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APPLIED RESEARCH

Multifractal Analysis of the Brazilian Electricity Market

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ABSTRACT In Brazil's wholesale electricity market, long-term contract prices are negotiated between power generators and large consumers. Unlike traditional markets, pricing is not driven by market forces but rather determined by complex computational models known as Hydrothermal Dispatch Optimization Models. These models calculate the Difference Settlement Price (DSP), serving as the short-term market price for electricity. The Brazilian market is divided into four interconnected submarkets: the Southeast, Northeast, North, and South. This study fills an existing research gap by examining the multifractality of these submarkets by applying Multifractal Detrended Fluctuation Analysis over a deseasonalized price return time series. Specifically, it aims to characterize the multifractal features of electricity prices, identify the underlying causes of this multifractality, and assess market efficiency indices over time. Our analysis of historical electricity prices revealed that all submarkets demonstrated anti-persistent behavior–also known as mean-reversion–and multifractality. This finding aligns with similar observations in global markets. The South submarket displayed the highest level of multifractality and the lowest market efficiency. Conversely, the North submarket had the lowest multifractality and the highest efficiency. Through sliding-window analysis, we investigated temporal variations in the Hurst exponent and Long Memory Magnitude, an index to compute market inefficiency. We found consistent anti-persistent behavior across all submarkets, with the South submarket showing greater volatility in its inefficiency index. While preliminary and requiring further in-depth analysis and consideration of other factors, these findings offer valuable insights for decision-makers and regulators pursuing new market arrangements to boost efficiency.

INDEX TERMS Brazilian electricity market, multifractal detrended fluctuation analysis, generalized hurst exponent, anti-persistence, singularity spectrum, market efficiency.

I. INTRODUCTION

The Brazilian electricity market consists of two commercial environments: (i) the regulated contracting environment (RCE) and the free contracting environment (FCE). In the RCE, distributors purchase electricity through public auctions to meet the demands of their captive consumers. Long-term contracts and fixed price structures govern this process. On the other hand, the FCE allows producers and large consumers to negotiate the terms of their supply contracts, where prices are influenced by factors such as supply and demand, cost of production, and risk evaluation [1]. The Chamber of Electric Energy Trading (CCEE in Portuguese) conducts a monthly settlement of agents' transactions in the short-term market (or spot market). It calculates the difference between what has been produced or consumed and what was initially contracted. These differences, or imbalances, are reconciled at the Difference Settlement Price (DSP). The DSP

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is an integral adjustment mechanism, ensuring equilibrium between supply and demand. Its dynamic nature provides invaluable insights into the efficiency and operational dynamics of the Brazilian electricity market, positioning it as a critical element in market analysis [2].

For over 20 years, the DSP was established every week by sophisticated computational models that considered technical information provided by the National Electric System Operator (ONS). Unlike other majority countries, Brazil's model is a *tight-pool* type, where the DSP is determined by the Operational Marginal Cost (OMC) without requiring generator bids. The OMC is calculated from a complex chain of hydrothermal dispatch optimization models, which aim to minimize the expected total operational cost while considering a series of constraints, as described by [3]. The DSP is calculated for each submarket (North, Northeast, South, and Southeast) and load level. The DSP is subject to a maximum and a minimum price, which the regulatory agency defines annually and applies to each settlement period. Since the DSP was calculated in advance (a week ahead), the models' input data are based on forecasts available before the system's real operation. These forecasts include values such as the declared generation availabilities and the anticipated demands for each submarket, detailed by [4] and [5].

In January 2021, Brazil officially implemented the hourly pricing system, calculated one day in advance (day-ahead) for every 24 hours of the following day. This significant transition was made possible by intense technical discussions between market entities and agents in 2017. The DESSEM model was chosen as the official model for calculating hydrothermal power plant dispatch, considering short-term network constraints for large-scale systems with very detailed hydroelectric, thermal, and network constraints, as outlined in [6]. From April 2018 to December 2020, a shadow operation was implemented to preview the potential impacts of the new hourly pricing system on the market. This significant trial period allowed the ONS, CCEE, and market agents to test the new model in real time. Furthermore, this phase enabled them to assess the impact on very short-term price formation and propose various adjustments to the calculation methodology.

Despite the progressive refinement of the pricing system in the Brazilian electricity sector, the link between price and the model chain continues to generate many discussions. In 2019, the Ministry of Mines and Energy (MME) established a Working Group to develop proposals for modernizing the Brazilian electricity sector. Among the various topics covered, the subgroup responsible for the "Price Formation Mechanisms" topic explored existing literature and international experiences to seek alternatives to the pricing model currently in effect in Brazil, as reported in [5]. However, implementing such a significant market change requires an in-depth understanding of the dynamics of spot electricity prices in Brazil.

Electricity price time series exhibit several stylized facts, including seasonality, mean reversion, price spikes, and multifractality. Seasonality in prices is an intrinsic statistical property in some markets, where the price may vary according to the hours of the day, days of the week, and seasons of the year (for example, higher prices in winter than in summer due to the use of energy resources for heating). Mean reversion or anti-persistence is a characteristic of spot prices in some countries, where upward (downward) movements are more likely to be followed by downward (upward) price movements. When upward and downward movements are more likely to be followed by the same type of movement, these series display persistence or long memory. The type of persistence in a time series can be characterized by the Hurst coefficient (H_2) that measures whether a series is random or does not display temporal correlation ($H_2 = 0.5$), anti-persistent ($0.0 < H_2 < 0.5$) or persistent ($0.5 < H_2 < 1.0$). Price spikes are one of the main features of spot electricity prices in various markets worldwide, characterized by a substantial increase and fall in price within a short period, [7]. Many studies connect price spikes to the fact that electricity is a non-storable commodity, implying more complex characteristics than other markets.

Multifractality in time series is recognized by a complex and heterogeneous distribution of self-similar behaviors, or fractals, manifested across various scales. This multifractal heterogeneity translates into various degrees of regularity and irregularity emerging in different segments of the same time series, reflecting the presence of multiple scaling exponents. Multifractal analysis has been used to decode the intricate scales' inherent complexities in unbalanced dynamic systems. Such systems span a wide range of domains, including, but not limited to, biological, geological, and hydrological fields, as well as various economic markets, such as financial markets, commodity markets, cryptocurrency markets, and electricity markets. In contrast to a monofractal analysis, multifractal analysis requires consideration of multiple scaling exponents to comprehensively capture the latent behaviors in a dynamic system, as revealed by its time series. In the literature, an excellent review of this methodology applied to financial and electricity markets can be found in [8].

Among the various methods used for multifractal analysis of time series, Multifractal Detrended Fluctuation Analysis (MFDFA) stands out, [9]. MFDFA is a sophisticated yet easy-to-implement statistical technique used to analyze the scaling behavior of non-stationary time series. The aim is to reveal and understand complex patterns typically hidden in these data sets. With this method, it is possible to calculate generalized Hurst exponents (Hq) and singularity spectrum, which reveal the series' multifractality. The existence of multiple Hurst exponents and a broader, asymmetrical shape of the singularity spectrum indicate that the time series is multifractal, showing an underlying complexity that includes multiple correlation regimes and intrinsic heterogeneity.

Numerous studies establish a significant correlation between multifractality and the efficient market theory. A higher degree of multifractality in market price returns indicates more significant inefficiency in that market. When evaluated through a sliding window technique, the indicators associated with market efficiency (or inefficiency) provide an effective means of quantifying the influence of external events on market efficiency. Such an approach allows for continuous monitoring of changes in market efficiency over time rather than a point analysis based on fixed periods, enabling a better understanding of the dynamic behavior of markets.

In our search in academic databases, such as Scopus and Web of Science, we did not identify studies investigating the multifractality of the electricity spot price in Brazil. This gap in the literature indicates that this particular aspect of the Brazilian market is yet to be adequately explored, creating a significant opportunity for future contributions in this study area.

Over the last 20 years, there has been a scarcity of studies focused on analyzing the behavior of the DSP. This gap in the literature can be partially justified by the reduced size of the weekly price series, which only surpassed the thousand value mark in 2021. The limited sample size can present a challenge, as depending on the type of analysis applied, small samples can lead to imprecise conclusions due to the insufficiency of statistical significance of the data. Therefore, it is essential to consider the limitations imposed by the sample size in interpreting the analysis results.

Therefore, in this work, we use MFDFA to probe the complexity of the Brazilian electricity market, using return series on deseasonalized spot market prices. This study offers an innovative contribution to the literature on the efficiency of electricity markets by analyzing multifractality in a distinct context - a market in which prices are generated from mathematical models rather than being exclusively determined by traditional supply and demand forces. This paper proposes a differentiated look and provides relevant insights about price dynamics in a less conventional market formation scenario.

We organize the article as follows: in section II, we present a bibliographic review that encompasses (i) the electricity market in Brazil, (ii) works on spot price evaluation in Brazil, and (iii) multifractality in electricity markets. In section III, we show the spot electricity price by submarket, its descriptive statistics, and probability distributions. In section IV, we present the MFDFA method. This section explains the algorithm and the necessary procedures and steps for its application. We discuss the two sources of multifractality: the presence of long-tail in the probability distribution of the time series values and the different long-range correlations of small and large fluctuations. We describe two widely used methods to identify these sources of multifractality. Moreover, we address the relationship between multifractality and efficient market theory, discussing how multifractal behaviors can impact market efficiency and the indexes used to measure it. In section V, we present the results of the application of MFDFA on the time series. We explain how to identify multifractality and its possible origins through various graphs. At the end of the section, we present a temporal evaluation of two indicators that help understand the evolution of multifractality over time. Finally, we present our conclusions in section VI.

II. BIBLIOGRAPHIC REVISON

A. BRAZILIAN ELECTRICITY MARKET

Electricity prices fundamentally depend on the balance between supply and demand, regardless of the model applied. High volatility of the spot price and price peaks are associated with the inability to store electricity and inelastic demand, as cited in [10] and [11]. However, the ongoing integration of intermittent renewable sources has increasingly affected the dynamics of price formation in the spot market over recent years. Alongside uncertainties about supply (hydrological variation and fuel prices) and demand (economic growth and temperature variation), intermittent plants have introduced new uncertainties tied to climatic factors such as wind direction, wind strength, and solar irradiance. Several studies, including [12], [13], [14], indicate that these sources lead to a reduction in prices in the spot market due to their near-zero or zero marginal cost. These studies also note an increase in spot market volatility, as we will observe low spot prices more frequently when renewable resources are abundant. However, price peaks will occur when these resources become scarce.

