IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 23 August 2023, accepted 31 August 2023, date of publication 6 September 2023, date of current version 13 September 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3312540

RESEARCH ARTICLE

Robust Control Strategy for an Uncertain Dual-Channel Closed-Loop Supply Chain With Process Innovation for Remanufacturing

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This work was supported in part by the Natural Science Foundation of Gansu Province under Grant 20JR10RA265 and Grant 20JR10RA269, in part by the Youth Natural Science Foundation of Lanzhou Jiaotong University under Grant 2020043, in part by the Tianyou Youth Talent Lift Program of Lanzhou Jiaotong University, and in part by the Higher Education Innovation Foundation of Gansu Province under Grant 2021B-112.

ABSTRACT The inventory and total cost control strategy are the core issue in all supply chains. When a supply chain includes parameter uncertainty, dual-channel and process innovation for remanufacturing (PIR), the system is complex so that its stability can't be guaranteed easily. Therefore, how to manage the complex uncertain dual-channel closed-loop supply chain (CLSC) with PIR is a big challenge. The inventory and total cost control strategy for the CLSC with PIR is studied in this article. First of all, the uncertain mathematical model for the the CLSC with PIR is established based on the current inventory and total operation cost. Secondly, the model is converted to an uncertain multi-model according to different initial operation cases which is then described with a set of switching subsystem based on discrete Takagi-Sugeno(T-S) fuzzy model. Thirdly, a fuzzy robust H_∞ control approach is introduced to restrain the bullwhip effect and deal with the impacts caused by parameter uncertainty. Meanwhile, the inventory and total operation cost control strategy of the CLSC system is also obtained by the H_∞ control method. At last, three simulation tests are conducted to illustrate the effectiveness of the inventory control strategy.

INDEX TERMS Dual-channel closed-loop supply chain, uncertain systems, robust H_∞ control, process innovation in remanufacturing, bullwhip effect, inventory management.

I. INTRODUCTION

In recent yeas, due to the economic benefits and increased preservation of the environment created by remanufacturing, the CLSC has been received widespread attention [1], [2], [3], [4], [5], [6], [7]. Compared to traditional channels, network channels have been applied rapidly and widely in sales of various products owing to the simpler purchase process, the shorter turn over cycle, the less trade costs and the more quickly information sharing [8], [9]. More manufacturers are paying attention to online and offline sales model which forms a dual channel CLSC. The CLSC is consisted of material flows, financial flows, and information flows around all the operation nodes of the whole CLSC which is an extended concept of traditional supply chain. Hence, there are more running nodes and more uncertainties included parameter uncertainty and customer's demand uncertainty in the CLSC [10], [11]. Furthermore, Process innovation in remanufacturing (PIR) refers to increasing technological innovation in the process of recycled products remanufacturing to increase the efficiency in production and reduce the cost of remanufactured products [12]. Therefore, a growing number of manufacturers increasingly focus on the recycling and PIR so as to further reduce the production cost and upgrade the degree of customer's satisfaction. However, the CLSC composed of dual channel, uncertain factors in every node and PIR is complex so that its operation can't be managed easily. How to manage the complex uncertain

The associate editor coordinating the review of this manuscript and approving it for publication was Jinquan Xu.

dual-channel CLSC with PIR has been an interesting research subject which is the core motivation of this article.

Therefore, the focus of this paper is to answer the following research questions:

1) How to ensure the stability of the uncertain CLSC with PIR and stochastic demand from customer?

2) How should the CLSC manage the manufacturer and retailer's inventories under different initial inventory levels?

To answer the questions above, the mathematical model of an uncertain dual-channel CLSC with process innovation for remanufacturing is established. For the purpose of addressing the above issues, motivated by [13], authors transfer the uncertain multi-model CLSC into a T-S fuzzy system. The fuzzy H_∞ control method proposed in [13] is introduced into our article to deal with the bullwhip effect and provide specific operational strategies for the uncertain dual-channel CLSC with PIR.

To sum up, the major contributions of this article are expressed as follows:

1) The uncertain model for a dual-channel CLSC with PIR is established. Base on the current inventory of manufacturer and retailer, the uncertain multi-model system is conducted and described by a discrete T-S fuzzy model

2) The PIR is taken into account in the CLSC system. By making a comparison with a CLSC system without PIR, the result shows that PIR can decrease the total cost of supply chain's operation which depends on the amount of old products recycling and the level of PIR investment.

3) The inventory and total operation cost control strategy are designed by applying the fuzzy robust H_∞ switching method. The robust fuzzy switching strategy is studied to achieve the switching actions between different inventory control strategies and the robust stability of the uncertain dual- channel CLSC system.

The paper is organized as follows: Section II reviews the related CLSC literature. Section III introduces a new type of dynamic uncertain dual-channel closed-loop battery supply chain with PIR, and the uncertain model for the dual-channel CLSC system with PIR is established. In Section IV, a T-S fuzzy model is utilized to described the dynamic multi-model dual-channel CLSC system. Section V introduces a fuzzy H_∞ control method to deal with the bullwhip effect and give the control strategy of inventory described in the CLSC system. Three type of simulation analysis are conducted in Section VI to illustrate the validity of the robust fuzzy approach. At last, the conclusion is drawn in Section VII.

II. LITERATURE REVIEW

In this section, a literature review for some research interests dealing with CLSC model is provided. This mainly analyzes three types of issues related to the paper:1) the inventory management using robust control method. 2) the uncertainty of the CLSC system, 3) the CLSC with remanufacturing.

First of all, control theory that regarded as a new effective research method is introduced into CLSC systems in resent years. In [14], a class of supply chain systems, including multi-echelons, time-delay impact, and a dual-channel mode with a B2B e-market is developed by a H_∞ control method. In [15], a dynamic closed-loop supply chain model for a class of linear discrete time systems consisting of the product return, the remanufacturing, the third party reverse logistic providers (3PRLP) collecting is set up, and then, the robust H_∞ control strategies are used to restrain uncertainties of the CLSC system. Ge and Huang [16] develop a novel type of switched inventory model for the CLSC, and the dynamic H_∞ control method is utilized to deal with the uncertainties. A lot of authors also focus on the robust control strategies for the CLSC [17], [18], [19]. However, the dual channel and PIR are not considered in the above literatures. In [13], a robust H_∞ fuzzy output-feedback control approach is proposed to deal with the uncertainties of a class of two-echelon non-linear supply chain systems which is described as a discrete T-S fuzzy system, then the simulation test is carried out to illustrate the effectiveness of the proposed approach. This article provides an effective method to attenuate the bullwhip effect of the CLSC. Furthermore, this method can be improved and expanded to restrain uncertainties, improve the robustness, and give an inventory management strategy for the CLSC.

In addition, many research on the CLSC have been carried out. Uncertainty is an important issue due to the existence in every supply chain system which have a significant impact on system stability. The multiple incommensurable goals of a multi-product, multi-stage supply chain including market's uncertain demands and supplies of raw materials in which the uncertainties are converted to a number of discrete scenarios with known probabilities is studied by using a dynamic robust operation model in [20]. In [21], a dynamic H_∞ control method is proposed to address the operation issue of a CLSC system with uncertainty of remanufacturing, disposal time-delays and demand. The integrated operations of the uncertain construction supply chain (CSC), which is converted to a multi-objective optimization model, has been addressed in a unique domain in [22]. A robust H_∞ control method is proposed in [23] for multiagent-based supply chain systems with uncertain demands under a switching topology. To extract the maximum value of high-salvage perishable products, a closed-loop supply chain network model with various uncertainties based on the product life cycle is established in [24]. There are other literatures which studies on the uncertainty of supply chain systems [25], [26], [27]. However, the research mentioned above only focus on one or two uncertainties of the whole CLSC. Up to present, few literatures can be found to improve the robustness of the uncertain dual-channel CLSC. The uncertainties included in the operation process of the whole CLSC should be considered.

Furthermore, the PIR considered in this article focuses on innovation in the production process stage of remanufacturing, such as dismantling, extraction, and manufacturing of old products [28], [29], [30], [31], [32]. The aim is to increase the efficiency of dismantling and remanufacturing old products through a series of approaches such as process optimization and technological innovation, thereby improving reuse efficiency and reducing remanufacturing cost [33]. In [34], authors focus on the link between remanufacturing and the opportunity to lower the variable remanufacturing cost via process innovation to study the manage strategy of a CLSC with PIR. A CLSC system model with PIR and its cost sharing mechanism is constructed in [35], in which a manufacturer who decides the effort sells new products and remanufactured old products with the help of a retailer to improve the PIR. In [36], a recovery quantity function related to market demand, recovery price and recovery effort level is constructed on the basis of considering PIR investment and sales effort to study the application of PIR in the whole process of closed-loop supply chain. According to the above literatures, few scholars consider the issue of remanufacturing with PIR in CLSC by referring lots of literatures. Jauhari et al. [37] proposes a closed-loop supply chain model consisting of a supplier, a manufacturer and a retailer under deterministic demand and quality dependent return rate which is developed by using multiple remanufacturing cycle and multiple production cycle policy. An iterative procedure is proposed to determine the optimal solutions. In [38], a closed-loop supply chain where the manufacturer manufactures products according to the demand and sells them through a retailer in the market is addressed. Authors construct four different scenarios-centralized and decentralized led by manufacturer, retailer, and third party. Jauhari and Wangsa [39] develops an integrated inventory model for a closed-loop supply chain (CLSC) system consisting of a manufacturer and a retailer. A mathematical model is proposed to minimize the joint total cost incurred by the supply chain. In [40], In this paper, a mathematical inventory model for a manufacturer-multi retailer system is proposed to optimize the inventory level and investments under a carbon tax policy. The model aims to minimize the joint total cost by simultaneously determining the shipment frequencies, shipment lot, safety factor, production rate, collection rate and the investment. Most of the available articles focus on static problem of the CLSC, but the specific operation strategy and the robustness of dynamic CLSC is not considered.

III. PROBLEM FORMULATION

Notations used in the model are listed in Table 1.

