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RESEARCH ARTICLE

Parallel Parking Path Planning in Narrow Space Based on a Three-Stage Curve Interpolation Method

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ABSTRACT A parallel parking path planning method aiming at the automatic parking application scene of parallel parking in narrow space of urban roadside parking lot is proposed in the paper, based on a three-stage curve interpolation method. After the vehicle finds and detects the idle parking space through the environment perception technology, the parking path is constructed using the clothoid curve, circular arc, and fifth-degree polynomial curve by performing reverse analysis starting from the parking termination point. Considering the constraints of vehicle technical parameters and collisions avoidance during the parking process, a feasible starting area for parking is established, and the constrained optimization problem is solved to generate a path curve that satisfies curvature continuity and parking constraints. Three different parking conditions are proposed for parking simulation experiments, and the path planning method proposed in the paper (method I), the circular arc and fifth-degree polynomial curve path planning method (method II), and the fifth-degree polynomial curve path planning method (method III) are compared and analyzed. The results show that the path planning method proposed in the paper can effectively eliminate the sudden change point of curvature, constrain the path curvature and ensure the collision-free requirement for obstacles when entering the parking space for different parking spaces or different starting parking points.

INDEX TERMS Narrow spaces, parallel parking, path planning, three-stage curve interpolation method.

I. INTRODUCTION

With the increase in car ownership, urban parking difficulties have become increasingly prominent. The parking spaces are becoming increasingly scarce, and the available space for parking is getting narrower. Especially in parallel parking of narrow spaces, the definition is to ensure that the length of the parking space is as short as possible when the vehicles stop into a parallel parking space in one step. This will be a significant challenge for drivers with insufficient driving experience, the drivers need to make multiple adjustments to the direction and position many times to complete the parking [1]. This increases the parking time and raises the risk of collisions with obstacles in the surroundings. The automatic parking path planning method in a narrow space can help the drivers park the vehicles safely and accurately into the parking space, save a certain parking time, and avoid collision with obstacles. Therefore, developing a path planning method that can safely and stably guide the vehicle into the narrow parking space has become one of the key technologies of the automatic parking system [2], [3], and [4].

Currently, automatic parking path planning methods are mainly based on mechanism, rule-based, and knowledge-based methods. References [5] and [6] proposed a method based on two circular arcs' tangents to plan the path for parallel parking. Reference [7] proposed a path-planning method based on circular arcs and straight lines for parallel parking, which reduces the tracking difficulty at the tangent point. References [8], [9], and [10] initial path planning for parallel parking is performed using the circular arc and straight-line method, followed by smoothing the initial path using the Bessel curve to eliminate sudden

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curvature changes. References [11], [12], and [13] proposed a path planning for parallel parking based on a combination of clothoid curves and straight lines, which can eliminate the phenomenon of sudden curvature change. References [14] and [15] proposed a path-planning method for parallel parking based on B-spline curves and used the vehicle kinematics model to analyze the possible collisions during parking and established a set of constraint equations to avoid collisions. Reference [16] proposed an automatic parking path planning algorithm based on the driver's experience, that is, the parking path is divided into five sections according to the driver's parking habits, and each section of the path is calculated differently according to the control requirements. References [17], [18], and [19] proposed a method based on a fifth-degree polynomial curve to plan the path for parallel parking. The path is a fifth-degree polynomial curve, which eliminates the phenomenon of sudden curvature change and ensures path continuity. Reference [20] proposed a path planning for parallel parking based on the fifth-degree polynomial curve by selecting constraint points to complete parallel parking of narrow spaces. Reference [21] proposed the use of a combination method involving clothoid curves, circular arcs, and straight lines for entering the parking space, while a combination of straight lines and circular arcs is used to adjust the parking path and enable straight-line parking in narrow spaces. Reference [22] proposed an entry section parking path of the clothoid curve, circular arc, and straight line combination method is designed, and an adjustment section parking path of a linear-arc combination method is designed for narrow parking spaces. Reference [23] proposed dividing the parallel parking path into two parts: the entry path, which guides the vehicle to approach the parking space closely, and the adjustment path, which guides the vehicle to achieve its target pose. The entry path is obtained by solving a constraint optimization problem using a combination of circular arcs, straight lines, and clothoid curves, while the adjustment path is planned based on a combination of circular arcs and straight lines. Reference [24] proposes the use of clothoid curves to eliminate sudden changes in curvature in the circular arc and straight line parking paths. This method offered a path planning approach for parking in narrow spaces. However, the calculation complexity increases due to multiple constraints. Therefore, considering collision constraints in advance and establishing a feasible starting area for parking can simplify the parking process calculations.

This paper proposes a path planning method of parallel parking based on the three-stage curve interpolation method, aiming at the automatic parking application scene of parallel parking in narrow space of urban roadside parking lot, the definition is to ensure that the length of the parking space is as short as possible when the vehicles stop into a parallel parking space in one step. Initially, the parking path curve is constructed through reverse analysis from the parking termination point, using the clothoid curve, circular arc, and fifth-degree polynomial curve. Then, the technical parameter constraints of the vehicle and the collision constraints during the parking process are analyzed, and the feasible starting area for parking is established. By solving the constrained optimization problem, the path curve is obtained. The path planning method proposed in this paper offers several advantages compared to other path planning methods, including effectively eliminating the sudden change point of curvature, satisfying curvature constraints, ensuring the collision-free requirement for obstacles, and requiring less parking space length.

II. PARALLEL PARKING SCENC ANALYSIS

A. VEHICLE KINEMATICS MODEL IN THE PARKING PROCESS

The low speed of the vehicle during parking can be regarded as the absence of lateral movement of the wheels, where the wheels only turn and roll. Therefore, based on the Ackerman steering principle, the vehicle kinematics model is established with the rectangle surrounded by the vehicle's maximum length and width as the vehicle contour boundary, as shown in Figure 1.

