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RESEARCH ARTICLE

A Knowledge-Based System for the Classification of Cancer Diseases in Human Body Using Hybrid Approaches Based on Bipolar Complex Fuzzy Information

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ABSTRACT Cancer is a collection of diseases in which cells separate without mechanism or order. The over-division of cells affects information recognized as a tumor. Not every tumor is considered cancer. To be identified as cancer, a tumor must damage neighboring muscles or tissues. International collaboration on cancer reporting (ICCR) information organization has been utilized to give a dominant, indication-based technique for the converging of carcinoma. The influence guarantees that the information organizations given for distinct tumor sorts have a dominant rule and style and includes all the parameters required to guide administration and prediction for separate cancers. This manuscript aims to expose certain new and valuable ideas with the help of bipolar complex fuzzy (BCF) information and power aggregation operators and evaluated their influential and dominant results and properties. Additionally, finding the most deadly and awkward sort of cancer in the world is very complicated. Thus, the main theme of the diagnosed operators is to evaluate the problem related to finding the deadliest type of cancer in human beings. Finally, we compare the final result with the certain prevailing result to enhance the worth of the evaluated approaches.

INDEX TERMS Bipolar complex fuzzy information, power averaging/geometric aggregation operators, classification of cancer diseases.

I. INTRODUCTION

Cancer indicates any one of a finite number of diseases described by the growth of unusual cells that separate unmanageable and can penetrate and damage typical physique tissue. The death rate of cancer in human beings is the second number but due to our valuable and intelligent doctors, nowadays the ratio of the death cases in cancer has reduced. Cancer-affected peoples have a lot of symptoms and all cancer symptoms differ on the sort and the stage of the cancer growth and also depend on the patient's overall health. In many cases, certain doctors have noticed that many types of cancer have no particular symptoms, based on the usefulness and feasibility of genuine cancer screening, as well as risk feature minimization (''means that the ratio of death is increasing or decreasing''). To facilitate the cancer patient, every doctor has followed the following procedure, first, they have found the stage of cancer, which means that the concerned doctor has to evaluate the size, location, and behavior of the cancer, then based on the above analysis doctor has suggested the treatment of cancer. Different types of cancer have needed different types of treatment, and most doctors have used the following strategy for the treatment of cancer patients chemotherapy, surgery, and radiation therapy. Certain cancer patients have used different therapies for their treatment. Most people have affected because of non-stop

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smoking addiction, eating an unhealthy diet, working in front of the sun and harmful ultraviolet rays from the sun can increase the risk of skin cancer, not doing exercise, increasing/decreasing weights without any diet, drinking alcohol, not properly checkup of cancer, etc. The main symptoms in a cancer patient are the form like pain, fatigue, complication in breathing, nausea, informal weight loss, brain dilemma, chemical changes in the skin, etc.

Cancer is an awkward and complex collection of diseases for certain possible reasons. Many intellectuals have analyzed the different forms of cancer using classical information and found a feasible and suitable solution. For analyzing the solution to cancer in the human body, every expert or doctor has needed to collect genuine data from many hospitals which is very complex work. But one of the most important and valuable questions, if we find the solution for the cancer patient by using fuzzy set theory instead of classical information is very complicated because for this it is necessary to develop someone the idea of fuzzy sets theory. Such sort of problem was faced by Zadeh and in 1965, Zadeh [1] invented the valuable idea of the fuzzy set (FS) by extending the 0, 1 "range of classical set" into [0, 1] "range of FS theory". Fuzzy logic is a valuable approach to variable evaluation that permits many possible supporting values to be evaluated through the same variable. Information cannot handle from the classical set; the theory of fuzzy set can easily evaluate them. Ohlan and Ohlan [2] investigated a bibliometric overview of FS and Abdullah et al. [3] discussed a fractional fuzzy decision-theoretical rough set. FS theory has been feasibly integrated into certain decision-making techniques to manage ambiguity and vague information [4], [5]. Various researchers worked in the modified structures of FS theory such as Yang et al. [6] discussed belief and plausibility measures on intuitionistic FS and developed the belief-plausibility TOPSIS approach, and Abid et al. [7] investigated similarity measures under the setting of T-spherical FS. However, the modeling techniques of FSs are restricted to evaluating ambiguity and awkward data in which two or massive information of ambiguity occurred continuously on two different sides. Bipolar FS (BFS) [8] gives a valuable definition of ambiguity and therefore permits experts to resolve it in a broader field because of the ambiguities that occurred in the dilemmas evaluated and the deficiencies of the decision-making procedure. The BFS has positive and negative supporting grades, where the value of the positive supporting grade is belonging to [0, 1] and the value of the negative supporting grade is contained in $[-1, 0]$. In this scenario, BFSs include massive data than FSs, which have more value. Certain scholars introduced various structures in the environment of BFS, like Jana et al. [9], Jana et al. [10], Yager and Rybalov [11], and Mesiar et al. [12] investigated various AOs for BFS, Jana [13] propounded MABAC approach for extending BFS, Singh [14] investigated bipolar fuzzy (BF) attribute implications, and Guterrez et al. [15] introduced BF measures. Also, certain scholars generalized the theory of BFS such as Naz et al. [16] described 2-tuple linguistic BF, Liu et al. [17] defined bipolar hesitant fuzzy linguistic, Mandal [18], propounded bipolar Pythagorean FS, Riaz and Tehrim [19] investigated geometric AOs for cubic BFS (CBFS), Riaz et al. [20] studied Einstein averaging AOs for CBFS. Mahmood [21] established a T-bipolar soft set (SS), Riaz and Tehrim [22] investigated CBFS and also Riaz and Tehrim [23] gave DM approach for CBFS.

The existing information explained above has certain limitations because the function contained in BFS managed with one-dimension information. None of these techniques are suitable to express the phase term and because of these issues, experts have lost a lot of data during the decision-making procedure. A well-known organization has done research on biometric information in many places and the researchers have decided to update the data time by time (periodicity). To engage periodic terms in the term of supporting grade, the theory of complex FS (CFS) was diagnosed by Ramot et al. [24]. After successfully constructing the theory of CFS, certain people have engaged in the field of certain circumstances such as Mahmood et al. [25] discussed the interdependency of the complex fuzzy neighborhood, Lui et al. [26] and Bi et al. [27] propounded entropy measures for CFS, and Dai et al. [28] and Song et al. [29] presented distance measures among interval-valued CFS, Alolaivan et al. [30] investigated complex fuzzy (CF) subgroups, and Ahsan et al. [31] introduced CF hypersoft mapping. Yang et al. [32] deduced partitioned Bonferroni mean operators in the setting of complex q-rung orthopair uncertain linguistic set. However, the modeling techniques of CFSs are restricted to evaluating ambiguity and awkward data in which two or massive information of ambiguity occurred continuously on two different sides. Bipolar CFS (BCFS) [33] gives a valuable definition of ambiguity and therefore permits experts to resolve it in a broader field because of the ambiguities that occurred in the dilemmas evaluated and the deficiencies of the decision-making procedure. The BCFS has positive and negative supporting grades, where the real and imaginary values of positive supporting grades is belonging to [0, 1] and the real and imaginary values of negative supporting grades are contained in $[-1, 0]$. In this scenario, BCFSs include massive data than FSs, CFSs, and BFS, which have value and consideration by certain scholars. We can describe that the BCFS is a valuable and appropriate tool for genuine decision-making requests [34], [35].

In many cases, certain doctors have noticed that many types of cancer have no particular symptoms, based on the usefulness and feasibility of genuine cancer screening, as well as risk feature minimization ("means that the ratio of death is increasing or decreasing"). To facilitate the cancer patient, every doctor has to follow the following procedure, first, they have found the stage of cancer, which means that the concerned doctor has evaluated the size, location, and behavior of the cancer, then based on the above analysis doctor has suggested the treatment of cancer. Different types of cancer have needed different types of treatment, and most doctors have used the following strategy for the treatment of cancer patients chemotherapy, surgery, and radiation therapy. Certain cancer patients have used different therapies for their treatment. Certain people have working one cancer by using FSs [36], [37], BFSs [38], [39], and CFSs [40]. Cancer is a collection of diseases in which cells separate without mechanism or order. The over-division of cells affects information recognized as a tumor. Not every tumor is considered cancerous. To be identified as cancer, a tumor must damage neighboring muscles or tissues. ICCR information organizations have been utilized to give a dominant, indication-based technique for the converging of carcinoma. The influence guarantees that the information organizations given for distinct tumor sorts have a dominant rule and style and include all the parameters required to guide administration and prediction for separate cancers. This manuscript aims to describe the following themes:

1. To expose the BCF power averaging (BCFPA), BCF power weighted averaging (BCFPWA), BCF power ordered weighted averaging (BCFPOWA), BCF power hybrid averaging (BCFPHA), BCF power geometric (BCFPG), BCF power weighted geometric (BCFPWG), BCF power ordered weighted geometric (BCFPOWG), BCF power hybrid geometric (BCFPHG) operators and evaluated their influential and dominant results and properties (''idempotency, boundedness, and monotonicity'').
2. To find the most deadly and awkward sort of cancer in the world is very complicated. The main theme of the diagnosed operators is to evaluate the problem related to finding the deadliest type of cancer (''using four important types of cancer and their related four useful and initial symptoms'') in human beings.
3. To compare the final result with the certain prevailing result is to enhance the worth of the evaluated approaches.

A small review of the constructed analysis: In section II, we provide a brief review of cancer and its types. Further, we recalled the theory of BCF information and their related theory. In section III, we exposed the BCFPA, BCFPWA, BCFPOWA, BCFPHA, BCFPG, BCFPWG, BCFPOWG, and BCFPHG operators and evaluated their influential and dominant results and properties (''idempotency, boundedness, and monotonicity''). In section IV, we find the most deadly and awkward sort of cancer in the world is very complicated. The main theme of the diagnosed operators is to evaluate the problem related to finding the deadliest type of cancer (''using four important types of cancer and their related four useful and initial symptoms'') in human beings. In section V, we try to compare the final result with the certain prevailing result to enhance the worth of the evaluated approaches. In section VI, we explained the concluding remarks.

