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RESEARCH ARTICLE

Optimizing Multi-Modal Electromagnetic Design Problems Using Quantum Particle Swarm Optimization With Differential Evolution

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ABSTRACT Many versatile and promising swarm intelligence evolutionary algorithms are being developed to solve engineering optimization problems. Although evolutionary algorithms have been implemented in various optimization fields, there is still potential for enhancement in the domain of complex, electromagnetic, and multimodal objective problems. To effectively address the shortcomings and slow convergence speed observed in both smart quantum particle swarm optimization (QPSO) and differential evolution (DE), a hybrid strategy is proposed. In the proposed QPSODE, apart from the smart strategy of QPSO for improving the exploration as a whole, more additional features such as non-linear adaptive control parameter, the partition of the swarm to apply smart and gaussian mutation mechanism, crossover and selection of best particle using Boltzmann strategy to avoid premature convergence are introduced. Consequently, applying the new design algorithm to several benchmark-constrained, mostly non-convex, and superconducting magnetic energy storage (SMES) electromagnetic problems shows a marked performance improvement. The performances of the QPSODE is compared with those of many other widely recognized population-based swarm intelligence optimizers. Experimental results and statistical analysis using Friedman test show that the search accuracy and the convergence of the hybrid QPSODE strategy are advantageous over other optimization approaches.

INDEX TERMS Smart particle, hybridization, QPSO, DE, Boltzmann selection strategy, energy storage device (SMES).

I. INTRODUCTION

In the study of engineering and artificial intelligence, optimization problems are ubiquitous, spontaneous, and associated with every real-world field, such as electromagnetics, digital computers, power engineering, telecommunications, control systems, robotics, and signal processing. The optimization design of electromagnetic devices has attracted the mainstream attention of researchers over the last two decades [1], [2], [3].

The design of electromagnetic devices generally incorporates elements of the optimization process, such as the restriction of constrained conditions, and the uncertainty of results. In the literature, different optimization methods, classical, linear programming, nonlinear programming (one-dimensional, constrained, unconstrained), and global optimization, are adapted to find the best-suited solution to a problem under a given set of constraints. Since the emergence of electromagnetic computing, especially with the development of the modern computer and the growth of numerical analysis techniques, the automated optimization of electromagnetic problems has increased exponentially. Therefore, there has been a tremendous improvement in the capabilities of computational methods and techniques for solving electromagnetic problems [4], [5].

The industry has increased its demand for device parameter optimization, and computational methods have evolved to meet this demand. New computing methods and models are required; only using more powerful computer hardware seems insufficient. The structure of the optimization problem

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not only places limitations on traditional optimization techniques like gradient descent and deterministic methods but also renders them incapable of solving these problems [6]. Classical optimization methods are, therefore, inadequate when the optimization problems cannot be formally characterized as continuous and differentiable functions. In contrast, stochastic optimization methods demonstrate that they require excellent initial value estimation and can converge to the optimal global solution, making them ideal for handling nonlinear optimization problems. Thus, stochastic algorithms and numerical techniques are the primary methods for handling electromagnetic design problems. As a result, significant work has been done to enhance the stochastic algorithms' fundamental formulation for resolving these problems. Many more real-world engineering optimization problems have been solved using all these stochastic techniques.

Numerous researchers have created a variety of compatible and user-friendly evolutionary intelligence optimization algorithms addressing objective functions that are challenging using traditional techniques [7]. It draws inspiration from nature and tries to build centralized, self-organizing systems by coordinating the behaviors of individual agents as they interact with their environments. It includes biological evolution and behaviors of flocks of birds and a swarm of ants such as artificial bee colony (ABC) [8], [9], [10], particle swarm optimization (PSO) [11], [12], [13], [14], differential evolution (DE) [15], [16], [17], grey wolf optimizer (GWO) [18], cuckoo search (CS) [19], [20], and others [21]. Particle swarm optimization was developed for optimization algorithms in the middle of the 1990s and was inspired by bird flocking; it is easy to implement and requires less computational time. It also provides a greater diversity of search trajectories, converges faster, and finds the best solutions to optimization problems [22], [23]. In PSO, particles that represent potential solutions move throughout a multidimensional search space at a velocity that is continuously being updated by the experience of the particle and the knowledge of the entire swarm. PSO has attracted the attention of many researchers worldwide, and they developed many improved versions of basic PSO and validated them through different optimization problems [24], [25]. Various enhanced versions of PSO have been presented by researchers in recent literature, such as terminal crossover and steering-based PSO [26], human behavior-based PSO [27], position-transitional PSO [28], and hybrid whale PSO [29]. Although PSO is straightforward and has successfully solved many optimization problems, it could encounter local minima when dealing with complex multimodal problems. In 2004, Sun developed the quantum-behaved version of PSO (QPSO), which uses wave function of time-dependent Schrodinger equation [30], to characterize particle solution candidates in the search space rather than the velocity and location of particles in Eq. (1):

$$i\hbar\psi\left(x_{ij}^{(t)}\right) = \frac{-\hbar}{2m}\frac{d^2\psi\left(x_{ij}^{(t)}\right)}{d(x_{ij}^{(t)})^2} + V\left(x_{ij}^{(t)}\right)\psi\left(x_{ij}^{(t)}\right) \tag{1}$$

To find an optimization solution where $\hbar$ is the Planck constant, potential energy $V\left(x_{ij}^{(t)}\right)$, a quantum state $\psi\left(x_{ij}^{(t)}\right)$ known as normalized wave state vector are used in this work's delta potential well model formulation:

$$\psi\left(x_{ij}^{(t)}\right) = \frac{1}{\sqrt{L_{ij}^{(t)}}}e^{\left(\frac{-\left|Z_{ij}^{(t)}-x_{ij}^{(t)}\right|}{L_{ij}^{(t)}}\right)} \tag{2}$$

In quantum mechanics particle appearance in a certain position in search space can be determined by Max Born from the probability density function:

$$\text{PDF} = \left|\psi\left(x_{ij}^{(t)}\right)\right|^2 = \frac{1}{L_{ij}^{(t)}}e^{\left(\frac{-2\left|Z_{ij}^{(t)}-x_{ij}^{(t)}\right|}{L_{ij}^{(t)}}\right)} \tag{3}$$

The position formulas for each particle, generated by the Monte Carlo method, are as follows:

$$x_{ij}^{(t+1)} = \begin{cases} p_{ij}^{(t)} + \beta * \left|M_{best}^{(t)} - x_{ij}^{(t)}\right| * \log(\frac{1}{u_{ij}^{(t)}}), & \textit{if } u_{ij}^{(t)} \geq 0.5 \\ p_{ij}^{(t)} - \beta * \left|M_{best}^{(t)} - x_{ij}^{(t)}\right| * \log(\frac{1}{u_{ij}^{(t)}}), & \textit{otherwise} \end{cases} \tag{4}$$

When compared to PSO, QPSO has a faster convergence capability and speed. As a result, numerous researchers from other disciplines have showed interest in QPSO. Many improved QPSO versions have been created by researchers so far, and they have already proven to handle challenging optimization problems. The authors of this article give a brief overview of QPSO and its variants, which have been used, tested, and verified for broad range of optimization problems in engineering fields. Numerous nonlinear, non-differentiable, and non-convex optimization problems are addressed with the QPSO technique. In [31], a levy flight strategy is used to find an optimal solution for the fuzzy portfolio with constraints. Li et al. utilized quantum-behaved discrete multi-objective PSO for complex network clustering [32]. In the medical field, the partition-cooperative QPSO is used for image segmentation to improve the quality and resolution [33]. In [34], the author introduces smart particles to the existing QPSO to improve the convergence speed for non-convex objective problems and in the electromagnetic field. Niu et al. use extreme learning machine-based QPSO to select the optimal input-hidden weight and hidden biases for hydrological time-series prediction [35]. Zhang et al. employed an improved QPSO with space transformation search and empirically verified its efficiency [36]. Numerous studies confirm that various optimization problems in multiple engineering fields have been solved using QPSO and DE techniques. The basic QPSO provides many benefits, including ease of implementation, fast convergence, low processing time, and robustness. However, for high-dimensional and multimodal functions, these algorithms encounter limited

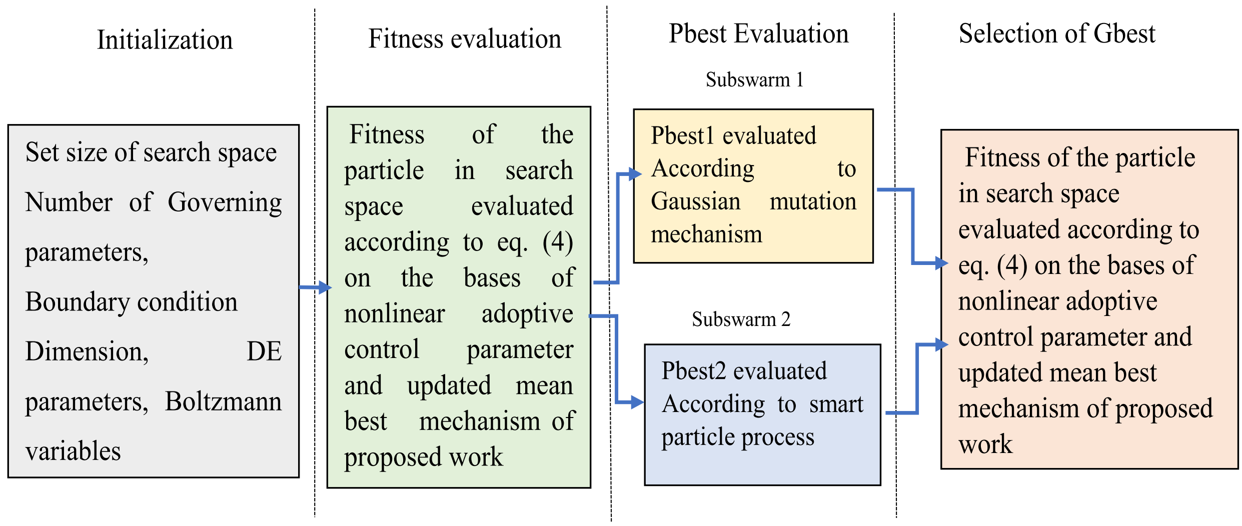

FIGURE 1. Schematic representation of the hybridized QPSO-DE method in the proposed work.

precision and slow convergence speed in the late evolutionary stage and stalling in local optima.

