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RESEARCH ARTICLE

Massive MIMO Detectors Based on Deep Learning, Stair Matrix, and Approximate Matrix Inversion Methods

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ABSTRACT Massive multiple-input multiple-output (MIMO) is an essential technology in fifth-generation (5G) and beyond 5G (B5G) communication systems. Massive MIMO is employed to meet the increasing request for high capacity in next-generation wireless communication networks. However, signal processing in massive MIMO incurs a high complexity due to a large number of transmitting and receiving antenna elements. In this paper, we propose low complexity massive MIMO data detection techniques based on zero-forcing (ZF) and vertical bell laboratories layered space-time (V-BLAST) method in combination with approximate matrix inversion techniques; Neumann series (NS) and Newton iteration (NI). The proposed techniques reduce the complexity of the ZF V-BLAST method since they avoid the exact matrix inverse computation. Initialization based on a stair matrix is also exploited to balance the performance and the complexity. In addition, we propose a massive MIMO detector based on approximate matrix inversion with a stair matrix initialization and deep learning (DL) based detector; MM Network (MMNet) algorithm. MMNet contains a linear transformation followed by a non-linear denoising stage. As signals propagate through the MMNet, the noise distribution at the input of the denoiser stages approaches a Gaussian distribution, form precisely the conditions in which the denoisers can attenuate noise maximally. We validated the performance of the proposed massive MIMO detection schemes in Gaussian and realistic channel models, i.e., Quadriga channels models. Simulations demonstrate that the proposed detectors achieve a remarkable improvement in the performance with a notable computational complexity reduction when compared to conventional ZF V-BLAST and the MMNET in both simple and real channel scenarios.

INDEX TERMS 5G, massive MIMO, Gauss-Seidel, successive overrelaxation, Neumann series, Newton iteration, deep learning, data detection, stair matrix, diagonal matrix, V-BLAST.

I. INTRODUCTION

Massive multiple-input multiple-output (MIMO) is a crucial technology in developing next-generation wireless communication networks, as it offers numerous advantages, such as increased network capacity and improved reliability. Over the past decade, a large number of transmitting and receiving antennas have been employed in massive MIMO networks to support the rapid increase in demand for bandwidth. Moreover, a large number of antennas in massive MIMO systems improve spectral efficiency and reliability, which are critical requirements for beyond fifth-generation (B5G) wireless networks. However, the computational complexity of massive MIMO detectors increases when a large number of antennas are utilized, making the conventional data detection techniques impractical for large-scale MIMO configurations. A survey of massive MIMO detectors is shown in [1] and [2], where a wide range of detectors are illustrated and criticized. The maximum likelihood (ML) based detector achieves the best performance [3]. Unfortunately, ML based detector uses an exhaustive search which causes the complexity to grow exponentially as the number of antennas increases in the

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massive MIMO systems [3]. Linear and non-linear massive MIMO detection techniques are studied to find near-optimal performance while reducing the computational complexity. Linear detectors, including the matched filter (MF) [4], the zero-forcing (ZF) [5], and the minimum mean-square error (MMSE) [6] are dependent on the matrix inversion, which is a computationally expensive mathematical operation in massive MIMO systems. Various iterative methods that use matrix inverse approximation are proposed to achieve good performance with low complexity when compared with MMSE based detector [7]. Using approximate matrix inversion in these detectors reduces the complexity required by avoiding the computation of the exact matrix inverse of the Gramian matrix [8]. These methods work well under specific configurations for massive MIMO detection, i.e., the number of antenna elements at the base station (BS) is larger than the number of transmitting users [1]. Neumann series (NS) is a popular matrix inverse approximation method implemented to reduce the complexity compared to the MMSE based detector [9]. The performance of the NS techniques suffers when the ratio of the receiving antennas and users is close to 1. Newton iteration (NI) is another iterative method that can obtain an approximate matrix inverse with good precision using a few iterations [10].

It is well known that the equalization matrix is diagonally dominant. Therefore, most existing detectors are utilizing the diagonal matrix in their detection. However, detection methods based on a diagonal matrix may not converge under certain circumstances, especially when the number of users approaches the number of BS antennas. In [11], stair matrices are discussed with a focus on their applications to iterative methods. Moreover, initialization is developed based on a stair matrix and is exploited for massive MIMO detectors through approximate matrix inversion methods. Different linear massive MIMO detectors, such as the Gauss-Seidel (GS) and the successive over-relaxation (SOR), are proposed to achieve a performance gain compared to other approximate matrix inversion methods [12].

However, linear detectors may suffer from a performance loss when the number of users approaches the number of BS antennas; hence, non-linear detectors are proposed. The vertical bell laboratories layered space-time (V-BLAST) is a non-linear detection algorithm based on linear detectors. The detection algorithm detects the transmitted symbols iteratively and utilizes linear detection techniques at each iteration [13].