In Brazil, the federal government has been encouraging the implementation of intermittent plants (wind and solar) since the beginning of the 2010s, reducing the prominence of hydroelectricity. Hydroelectric power represented over 90% of the installed capacity in 2001 and currently accounts for less than 70%, according to [15]. Like other countries, the increase in energy production from intermittent sources has led to increased uncertainties in the operation of the electrical system and price formation in the market, as discussed in [16].

The formation of electricity prices in the market is linked to the system's operation coordinated by the National Electric System Operator (ONS). The Electric Energy Trading Chamber (CCEE) is responsible for calculating the spot price, known as the DSP, using the same mathematical models the operator uses for optimizing hydrothermal dispatch, as explained in [3]. A series of models use dynamic programming to optimize hydrothermal dispatch in the medium term (up to five years), short term (up to two months), and very short term (up to seven days).

Since Brazil's electric sector relies predominantly on hydroelectric power, the flows and stored volumes in the power plants' reservoirs play an essential role in the system's stochasticity. The medium and short-term models calculate the future value of water, indicating the intrinsic cost of water for the system's operation. Future periods more likely to lack this resource have higher future water values, consequently making the system operation more costly due to the dispatch of thermal power plants with high unit variable costs. On the other hand, when there is an abundant water supply in the future, the future water value will be lower, reducing thermal dispatch and the Marginal Operation Cost (MOC).

The future value of water is passed along the chain of models, from long/medium-term to short and very short-term, ensuring that models with less stochasticity (short term) or deterministic (very short term) consider this uncertainty source. After the problem converges, the MOC is given by the Lagrange multiplier associated with demand constraints, according to [3], [6], and [17]. The DSP value for each submarket equals the MOC, limited by a maximum and minimum price defined by the regulator and valid for each verification period. Thus, the spot price in Brazil does not form through a competitive process between supply and demand but is calculated through the complex chain of mathematical models previously explained. For about 20 years (from 2001 to December 2020), the DSP was calculated every week per submarket (Southeast, South, Northeast, and North) and per load level (representing light, medium, and heavy loads), as discussed in [18] and [19]. The price began to be calculated one day ahead for the next 24 hours on January 01, 2021, as shown in [6], [20], and [21].

B. BRAZILIAN ELECTRICITY SPOT PRICES

The behavior and main stylized facts characterizing the DSP have been relatively unexplored over the past 20 years. Queiroz et al. [22] proposed a neural network model to simulate monthly spot electricity prices in Brazil. According to the authors, this model could replace the official model (optimizing hydrothermal dispatch using stochastic dual dynamic programming) with advantages in terms of computational effort without yielding deviations that would compromise risk analysis algorithms.

Oliveira et al. [23] introduced a methodology to estimate the prices of options contracts for electricity markets in Brazil. As per the authors, a significant theoretical contribution made in the work was demonstrating that a mean-reversion stochastic process (Ornstein–Uhlenbeck Vasicek) accurately represents the spot price in Brazil when compared to geometric Brownian models.

Gontijo et al. [24] used Dynamic Time Scan Forecasting (DTSF) for predicting spot electricity prices in Brazil, a methodology initially formulated for predicting wind and power generation in industrial plants. The method involves scanning a time series and identifying past patterns (called "matches") similar to the latest available observations. Future values are predicted from the most similar matches using aggregation functions such as the median. The authors compared the results obtained with DTSF to eight other methodologies from the M4-Competition. According to them, DTSF surpassed all other methodologies in performance except when dividing the price series into seasons.

Daglish et al. [25] analyzed the impact of the 2004 electricity sector reform on the volatility of spot electricity prices in Brazil. They fit a Markov Switching model to the monthly price time series sampled from January 2000 to October 2016. The authors considered the Markov Switching ideal to model spot prices in Brazil because the prices behave in two distinct volatility regimes.

Lauro et al. [26] proposed a methodology for simulating the decision-making process for energy contracting from a hydroelectric power plant, considering uncertainties about the spot price, forward contract prices, and generation scaling factor. The authors considered the monthly prices and two-stage stochastic dynamic programming to model the uncertainty in forward prices.

With the initiation of hourly price publication in 2021, new research on the price of electricity in the Brazilian market has emerged. Marchetti and Rego [20] demonstrated the impact of adopting the new price formation methodology (weekly to hourly) on the fair value of a generic wind and solar plant. According to the authors, the adoption of hourly prices negatively impacted the fair value of a wind farm. Some projects showed a depreciation of about 9% of the total value of the enterprise due to the trading of contracts with flat seasonality and modulation. Solar plants experienced positive impacts in the Southeast but without significant impacts in the Northeast. Nametala et al. [21] used the hourly prices for the year 2021 (arithmetic mean of all submarkets) and investigated statistical aspects of the time series, such as regime-switching characterization and the sources of price spike formations. In addition, they investigated the price relationship with exogenous variables and finally compared electricity markets from other countries. Gontijo et al. [27] applied DTSF for hourly price forecasting in Brazil. They considered hourly prices from 2019, taking into account the testing period until the end of 2021. The DTSF methodology had already been applied in the other paper by the same authors for the forecasting of weekly electricity prices in Brazil, [24]. DTSF showed better predictive performance and less variability when compared to statistical models and machine learning.

C. FRACTAL AND MULTIFRACTAL BEHAVIOR OF ELECTRICITY MARKETS

Fractal and multifractal systems can be found across various fields, including physics, geology, hydrology, biology, social sciences, psychology, economics, and computer science. Fractals are geometric structures that exhibit self-similarity, meaning their complex structure repeats across all scales. People often describe this character as a fractal dimension, which can be considered a measure of the fractal's roughness or complexity [28].

Though we can apply these concepts to objects and images, in this study, we are particularly interested in observing fractal and multifractal behaviors in time series. A time series' self-similarity can manifest through the power law describing the relationship between the series' fluctuations at different time scales. This means that a single scaling exponent, which measures the time series' roughness, irregularity, or complexity, links the variance or other series statistics measured at various time scales by a power law. Mandelbrot, besides introducing the concept of fractal geometry, also published the pioneer studies applying fractal analysis to time series, as presented in [29] and [30].

The concept of multifractality emerged when the French mathematician was studying turbulence and noticed that a single scaling exponent could not adequately describe this phenomenon, as is typical of regular fractal objects. Instead, he observed that turbulence displayed a distribution of fractal dimensions across a range of scales, as noted in [31]. Multifractal time series show different autocorrelations of large fluctuations from the autocorrelation of small fluctuations, requiring more than one scaling exponent to describe their behavior fully.

Complex systems like the electricity market can be described by analyzing price, return, or volatility time series that present stylized facts such as seasonality, long memory, price spikes, and multifractality. Using the Rescaled Range (R/S) method, Weron and Przybyłowicz [32] assessed the daily return series of the electricity markets in California and Switzerland. They calculated the Hurst coefficients for both markets and concluded that both exhibit mean reversion. The Hurst coefficient of CalPX ($H_2 = 0.4193$) is lower than that of the Swiss market ($H_2 = 0.4391$), leading Weron and Przybyłowicz to conclude that the former has more pronounced anti-persistence compared to the latter. In addition, they confirmed a conclusion from another paper [11], where they modeled the spot electricity price return in California (CalPX) as a mean reversion process.

Simonsen [33] applied the Average Wavelet Coefficient (AWC) method to data from the Nordic Spot Electricity Market from 1992 to 2000, concluding that hourly spot prices approximate an anti-persistent (mean-reverting) process, characterized by a Hurst coefficient of 0.41. The presence of crossover near the 1-day scale indicates multiple scaling exponents, leading the author to consider evaluating the Nordic market using methods that allow for estimating multiple exponents (multifractality), such as the continuous wavelet transform. The price exhibits a persistent process for scales less than one day, with $H_2 > 0.5$, and anti-persistence appears for scales larger than one day.

Norouzzadeh et al. [34] analyzed the return on the hourly price of the Spanish electricity market using Multifractal Detrended Fluctuation Analysis (MFDFA), obtaining a Hurst coefficient $H = 0.16 \pm 0.01$, indicating strong anti-persistence.

Serletis and Bianchi [35] used the Detrended Moving Average (DMA) method to analyze the informational efficiency of the Alberta electricity market and check whether power exchange transactions (energy flows between markets) are becoming increasingly significant in electricity markets. The results showed that the Alberta electricity market is highly inefficient (anti-persistent), and cross-border electricity trade between Alberta and neighboring jurisdictions helps predict price dynamics in the Alberta electricity market.

Erzgräber et al. [36] analyzed system prices in the Nordic market using different techniques to estimate the Hurst exponent. They concluded that the observed variation in Hurst exponents could be considered a signal of multifractality in electricity prices.

Uritskaya and Serletis [37] applied the DFA method at different scales to confirm the presence of multiple Hurst coefficients (multifractality) in the daily electricity prices in the Alberta (Canada) and Mid-Columbia (United States) markets, as well as in the Alberta natural gas market. Using the DFA results for the three markets, Uristkaya et al. analyzed market efficiency, considering those that exhibited long memory, $H_2 = 0.5 \quad (fGn) \quad and \quad H_2 = 1.5 \quad (fBm)$, to be less efficient. They confirmed the anti-persistence and consequent inefficiency of the Alberta and Mid-Columbia markets, with $H_2(Alberta) = 1.22 \pm 0.01 \quad and \quad H_2(Mid - C) = 1.32 \pm 0.05$.

Malo [38] proposed modeling the dynamics of Nordpool's spot and future prices using Copula-MSM (Markov Switching Multifractal). Although the proposal of this article is outside our objective, Malo did analyze the scale exponents (Hurst exponent) of the Nord Pool's daily closing prices for spot contracts between March 1998 and January 2006. He used several methods to calculate the exponent, and the average value found was $H = 0.32$.

Serinaldi [39] conducted a study on the precautions to take when applying the methodology for calculating the Hurst coefficient, especially concerning the signal nature: fBm (Fractional Brownian Motion) or fGn (Fractional Gaussian Noise). In addition to financial series, Serinaldi used price series from four electricity markets: daily average price for Alberta (persistent, $H_2 > 0.5$) and Mid-Columbia (anti-persistent, $H_2 < 0.5$), hourly prices for Alberta (persistent, $H_2 > 0.5$), hourly prices for Ontario (persistent, $H_2 > 0.8$), and hourly prices for the Italian market (persistent, $H > 0.5$).