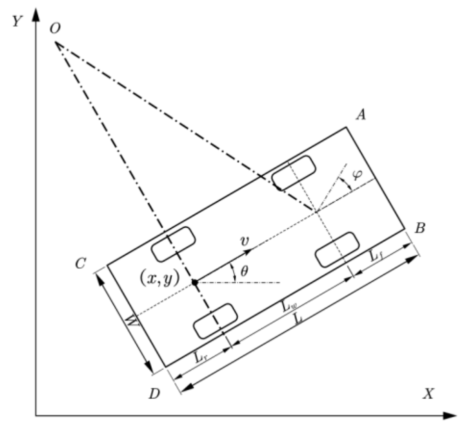

FIGURE 1. Vehicle kinematics model.

where x and y are the coordinates of the center point of the rear axis; v is the speed of the center point of the rear axle, m/s; θ is the yaw angle, °; φ is the equivalent rotation angle of the front wheel, °; L is the vehicle length, m W is the vehicle width, m L_w is the wheelbase of the vehicle, m; L_f and L_r are the length of the front suspension and rear suspension of the vehicle, m; The kinematics equations of the vehicle can be derived as:

$$\begin{cases} \dot{x} = v \cdot cos\theta \\ \dot{y} = v \cdot sin\theta \\ \dot{\theta} = v \cdot tan\varphi / L_w \end{cases} \tag{1}$$

When the vehicle travels along the planned path, according to the kinematic relationship of the vehicle, the instantaneous steering radius r of the center point of the rear axle of the vehicle is equal to the radius of curvature R of the path curve. The relationship between the radius of curvature of the path curve and the instantaneous steering radius of the center point of the rear axle can be described by equation (2). The first derivative of the path curve to x is equal to the tangent value of the vehicle's body attitude angle θ, as shown in equation (3).

$$\frac{tan\varphi}{L_w} = \frac{\ddot{y}}{\left(1+\dot{y}^2\right)^{\frac{3}{2}}} \tag{2}$$

$$\dot{y} = tan\theta \tag{3}$$

The curvature change rate is determined by the steering angle of the front wheel, the maximum steering angular velocity of the steering wheel, and the curvature of the planned path. These factors are limited by the mechanical structure of the vehicle. If the steering angle, steering angular velocity, or path curvature is too large, the vehicle cannot accurately track the planned path. Therefore, the constraints on the steering angle, path curvature, and curvature change rate of the front wheel are as follows:

$$\begin{cases} x_A = x + \left(L_f + L_w\right) \cdot cos\theta - \frac{W}{2} \cdot sin\theta \\ y_A = y + \left(L_f + L_w\right) \cdot sin\theta + \frac{W}{2} \cdot cos\theta \end{cases} \tag{4}$$

$$\begin{cases} x_B = x + \left(L_f + L_w\right) \cdot cos\theta + \frac{W}{2} \cdot sin\theta \\ y_B = y + \left(L_f + L_w\right) \cdot sin\theta - \frac{W}{2} \cdot cos\theta \end{cases} \tag{5}$$

$$\begin{cases} x_C = x - L_r \cdot cos\theta + \frac{W}{2} \cdot sin\theta \\ y_C = y - L_r \cdot sin\theta - \frac{W}{2} \cdot cos\theta \end{cases} \tag{6}$$

$$\begin{cases} x_D = x - L_r \cdot cos\theta - \frac{W}{2} \cdot sin\theta \\ y_D = y - L_r \cdot sin\theta + \frac{W}{2} \cdot cos\theta \end{cases} \tag{7}$$

B. PARALLEL PARKING SCENE REQUIREMENTS

The Autonomous vehicle is located on a two-way single-lane road when performing parallel parking operations. The vehicle performs parking operations along the planned plan from the parking starting point until it reaches the parking termination point after the vehicle finds and detects the idle parking space through the environment perception technology. The parking scene can be established as shown in Figure 2.

where O is the coordinate origin, the parking space length direction is the x-axis, the parking space width direction is the y-axis, W_r is the single lane road width, m; L_p is the length of the parking space, m; W_p is the width of parking space, m. According to the standard of "Specification for parking spaces on urban roads," the size of parallel parking spaces should meet the following conditions: 1) $W_p \geq 2.4$ meters 2) $L_p \geq 6.0$ meters 3) $W_r \geq 2.4$ meters. When the vehicle enters the parking space with an arc with the minimum turning radius as the radius, it takes about 6.25 meters to enter the parallel parking space with the vehicle parameters in this paper. Therefore, the length of the parallel parking space in the narrow space is defined as $L_p \leq 6.5$ meters. Then, the width of single lane W_r is set to 3.6 meters, the length of parking space L_p is 6.5 meters, and the width of parking space W_p is 2.2 meters.

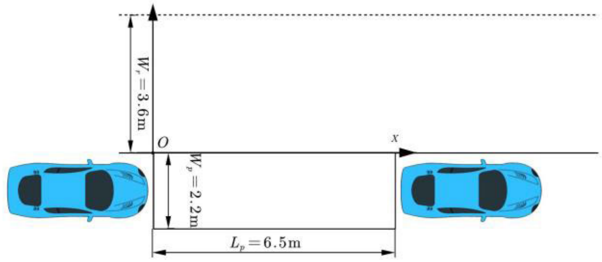

FIGURE 2. Parallel parking scene.

The parallel parking path is designed to be composed of a clothoid curve C_1C_2, a circular arc C_2C_3, and a fifth-degree polynomial curve C_3C_4, as shown in Figure 3. If the vehicle drives out of the parking space with the circular arc with the minimum turning radius, it satisfies the condition of minimizing the length of the parking space. However, curvature mutation occurs at the end of the circular arcs, resulting in the wheels steering in place and aggravating tire wear. The selection of C_1 is based on the size of the parking space as shown in the clothoid curve path planning. The clothoid curve has the characteristic that the curvature can change continuously, which can be used as a transition curve to connect the parking termination point C_1 to the circular arc starting point C_2 where the curvature of the clothoid curve at point C_2 is equal to the curvature of the circular arc of the minimum turning radius, so that the path curvature is continuous, and the curvature mutation phenomenon is eliminated. The C_2C_3 curve is a circular arc curve with the minimum turning radius of the vehicle as the radius. The point C_3 represents the position of the center point of the rear axle of the vehicle when the front of the vehicle is completely out of the parking space. To obtain a continuous curvature trajectory, appropriate initial and termination conditions for position and velocity, as well as appropriate initial and termination curvature values, are required. This involves six constraints, necessitating the use of a fifth-degree polynomial curve to connect the parking starting point C_4 $(x_4, y_4, \theta_4, \varphi_4)$ and the circular arc termination point C_3 $(x_3, y_3, \theta_3, \varphi_3)$. The fifth-degree polynomial curve only needs to determine the pose of the starting point and termination point, which reduces the computational complexity and saves path planning time, compared to the clothoid curve and circular arc that require determining the pose of more than two standard points and calculating the drawing parameters of the clothoid curve and circular arc.