Studies on hybrid systems in swarm intelligence have recently gained popularity in tackling more complex and challenging problems. More recent advancements in the machine learning and artificial intelligence (AI) domains have led to the development of more efficient addition to assembling quantum particle swarm optimization and other evolutionary optimization algorithms. Nirmal et al. use a hybridized Gaussian QPSO with the cuckoo search for solving first and second-order differential equations by converting these into unconstrained optimization problems [37]. Protopopescu et al. utilized quantum-inspired algorithms to tackle various optimization problems [38]. Yuanyuan published a quantum evolutionary algorithm for identifying communities in complex social networks [39].

The application of QPSO with a mutation operator (QPSO-MO) has been endorsed to enhance diversity and avoid the local optimum in search space [40]. However, a limited number of studies have explored these advanced techniques for optimization in electromagnetics and storage components, especially in large-size devices. The experimental results on a low-dimensional unimodal show that these enhanced QPSOs have superior convergence performance. However, these algorithms still have a few limitations, such as early convergence and low precision in the late evolutionary stage, as well as falling into local optima on high-dimensional multimodal functions.

Specified the drawbacks, a hybridized QPSODE algorithm will be proposed and applied to non-linear and multimodal problems. The study enhances previous hybrid approaches, used in QPSO research, that integrated smart particle behaviors QPSO with DE [41]. The study analyzes how the contraction-expansion coefficient impacts an algorithm's ability to find the optimal solutions by influencing particle mobility during optimization. In the current work, first we used the reliability over dynamic contraction expansion coefficient with particle fitness. Secondly, we then selected the best particle from subswarms amongst the Gaussian mutated and smart particle mechanism for enhancing global convergence. Thirdly, DE selection under the Boltzmann probability and crossover strategy are adapted. A hybridized algorithm is typically created by integrating the features of smart QPSO and DE algorithms to boost the efficiency and convergence of an algorithm to solve constrained optimization problems. The QPSODE algorithm has been incorporated into the proposed work, as depicted in the block diagram in Fig. 1.

The rest of this paper is organized as follows. The introduction section discusses some relevant literature on general optimization, evolutionary algorithms, PSO, and its variants as well as its advanced version QPSO. Section II develops a hybridized algorithm (QPSODE) that considers some methods of improving performance, such as nonlinear parameter adjustment structure, smart particle, and Bultmann selection procedure for premature prevention mechanism. Section III then goes over the performance test results for 25 benchmark functions, statistical test and shows how to use QPSODE to solve these constrained functions as well as relevant comparison experiments of various metaheuristic algorithms followed by electromagnetic problems application are reported in section IV. Finally, section V presents the conclusion and future work of the paper.

II. METHODOLOGY OF HYBRID QPSODE

The basic idea behind the proposed QPSODE hybridization is to integrate QPSO and DE with a smart strategy, adaptive parameter control, fusion of DE mechanism, Gaussian mutation, and Boltzmann selection to improve the effectiveness of global search and convergence. The essential procedures that follow a new hybrid algorithm are stressed below.

First, a nonlinear adjustment of the contraction–expansion coefficient is adapted in the proposed strategy taken exponential function values according to Eq. (5). QPSODE used dynamic nonlinear function values rather than con-

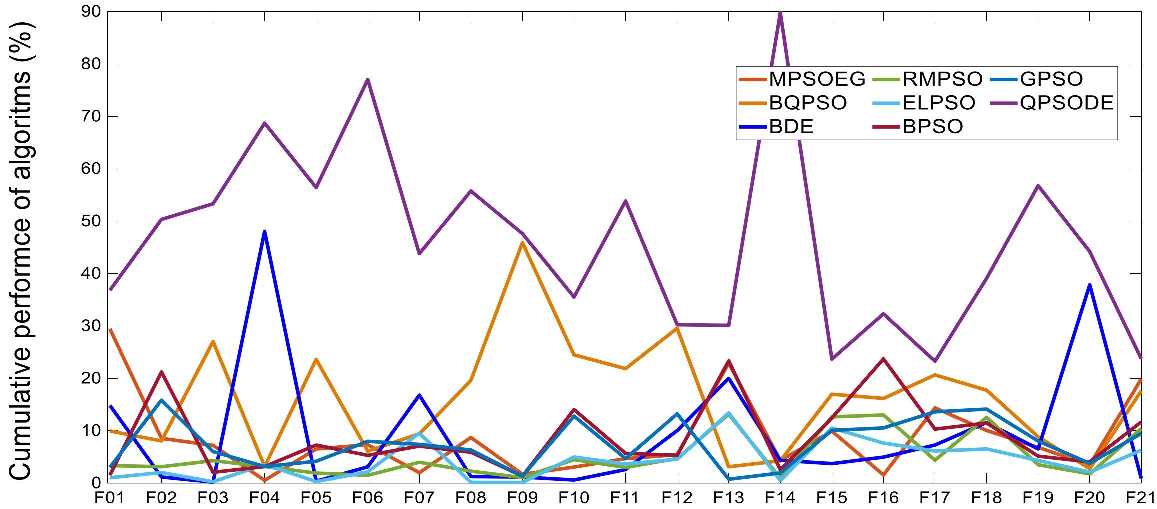

FIGURE 2. Cumulative performance of all competitors during optimization process.

TABLE 1. Numerical results of best objective values for all problems.

No	QPSODE	MPSOEG	BQPSO	BDE	RMPSO	ELPSO	BPSO	GPSO
F01	-89.9998	-71.9998	-24.3201	-36.285	-8.19852	-2.54536	-3.76583	-7.39577
F02	-57.1180	-9.69436	-9.14699	-1.33053	3.563371	2.304348	-24.0788	-17.9748
F03	-59.4576	-8.05024	-30.1444	0.21963	-4.78311	-0.28691	-2.30371	-6.71921
F04	-10.6616	-6.57958	-9.46178	-3.33283	-2.00909	-2.80958	-4.72526	-6.23707
F05	-144.234	-16.6789	-60.4085	-0.97899	-4.91561	0.757796	-18.5745	-10.6718
F06	-70.1677	-6.69331	-5.57310	2.835874	-1.36528	1.973708	-4.83411	-7.27401
F07	-111.463	-9.67933	-45.2239	-5.37852	-6.21008	-7.57451	-11.7292	-9.70235
F08	-296.733	-46.3454	-104.379	-6.70325	-12.1782	-0.81597	-31.4109	-33.6747
F09	-744.446	-26.5472	-717.906	-18.2780	-16.5109	2.143096	-20.9419	-21.0534
F10	-739.397	-63.9003	-509.627	-12.5516	-93.5867	-103.724	-292.402	-266.224
F11	-72.9665	-3.33223	-15.6676	-27.9813	-6.64587	-15.8997	-11.7828	-12.3251
F12	-0.29374	1.039155	-0.32042	0.089676	0.495613	0.91092	0.508509	0.074271
F13	-0.89315	-0.67974	-0.09274	-0.59312	-0.39024	0.397656	-0.69247	-0.02221
F14	-149.971	-7.23492	-7.11915	7.287317	-1.59707	-0.85612	-4.26079	-3.25260
F15	-10.0644	-4.21977	-7.21827	-1.58525	-5.38002	-4.47078	-5.29167	-4.28386
F16	-10.7058	-0.52211	-5.35824	1.645415	-4.30934	-2.53613	-7.85798	-3.49127
F17	-100.002	0.78153	4.605174	-69.9412	4.605172	5.199633	4.605179	4.60517
F18	-17.5979	-4.52838	-7.99157	5.212526	-5.65716	-2.93808	-5.16982	-6.36442
F19	-101.823	-12.2497	-15.8591	-11.7051	-6.29356	-7.76531	-9.23755	-14.4004
F20	-518.086	-40.8925	-32.1669	-443.747	-20.7948	-24.7139	-47.4309	-44.3142
F21	-81.3423	-68.2065	-60.3624	-3.08529	-35.7313	-21.4727	-40.0508	-32.4914

stant values which plays an imperative role in the QPSO's convergence, for that reason algorithm will be capable to reach optimal solution. Then, the contraction-expansion coefficient is adjusted to regulate particle speeds. The contraction-expansion coefficient in this proposed approach changes as follows:

$$\beta = \left(\frac{t}{T_{max}}\right)^2 \times \alpha_{ij}^t \tag{5}$$

where α_i in Eq. (6) is the fraction of the absolute difference between the fitness of the i^{th} particle and that of its neighbors divided by the square of the worse performance within the same swarm generation.

$$\alpha_{ij}^t = \frac{\left|f(x_{ij}^{(t)}) - f(x_{(i-1)j}^{(t)})\right|}{\left(f(x_{ij}^{(t)}{}_{worst})\right)^2} + rand() \tag{6}$$

Second, the QPSODE procedure obtains the particle from updated population by partitioning and combining after using two different strategies to pick up the best leading particle from subswarms. The partition mechanism uses a random selection technique to split the entire swarm into two subswarms, increasing the diversity and enabling particles escape from local minima. The application of Gaussian mutation

TABLE 2. Numerical results of standard deviation objective values for all problems.