In this section, a new kind of dynamic uncertain dual-channel closed-loop battery supply chain with PIR is considered, in which it composes of three echelons: a manufacturer, a retailer, and customers. The structure of the dynamic CLSC system is shown in Figure 1. The manufacturer sells its products both through the tradition channel and the internet channel. By traditional channel, manufacturer sells products offline through a retailer. The manufacturer and the retailer all accept the new products recovered from the customers gratuitously. In addition, the manufacturer collects the recycled products from the customers for remanufacturing.

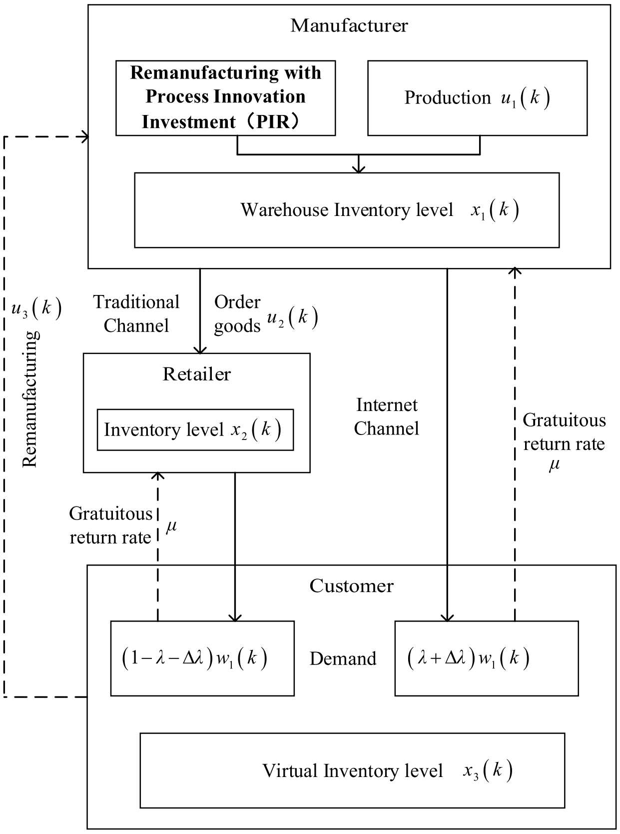


FIGURE 1. The structural sketch of the CLSC with the PIR investment and the dual-channel.

The lead acid battery is a real example, and its sale process is consistent with the supply chain system model mentioned above. In this article, due to our main purpose is to investigate the effectiveness of robust H_∞ control methods on the CLSC system, we have simplified some of the operational processes of the CLSC system. When the performance of batteries decreases or their lifespan completely ends after a period of use, these batteries are the goal for manufacturers to recycle.

In this article, PIR investment is considered in the CLSC system. PIR improves the efficiency of remanufacturing recycled products by technological investment, reduces the cost of remanufacturing recycled products, and thus reduces the total production cost. The total investment cost of PIR is:$T_i = K_i c_p^2$, this quadratic cost function is extensively cited in research and development literatures [41], [42]. K_i represents enterprise's R&D investment capability, set to 1 here. $c_p \in (0, 1)$ is the PIR investment level. The higher the value, the higher the PIR investment level is. $c_r - c_p > 0$ represents the unit remanufacturing cost savings which need to meet in the CLSC. We assume that the processes of the retailer's ordering, the new production of manufacturer and the recycling from customers can be finished in one iteration, so the time-delay is not be taken into account in this article.

To develop the mathematical model, we use the following assumptions:

TABLE 1. Description of symbols in the CLSC model.

Symbol	Unit	Description
$x_1(k)$	set	The manufacturer's current inventory
$x_2(k)$	set	The retailer's current inventory
$x_3(k)$	set	The customer's virtual inventory
$u_1(k)$	set	The production of manufacturer
$u_2(k)$	set	The retailer's order quantity from the manufacturer
$u_3(k)$	set	The quantity of old products recycled from customers
$w_1(k)$	set	The current uncertain demand of customer
μ	/	The gratuitous return rate
λ	/	The consumer preference rate for online channel
$z(k)$	yuan	The total operation cost of the CLSC at moment k.
c_{h_1}	yuan/set	The unit inventory cost of the manufacturer
c_{h_2}	yuan/set	The unit inventory cost of the retailer
c_n	yuan/set	The unit cost of new production
c_r	yuan/set	The unit cost of remanufacturing without PIR
c_p	yuan/set	The the PIR investment level
c_m	yuan/set	The unit cost of manufacturer's recycling from customer
c_q	yuan/set	The unit cost of gratuitous return of the new products from customer
c_s	yuan/set	The unit cost of retailer's ordering from manufacturer

1) The manufacturer leads the recycling process, the quantity of recycling is unrestricted, and all recycled batteries can be used for remanufacturing.

2) Manufacturers add PIR investment in the remanufacturing process to improve remanufacturing efficiency, and there is no waste generated during the remanufacturing process.

3) The quality of remanufactured products is the same as that of new products and can be sold at the same price.

4) Customers can return products gratuitously through both online and traditional channels.

5) Products that are returned without reason are not damaged and can be sold directly with new products, but cross channel return behavior is not considered.

6) Within the same iteration cycle, all supply chain node activities are completed, the time delay problem of supply chain nodes is not considered in this article.

From Figure 1, the dynamic mathematical model of the battery CLSC system considered uncertainty of demand from customer and parameters is established, in which the inventory states and the total operation cost of the CLSC is included. The Eq. (1) is set up based on the following principles: 1) the inventory at moment $k+1$ is equal to the algebraic sum of the inventory at moment k, inflow and outflow of inventory at moment k. 2) the total operation cost at moment k is equal to the sum of manufacturer and retailer's inventory cost, the cost of new production, the cost of remanufacturing with PIR, the cost of manufacturer's recycling from customer, the cost of gratuitous return of the new products from customer and the cost of retailer's ordering from manufacturer at moment k. The Eq. (1) is shown as follow:

$$\begin{cases} x_1(k+1) = x_1(k) + u_3(k) + (\mu+\Delta\mu)(\lambda+\Delta\lambda)x_3(k) \\ \qquad +u_1(k) - u_2(k) - (\lambda+\Delta\lambda)w_1(k) \\ x_2(k+1) = x_2(k) + (\mu+\Delta\mu)(1-\lambda-\Delta\lambda)x_3(k) \\ \qquad +u_2(k) - (1-\lambda-\Delta\lambda)w_1(k) \\ x_3(k+1) = x_3(k) + w_1(k) - (\mu+\Delta\mu)x_3(k) - u_3(k) \\ z(k) = \left(c_{h_1}+\Delta c_{h_1}\right)x_1(k) + \left(c_{h_2}+\Delta c_{h_2}\right)x_2(k) \\ \qquad +(c_n+\Delta c_n)u_1(k) + (c_r+\Delta c_r - c_p - \Delta c_p \\ \qquad +\left(c_p+\Delta c_p\right)^2)u_3(k) + (c_m+\Delta c_m)u_3(k) \\ \qquad +(c_q+\Delta c_q)(\mu+\Delta\mu)x_3(k) + (c_s+\Delta c_s)u_2 \end{cases} \tag{1}$$

where $\Delta\mu$, $\Delta\lambda$, Δc_{h_1}, Δc_{h_2}, Δc_n, Δc_r, Δc_p, Δc_m, Δc_q, Δc_s represent the uncertainty item of the uncertain parameters. $x_1(k+1)$,$x_2(k+1)$, $x_3(k+1)$ are the inventories at moment $k+1$.

Remark 2.1 All the states in Eq. (1) are the deviation values. For different initial inventory of manufacturer and retailer, the Eq. (1) will be represented as different forms.

The different initial inventory levels of the CLSC will result in complex inventory control issue. Therefore, the dynamic dual channel battery CLSC system can't be expressed by a single definite model. Based on the basic dynamic model Eq. (1), a dynamic multi-model switching CLSC system is established, whose ith subsystem is described as follows:

$$\begin{cases} X(k+1) = (A_i+\Delta A_i)X(k) + (B_i+\Delta B_i)U(k) \\ \qquad +(B_{wi}+\Delta B_{wi})W(k) \\ z(k) = (C_i+\Delta C_i)X(k) + (D_i+\Delta D_i)U(k) \\ \qquad +(D_{wi}+\Delta D_{wi})W(k) \end{cases} \tag{2}$$

where $X(k+1) = [x_1(k), x_2(k), x_3(k)]^T$; $U(k) = [u_1(k), u_2(k), u_3(k)]^T$; $W(k) = [w_1(k), w_1(k), w_1(k)]^T$; $A_i, B_i, C_i, D_i, B_{wi}, D_{wi}$ are the real matrices witch appropriate demension; ΔA_i, ΔB_i, ΔC_i, ΔD_i, ΔB_{wi}, ΔD_{wi} are the uncertainty, correspondingly. It should be noted that the CLSC is represented by deviation variables, which are different from the actual value and the nominal value.

IV. T–S FUZZY MODEL

In this section, a T-S fuzzy model is utilized to described the dynamic dual-channel CLSC system, which is consisted of a set of if-then fuzzy rules. For the purpose of fully describing the inventory control strategies under initial inventory levels, the current inventory and the corresponding control strategies are used in the T-S fuzzy system to represent the CLSC as a multi-model switching system. Eq. (2) is described by the T-S fuzzy model as below:

$R_i : if\ x_1(k)\ is\ M_1^i, \cdots, x_j(k)\ is\ M_j^i,\ \cdots, and\ x_n(k)\ is\ M_n^i,$

then

$$\begin{cases} X(k+1) = (A_i+\Delta A_i)X(k) + (B_i+\Delta B_i)U(k) \\ \qquad + (B_{wi}+\Delta B_{wi})W(k) \\ z(k) = (C_i+\Delta C_i)X(k) + (D_i+\Delta D_i)U(k) \\ \qquad + (D_{wi}+\Delta D_{wi})W(k) \\ X(k) = \varphi(k), i = 1, 2, \ldots, r, k = \{0, 1, 2\ldots, N\} \end{cases} \tag{3}$$

where $R_i\,(i = 1, 2, \ldots, r)$ represents the ith fuzzy rule; r represents the number of fuzzy rules; M_j^i represents the fuzzy set of the jth manufacturer or retailer under the ith rule, $n$is the quantity of manufacturer, retailer and customers; $W(k) \in l2[0, +\infty]$; $\varphi(k)$ is the initial inventories of manufacturer, retailer and customers; $X(k) = [x_1(k), x_2(k), \cdots, x_n(k)]^T$, and $U(k) = [u_1(k), u_2(k), \cdots, u_n(k)]^T$.