III. PARALLEL PARKING PATH PLANNING

A. CLOTHOID CURVE PATH PLANNING

By reverse analysis, the safe distance between the vehicle and the parking space boundary is set to $L_{safe} = 0.2m$, and the coordinate of the parking termination point is determined as $C_1\left(L_f, -W/2 - L_{safe}\right)$.

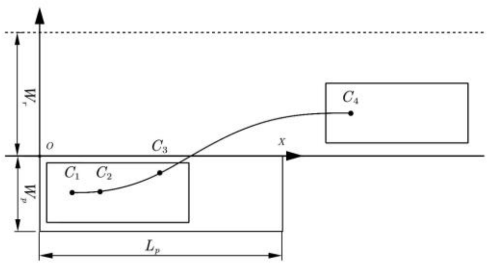

FIGURE 3. Parallel parking path.

Set the curvature change rate of the clothoid curve to be c, the initial point to be the parking termination point C_1, the initial azimuth angle to be zero, and the arc length of the curve to be $s = 1/(R_{min} \cdot c)$. The coordinate of point i with a radius of curvature ρ on the clothoid curve and the azimuth angle α_i be derived as:

$$x_i = \sum_{i=0}^{N} (-1)^i \frac{c^{2i} s^{4i+1}}{2^{2i}(4i+1)(2i)!} \tag{8}$$

$$y_i = \sum_{i=0}^{N} (-1)^i \frac{c^{2i+1} s^{4i+3}}{2^{2i+1}(4i+3)(2i+1)!} \tag{9}$$

$$\alpha_i = \frac{c}{2} s^2 \tag{10}$$

The starting point of the clothoid curve is defined as the parking termination point C_1. The length of the clothoid curve is determined by the curvature change rate and the minimum turning radius. Since the minimum turning radius is already known, the curvature change rate only needs to be less than or equal to the maximum curvature change rate σ_{max} limited by the wheel steering mechanism structure. By inputting the parking termination point C_1 and the appropriate turning curve curvature change rate c into the equations (8), (9), and (10), the termination point $C_2(x_2, y_2)$ of the clothoid curve and the vehicle azimuth α_2 can be obtained.

B. CIRCULAR ARC PATH PLANNING

The minimum turning radius of the vehicle is defined as the radius of the circular arc segment. The center of the circular arc can be expressed by the coordinate at the end point of the clothoid curve $C_2(x_2, y_2)$ and the azimuth angle α_2 of the vehicle, which is $C_0(x_0, y_0) = (x_2 - R_{min} \cdot sin\alpha_2, y_2 + R_{min} \cdot cos\alpha_2)$.

According to the geometric principle, the circular arc equation can be obtained:

$$(x - x_0)^2 + (y - y_0)^2 = R_{min}^2 \tag{11}$$

The starting point of the circular arc is the termination point of the clothoid curve. When the right front point of the vehicle (x_b, y_b) coincides with the parking space boundary line, the coordinate of the center point of the rear axle is the termination point of the circular arc. To obtain the coordinate of the termination point of the circular arc, the turning radius R_b of the right front point of the vehicle is obtained by the geometric parameters of the vehicle,

$$R_b = \sqrt[2]{(L_f + L_w)^2 + \left(R_{min} + \frac{W}{2}\right)^2} \tag{12}$$

Then the steering circular arc equation of the right front point of the vehicle is obtained:

$$(x_b - x_0)^2 + (y_b - y_0)^2 = R_b^2 \tag{13}$$

When the right front point of the vehicle coincides with the boundary line of the parking space, the ordinate of the right front point of the vehicle $y_b = 0$. The right point abscissa x_b is expressed as follows.

$$x_b = x + (L_f + L_w) \cdot cos\theta_3 + \frac{W}{2} \cdot sin\theta_3 \tag{14}$$

where θ_3 is the angle between the direction of motion of the right front point of the vehicle and the x-axis when it travels to the boundary line of the parking space. From equation (2), when the right front point of the vehicle rotates θ angle around the center of the circular arc to coincide with the boundary line of the parking space, the center point of the rear axle also rotates θ angle from C_2 to C_3, the θ_3 can be described as follows:

$$\theta_3 = arctan(\dot{y}_{b1}) - arctan(\dot{y}_{b0}) + \alpha_2 \tag{15}$$

where y_{b0} and y_{b1} are the ordinates of the right front point of the vehicle when the coordinate of the center point of the rear axle is located at the termination point C_2 of the clothoid curve and the termination point C_3 of the circular arc respectively.

By inputting θ_3 into equation (14), the abscissa x_b of the right front point of the vehicle at this time can be obtained. The coordinate of the right front point of the vehicle (x_b, y_b) and the vehicle direction angle θ_3 at this time can be inputted into equation (5) to obtain the termination point coordinate of the circular arc segment $C_3(x_3, y_3)$.

C. FIFTH-DEGREE POLYNOMIAL CURVE PATH PLANNING

The last section of the path is the curve C_3C_4 that connects the termination point of the circular arc $(x_3, y_3, \theta_3, \varphi_3)$ and the starting point of the parking $(x_4, y_4, \theta_4, \varphi_4)$. It is required that the calculation is convenient, and the path curvature is continuous. Therefore, this paper chooses the fifth-degree polynomial curve, and its equation is expressed as:

$$y = c_1x^5 + c_2x^4 + c_3x^3 + c_4x^2 + c_5x + c_6 \tag{16}$$

where x and y are the coordinates of the center point of the rear axis; c_1, c_2, c_3, c_4, c_5 and c_6 are coefficients of fifth-degree polynomial, and $c_1 \neq 0$.