II. PRELIMINARIES

Cancer is an awkward and complex collection of diseases for certain possible reasons. Many intellectuals have analyzed the different forms of cancer using classical information and found a feasible and suitable solution. For analyzing the solution to cancer in the human body, every expert or doctor has needed to collect genuine data from many hospitals which is very complex work. But one of the most important and valuable questions, if we find the solution for the cancer patient by using fuzzy set theory instead of classical information is very complicated because for this it is necessary to develop someone the idea of fuzzy sets theory. Cancer is a collection of diseases in which cells separate without mechanism or order. The over-division of cells affects information recognized as a tumor. Not every tumor is considered cancerous. To be identified as cancer, a tumor must damage neighboring muscles or tissues. ICCR information organization has been utilized to give a dominant, indication-based technique for the converging of carcinoma. The influence guarantees that the information organizations given for distinct tumor sorts have a dominant rule and style and include all the parameters required to guide administration and prediction for separate cancers. Figure 1 shows the overall ten types of cancer.

Cancer indicates any one of a finite number of diseases described by the growth of unusual cells that separate unmanageable and can penetrate and damage typical physique tissue. The death rates of cancer in human beings are in the second number but due to our valuable and intelligent doctors, nowadays the ratio of the death cases in cancer has reduced. Cancer-affected peoples have a lot of symptoms and all cancer symptoms differ on the sort and the stage of the cancer growth and also depend on the patient's overall health. In many cases, certain doctors have noticed that many types of cancer have no particular symptoms, based on the usefulness and feasibility of genuine cancer screening, as well as risk feature minimization (''means that the ratio of death is increasing or decreasing''). To facilitate the cancer patient, every doctor has followed the following procedure, first, they have found the stage of cancer, which means that the concerned doctor has evaluated the size, location, and behavior of cancer, then based on the above analysis doctor has suggested the treatment of cancer. Different types of cancer have needed different types of treatment, and most doctors have used the following strategy for the treatment of cancer patients chemotherapy, surgery, and radiation therapy. Certain cancer patients have used different therapies for their treatment. Most people have affected because of non-stop smoking addiction, eating an unhealthy diet, working in front of the sun and harmful ultraviolet rays from the sun can increase the risk of skin cancer, not doing exercise, increasing/decreasing weights without any diet, drinking alcohol, not properly checkup of cancer, etc. The main symptoms in a cancer patient are the form like pain, fatigue, a complication in breathing, nausea, informal weight loss, brain dilemma, chemical changes in the skin, etc. In this theory, we focused on the main four types of cancer and try to evaluate the most critical type in all of them. Further, we recalled the theory of BCF information and their related theory and power aggregation operator.

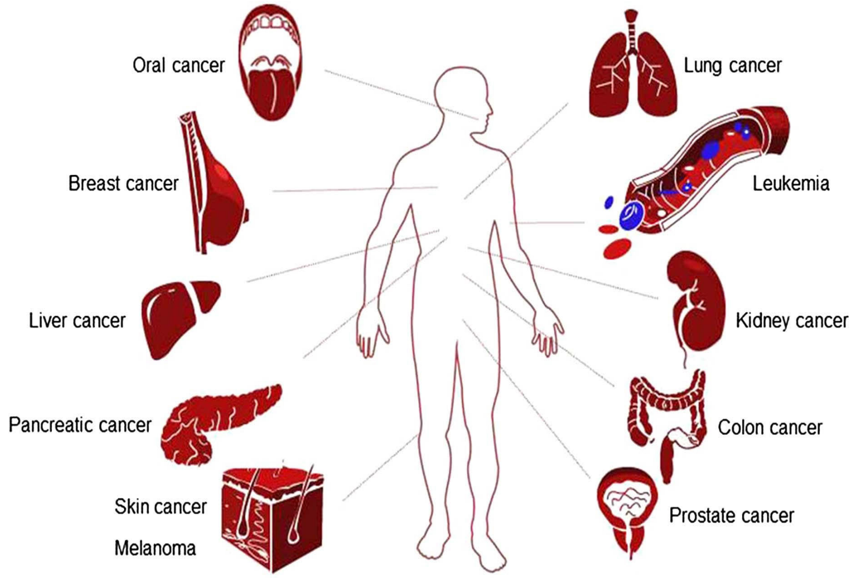

FIGURE 1. **Represented the framework of cancer in human body.**

Definition 1 [33]: A mathematical form of set:

$$\check{D} = \left\{\left(\ell \mathring{A}_{P-\check{D}}(\ell),\ \mathring{A}_{N-\check{D}}(\ell)\right) | \ell \in \mathfrak{L}\right\} \tag{1}$$

Expressed the BCF set with positive and negative supporting grade $\mathring{A}_{P-\check{D}}(\ell) = \mathring{A}_{RP-\check{D}}(\ell) + \iota\, \mathring{A}_{IP-\check{D}}(\ell)$ and $\mathring{A}_{N-\check{D}}(\ell) = \mathring{A}_{RN-\check{D}}(\ell) + \iota\, \mathring{A}_{IN-\check{D}}(\ell)$ with $\mathring{A}_{RP-\check{D}}(\ell), \mathring{A}_{IP-\check{D}}(\ell) \in [0,\ 1]$ and $\mathring{A}_{RN-\check{D}}(\ell), \mathring{A}_{IN-\check{D}}(\ell) \in [-1,\ 0]$. Further, $\check{D} = \left(\mathring{A}_{P-\check{D}}(\ell), \mathring{A}_{N-\check{D}}(\ell)\right) = \left(\mathring{A}_{RP-\check{D}}(\ell) + \iota \mathring{A}_{IP-\check{D}}(\ell),\ \mathring{A}_{RN-\check{D}}(\ell) + \iota\, \mathring{A}_{IN-\check{D}}(\ell)\right)$, explained the BCF numbers (BCFNs).

Definition 2 [34]: Using the BCFNs, the set:

$$\dot{S}_{SF}\left(\check{D}\right) = \frac{1}{4}\begin{pmatrix} 2 + \mathring{A}_{RP-\check{D}}(\ell) + \mathring{A}_{IP-\check{D}}(\ell) \\ + \mathring{A}_{RN-\check{D}}(\ell) + \mathring{A}_{IN-\check{D}}(\ell) \end{pmatrix}, \quad \dot{S}_{SF} \in [0,\ 1] \tag{2}$$

Expressed the score value and the set:

$$\dot{H}_{AF}\left(\check{D}\right) = \frac{\mathring{A}_{RP-\check{D}}(\ell) + \mathring{A}_{IP-\check{D}}(\ell) - \mathring{A}_{RN-\check{D}}(\ell) - \mathring{A}_{IN-\check{D}}(\ell)}{4}, \quad \dot{H}_{AF} \in [0,\ 1] \tag{3}$$

Expressed the accuracy value. Further, in the availability of Eq. (2) and Eq. (3), we obtained: $\dot{S}_{SF}\left(\check{D}_1\right) < \dot{S}_{SF}\left(\check{D}_2\right)$, then $\check{D}_1 < \check{D}_2$; if $\dot{S}_{SF}\left(\check{D}_1\right) > \dot{S}_{SF}\left(\check{D}_2\right)$, then $\check{D}_1 > \check{D}_2$; if $\dot{S}_{SF}\left(\check{D}_1\right) = \dot{S}_{SF}\left(\check{D}_1\right)$, then if $\dot{H}_{AF}\left(\check{D}_1\right) < \dot{H}_{AF}\left(\check{D}_2\right)$, then $\check{D}_1 < \check{D}_2$; if $\dot{H}_{AF}\left(\check{D}_1\right) > \dot{H}_{AF}\left(\check{D}_2\right)$, then $\check{D}_1 > \check{D}_2$; if $\dot{H}_{AF}\left(\check{D}_1\right) = \dot{H}_{AF}\left(\check{D}_2\right)$, then $\check{D}_1 = \check{D}_2$. For $\check{D}_1 = \left(\mathring{A}_{P-\check{D}_1}, \mathring{A}_{N-\check{D}_1}\right) = \left(\mathring{A}_{RP-\check{D}_1} + \iota \mathring{A}_{IP-\check{D}_1}, \mathring{A}_{RN-\check{D}_1} + \iota\, \mathring{A}_{IN-\check{D}_1}\right)$ and $\check{D}_2 = \left(\mathring{A}_{P-\check{D}_2}, \mathring{A}_{N-\check{D}_2}\right) = \left(\mathring{A}_{RP-\check{D}_2} + \iota \mathring{A}_{IP-\check{D}_2}, \mathring{A}_{RN-\check{D}_2} + \iota\, \mathring{A}_{IN-\check{D}_2}\right)$ with $\lambda > 0$, we have

1.
$$\check{D}_1 \oplus \check{D}_2 = \begin{pmatrix} \mathring{A}_{RP-\check{D}_1} + \mathring{A}_{RP-\check{D}_2} - \mathring{A}_{RP-\check{D}_1}\mathring{A}_{RP-\check{D}_2} \\ +\iota\left(\mathring{A}_{IP-\check{D}_1} + \mathring{A}_{RP-\check{D}_2} - \mathring{A}_{IP-\check{D}_1}\mathring{A}_{IP-\check{D}_2}\right), \\ -\left(\mathring{A}_{RN-\check{D}_1}\mathring{A}_{RN-\check{D}_2}\right) + \iota\left(-\left(\mathring{A}_{IN-\check{D}_1}\mathring{A}_{IN-\check{D}_2}\right)\right) \end{pmatrix}$$

2.
$$\check{D}_1 \otimes \check{D}_2 = \begin{pmatrix} \mathring{A}_{RP-\check{D}_1}\mathring{A}_{RP-\check{D}_2} + \iota\, \mathring{A}_{IP-\check{D}_1}\mathring{A}_{IP-\check{D}_2}, \\ \mathring{A}_{RN-\check{D}_1} + \mathring{A}_{RN-\check{D}_2}\mathring{A}_{RN-\check{D}_1} + \mathring{A}_{RN-\check{D}_2} \\ +\iota\left(\mathring{A}_{IN-\check{D}_1} + \mathring{A}_{IN-\check{D}_2}\mathring{A}_{IN-\check{D}_1} + \mathring{A}_{IN-\check{D}_2}\right) \end{pmatrix}$$

3.
$$\lambda\check{D}_1 = \begin{pmatrix} 1 - \left(1 - \mathring{A}_{RP-\check{D}_1}\right)^{\lambda} \\ +\iota\left(1 - \left(1 - \mathring{A}_{IP-\check{D}_1}\right)^{\lambda}\right), \\ -\left|\mathring{A}_{RN-\check{D}_1}\right|^{\lambda} + \iota\left(-\left|\mathring{A}_{IN-\check{D}_1}\right|^{\lambda}\right) \end{pmatrix}$$

4.
$$\check{D}_1^{\lambda} = \left(\begin{pmatrix} \left(\mathring{A}_{RP-\check{D}_1}\right)^{\lambda} + \iota\left(\mathring{A}_{IP-\check{D}_1}\right)^{\lambda}, \\ -1 + \left(1 + \mathring{A}_{RN-\check{D}_1}\right)^{\lambda} \\ +\iota\left(-1 + \left(1 + \mathring{A}_{IN-\check{D}_1}\right)^{\lambda}\right) \end{pmatrix}\right)$$

Definition 3 [41]: For any positive collection of numbers, the structure:

$$PA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \sum_{\check{j}=1}^{n} \frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}\check{D}_{\check{j}} \tag{4}$$

Expressed the power aggregation operator with $\mathrm{T}\left(\check{D}_{\check{j}}\right) = \sum_{\substack{\check{j}=1 \\ \check{j}\neq\varsigma}}^{n} Sup\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right)$ and $Sup\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right)$ signifies the support for $\check{D}_{\check{j}}$ from $\check{D}_{\varsigma}$ along with the following conditions:

1. $Sup\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right) \in [0, 1]$;
2. $Sup\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right) = Sup\left(\check{D}_{\varsigma}, \check{D}_{\check{j}}\right)$;

$Sup\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right) \geq Sup\left(\check{D}_p, \check{D}_q\right)$ if $\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right) < \left(\check{D}_p, \check{D}_q\right)$, d is a distance measure.