After using a standard singleton fuzzifier, fuzzy inference, and weighted defuzzifier, Eq. (2) is converted as below: (4), as shown at the bottom of the next page, where $\mu_i(X(k)) = \frac{h_i(X(k))}{\sum_{i=1}^{r} X(k)}$, $h_i(X(k)) = \prod_{j=1}^{n} M_j^i(x_j(k))$, $M_j^i(x_j(k))$ represents the membership function of $x_j(k)$ with regard to the fuzzy set M_j^i, and $h_i(X(k))$ represents the normalized weight in rule i.

V. ROBUST CONTROL STRATEGY OF THE UNCERTAIN CLSC

The Lebesgue measurable form is introduced to describe the time-varying parameter uncertainties of the fuzzy CLSC system. A set of constant matrices are introduced as: H_{1i}, H_{2i}, E_{11i}, E_{12i}, E_{13i}, E_{21i}, E_{22i}, and E_{23i} are known constant matrices with suitable demension. $F_{1i}(k)$ and $F_{2i}(k)$ are time-varying matrices. The uncertainty in Eq. (4) can be represented as follows:

$$\begin{cases} \left[\Delta A_i \;\, \Delta B_i \;\, \Delta B_{wi}\right] = H_{1i}F_{1i}(k)[E_{11i} \quad E_{12i} \quad E_{13i}] \\ \\ \left[\Delta C_i \;\, \Delta D_i \;\, \Delta D_{wi}\right] = H_{2i}F_{2i}(k)[E_{21i} \quad E_{22i} \quad E_{23i}] \\ \end{cases} \tag{5}$$

where $i = 1, 2, \ldots, r$, $F_{1i}^T(k)F_{1i}(k) \le I$ and $F_{2i}^T(k)F_{2i}(k) \le I$;$F_{1i}(k)$ and $F_{2i}(k)$ represent unknown but bounded time-varying matrices.

In this section, the H_∞ criterion is introduced to suppress the uncertainty of the CLSC, which has the discrete form as follow:

$$\sum\nolimits_{k=0}^{\infty} z^T(k)z(k) < \gamma^2 \sum\nolimits_{k=0}^{\infty} w_1^T(k)w_1(k) \tag{6}$$

The attenuation level γ is used to describe the maximum upper bound of the bullwhip effect. The smaller the value of γ, the smaller the bullwhip effect is. In order to express it more clearly, Eq. (6) can be rewritten as follows:

$$\frac{\|Total\ cost\ of\ CLSC\|_2}{\|Customer's\ demand\|_2} \le \gamma \tag{7}$$

where $\|\cdot\|_2$ represents the $l2$ norm. Mean while, Eq. (7) describes the robust control performance of the CLSC. Generally speaking, for the purpose of suppressing the bullwhip effect, we hope that the attenuate level γ is set as small as possible so that the better performance of the CLSC system can be obtained. However, the smaller γ will result in poor robustness of the system. The attenuation level γ should be selected appropriately based on specific calculation process.

For the purpose of minimizing the total operation cost of the CLSC system and guaranteeing the inventories converge to the nominal value, fuzzy rules are given as below.

If $x_1(k)$ is $M_1^i, \cdots, x_j(k)$ is $M_j^i, \cdots$,and $x_n(k)$ is M_n^i, then $U(k) = -K_iX(k), i = 1, 2, \ldots, r$, where K_i represents the gain matrix of the current inventories.

Now, the follow PDC control law is considered in this article. The fuzzy control law of the whole CLSC can be shown as follows:

$$U(k) = -\sum\nolimits_{i}^{r} \mu_i(X(k))K_iX(k) \tag{8}$$

In order to obtain main results, the follow two definitions are used in this section which was shown as below:

Definition 1 ([43]): A group of fuzzy sets $\{F_j^m, m = 1, 2, \cdots, q_i\}$ are defined to be a standard fuzzy partition (SFP) in the universe X if each F_j^m is a normal fuzzy set and $\{F_j^m, m = 1, 2, \cdots, q_i\}$ are full-overlapped in the universe X. q_i is the number of fuzzy partitions on X.

Definition 2 ([43]): In a designed fuzzy system, if an overlapped rules group has the maximum number of fuzzy rules, which is defined as a maximal overlapped-rules group (MORG).

Now, the issue of restraining the bullwhip effect is converted to the stability problem of Eq. (4). Our purpose is to find the gain matric K_i which can ensure the closed-loop stability at attenuation level γ. The theorem 1 developed in [13] is introduced to increase the control performance of the uncertain CLSC system, deal with bullwhip effect and the uncertainty of parameter and customer's demand. The solution for gain matric K_i can be obtained by solving the LMIs described in theorem 1.

Theorem 1 ([13]): For a known constant $\gamma > 0$, if constants $\varepsilon_{ijc} > 0$, local matrix $X_c > 0$, and matrices $Y_{ic}$with the appropriate dimensions in G_c exist, such that (9) and (10), as shown at the bottom of page 7,
where $LS_{ij} = A_iX_c - B_iY_{jc}$, $TS_{ij} = C_iX_c - D_iY_{jc}$; $U_{ij} = E_{11i}X_c - E_{12i}Y_{jc}$, $V_{ij} = E_{21i}X_c - E_{22i}Y_{jc}$; $\Omega_{33} = -X_c + \varepsilon_{ijc}H_{1i}H_{1i}^T + \varepsilon_{jic}H_{1j}H_{1j}^T$; $\Omega_{44} = -I + \varepsilon_{ijc}H_{2i}H_{2i}^T + \varepsilon_{jic}H_{2j}H_{2j}^T$;$I$ is identity matrix the with appropriate dimension, Gc represents the cth MORG, $c = 1, 2, \cdots, \prod_{j=1}^{n}(m_j - 1)$, m_j is the number of fuzzy partitions of the jth state variable. Then, the asymptotic stable of CLSC system Eq. (4) is guaranteed. The gain matrices K_i can be calculated by $K_i = Y_{ic}X_c^{-1}$. In fact, the robust control strategy of the CLSC is consisted of the inventory control strategies and the robust switching strategy.

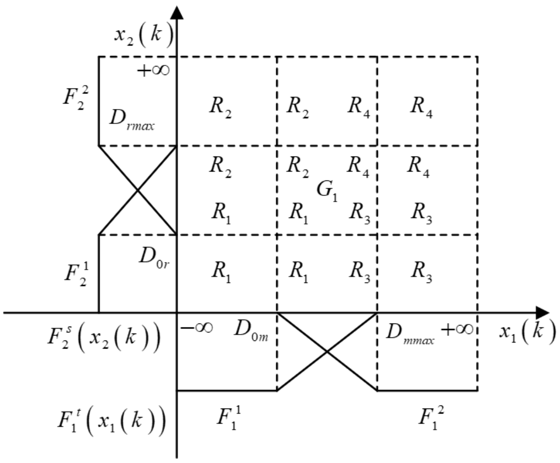


FIGURE 2. Fuzzy partitions of current inventory of the manufacturer and the retailer.

VI. SIMULATION TEST

The fuzzy robust H_∞ control method introduced in Theorem 1 is used to eliminated the bullwhip effect and deal with the inventory control issue in this section which is depicted in Section II. In the simulation tests, a three echelons battery CLSC system consisted of a manufacturer, a retailer and customers is selected as an research target.

The fuzzy patitions represented the inventories for the manufacturer and the retailer are shown in Figure 2. $x_1(k)$ and $x_2(k)$ represent the current inventories of the manufacturer and the retailer, respectively. $F_1^t(x_1(k))\,(t=1,2)$ and $F_2^s(x_1(k))\,(s=1,2)$ are the fuzzy patitions of $x_1(k)$ and $x_2(k)$,which conform the critera of SFP. In the CLSC system, G_1 (consisting of R_1, R_2, R_3andR_4) is the only MORG. D_{0m} is the battery manufacturer's safe inventory. D_{mmax} represents the battery manufacturer's maximum inventory. D_{0r} is the battery retailer's safe inventory. D_{rmax} represents the battery retailer's maximum inventory. $D_{0m}=80$, $D_{mmax}=120$, $D_{0r}=60$, $D_{rmax}=90(\times 10^3$sets).

By considering the initial inventory levels, the battery production strategies of the manufacturer and the ordering strategies of the retailer can be expressed as follow:

R_1 : The battery manufacturer produces new batteries and recycles old batteries from customers for remanufacturing. Meanwhile, the retailer place orders from manufacturers and accept the gratuitous return from customers.

R_2 : The battery manufacturer produces new battery and recycles old batteries for remanufacturing. Meanwhile, the retailer does not place orders from manufacturers, only accept the gratuitous return from customers.

R_3 : The battery manufacturer does not produce new batteries and only recycles old batteries for remanufacturing. Meanwhile, the retailer places orders from manufacturers and accept the gratuitous return from customers.

R_4 : The battery manufacturer does not produce new battery and only recycles old batteries for remanufacturing. Meanwhile, the retailer does not place orders from manufacturers and only accept the gratuitous return from customers.