No	QPSODE	MPSOEG	BQPSO	BDE	RMPSO	ELPSO	BPSO	GPSO
F01	7.097977	15.31565	4.595857	10.28359	2.436131	1.477287	0.773909	2.948883
F02	14.59426	1.138052	3.164314	1.47E-14	0.625425	0.83911	7.727046	6.645762
F03	16.20093	2.119836	8.665256	0.160425	1.570138	0.196711	0.144275	1.736857
F04	2.051761	0.720318	1.391712	1.459872	0.52257	0.640771	2.051736	0.965469
F05	48.02035	2.043671	15.8292	3.674423	0.684398	1.214449	5.738167	2.386606
F06	20.96134	1.570065	2.346053	2.870197	0.885138	0.641807	1.177008	0.976074
F07	31.62734	1.731472	13.78790	3.470495	3.223932	1.767729	2.073739	1.835883
F08	103.5585	8.643339	29.32355	3.385903	1.286646	2.616118	6.963894	7.020342
F09	230.5535	4.936861	230.9902	8.077669	5.628924	1.070656	2.93445	3.849772
F10	210.0313	7.154677	133.0212	9.96E-14	14.50147	25.09072	28.24139	36.89919
F11	6.868675	0.810837	4.45157	10.12418	1.749248	1.530345	2.344333	2.473453
F12	0.296427	0.011466	0.428673	1.346547	0.128272	0.072247	0.130679	0.154386
F13	0.092055	0.051025	0.261605	1.126589	0.582719	0.409988	0.339543	0.317953
F14	36.20174	1.920582	1.783684	3.20E-14	0.918602	0.849066	0.770637	1.099938
F15	0.811643	0.595124	1.458918	0.073195	1.025567	0.697916	0.932448	0.910569
F16	2.554925	1.076073	1.532744	0.225597	2.585732	0.615358	3.022508	0.945586
F17	0.643223	1.407552	0.014997	25.42995	0.073453	0.412137	0.124287	0.058304
F18	0.731886	0.860357	1.171346	1.60E-14	2.019998	0.939853	1.014228	2.217034
F19	31.23072	1.488178	5.052127	4.358042	1.629499	3.048692	2.180963	4.320327
F20	175.2905	5.493195	6.743029	133.6508	5.735513	6.815838	8.528378	7.806694
F21	12.73767	6.499063	7.193879	0.474387	7.777851	3.120927	8.808345	6.544742

TABLE 3. Numerical results of worst objective values for all problems.

No	QPSODE	MPSOEG	BQPSO	BDE	RMPSO	ELPSO	BPSO	GPSO
F01	4.359226	-11.883	2.104024	10.0010	2.336842	4.353856	4.311099	3.360958
F02	1.793335	0.243774	2.626408	-1.33053	7.650185	5.027441	5.850827	1.270213
F03	-2.96591	-0.99102	-0.06948	0.708162	-0.65137	0.192192	-0.68416	0.680391
F04	-2.06573	-2.85223	-0.78687	1.495317	0.434727	0.513171	1.423708	0.231648
F05	-1.30292	-0.75544	-2.87495	10.6001	0.006316	3.207023	2.516461	2.541211
F06	-0.99411	0.024469	1.948667	10.0000	2.751528	3.268063	2.573347	1.358371
F07	-4.17386	-2.90399	0.016261	4.099921	7.941983	-1.86464	5.180778	5.190791
F08	-13.9993	-1.61784	-5.84881	4.02331	-2.51025	4.460052	2.110826	0.323146
F09	-1.32099	-1.68097	1.772053	10.0443	10.36203	4.302315	3.325622	3.746374
F10	-30.4012	-34.2601	-35.5141	-12.5516	-7.57634	-7.13337	-22.6244	-21.9449
F11	-10.6952	7.023184	1.135755	3.10000	2.612616	-5.71783	7.937198	2.534325
F12	1.112528	1.111074	1.104032	4.003249	1.141093	1.157452	1.166741	1.157309
F13	-0.18247	-0.08421	1.746308	2.00023	2.134921	1.551017	2.236082	1.850235
F14	-2.99559	1.850922	2.503673	7.287317	2.137744	5.138684	3.595108	2.760563
F15	-1.35714	-0.30056	0.724779	-0.42677	1.702412	2.559606	0.980497	0.300942
F16	2.099891	2.290396	-0.91054	4.514865	4.145606	1.472109	2.415598	2.188304
F17	-85.6183	4.605174	4.740055	9.98322	5.187236	6.030804	6.205965	5.504721
F18	-8.13032	2.731477	-0.08081	5.212526	2.492634	2.863093	1.026928	2.541147
F19	-3.56487	-2.00375	0.528321	3.82572	2.480162	2.063349	1.989971	2.778035
F20	-1.64603	-17.8806	-1.50743	-26.5287	2.454626	5.618011	0.391215	1.996578
F21	-15.8874	-44.2741	-10.0886	-1.87589	2.143291	-4.89066	2.223952	-0.20190

to $p_{ij}^{(t)}$ is realized. According to the suggested mutation mechanism represented in Eq. (7), the particle's worst and best positions inside the subswarm are defined by $x_{ij}^{(t)}{}_{max}$ and $x_{ij}^{(t)}{}_{min}$. According to Eq. (8), the Gaussian distribution parameter 'h' grows non-linearly.

$$p_{1j}^{(t)} = P_{1j}^{(t)} + \left(x_{ij}^{(t)}{}_{max} - x_{ij}^{(t)}{}_{min}\right).Gausian(o,h) \quad (7)$$

TABLE 4. Numerical results of mean best objective values for all problems.

No	QPSODE	MPSOEG	BQPSO	BDE	RMPSO	ELPSO	BPSO	GPSO
F01	-89.2369	-56.3867	-23.1858	-29.9206	-3.7876	-1.11108	-2.82778	-4.52695
F02	-39.8181	-9.49712	-7.4488	-1.33053	3.759072	2.592996	-17.4249	-11.2714
F03	-47.1973	-6.32749	-22.7274	0.401621	-2.23543	-0.09295	-2.28864	-6.02329
F04	-9.99605	-5.42571	-5.59171	-2.49412	-1.65025	-2.23178	-3.41095	-5.97996
F05	-91.7434	-13.7094	-43.4955	5.301618	-4.45829	2.148955	-13.7472	-8.93602
F06	-53.0686	-5.91508	-2.9296	5.617177	-0.81721	2.708902	-4.05145	-3.88064
F07	-86.6459	-6.84706	-31.8001	-2.17579	-4.08976	-6.61227	-10.7228	-7.97179
F08	-173.451	-26.1492	-72.5223	1.647369	-12.0041	2.180809	-24.5364	-30.9478
F09	-505.435	-18.3312	-422.316	-15.0141	-9.33776	3.369532	-19.7132	-16.9242
F10	-601.054	-58.3988	-386.225	-12.5516	-84.9482	-70.7665	-283.511	-242.772
F11	-68.7387	-2.80489	-9.44279	-11.0661	-5.49772	-14.9999	-10.7255	-10.5224
F12	0.155469	1.043235	0.211435	1.123934	0.609755	0.958917	0.661719	0.167099
F13	-0.69801	-0.66502	0.146572	0.941833	-0.09392	0.842114	-0.62995	0.338681
F14	-134.075	-6.28317	-4.44575	7.287317	-0.99789	-0.58142	-4.17125	-1.98855
F15	-9.98309	-1.97552	-4.57641	-1.58061	-4.91609	-4.36525	-4.66793	-3.76122
F16	-1.67333	0.428987	-3.40662	1.676178	-2.63305	-2.36123	-5.28567	-2.55465
F17	-99.9696	1.542826	4.607169	-3.89382	4.614885	5.671738	4.615394	4.609501
F18	-17.5411	-3.53912	-4.99979	5.212526	-3.54289	-1.91676	-4.43972	-3.87701
F19	-73.9323	-11.4063	-9.75046	-4.00504	-5.13102	-5.65761	-8.37938	-12.7002
F20	-326.535	-39.0938	-25.3992	-349.593	-17.8155	-18.7858	-41.2071	-42.5732
F21	-53.2388	-66.2914	-48.6146	-2.73788	-32.3352	-17.483	-27.0108	-29.7914

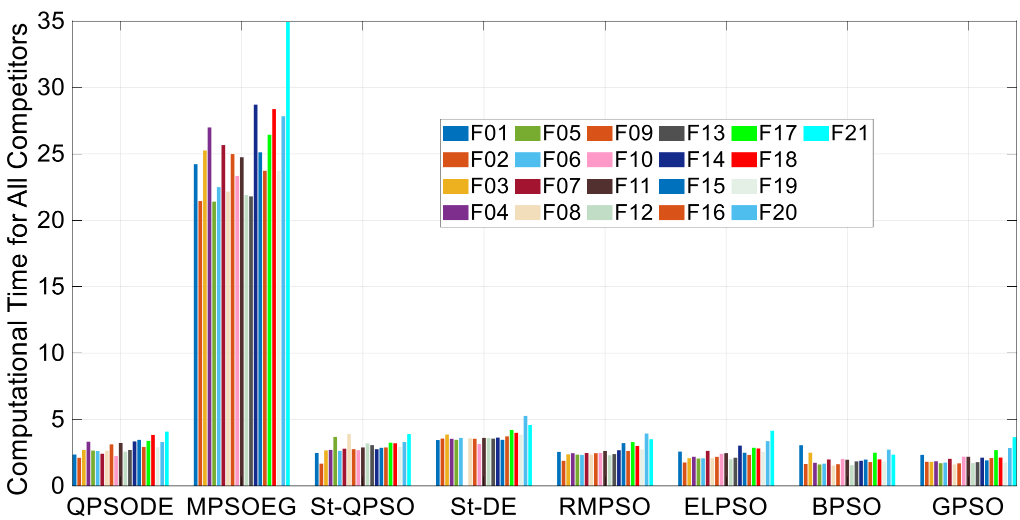

FIGURE 3. Computational time comparison of all algorithms taken during problems evaluation.