III. POWER AVERAGING/GEOMETRIC AGGREGATION OPERATORS FOR BCF INFORMATION

This section aims to expose certain new and valuable ideas with the help of BCF information and power aggregation operators, called BCFPA, BCFPWA, BCFPOWA, BCFPHA, BCFPG, BCFPWG, BCFPOWG, BCFPHG operators, and evaluated their influential and dominant results and properties ("idempotency, boundedness, and monotonicity"). Further, $\lambda = (\lambda_1, \lambda_1, , .. \lambda_n)^T$ be the weight vector (WV) with $\lambda_{\check{j}} \in [0, 1]$ for all $\check{j}$ and $\sum_{\check{j}=1}^{n} \lambda_{\check{j}} = 1$ in this manuscript.

Definition 4: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1, 2, \ldots, n)$, the BCFPA operator is offered as

$$BCFPA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \bigoplus_{\check{j}=1}^{n} \frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}\check{D}_{\check{j}} \tag{5}$$

where $\mathrm{T}\left(\check{D}_{\check{j}}\right) = \sum_{\substack{\check{j}=1 \\ \check{j}\neq\varsigma}}^{n} Sup\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right)$ and $Sup\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right)$ signifies the support for $\check{D}_{\check{j}}$ from $\check{D}_{\varsigma}$ along with the following conditions:

1. $Sup\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right) \in [0, 1]$;
2. $Sup\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right) = Sup\left(\check{D}_{\varsigma}, \check{D}_{\check{j}}\right)$;
3. $Sup\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right) \geq Sup\left(\check{D}_p, \check{D}_q\right)$ if $\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right) < \left(\check{D}_p, \check{D}_q\right)$, d is any distance measure among BCFSs.

Theorem 1: The outcome achieved by Eq (5), is a BCFN and

$$BCFPA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \begin{pmatrix} 1-\prod_{\check{j}=1}^{n}\left(1-\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}(\check{D}_{\check{j}})}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}} \\ +\iota\left(1-\prod_{\check{j}=1}^{n}\left(1-\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}(\check{D}_{\check{j}})}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}}\right), \\ -\prod_{\check{j}=1}^{n}\left|\mathring{A}_{RN-\check{D}_{\check{j}}}\right|^{\frac{1+\mathrm{T}(\check{D}_{\check{j}})}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}} \\ +\iota\left(-\prod_{\check{j}=1}^{n}\left|\mathring{A}_{IN-\check{D}_{\check{j}}}\right|^{\frac{1+\mathrm{T}(\check{D}_{\check{j}})}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}}\right) \end{pmatrix} \tag{6}$$

Proof: For easiness, let $\frac{1+\mathrm{T}(\check{D}_{\check{j}})}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)} = \hat{Y}$. Now by employing the operational laws in Def (1), we have

$$\hat{Y}_1\check{D}_1 = \begin{pmatrix} 1-\left(1-\mathring{A}_{RP-\check{D}_1}\right)^{\hat{Y}_1} + \iota\left(1-\left(1-\mathring{A}_{IP-\check{D}_1}\right)^{\hat{Y}_1}\right), \\ -\left|\mathring{A}_{RN-\check{D}_1}\right|^{\hat{Y}_1} + \iota\left(-\left|\mathring{A}_{IN-\check{D}_1}\right|^{\hat{Y}_1}\right) \end{pmatrix}$$

$$\hat{Y}_2\check{D}_2 = \begin{pmatrix} 1-\left(1-\mathring{A}_{RP-\check{D}_2}\right)^{\hat{Y}_2} + \iota\left(1-\left(1-\mathring{A}_{IP-\check{D}_2}\right)^{\hat{Y}_2}\right), \\ -\left|\mathring{A}_{RN-\check{D}_2}\right|^{\hat{Y}_2} + \iota\left(-\left|\mathring{A}_{IN-\check{D}_2}\right|^{\hat{Y}_2}\right) \end{pmatrix}$$

$$\hat{Y}_1\check{D}_1 \oplus \hat{Y}_2\check{D}_2 = \begin{pmatrix} 1-\left(1-\mathring{A}_{RP-\check{D}_1}\right)^{\hat{Y}_1}\left(1-\mathring{A}_{RP-\check{D}_2}\right)^{\hat{Y}_2} \\ +\iota\left(1-\left(1-\mathring{A}_{IP-\check{D}_1}\right)^{\hat{Y}_1}\left(1-\mathring{A}_{IP-\check{D}_2}\right)^{\hat{Y}_2}\right) \\ -\left(\left(-\left|\mathring{A}_{RN-\check{D}_{\check{j}}}\right|^{\hat{Y}_2}\right)\left(-\left|\mathring{A}_{IN-\check{D}_{\check{j}}}\right|^{\hat{Y}_2}\right)\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1-\prod_{\check{j}=1}^{2}\left(1-\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(1-\prod_{\check{j}=1}^{2}\left(1-\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right), \\ -\prod_{\check{j}=1}^{2}\left|\mathring{A}_{RN-\check{D}_{\check{j}}}\right|^{\hat{Y}_{\check{j}}} + \iota\left(-\prod_{\check{j}=1}^{2}\left|\mathring{A}_{IN-\check{D}_{\check{j}}}\right|^{\hat{Y}_{\check{j}}}\right) \end{pmatrix}$$

$\Rightarrow$ Eq (6) holds for $n = 2$. Next, assume that Eq (6) is true for $n = w$ i.e.

$$BCFPA\left(\check{D}_1, \check{D}_2, .., \check{D}_w\right)$$

$$= \begin{pmatrix} 1-\prod_{\check{j}=1}^{w}\left(1-\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(1-\prod_{\check{j}=1}^{w}\left(1-\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right), \\ -\prod_{\check{j}=1}^{w}\left|\mathring{A}_{RN-\check{D}_{\check{j}}}\right|^{\hat{Y}_{\check{j}}}+\iota\left(-\prod_{\check{j}=1}^{w}\left|\mathring{A}_{IN-\check{D}_{\check{j}}}\right|^{\hat{Y}_{\check{j}}}\right) \end{pmatrix}$$

For $n = w + 1$, we have

$$BCFPA\left(\check{D}_1,\ \check{D}_2,\ ..,\check{D}_w,\ \check{D}_{w+1}\right)$$

$$= \begin{pmatrix} 1-\prod_{\check{j}=1}^{w}\left(1-\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(1-\prod_{\check{j}=1}^{w}\left(1-\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right), \\ -\prod_{\check{j}=1}^{w}\left|\mathring{A}_{RN-\check{D}_{\check{j}}}\right|^{\hat{Y}_{\check{j}}}+\iota\left(-\prod_{\check{j}=1}^{w}\left|\mathring{A}_{IN-\check{D}_{\check{j}}}\right|^{\hat{Y}_{\check{j}}}\right) \end{pmatrix}$$

$$\oplus \begin{pmatrix} 1-\left(1-\mathring{A}_{RP-\check{D}_{w+1}}\right)^{\hat{Y}_{w+1}} \\ +\iota\left(1-\left(1-\mathring{A}_{IP-\check{D}_{w+1}}\right)^{\hat{Y}_{w+1}}\right), \\ -\left(\left|\mathring{A}_{RN-\check{D}_{w+1}}\right|^{\hat{Y}_{w+1}}\right)+\iota\left(-\left(\left|\mathring{A}_{IN-\check{D}_{w+1}}\right|^{\hat{Y}_{w+1}}\right)\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1-\prod_{\check{j}=1}^{w+1}\left(1-\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(1-\prod_{\check{j}=1}^{w+1}\left(1-\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right), \\ -\prod_{\check{j}=1}^{w+1}\left|\mathring{A}_{RN-\check{D}_{\check{j}}}\right|^{\hat{Y}_{\check{j}}} \\ +\iota\left(-\prod_{\check{j}=1}^{w+1}\left|\mathring{A}_{IN-\check{D}_{\check{j}}}\right|^{\hat{Y}_{\check{j}}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1-\prod_{\check{j}=1}^{w+1}\left(1-\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}} \\ +\iota\left(1-\prod_{\check{j}=1}^{w+1}\left(1-\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}}\right), \\ -\prod_{\check{j}=1}^{w+1}\left|\mathring{A}_{RN-\check{D}_{\check{j}}}\right|^{\frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}} \\ +\iota\left(-\prod_{\check{j}=1}^{w+1}\left|\mathring{A}_{IN-\check{D}_{\check{j}}}\right|^{\frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}}\right) \end{pmatrix}$$

$\Rightarrow$ Eq (6) holds for all n.

Property 1: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}},\ \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}},\ \mathring{A}_{IP-\check{D}_{\check{j}}},\ \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1,\ 2,\ \ldots,\ n)$, if $\check{D}_{\check{j}} = \check{D}\ \forall\ \check{j}$ then

$$BCFPA\left(\check{D}_1,\ \check{D}_2,\ ..,\ \check{D}_n\right) = \check{D} \tag{7}$$

This property is said to be Idempotency.