By inputting the coordinate of the termination point of circular arc $C_3(x_3, y_3)$ and the coordinate of parking starting point $C_4(x_4, y_4)$ into equation (16), the fifth-degree polynomial equation of C_3 and C_4 can be obtained:

$$y_3 = c_1x_3^5 + c_2x_3^4 + c_3x_3^3 + c_4x_3^2 + c_5x_3 + c_6 \tag{17}$$

$$y_4 = c_1x_4^5 + c_2x_4^4 + c_3x_4^3 + c_4x_4^2 + c_5x_4 + c_6 \tag{18}$$

The first derivative of the fifth-degree polynomial equation to x can be obtained:

$$\dot{y} = 5c_1x^4 + 4c_2x^3 + 3c_3x^2 + 2c_4x + c_5 \tag{19}$$

According to equation (3), the following equation can be obtained:

$$tan\theta_3 = 5c_1x_3^4 + 4c_2x_3^3 + 3c_3x_3^2 + 2c_4x_3 + c_5 \tag{20}$$
$$tan\theta_4 = 5c_1x_4^4 + 4c_2x_4^3 + 3c_3x_4^2 + 2c_4x_4 + c_5 \tag{21}$$

When the vehicle is at the parking starting point, the path curvature is zero, that is, the equivalent rotation angle of the front wheel is zero, and $\theta_4 = 0$.

The second derivative of the fifth-degree polynomial equation to x can be obtained:

$$\ddot{y}_3 = 20c_1x_3^3 + 12c_2x_3^2 + 6c_3x_3 + c_4 \tag{22}$$
$$\ddot{y}_4 = 20c_1x_4^3 + 12c_2x_4^2 + 6c_3x_4 + c_4 \tag{23}$$

The calculation equation of path curvature is:

$$\rho = \frac{\ddot{y}}{(1+\dot{y}^2)^{\frac{3}{2}}} \tag{24}$$

When the vehicle is at the termination point of the circular arc, the path curvature is the curvature of the circular arc $\rho = 1/R_{min}$, then $\ddot{y}_3$ can be calculated by the above formulas. When the vehicle is at the parking starting point, the path curvature is zero, that is, $\ddot{y}_4 = 0$.

By inputting the pose of point C_3 $(x_3, y_3, \theta_3, \varphi_3)$ and point C_4 $(x_4, y_4, \theta_4, \varphi_4)$ into and solve equations (17), (18), (20), (21), (22) and (23), the fifth-degree polynomial coefficients c_1, c_2, c_3, c_4, c_5 and c_6 can be obtained.

D. DETERMINE THE PARKING FEASIBLE STARTING AREA

The clothoid curve segment and the circular arc segment are determined by fixed parameters such as the maximum curvature change rate and the minimum turning radius, that is, the path followed by these segments remains constant for a given set of vehicle conditions. It allows for pre-calculation of the clothoid curve and circular arc segments, reducing the overall path planning time. Therefore, it is only necessary to calculate the feasible starting area of parking that satisfies the collision constraints, and then select the parking starting point according to the factors such as path length, maximum path curvature, and occupied lane width. The path planning of parallel parking in narrow spaces can be completed by connecting the termination point of the circular arc C_3 with the parking starting point C_4 using the fifth-degree polynomial curve.

When the circular arc is used as the path curve to enter and exit the parking space, the length of the parking space is the shortest. Therefore, when solving the parking starting area, a combination of a straight line a circular arc can be used instead of the fifth-degree polynomial to calculate the coordinates of upper and lower limit points of the parking starting area O_{max} and O_{min}.

The solution of the lower limit point is shown in Figure 4. When the vehicle reaches the termination point of the circular arc C_3, the vehicle still maintains its current body attitude angle and continues to drive in a straight line. When it reaches the limit position, the vehicle drives out of the parking space with a circular equal to the minimum turning radius, to analyze the upper and lower limit points of the parking starting area.

The point where the rear axle intersects the right boundary of the body has the smallest radius of curvature during the parking process. When the vehicle reaches the point where the rear axle aligns with the right boundary of the body and intersects with the parking space line, the body boundary will not collide with any obstacles. At this position, the center point of the rear axle represents the lower limit point of the straight line. The vehicle will turn at the minimum turning radius until its the body attitude angle reaches 0 °, that is, the vehicle is parallel to the road, and the position of the center point of the rear axle represents the lower limit point of the parking starting area.

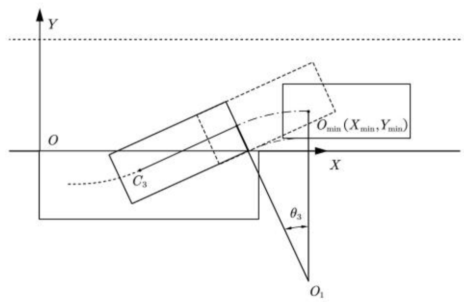

FIGURE 4. Lower limit point calculation diagram.

The horizontal and vertical coordinates of the lower limit point calculated by equation (25) are:

$$\begin{cases} X_{min} = x_b + \left(R_{min} - \frac{W}{2}\right) \cdot sin\theta_3 \\ Y_{min} = R_{min} + \left(R_{min} - \frac{W}{2}\right) \cdot cos\theta_3 \end{cases} \tag{25}$$

The solution for the upper pole is shown in Figure 5. The maximum width of the vehicle occupying the lane is determined by the left front point of the vehicle. To ensure that the vehicle does not collide with the upper boundary, specifically, the left front point of the vehicle,

$$Y_{amax} \leq L_{road} \tag{26}$$

The driving radius of the right front point of the vehicle is:

$$R_a = \sqrt[2]{\left(R_{min} + \frac{W}{2}\right)^2 + \left(L_f + L_w\right)^2} \tag{27}$$

Furthermore, the ordinate of the upper limit point can be calculated as:

$$Y_{max} = L_{road} + R_{min} - R_a \tag{28}$$

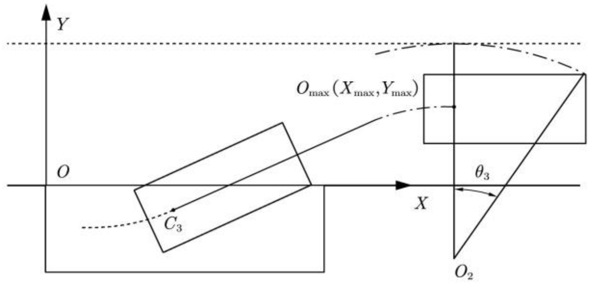

FIGURE 5. Upper limit point calculation diagram.