$$h = \left(\frac{t}{T_{max}}\right)^2 \tag{8}$$

To increase the searching ability of particles, the smart particle phenomenon is introduced in the second subswarm in Eq. (9), where previous best particle position is used from memory-set to better explore the given search space.

$$p_{2j}^{(t)} = \begin{cases} x_{ij}^{(t)} & \text{if } x_{ij}^{(t)} < p_{x(i-1)j}^{(t)} \\ p_{x(i-1)j}^{(t)} & \text{hold previous best} \end{cases} \tag{9}$$

After successfully compilation of Gaussian mutation and the smart particle process, we will get $p_{ij}^{(t)}$ from each swarm, and best one will be chosen according to Eq. (10), to lead the swarm for enhancing the performance of the algorithm to quickly converge to global minima.

$$G_1^{(t)} = \begin{cases} p_{1j}^{(t)} & \text{if } p_{1j}^{(t)} < p_{2j}^{(t)} \\ p_{2j}^{(t)} & \text{othewise} \end{cases} \tag{10}$$

Third. After the mutation and smart operation, the crossover of DE is applied to each pair of target vector $x_{ij}^{(t)}$ and its

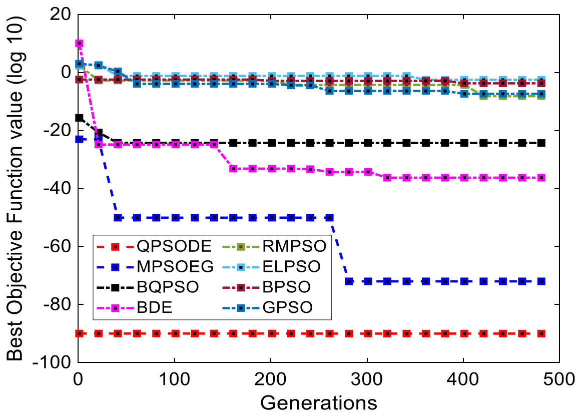

FIGURE 4. Optimization process of problem F01.

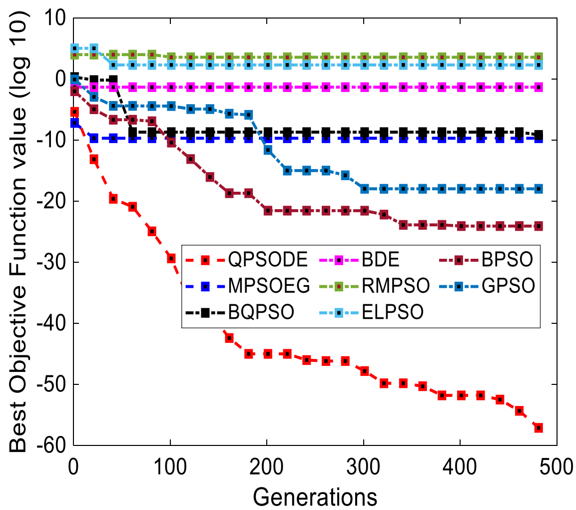

FIGURE 5. Optimization process of problem F02.

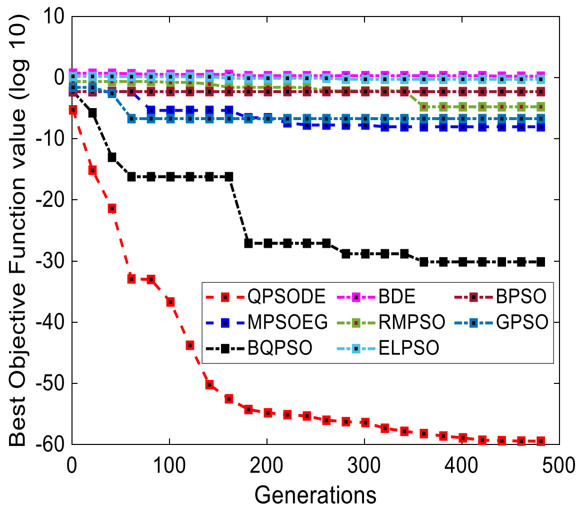

FIGURE 6. Optimization process of problem F03.

corresponding mutant vector $V_{ij}^{(t)}$ to generate a trail vector $U_i(t)$. In the basic version, DE employs the binomial crossover defined as follows:

$$U_{ij}^{(t)} = \begin{cases} V_{ij}^{(t)} & if\ (P_{boltzman} \leq Cr)\ or\ (i = i_{rand}) \\ x_{ij}^{(t)} & otherwise \end{cases} \tag{11}$$

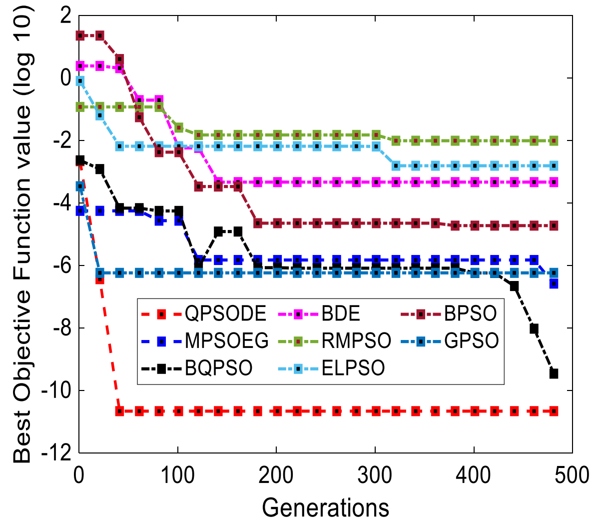

FIGURE 7. Optimization process of problem F04.

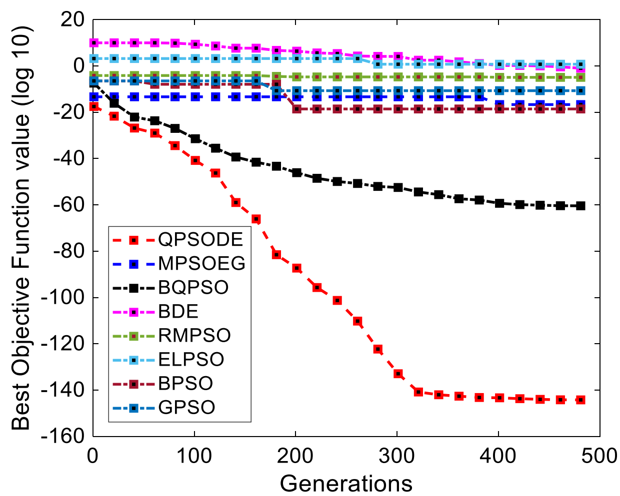

FIGURE 8. Optimization process of problem F05.

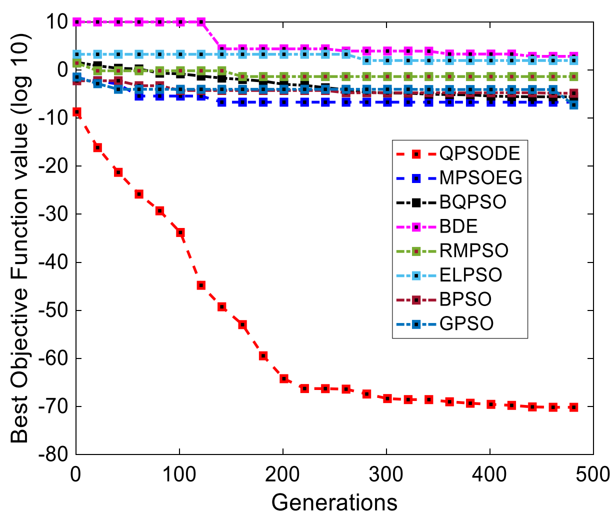

FIGURE 9. Optimization process of problem F06.

Fourth, to prevent a premature convergence systematically we introduce Boltzmann selection in this proposed work. To implement the Boltzmann selection for crossover property

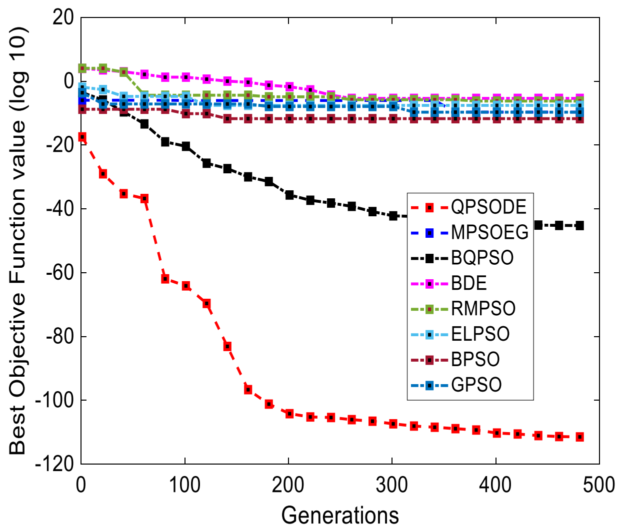

FIGURE 10. Optimization process of problem F07.