A. RELATED WORK

Deep learning (DL) is one of the most auspicious technologies for artificial intelligence (AI) and achieved enormous success in signal and image processing, speech recognition, and biomedical sciences. The DL approach uses artificial neural networks to solve complex problems by modeling these problems into layers of neurons that are inspired by the human brain [14]. In wireless communication systems, it can be trained effectively to predict/approximate the transmitted signal vectors [15]. DL based detectors can be classified as: the single channel realization architectures and varying channel realization architectures [2]. The DL based detectors were used to attain an optimal performance while minimizing the complexity. However, the results of the detection techniques degrade when dealing with real channel models and high modulation schemes. Additionally, some of the DL techniques suffer from high computational complexity in specific scenarios preventing their implementation in real applications. Significant research in recent years used the DL approach such as [3], [16], [17], [18], [19], [20], and [21].

Since 2017, there has been a prominent trend in the research and industry communities to employ DL in massive MIMO detectors. DL is recruited to shift the computational complexity of the massive MIMO detectors to the offline phase, enabling faster run-time in the online data detection phase. The deep network in a massive MIMO detector is employed in DetNet [22] where a projected gradient descent method is used. DetNet performs well in simple channel scenarios and with low-order modulation schemes. A modified DetNet requires a small number of parameters to be optimized; unfortunately, DetNet training is unstable in the case of realistic and correlated channels. In addition, the scalability of the DetNet algorithm is miserable because of a huge number of training parameters. In recent years (2018-2023), a substantial stream in the research sector to avail DL to create a robust massive MIMO detector is observed.

A model-driven DL network is demonstrated based on the orthogonal approximate message passing network (OAMP-Net) [23]. The detector based on OAMP-Net adds some adjustable parameters to the existing OAMP method, but a strict assumption has to exist. The OAMP-Net is dominated by the matrix inverse in each layer. Hence, it is not feasible for real-life applications/implementation due to high computational complexity. The OAMP-Net performance is improved, and OAMP-Net2 is proposed in [24], where new training parameters are demonstrated. Unlike the OAMP-Net, imperfect channels are taken into consideration. However, similar to the original OAMP-Net, it is dominated by the matrix inverse. The MMNet algorithm is proposed to conquer challenges in both the DetNet and the OAMPNet [25]. MMNet is designed to be trained online for each channel matrix and an iterative soft thresholding algorithm is used. Although it obtains a satisfactory performance when implemented in a realistic channel simulator, MMNet incurs high latency due to sequential online training. In addition, the performance significantly deteriorates when high modulation schemes are employed.

A HyperMIMO based detector displaces the MMNet training process by a single inference over a trained hyper-network where the number of MMNet parameters are reduced [26]. In comparison with the MMNet, HyperMIMO performs slightly worse. It also has to be re-trained when serious changes in the channel statistics are observed. In [27], a massive MIMO detector based on an efficient data-driven DL

TABLE 1. Advantages and disadvantages of existing DL based detectors.

DL Technique	Advantages	Disadvantages
DetNet [22]	• It has adequate performance when tested on constant and Rayleigh fading channels. • It performs well in low-order modulation such as BPSK and 4-QAM.	• It contains a large number of trainable parameters, which makes the technique un-scalable for high-order modulation and large MIMO systems. It uses a single detection model for different channel models. • The performance is not stable for realistic and correlated channels.
OAMP-Net [23]	• It achieves higher SER performance than DetNet on i.i.d. Gaussian channels. • OAMP-Net solves the issue of the high number of trainable parameters that exist in other DL algorithms. • OAMP-Net uses only two trainable parameters in each layer and performs well while using low-order modulation schemes.	• It experiences high complexity because of the matrix inverse operation required in each layer. • The performance is degraded in realistic and correlated channels. • It uses a single detection model for different channel models and does not perform well in massive MIMO systems.
MMNet [25]	• It addresses the bad performance of OAMPNet and DetNet in realistic channels. • It offers online training for each realization of $\boldsymbol{H}$ and does not use matrix inversion in each layer. • It has lower complexity than OAMPNet and DetNet architectures. • It has better performance than the MMSE, DetNet, and OAMPNet in realistic channels. It provides, in the linear and denoising stages of each layer, a balance between flexibility and complexity.	• The performance is reduced when using high-order modulation schemes. • It requires the neural network to be retained for each realization of the channel matrix ($\boldsymbol{H}$), which causes the real practical implementation to be complicated. • It experiences latency because of the sequential online training.
HyperMIMO [32]	• It has lower complexity than the MMNet. This is realized by minimizing the number of trainable parameters in each layer. • It has superior performance compared to the OAMP-Net DL algorithm. • It was practically implemented and has good robustness against user mobility.	• It has a lower performance than the MMNet algorithm. • It requires to be retained when the channel matrix is changed considerably.
DLNet [16], [17]	• A new loss function is proposed based on the sum of mean squared error between the transmitted and estimated signals at each layer. • It avoids matrix inversion. • It can be used in a variety of channel distributions and statistics.	• It is not tested in real channel scenarios.

network is proposed. An extrapolation factor is exploited as a learnable parameter for the iterative sequential detector. Moreover, additional learnable parameters and multiple soft-sign activation functions are demonstrated to host different high modulation orders.