Property 2: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}},\ \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}},\ \mathring{A}_{IP-\check{D}_{\check{j}}},\ \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1,\ 2,\ \ldots,\ n)$, suppose $\check{D}^{-} = \left(\min_{\check{j}}\left\{\mathring{A}_{RP-\check{D}_{\check{j}}}\right\} + \iota\ \min_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\},\ \max_{\check{j}}\left\{\mathring{A}_{RN-\check{D}_{\check{j}}}\right\} + \iota\ \max_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}\right)$, and $\check{D}^{+} = \left(\max_{\check{j}}\left\{\mathring{A}_{RP-\check{D}_{\check{j}}}\right\} + \iota\ \max_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\},\ \min_{\check{j}}\left\{\mathring{A}_{RN-\check{D}_{\check{j}}}\right\} + \iota\ \min_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}\right)$, then

$$\check{D}^{-} \leq BCFPA\left(\check{D}_1,\ \check{D}_2,\ ..,\ \check{D}_n\right) \leq \check{D}^{+} \tag{8}$$

This property is said to be boundedness.

Property 3: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}},\ \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}},\ \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, if $\check{D}'_{\check{j}} = \left(\mathring{A}_{P-\check{D}'_{\check{j}}},\ \mathring{A}_{N-\check{D}'_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}'_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}'_{\check{j}}},\ \mathring{A}_{RN-\check{D}'_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}'_{\check{j}}}\right)$, $(\check{j} = 1,\ 2,\ \ldots,\ n)$, is any permutation, then

$$BCFPA\left(\check{D}_1,\ \check{D}_2,\ ..,\ \check{D}_n\right) = BCFPA\left(\check{D}'_1,\ \check{D}'_2,\ ..,\ \check{D}'_n\right) \tag{9}$$

This property is said to be commutativity.

In the definition of the BCFPA operator, it is viewed as that every value is similarly significant, in any case, in real-life various attributes assume various parts during the aggregation, so for this below we describe the BCFPWA operator.

Definition 5: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}},\ \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}},\ \mathring{A}_{IP-\check{D}_{\check{j}}},\ \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1,\ 2,\ \ldots,\ n)$, the BCFPWA operator is offered as

$$BCFPWA\left(\check{D}_1,\ \check{D}_2,\ ..,\ \check{D}_n\right) = \bigoplus_{\check{j}=1}^{n} \frac{\gimel_{\check{j}}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}{\sum_{\check{j}=1}^{n}\gimel_{\check{j}}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}\check{D}_{\check{j}} \tag{10}$$

where $\mathrm{T}\left(\check{D}_{\check{j}}\right) = \sum_{\substack{\check{j}=1 \\ \check{j}\neq\kappa}}^{n} \mathrm{Sup}\left(\check{D}_{\check{j}},\ \check{D}_{\kappa}\right)$.

Theorem 2: The outcome achieved by Eq (10), is a BCFN and

$$BCFPWA\left(\check{D}_1,\ \check{D}_2,\ ..,\ \check{D}_n\right)$$

$$= \begin{pmatrix} 1-\prod_{\check{j}=1}^{n}\left(1-\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\frac{\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\check{j}}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\check{j}}\right)\right)}} \\ +\iota\left(1-\prod_{\check{j}=1}^{n}\left(1-\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\frac{\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\check{j}}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\check{j}}\right)\right)}}\right), \\ -\prod_{\check{j}=1}^{n}\left|\mathring{A}_{RN-\check{D}_{\check{j}}}\right|^{\frac{\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\check{j}}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\check{j}}\right)\right)}} \\ +\iota\left(-\prod_{\check{j}=1}^{n}\left|\mathring{A}_{IN-\check{D}_{\check{j}}}\right|^{\frac{\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\check{j}}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\check{j}}\right)\right)}}\right) \end{pmatrix}$$

Likewise, the BCFPA operator and BCFPWA operator hold the following properties.

Property 1: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1, 2, \ldots, n)$, if $\check{D}_{\check{j}} = \check{D}\ \forall\,\check{j}$ then

$$BCFPWA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \check{D} \tag{11}$$

This property is said to be Idempotency.

Property 2: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1, 2, \ldots, n)$, suppose

$$\check{D}^{-} = \left(\min_{\check{j}}\left\{\mathring{A}_{RP-\check{D}_{\check{j}}}\right\} + \iota\,\min_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}, \max_{\check{j}}\left\{\mathring{A}_{RN-\check{D}_{\check{j}}}\right\} + \iota\,\max_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}\right),$$

and

$$\check{D}^{+} = \left(\max_{\check{j}}\left\{\mathring{A}_{RP-\check{D}_{\check{j}}}\right\} + \iota\,\max_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}, \min_{\check{j}}\left\{\mathring{A}_{RN-\check{D}_{\check{j}}}\right\} + \iota\,\min_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}\right),$$

then

$$\check{D}^{-} \leq BCFPWA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) \leq \check{D}^{+} \tag{12}$$

This property is said to be boundedness.

Property 3: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, if $\check{D}_{\check{j}}' = \left(\mathring{A}_{P-\check{D}_{\check{j}}'}, \mathring{A}_{N-\check{D}_{\check{j}}'}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}'} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}'}, \mathring{A}_{RN-\check{D}_{\check{j}}'} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}'}\right)$, $(\check{j} = 1, 2, \ldots, n)$, is any permutation, then

$$BCFPWA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = BCFPWA\left(\check{D}_1', \check{D}_2', .., \check{D}_n'\right) \tag{13}$$

This property is said to be commutativity.

Definition 6: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1, 2, \ldots, n)$, the BCFPOWA operator is offered as

$$BCFPOWA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \bigoplus_{\check{j}=1}^{n} \frac{\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\varrho(\check{j})}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\varrho(\check{j})}\right)\right)}\check{D}_{\varrho(\check{j})} \tag{14}$$

where, $(\varrho(1), \varrho(2), .., \varrho(n))$ is a permutation of $(1, 2, .., n)$ with $(1, 2, .., n)$ with $\check{D}_{\varrho(\check{j}-1)} \geq \check{D}_{\varrho(\check{j})}\ \forall\,\check{j}$.

Theorem 3: The outcome achieved by Eq (14), is a BCFN and

$$BCFPOWA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \begin{pmatrix} 1-\prod_{\check{j}=1}^{n}\left(1-\mathring{A}_{RP-\check{D}_{\varrho(\check{j})}}\right)^{\frac{\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\varrho(\check{j})}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\varrho(\check{j})}\right)\right)}} \\ +\iota\left(1-\prod_{\check{j}=1}^{n}\left(1-\mathring{A}_{IP-\check{D}_{\varrho(\check{j})}}\right)^{\frac{\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\varrho(\check{j})}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\varrho(\check{j})}\right)\right)}}\right), \\ -\prod_{\check{j}=1}^{n}\left|\mathring{A}_{RN-\check{D}_{\varrho(\check{j})}}\right|^{\frac{\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\varrho(\check{j})}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\varrho(\check{j})}\right)\right)}} \\ +\iota\left(-\prod_{\check{j}=1}^{n}\left|\mathring{A}_{IN-\check{D}_{\varrho(\check{j})}}\right|^{\frac{\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\varrho(\check{j})}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\text{T}\left(\check{D}_{\varrho(\check{j})}\right)\right)}}\right) \end{pmatrix} \tag{15}$$

The BCFPOWA operator holds the following properties.

Property 1: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1, 2, \ldots, n)$, if $\check{D}_{\check{j}} = \check{D}\ \forall\,\check{j}$ then

$$BCFPOWA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \check{D} \tag{16}$$

This property is said to be Idempotency.

Property 2: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota\mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\,\mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1, 2, \ldots, n)$, suppose $\check{D}^{-} = \left(\min_{\check{j}}\left\{\mathring{A}_{RP-\check{D}_{\check{j}}}\right\} + \right.$

$\iota\ \min_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}, \max_{\check{j}}\left\{\mathring{A}_{RN-\check{D}_{\check{j}}}\right\}+\iota\ \max_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}\Big)$, and $\check{D}^{+}=\Big(\max_{\check{j}}\left\{\mathring{A}_{RP-\check{D}_{\check{j}}}\right\}+\iota\ \max_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}, \min_{\check{j}}\left\{\mathring{A}_{RN-\check{D}_{\check{j}}}\right\}+\iota\ \min_{\check{j}}\left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}\Big)$, then

$$\check{D}^{-}\leq BCFPOWA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right)\leq\check{D}^{+} \tag{17}$$

This property is said to be boundedness.

Property 3: $\check{D}_{\check{j}}=\left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right)=\left(\mathring{A}_{RP-\check{D}_{\check{j}}}+\iota\mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}}+\iota\ \mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, if $\check{D}'_{\check{j}}=\left(\mathring{A}_{P-\check{D}'_{\check{j}}}, \mathring{A}_{N-\check{D}'_{\check{j}}}\right)=\left(\mathring{A}_{RP-\check{D}'_{\check{j}}}+\iota\mathring{A}_{IP-\check{D}'_{\check{j}}}, \mathring{A}_{RN-\check{D}'_{\check{j}}}+\iota\ \mathring{A}_{IN-\check{D}'_{\check{j}}}\right)$, $(\check{j}=1,2,\ldots,n)$, is any permutation, then

$$BCFPOWA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right)=BCFPOWA\left(\check{D}'_1, \check{D}'_2, .., \check{D}'_n\right) \tag{18}$$

This property is said to be commutativity.

Following we interpret the BCFPHA operator

Definition 7: For a group of BCFNs $\check{D}_{\check{j}}=\left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right)=\left(\mathring{A}_{RP-\check{D}_{\check{j}}}+\iota\mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}}+\iota\ \mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j}=1, 2, \ldots, n)$, the BCFPHA operator is offered as

$$BCFPHA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right)=\bigoplus_{\check{j}=1}^{n}\frac{\lambda_{\check{j}}\left(1+\mathrm{T}\left(\check{D}^{\#}_{\varrho(\check{j})}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\mathrm{T}\left(\check{D}^{\#}_{\varrho(\check{j})}\right)\right)}\check{D}^{\#}_{\varrho(\check{j})}$$
$$=\left(\begin{array}{c}1-\prod_{\check{j}=1}^{n}\left(1-\mathring{A}_{RP-\check{D}^{\#}_{\varrho(\check{j})}}\right)^{\frac{\lambda_{\check{j}}\left(1+\mathrm{T}\left(\check{D}^{\#}_{\varrho(\check{j})}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\mathrm{T}\left(\check{D}^{\#}_{\varrho(\check{j})}\right)\right)}}\\ +\iota\left(1-\prod_{\check{j}=1}^{n}\left(1-\mathring{A}_{IP-\check{D}^{\#}_{\varrho(\check{j})}}\right)^{\frac{\lambda_{\check{j}}\left(1+\mathrm{T}\left(\check{D}^{\#}_{\varrho(\check{j})}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\mathrm{T}\left(\check{D}^{\#}_{\varrho(\check{j})}\right)\right)}}\right),\\ -\prod_{\check{j}=1}^{n}\left|\mathring{A}_{RN-\check{D}^{\#}_{\varrho(\check{j})}}\right|^{\frac{\lambda_{\check{j}}\left(1+\mathrm{T}\left(\check{D}^{\#}_{\varrho(\check{j})}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\mathrm{T}\left(\check{D}^{\#}_{\varrho(\check{j})}\right)\right)}}\\ +\iota\left(-\prod_{\check{j}=1}^{n}\left|\mathring{A}_{IN-\check{D}^{\#}_{\varrho(\check{j})}}\right|^{\frac{\lambda_{\check{j}}\left(1+\mathrm{T}\left(\check{D}^{\#}_{\varrho(\check{j})}\right)\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\mathrm{T}\left(\check{D}^{\#}_{\varrho(\check{j})}\right)\right)}}\right)\end{array}\right) \tag{19}$$

where, $\check{D}^{\#}_{(\check{j})}\left(\check{D}^{\#}_{(\check{j})}=n\Im_{\check{j}}\check{D}_{\check{j}}\right)$, $(\varrho(1), \varrho(2), .., \varrho(n))$is a permutation of $(1, 2, .., n)$with $\check{D}_{\varrho(\check{j}-1)}\geq\check{D}_{\varrho(\check{j})}\ \forall\check{j}$. Additionally, $\Im_{\check{j}}\ (\check{j}=1, 2, \ldots n)$, $0\leq\Im_{\check{j}}\leq1$and $\sum_{\check{j}=1}^{n}\Im_{\check{j}}=1$.