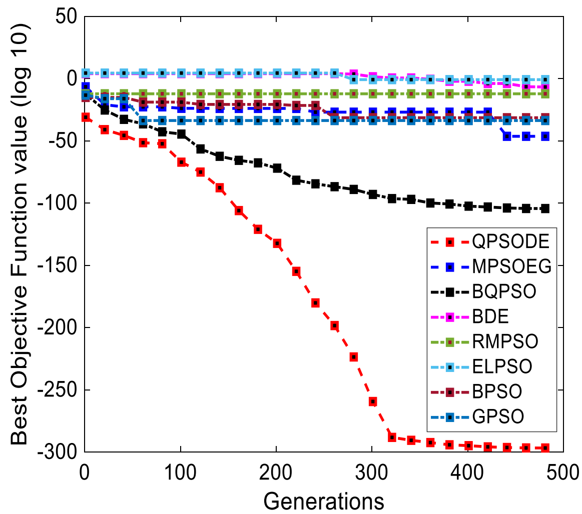

FIGURE 11. Optimization process of problem F08.

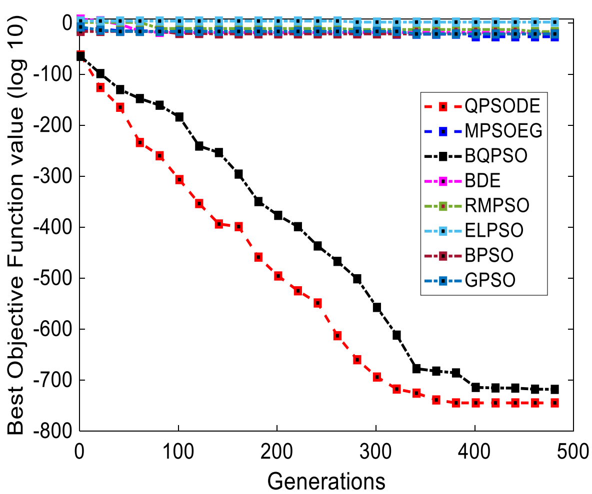

FIGURE 12. Optimization process of problem F09.

we must know that $P_{boltzman}$ uses a function of particle fitness. To find the best solution of the particle in the search

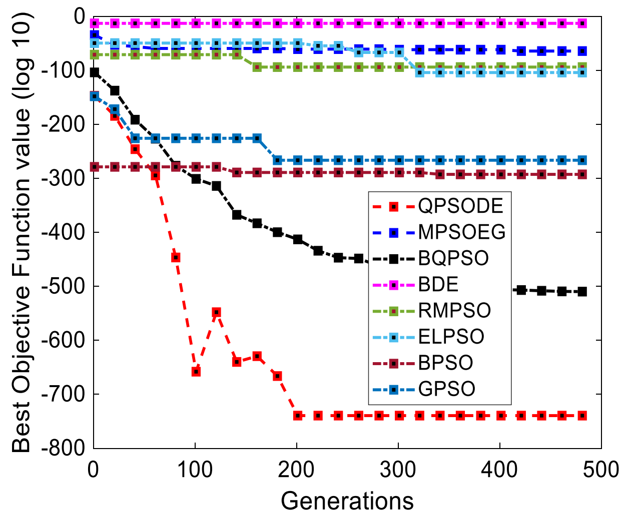

FIGURE 13. Optimization process of problem F10.

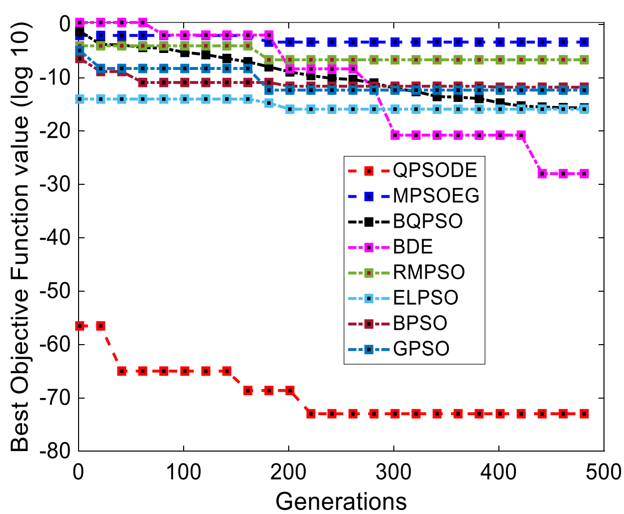

FIGURE 14. Optimization process of problem F11.

space $P_{boltzman}$ should be used as measure of the occurrence frequency for each generation.

$$P_{boltzman} = 1 - \frac{1}{D}\frac{exp(\sqrt{f(x_{ij}^{(t)})}/T)}{\sum_{i=1}^{N} exp(f(x_{ij}^{(t)}{}_{w})^{2}/T)}$$

$$T = T_0(\mu^t) \tag{12}$$

$P_{boltzman}$ determines $x_{ij}^{(t)}$'s selection probability based on fitness, $f(x_{ij}^{(t)})$, with lower fitness leading to higher probability. $\mu < 1$ is a control parameter and temperature T ranges from 100 to 200 with initial T_0.

According to the Boltzmann process the selection probability of particles will be maximized whose fitness value is minimum to enhance the performance of the proposed hybrid algorithm QPSODE.

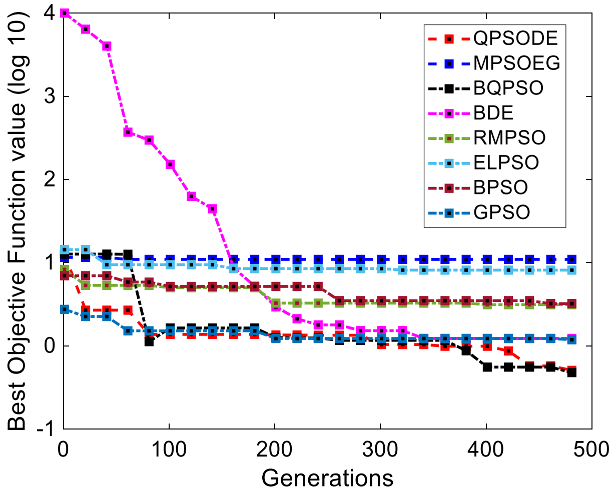

FIGURE 15. Optimization process of problem F12.

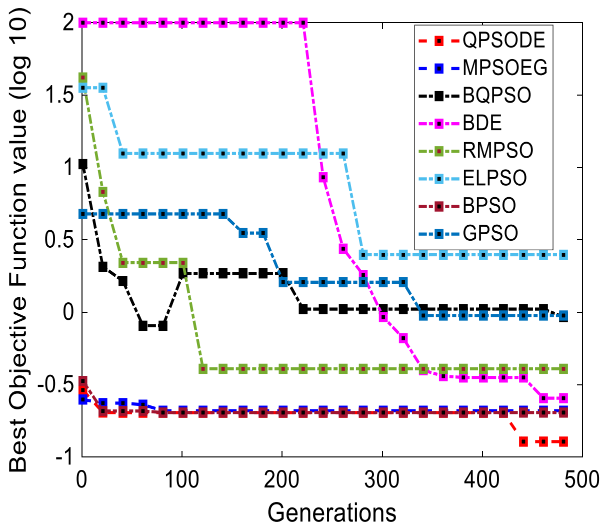

FIGURE 16. Optimization process of problem F13.

III. ANALYSIS OF PROPOSED HYBRIDIZED QPSODE ALGORITHM

To validate the performance of the proposed QPSODE hybridization strategy, twenty-one minimization benchmark problems and the engineering electromagnetic workshop problem 22 are selected as case studies, and their optimal solution is presented in this section. These benchmark problems are used in researches in different optimization fields of engineering. The aim is to assess how well our proposed QPSODE perform and also how effectively they were to implement. We have made sure that all algorithms are evaluated using the same requirements for fair comparison, a population size of 40, 500 generations, and identical dimensions for all benchmark functions, while PSO modifications use C_1 (cognitive) and C_2 (social) values of 2, while DE and QPSODE use a crossover rate of 0.9 and a mutation factor of 0.6. The performance of QPSODE was compared with several representative swarm intelligence algorithms, including classical QPSO [30], the modified particle swarm optimization with an effective guidance (MPSOEG) [42], the simple and efficient heuristic differential evolution (DE) [43], the repository and mutation based particle swarm optimization (RMPSO) [25], the enhanced leader particle swarm optimization (ELPSO) [44], the global particle swarm optimization (GPSO) [45], the standard particle swarm optimization (BPSO) [11], and the Lévy flight based QPSO in Table 6 [31].

A. NUMERICAL ANALYSIS

In this section of the paper, we will evaluate the QPSODE algorithm's performance using 21 well-known benchmark functions. These functions capture the complexity and non-linearity often encountered in real-world engineering problems, such as system optimization, parameter estimation, or design optimization. Nonlinear unimodal includes the Sphere, Chung Reynolds, Schwefel's 2.22, Hyper-Ellipsoid, and Csendes functions. Weierstrass, Styblinski-Tang, Rosenbrock, Qing, Rastrigin, Giunta, Ackley's, Cosine Mixture, Csendes, HappyCat, HGBat, Salomon, Zakharov, Griewank's, Deb 3, and Michalewicz all are nonlinear multimodal functions [46]. All of these functions are minimization problems, with a minimum value of 0. These benchmark functions are denoted as F01-F21 in our experiment results. To analyze QPSODE's performance, we perform a numerical comparison against the results of seven different methods.

Tables 1-4 display all numerical results, including the best, standard deviation, worst, and mean objective values for each algorithm. The numerical results show that, the QPSODE algorithm can determine the global optimal result for multimodal fifteen benchmark functions except F12. On the other hand, standard QPSO algorithms perform excellent on four benchmark functions, but their results are different and worse compared to the QPSODE algorithm. Figure 2 depicts the overall computational performance of all algorithms, with the contribution under various types of problems analyzed. We compared how quickly the algorithms converged in each function. The standard deviation of the values of the objective functions is used to calculate consistency. The number of successful function evaluations is used to measure reliability and the faster convergence determine effectiveness. Moreover, Fig. 3. presents a graphical comparison of the time taken by the proposed algorithm and the other algorithms used in the study. The comparison includes all benchmark functions implemented in Matlab. By examining this graph, we can assess the efficiency and performance of the proposed algorithm in terms of its execution time compared to the other methods.