A model-driven DL structure for data detection and channel estimation is proposed in [28]. The proposed special DL structure takes advantage of domain knowledge in the few-bit quantization process and relies on the original quantized system model. Unfortunately, the performance of this detector significantly deteriorates when spatially correlated channels are used. In [16], a massive MIMO decoder based on DL with 40 layers has been proposed. At each layer, a loss function is demonstrated based on the sum of mean-squared errors between transmitted and estimated signals. This decoder achieves a significant computational complexity reduction due to the non-existence of matrix inversion. However, this method is not tested in the case of realistic channel scenarios. In [29], a ZF-ML framework with DL approach is utilized to design an efficient massive MIMO detector. The DL is employed to suppress the interfering signals. The work in [30] considers the DL approach for a small-scale MIMO detectors and shows the benefits of DL employment in the performance-complexity profile of MIMO detector.

The work in [15] provides insight into how to leverage the DL scheme for massive MIMO detector design. In [31], approximate matrix inversion methods are exploited to enhance the performance of the MMNet while attaining low computational complexity. The utilization of approximate matrix inversion methods such as the GS and the SOR with MMNet method provided remarkable performance enhancement for the MMNet detection architecture [31].

Table 1 presents a brief overview of the DL-based detectors such as the DetNet, the OAMP-Net, the MMNet, and the HyperMIMO. It is clear that most existing DL based detectors are unstable for realistic channel scenarios or suffer a significant performance deterioration compared to the ML based detectors. However, they perform well in Gaussian channels and low modulation schemes. Motivated by this challenge, we revisit large-scale MIMO detection to strike a balance between performance and complexity in realistic channel scenarios.

B. CONTRIBUTION AND ORGANIZATION

Massive MIMO technology is employed in 5G communication systems to achieve a high quality-of-service (QoS) and increase capacity. However, massive MIMO receiver design is not a trivial task. One of the major problems in massive MIMO is achieving a satisfactory trade-off in the detector's performance-complexity profile. This challenge is becoming imperious when the number of transmitting antennas is approaching the number of receiving antennas. In addition, this problem is more substantial in the existence of a practical channel scenario. The motivation for this paper originates from obtaining an attractive trade-off between the computational complexity and the performance of massive MIMO detectors based on DL and approximate matrix inversion methods in realistic channel scenarios.

This paper also introduces a low-complexity massive MIMO detection method based on a modified version of the V-BLAST algorithm and the approximate matrix inversion methods such as the NS and NI. It should be noted that the initialization stage has a great impact on the convergence rate, and hence the detectors performance and computational complexity. Therefore, the stair matrix initialization is employed for both the NS and NI methods to improve the performance of the proposed V-BLAST massive MIMO detector. Design of the proposed algorithm ensures that good performance is maintained and a considerable reduction in complexity is achieved since matrix inversion is avoided in the ZF V-BLAST algorithm by using proposed iterative methods. Moreover, a stair matrix initialization with approximate matrix inversion is leveraged to improve the performance of a DL based detector.

The main contributions of the paper are summarized as follows:

- We propose massive MIMO detection techniques that have good performance and low complexity based on ZF V-BLAST and the approximate matrix inversion techniques, such as the NS and NI methods. The proposed techniques avoid exact matrix inversion in the ZF V-BLAST algorithm by using iterative methods. Moreover, the stair matrix initialization is utilized for both NS and NI methods to reduce the computational complexity of the proposed detectors with a high-performance gain.
- We also propose a hybrid structure between the DL and approximate matrix inversion methods. The initialization stage is also developed based on the stair matrix structure while the detection process exploits the MMNet structure.
- We use extensive simulations to investigate the performance and the computational complexity of the proposed massive MIMO detectors in different configurations. In order to avoid any misleading conclusion, perfect channel state information (CSI) and real channel scenarios are considered.

The rest of the paper is organized as follows. Section II introduces the massive MIMO system model. In Section III, we discuss different relevant linear and non-linear massive MIMO detection methods. Section IV describes the proposed massive MIMO detection technique. Section V provides the complexity analysis. Section VI presents the implementation details and performance results of the proposed techniques. Finally, summary and future research directions are provided in Section VII.

Notation: This list demonstrates the notations which are used in the paper:

a, μ, A	Scalar. Italics characters of standard weight.
$\boldsymbol{v}$	Vector. Boldface, lower case.
$\boldsymbol{M}$	Matrix. Boldface, upper case alphabetical.
$\boldsymbol{v}_i$	Element i of vector $\boldsymbol{v}$.
$\boldsymbol{M}_{i,j}$	Element at row i and column j of matrix, or matrix expression, $\boldsymbol{M}$.
$[\boldsymbol{M}]_{mm}$	The $(m, m)^{\text{th}}$ element of a matrix $\boldsymbol{M}$.
$\boldsymbol{M}^T, \boldsymbol{v}^T$	Matrix and vector transpose.
$\boldsymbol{M}^{-1}$	Matrix inverse.
$\boldsymbol{H}^+$	Moore-Penrose generalized inverse of $\boldsymbol{H}$.
$\|\boldsymbol{v}\|$	Euclidean norm of vector $\boldsymbol{v}$.
$\{a_i\}$	The set containing elements a_i.
$\max\{\cdot\}$	The maximum value taken by the argument.
$\min\{\cdot\}$	The minimum value taken by the argument.
$E[x]$	Expectation of random variable x.
$\boldsymbol{I_K}$	$K \times K$ identity matrix.