Based on Eq. (4) and the inventory control strategies described above, by considering the parameters uncertainties and the uncertain customer's demand of the CLSC system, the multi-model of uncertain dual-channel CLSC is expressed as follows:

$$R_1:\begin{cases}x_1(k+1)=x_1(k)+u_3(k)\\ \qquad +(\mu+\Delta\mu)(\lambda+\Delta\lambda)x_3(k)+u_1(k)\\ \qquad -u_2(k)-(\lambda+\Delta\lambda)w_1(k)\\ x_2(k+1)=x_2(k)\\ \qquad +(\mu+\Delta\mu)(1-\lambda-\Delta\lambda)x_3(k)\\ \qquad +u_2(k)-(1-\lambda-\Delta\lambda)w_1(k)\\ x_3(k+1)=x_3(k)+w_1(k)-(\mu+\Delta\mu)x_3(k)\\ \qquad -u_3(k)\\ z(k)=\left(c_{h_1}+\Delta c_{h_1}\right)x_1(k)+\left(c_{h_2}+\Delta c_{h_2}\right)x_2(k)\\ \qquad +(c_n+\Delta c_n)u_1(k)+(c_r+\Delta c_r-c_p-\Delta c_p\\ \qquad +\left(c_p+\Delta c_p\right)^2)u_3(k)+(c_m+\Delta c_m)u_3(k)\\ \qquad +(c_q+\Delta c_q)(\mu+\Delta\mu)x_3(k)+(c_s+\Delta c_s)\end{cases}$$

$$R_2:\begin{cases}x_1(k+1)=x_1(k)+u_3(k)\\ \qquad +(\mu+\Delta\mu)(\lambda+\Delta\lambda)x_3(k)+u_1(k)\\ \qquad -(\lambda+\Delta\lambda)w_1(k)\\ x_2(k+1)=x_2(k)+(\mu+\Delta\mu)(1-\lambda-\Delta\lambda)x_3(k)\\ \qquad -(1-\lambda-\Delta\lambda)w_1(k)\\ x_3(k+1)=x_3(k)+w_1(k)-(\mu+\Delta\mu)x_3(k)\\ \qquad -u_3(k)\\ z(k)=\left(c_{h_1}+\Delta c_{h_1}\right)x_1(k)+\left(c_{h_2}+\Delta c_{h_2}\right)x_2(k)\\ \qquad +(c_n+\Delta c_n)u_1(k)+(c_r+\Delta c_r-c_p-\Delta c_p\\ \qquad +\left(c_p+\Delta c_p\right)^2)u_3(k)+(c_m+\Delta c_m)u_3(k)\\ \qquad +(c_q+\Delta c_q)(\mu+\Delta\mu)x_3(k)\end{cases}$$

$$\begin{cases}X(k+1)=\sum_{i=1}^{r}\mu_i(X(k))[(A_i+\Delta A_i)X(k)+(B_i+\Delta B_i)U(k)\\ +(B_{wi}+\Delta B_{wi})W(k)]\\ z(k)=\sum_{i=1}^{r}\mu_i(X(k))[(C_i+\Delta C_i)X(k)+(D_i+\Delta D_i)U(k)\\ +(D_{wi}+\Delta D_{wi})W(k)]\end{cases}\tag{4}$$

$$R_3:\begin{cases} x_1(k+1) = x_1(k) + u_3(k) \\ \qquad + (\mu+\Delta\mu)(\lambda+\Delta\lambda)x_3(k) - u_2(k) \\ \qquad -(\lambda+\Delta\lambda)w_1(k) \\ x_2(k+1) = x_2(k) + (\mu+\Delta\mu)(1-\lambda-\Delta\lambda)x_3(k) \\ \qquad + u_2(k) - (1-\lambda-\Delta\lambda)w_1(k) \\ x_3(k+1) = x_3(k) + w_1(k) \\ \qquad - (\mu+\Delta\mu)x_3(k) - u_3(k) \\ z(k) = \left(c_{h_1}+\Delta c_{h_1}\right)x_1(k) + \left(c_{h_2}+\Delta c_{h_2}\right)x_2(k) \\ \qquad + \left(c_r+\Delta c_r - c_p - \Delta c_p \right. \\ \qquad \left. + \left(c_p+\Delta c_p\right)^2\right)u_3(k) \\ \qquad + (c_m+\Delta c_m)u_3(k) \\ \qquad + (c_q+\Delta c_q)(\mu+\Delta\mu)x_3(k) \\ \qquad + (c_s+\Delta c_s)u_2 \end{cases}$$

$$R_4:\begin{cases} x_1(k+1) = x_1(k) + u_3(k) \\ \qquad + (\mu+\Delta\mu)(\lambda+\Delta\lambda)x_3(k) \\ \qquad - (\lambda+\Delta\lambda)w_1(k) \\ x_2(k+1) = x_2(k) + (\mu+\Delta\mu)(1-\lambda-\Delta\lambda)x_3(k) \\ \qquad -(1-\lambda-\Delta\lambda)w_1(k) \\ x_3(k+1) = x_3(k) + w_1(k) - (\mu+\Delta\mu)x_3(k) \\ \qquad - u_3(k) \\ z(k) = \left(c_{h_1}+\Delta c_{h_1}\right)x_1(k) + \left(c_{h_2}+\Delta c_{h_2}\right)x_2(k) \\ \qquad + \left(c_r+\Delta c_r - c_p - \Delta c_p \right. \\ \qquad \left. + \left(c_p+\Delta c_p\right)^2\right)u_3(k) \\ \qquad + (c_m+\Delta c_m)u_3(k) \\ \qquad + (c_q+\Delta c_q)(\mu+\Delta\mu)x_3(k) \end{cases}$$

In addition, the T-S fuzzy model of battery supply chain inventory system is conducted as below:

R_i : if $x_1(k)$ is F_1^t, and $x_2(k)$ is F_2^s, then

$$\begin{cases} X(k+1) = (A_i+\Delta A_i)X(k) + (B_i+\Delta B_i)U(k) \\ \qquad + (B_{wi}+\Delta B_{wi})W(k) \\ z(k) = (C_i+\Delta C_i)X(k) + (D_i+\Delta D_i)U(k) \\ \qquad + (D_{wi}+\Delta D_{wi})W(k) \end{cases}$$

where $t = 1, 2$ and $s = 1, 2$.

For the multi-model of uncertain dual-channel CLSC, the switching fuzzy rules are described as follows:

R_i : if $x_1(k)$ is M_1^i, and $x_2(k)$ is M_2^i, then

$$U(k) = \sum_i^4 u_i(X(k))K_{ic}X(k)$$

In simulation experiment, the battery production enterprise named "Chaowei Power Group Co., Ltd" (hereinafter referred to as Chaowei) as a manufacturer in the CLSC which is a manufacturer of electric vehicle battery in China. According to the battery manufacturer and the retailer's inventory control strategies, based on the actual investigation for Chaowei, the parameters are chosen as follows: $c_{h1} = 0.015$, $c_{h2} = 0.02$, $c_q = 0.02$, $c_s = 0.18$, $c_m = 0.1$, $c_n = 0.3$, $c_r = 0.1$, $c_p = 0.05(\times 10^3 yuan)$.

$$\mu = 0.01, \lambda = 0.5,$$

$$A_1 = A_2 = A_3 = A_4 = \begin{bmatrix} 1 & 0 & \mu\lambda \\ 0 & 1 & \mu(1-\lambda) \\ 0 & 0 & 1-\mu \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

$$\begin{bmatrix} -X_c & * & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * & * \\ LS_{ii} & B_{wi} & -X_c + \varepsilon_{iic}H_{1i}H_{1i}^T & * & * & * \\ TS_{ii} & D_{wi} & 0 & -I + \varepsilon_{iic}H_{2i}H_{2i}^T & * & * \\ U_{ii} & E_{13i} & 0 & 0 & -\varepsilon_{iic}I & * \\ V_{ii} & E_{23i} & 0 & 0 & 0 & -\varepsilon_{iic}I \end{bmatrix} < 0, i \in I_c \tag{9}$$

$$\begin{bmatrix} -4X_c & * & * & * & * & * & * & * \\ 0 & -4\gamma^2 I & * & * & * & * & * & * \\ LS_{ij}+LS_{ji} & B_{wi}+B_{wj} & \Omega_{33} & * & * & * & * & * \\ TS_{ij}+TS_{ji} & D_{wi}+D_{wj} & 0 & \Omega_{44} & * & * & * & * \\ U_{ij} & E_{13i} & 0 & 0 & -\varepsilon_{ijc}I & * & * & * \\ V_{ij} & E_{23i} & 0 & 0 & 0 & -\varepsilon_{ijc}I & * & * \\ U_{ji} & E_{13j} & 0 & 0 & 0 & 0 & -\varepsilon_{jic}I & * \\ V_{ji} & E_{23j} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{jic}I \end{bmatrix} < 0, i < j; i, j \in I_c \tag{10}$$

$$B_{w1} = B_{w2} = B_{w3} = B_{w3} = \begin{bmatrix} -\lambda & 0 & 0 \\ 0 & \lambda - 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$C_1 = C_2 = C_3 = C_4 = \begin{bmatrix} c_{h1} & c_{h2} & \mu c_q \end{bmatrix},$$
$$D_1 = \begin{bmatrix} c_n & c_s & c_r - c_p + c_p^2 + c_m \end{bmatrix},$$
$$D_2 = \begin{bmatrix} c_n & 0 & c_r - c_p + c_p^2 + c_m \end{bmatrix},$$
$$D_3 = \begin{bmatrix} 0 & c_s & c_r - c_p + c_p^2 + c_m \end{bmatrix},$$
$$D_4 = \begin{bmatrix} 0 & 0 & c_r - c_p + c_p^2 + c_m \end{bmatrix}$$
$$E_{11i} = \begin{bmatrix} 0 & 0 & 0.01 \\ 0 & 0 & 0.01 \\ 0 & 0 & -0.01 \end{bmatrix}, \; E_{12i} = E_{13i} = E_{23i} = 0$$
$$E_{21i} = [0, 0.004, 0.002],$$
$$E_{22i} = [0.003, 0.002, 0.0015]$$
$$H_{1i} = 0.1, \; H_{2i} = 0.2, F_{1i} = F_{2i} = \sin(k)$$