Qian et al. [40] proposed a modification to the MFDFA algorithm, replacing the process of removing local trends using a degree m polynomial with Empirical Mode Decomposition (EMD). To prove its effectiveness, they applied the proposed method to the Shanghai Stock Exchange Composite index, with a frequency of 1 minute, and to the daily prices of the Australian Electricity Market, confirming anti-persistence.

Rypdal and Løvsletten [41] introduced two mean-reversion models based on the multifractal random walk (MRW). The first model describes the anti-persistence of Norway's (Nordpool) spot electricity prices through exponential decay of correlations (damped MRW), while the second model describes the decay of correlations by power law (fractional MRW). The data set consisted of the hourly spot price measured in Norwegian Kroner (NOK) from May 4, 1992, to August 27, 2011. The authors presented maximum likelihood methods for estimating the parameters of these models. They concluded that the damped MRW model is more suitable for forecasting spot prices than the fractional MRW model. However, the multifractal models more effectively exploit memory effects in volatility for future price prediction.

Liu et al. [42] investigated the feasibility of applying multifractal theory to analyze electricity price fluctuations. They used hourly price series from the Pennsylvania-New Jersey-Maryland (PJM) electricity market to demonstrate the effectiveness of the proposed VaR estimation method for assessing short-term electricity price volatility risk.

Ghosh et al. [43] analyzed the multifractal behavior of energy bid prices in five different areas of India, sampled at 15-minute intervals, using the MFDFA method. The analysis was done monthly from April 2012 to March 2014, confirming the multifractality of prices estimated by the width of the singularity spectrum, $\Delta\alpha$. The multifractal analysis used prices, not returns.

Ali et al. [44] investigated and compared the overall multifractality, time-varying, and efficiency of four electric regions in the United States. They applied the MFDFA method to the daily logarithmic return of electricity prices from the MASS Hub, Mid-C, Palo Verde, and PJM West markets, sampled from 2001 to 2021. The efficiency of the markets was estimated using the MLM (Magnitude of Long Memory) index, also known as MDM (Market Deficiency Measure). As a result, they showed that all markets exhibited anti-persistence ($H(q = 2) < 0.5$) and multifractal behavior estimated through the strength of multifractality Δh, with PJM West showing the highest strength and Mass Hub the lowest. MLM confirms PJM West as the least efficient market and Mass Hub as the most efficient. They used sliding window analysis over H_2, ΔH, and MLM. All markets maintained anti-persistence behavior overall period.

Han et al. [45] analyzed the time series of electricity prices in Germany and Austria indexed on the European market (EPEX), focusing specifically on day-ahead hourly, intraday hourly, and intraday 15-minute market prices. They used MFDFA to confirm strong anti-persistence in the price series for time scales larger than 12 hours ($H_2 \sim 0.16$). For time scales shorter than 12 hours, both intraday hourly and day-ahead hourly prices showed persistence, with $H_2 \sim 0.63$ and $H_2 \sim 0.61$, respectively. In contrast, the 15-minute intraday prices remained anti-persistent with $H_2 \sim 0.31$. The strength of multifractality, calculated through $\Delta\alpha$, was more intense for the smallest time scales in all price series. Finally, they found that long-term behavior is strongly influenced by the evolution of large-scale weather patterns, with a typical time scale of four days.

Cramer et al. [46] suggested using multifractal analysis (MFDFA) as an additional validation method for more complex features in scenarios generated by generative models. Synthetic series produced by Generative Adversarial Networks (GANs), Wasserstein GANs (WGANs), and Variational Autoencoders (VAEs) trained with time series of generation from photovoltaic and wind power plants in Germany (between 2013 and 2015), and intraday electricity price time series from the European Energy Market (between 2017 and 2019) underwent various types of validation, including multifractal features such as the width of the singularity spectrum.

Čuperk [47] explored the maturity of the Czech intraday electricity market during the COVID-19 pandemic, using multifractal analysis (MFDFA) on intraday hourly average prices and the Magnitude of Long Memory (MLM) index. They observed a nonlinear relationship between the Czech government's COVID-19 policy and the Hurst exponent at long time scales, the width of the singularity spectrum, and the MLM index at short time scales, indicating that flexible anti-COVID policies are associated with a more mature market and vice versa.

III. DATA

This article uses the weekly spot prices of the four submarkets: Southeast, South, Northeast, and North. Data are available on the CCEE website [48], as of June 30, 2001. Until 2021, the price was calculated based on the Marginal Cost generated by DECOMP and measured in BRL/MWh. As of January 1, 2021, with the publication of hourly prices calculated using the DESSEM model, [6] the weekly price is the arithmetic mean of the hourly price values throughout the operating week, which begins on Saturday and ends on the following Friday. Electricity purchase and sale transactions in the Brazilian market are settled monthly, hence the need to maintain weekly and monthly price calculations.

The analyses carried out a historical series of average weekly prices per submarket, covering the period from June 30, 2001, to the last operational week of 2022, which covered the period from December 24 to 30, 2022. The graphs (A1), (B1), (C1), and (D1) of Figure 1 present the behaviors of these series.

In Brazil, excess production capacity and low demand influence the price of electricity, characterizing it by long periods of low prices. A good example is the period that started in 2002 and lasted about five years, resulting from a 20% decreased demand during the energy rationing. The need for consumers to adapt to this reduction led many to invest in more efficient processes, influencing the post-rationing demand recovery.

In 2008, there was an imbalance between supply and demand due to problems related to the availability of natural gas, which resulted in a new escalation of prices, reaching its peak in January of that year. A new period of an unfavorable hydrological regime began in 2013, and prices gradually escalated, reaching their maximum value in 2014, about R$ 800.00/MWh.

In January 2015, political and regulatory interference led to an artificial price reduction due to the regulatory agency's implementation of a new ceiling price calculation method. Between 2015 and 2021, the price exhibited high volatility due to hydrological issues. From November 2020 to April 2021, the country faced the worst drought since flow data began to be recorded (1931), affecting the regions with large hydroelectric plant reservoirs. As a result, prices remained

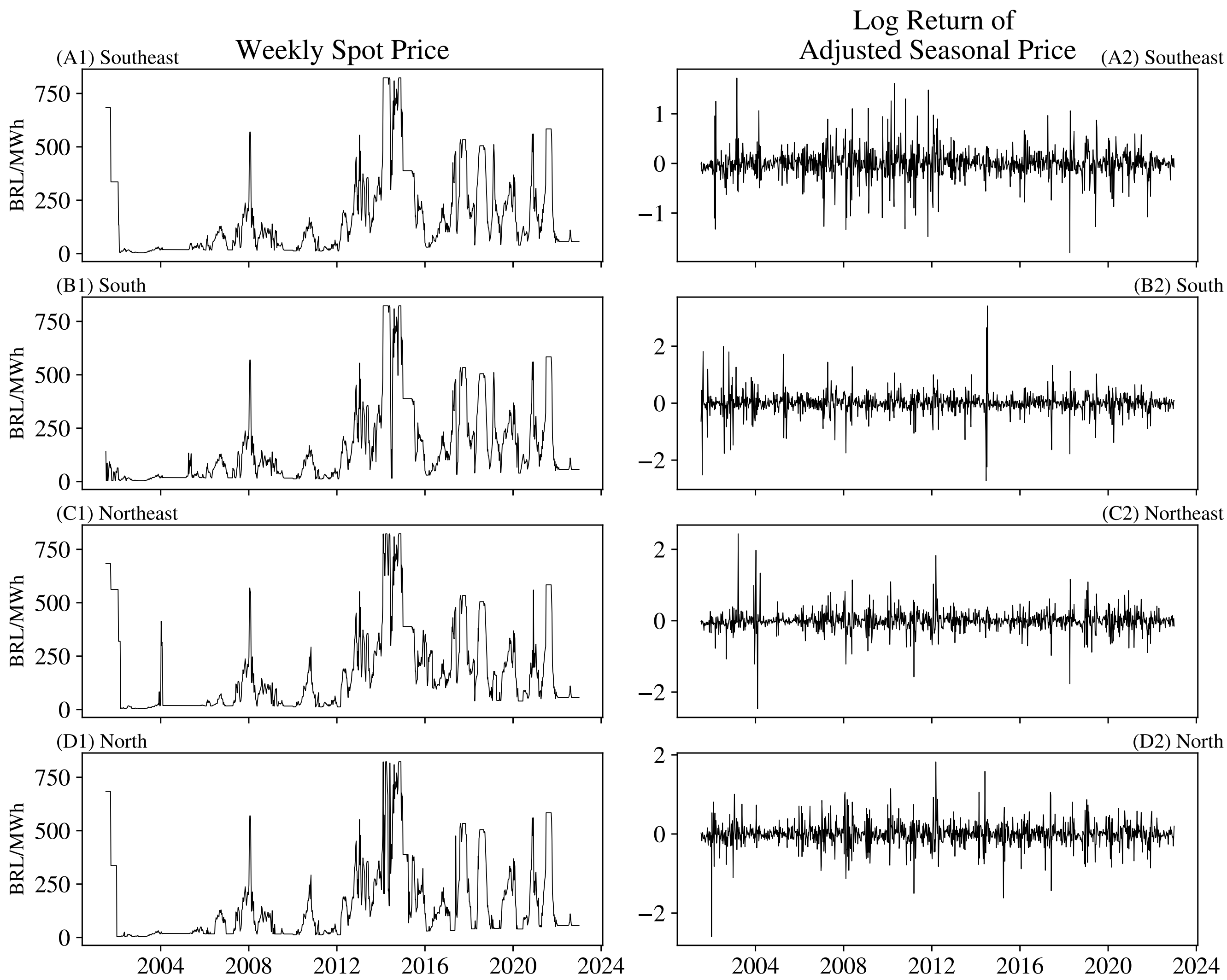

FIGURE 1. **Historical time series of weekly difference settlement price and log return of adjusted seasonal price.**

high for an extended period until the 2021/2022 rainy season, when there was a significant increase in the amount of energy stored in the reservoirs. Additionally, the market was still recovering from the impact of the COVID-19 pandemic. These combined factors led to a drastic price reduction, reaching their lowest value in 2022 and remaining at that level.