As shown in Figure 6, the vehicle rotates θ_3 angle at the upper and lower limit points of the straight line with the minimum turning radius. The angle between the line connecting the upper and lower limit points in the parking starting area and the horizontal direction is also θ angle, so the abscissa of the upper limit point can be calculated as:

$$X_{max} = X_{min} + \frac{Y_{max} - Y_{min}}{tan\theta_3} \tag{29}$$

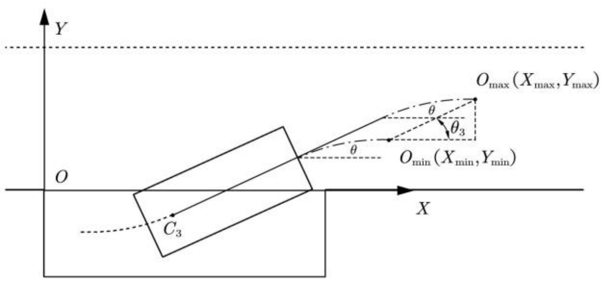

FIGURE 6. Relationship between upper and lower limit point.

Combining equations (25), (28), and (29), it can be calculated by the formula (30) to find the scope of the parking starting area, as shown in Figure 7:

$$\begin{cases} Y = Y_{min} \\ Y = Y_{max} \\ Y = \dfrac{(Y_{max} - Y_{min})}{(X_{max} - X_{min})} \cdot X + Y_{min} - \dfrac{(Y_{max} - Y_{min})}{(X_{max} - X_{min})} \cdot X_{min} \end{cases} \tag{30}$$

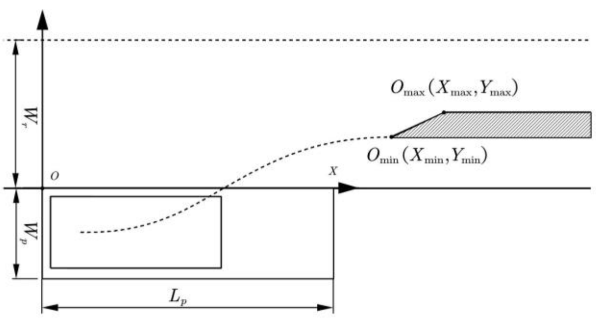

FIGURE 7. The feasible starting area of the parking diagram.

IV. SIMULATION VERIFICATION

To simulate and verify the adaptability and feasibility of the parking path planning algorithm proposed in this paper, the parking scene is established by MATLAB, the parking termination point (0.9225, -1.0695) is fixed, and three parking starting points (8.5,1.2503), (9.6,1.85), (9.6,1.2503) in the parking start area are set up for parking path planning. Three methods of path planning are compared and analyzed: the three-stage curve planning method proposed in this paper (method I), the circular arc and fifth-degree polynomial planning method (method II), and the fifth-degree polynomial planning method (method III).

In the first parking condition (8.5, 1.2503), the horizontal position and vertical position of the parking starting point are the smallest among the parking feasible starting area, resulting in the shortest parking path for the vehicle.

In the second parking condition (9.6, 1.85), the horizontal position of the parking starting point is at its minimum when the vertical position is at its maximum within the feasible parking starting area, resulting in the widest lane occupied by the vehicle during parking.

In the third parking condition (9.6, 1.2503), the horizontal position of the parking starting point is the same as in condition two, and the vertical position is the same as in condition one, resulting in a smaller curvature of the parking path compared to the other two conditions.

According to a common passenger car, the basic parameters of the selected vehicle are shown in Table 1.

TABLE 1. Basic parameters of the vehicle.

Parameters	Value
Vehicle length L/m	4.1
Vehicle width W/m	1.739
Wheelbase L_w/m	2.579
Front overhang L_f/m	0.75
Front overhang L_r/m	0.7225
Maximum equivalent front wheel angle $\delta/°$	37.5°
Minimum turning radius R_{min}/m	5

A. THE FIRST PARKING CONDITION

The contour envelope diagram of the vehicle driving along the planned path is shown in Figure 8, 9, and 10 (The motion trajectory of A, B, C, and D represents the left front, right front, left back, and right back points of the vehicle, respectively, in the figures.). By comparing the path curve and vehicle contour envelope diagram of the three path planning methods, it can be observed that, during the parking process, the vehicle contour line of three methods does not intersect with the red parking space boundary line, which can safely, accurately, and smoothly guide the vehicle into the parking space.

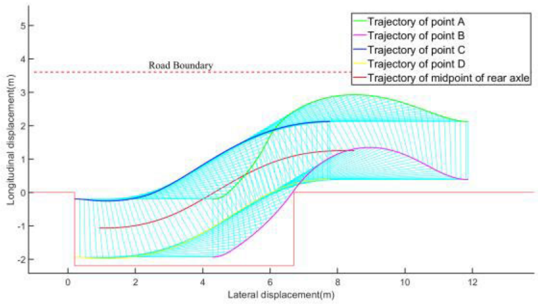

FIGURE 8. **The contour envelope diagram of the vehicle driving along the method I with a starting point of (8.5,1.2503).**

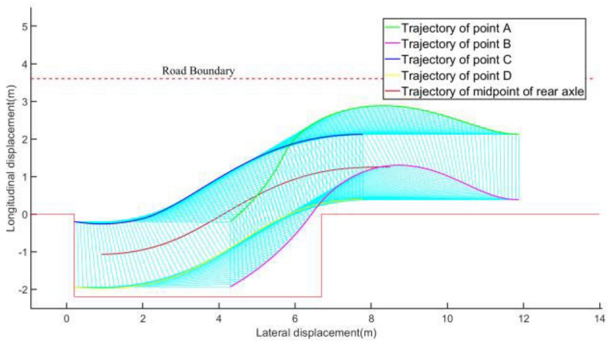

FIGURE 9. **The contour envelope diagram of the vehicle driving along the method II with a starting point of (8.5,1.2503).**