By solving the LMI Tool box described in Theorem 1, the feasp solution can be obtained, and the stability of CLSC system is guaranteed. In order to illustrate the effect of the PIR investment, the solution is calculated in two different scenarios. $c_p = 0$ means that there is no PIR investment include into the CLSC. In this article, we chosen $c_p = 0.05$ as the PIR investment coefficient which presents the PIR investment level. When $c_p = 0$, the gain matrix can be obtained as follow:

$$X_c = \begin{bmatrix} 29.6344 & -2.6797 & -5.2858 \\ -2.6797 & 19.9247 & -0.5583 \\ -5.2858 & -0.5583 & 26.3995 \end{bmatrix}$$
$$K_1 = \begin{bmatrix} 0.4503 & -0.1352 & 0.4503 \\ -0.0997 & 0.3046 & -0.0093 \\ 0.2351 & 0.0186 & -0.6646 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} 0.4025 & -0.2403 & 0.4861 \\ -0.1374 & 0.7284 & 0.0220 \\ 0.2372 & 0.0561 & -0.7488 \end{bmatrix}$$
$$K_3 = \begin{bmatrix} 0.5750 & -0.0864 & 0.7761 \\ -0.1091 & 0.4014 & -0.1156 \\ 0.4465 & -0.0967 & -0.7578 \end{bmatrix}$$
$$K_4 = \begin{bmatrix} 0.6134 & -0.2060 & 0.7732 \\ -0.1501 & 0.7537 & -0.0545 \\ 0.4506 & -0.0136 & -0.7367 \end{bmatrix}$$

when $c_p = 0.05$, the gain matrix can be obtained as follow:

$$X_c = \begin{bmatrix} 29.1131 & -2.9526 & -5.1064 \\ -2.9526 & 20.0693 & 0.0983 \\ -5.1064 & 0.0983 & 27.8929 \end{bmatrix}$$
$$K_1 = \begin{bmatrix} 0.2721 & -0.1374 & 0.4900 \\ -0.0943 & 0.3059 & -0.0267 \\ 0.2971 & 0.0357 & -0.7284 \end{bmatrix}$$
$$K_2 = \begin{bmatrix} 0.3842 & -0.2549 & 0.5146 \\ -0.1376 & 0.7231 & 0.0115 \\ 0.2996 & 0.1024 & -0.7729 \end{bmatrix}$$
$$K_3 = \begin{bmatrix} 0.5529 & -0.1299 & 0.8149 \\ -0.1287 & 0.3455 & -0.1193 \\ 0.5442 & -0.0476 & -0.7658 \end{bmatrix}$$
$$K_4 = \begin{bmatrix} 0.5847 & -0.2112 & 0.8047 \\ -0.1386 & 0.7576 & -0.0758 \\ 0.5328 & 0.0159 & -0.7782 \end{bmatrix}$$

In order to illustrate the effectiveness of the inventory control strategy, simulation tests are conducted based on different initial current inventory level. The simulation tests are carryed out to illustrate three main performances: A) the bullwhip effect restraint; B) comparison of the CLSC's total cost; C) inventory control strategy.

It should be noted that all the variables in the simulation results are the sum of nominal values and deviation values. The specific simulation process is done as follows:

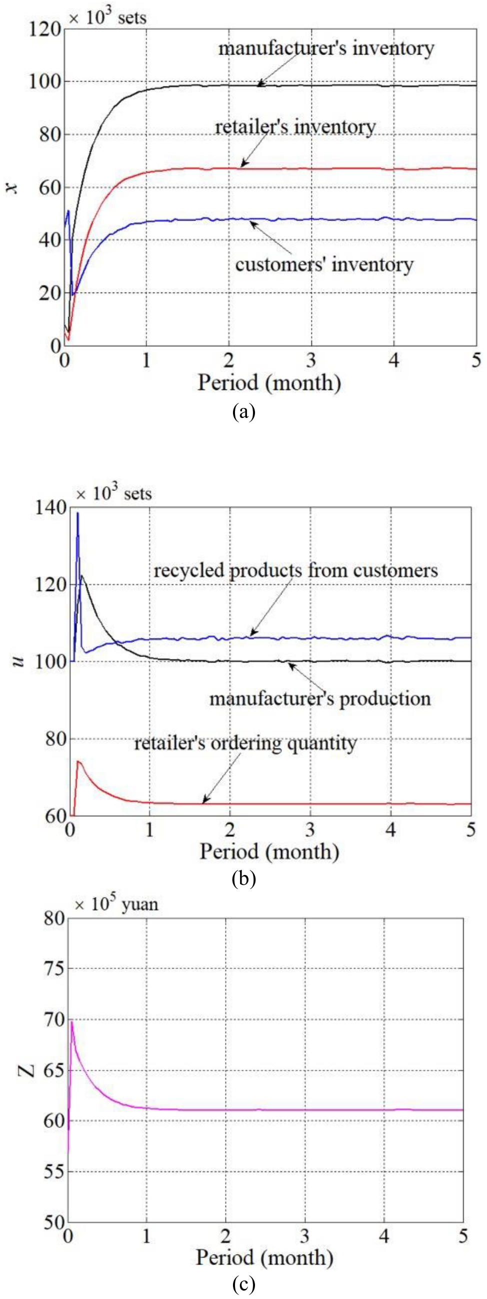
A. SIMULATION TESTS FOR THE BULLWHIP EFFECT RESTRAINT

The simulation 1 is carryed out to verify the effectiveness of restraint for bullwhip effect of the dual channel CLSC with PIR. The initial inventory values are chosen as: $x_1(0) = -102, x_2(0) = -75, x_3(0) = 0$. The norminal values are set as:$\bar{x}_1(k) = 110, \bar{x}_2(k) = 80, \bar{x}_3(k) = 45, \bar{u}_1(k) = 100, \bar{u}_2(k) = 60, \bar{u}_3(k) = 100(\times 10^3\text{sets})$. The external uncertain demand is set as normal distribution, uniform distribution and mixed periodic disturbance, respectively. The verification simulation in this part is conducted as three Cases.

In Case1, the $d(t)$ was selected as a normal distribution where the demand satifies $d(t) \sim N(6, 0.3^2)$. Figure 3 shows the evolution process of the invertories, control input and the total operation cost of the CLSC. Due to the initial values of current inventory for both manufacturer and retailer are all smaller than D_{0m} and D_{0r}, the fuzzy control rule R_1 is activated at this mement. In order to increase inventory as quickly as possible, the manufacturer produces new batteries and recycles old batteries for remanufacturing. Meanwhile, the retailer place orders from manufacturer. Figure 3(a) and Figure 3(b) show that the inventories of manufancturer and retailer rise rapidly. In the same way, the total cost of the CLSC also rises dramatically, which is described in Figure 3(c). In addition, it is obviously noted that the bullwhip effect has been well suppressed by the fuzzy roubust H_∞control method. The system states are all converged to stable values. In this Case, the manufacturer's inventory is converted to about 100×10^3sets, the retailer's inventory is about 70×10^3sets, the customer's inventory is about 50×10^3sets which is equal to the norminal values mentioned above.

In Case 2 and Case 3, the uncertain demands $d(t)$ are set as uniform distribution ($d(t) \sim U(6, 7)$) and mixed periodic disturbance ($d(t) = 6 + 0.5\sin(k) + 0.5\cos(k)$), respectively. In Figure 4 and Figure 5, it is obviously seen that the bullwhip effect of the CLSC is almost eliminated. Although small oscillation is occurred in Case 3, the system states all converge to the stable values quickly. The stable values of inventories are the same as in Case 1.

Remark 5.1: Theoretically, it should be paid attention to Case 1, Case 2 and Case 3 of Simulation 1 that any form of

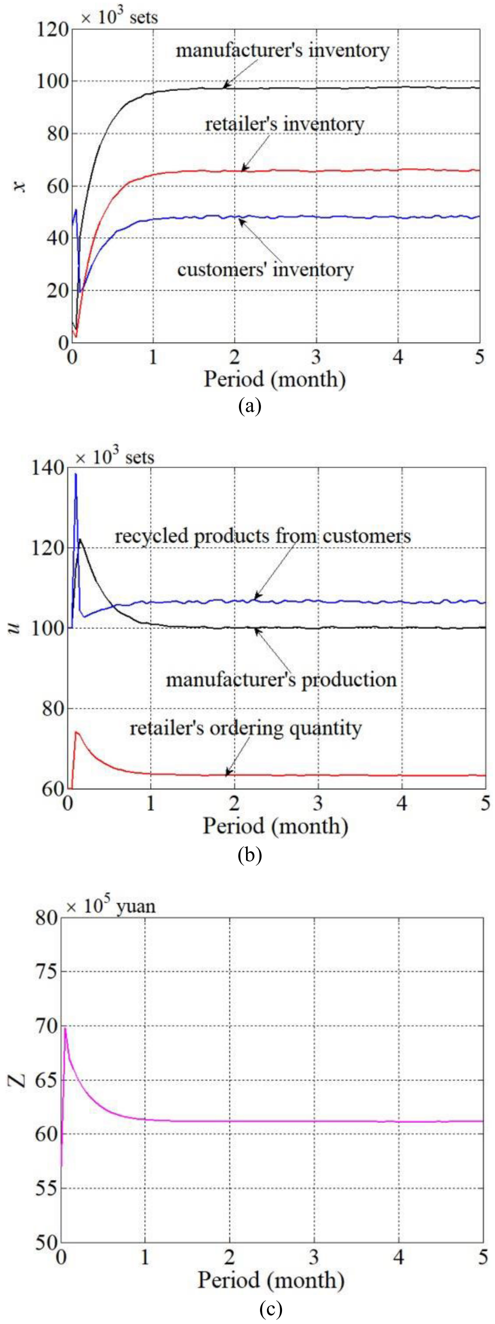


(a)

(b)

(c)

FIGURE 3. Bullwhip effect restraint results in Case 1 of Simulation 1; black solid line represents manufacturer, red solid line represents retailer, blue solid line represents customer, magenta solid line represents total operation cost of the CLSC: (a) inventory, (b) control input, (c) total operation cost.