The data in Table 1 shows that QPSODE has achieved a high level of success in the study, indicating effective implementation and yielding the best objective function value for benchmark problems F01, 2, 3, 4, 5, 6, 7, 8,9, 10, 11,13,14,15,17,18,19; however, according the results F12 BQPSO, F20 BDE, and F21 MPSOGE also perform well. In Table 2, QPSODE outperforms problems F02, F04, F07, F12, F14, F15, F17, and F18 in terms of standard deviation. As a result, we can conclude that QPSODE is the best of all

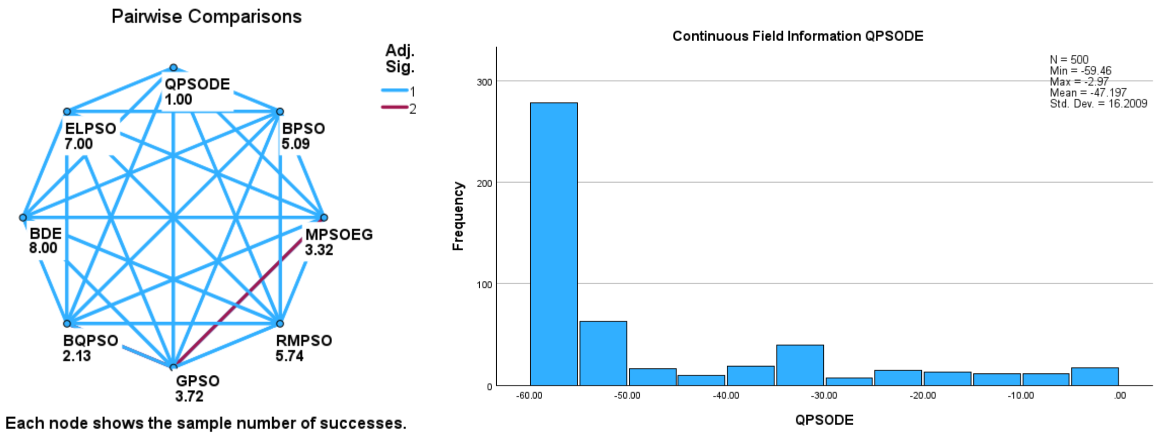

FIGURE 17. (a) Pairwise comparison of all algorithms (left). (b) distribution of the data comparison of all algorithms (right).

TABLE 5. Friedman test for QPSODE and other algorithms for all problems.

No	QPSODE	MPSOEG	BQPSO	BDE	RMPSO	ELPSO	BPSO	GPSO
F01	1	2	4	3	6	8	7	5
F02	1	3	5	6	8	7	2	4
F03	1	3	2	8	6	7	5	4
F04	1	4	3	6	8	7	5	2
F05	1	4	2	8	6	7	3	5
F06	1	2	5	8	6	7	3	4
F07	1	6	2	8	7	5	3	4
F08	1	4	2	7	6	8	5	3
F09	1	4	2	6	7	8	3	5
F10	1	7	2	8	5	6	3	4
F11	1	8	5	6	7	2	4	3
F12	2	8	1	6	4	7	5	3
F13	1	3	5	7	4	8	2	6
F14	1	2	3	8	6	7	4	5
F15	1	7	4	8	2	5	3	6
F16	5	7	2	8	4	5	1	3
F17	1	2	3	8	6	7	4	5
F18	1	5	2	8	6	7	4	3
F19	1	3	4	7	8	6	5	2
F20	1	5	6	2	8	7	4	3
F21	2	1	3	8	4	7	6	5
Average rank	1.28	4.28	3.19	6.85	5.90	6.57	3.85	4.00

algorithms. To put our proposed method to the test, we compared best objective function values graphically.

B. GRAPHICAL ANALYSIS

This section presents graphical comparisons of the performance of each algorithm for problems F01-F13 in Figs. 4-16. The assessment considers reliability, efficiency, and consistency. Additionally, graphical representations for problems F14-F25 in Figs. 21-32 are included in the Appendix section. The results reported in the figures show that the proposed QPSODE performs significantly superior for non-convex benchmark problems.

Based on the simulation results, we can conclude that by incorporating dynamic nonlinear adjustment of β, smart particle nomination in swarm distribution, and different selections, the QPSODE's convergence speed has significantly improved, which could help the algorithm perform well on both uni and multimodal functions. The findings for Chung Reynolds and Schwefel's 2.22 parts indicate that a faster convergence rate produces a more excellent local capability of QPSODE. The balance between global and local search capability on complex multimodal function optimization problems is essential to the algorithms' effectiveness. As a result, the QPSODE algorithm's output values are more accurate and fluctuate less, indicating that it has higher stability and faster convergence than other algorithms.

The adaptive control parameter selection enables the algorithm to dynamically adjust its behavior based on the problem characteristics, resulting in improved optimization performance. This combination allows and strengthen

TABLE 6. Numerical results of mean/std/best objective values for all problems.

Functions		QPSODE	MPSOEG	BQPSO	BDE	RMPSO	ELPSO	BPSO	GPSO	LQPSO[31]
F01	Best	-89.9998	-71.9998	-24.3201	-36.285	-8.19852	-2.54536	-3.76583	-7.39577	-0.025
	Std	7.097977	15.31565	4.59585	10.2835	2.43613	1.477287	0.77390	2.948883	2.112
	Mean	-89.2369	-56.3867	-23.1858	-29.9206	-3.7876	-1.11108	-2.82778	-4.52695	-2.518
F02	Best	-57.1180	-9.69436	-9.14699	-1.33053	3.56337	2.304348	-24.0788	-17.9748	0.878
	Std	14.59426	1.138052	3.16431	1.4714	0.62542	0.83911	7.72704	6.645762	0.899
	Mean	-39.8181	-9.49712	-7.4488	-1.33053	3.75907	2.592996	-17.4249	-11.2714	0.900
F03	Best	-59.4576	-8.05024	-30.1444	0.21963	-4.78311	-0.28691	-2.30371	-6.71921	-1.004
	Std	16.20093	2.119836	8.66525	0.16042	1.57013	0.196711	0.14427	1.736857	. 6.514
	Mean	-47.1973	-6.32749	-22.7274	0.40162	-2.23543	-0.09295	-2.28864	-6.02329	-1.004
F04	Best	-10.6616	-6.57958	-9.46178	-3.33283	-2.00909	-2.80958	-4.72526	-6.23707	-14.352
	Std	2.051761	0.720318	1.39171	1.45987	0.52257	0.640771	2.05173	0.965469	13.407
	Mean	-9.99605	-5.42571	-5.59171	-2.49412	-1.65025	-2.23178	-3.41095	-5.97996	-13.485
F05	Best	-144.234	-16.6789	-60.4085	-0.97899	-4.91561	0.757796	-18.5745	-10.6718	-28.133
	Std	48.02035	2.043671	15.8292	3.67442	0.68439	1.214449	5.73816	2.386606	23.593
	Mean	-91.7434	-13.7094	-43.4955	5.30161	-4.45829	2.148955	-13.7472	-8.93602	-25.093
F06	Best	-70.1677	-6.69331	-5.57310	2.83587	-1.36528	1.973708	-4.83411	-7.27401	-17.723
	Std	20.96134	1.570065	2.34605	2.87019	0.88513	0.641807	1.17700	0.976074	15.146
	Mean	-53.0686	-5.91508	-2.9296	5.61717	-0.81721	2.708902	-4.05145	-3.88064	-15.384
F07	Best	-111.463	-9.67933	-45.2239	-5.37852	-6.21008	-7.57451	-11.7292	-9.70235	4.195
	Std	31.62734	1.731472	13.7879	3.47049	3.22393	1.767729	2.07373	1.835883	2.862
	Mean	-86.6459	-6.84706	-31.8001	-2.17579	-4.08976	-6.61227	-10.7228	-7.97179	3.418

FIGURE 18. View of superconducting magnetic energy storage (SMES) [48].

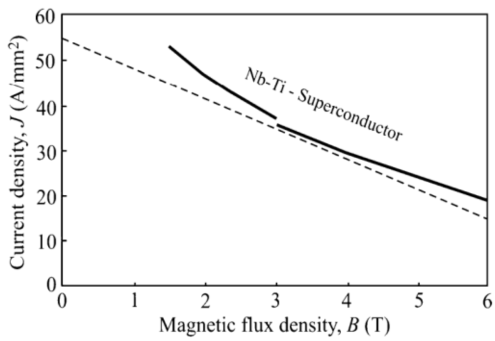

FIGURE 19. Critical curve of the superconductor is in continuous black and dotted line which represent quench condition.

the exploitation of the algorithm. Gaussian mutation phenomenon and Boltzmann process enhancement aims to provide an additional mechanism for exploration and exploitation of the search space, promoting a more comprehensive exploration of the solution landscape. From the above numerical and graphical results, QPSODE outperforms other techniques and achieved global minima more quickly as compared to other competitors.