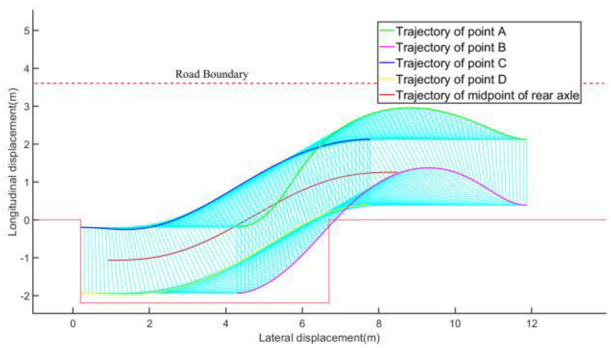

FIGURE 10. **The contour envelope diagram of the vehicle driving along the method III with a starting point of (8.5,1.2503).**

The curvature of the path planned by the three path planning methods is shown in Figure 11. The path curvature planned based on the fifth-degree polynomial curve exceeds the constraints of vehicle technical parameters (the curvature is not larger than $1/R_{min} = 0.2$ as shown in the dotted line in the figure) so that the vehicle cannot track the path, while the other two methods satisfy the constraints of vehicle technical parameters. Compared with the path planning method based on circular arc and fifth-degree polynomial, the path planning method proposed in this paper eliminates the phenomenon of sudden changes in curvature, that is, it avoids steering in place at the parking termination point, thereby preventing tire wear. Therefore, the method in this paper can not only ensure the continuity of curvature, but also satisfy the curvature constraint, which is better than the other two methods.

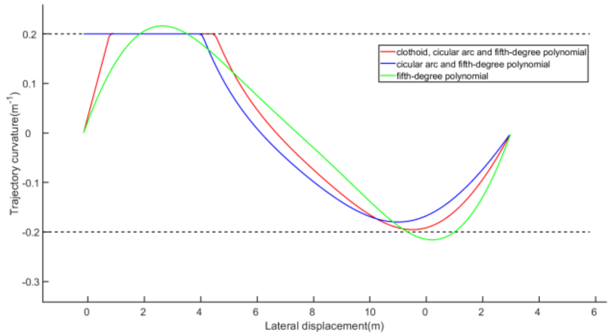

FIGURE 11. **The path curvature comparison diagram with a starting point of (8.5,1.2503).**

The yaw angle of the vehicle traveling along the planned path of the three path planning methods is shown in Figure 12. When the vehicle follows the path planned based on the fifth-degree polynomial curve, the maximum yaw angle of the vehicle is smaller compared to the other two methods. Additionally, the longitudinal displacement at which the maximum yaw angle occurs is larger. This implies that when the vehicle follows the path planned based on the fifth-degree polynomial curve, it needs to initiate the steering maneuver earlier, resulting in a longer required parking space length.

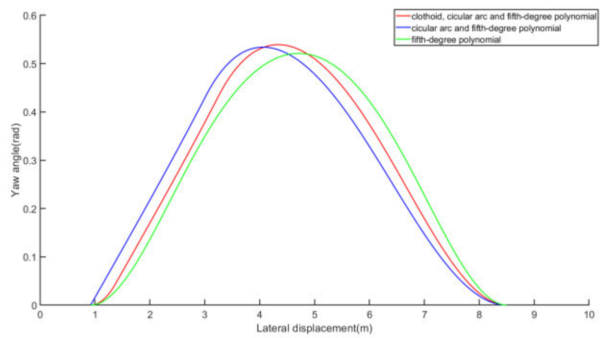

FIGURE 12. **The vehicle yaw angle comparison diagram with the starting point of (8.5,1.2503).**

B. THE SECOND PARKING CONDITION

The contour envelope diagram of the vehicle driving along the planned path is shown in Figure 13, 14, and 15 (The motion trajectory of A, B, C, and D represents the left front, right front, left back, and right back points of the vehicle, respectively, in the figures.). By comparing the path curve and vehicle contour envelope diagram of the three path planning methods, it is evident that the contour line of the vehicle parking along the path planned based on the fifth-degree polynomial curve intersects with the red boundary line of the parking space. This indicates that the vehicle would collide with the obstacles in the surrounding environment. In contrast, the contour lines of the vehicle driving along the planned path of the other methods do not intersect with the red parking space boundary line and can guide the vehicle into the parking space safely, accurately, and smoothly.

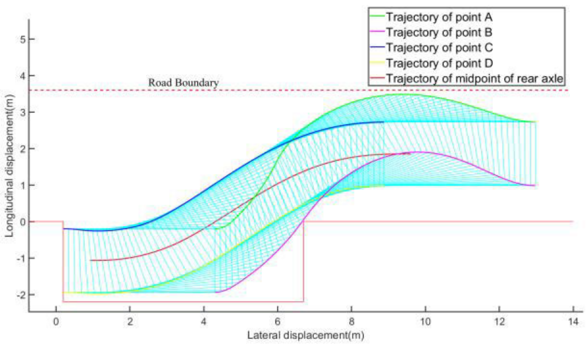

FIGURE 13. **The contour envelope diagram of the vehicle driving along the method I with a starting point of (9.6,1.85).**

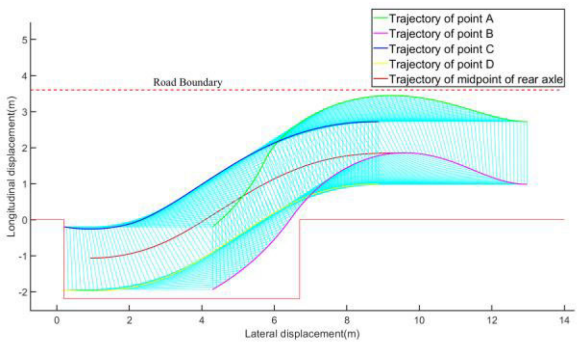

FIGURE 14. **The contour envelope diagram of the vehicle driving along the method II with a starting point of (9.6,1.85).**