Remark 1: By taking $\lambda=\left(\frac{1}{n}, \frac{1}{n}, .., \frac{1}{n}\right)^{t}$ then the BCFPHA is transformed to BCFPWA and by taking $\Im=\left(\frac{1}{n}, \frac{1}{n}, .., \frac{1}{n}\right)^{t}$ then the BCFPHA operator is transformed into the BCFPOWA operator.

Definition 8: For a group of BCFNs $\check{D}_{\check{j}}=\left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right)=\left(\mathring{A}_{RP-\check{D}_{\check{j}}}+\iota\mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}}+\iota\ \mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j}=1, 2, \ldots, n)$, the BCFPG operator is offered as

$$BCFPGA\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right)=\bigotimes_{\check{j}=1}^{n}\check{D}_{\check{j}}^{\frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}} \tag{20}$$

where $\mathrm{T}\left(\check{D}_{\check{j}}\right)=\sum_{\check{j}=1, \check{j}\neq\kappa}^{n}\mathrm{Sup}\left(\check{D}_{\check{j}}, \check{D}_{\kappa}\right)$

Theorem 4: The outcome achieved by Eq (20), is a BCFN and

$$BCFPG\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right)=\left(\begin{array}{c}\prod_{\check{j}=1}^{n}\left(\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}}+\\ \iota\left(\prod_{\check{j}=1}^{n}\left(\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}}\right),\\ -1+\prod_{\check{j}=1}^{n}\left(1+\mathring{A}_{RN-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}}\\ +\iota\left(-1+\prod_{\check{j}=1}^{n}\left(1+\mathring{A}_{IN-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}}\right)\end{array}\right) \tag{21}$$

Proof: For easiness, let $\frac{1+\mathrm{T}\left(\check{D}_{\check{j}}\right)}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}\left(\check{D}_{\check{j}}\right)\right)}=\hat{Y}$. Now by employing the operational laws in Def (1), we have

$$\check{D}_1^{\hat{Y}_1}=\left(\left(\begin{array}{c}\left(\mathring{A}_{RP-\check{D}_1}\right)^{\hat{Y}_1}+\iota\left(\mathring{A}_{IP-\check{D}_1}\right)^{\hat{Y}_1},\\ -1+\left(1+\mathring{A}_{RN-\check{D}_1}\right)^{\hat{Y}_1}\\ +\iota\left(-1+\left(1+\mathring{A}_{IN-\check{D}_1}\right)^{\hat{Y}_1}\right)\end{array}\right)\right)$$

$\check{D}_2^{\hat{Y}_2}$

$$= \left(\left(\begin{array}{c}\left(\mathring{A}_{RP-\check{D}_2}\right)^{\hat{Y}_2} + \iota\left(\mathring{A}_{IP-\check{D}_2}\right)^{\hat{Y}_2}, \\ -1+\left(1+\mathring{A}_{RN-\check{D}_2}\right)^{\hat{Y}_2} \\ +\iota\left(-1+\left(1+\mathring{A}_{IN-\check{D}_2}\right)^{\hat{Y}_2}\right)\end{array}\right)\right)$$

$\check{D}_1^{\hat{Y}_1} \otimes \check{D}_2^{\hat{Y}_2}$

$$= \left(\left(\begin{array}{c}\left(\mathring{A}_{RP-\check{D}_1}\right)^{\hat{Y}_1} \\ +\iota\left(\mathring{A}_{IP-\check{D}_1}\right)^{\hat{Y}_1}, \\ -1+\left(1+\mathring{A}_{RN-\check{D}_1}\right)^{\hat{Y}_1} \\ +\iota\left(-1+\left(1+\mathring{A}_{IN-\check{D}_1}\right)^{\hat{Y}_1}\right)\end{array}\right)\right)$$

$$\otimes \left(\left(\begin{array}{c}\left(\mathring{A}_{RP-\check{D}_2}\right)^{\hat{Y}_2} \\ +\iota\left(\mathring{A}_{IP-\check{D}_2}\right)^{\hat{Y}_2}, \\ -1+\left(1+\mathring{A}_{RN-\check{D}_2}\right)^{\hat{Y}_2} \\ +\iota\left(-1+\left(1+\mathring{A}_{IN-\check{D}_2}\right)^{\hat{Y}_2}\right)\end{array}\right)\right)$$

$$= \left(\begin{array}{c}\left(\mathring{A}_{RP-\check{D}_1}\right)^{\hat{Y}_1}\left(\mathring{A}_{RP-\check{D}_2}\right)^{\hat{Y}_2} \\ +\iota\left(\left(\mathring{A}_{IP-\check{D}_1}\right)^{\hat{Y}_1}\left(\mathring{A}_{IP-\check{D}_2}\right)^{\hat{Y}_2}\right) \\ -1+\left(1+\mathring{A}_{RN-\check{D}_1}\right)^{\hat{Y}_1}\left(1+\mathring{A}_{RN-\check{D}_2}\right)^{\hat{Y}_2} \\ +\iota\left(-1+\left(1+\mathring{A}_{IN-\check{D}_1}\right)^{\hat{Y}_1}\left(1+\mathring{A}_{IN-\check{D}_2}\right)^{\hat{Y}_2}\right)\end{array}\right)$$

$$= \left(\begin{array}{c}\prod_{\check{j}=1}^{2}\left(\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(\prod_{\check{j}=1}^{2}\left(\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right), \\ -1+\prod_{\check{j}=1}^{2}\left(1+\mathring{A}_{RN-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(-1+\prod_{\check{j}=1}^{2}\left(1+\mathring{A}_{IN-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right)\end{array}\right)$$

$\Rightarrow$ Eq (21) holds for $n = 2$. Now assume that Eq (21) holds $n = w$ i.e.

$$BCFPG\left(\check{D}_1, \check{D}_2, .., \check{D}_w\right)$$

$$= \left(\begin{array}{c}\prod_{\check{j}=1}^{w}\left(\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(\prod_{\check{j}=1}^{w}\left(\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right), \\ -1+\prod_{\check{j}=1}^{w}\left(1+\mathring{A}_{RN-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(-1+\prod_{\check{j}=1}^{w}\left(1+\mathring{A}_{IN-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right)\end{array}\right)$$

Now for $n = w + 1$, we have

$$BCFPG\left(\check{D}_1, \check{D}_2, .., \check{D}_w, \check{D}_{w+1}\right)$$

$$= \left(\begin{array}{c}\prod_{\check{j}=1}^{w}\left(\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(\prod_{\check{j}=1}^{w}\left(\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right), \\ -1+\prod_{\check{j}=1}^{w}\left(1+\mathring{A}_{RN-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(-1+\prod_{\check{j}=1}^{w}\left(1+\mathring{A}_{IN-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right)\end{array}\right)$$

$$\otimes \left(\begin{array}{c}\left(\mathring{A}_{RP-\check{D}_{w+1}}\right)^{\hat{Y}_{w+1}} \\ +\iota\left(\mathring{A}_{IP-\check{D}_{w+1}}\right)^{\hat{Y}_{w+1}}, \\ -1+\left(1+\mathring{A}_{RN-\check{D}_{w+1}}\right)^{\hat{Y}_{w+1}} \\ +\iota\left(-1+\left(1+\mathring{A}_{IN-\check{D}_{w+1}}\right)^{\hat{Y}_{w+1}}\right)\end{array}\right)$$

$$= \left(\begin{array}{c}\prod_{\check{j}=1}^{w+1}\left(\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(\prod_{\check{j}=1}^{w+1}\left(\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right), \\ -1+\prod_{\check{j}=1}^{w+1}\left(1+\mathring{A}_{RN-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}} \\ +\iota\left(-1+\prod_{\check{j}=1}^{w+1}\left(1+\mathring{A}_{IN-\check{D}_{\check{j}}}\right)^{\hat{Y}_{\check{j}}}\right)\end{array}\right)$$

$$= \left(\begin{array}{c} \prod_{\check{j}=1}^{n} \left(\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}(\check{D}_{\check{j}})}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}} + \iota \left(\prod_{\check{j}=1}^{n} \left(\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}(\check{D}_{\check{j}})}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}} \right), \\ -1 + \prod_{\check{j}=1}^{n} \left(1+\mathring{A}_{RN-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}(\check{D}_{\check{j}})}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}} + \iota \left(-1 + \prod_{\check{j}=1}^{n} \left(1+\mathring{A}_{IN-\check{D}_{\check{j}}}\right)^{\frac{1+\mathrm{T}(\check{D}_{\check{j}})}{\sum_{\check{j}=1}^{n}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}} \right) \end{array} \right)$$

$\Rightarrow$ Eq (21) holds for all n.

Property 1: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota \mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\, \mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1, 2, \ldots, n)$, if $\check{D}_{\check{j}} = \check{D}$ $\forall\, \check{j}$ then

$$BCFPG\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \check{D} \tag{22}$$

This property is said to be Idempotency.