(a)

(b)

(c)

FIGURE 4. Bullwhip effect restraint results in Case 2 of Simulation 1; black solid line represents manufacturer, red solid line represents retailer, blue solid line represents customer, magenta solid line represents total operation cost of the CLSC: (a) inventory, (b) control input, (c) total operation cost.

unknown but bounded demand disturbances in the CLSC can be attenuated by the fuzzy robust H_∞ control method. In practice, the attenuate coefficient γ is chosen as small as possible to obtain better control performance. However, a smaller γ will result in worse robustness of the system. Hence, the suitable γ is chosen based on the specific calculation process.

B. THE COMPARISON OF THE CLSC'S TOTAL COST

The comparison Simulation 2 is conducted to illustrate the total operation cost reduction of the CLSC in this part, in which the PIR is introduced. The initial values are chosen the same as Part A: $x_1(0) = -102, x_2(0) = -75, x_3(0) = 0$, $\bar{x}_1(k) = 110, \bar{x}_2(k) = 80, \bar{x}_3(k) = 45, \bar{u}_1(k) = 100, \bar{u}_2(k) = 60, \bar{u}_3(k) = 100(\times 10^3\text{sets})$. The PIR investment cost coefficient $C_p = 0$ means that it is no PIR investment in the CLSC. In Figure 6(a), the black solid line, red solid line and bule solid line presents the manufacturer, retailer and customer's inventories when $C_p = 0.05$. the

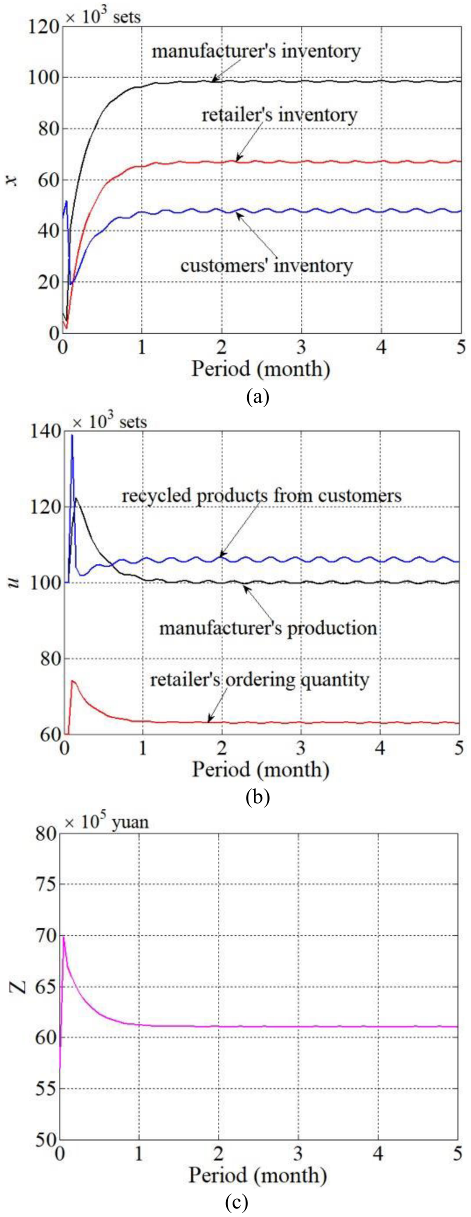


FIGURE 5. Bullwhip effect restraint results in Case 3 of Simulation 1; black solid line represents manufacturer, red solid line represents retailer, blue solid line represents customer, magenta solid line represents total operation cost of the CLSC: (a) inventory, (b) control input, (c) total operation cost.

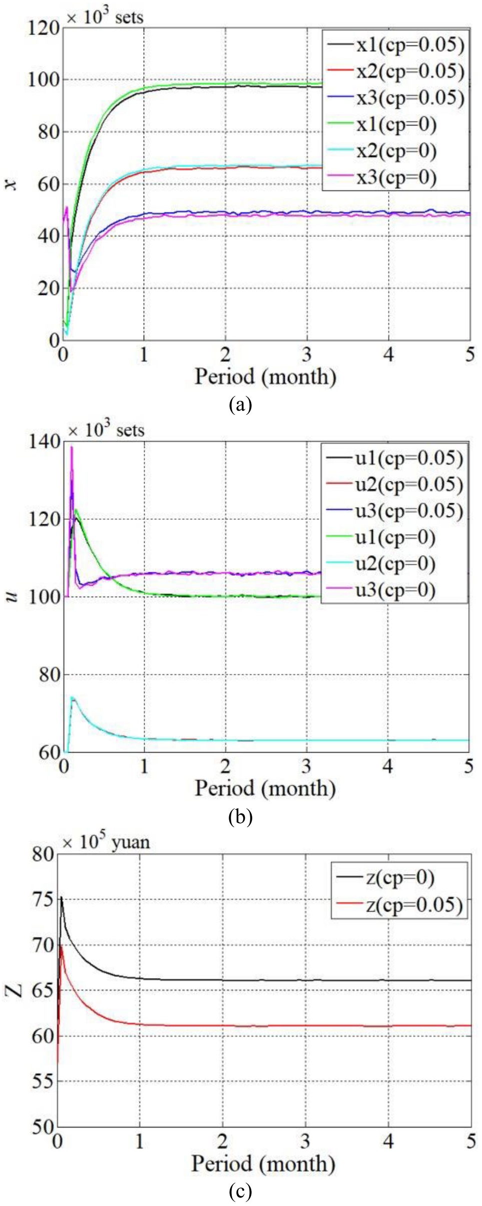


FIGURE 6. The comparison of total operation cost in Simulation 2: (a) inventory, (b) control input, (c) total operation cost: red solid line represents total cost with PIR, black solid line represents total cost without PIR.

green solid line, cyan solid line and magenta solid line are the inventories when $C_p = 0$. In Figure 6(c), the black solid line is the total cost when $C_p = 0$, and the red solide line represents the total cost when $C_p = 0.05$. It is obviously noted that the total cost of CLSC is decreased by 8%, when PIR is used in the CLSC proposed in this article. Figure 6(b) is the control input.

Remark 5.2 It must be noted that the total cost of the CLSC is reduce by 8% when the PIR investment cost coefficient C_p is set to 0.05, meanwhile the quantity of recycling and new products are set to the same. Different C_p indicates different level of PIR investment. The higher the PIR investment, the more products are recovered, which can reduce the total cost of supply chain operation. Therefore, in the CLSC system proposed in this article, the reduction in total operating costs depends on the amount of old product recycling and the level of PIR investment.

C. INVENTORY CONTROL STRATEGY IN DIFFERENT CASES

In this part, Simulation 3 is provided in three different cases to illustrate the strategy how to control the inventory and total cost of the CLSC at different initial inventory levels. In the three simulations, the nominal values are all the same

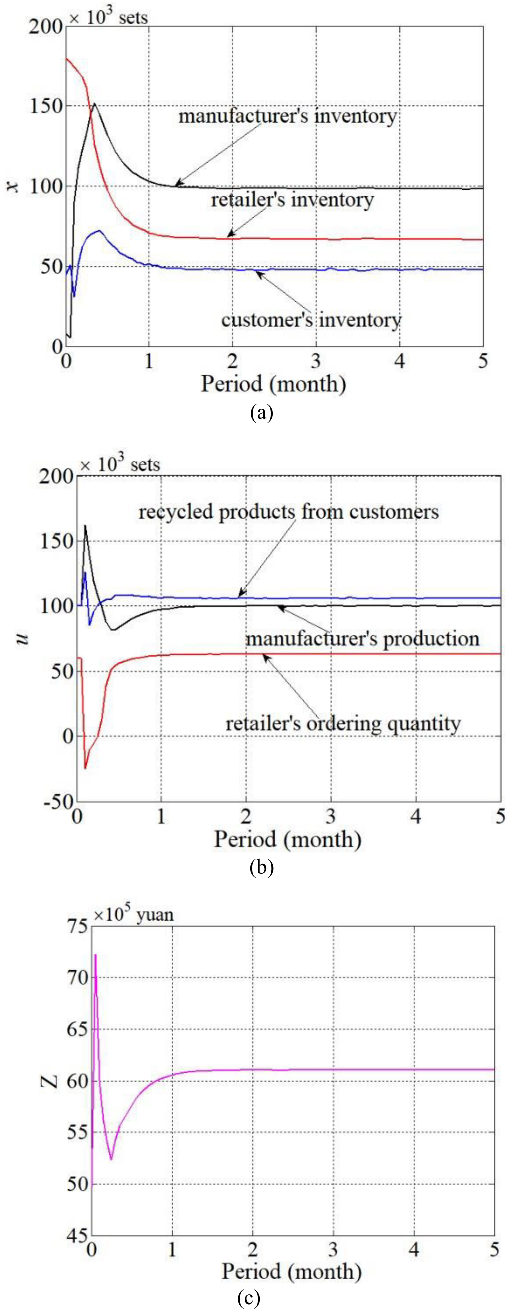


FIGURE 7. Inventory control results in Case 1 of Simulation 3; black solid line represents manufacturer, red solid line represents retailer, blue solid line represents customer, magenta solid line represents total operation cost of the CLSC: (a) inventory, (b) control input, (c) total operation cost.