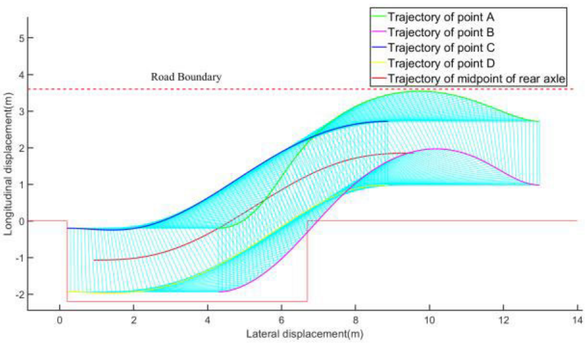

FIGURE 15. **The contour envelope diagram of the vehicle driving along the method III with a starting point of (9.6,1.85).**

The path curvature of the vehicle traveling along the planned path of the three path planning methods is shown in Figure 16. The path curvature based on the fifth-degree polynomial curve planning still exceeds the constraints of vehicle technical parameters (the curvature is not larger than $1/R_{min} = 0.2$ as shown in the dotted line in the figure), and the vehicle collides with the obstacle. Under this condition, this method still cannot achieve parallel parking in a narrow space. The other two methods satisfy the constraints of vehicle technical parameters. The path planning method based on circular arc and fifth-degree polynomial also appears in the problem of curvature mutation at the end point of parking. Therefore, the method proposed in this paper can not only ensure the continuity of curvature, but also satisfy the curvature constraint, which is better than the other two methods.

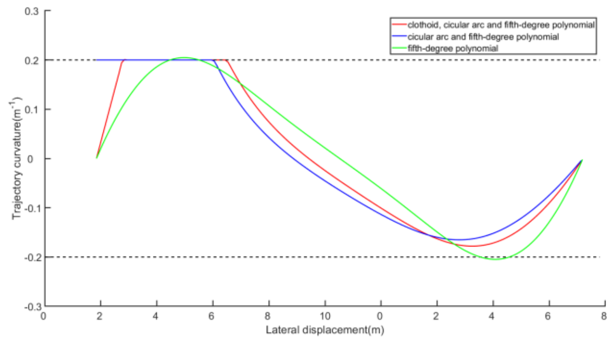

FIGURE 16. **The path curvature comparison diagram with a starting point of (9.6,1.85).**

The yaw angle of the vehicle traveling along the planned path of the three path planning methods is shown in Figure 17. Although the maximum yaw angles of the vehicle along the planned paths are similar for all three methods, there is a significant difference in the longitudinal displacement at which the maximum yaw angle occurs. It is concluded that the steering center of the three-stage curve planning path in this paper occurs earlier compared to the path based on circular arc and fifth-degree polynomial planning, and later compared to the path based on fifth-degree polynomial planning. This observation highlights the impact on the required parking space length. In other words, the three-stage path planning method requires a shorter parking space length compared to the fifth-degree polynomial planning method, but a larger length compared to the circular arc and fifth-degree polynomial planning method.

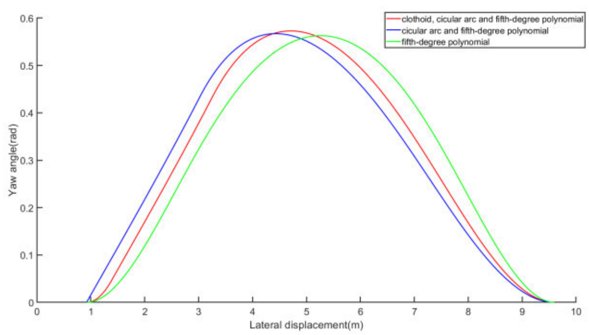

FIGURE 17. **The vehicle yaw angle comparison diagram with the starting point of (9.6,1.85).**

C. THE THIRD PARKING CONDITON

The contour envelope diagram of the vehicle driving along the planned path is shown in Figure 18, 19, and 20 (The motion trajectory of A, B, C, and D represents the left front, right front, left back, and right back points of the vehicle, respectively, in the figures.). By comparing the path curve and vehicle contour envelope diagram of the three path planning methods, it is evident that the contour line of the vehicle parking along the path planned based on the fifth-degree polynomial curve intersects multiple times with the boundary line of the parking space. This indicates that the vehicle would experience severe collisions with the obstacles in the surrounding environment, so it unable to enter the parking space. In contrast, the contour line of the vehicle parking along the rest of the path does not intersect with the red parking space boundary line, which can guide the vehicle into the parking space safely, accurately, and smoothly.

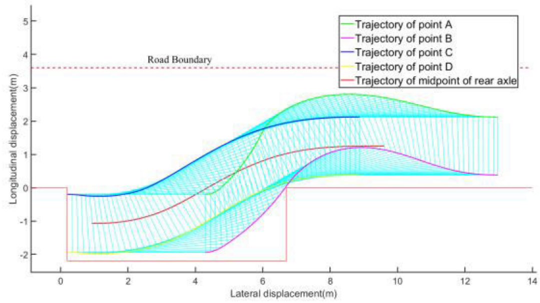

FIGURE 18. **The contour envelope diagram of the vehicle driving along the method I with a starting point of (9.6,1.2503).**

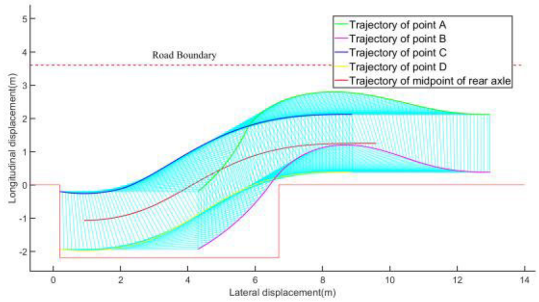

FIGURE 19. **The contour envelope diagram of the vehicle driving along the method II with a starting point of (9.6,1.2503).**

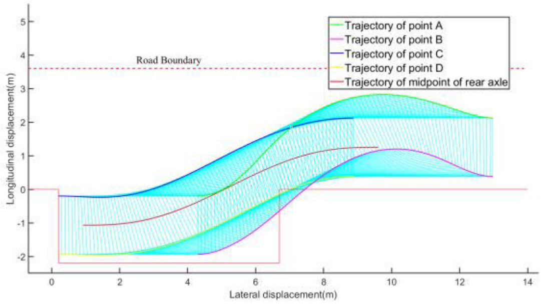

FIGURE 20. **The contour envelope diagram of the vehicle driving along the method III with a starting point of (9.6,1.2503).**

The path curvature of the vehicle traveling along the planned path of the three path planning methods is shown in Figure 21. The three path planning methods all satisfy the constraints of the vehicle technical parameter (the curvature is not larger than $1/R_{min} = 0.2$ as shown in the dotted line in the figure). The curvature of the path planned based on the fifth-degree polynomial curve near the parking termination point is significantly smaller compared to the other two path planning methods. Moreover, the overall curvature of the path is more continuous, resulting in smoother vehicle motion and enhanced driving comfort for the occupants. However, this path planning method requires excessive parking space length and does not meet the requirements for parallel parking in narrow spaces. The path planning method based on circular arc and fifth-degree polynomial curve also has the problem of curvature mutation at the termination point of parking. Therefore, the method proposed in this paper can not only ensure the continuity of curvature but also satisfy the curvature constraint, which is better than the other two methods.