II. SYSTEM MODEL

The system model for the detection of uplink massive MIMO system is provided in (1) [32]. We assume a massive MIMO system with N_r total number of receiving antennas at the BS and N_t users with a single antenna. The transmitted symbols by the transmitters are considered as a $N_t \times 1$ vector denoted by $\boldsymbol{x}$ while the received signal is a $N_r \times 1$ vector represented by $\boldsymbol{y}$. The relationship between the received signal

$\boldsymbol{y}$ and the transmitted symbols $\boldsymbol{x}$ is described as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{n}$ is $N_r \times 1$ noise vector that is assumed as additive white Gaussian noise (AWGN) $n_i \sim \mathcal{CN}\left(0, \sigma^2\right)$ with independent and identically distributed (i.i.d) entries.

The channel matrix ($\boldsymbol{H}$) represents the channel coefficients between the N_t single antenna users and the N_r receiving antennas at the (BS). The channel matrix ($\boldsymbol{H}$) for an $N_r \times N_t$ massive MIMO system is represented as

$$\boldsymbol{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1j} \\ h_{21} & h_{22} & \cdots & h_{2j} \\ \vdots & \vdots & \vdots & \vdots \\ h_{i1} & h_{i2} & \cdots & h_{ij} \end{bmatrix}. \tag{2}$$

The power for the channel coefficient h_{ij} between the jth user and ith receiving antenna at the BS is provided by $P_{ij} = E\left[\left|h_{ij}\right|^2\right]$. The noise power is given by $E\left[|n_i|^2\right] = \sigma^2$ and transmitted signals are normalized where $E\left[\left|x_j\right|^2\right] = 1$. The transmitted symbols $\boldsymbol{x}$ are selected randomly from the predefined constellation set by the transmitting antennas. The signal-to-noise ratio (SNR) is represented by $\text{SNR} = E\left[\left|x_j\right|^2\right]/\sigma^2$.

Gaussian and realistic channel models, in addition to channels with imperfect channel estimation, are considered to evaluate the proposed detection techniques. The Gaussian channels assume variables that have zero mean and unity variance. Moreover, the channel coefficients are considered to be i.i.d complex Gaussian random variables in addition to having a power spectral density (PSD) of $N_0/2$ [33]. In the case of realistic channels, we consider the quasi-deterministic radio channel generator (Quadriga) model provided in [34].

The Quadriga simulator is utilized for the simulation of realistic channels in massive urban MIMO system configurations. The Quadriga channel model based on the 3^{rd} generation partnership project (3GPP) can generate realistic channel impulse responses that consider various factors such as the users' variable speeds, arbitrary length of the channel traces, and multi-dimensional propagation [34]. We also consider the model presented in [35] for the imperfect channel estimation. In practical systems, the receiver has an estimate $\hat{\boldsymbol{H}}$ of the actual channel $\boldsymbol{H}$. The channel state information (CSI) for the imperfect channel is given by $\hat{\boldsymbol{H}} = r_0\boldsymbol{H} + \sqrt{1 - r_0^2}\boldsymbol{E}$, [35] where r_0 represents the correlation coefficient between the actual channel $\boldsymbol{H}$ and the estimated channel $\hat{\boldsymbol{H}}$. The error matrix $\boldsymbol{E}$ is statistically identical to $\boldsymbol{H}$ and is independent of $\boldsymbol{H}$.

III. MASSIVE MIMO DETECTION

A. CLASSICAL LINEAR MASSIVE MIMO DETECTORS

The purpose of massive MIMO detection is to determine the transmitted symbols vector sent by the users using the received signal and the channel matrix [36].

The maximum likelihood (ML) achieves the best performance. However, it performs an exhaustive search to find all the possible solutions for the received signal. The ML algorithm is represented as [3]

$$\hat{\boldsymbol{x}}_{ML} = \arg \min_{\boldsymbol{x} \in O^K} \|\boldsymbol{y} - \boldsymbol{H}\|_2^2, \tag{3}$$

where $\hat{\boldsymbol{x}}_{ML}$ is received signal estimated by the ML detector. However, the computational complexity of the ML based detector increases exponentially as the number of antennas increases O^{N_t} [3]. Therefore, the ML algorithm is unsuitable for practical implementation due to a high computational complexity.

1) LINEAR ZF DETECTOR

The ZF detector has better performance than the MF detector [5]. It tries to maximize the received signal-to-interference-plus-noise ratio (SINR) by inverting the channel matrix. The equalization matrix for linear ZF detector is given as [5]

$$\boldsymbol{A}_{ZF}^H = \left(\boldsymbol{H}^H\boldsymbol{H}\right)^{-1}\boldsymbol{H}^H = \boldsymbol{H}^+, \tag{4}$$

where $\boldsymbol{H}^+$ represent the Moore-Penrose pseudo-inverse of the matrix $\boldsymbol{H}$. It should be noted that the ZF detector ignores the noise and works well when the interferences are limited.