Property 2: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota \mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\, \mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1, 2, \ldots, n)$, suppose $\check{D}^{-} = \left(\min_{\check{j}} \left\{\mathring{A}_{RP-\check{D}_{\check{j}}}\right\} + \iota \min_{\check{j}} \left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}, \max_{\check{j}} \left\{\mathring{A}_{RN-\check{D}_{\check{j}}}\right\} + \iota \max_{\check{j}} \left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}\right)$, and $\check{D}^{+} = \left(\max_{\check{j}} \left\{\mathring{A}_{RP-\check{D}_{\check{j}}}\right\} + \iota \max_{\check{j}} \left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}, \min_{\check{j}} \left\{\mathring{A}_{RN-\check{D}_{\check{j}}}\right\} + \iota \min_{\check{j}} \left\{\mathring{A}_{IP-\check{D}_{\check{j}}}\right\}\right)$, then

$$\check{D}^{-} \leq BCFPG\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) \leq \check{D}^{+} \tag{23}$$

This property is said to be boundedness.

Property 3: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota \mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\, \mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, if $\check{D}'_{\check{j}} = \left(\mathring{A}_{P-\check{D}'_{\check{j}}}, \mathring{A}_{N-\check{D}'_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}'_{\check{j}}} + \iota \mathring{A}_{IP-\check{D}'_{\check{j}}}, \mathring{A}_{RN-\check{D}'_{\check{j}}} + \iota\, \mathring{A}_{IN-\check{D}'_{\check{j}}}\right)$, $(\check{j} = 1, 2, \ldots, n)$, is any permutation, then

$$BCFPG\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = BCFPA\left(\check{D}'_1, \check{D}'_2, .., \check{D}'_n\right) \tag{24}$$

This property is said to be commutativity.

In the definition of the BCFPG operator, it is viewed as that every value is similarly significant, in any case, in real-life various attributes assume various parts during the aggregation, so for this below we describe the BCFPWG operator.

Definition 9: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota \mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\, \mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1, 2, \ldots, n)$, the BCFPWG operator is offered as

$$BCFPWG\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \bigotimes_{\check{j}=1}^{n} \check{D}_{\check{j}}^{\frac{\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}} \tag{25}$$

where $\mathrm{T}\left(\check{D}_{\check{j}}\right) = \sum_{\substack{J=1 \\ \check{j}\neq \varsigma}}^{n} \mathrm{Sup}\left(\check{D}_{\check{j}}, \check{D}_{\varsigma}\right)$.

Theorem 5: The outcome achieved by Eq (25), is a BCFN and

$$BCFPWG\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \left(\begin{array}{c} \prod_{\check{j}=1}^{n} \left(\mathring{A}_{RP-\check{D}_{\check{j}}}\right)^{\frac{\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}} + \iota \left(\prod_{\check{j}=1}^{n} \left(\mathring{A}_{IP-\check{D}_{\check{j}}}\right)^{\frac{\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}} \right), \\ -1 + \prod_{\check{j}=1}^{n} \left(1+\mathring{A}_{RN-\check{D}_{\check{j}}}\right)^{\frac{\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}} + \iota \left(-1 + \prod_{\check{j}=1}^{n} \left(1+\mathring{A}_{IN-\check{D}_{\check{j}}}\right)^{\frac{\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\check{j}})\right)}} \right) \end{array} \right) \tag{26}$$

Remark 2: The BCFPWG operator holds the same properties which the BCFPG operator satisfies.

Below we explore the BCFPOWG operator

Definition 10: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}}\right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota \mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\, \mathring{A}_{IN-\check{D}_{\check{j}}}\right)$, $(\check{j} = 1, 2, \ldots, n)$, the BCFPOWG operator is offered as

$$BCFPOWG\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \bigotimes_{\check{j}=1}^{n} \check{D}_{\varrho(\check{j})}^{\frac{\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\varrho(\check{j})})\right)}{\sum_{\check{j}=1}^{n}\lambda_{\check{j}}\left(1+\mathrm{T}(\check{D}_{\varrho(\check{j})})\right)}}$$

$$= \left(\begin{array}{c} \prod_{\check{j}=1}^{n} \left(\mathring{A}_{RP-\check{D}_{\varrho(\check{j})}} \right)^{\frac{\lambda_{\check{j}}(1+T(\check{D}_{\varrho(\check{j})}))}{\sum_{\check{j}=1}^{n} \lambda_{\check{j}}(1+T(\check{D}_{\varrho(\check{j})}))}} + \iota \left(\prod_{\check{j}=1}^{n} \left(\mathring{A}_{IP-\check{D}_{\varrho(\check{j})}} \right)^{\frac{\lambda_{\check{j}}(1+T(\check{D}_{\varrho(\check{j})}))}{\sum_{\check{j}=1}^{n} \lambda_{\check{j}}(1+T(\check{D}_{\varrho(\check{j})}))}} \right), \\ -1 + \prod_{\check{j}=1}^{n} \left(1 + \mathring{A}_{RN-\check{D}_{\varrho(\check{j})}} \right)^{\frac{\lambda_{\check{j}}(1+T(\check{D}_{\varrho(\check{j})}))}{\sum_{\check{j}=1}^{n} \lambda_{\check{j}}(1+T(\check{D}_{\varrho(\check{j})}))}} + \iota \left(-1 + \prod_{\check{j}=1}^{n} \left(1 + \mathring{A}_{IN-\check{D}_{\varrho(\check{j})}} \right)^{\frac{\lambda_{\check{j}}(1+T(\check{D}_{\varrho(\check{j})}))}{\sum_{\check{j}=1}^{n} \lambda_{\check{j}}(1+T(\check{D}_{\varrho(\check{j})}))}} \right) \end{array} \right) \quad (27)$$

where, $(\varrho(1), \varrho(2), .., \varrho(n))$ is a permutation of $(1, 2, .., n)$ with $\check{D}_{\varrho(\check{j}-1)} \geq \check{D}_{\varrho(\check{j})}\ \forall\, \check{j}$.

Remark 3: The BCFPOWG operator also holds the idempotency, boundedness, and commutativity properties as we defined for BCFPOWA.

Following we interpret the BCFPHG operator

Definition 11: For a group of BCFNs $\check{D}_{\check{j}} = \left(\mathring{A}_{P-\check{D}_{\check{j}}}, \mathring{A}_{N-\check{D}_{\check{j}}} \right) = \left(\mathring{A}_{RP-\check{D}_{\check{j}}} + \iota \mathring{A}_{IP-\check{D}_{\check{j}}}, \mathring{A}_{RN-\check{D}_{\check{j}}} + \iota\, \mathring{A}_{IN-\check{D}_{\check{j}}} \right)$, $(\check{j} = 1, 2, \ldots, n)$, the BCFPHG operator is offered as

$$BCFPHG\left(\check{D}_1, \check{D}_2, .., \check{D}_n\right) = \bigotimes_{\check{j}=1}^{n} \left(\check{D}^{\#}_{\varrho(\check{j})}\right)^{\frac{\lambda_{\check{j}}(1+T(\check{D}^{\#}_{\varrho(\check{j})}))}{\sum_{\check{j}=1}^{n} \lambda_{\check{j}}(1+T(\check{D}^{\#}_{\varrho(\check{j})}))}}$$

$$= \left(\begin{array}{c} \prod_{\check{j}=1}^{n} \left(\mathring{A}_{RP-\check{D}^{\#}_{\varrho(\check{j})}} \right)^{\frac{\lambda_{\check{j}}(1+T(\check{D}^{\#}_{\varrho(\check{j})}))}{\sum_{\check{j}=1}^{n} \lambda_{\check{j}}(1+T(\check{D}^{\#}_{\varrho(\check{j})}))}} + \iota \left(\prod_{\check{j}=1}^{n} \left(\mathring{A}_{IP-\check{D}^{\#}_{\varrho(\check{j})}} \right)^{\frac{\lambda_{\check{j}}(1+T(\check{D}^{\#}_{\varrho(\check{j})}))}{\sum_{\check{j}=1}^{n} \lambda_{\check{j}}(1+T(\check{D}^{\#}_{\varrho(\check{j})}))}} \right), \\ -1 + \prod_{\check{j}=1}^{n} \left(1 + \mathring{A}_{RN-\check{D}^{\#}_{\varrho(\check{j})}} \right)^{\frac{\lambda_{\check{j}}(1+T(\check{D}^{\#}_{\varrho(\check{j})}))}{\sum_{\check{j}=1}^{n} \lambda_{\check{j}}(1+T(\check{D}^{\#}_{\varrho(\check{j})}))}} + \iota \left(-1 + \prod_{\check{j}=1}^{n} \left(1 + \mathring{A}_{IN-\check{D}^{\#}_{\varrho(\check{j})}} \right)^{\frac{\lambda_{\check{j}}(1+T(\check{D}^{\#}_{\varrho(\check{j})}))}{\sum_{\check{j}=1}^{n} \lambda_{\check{j}}(1+T(\check{D}^{\#}_{\varrho(\check{j})}))}} \right) \end{array} \right) \quad (28)$$

where, $\check{D}^{\#}_{(\check{j})}\left(\check{D}^{\#}_{(\check{j})} = n\Im_{\check{j}}\check{D}_{\check{j}}\right)$, $(\varrho(1), \varrho(2), .., \varrho(n))$is a permutation of $(1, 2, .., n)$with $\check{D}_{\varrho(\check{j}-1)} \geq \check{D}_{\varrho(\check{j})}\ \forall\, \check{j}$. Additionally, $\Im_{\check{j}}\ (\check{j} = 1, 2, \ldots n)$, $0 \leq \Im_{\check{j}} \leq 1$and $\sum_{\check{j}=1}^{n} \Im_{\check{j}} = 1$.

Remark 4: By taking $\lambda = \left(\frac{1}{n}, \frac{1}{n}, .., \frac{1}{n}\right)^t$ then the BCFPHG is transformed to BCFPWG and by taking $\Im = \left(\frac{1}{n}, \frac{1}{n}, .., \frac{1}{n}\right)^t$ then the BCFPHG operator is transformed into the BCFPOWG operator.