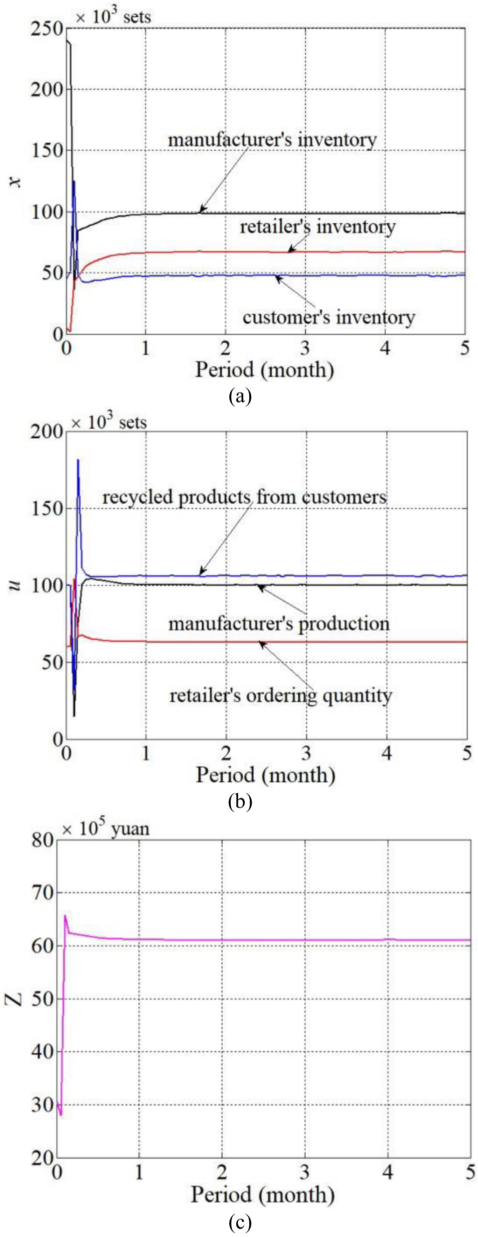
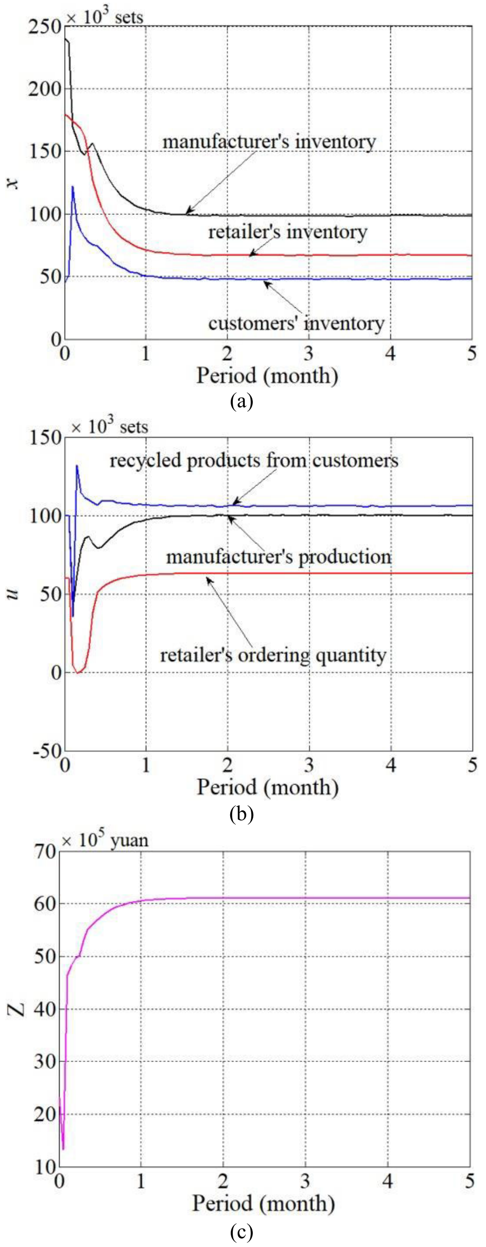


FIGURE 8. Inventory control results in Case 2 of Simulation 3; black solid line represents manufacturer, red solid line represents retailer, blue solid line represents customer, magenta solid line represents total operation cost of the CLSC: (a) inventory, (b) control input, (c) total operation cost.

as Part A and Part B: $\bar{x}_1(k) = 110, \bar{x}_2(k) = 80, \bar{x}_3(k) = 45, \bar{u}_1(k) = 100, \bar{u}_2(k) = 60, \bar{u}_3(k) = 100(\times 10^3\text{sets})$, but the initial inventory values are different.

In Case 1, the initial inventory value are chosen as: $x_1(0) = -102,\ x_2(0) = 100, x_3(0) = 0$. At this moment, the manufacturer's inventory is at a lower level, but the retailer's inventory is at a higher level. In order to reach to higher inventory level rapidly, the manufacturer must increase the output of new products and recycling of old products. Similarly, for the purpose of obtaining a lower inventory level quickly, the retailer doesn't place orders from manufacturers, only accept the gratuitous return. The fuzzy control rule R_2 of the MORG is activated at this moment. Figure 7(a) shows that manufacturer's inventory rises sharply and then falls to the nominal value, and the retailer's inventory falls to its nominal value from a high value. Correspondingly, the control input is shown in Figure 7(b) which illustrates the control strategy for the manufacturer and the retailer' inventory in this case. Figure 7(c) shows the total operation cost of the CLSC in this case.



(a)

(b)

(c)

FIGURE 9. Inventory control results results in Case 3 of Simulation 3; black solid line represents manufacturer, red solid line represents retailer, blue solid line represents customer, magenta solid line represents total operation cost of the CLSC: (a) inventory, (b) control input, (c) total operation cost.

In Case 2, the initial inventory values are set as: $x_1(0) = 130$, $x_2(0) = -75, x_3(0) = 0$. At this point, the situation is exactly the opposite of Case1. The manufacturer's inventory is at a higher level, but the retailer's inventory are at a lower level. For the purpose of obtaining the desired inventory, the manufacturer doesn't product new batteries, only recycles old batteries for remanufacturing. Meanwhile, the retailer is not only place orders from manufacturers but also accept the gratuitous return. The fuzzy control rule R_3 of the MORG is activated at this time. It can be seen that manufacturer's inventory falls to 100×10^3sets, and retailer's inventory rises to 70×10^3sets. Accordingly, the control input is shown in Figure 8(b). Figure 8(b) gives the control strategy that the manufacturer's production and the recycling should be dropped quickly at first, and rose to a stable value. The retailer's order from manufacturer should be risen rapidly first, and converged to a nominal value. Figure 8(c) is the total cost of the CLSC in this case.

In Case 3, we set the initial inventory value as: $x_1(0) = 130$, $x_2(0) = 100, x_3(0) = 0$. At this time, the manufacturer's inventory and the retailer's inventory are all at a higher level. To obtain the nominal inventory, the manufacturer doesn't product new batteries, only recycles old batteries for remanufacturing. Meanwhile, the retailer doesn't place orders from manufacturers, only accept the gratuitous return. The fuzzy control rule R_4 of the MORG is activated at this time. It can be obviously noted that manufacturer's inventory falls to 100×10^3sets, and retailer's inventory rises to 70×10^3sets in Figure 9(a). Figure 9(b) shows the specific inputs of the control strategy in this case. The manufacturer's production and the recycling should dropped quickly first and then rose to their nominal value. The retailer's order from manufacturer should be also declined rapidly first, and converged to its nominal value. Figure 9(c) shows the total cost of the CLSC in this case.

As shown in Figure 7-9, the following conclusions can be obtained.

1) By using the robust fuzzy inventory control strategies, the manufacturer and the retailer can eventually organize production and place ordering from the manufacturer normally.

2) By introducing the robust switching strategy, the states of the uncertain dual-channel CLSC with PIR can eventually operate steadily.

3) Under the different initial inventory and the same nominal value, all states in the uncertain dual-channel closed-loop supply chain can eventually converge to the same nominal value. It is indicated that the robust switching control strategy can availably deal with the influence of the uncertainties and manage the dual-channel CLSC system effectively.

VII. CONCLUSION

In this article, the inventory and total operation cost management strategy for a dual-channel uncertain CLSC with PIR is addressed. First of all, the mathematical model of the uncertain dual-channel CLSC system with PIR is established based on the current inventory and total operation cost. Secondly, the model is converted to an uncertain multi-model according to different initial operation cases which is then described with a set of switching subsystem by using a discrete T-S fuzzy model. Thirdly, a fuzzy robust H_∞ method is introduced to manage the CLSC included bullwhip effect restraint, handling of PIR impact and inventory control strategy. At last, three simulation test results show that: 1) the bullwhip effect in three different customer's demand can be restrained well. 2) the total cost of the CLSC is reduce by 8% when the PIR investment cost coefficient C_p is set to 0.05 and the number of recycling and new products are set to the same at this time. The reduction in total operating costs depends on the amount of old product recycling and the level of PIR investment. 3) The robust switching control strategy can availably attenuate the effect of the uncertainties and manage the CLSC effectively under the different initial inventory.

The management insights we have summarized:1) the fuzzy robust H_∞ method can effectively suppress inventory backlog, severe shortages, and unstable supply and demand caused by dual channel closed-loop supply chain systems. When the customer demand with significant fluctuations, the system's inventory, ordering/production, and costs can converge to the nominal values in a short period of time.2) the PIR investment can improve the flexibility of enterprise operations and reduce production costs. From a long-term perspective, enterprises should consider to add the PIR investment efforts in their production. 3) the fuzzy robust H_∞ method provides the management strategies of inventory and total cost control for the managers of supply chain at different initial inventories, and these management strategies are able to apply as a monitoring system of current inventory. Through the monitoring system, management strategies can guarantee the stability of the CLSC at low total cost which can be extended to other CLSC systems.

The time-delay is not be taken into account in this article, however, in practice, each node in the supply chain system has a time-delay. In addition, the operation process and the structure of the CLSC have been simplified. In our furture work, the time-delay during the operation of the CLSC will be considered firstly. Secondly, future research may focus on studying the collaborative PIR investment between upstream suppliers and downstream manufacturers. Finally, future research may investigate a more complex CLSC structure which includes cross channel return and free-riding behavior between dual channel.

REFERENCES

[1] Y. Liang, S. Pokharel, and G. H. Lim, "Pricing used products for remanufacturing," *Eur. J. Oper. Res.*, vol. 193, no. 2, pp. 390–395, Mar. 2009.