2) LINEAR MMSE DETECTOR

Minimum mean-square estimation (MMSE) is a linear detection technique that takes the effects of the noise into consideration. Hence, it outperforms the MF and ZF linear detectors at lower SNR [37]. It works by minimizing the mean-square error between the transmitted signal ($\boldsymbol{x}$) and $\boldsymbol{H}^H\boldsymbol{y}$ which is presented as [37]

$$\boldsymbol{A}_{\text{MMSE}}^H = \arg \min_{\boldsymbol{H} \in^{N_r \times N_t}} E\left\|\boldsymbol{x} - \boldsymbol{H}^H\boldsymbol{y}\right\|^2. \tag{5}$$

The MMSE detector takes the effects of noise into consideration and is represented as [6]

$$\boldsymbol{A}_{\text{MMSE}}^H = \left(\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I}\right)^{-1}\boldsymbol{H}^H. \tag{6}$$

The MMSE has high computational complexity since it depends on high-order matrix inversion. Moreover, the algorithm suffers from unstable performance for higher-order matrices [6]. Additionally, the performance of the MMSE based detector is severely degraded when working under ill-conditioned channels [2].

B. LINEAR DETECTORS BASED ON APPROXIMATE MATRIX INVERSION

1) NEUMANN SERIES

The NS is an approximate matrix inversion method used to reduce the complexity of linear detection [8]. The Gramian matrix $\boldsymbol{G} = \boldsymbol{H}^H\boldsymbol{H}$ inversion is a highly computational task. The NS simplifies the matrix inversion by decomposing the matrix $\boldsymbol{G}$ into $\boldsymbol{G} = \boldsymbol{D} + \boldsymbol{E}$, where $\boldsymbol{D}$ is the diagonal matrix, and $\boldsymbol{E}$ is presenting the non-diagonal elements. The Gramian matrix inversion based on the NS method can be represented as [8]

$$\boldsymbol{G}^{-1} = \sum_{n=0}^{\infty} \left(-\boldsymbol{D}^{-1}\boldsymbol{E}\right)^n \boldsymbol{D}^{-1}, \tag{7}$$

where the condition for convergence of the NS algorithm to the matrix inverse is provided by [8]

$$\lim_{n\to\infty} \left(-\boldsymbol{D}^{-1}\boldsymbol{E}\right)^n = 0. \tag{8}$$

It should be noted that the NS method suffers from a considerable performance loss when the ratio of the number of receiving antennas to users is close to 1. When the number of iterations is low ($n \leq 2$), the NS method has a computational complexity of $O\left(N_t^2\right)$.

2) NEWTON ITERATION METHOD

The NI method, also called the Newton-Raphson method, is an iterative method used to find the approximate matrix inverse [38]. The estimate of the matrix inverse at a specific iteration (n) is provided as [38]

$$\boldsymbol{X}_n^{-1} = \boldsymbol{X}_{n-1}^{-1}\left(2\boldsymbol{I} - \boldsymbol{G}\boldsymbol{X}_{n-1}^{-1}\right). \tag{9}$$

The NI method was shown to converge faster when compared to the NS method, even though it requires one additional multiplication in each iteration [10]. The NI method has a computational complexity of $O\left(N_t^2\right)$ [7].

3) GAUSS-SEIDEL METHOD

The GS method is an iterative technique that is employed to estimate the matrix inversion, and it has been employed in linear detectors [39]. The Gram matrix $\boldsymbol{G}$ can be decomposed as $\boldsymbol{G} = \boldsymbol{D} + \boldsymbol{U} + \boldsymbol{L}$, where $\boldsymbol{U}$ is the upper triangular matrix of $\boldsymbol{G}$, and $\boldsymbol{L}$ is the lower triangular matrix of $\boldsymbol{G}$. The detected signal for the GS detector is estimated as [40]

$$\hat{\boldsymbol{x}}^{(n)} = (\boldsymbol{D}+\boldsymbol{L})^{-1}\left(\hat{\boldsymbol{x}}_{MF} - \boldsymbol{U}\hat{\boldsymbol{x}}^{(n-1)}\right). \tag{10}$$

The computational complexity for the GS based detector is $O(N_t^2)$ [40]. The GS detector's initial estimate could be considered a zero vector if it is unknown [41]. However, a low convergence rate is noticed when zero vector is used for initialization.

4) SUCCESSIVE OVER-RELAXATION METHOD

A relaxation factor ω can be used to improve the convergence and performance of the GS method. The SOR method has a $O(N_t^2)$ computational complexity that is similar to the GS method [42]. The GS is considered as a special case of the SOR method where $\omega = 1$ [43]. The SOR is reliant on having a suitable value for the relaxation parameter. Since the prediction of the optimal relaxation parameter is a challenging task, performance degradation could occur if an inappropriate value of ω is used. The estimated signal for the SOR detector is given as [42]

$$\hat{\boldsymbol{x}}^{(n)} = \left(\frac{1}{\omega}\boldsymbol{D}+\boldsymbol{L}\right)^{-1}\left(\hat{\boldsymbol{x}}_{\text{MF}} + ((\frac{1}{\omega}-1)\boldsymbol{D}-\boldsymbol{U})\hat{\boldsymbol{x}}^{(n-1)}\right). \tag{11}$$

It should be noted that all above-mentioned iterative methods are convergent for any initial solution ($\hat{x}^{(0)}$) if the the equalization matrix is strictly diagonally dominant (SDD). However, the convergence rate is related to the spectral radius of the iterative method. Hence, each iterative method has it is own convergence rate. Moreover, the convergence rate depends on the selection of $\hat{x}^{(0)}$ [12]. In [11] and [44], the relationship between the convergence rate, maximum eigenvalue, and minimum eigenvalue is comprehensively discussed. It is also shown that the convergence conditions can be satisfied when the number of BS antennas is sufficiently large. Table 2 summarizes the pros and cons of conventional massive MIMO detectors.