IV. APPLICATION "CLASSIFICATION OF CANCER DISEASES IN HUMAN BODY USING HYBRID APPROACHES BASED ON BIPOLAR COMPLEX FUZZY INFORMATION"

ICCR information organizations have been utilized to give a dominant, indication-based technique for the converging of carcinoma. The influence guarantees that the information organizations given for distinct tumor sorts have a dominant rule and style and include all the parameters required to guide administration and prediction for separate cancers. Cancer indicates any one of a finite number of diseases described by the growth of unusual cells that separate unmanageable and can penetrate and damage typical physique tissue. The death rate of cancer in human beings is the second number but due to our valuable and intelligent doctors, nowadays the ratio of the death cases in cancer has reduced. Cancer-affected peoples have a lot of symptoms and all cancer symptoms differ on the sort and the stage of the cancer growth and also depend on the patient's overall health. In many cases, certain doctors have noticed that many types of cancer have no particular symptoms, based on the usefulness and feasibility of genuine cancer screening, as well as risk feature minimization ("means that the ratio of death is increasing or decreasing"). To facilitate the cancer patient, every doctor has followed the following procedure, first, they have found the stage of cancer, which means that the concerned doctor has to evaluate the size, location, and behavior of the cancer, then based on the above analysis doctor has suggested the treatment of cancer. Different types of cancer have needed different types of treatment, and most doctors have used the following strategy for the treatment of cancer patients chemotherapy, surgery, and radiation therapy. Certain cancer patients have used different therapies for their treatment. Most people have affected because of non-stop smoking addiction, eating an unhealthy diet, working in front of the sun and harmful ultraviolet rays from the sun can increase the risk of skin cancer, not doing exercise, increasing/decreasing weights without any diet, drinking alcohol, not properly checkup of cancer, etc. The main symptoms in a cancer patient are the form like pain, fatigue, a complication in breathing, nausea, informal weight loss, brain dilemma, chemical changes in the skin, etc. In this theory, we focused on the main four types of cancer and try to evaluate the most critical type in all of them. Further, we recalled the theory of BCF information and their related theory and power aggregation operator. Cancer is an awkward and complex collection of diseases for certain

TABLE 1. Represented BCF information.

	$\mathfrak{X}_1$	$\mathfrak{X}_2$	$\mathfrak{X}_3$	$\mathfrak{X}_4$
$\check{D}_1$	$\begin{pmatrix}0.41+\iota\,0.61,\\-0.42-\iota\,0.62\end{pmatrix}$	$\begin{pmatrix}0.82+\iota\,0.53,\\-0.84-\iota\,0.51\end{pmatrix}$	$\begin{pmatrix}0.31+\iota\,0.11,\\-0.32-\iota\,0.12\end{pmatrix}$	$\begin{pmatrix}0.75+\iota\,0.36,\\-0.1-\iota\,0.17\end{pmatrix}$
$\check{D}_2$	$\begin{pmatrix}0.43+\iota\,0.63,\\-0.44-\iota\,0.64\end{pmatrix}$	$\begin{pmatrix}0.86+\iota\,0.57,\\-0.88-\iota\,059.\end{pmatrix}$	$\begin{pmatrix}0.33+\iota\,0.13,\\-0.34-\iota\,0.14\end{pmatrix}$	$\begin{pmatrix}0.6+\iota\,0.3,\\-0.8-\iota\,0.7\end{pmatrix}$
$\check{D}_3$	$\begin{pmatrix}0.45+\iota\,0.65,\\-0.46-\iota\,0.66\end{pmatrix}$	$\begin{pmatrix}0.81+\iota\,0.58,\\-0.83-\iota\,0.56\end{pmatrix}$	$\begin{pmatrix}0.35+\iota\,0.15,\\-0.36-\iota\,0.16\end{pmatrix}$	$\begin{pmatrix}0.5+\iota\,0.25,\\-0.9-\iota\,0.17\end{pmatrix}$
$\check{D}_4$	$\begin{pmatrix}0.47+\iota\,0.67,\\-0.48-\iota\,0.68\end{pmatrix}$	$\begin{pmatrix}0.85+\iota\,0.54,\\-0.89-\iota\,0.52\end{pmatrix}$	$\begin{pmatrix}0.37+\iota\,0.17,\\-0.38-\iota\,0.18\end{pmatrix}$	$\begin{pmatrix}0.25+\iota\,0.67,\\-0.5-\iota\,0.42\end{pmatrix}$

possible reasons. Many intellectuals have analyzed the different forms of cancer using classical information and found a feasible and suitable solution. For analyzing the solution to cancer in the human body, every expert or doctor has needed to collect genuine data from many hospitals which is very complex work. But one of the most important and valuable questions, if we find the solution for the cancer patient by using fuzzy set theory instead of classical information is very complicated because for this it is necessary to develop someone the idea of fuzzy sets theory. Cancer is a collection of diseases in which cells separate without mechanism or order. The over-division of cells affects information recognized as a tumor. Not every tumor is considered cancerous. To be identified as cancer, a tumor must damage neighboring muscles or tissues.

A. DECISION-MAKING PROCEDURE

Assume a collection of alternatives $\check{D} = \left\{\check{D}_1,\ \check{D}_2,\ \ldots,\ \check{D}_m\right\}$ and their attributes $\mathfrak{X} = \{\mathfrak{X}_1,\ \mathfrak{X}_2,\ \ldots, \mathfrak{X}_n\}$ with weight vectors $\sum_{j=1}^{n} \mathfrak{I}_j = 1$, where $\mathfrak{I} = \{\mathfrak{I}_1, \mathfrak{I}_2, \ldots, \mathfrak{I}_n\}^T$ and $\mathfrak{I}_j, j = 1, 2, \ldots, n \in [0, 1]$. To find the most deadly and awkward sort of cancer in the world is very complicated. The main theme of the diagnosed operators is to evaluate the problem related to finding the deadliest type of cancer ("using four important types of cancer and their related four useful and initial symptoms") in human beings and try to compare the final result with the certain prevailing result is to enhance the worth of the evaluated approaches. For this, we computed the matrix which includes the BCFNs $\check{D} = \left(\mathring{A}_{P-\check{D}}(\ell), \mathring{A}_{N-\check{D}}(\ell)\right) = \left(\mathring{A}_{RP-\check{D}}(\ell) + \iota\mathring{A}_{IP-\check{D}}(\ell),\ \mathring{A}_{RN-\check{D}}(\ell) + \iota\,\mathring{A}_{IN-\check{D}}(\ell)\right)$, expressed the BCF set with positive and negative supporting grade $\mathring{A}_{P-\check{D}}(\ell) = \mathring{A}_{RP-\check{D}}(\ell) + \iota\,\mathring{A}_{IP-\check{D}}(\ell)$ and $\mathring{A}_{N-\check{D}}(\ell) = \mathring{A}_{RN-\check{D}}(\ell) + \iota\,\mathring{A}_{IN-\check{D}}(\ell)$ with $\mathring{A}_{RP-\check{D}}(\ell), \mathring{A}_{IP-\check{D}}(\ell) \in [0,\ 1]$ and $\mathring{A}_{RN-\check{D}}(\ell), \mathring{A}_{IN-\check{D}}(\ell) \in [-1,\ 0]$. Then, we organized the following procedure for decision-making, such that

Stage 1: We summarized the BCF information in the shape of a matrix, where the BCF information is represented by: $\check{D} = \left(\mathring{A}_{P-\check{D}}(\ell), \mathring{A}_{N-\check{D}}(\ell)\right) = \left(\mathring{A}_{RP-\check{D}}(\ell) + \iota\mathring{A}_{IP-\check{D}}(\ell),\ \mathring{A}_{RN-\check{D}}(\ell) + \iota\,\mathring{A}_{IN-\check{D}}(\ell)\right)$.

Stage 2: Aggregate the information given in the matrix, using the theory of BCFPA, BCFPWA, BCFPOWA, BCFPG, BCFPWG, and BCFPOWG operators.

Stage 3: Compute the score value of the obtained information, if the score value failed, use the accuracy value.

Stage 4: Rank all alternatives and find the better decision.

B. ILLUSTRATED EXAMPLE

Different types of cancer have needed different types of treatment, and most doctors have used the following strategy for the treatment of cancer patients chemotherapy, surgery, and radiation therapy. Certain cancer patients have used different therapies for their treatment. Most people have affected because of non-stop smoking addiction, eating an unhealthy diet, working in front of the sun and harmful ultraviolet rays from the sun can increase the risk of skin cancer, not doing exercise, increasing/decreasing weights without any diet, drinking alcohol, not properly checkup of cancer, etc. The main symptoms in a cancer patient are the form like pain, fatigue, a complication in breathing, nausea, informal weight loss, brain dilemma, chemical changes in the skin, etc. In this theory, we focused on the main four types of cancer and try to evaluate the most critical type in all of them. Here we consider four main and important types of cancer represented in the shape of alternatives, whose brief information is explained below:

$\check{D}_1$: Bladder cancer.
$\check{D}_2$: Breast cancer.
$\check{D}_3$: Colorectal cancer.
$\check{D}_4$: Kidney cancer.

For every type of cancer, we used four symptoms in the shape of the attribute, such that

$\mathfrak{X}_1$: Smoking Addiction.
$\mathfrak{X}_2$: Drinking Alcohol.
$\mathfrak{X}_3$: Genetic Mutations.
$\mathfrak{X}_4$: Exposure to many Chemicals.

Then by using invented algorithms, we diagnosed the beneficial decision with $\gimel = (0.24,\ 0.26,\ 0, 15,\ 0.35)$, representing the weight vector. Then, we organized the following procedure for decision-making, such that

Stage 1: We summarized the BCF information in the shape of a matrix, where the BCF information is represented by: $\check{D} = \left(\mathring{A}_{P-\check{D}}(\ell), \mathring{A}_{N-\check{D}}(\ell)\right) = \left(\mathring{A}_{RP-\check{D}}(\ell) + \iota\mathring{A}_{IP-\check{D}}(\ell),\ \mathring{A}_{RN-\check{D}}(\ell) + \iota\,\mathring{A}_{IN-\check{D}}(\ell)\right)$, see Table 1.

TABLE 2. Represented the accumulated values.