C. STATISTICAL TEST

Statistical analyses were conducted across a range of benchmark problems to comprehensively assess the relative performance of QPSODE in comparison to other optimization approaches. In the analyses, the QPSODE method was

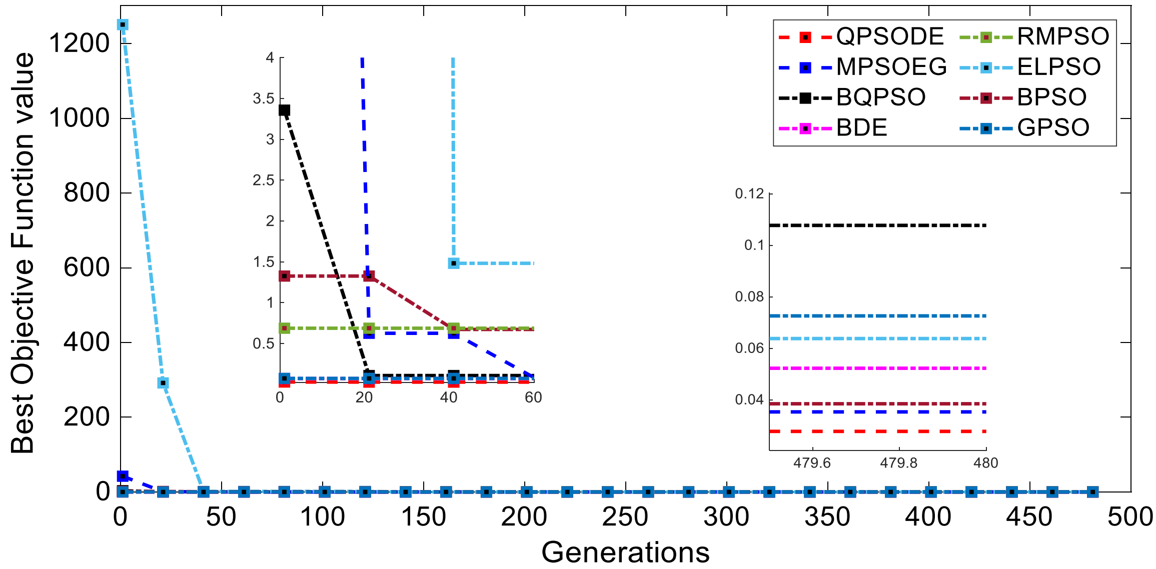

FIGURE 20. Performance comparison of different optimal algorithms on team workshop problem 22.

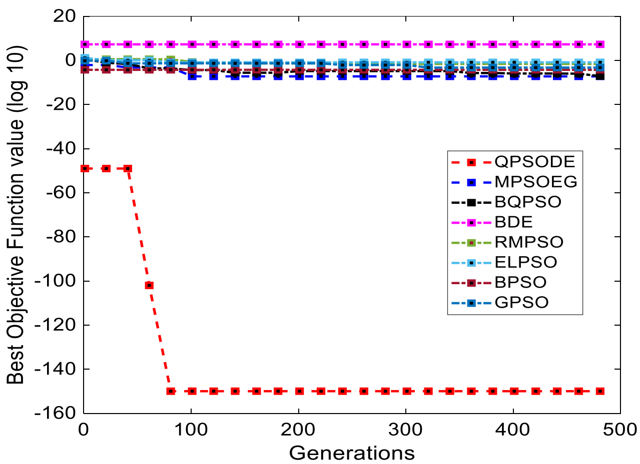

FIGURE 21. Optimization process of problem F14.

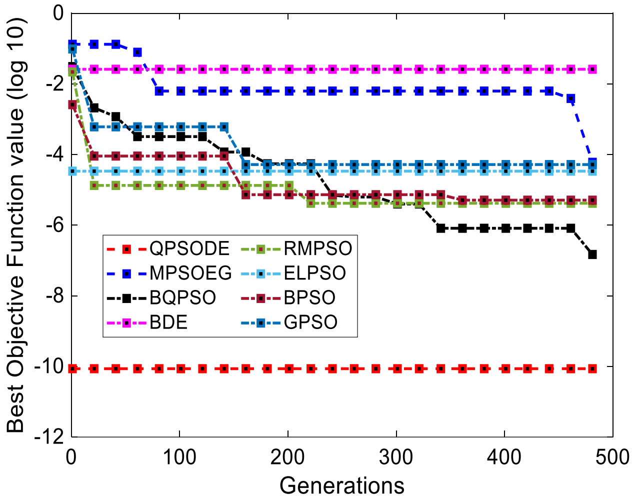

FIGURE 22. Optimization process of problem F15.

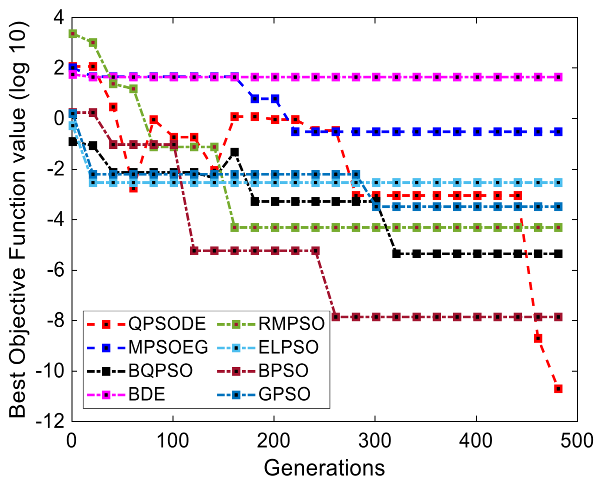

FIGURE 23. Optimization process of problem F16.

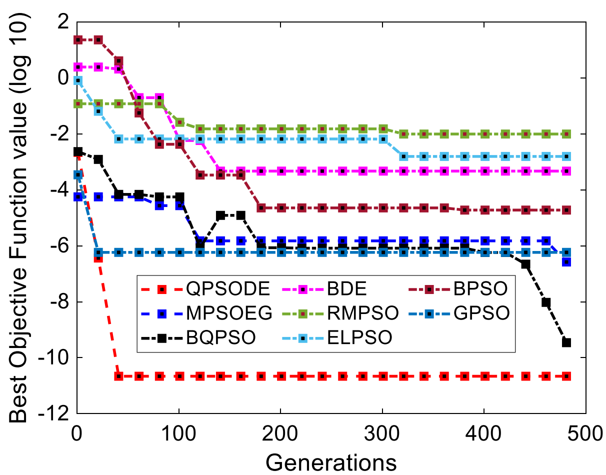

FIGURE 24. Optimization process of problem F17.

meticulously compared with each selected algorithm. The comparison results obtained using the Friedman test as the evaluation metric are presented in Table 5. By Thoroughly examining the contents of Table 5, it becomes obvious

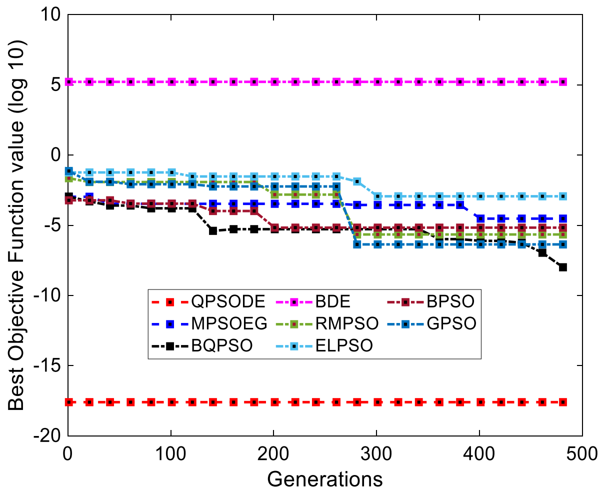

FIGURE 25. Optimization process of problem F18.

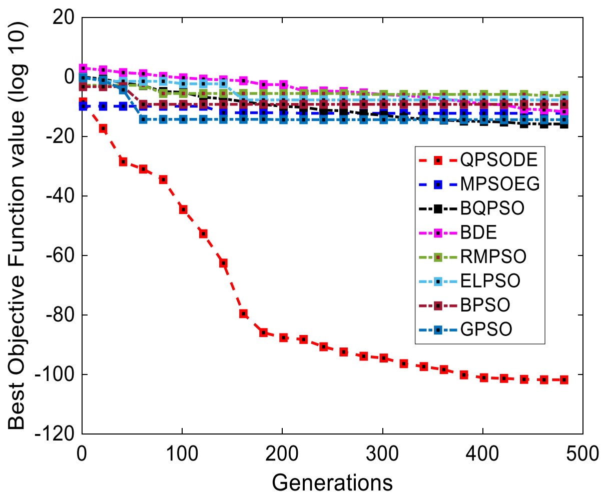

FIGURE 26. Optimization process of problem F19.

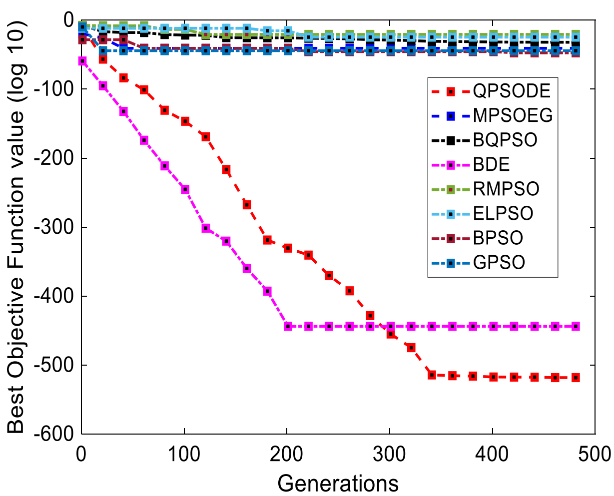

FIGURE 27. Optimization process of problem F20.

that the experimental evaluations encompassed a total of 21 benchmark functions. Noteworthy is the consistent trend observed in these assessments, which unequivocally highlights QPSODE's clear superiority over the other algorithms.