5) STAIR MATRIX INITIALIZATION FOR APPROXIMATE MATRIX INVERSION DETECTORS

The performance, complexity, and convergence rate of the approximate matrix inversion methods used in linear massive MIMO detection is highly dependent on the initial values. The equalization matrix is considered to be diagonally dominant in MMSE based detector. Hence, most of the existing approximate matrix inverse iterative detectors use the diagonal matrix. However, the solution of the linear iterative methods may not converge when the diagonal matrix is adopted [44]. The stair matrix is a special case of the tridiagonal matrix that has zero off-diagonal elements in either the odd or even rows [11]. Tridiagonal matrix can be classified as a stair matrix ($\boldsymbol{S}$) if it satisfies one of the following conditions [12]:

$$\begin{aligned}&\boldsymbol{S}_{(i,i-1)} = 0,\\ &\boldsymbol{S}_{(i,i+1)} = 0, \quad i = 2,4,\ldots,2\left|\frac{N_t}{2}\right|.\end{aligned} \tag{12}$$

$$\begin{aligned}&\boldsymbol{S}_{(i,i-1)} = 0,\\ &\boldsymbol{S}_{(i,i+1)} = 0, \quad i = 1,3,\ldots,2\left\lfloor\frac{N_t-1}{2}\right\rfloor+1.\end{aligned} \tag{13}$$

For example, a stair matrix with a dimension of 6×6 can be represented in the following form [12]:

$$\boldsymbol{S} = \begin{bmatrix} \times & \times & & & & \\ & & \times & & & \\ & & \times & \times & \times & \\ & & & \times & & \\ & & & \times & \times & \times \\ & & & & & \times \end{bmatrix}. \tag{14}$$

In [12], the stair matrix is exploited instead of the diagonal matrix to initialize several approximate matrix inversion massive MIMO detectors. Most approximate matrix inversion methods utilize $\hat{\boldsymbol{x}}_{(0)} = \boldsymbol{D}^{-1}\boldsymbol{y}_{MF}$ for selection of the

TABLE 2. Pros and cons of conventional massive MIMO detection techniques.

Technique	Pros	Cons
MF	It works well when the users count is much lower than the receiving antennas [4]. It has low complexity [4].	The method exhibits bad performance in ill-conditioned channels [4].
ZF	It has low complexity [5]. It has good performance in environments where interference is limited [5].	It has bad performance in ill-conditioned channels [2] [5].
MMSE	It has better performance than MF and ZF techniques [37]. It considers the noise effects and reduces noise enhancements.	It has bad performance in ill-conditioned channels. The MMSE has high complexity and suffers from unstable performance for higher-order matrices [6].
GS	The technique performs well when β is close to 1.	The algorithm is difficult for parallel implementation because it is reliant on sequential iterations [45].
SOR	The technique performs well when β is close to 1. The algorithm managed to outperform NS and CG-based detectors [43].	It is dependent on the relaxation parameter ω that is difficult to predict [1].
CG	It has satisfactory performance when the ratio of receiving antennas and users is high [46]. The CG can provide numerically stable performance [47].	The number of operations is high [46]. The method needs a large number of iterations [46].
NS	It has low complexity [8].	It has bad performance when β is close to 1 [8]. The algorithm is slower compared with NI method [38].
NI	It has a faster convergence rate than NS [38].	It requires a higher number of multiplications than NS [38].
Local search	It can achieve near-optimal performance by using the fixed neighborhood to minimize the ML cost [48].	It has prohibitive complexity and reduced performance when using high modulation schemes [48].
Belief propagation	It can attain near-optimal results in cases where the number of antennas is high and with low correlated channels [49].	This iterative technique does not always ensure the solution will converge [49]. It is dependent on finding the optimal damping factor which is difficult to predict [49].
BOX Detection	It can achieve satisfactory performance with inexpensive hardware complexity [50].	The performance suffers when the ratio of antennas at BS and users is high [51].
V-BLAST (SIC)	It has superior performance compared to ZF and MMSE techniques [33].	The method requires more complexity compared to ZF and MMSE [33]. The results of the algorithm are influenced by the first detected signal [33].

initial solution. The detector based on approximate matrix inversion methods such as the GS and SOR with stair matrix initialization is discussed in [12]. The algorithm uses the stair matrix to obtain the initial solution that is given by $\hat{\boldsymbol{x}}_{(0)} = \boldsymbol{S}^{-1}\boldsymbol{y}_{MF}$ for the GS and SOR linear iterative methods. The operators diag($\boldsymbol{A}$), triu($\boldsymbol{A}$), tril($\boldsymbol{A}$), and stair($\boldsymbol{A}$) is used to represent the commands used to obtain the diagonal, upper triangular, lower triangular, and stair matrix components of $\boldsymbol{A}$, respectively. In addition, massive MIMO detectors based on NI and NS with stair matrix are proposed in [12]. The GS and SOR methods achieved the best performance with the lowest complexity compared to NS and NI based detectors [12]. Moreover, the complexity involved in computing $\boldsymbol{S}^{-1}$ and $\boldsymbol{D}^{-1}$ is the same, with both operations having a computational cost of $O(N_t)$ [12].