Operators	$\check{D}_1$	$\check{D}_2$	$\check{D}_3$	$\check{D}_4$
BCFPA	$\begin{pmatrix} 0.639+\iota\,0.455, \\ -0.326-\iota\,0.289 \end{pmatrix}$	$\begin{pmatrix} 0.539+\iota\,0.467, \\ -0.569-\iota\,0.445 \end{pmatrix}$	$\begin{pmatrix} 0.576+\iota\,0.466, \\ -0.601-\iota\,0.322 \end{pmatrix}$	$\begin{pmatrix} 0.565+\iota\,0.565, \\ -0.539-\iota\,0.412 \end{pmatrix}$
BCFPWA	$\begin{pmatrix} 0.461+\iota\,0.288, \\ -0.509-\iota\,0.514 \end{pmatrix}$	$\begin{pmatrix} 0.409+\iota\,0.293, \\ -0.771-\iota\,0.697 \end{pmatrix}$	$\begin{pmatrix} 0.387+\iota\,0.29, \\ -0.797-\iota\,0.537 \end{pmatrix}$	$\begin{pmatrix} 0.363+\iota\,0.392, \\ -0.725-\iota\,0.641 \end{pmatrix}$
BCFPOWA	$\begin{pmatrix} 0.377+\iota\,0.286, \\ -0.522-\iota\,0.506 \end{pmatrix}$	$\begin{pmatrix} 0.04+\iota\,0.262, \\ -0.758-\iota\,0.639 \end{pmatrix}$	$\begin{pmatrix} 0.371+\iota\,0.253, \\ -0.778-\iota\,0.489 \end{pmatrix}$	$\begin{pmatrix} 0.311+\iota\,0.376, \\ -0.684-\iota\,0.621 \end{pmatrix}$
BCFPG	$\begin{pmatrix} 0.537+\iota\,0.406, \\ -0.517-\iota\,0.398 \end{pmatrix}$	$\begin{pmatrix} 0.519+\iota\,0.404, \\ -0.672-\iota\,0.558 \end{pmatrix}$	$\begin{pmatrix} 0.508+\iota\,0.392, \\ -0.501-\iota\,0.437 \end{pmatrix}$	$\begin{pmatrix} 0.441+\iota\,0.505, \\ -0.641-\iota\,0.486 \end{pmatrix}$
BCFPWG	$\begin{pmatrix} 0.748+\iota\,0.625, \\ -0.322-\iota\,0.243 \end{pmatrix}$	$\begin{pmatrix} 0.723+\iota\,0.615, \\ -0.494-\iota\,0.394 \end{pmatrix}$	$\begin{pmatrix} 0.705+\iota\,0.598, \\ -0.378-\iota\,0.268 \end{pmatrix}$	$\begin{pmatrix} 0.626+\iota\,0.723, \\ -0.44-\iota\,0.315 \end{pmatrix}$
BCFPOWG	$\begin{pmatrix} 0.675+\iota\,0.609, \\ -0.271-\iota\,0.249 \end{pmatrix}$	$\begin{pmatrix} 0.71+\iota\,0.581, \\ -0.486-\iota\,0.363 \end{pmatrix}$	$\begin{pmatrix} 0.688+\iota\,0.561, \\ -0.381-\iota\,0.224 \end{pmatrix}$	$\begin{pmatrix} 0.613+\iota\,0.693, \\ -0.369-\iota\,0.318 \end{pmatrix}$

TABLE 3. Expressed the score values.

Operators	$\dot{S}_{SF}(\check{D}_1)$	$\dot{S}_{SF}(\check{D}_2)$	$\dot{S}_{SF}(\check{D}_3)$	$\dot{S}_{SF}(\check{D}4)$
BCFPA	0.619	0.511	0.53	0.545
BCFPWA	0.432	0.308	0.336	0.347
BCFPOWA	0.409	0.316	0.339	0.346
BCFPG	0.507	0.423	0.491	0.455
BCFPWG	0.702	0.613	0.654	0.648
BCFPOWG	0.691	0.611	0.661	0.655

TABLE 4. Shown the ranking values.

Operators	Ranking
BCFPA	$\check{D}_1 > \check{D}_4 > \check{D}_3 > \check{D}_2$
BCFPWA	$\check{D}_1 > \check{D}_4 > \check{D}_3 > \check{D}_2$
BCFPOWA	$\check{D}_1 > \check{D}_4 > \check{D}_3 > \check{D}_2$
BCFPG	$\check{D}_1 > \check{D}_3 > \check{D}_4 > \check{D}_2$
BCFPWG	$\check{D}_1 > \check{D}_3 > \check{D}_4 > \check{D}_2$
BCFPOWG	$\check{D}_1 > \check{D}_3 > \check{D}_4 > \check{D}_2$

Stage 2: Aggregate the information given in a matrix, using the theory of BCFPA, BCFPWA, BCFPOWA, BCFPG, BCFPWG, and BCFPOWG operators, and using distance measures investigated by Mahmood and Ur Rehman [29], see Table 2.

Stage 3: Compute the score value of the obtained information, if the score value is failed, use the accuracy value, see Table 3.

Stage 4: Rank all alternatives and find the better decision, see Table 4.

We learned from the well-known research, we noticed that in 2021, the most viral and affected type of cancer is called bladder cancer. Similarly, by using the diagnosed operators, we obtained the final result in the shape of $\check{D}_1$, represented bladder cancer.

V. COMPARATIVE ANALYSIS

To compute the comparative analysis of the diagnosed operators, we find the most deadly and awkward sort of cancer in the world is very complicated. The main theme of the diagnosed operators is to evaluate the problem related to finding the deadliest type of cancer ("using four important types of cancer and their related four useful and initial symptoms") in human beings and try to compare the final result with the certain prevailing result is to enhance the worth of the evaluated approaches. In this scenario, BCFSs include massive data than FSs, CFSs, and BFS, which have value and consideration by certain scholars. We can describe that the BCFS is a valuable and appropriate tool for genuine decision-making requests. For comparison, we used the Power AOs (PAOs) for intuitionistic fuzzy information diagnosed by Xu [42], Dombi prioritized AOs diagnosed by Jana et al. [10], and PAOs for complex fuzzy information investigated by Hu et al. [43], geometric AOs for cubic BFS invented by Riaz and Tehrim [19], Einstein averaging AOs for cubic BFS diagnosed by Riaz et al. [20], Hamacher aggregation operators for BCF information diagnosed by Mahmood et al. [34], and Dombi aggregation oper-

TABLE 5. Exposed the comparative analysis using the data in Table 1.

Operators	$\dot{S}_{SF}(\check{D}_1)$	$\dot{S}_{SF}(\check{D}_2)$	$\dot{S}_{SF}(\check{D}_3)$	$\dot{S}_{SF}(\check{D}4)$	Ranking
Xu [42]	Crashed	Crashed	Crashed	Crashed	Crashed
Jana et al. [10]	Crashed	Crashed	Crashed	Crashed	Crashed
Hu et al. [43]	Crashed	Crashed	Crashed	Crashed	Crashed
Riaz and Tehrim [19]	Crashed	Crashed	Crashed	Crashed	Crashed
Riaz et al. [20]	Crashed	Crashed	Crashed	Crashed	Crashed
BCFHWA [34]	0.637	0.489	0.519	0.54	$\check{D}_1 > \check{D}_4 > \check{D}_3 > \check{D}_2$
BCFHWG [34]	0.363	0.511	0.481	0.46	$\check{D}_2 > \check{D}_3 > \check{D}_4 > \check{D}_1$
BCFDWA [35]	0.686	0.538	0.601	0.584	$\check{D}_1 > \check{D}_3 > \check{D}_4 > \check{D}_2$
BCFDWG [35]	0.28	0.338	0.341	0.304	$\check{D}_3 > \check{D}_2 > \check{D}_4 > \check{D}_1$
BCFPA	0.619	0.511	0.53	0.545	$\check{D}_1 > \check{D}_4 > \check{D}_3 > \check{D}_2$
BCFPWA	0.432	0.308	0.336	0.347	$\check{D}_1 > \check{D}_4 > \check{D}_3 > \check{D}_2$
BCFPOWA	0.409	0.316	0.339	0.346	$\check{D}_1 > \check{D}_4 > \check{D}_3 > \check{D}_2$
BCFPG	0.507	0.423	0.491	0.455	$\check{D}_1 > \check{D}_3 > \check{D}_4 > \check{D}_2$
BCFPWG	0.702	0.613	0.654	0.648	$\check{D}_1 > \check{D}_3 > \check{D}_4 > \check{D}_2$
BCFPOWG	0.691	0.611	0.661	0.655	$\check{D}_1 > \check{D}_3 > \check{D}_4 > \check{D}_2$

ators using the BCF information invented by Mahmood and Ur Rehman [35] and diagnosed operators. We applied the existing and investigated operators and methods to the information described in Table 1. The comparative analysis is mentioned in Table 5.

Table 5 provides that the information in Table 1 is beyond the ability of the operators and methods diagnosed by Xu [42], Jana et al. [10], Hu et al. [43], Riaz and Tehrim [19], and Riaz et al. [20] as the information of Table 1, is in the model two-dimensional information with both poles and these above-mentioned theories can't cope with two-dimensional information along with both poles (positive and negative). Further, the operators and methods diagnosed by Mahmood et al. [34] and Mahmood and Ur Rehman [35] are in the structure of BCFS. Thus, they can solve the information in Table 1 and the outcome along with ranking order is described in Table 5. Table 5 provides three different types of results in the shape of $\check{D}_1, \check{D}_2$ and $\check{D}_3$, but most operators gave their results in the shape of $\check{D}_1$, represented bladder cancer.

The theories invented by Riaz and Tehrim [19], and Riaz et al. [20] are in the model of cubic BFS, this means that these theories are merely capable of handling the information with both poles or sides and can't handle the 2^{nd} dimension. Moreover, the theories invented by Mahmood et al. [34] and Mahmood and Ur Rehman [35], can tackle the information under BCFS but in these theories, the weight of the attributes is provided by the decision expert or analyst by his/her own choice and which can be biased. However, in the invented operators, the weight is determined through proper formula by considering the importance of attributes. Also, the invented operators cope with both pole and 2^{nd} dimension at a time. Therefore, the diagnosed operators have massive dominance and benefits over the mentioned prevailing theories.

VI. CONCLUSION

When the call gains out of the influence, they become cancer cells. These cancer cells develop and separate more rapidly than typical cells and they also do not die such typical cells. Cancer research is critical to work on the anticipation, discovery, and therapy of these cancers, and guarantee that survivors live longer better-quality lives. Cancer research additionally distinguishes the reasons for cancer and is directing the way toward further developed techniques for conclusion and treatment. Thus, the main aim of this article was to study cancer diseases in the human body with the help of power AOs in the setting of BCFS. Further, we exposed the BCFPA, BCFPWA, BCFPOWA, and BCFPHA operators and evaluated their influential and dominant results and properties ("idempotency, boundedness, and monotonicity") and diagnosed the BCFPG, BCFPWG, BCFPOWG, and BCFPHG operators and evaluated their influential and dominant results and properties ("idempotency, boundedness, and monotonicity"). After that, we found the most deadly and awkward sort of cancer in the world is very complicated. The main theme of the diagnosed operators is to evaluate the problem related to finding the deadliest type of cancer ("using four important types of cancer and their related four useful and initial symptoms") in human beings. After applying the initiated method based on the diagnosed operators, we obtained that $\check{D}_1$ that is, bladder cancer is the most viral and affected type of cancer. Finally, we compared the final result with the certain prevailing result to enhance the worth of the evaluated approaches and portrayed the advantages and supremacy of the diagnosed work.

In the future, our goal is to expand our presented work in various domains like hesitant fuzzy linguistics [44] and aggregation operators [45], [46].

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