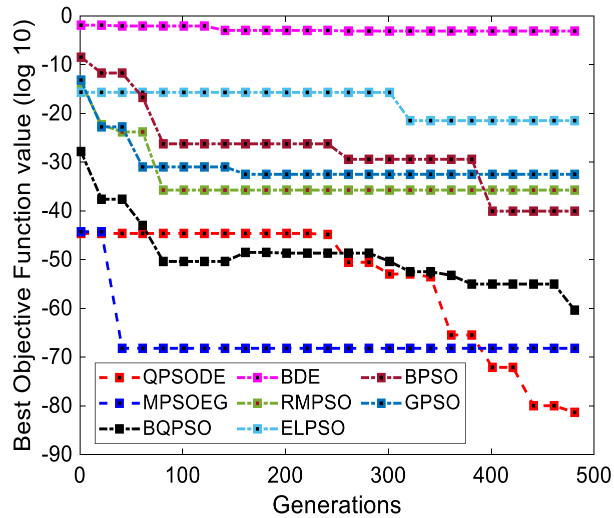

FIGURE 28. Optimization process of problem F21.

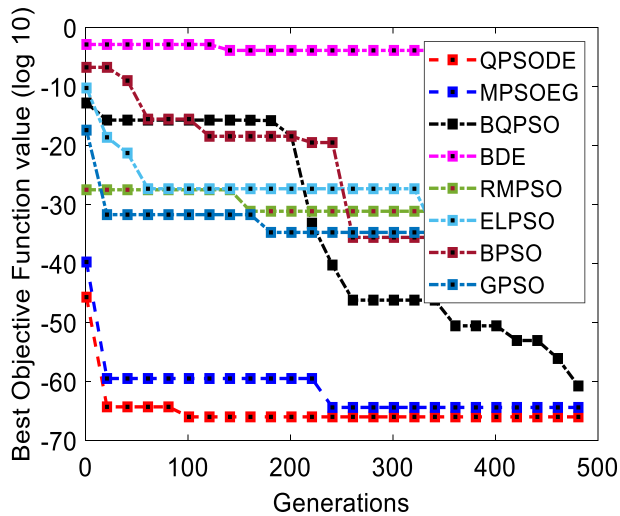

FIGURE 29. Optimization process of problem F22.

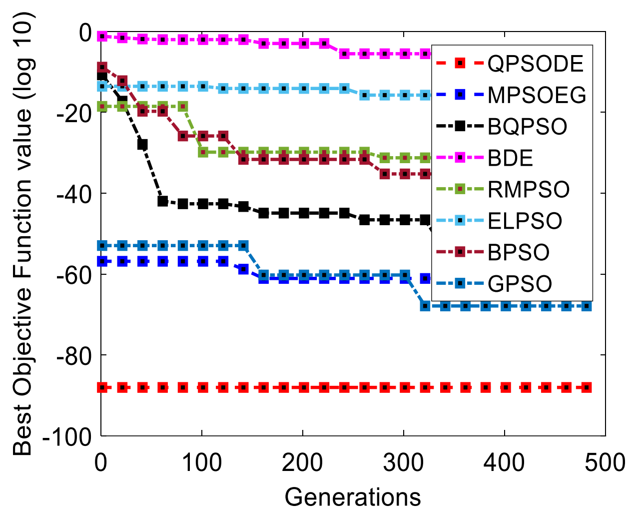

FIGURE 30. Optimization process of problem F23.

This robust evidence underscores the algorithm's significant effectiveness and statistical significance. Figure 17(a)

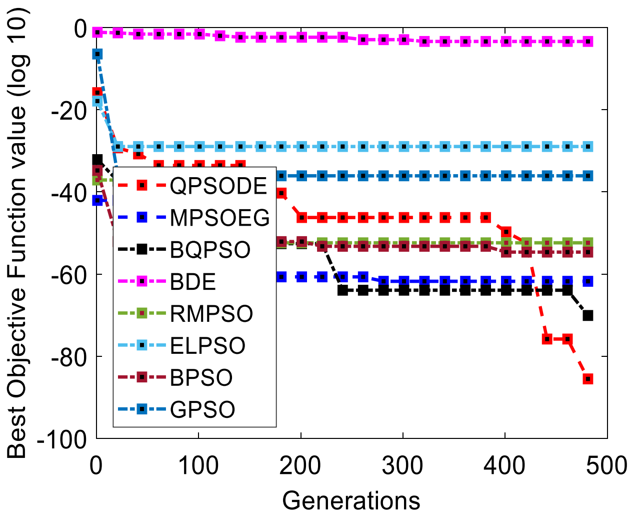

FIGURE 31. Optimization process of problem F24.

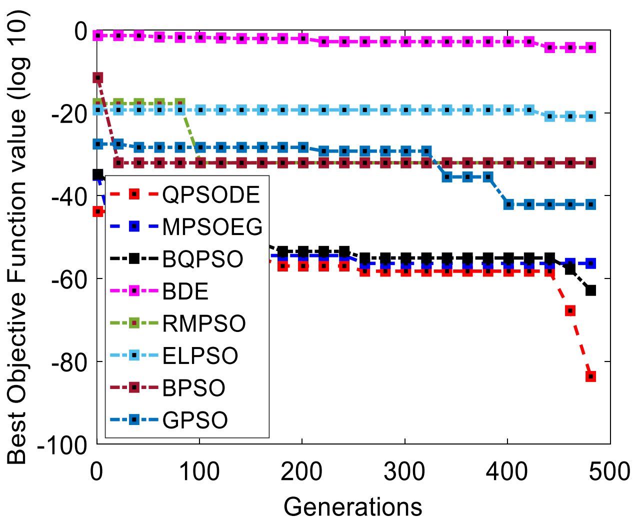

FIGURE 32. Optimization process of problem F25.

show pairwise comparison of all algorithms and Fig. 17(b) illustrates the spread of data and highlights the performance of QPSODE algorithms.

IV. APPLICATION

In general, an optimization algorithm is considered credible when it can address a variety of real-world problems. In order to validate the performance of proposed hybridized QPSODE, the superconducting magnetic energy storage (SMES) known as TEAM workshop problem 22 is then selected as a case study for engineering application. Detailing the configuration of the TEAM problem 22 in Fig. 18 like a typical storage device, the device has two concentric coils arranged so that the outer coil surrounds the inner coil.

However, the problem formulation is substantially different. The parameters of inner coil are known and well-defined. They calculate the amount of energy stored in magnetic fields and the amount of current density J_1 produced. The parameters of coil two must be changed to provide the current density J_2 required to minimize the stray field caused by J_1. Finding the SMES device's optimal configuration to retain the stored E=180MJ while minimizing the stray field to the minimum is the proposed QPSODE objective, which has been previously established in the literature [47]. The stray field's magnetic flux density, B_{stray} is examined at 22 evenly spaced points.

The objective of this work is to minimize the stray field outside to external solenoid to improve the safety issue.

$$\min f = \frac{B_{stray}^2}{B_{norm}^2} + \frac{|Energy - E_{ref}|}{E_{ref}} \tag{13}$$

$$|B_{\max}|_{1,2} \leq 4.92T$$

$$B_{stray}^2 = (\sum_{i=1}^{22} |B_{stray,i}|^2)/22 \tag{14}$$

Moreover, the electromagnetic field generated inside the solenoids must not violate specific existing situation in order to guarantee the coils' superconductivity: To maintain the superconducting property, the materials used in the coils must

$$|J_i| \leq (-6.4|B_i+56)(A/\mathrm{mm}^2)(i=1,2)$$

not exceed the bounds established by the quench condition as shown in Fig. 19. The equation yields the dotted curve. Within this application, the 2-D finite-element technique is utilized to determine the electromagnetic field and the required performance parameters, as defined in equations (13) and (14). This case study also examines the previous algorithms for comparing them to the proposed hybrid QPSODE. In Fig. 19, the final results of different algorithms are presented. Therefore, the enhanced QPSODE is clearly superior in convergence speed and minimization, shown in sub plots results on the SMES problem, proving it to be a more effective and valuable search technique for addressing engineering optimization problems.

The performance profile for engineering problem is plotted Fig. 20. As it can be seen, the BQPSO and MPSOEG approaches have demonstrated superior performances along with the QPSODE compared to traditional other optimization methods in this research study. On the other hand, QPSODE algorithm has shown particular effectiveness for multimodal and nonlinear engineering optimization problems, as well as exhibited broad applicability across various engineering optimization problem categories. These findings emphasize the potential of these novel approaches to enhance engineering optimization practices and contribute to advancements in the field.

This analysis helps in selecting appropriate optimization algorithms and strategies for structural optimization, system optimization, parameter estimation, or design optimization problems where finding the global optimum is critical for achieving efficient and robust operational designs.

V. CONCLUSION AND FUTURE WORK

In this paper, our objective is to propose a hybrid QPSODE algorithm with different features, non-linear adaptive control parameter, swarm partitioning to pertain smart and Gaussian mutation mechanism, crossover and selection of best particle using Boltzmann strategy to avoid local optima. Several experiments are conducted on an array of benchmark problems including constrained multimodal objective problems and a prototype engineering problem. The proposed algorithm can maintain the population diversity in the whole search process and enhance the convergence speed, performance, and effectively optimized multimodal and engineering problems. The number of reported results show that the proposed QPSODE class of procedures has good response for unimodal, multimodal and hybrid problems. QPSODE has features such as the proper balance of local and global search abilities and can be reliable and effective for a wide spectrum of applications in many fields of science and engineering.

The proposed hybrid QPSODE approach is significant for optimizing engineering and multimodal problems. However, its generalizability, scalability, and computational effectiveness require further study. Future research should concentrate on comprehensive testing of various benchmark functions and practical applications, as well as enhancing parameter tuning, managing challenging constraints, and conducting theoretical analysis. Comparing the algorithm with other innovative techniques can provide a better understanding of its strengths and weaknesses, potentially leading to improvements and optimizations.

APPENDIX: GRAPHICAL REPRESENTATION OF NONLINEAR MULTIMODAL BENCHMARK PROBLEMS F14-F25

See Figures 21–32, and Table 6.

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