C. VERTICAL BELL LABORATORIES LAYERED SPACE-TIME (V-BLAST) ALGORITHM

V-BLAST is a non-linear detection technique that is based on linear detectors such as the linear ZF detector. It is also called order successive interference cancellation (OSIC). The V-BLAST detection algorithm is shown in [32]. The interference introduced by the different antennas is canceled using the V-BLAST algorithm. The algorithm performs the detection and the cancellation sequentially in a serial fashion. First, it detects, using the ZF detector, the signal with the highest SINR to ensure the best performance. After that, the algorithm detects the second strongest signal and arranges for the cancellation of the signal from the remaining signal set. The algorithm is repeated iteratively until the detection of all the signals is completed. The ZF V-BLAST algorithm has better performance than classical linear detectors such as the ZF. However, the ZF V-BLAST method has higher computational complexity [32]. The two main factors that increase the computational complexity of the ZF V-BLAST algorithms are the ordering and cancellation process in addition to the matrix inversion computation for the equalization matrix $\boldsymbol{A}$ [32].

In the following, we compare the effect of interference of linear detectors such as ZF and maximum ratio combiner

(MRC) and compare them with VBLAST. Following [52], the SINR for the m^{th} user is

$$\mathrm{SINR_m} = \frac{[|\vec{w}_m^H \vec{h}_m x_m|^2]}{[|\vec{w}_m^H (\sum_{j \neq m} \vec{h}_j x_j + \vec{n})|^2]} \tag{15}$$

where $\vec{w}_m$ is the m^{th} column of the linear detecting matrix, W.

With perfect CSI, ZF removes the interference while interference increases for MRC. As discussed in [53], the sum of the interference is required to be small before the MRC receiver is similar to the ZF receiver because the aggregate interference corrupts MRC while ZF deletes it. In this paper, we assume self-interference is negligible because each user has a single antenna. ZF, in general, suppresses the effect of interference and suffers from noise enhancement under the assumption of the perfect channel. However, VBLAST is based on successive interference cancelation and detection, so if there are no other users, we come back to the case of the linear detector, and hence if we use ZF VBLAST, we will have a match of the ZF case [53]. Note that if we assume, each user has multiple antenna and we have multiple users, self-interference will occur in case no other users appears. This can be explored for future work.

D. MMNet DEEP LEARNING ARCHITECTURE

The MMNet is a massive MIMO DL network architecture with a varying channel realization architecture [25]. The MMNet addresses the poor performance of the DetNet and the OAMP-Net in realistic channel models. The MMNet architecture balances flexibility and complexity within each layer of the neural network. Earlier DL detection techniques such as the DetNet and OAMP-Net are trained offline and have single channel realization architecture. However, the MMNet is designed with varying channel realization and supports both online and offline training. For the MMNet, training and testing data are generated through the massive MIMO model illustrated in (1) where the signal ($\boldsymbol{x}$), the channel matrix ($\boldsymbol{H}$) and the noise ($\boldsymbol{n}$) are the sources of randomness. In addition, each $\boldsymbol{x}$ is generated randomly and uniformly over the corresponding constellation set. Adam optimizer is utilized with a learning rate of 10^{-3}. In order to have a fair comparison, we used the training parameters and hyperparameters as mentioned in [25]. The MMNet architecture that is implemented for realistic channels has ten layers with $2N_t(N_r+1)$ trainable parameters per layer. For arbitrary channels, the MMNet architecture model is designed to be suitable as in [25]

$$\boldsymbol{z}_t = \hat{\boldsymbol{x}}_t + \boldsymbol{\Theta}_t^{(1)} \left(\boldsymbol{y} - \boldsymbol{H}\hat{\boldsymbol{x}}_t\right), \tag{16}$$

where

$$\hat{\boldsymbol{x}}_{t+1} = \eta_t \left(\boldsymbol{z}_t; \sigma_t^2\right). \tag{17}$$

The MMNet has a complex trainable matrix ($\boldsymbol{\Theta}_t^{(1)}$) that enables the model to work in scenarios where several noise levels influence the transmitted signal. This enables the MMNet algorithm to perform well in realistic environments

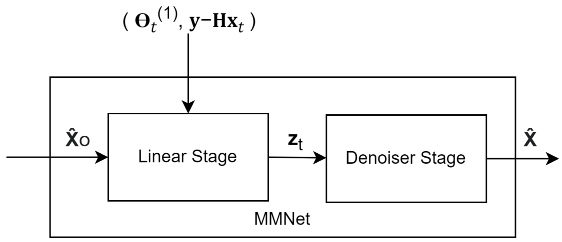


FIGURE 1. Block diagram of the MMNet architecture [25].

