for power system reliability evaluation," *IEEE Trans. Power Syst.*, vol. 32, no. 2, pp. 1568–1569, Mar. 2017.
- [21] Z. Shu, P. Jirutitijaroen, A. M. L. da Silva, and C. Singh, "Accelerated state evaluation and Latin hypercube sequential sampling for composite system reliability assessment," *IEEE Trans. Power Syst.*, vol. 29, no. 4, pp. 1692–1700, Jul. 2014.
- [22] X. He, T. Ding, X. Zhang, Y. Huang, L. Li, Q. Zhang, and F. Li, "A robust reliability evaluation model with sequential acceleration method for power systems considering renewable energy temporal-spatial correlation," *Appl. Energy*, vol. 340, Jun. 2023, Art. no. 120996.
- [23] J. Cai, Q. Xu, M. Cao, and B. Yang, "A novel importance sampling method of power system reliability assessment considering multi-state units and correlation between wind speed and load," *Int. J. Electr. Power Energy Syst.*, vol. 109, pp. 217–226, Jul. 2019.
- [24] X. Zhuang, C. Ye, Y. Ding, L. Cheng, Y. Song, S. Ye, S. Tian, and R. Chen, "Data-driven efficient reliability evaluation of power systems with wind penetration: An integrated GANs and CE method," *IET Gener., Transmiss. Distrib.*, vol. 14, no. 4, pp. 577–584, Feb. 2020.
- [25] L. Geng, Y. Zhao, and W. Li, "Enhanced cross entropy method for composite power system reliability evaluation," *IEEE Trans. Power Syst.*, vol. 34, no. 4, pp. 3129–3139, Jul. 2019.
- [26] Y. Zhao, Y. Han, Y. Liu, K. Xie, W. Li, and J. Yu, "Cross-Entropy-Based composite system reliability evaluation using subset simulation and minimum computational burden criterion," *IEEE Trans. Power Syst.*, vol. 36, no. 6, pp. 5198–5209, Nov. 2021.
- [27] A. M. L. da Silva, J. F. d. Costa Castro, and R. Billinton, "Probabilistic assessment of spinning reserve via cross-entropy method considering renewable sources and transmission restrictions," *IEEE Trans. Power Syst.*, vol. 33, no. 4, pp. 4574–4582, Jul. 2018.
- [28] Q. Chen and L. Mili, "Composite power system vulnerability evaluation to cascading failures using importance sampling and antithetic variates," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2321–2330, Aug. 2013.

- [29] O. A. Ansari, Y. Gong, W. Liu, and C. Y. Chung, "Data-driven operation risk assessment of wind-integrated power systems via mixture models and importance sampling," *J. Mod. Power Syst. Clean Energy*, vol. 8, no. 3, pp. 437–445, May 2020.
- [30] J. Cai and Q. Xu, "Capacity credit evaluation of wind energy using a robust secant method incorporating improved importance sampling," *Sustain. Energy Technol. Assessments*, vol. 43, Feb. 2021, Art. no. 100892.
- [31] P. Subcommittee, "IEEE reliability test system," *IEEE Trans. Power App. Syst.*, vol. PAS-98, no. 6, pp. 2047–2054, Nov. 1979.



FANG WANG received the B.S. and M.S. degrees from the Harbin Institute of Technology, Harbin, China, in 2001 and 2003, respectively. She is currently a Lecturer with the Nanjing Institute of Technology. Her research interests include power system operation and control.



DONGLI XU received the B.S. and M.S. degrees from Hohai University, Nanjing, China, in 2006 and 2009, respectively. He is currently a Research Associate with the Nanjing Institute of Technology. His research interests include power system operation and control.



YUQI WANG received the B.S. degree from Yangzhou University, Yangzhou, China, in 2018, and the M.S. degree from the Nanjing Institute of Technology, China, in 2022. He is currently with State Grid Jiangsu Electric Power Company Ltd., Nanjing Gaochun District Power Supply Branch, Nanjing, China. His research interests include power system planning and reliability.



FAN CHEN received the B.S. and M.S. degrees from Wuhan University, Wuhan, China, in 2004 and 2006, respectively, and the Ph.D. degree from Hohai University, Nanjing, China, in 2016. She is currently a Professor with the Nanjing Institute of Technology. Her research interests include power system planning and reliability, renewable energy integration in power systems, and power system optimization.

...