Table 1 presents the descriptive statistics of the weekly price series. Each series consists of 1127 values, covering more than 20 years of operation in Brazil's electricity market. When analyzing the probability distribution density function of the four submarkets, we can observe a positive asymmetry, which is visually evidenced in the histograms of Figure 2 and also confirmed by the relation Mode < Median < Mean in Table 1. Approximately 60% to 70% of the sample values are below BRL 150/MWh, while the rest of the sample is distributed in a price range that varies from 150 to 822.30 BRL/MWh, as shown in Table 2 of the percentiles of weekly prices.

We used the formulation that computes their strengths to analyze the unobserved trend and seasonality components, as described in Hyndman and Athanasopoulos [49]. The decomposition was applied to the logarithm of the prices employing the Seasonal-Trend Decomposition using LOESS (STL) approach, as described in Cleveland et al. [50] and implemented in Seabold and Perktold [51]. STL and other time series decomposition methods are described in detail in a recent survey article, [52]. We considered the annual seasonality, which has a period of 54 weeks, in order to capture the seasonal patterns in the data. Decomposition into trend and seasonality is a powerful tool for describing and understanding the behavior of energy prices over time. We can isolate the seasonal effect of prices and analyze the behavior of the underlying trend more clearly, contributing to a better understanding of the factors affecting energy prices.

The strength of seasonality and trend are presented in Table 1 in the "Season" and "Trend" lines, respectively.

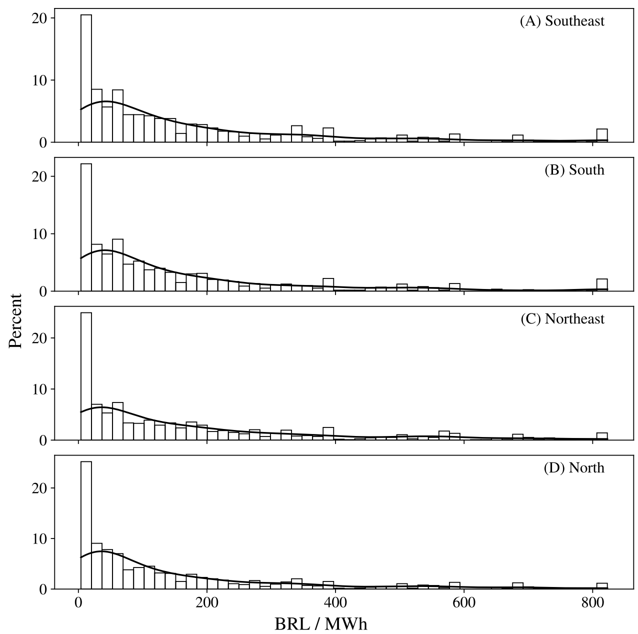

FIGURE 2. Histogram of weekly spot price by submarket.

TABLE 1. Descriptive statistics of weekly electricity price time series for each submarket (Prices are measured in BRL/MWh).

Statistics	Hourly Price			
	Southeast	South	Northeast	North
Minimum	4.00	4.00	4.00	4.00
Moda	18.59	18.59	18.59	18.59
Median	95.54	83.69	95.85	74.19
Mean	169.25	155.38	171.70	150.51
Maximum	822.83	822.83	822.83	822.83
Std. Dev.	191.95	183.19	194.82	182.39
Skewness	1.66	1.84	1.51	1.77
Kurtosis	2.25	3.11	1.57	2.60
Season	0.1349	0.1551	0.2239	0.4534
Trend	0.7636	0.7675	0.7722	0.7904
Count	1127	1127	1127	1127

We observed a significantly higher trend strength than seasonality, suggesting long-term factors may substantially influence prices. These factors may include structural changes in energy supply and demand, energy policies, or macroeconomic aspects. However, seasonality underscores the importance of seasonal patterns in energy prices.

We obtain the return series r_{t+1} by applying the logarithmic difference to the prices after removing the seasonal component, as described in equation (1). The graphs (A2), (B2), (C2), and (D2) in Figure 1 illustrate the variations of the returns for each submarket. Throughout the article, we will use the term "return" or "return series" to refer to this measure.

$$r_{t+1} = \log p_{t+1} - \log p_t \tag{1}$$

IV. METHODOLOGY

One of the goals of this article is to contribute to discussions about electricity pricing in Brazil through the analysis of spot prices from a perspective that has yet to be approached: multifractality. The multifractal analysis of historical price return series allows for the verification of self-similarity of the underlying stochastic process; that is, it allows for analyzing the behavior of the series at different scales. In this article, we investigate the multifractal properties in the time series of deseasonalized weekly spot price returns in each submarket. In addition, the relationship between multifractality and market efficiency will be studied temporally, showing even the impact of adopting hourly prices from 2021.

TABLE 2. Percentiles of weekly price time series for each submarket (BRL/MWh).

Percentil	Southeast	South	Northeast	North
10%	16.310	16.216	16.310	15.558
20%	18.846	18.590	18.590	18.330
30%	38.250	35.808	30.374	30.062
40%	59.016	55.700	55.700	43.100
50%	95.540	83.690	95.850	74.190
60%	134.760	120.890	145.272	111.836
70%	191.848	176.912	199.786	170.596
80%	301.248	247.066	307.782	254.738
90%	472.510	405.454	505.180	393.592
100%	822.830	822.830	822.830	822.830

Multifractal Detrended Fluctuation Analysis (MFDFA) is an advanced statistical technique used to analyze the scaling behavior of non-stationary time series. This approach aims to reveal and understand complex patterns, such as multifractality, which are usually hidden in these datasets. Developed by Kantelhardt et al. [9] as a generalization of the Detrended Fluctuation Analysis proposed by Peng et al. [53], MFDFA calculates the generalized Hurst exponent $H(q)$ and the Rényi exponent $\tau(q)$ for different moments q. These exponents capture the statistical properties of fluctuations at different scales, allowing for the separate investigation of the contribution of smaller scales ($q < 0$) and larger scales ($q > 0$).

Multifractal analysis using the MFDFA method has been applied in various areas of knowledge that present long-range power-law correlations. For a comprehensive review of the methodologies and application areas of multifractal analysis, we recommend reading the work of Kantelhardt [54]. Moreover, in the context of the financial market, multifractal analysis has been widely explored, and an excellent resource for a review on the subject can be found in the article by Jiang et al. [8].

A. MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS

According to [9], the MFDFA method consists of 5 steps, where the first three steps are based on the DFA method, [53].

Let p_t be a time series where $t = 1, \cdots, N$ being N the total number of observations. We define the return series r_{t+1} by the logarithmic difference of the price, according to Equation (1).

1) Determine the integrated series Y through the accumulated deviation as per equation 2, where $\bar{r}$ is the mean of

the return series r.

$$Y(i)=\sum_{k=1}^{i}[r_k-\bar{r}],\quad i=1,\cdots,N \tag{2}$$

2) Divide the integrated series into non-overlapping intervals of equal size s, where $N_s=\lfloor N/s\rfloor$. When N is not an integer multiple of the scale s, then some points at the end of the series will not be computed. The integrated series is divided again from end to start to avoid this loss of information. This leads to having $2N_s$ segments.
3) At this step, the removal of local trend in each of the $2N_s$ segments is done by means of a polynomial fit of degree m, chosen considering the capability of trend elimination. For each segment $i=1,\cdots,s$, the variance is calculated according to Equation (3), where $y_v(i)$ is the polynomial of degree m calculated through the least squares method.

$$F^2(s,v)=\frac{1}{s}\sum_{i=1}^{s}Y[(v-1)s+i]-y_v(i)^2 \tag{3}$$

for any v, $v\in(1,\cdots,N_s)$

$$F^2(s,v)=\frac{1}{s}\sum_{i=1}^{s}Y[N-(v-N_s)s+i]-y_v(i)^2 \tag{4}$$

where $v\in(N_s+1,\cdots,2N_s)$

4) The fluctuation function of order q for each of the $2N_S$ segments is given by Equation (5) for any real q different from zero and by Equation (6) when $q=0$.

$$F_q(s)=\left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F^2(s,v)]^{\frac{q}{2}}\right\}^{\frac{1}{q}} \tag{5}$$

$$F_0(s)=exp\left\{\frac{1}{4N_s}\sum_{v=1}^{2N_s}ln[F^2(s,v)]\right\} \tag{6}$$

5) Determine the scaling behavior of the fluctuation functions by analyzing the log-log plot $F_q(s)\times s$ for each value of q. If the series are correlated by the long-range power law, then $F_q(s)$ increases for large values of s by the power law as shown in Equation (7), where the scaling exponent is calculated as the slope of the linear regression of $log\ F_q(s)$ against $log\ s$, for each q.

$$F_q(s)\sim s^{h(q)} \tag{7}$$

The singularity spectrum, also known as the multifractal spectrum, provides an alternative way to describe the multifractality of a time series. Kantelhardt et al. [9] demonstrated that the scaling exponent or Rényi coefficient, τ_q, can be obtained via the relationship between box counting formalism and partition function $Z_q(s)$, as shown in Equation (8). This approach delivers a detailed characterization of fluctuations at different scales and allows for a more comprehensive analysis of the time series multifractal properties.

$$\tau(q)=qh(q)-1 \tag{8}$$

Equation (8) illustrates the relationship between the generalized Hurst coefficient $h(q)$ and the scaling exponent $\tau(q)$. This relationship is critical, as it allows the multifractal formalism to demonstrate a link between $\tau(q)$ and the multifractal spectrum $f(\alpha)$. By applying the Legendre transform to Equation (8), we obtain $f(\alpha)$:

$$\text{Singularity Spectrum}=\begin{cases}\alpha=\tau'(q)\\ f(\alpha)=q\alpha-\tau(q)\end{cases} \tag{9}$$

where α is the Hölder coefficient and $f(\alpha)$ is the dimension of the series subset characterized by α.