[2] S. Vadde, A. Zeid, and S. V. Kamarthi, "Pricing decisions in a multi-criteria setting for product recovery facilities," *Omega*, vol. 39, no. 2, pp. 186–193, Apr. 2011.

[3] J.-P. Kenné, P. Dejax, and A. Gharbi, "Production planning of a hybrid manufacturing-remanufacturing system under uncertainty within a closed-loop supply chain," *Int. J. Prod. Econ.*, vol. 135, no. 1, pp. 81–93, Jan. 2012.

[4] G. C. Souza, "Closed-loop supply chains: A critical review, and future research," *Decis. Sci.*, vol. 44, no. 1, pp. 1–38, Feb. 2013.

[5] Q. Qiang, K. Ke, T. Anderson, and J. Dong, "The closed-loop supply chain network with competition, distribution channel investment, and uncertainties," *Omega*, vol. 41, no. 2, pp. 186–194, Apr. 2013.

[6] S. Zhang, X. Zhao, and J. Zhang, "Dynamic model and fuzzy robust control of uncertain closed-loop supply chain with time-varying delay in remanufacturing," *Ind. Eng. Chem. Res.*, vol. 53, no. 23, pp. 9805–9811, May 2014.

[7] Y. Guan and Q. Hou, "Dynamic strategy of power battery closed-loop supply chain considering cascade utilization," *IEEE Access*, vol. 10, pp. 21486–21496, 2022.

[8] D. Gao, N. Wang, Z. He, and T. Jia, "The bullwhip effect in an online retail supply chain: A perspective of price-sensitive demand based on the price discount in e-commerce," *IEEE Trans. Eng. Manag.*, vol. 64, no. 2, pp. 134–148, May 2017.

[9] D. Gao, N. Wang, B. Jiang, J. Gao, and Z. Yang, "Value of information sharing in online retail supply chain considering product loss," *IEEE Trans. Eng. Manag.*, vol. 69, no. 5, pp. 2155–2172, Oct. 2022.

[10] L. Xu, J. Wei, T.-M. Choi, and W. Wang, "Managing online channel and optimization in supply chain systems with different channel leaderships," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 11, pp. 7177–7190, Nov. 2021.

[11] H. Pei, Y. Liu, and H. Li, "Robust pricing for a dual-channel green supply chain under fuzzy demand ambiguity," *IEEE Trans. Fuzzy Syst.*, vol. 31, no. 1, pp. 53–66, Jan. 2023.

[12] Z. Bao, W. Li, M. Gao, C. Liu, X. Zhang, and W. Cai, "An integrated optimization decision method for remanufacturing process based on conditional evidence theory under uncertainty," *IEEE Access*, vol. 8, pp. 221119–221126, 2020.

[13] S. Zhang, Y. Hou, and X. Zhao, "Robust stabilization for discrete uncertain Takagi–Sugeno fuzzy systems based on a piecewise Lyapunov function," *Ind. Eng. Chem. Res.*, vol. 53, no. 17, pp. 7132–7140, Apr. 2014.

[14] X. Y. Huang, N. N. Yan, and H. F. Guo, "An H_∞ control method of the bullwhip effect for a class of supply chain system," *Int. J. Prod. Res.*, vol. 45, no. 1, pp. 207–226, Jun. 2007.

[15] X.-Y. Huang, N.-N. Yan, and R.-Z. Qiu, "Dynamic models of closed-loop supply chain and robust H_∞ control strategies," *Int. J. Prod. Res.*, vol. 47, no. 9, pp. 2279–2300, May 2009.

[16] R. G. Ge and X. Y. Huang, "Uncertain supply chain model based on switched inventory and robust H_∞ control," (in Chinese), *Syst. Eng.*, vol. 27, no. 3, pp. 119–122, Mar. 2009.

[17] H. Sarimveis, P. Patrinos, C. D. Tarantilis, and C. T. Kiranoudis, "Dynamic modeling and control of supply chain systems: A review," *Comput. Oper. Res.*, vol. 35, no. 11, pp. 3530–3561, Nov. 2008.

[18] R. G. Ge and X. Y. Huang, "Closed-loop supply chain switching model with outsourcing selection and its robust control," (in Chinese), *Comput. Integr. Manuf. Syst.*, vol. 15, no. 10, pp. 2012–2016, Oct. 2009.

[19] R. G. Ge and X. Y. Huang, "Switching model for scheduled advertisements on e-commerce website and robust control," (in Chinese), *J. Northeastern Univ.*, vol. 30, no. 7, pp. 1055–1057, Jul. 2009.

[20] J. Xu and X. Huang, "A multi-objective robust operating model for supply chain with uncertain market supplies and demands," (in Chinese), in *Proc. 6th World Congr. Intell. Control Automat.*, Dalian, China, pp. 1388–1392, Jun. 2006.

[21] X. Huang and R. Qiu, "Supply chain dynamic model based on remanufacturing and its robust HH_∞ control," *Contr. Deci.*, vol. 22, no. 6, pp. 667–674, 2007.

[22] Q. Liu, J. Xu, and F. Qin, "Optimization for the integrated operations in an uncertain construction supply chain," *IEEE Trans. Eng. Manag.*, vol. 64, no. 3, pp. 400–414, Aug. 2017.

[23] Q.-K. Li, H. Lin, X. Tan, and S. Du, "HH_∞ consensus for multiagent-based supply chain systems under switching topology and uncertain demands," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 50, no. 12, pp. 4905–4918, Dec. 2020.

[24] Y. Yang and Y. Chen, "Designing closed-loop supply chain network for high-salvage perishable products under uncertain environment," *IEEE Access*, vol. 9, pp. 102236–102250, 2021.

[25] S. Asian and X. Nie, "Coordination in supply chains with uncertain demand and disruption risks: Existence, analysis, and insights," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 44, no. 9, pp. 1139–1154, Sep. 2014.

[26] J. Gao, X. Yang, and D. Liu, "Uncertain Shapley value of coalitional game with application to supply chain alliance," *Appl. Soft Comput.*, vol. 56, pp. 551–556, Jul. 2017.

[27] Y. Deng, M. Jiang, and C. Ling, "An improved diffusion model for supply chain emergency in uncertain environment," *Soft Comput.*, vol. 24, no. 9, pp. 6385–6394, May 2020.

[28] Z. Griliches, "Patent statistics as economic indicators: A survey," in *R&D and Productivity: The Econometric Evidence*. Chicago, IL, USA: Univ. Chicago Press, 1998, pp. 287–343.

[29] J. M. Utterback and W. J. Abernathy, "A dynamic model of process and product innovation," *Omega*, vol. 3, no. 6, pp. 639–656, May 1975.

[30] P. D. Collins, J. Hage, and F. M. Hull, "Organizational and technological predictors of change in automaticity," *Acad. Manag. J.*, vol. 31, no. 3, pp. 512–543, Sep. 1988.

[31] S. Rosenkranz, "Simultaneous choice of process and product innovation when consumers have a preference for product variety," *J. Econ. Behav. Org.*, vol. 50, no. 2, pp. 183–201, Feb. 2003.

[32] *Proposed Guidelines for Collecting and Interpreting Technological Innovation Data*, Org. Econ. Co-Oper. Develop. (OECD), Paris, France, Sep. 2005, pp. 12–23.

[33] Q. Zhu, J. Sarkis, and K.-H. Lai, "Supply chain-based barriers for truck-engine remanufacturing in China," *Transp. Res. E, Logistics Transp. Rev.*, vol. 68, pp. 103–117, Aug. 2014.

[34] M. Reimann, Y. Xiong, and Y. Zhou, "Managing a closed-loop supply chain with process innovation for remanufacturing," *Eur. J. Oper. Res.*, vol. 276, no. 2, pp. 510–518, Jul. 2019.

[35] H. Chen, Z. Dong, G. Li, and K. He, "Remanufacturing process innovation in closed-loop supply chain under cost-sharing mechanism and different power structures," *Comput. Ind. Eng.*, vol. 162, Dec. 2021, Art. no. 107743.

[36] J. Yao and D. Chen, "Pricing and coordination strategy of closed-loop supply chain based on market effort and process innovation for remanufacturing," *Comput. Integr. Manuf. Syst.*, vol. 27, no. 11, pp. 3329–3340, Nov. 2021.

[37] W. A. Jauhari, A. R. Dwicahyani, and N. A. Kurdhi, "Lot sizing decisions in a closed-loop supply chain system with remanufacturing," *Int. J. Proj. Manag.*, vol. 10, no. 3, pp. 381–409, Apr. 2017.

[38] A. Y. Christy, B. N. Fauzi, N. A. Kurdi, W. A. Jauhari, and D. R. S. Saputro, "A closed-loop supply chain under retail price and quality dependent demand with remanufacturing and refurbishing," in *Proc. 1st Int. Conf. Math.-Educ., Theory Appl. (ICMETA)*, Surakarta, Indonesia, Dec. 2017, pp. 1–7.

[39] W. A. Jauhari and I. D. Wangsa, "A manufacturer-retailer inventory model with remanufacturing, stochastic demand, and green investments," *Process Integr. Optim. Sustainability*, vol. 6, no. 2, pp. 253–273, Jan. 2022.

[40] W. A. Jauhari, "Sustainable inventory management for a closed-loop supply chain with energy usage, imperfect production, and green investment," *Cleaner Logistics Supply Chain*, vol. 4, Jul. 2022, Art. no. 100055.

[41] J. Hu, Q. Hu, and Y. Xia, "Who should invest in cost reduction in supply chains?" *Int. J. Prod. Econ.*, vol. 207, pp. 1–18, Jan. 2019.

[42] Q. Zhang, J. Zhang, and G. Zaccour, "Strategic technology licensing in a supply chain," *Eur. J. Oper. Res.*, vol. 267, no. 1, pp. 162–175, Mar. 2018.

[43] Z.-H. Xiu and G. Ren, "Stability analysis and systematic design of Takagi–Sugeno fuzzy control systems," *Fuzzy Sets Syst.*, vol. 151, no. 1, pp. 119–138, Apr. 2005.

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