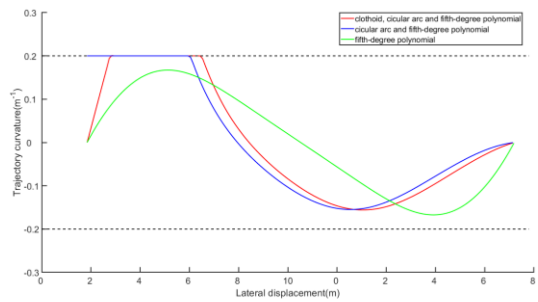

FIGURE 21. **The path curvature comparison diagram with a starting point of (9.6,1.2503).**

The yaw angle of the vehicle driving along the planned path of the three path planning methods is shown in Figure 22. When the vehicle follows the planned path based on the fifth-degree polynomial curve path planning method, and the maximum yaw angle of the vehicle is significantly smaller than the other two methods. The position of the maximum yaw angle is also different from the above conditions, which deviates from the longitudinal displacement of the maximum yaw angle of the other two methods. The vehicle needs to initiate steering maneuvers in advance when driving along the path planned based on the fifth-degree polynomial curve. Therefore, the vehicle cannot successfully enter the parking space when parking in a narrow space. In contrast, the three-stage path planning method proposed in this paper and the path planning method based on the circular arc and fifth-degree polynomial curve are replaced by other curves at the parking space, which can delay the steering operation of the vehicle and reduce the demand for the length of the parking space. Due to the phenomenon of curvature mutation at the end of the circular arc, the combination of the clothoid curve and the circular arc used in this paper effectively solves this problem.

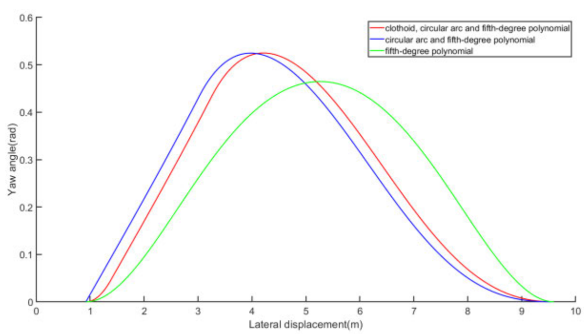

FIGURE 22. **The vehicle yaw angle comparison diagram with the starting point of (9.6,1.2503).**

TABLE 2. The comparative analysis of three path planning methods in three parking conditions.

Conditions	The path planning method	The length of parking space occupied	Whether the vehicle collides with surrounding obstacles	Whether the path curvature is continuous	Whether the curvature constraint is satisfied	The position where the maximum course Angle occurs
The first	Method I	6.47m	No	Yes	Yes	4.34m
	Method II	6.24m	No	No	Yes	4.08m
	Method III	6.67m	Yes	Yes	No	4.70m
The second	Method I	6.47m	No	Yes	Yes	4.71m
	Method II	6.24m	No	No	Yes	4.43m
	Method III	6.98m	Yes	Yes	No	5.27m
The third	Method I	6.47m	No	Yes	Yes	4.22m
	Method II	6.24m	No	No	Yes	3.98m
	Method III	7.45m	Yes	Yes	Yes	5.26m

D. SUMMARY OF COMPARATIVE ANALYSIS

According to the above comparison analysis based on the evaluation criteria, the three-stage path planning method combining fifth-degree polynomial, circular arc, and clothoid curve demonstrates advantages over the fifth-degree polynomial path planning method by ensuring curvature continuity, occupying less parking space length, and reducing the parking space length requirement. Compared to the path planning method that combines fifth-degree polynomial curve and circular arc, the path planned using the three-stage method is curvature continuous, without abrupt changes in curvature, and satisfies the curvature constraints. Therefore, it can be concluded that the three-stage path planning method possesses the advantages of requiring less parking space, maintaining curvature continuity, and satisfying curvature constraints. Moreover, under three different parking conditions, the vehicle can park in narrow parallel spaces without collisions, thereby demonstrating the adaptability and feasibility of the three-segment path planning method.

V. CONCLUSION

(1) Aiming at the path planning problem of parallel parking in narrow spaces, this paper analyzes and establishes the parallel parking scene in narrow spaces, and determines the parking termination point and the designed parking path.

(2) A parallel parking path planning method based on the three-stage curve interpolation method is proposed. Firstly, through reverse analysis, a parallel parking path curve is designed by using the clothoid curve, circular arc, and fifth-degree polynomial curve. Constraints on the vehicle's technical parameters and parking collision constraints are analyzed and established, along with a feasible starting area for parking. And the path planning of parallel parking in narrow spaces is completed. The planned path not only ensures the continuous curvature of the parallel parking path but also reduces the demand for parking space length.

(3) Three different parking starting conditions are designed. The path planning method, the circular arc and fifth-degree polynomial curve path planning method, and the fifth-degree polynomial curve path planning method proposed in this paper are verified through group simulation. The verification results show that the path planning method proposed in this paper can satisfy the constraints of the vehicle technical parameter and collision constraints compared with the path planning method based on the fifth-degree polynomial curve and the path planning method based on the circular arc and fifth-degree polynomial curve. The method proposed in this paper offers the advantages of simple and convenient calculations as well as a wide range of applications. The vehicle can safely, accurately, and smoothly enter the parking space following the planned trajectory, thereby addressing the challenge of parallel parking in narrow spaces.

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