Some important points that can be highlighted about the MFDFA algorithm are:

(a) Degree m polynomials are used to remove order $m-1$ trends in the original series.
(b) The choice of the polynomial degree for local trend removal should be made by comparing different orders. In most cases, the trend can be eliminated with degree 3 or less polynomials.
(c) The results obtained by the DFA method are reproduced for $q=2$ in equation (3), which is the quadratic fluctuation.
(d) By construction, the function $F_q(s)$ is defined only for values of $s\geq m+2$.
(e) For very large scales, $s>N/4$, the function $F_q(s)$ becomes statistically unreliable due to the limited number of segments N_s. Therefore, scales larger than $N/4$ should be excluded from the fitting procedure to determine the scaling coefficient $h(q)$.
(f) The scaling coefficient $h(q)$ is also known as the generalized Hurst exponent.
(g) For stationary time series, the exponent $h(2)$ is known as the Hurst exponent (H_2) and ranges between 0 and 1. The time series is considered uncorrelated when $H_2=0.5$, anti-persistent when $0<H_2<0.5$, and persistent (long memory) when $0.5<H_2<1$.
(h) A time series is considered monofractal when the scaling coefficient $h(q)$ is constant and independent of q.
(i) A time series is considered multifractal when there is a strong dependence between $h(q)$ and q. For $q>0$, $h(q)$ describes the scaling behavior of the segments with large fluctuations, while for $q<0$, $h(q)$ describes the scaling behavior of the segments with small fluctuations.
(j) The MFDFA method only calculates positive generalized Hurst exponents, becoming inaccurate for strongly anti-persistent time series, i.e., when $h(q)$ is close to zero. In these cases, it is recommended to integrate the time series before applying the MFDFA algorithm, resulting in $\bar{h}(q)=h(q)+1$ for the new integrated series.
(k) The accuracy of the MFDFA method depends on the size N of the time series. Series with fewer than 10,000 values may exhibit apparent multifractality, as discussed in [55].

B. SOURCES OF MULTIFRACTALITY

Kantelhardt et al. [9] presented a procedure to analyze two different types of multifractality: (i) multifractality due to a broad probability density function for the values of the return series and (ii) multifractality due to differing long-range correlations (linear and nonlinear) for small and large fluctuations.

When we shuffle time series data, we destroy temporal correlations (short and long-range memories), with the probability distribution remaining intact. According to [9], Equation (10) gives the impact of the correlation on the apparent multifractality.

$$h_{corr}(q) = h(q) - h_{shuf}(q) \tag{10}$$

where $h_{shuf}(q)$ indicates the generalized Hurst coefficient for the shuffled series and $h_{cor}(q)$ is the generalized Hurst coefficient due to the linear and nonlinear correlations of the series. If only type (i) multifractality is present, then $h_{corr}(q) = 0$ and $h_{shuf}(q) = h(q)$. When $h_{corr}(q) \neq 0$, it indicates the presence of correlations, and if $h_{corr}(q)$ depends on q, then we have the presence of type (ii) multifractality. If only type (ii) correlation exists, then $h_{shuf}(q) = 0.5$ and $h(q) = 0.5 + h_{corr}(q)$. If both types of multifractality are present, then $h_{corr}(q)$ and $h_{shuf}(q)$ depend on q.

Other works have shown an equivalent formulation considering Δh, Equation (13), and $\Delta\alpha$, Equation (14), instead of $h(q)$, as in [8] Section 7.2.

$$\begin{aligned} \Delta h_{corr} &= \Delta h - \Delta h_{shuf} \\ \Delta\alpha_{corr} &= \Delta\alpha - \Delta\alpha_{shuf} \end{aligned} \tag{11}$$

We employ the method of the Amplitude Adjusted Fourier Transform (AAFT) [56] to analyze the impact of a broad probability density function on the multifractality of return series. However, various methods exist to eliminate the nonlinear component of time series, as discussed in [8]. The AAFT method involves creating surrogate series, preserving the temporal correlation structure but eliminating the nonlinear component. We applied the following procedure: (i) transforms the original time series into the frequency domain using the Fourier transform; (ii) shuffled the phases of these components randomly; (iii) applies the inverse Fourier transform to obtain the surrogate time series. This technique preserves the linear statistical characteristics of the original series, such as mean and variance, while eliminating nonlinear temporal correlation. So, we use surrogate series to investigate the influence of the probability density function on the multifractality of the return series. Suppose the surrogate series results exhibit less multifractality than the original series, suggesting that the broad probability density function plays a significant role in the multifractality of the return series.

C. EFFICIENCY OF BRAZILIAN ELECTRICITY MARKET

The Efficient Market Theory, proposed by Fama in 1970 [57], establishes a relationship between the availability of information and market price. In an efficient market, the price reflects all available information, making it unpredictable, as it behaves randomly. According to Fama, the price in an efficient market follows a Markovian stochastic process, in which the probabilities associated with the process at a given future time depend only on the present state, independent of past events. Therefore, a Markovian process does not exhibit a temporal correlation between its data and is considered memoryless.

Recently, studies conducted by Cajueiro and Tabak [58], [59] analyzed market efficiency using the Hurst coefficient (H_2), calculated over the sampling period in a sliding window, using R/S or DFA methodologies as proxies. However, these studies adopted monofractal methods to calculate the Hurst coefficient without considering the possible multifractal characteristics of the markets analyzed. By utilizing multifractal methods, like MFDFA, we can capture additional nuances in the scale structure of market prices and explore the presence of multifractality, which can provide valuable insights into the complexity and efficiency of the electricity market.

Kristoufek and Vosvrda [60] employed the Hurst coefficient, the fractal dimension, and lag one autocorrelation to propose a combined measure of market inefficiency given by Equation (12):

$$IE = \sqrt{\sum_{i=1}^{m} \left(\frac{M_i - M_i^*}{M_{i,max} - M_{i,min}} \right)^2} \tag{12}$$

where M_i is the *i-th* measure estimated by the method i, M_i^* is the theoretical value for uncorrelated series, and the distance $M_i - M_i^*$ is a market inefficiency index. Several publications applied this index in various markets, and the results consistently showed that emerging markets were less efficient. In contrast, developed markets were more efficient, i.e., they presented a lower level of inefficiency. This observation aligns with the Efficient Market Hypothesis, which postulates that prices reflect all available information in efficient markets, while in less efficient markets, information may not be fully incorporated into prices, creating opportunities for arbitrage and potentially resulting in greater inefficiency. These findings underscore the importance of investigating the efficiency of financial, commodities, and electricity markets and their impact on asset prices, providing valuable insights for investors, regulators, and policymakers.

As seen in section IV-A, $h(q)$ is independent of q for monofractal time series with compact support. If small and large fluctuations scale differently, then $h(q)$ shows a strong dependence on q, characterizing the time series as multifractal. The variability of $h(q)$ is directly related to the degree of multifractality that the signal presents and can be measured by Equation (13), as per [61]:

$$\Delta h = h(q_{min}) - h(q_{max}) \tag{13}$$

The strength of multifractality, measured through Δh, was applied to the returns of indices from 32 stock markets in different countries. In addition to this measure, [61] used another measure of multifractality strength, based on the length of the multifractal spectrum $f(\alpha)$, called $\Delta\alpha$, where α represents the Hölder coefficient, as can be seen in equation (14). These measures provide a comprehensive assessment of the multifractality present in stock markets, allowing an understanding of these time series' complex and nonlinear characteristics. By considering different measures of multifractality strength, it is possible to gain a complete perspective on the structure and dynamics of these markets, contributing to the understanding of the underlying processes and aiding in investment decision-making.

$$\Delta\alpha = \alpha(q_{min}) - \alpha(q_{max}) \tag{14}$$

Numerous articles studied the efficiency of electricity markets. For a review of articles that addressed this issue, see [62]. More recently, [44] estimated the efficiency of the US electricity market (MASS, MIDC, PALO, and PJM) through the Magnitude of Long Memory index ([63]) given by Equation (15)

$$MLM = \frac{1}{2}\left(|h(q_{min}) - 0.5| + |h(q_{max}) + 0.5|\right) = \frac{1}{2}\Delta h \tag{15}$$

where a market is efficient when the small fluctuations ($q = q_{min} < 0$) and the large fluctuations ($q = q_{max} > 0$) follow Markovian stochastic processes (random walk) and the value of MLM tends towards zero. The larger the value of MLM, the higher the degree of multifractality of the process and the lower the efficiency of the underlying market. Conversely, the lower the MLM, the more efficient the market is.

In the Brazilian electricity market, the spot price is not determined by equilibrium between supply and demand, as in other markets. Instead, computational models that simulate this equilibrium by optimizing hydrothermal dispatch calculate the price. The efficiency of this market is related to the randomness of the marginal cost of operation and, consequently, the settlement price of differences generated by the models. The further the Hurst coefficient H_2 is from 0.5, the more predictable and less random the price will be.

Unlike the financial market, where the presence of noise traders can cause irregularities in price fluctuations, in the Brazilian electricity market, irregularities are related to extreme events, such as the impact of COVID-19 on demand, the influence of climatic phenomena like El Niño and La Niña on renewable energy generation, structural imbalances between supply and demand due to excess or lack of capacity, and regulatory interferences, such as the reduction of the maximum DSP in January 2015. These factors contribute to the complexity and non-linearity of prices in the Brazilian electricity market, and analysis of its dynamics represents a challenging task of great relevance for understanding the functioning of this market.

V. RESULTS

The algorithms used in this work were written by the authors in Python (Python Software Foundation. Python Language Reference, version 3.9. available at http://www.python.org), based on MFDFA, [9] and its implementation in Matlab available at [64]. A flowchart delineating the key stages involved in the assessment of spot price time series in the Brazilian market through MFDFA is presented in Figure 3.

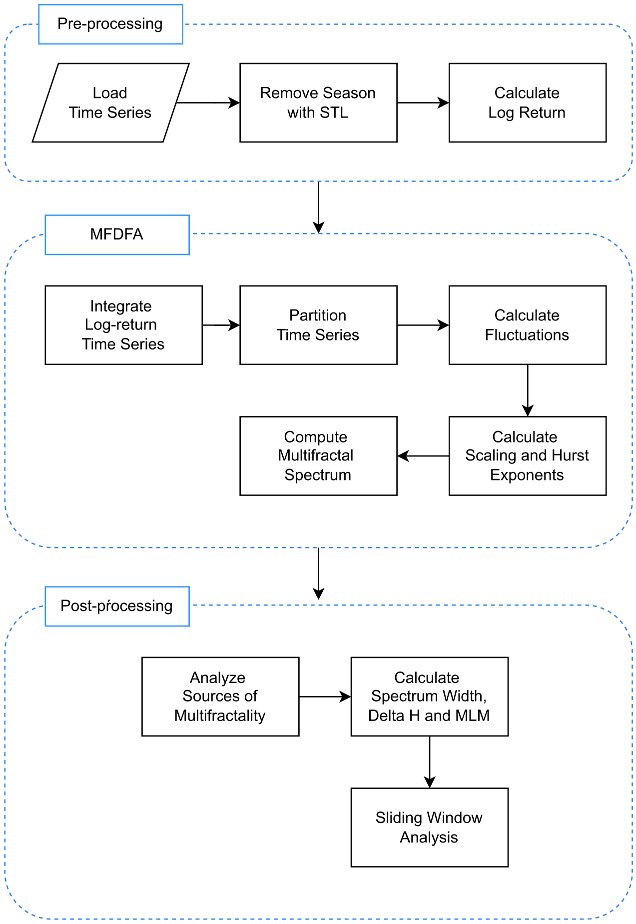

FIGURE 3. Flowchart of the major steps to analyze time series with MFDFA.

After conducting a series of tests, we chose a fourth-degree polynomial ($m = 4$) to eliminate local trends within the data. Although the scale set s suggested by [9] is generally confined to the range $10 \leq s \leq \lfloor N/4 \rfloor$, we heuristically adjusted this to the range $16 \leq s \leq 128$. In this context, $N = 1127$ denotes the total number of observations in our weekly return series, as outlined in Table 1. Regarding the parameter q, we followed the recommendations by [9], employing a range of $-5 \leq q \leq 5$.

A. FLUCTUATION FUNCTIONS ANALYSIS

We analyzed the log-log plot of $F_q(s)$ versus s for each q, as shown in Figure 4 to determine the scaling behavior of the fluctuation functions. If the time series x_i is long-range power-law correlated, $F_q(s)$ increases for large values of s by the power law, according to equation (7).

Although the polynomial fit of order m used by the MFDFA method removes trends of order $m - 1$ in the original series, it is essential to note that this procedure does not guarantee

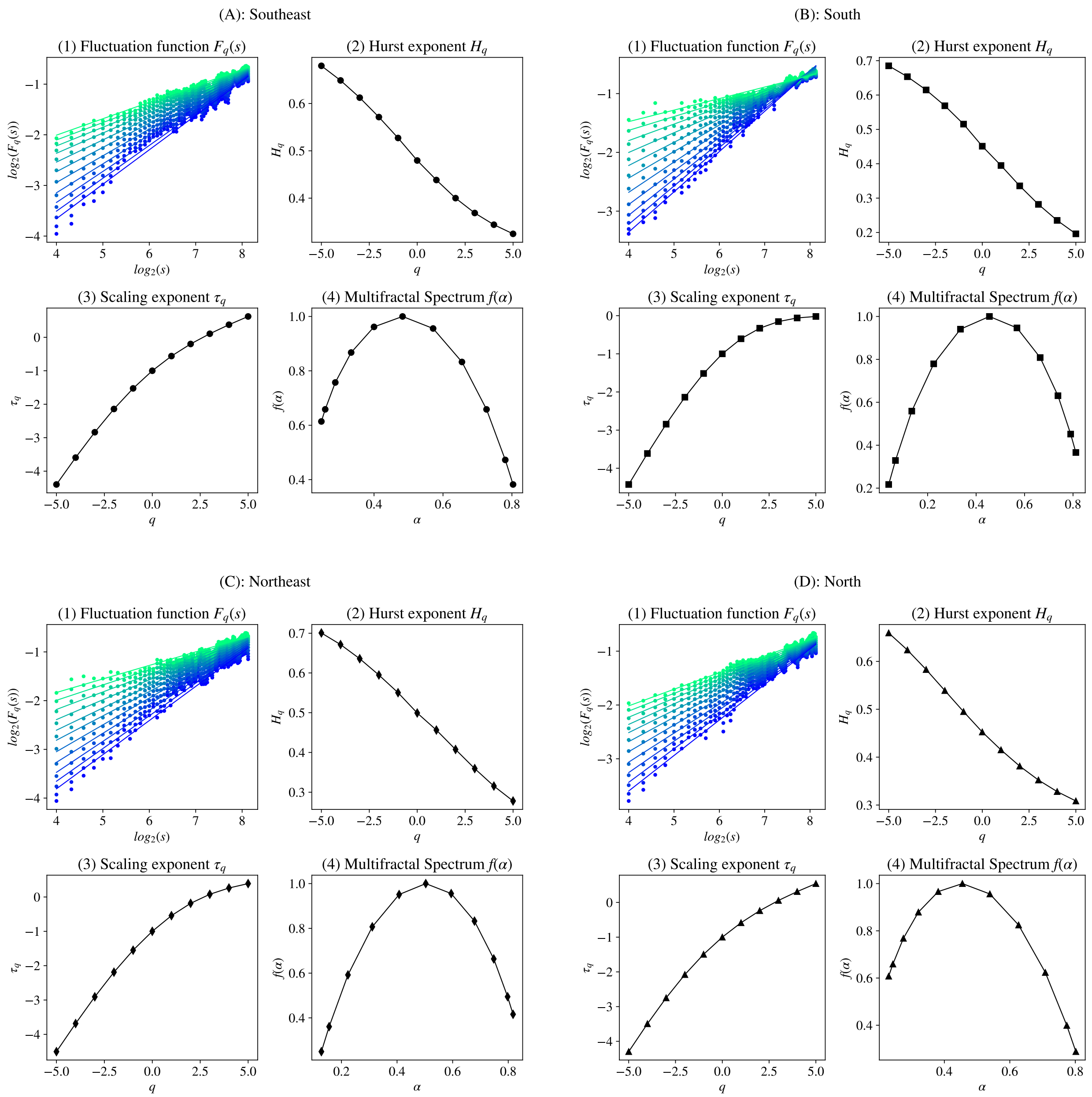

FIGURE 4. **MFDFA plots applied to the Brazilian market's weekly return index of spot electricity prices. The figure presents a set of 4 graphs for each submarket, showing (1) the scaling behavior of fluctuation functions, (2) the generalized Hurst exponent, (3) the scaling exponent or Reyni exponent, and (4) the multifractal spectrum. In the fluctuation function graph (1), colors varying from blue to green indicate values of q ranging from -5 to $+5$.**

the elimination of oscillatory trends, such as seasonality. The presence of intrinsic seasonal phenomena in the time series analyzed can result in a nonlinear relationship in the log-log plot of $F_q(s)$ as a function of s. We should observe this non-linearity through crossovers that separate regions with different slopes.

It is crucial to rigorously address seasonal variations in the data prior to employing MFDFA, as neglecting to do so could lead to erroneous conclusions about the time series' multifractality. In this study, we leverage the Seasonal-Trend Decomposition using LOESS (STL) methodology, which was initially proposed by [50] and has been implemented in [51]. We specifically focus on annual seasonality, characterized by a 54-week cycle, to effectively isolate seasonal components from long-term trends and fluctuations.

This pre-processing step is critical for ensuring the accurate application of MFDFA and a reliable assessment of the time series' multifractal nature. By meticulously eliminating seasonal variations, we can better examine the data's intrinsic fluctuations and better understand its multifractal characteristics.

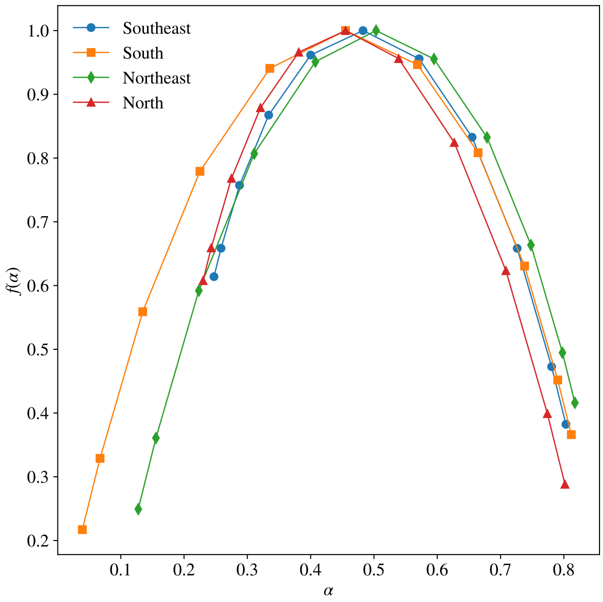

FIGURE 5. Multifractal spectrum for each submarket.

After analyzing the plots in Figure 4, we observe indications of multifractality in the return series of the four submarkets. However, it is important to approach this conclusion with caution, considering the small sample size, composed of only 1127 observations.

According to [55], when applying the MFDFA method to short and uncorrelated time series, mistaken detection of multifractality instead of mono fractality may occur (known as the finite size effect). Therefore, we should consider the multifractality of the weekly spot price returns as apparent, subject to this caveat.

Even though we obtained suggestive results of multifractality, the analysis of time series with a limited size requires careful interpretation. A larger data set would be needed for a more robust conclusion about the presence of multifractality, allowing a more reliable analysis of the statistical properties of the time series at different scales.

B. GENERALIZED HURST EXPONENTS

In the fluctuation function plots presented in Figure 4, we can observe the slope coefficients of the lines that best fit the $log_2(s)$ and $log_2(F_q)$ points for a given moment q. These slope coefficients correspond to the generalized Hurst exponents H_q. Analyzing the curves, we can observe that the value of H_q varies according to the value of q, which indicates the presence of apparent multifractality in the time series. The Hurst coefficients are calculated when $q = 2$ and are represented by $H_q(q = 2)$ or simply H_2.

Although the spot price formation process in the Brazilian energy market does not follow a traditional market equilibrium but is instead a result of hydrothermal dispatch optimization models, the Hurst coefficients of the submarkets indicate the presence of anti-persistence or mean reversion ($H_2 < 0.5$). This result corroborates mean reversion as a stylized fact found in other energy markets around the world, as mentioned in previous studies ([7], [11], [32], [33], [34], [35], [41]). The "hurst" column in Table 3 presents the H_2 values for all submarkets, measuring the degree of mean reversion in the analyzed time series.

As seen in the previous section, we calculated the strength of the apparent multifractality by applying equation (13) to the generalized Hurst exponents of each submarket. The *delta_h* column in Table 3 presents the results indicating that the South submarket exhibits the highest multifractality strength ($\Delta h = 0.489037$), followed by the Northeast ($\Delta h = 0.422751$), Southeast ($\Delta h = 0.355185$), and North ($\Delta h = 0.350836$).

TABLE 3. The table displays the results of applying the MFDFA to the original, shuffled, and surrogate series for each submarket. Each column represents an indicator (Hurst coefficient, width of the multifractal spectrum, strength of multifractality, and magnitude of long memory), and the rows represent the type of series (original, shuffled, and surrogate). We grouped the types of series by submarkets (Southeast, South, Northeast, and North). The values corresponding to each indicator and type of series are filled in the appropriate cells. For example, in the "hurst_original" cell of the Southeast submarket, we have the value of the Hurst coefficient for the original series; in the "hurst_shuffled" cell, we have the value of the Hurst coefficient for the shuffled series; and in the "hurst_surrogate" cell we have the value of the Hurst coefficient for the surrogate series. We applied the same principles to the other indicators and types of series.

	hurst H_2	width $\Delta\alpha$	delta_h Δh	mlm MLM
		Southeast		
original	0.400	0.556	0.355	0.178
shuffled	0.509	0.284	0.167	0.083
surrogate	0.396	0.307	0.182	0.091
		South		
original	0.336	0.772	0.489	0.244
shuffled	0.502	0.399	0.239	0.120
surrogate	0.320	0.424	0.256	0.128
		Northeast		
original	0.407	0.690	0.423	0.211
shuffled	0.508	0.355	0.209	0.104
surrogate	0.415	0.394	0.232	0.116

C. MULTIFRACTAL SPECTRUM

The plot in Figure 5 presents the singularity spectrum through the relationship between the Hölder coefficient α and $f(\alpha)$, the dimension of the subset of the series characterized by α, according to Equation (9).

The length of the singularity spectrum, represented by $\Delta\alpha$ according to equation (14), measures the strength of multifractality in each time series. As observed in Table 3, the South submarket presents the highest value of $\Delta\alpha = 0.772394$, followed by the Northeast ($\Delta\alpha = 0.689705$), North ($\Delta\alpha = 0.571663$), and Southeast ($\Delta\alpha = 0.555934$). This relationship indicates that the South submarket has the highest multifractality among the four Brazilian submarkets,

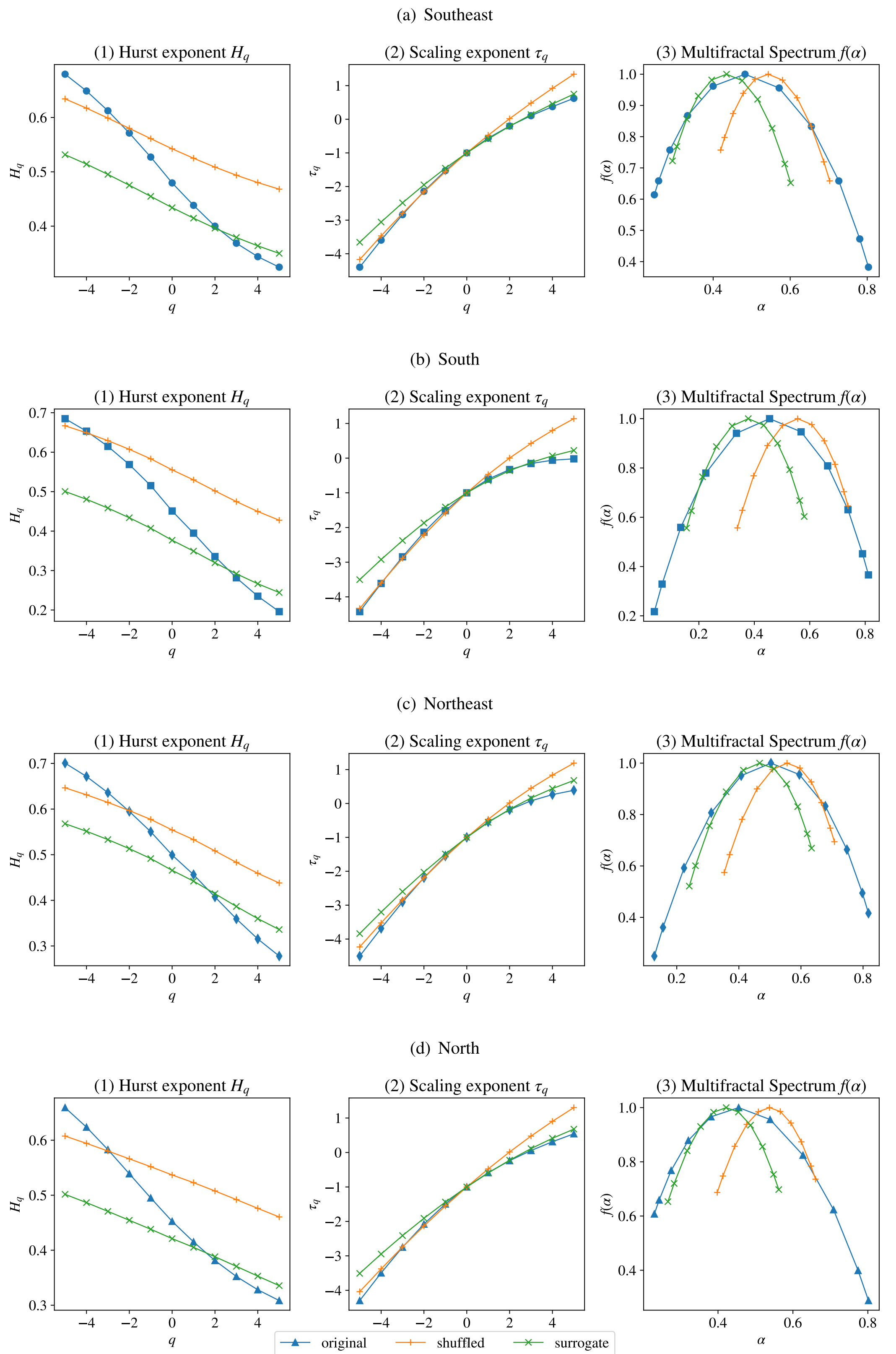

FIGURE 6. **Graphs displaying the potential causes of multifractality in the return series of spot electricity prices for the four Brazilian markets through the plots of the generalized Hurst exponent (1), scaling exponent (2), and multifractal spectrum (3). We used the MFDFA algorithm in the shuffled and surrogate series and extracted the generalized Hurst exponents and singularity spectra. To minimize the influence of the initial seed for pseudo-random number generation, we carried out 1000 runs of MFDFA (for each type of surrogate series and each submarket), changing the initial seed. The orange (shuffled) and green (surrogate) curves represent the averages of MFDFA results for each parameter. The blue curves present the results for the original series.**

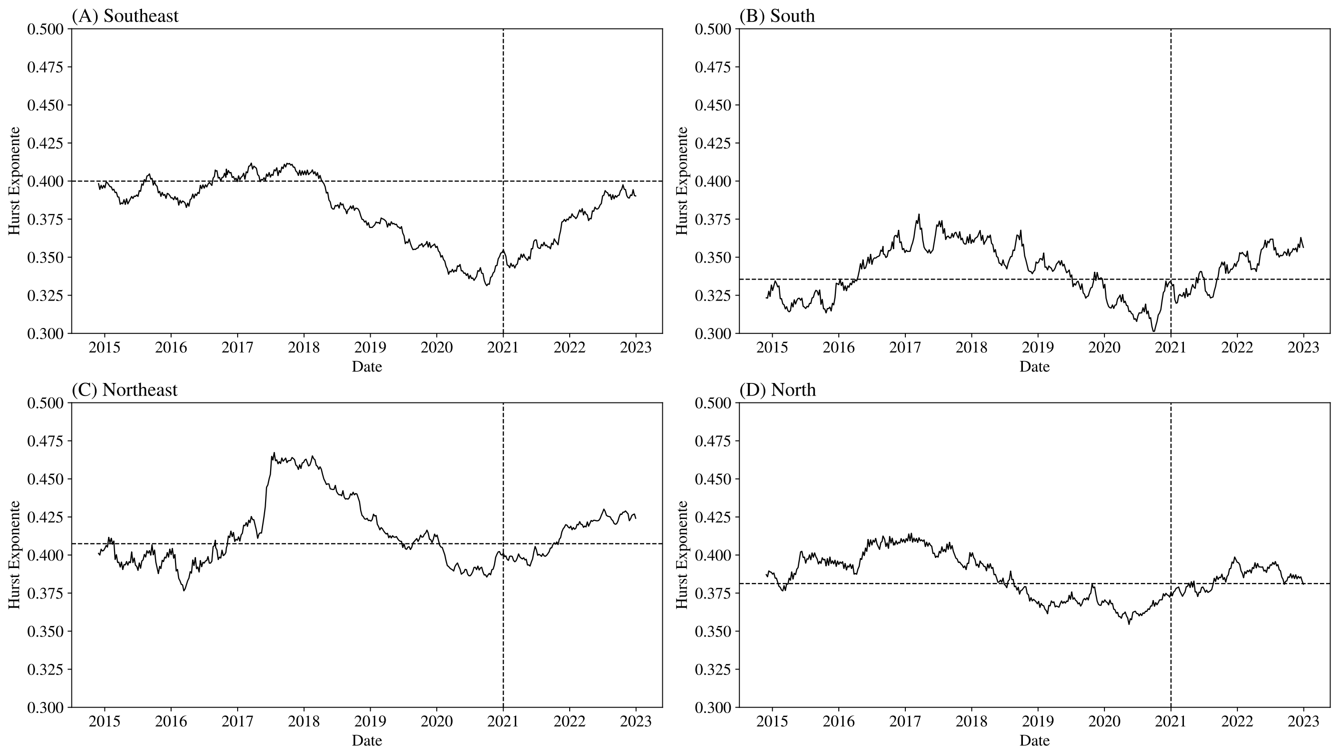

FIGURE 7. The graphs present the dynamic behavior of the Hurst coefficient (H_2) for the four submarkets using a window of 702 weeks and a unit step. The dashed vertical line indicates when there was a transition in how the price was calculated, with the introduction of the hourly hydrothermal dispatch model on January 1, 2021. Before this date, the price was calculated weekly through a weekly hydrothermal dispatch model. From this landmark, the weekly price started to be obtained by the arithmetic average of the hourly prices throughout the week. The dashed horizontal line represents the value of H_2 calculated for the complete series, as presented in Table 3.

confirming the same conclusion obtained when the Δh measured the strength of multifractality.

The higher multifractality observed in the South submarket reflects a more complex system regarding spot price fluctuations. This characteristic can be attributed to the specificities of this submarket, such as a more variable hydrological regime and a lower seasonality. Furthermore, the hydroelectric power plants in this submarket do not have storage capacity, and their production is directly related to river flow variations. This more significant variability in hydroelectric energy production is captured by price formation models, resulting in a more volatile operating marginal cost and, consequently, a more volatile spot price.

D. SOURCES OF MULTIFRACTALITY

Intending to investigate the origins of multifractality in the Brazilian electricity market, we applied the shuffling procedure to the return series, as described in [9] and [65]. As mentioned in Section IV-B, we eliminate linear and nonlinear temporal correlations when shuffling the series, preserving the probability distribution. If the multifractality was exclusively type (ii), then we would expect $H_{shuf}(q)$ to be independent of q, with a value of $H_{shuf}(q) = 0.5$. However, in the plots in Figure 6, we can observe that shuffling the series did not eliminate multifractality, as $\Delta h_{shuf} \neq 0$ and $\Delta \alpha_{shuf} \neq 0$. This result indicates that both types of multifractality mentioned in [9] are relevant to the Brazilian electricity market.

Furthermore, we analyzed type (i) multifractality by applying the surrogate method to the original series, as described in [65]. Similarly, the surrogate procedure did not eliminate the series' multifractality since $\Delta h_{surr} \neq 0$ and $\Delta \alpha_{surr} \neq 0$. Table 3 presents the values of H_2, $\Delta \alpha$, Δh, and MLM for the original, shuffled, surrogate series and by submarket. For all submarkets, the values of Δh and $\Delta \alpha$ of the surrogate weekly return series are higher than those of the shuffled series. This result indicates that the multifractality of weekly returns is more influenced by the long-range correlations between small and large fluctuations (temporal relation of the data) than by the broad probability density function.

E. TIME VARYING ANALYSIS

In previous sections, we conducted calculations of various indices derived from the application of MFDFA in the return series of deseasonalized spot prices in the four Brazilian submarkets. The analyses showed that all submarkets present anti-persistence and multifractal behavior. In this section, we will use the sliding window technique to investigate the dynamic behavior of the Hurst coefficient (H_2) and the magnitude of long memory (MLM). This approach, proposed by Cajueiro and Tabak [59], has been widely applied in the financial market to analyze the behavior of stock markets

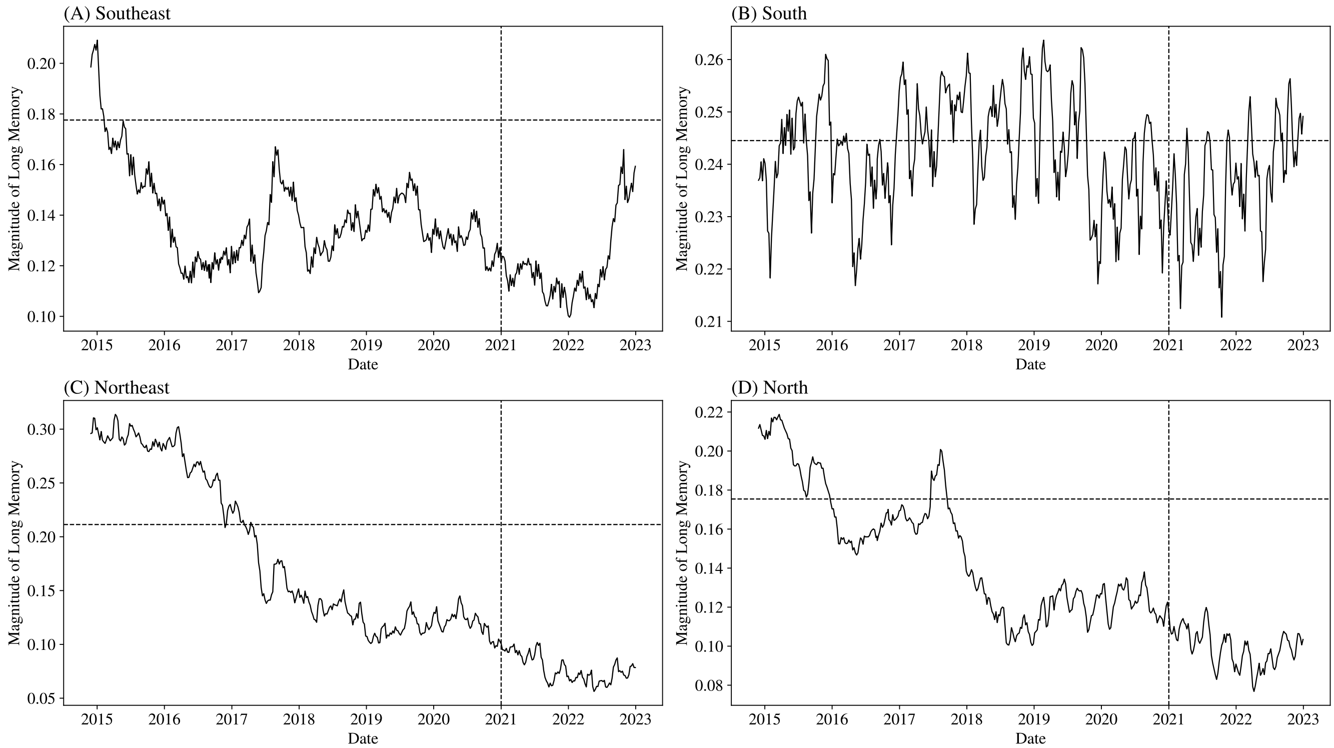

FIGURE 8. The graphs display the evolution of the multifractal spectrum length for each submarket over the evaluation period. The vertical dashed line indicates the start of hourly price disclosure from January 1, 2021. The dashed horizontal line marks the $\Delta\alpha$ value calculated for the complete series and presented in Table 3.

in different countries. Recently, this technique has also been applied to the electricity markets in the United States by Ali et al. [44] and in the Czech Republic by Čurpek [47].

The sliding window technique is commonly used in time series analysis, signal processing, machine learning, and other disciplines dealing with sequential data. The basic concept is simple: instead of processing all data at once, a ''window'' of fixed size is ''slide'' along the data, processing only the data within that window at each point. This technique can be helpful in various tasks, such as data smoothing, anomaly detection, calculating moving statistics, etc. The ''step'' in the sliding window technique refers to how many data points the window moves each time. A smaller step provides a more detailed analysis but may have a higher computational cost when compared to choosing a larger step. Given our small sample size, we conducted several experiments with window values between 700 and 800 weeks before settling on the final value of 702 weeks (equivalent to 13 years) and a step of one week. Thus, the calculations of the indices begin on December 19, 2014, and are redone for each week until the end of the period, keeping the window size fixed.

The graphs in Figure 7 illustrated the evolution of the Hurst coefficient (H_2) for each submarket over the evaluation period. The H_2 values in all submarkets showed no significant deviations, and anti-persistence behavior was maintained. There is an observable upward trend in the Hurst coefficients for all submarkets following the introduction of the hourly dispatch model. This trend suggests decreased anti-persistence and increased randomness in weekly spot prices. This behavior could be related to the hourly model's greater granularity and flexibility, allowing for better adaptation to short-term supply and demand conditions. However, a more thorough study is needed to understand the causes and implications of this change in the growth rate of H_2 at the end of 2020. Other factors, such as regulatory changes, alterations in energy supply and demand, and economic events, can also influence the behavior of the electricity market and should be considered in future analyses.

The graphs in Figure 8 showed the evolutions of the magnitude index of long memory (MLM) for each submarket over the evaluation period. Examining the results, we observed different behaviors in the submarkets. The Northeast and North submarkets showed a trend of reduction in MLM values over time. This reduction trend indicates an increase in the efficiency of these markets, as a lower magnitude of long memory is associated with higher market efficiency.

In the case of the South submarket, we did not observe a clear trend, as the MLM values fluctuated strongly over the evaluation period. However, from 2021 onwards, we observed an increase in the growth rate of MLM, indicating a possible decrease in market efficiency.

The Southeast submarket presented an interesting behavior. Initially, the growth rate of MLM was negative, indicating an improvement in market efficiency. However, from mid-2022 onwards, there was a change in the growth rate, which became positive, suggesting a possible decrease in market efficiency.

These results indicate that the submarkets exhibit distinct dynamics in terms of market efficiency over time. The variations in *MLM* values may reflect changes in market structure, energy supply and demand, and regulatory and climatic factors. However, it is important to highlight that more in-depth analyses and consideration of other factors are necessary for a complete understanding of the trends and implications of these results on the efficiency of the electricity market in Brazil.

VI. CONCLUSION

In this study, we presented an analysis of multifractality in the Brazilian electricity market, using the Multifractal Detrended Fluctuation Analysis (MFDFA) method on time series of spot prices for each of the four submarkets. We found that, despite the atypical price formation process, based on optimization models of hydrothermal dispatch, the submarkets share an anti-persistence behavior (or mean reversion) and exhibit multifractal characteristics. Such results converge with the stylized facts observed in other global electricity markets, indicating a universal nature of these attributes.

We detected variations in multifractality and efficiency among the submarkets attributed to their intrinsic characteristics. In particular, the South submarket revealed the highest multifractality and lowest efficiency, possibly due to a more unstable hydrological regime and lower seasonality. In contrast, the North submarket exhibited the lowest multifractality and highest efficiency. Using the Long Memory Magnitude (MLM) as an efficiency index, we evidenced a complex framework of variability and multifractality in the Brazilian submarkets.

Furthermore, we explored the origin of multifractality using shuffled and surrogate series techniques. We discovered that the multifractality of weekly returns is more dominated by long-term correlations among fluctuations of variable magnitude than by the broad probability density function.

The investigation of the dynamic behavior of the Hurst coefficient and the MLM index through the sliding window technique revealed the constancy of anti-persistence and a trend of increase in the Hurst coefficient after introducing the hourly dispatch model. These observations suggest a decrease in anti-persistence and greater randomness in weekly spot prices, potentially related to the higher granularity and flexibility of the hourly model.

Submarkets exhibited variable dynamics over time in terms of market efficiency. Changes in H_2 and *MLM* values may reflect changes in market structure, energy supply and demand, and economic, regulatory, and climatic factors. However, it is crucial to highlight the need for more in-depth analyses and the consideration of other factors for a more comprehensive understanding of the trends and implications of these results.

Although our findings suggest a presence of multifractality, we emphasize that the analysis of time series of limited size requires caution in the interpretation of the results. A more extensive data set would be necessary for a more robust conclusion, such as the hourly prices available from 2021, allowing more precise statistical analysis of the time series at different scales.

In summary, this study provides a valuable contribution to understanding the efficiency and dynamics of the Brazilian electricity market.

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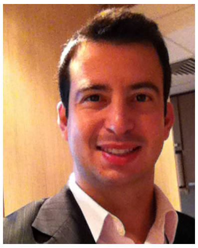

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