with arbitrary channels. Fig. 1 shows the block diagram for the MMNet architecture. MMNet neural network contains a linear transformation followed by a non-linear denoising stage. It repeatedly refines an estimate of the signal by alternating between a linear detection stage and a non-linear denoising stage. When signals propagate through the MMNet stages, the noise distribution at the input of the non-linear denoiser stages approaches a Gaussian distribution, creating precisely the conditions in which the denoisers can attenuate noise maximally. An efficient massive MIMO detection technique has been proposed based on the initialization of the MMNet algorithm using several approximate matrix inversion methods such as GS, SOR, and conjugate gradient (CG) [31]. The massive MIMO detector based on the initialization of the MMNet with the GS method achieved the best performance, followed by SOR and CG. Moreover, all proposed detectors significantly improve performance compared to MMNet while maintaining low complexity [31]. The loss function of MMNet is calculated as $Loss = \frac{1}{L}\sum_{l=1}^{L} \left\|\hat{\boldsymbol{x}}_t - \boldsymbol{x}\right\|_2^2$ where L presents the number of layers.

IV. PROPOSED MASSIVE MIMO DETECTOR

The ZF V-BLAST algorithm outperforms linear detection techniques such as ZF for massive MIMO systems. However, the ZF V-BLAST algorithm is highly complex due to the matrix inversion, which is computationally demanding. We propose a low-complexity massive MIMO detection algorithm based on the linear ZF detection method and the V-BLAST algorithm. The ZF V-BLAST algorithm is enhanced by using two iterative approximate matrix inversion methods: the NS and NI.

Matrix inversion is used to sustain good performance while reducing computational complexity for massive MIMO detection. In Fig. 2, we provide the block diagram of the proposed massive MIMO detection technique based on the low complexity V-BLAST algorithm. In the proposed technique, the exact matrix inverse in the iterative process of the ZF V-BLAST algorithm is replaced with matrix inverse approximation using NS and NI methods.

One of the major bottlenecks for the ZF V-BLAST algorithm is the computational complexity required to calculate the exact matrix inverse for the Gramian matrix. We consider two approximate matrix inversion methods, the NS and NI, for estimating the matrix inverse. The equations for the NS and NI methods to avoid the calculation of matrix inverse are provided in (7) and (9), respectively. We use a small

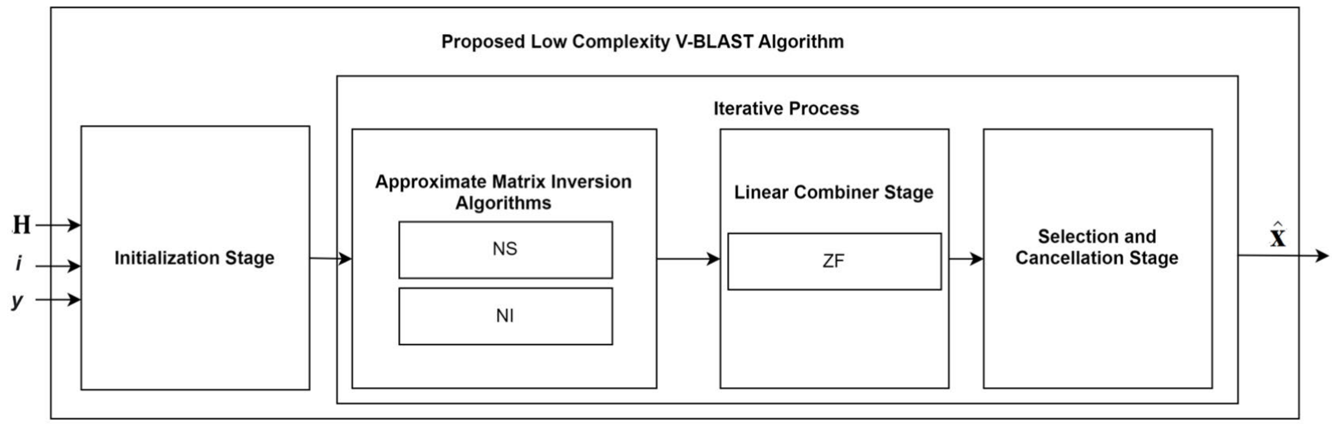


FIGURE 2. Block diagram for the proposed low complexity V-BLAST massive MIMO detection algorithm.

number of iterations ($n \leq 2$) for the NS and NI approximate matrix inversion methods in order to ensure that the computational complexity remains low for the proposed algorithms. Algorithm 4 shows the proposed massive MIMO detection technique.

In the proposed algorithm, the Gramian matrix [54] is obtained first for ZF linear detector, which is given by $\boldsymbol{B_{ZF}} = \boldsymbol{H}^H\boldsymbol{H}$. The matrix inverse is approximated using the NS and NI methods. The number of iterations is kept equal to or less than 2 in order to ensure that the complexity of the approximate methods is $O(N_t^2)$. The approximated matrix inverse is then used in the equalization matrix for the classical linear detection techniques. The algorithm detects the signal with the highest SINR using ZF to ensure the best performance. After that, the second strongest signal is detected and arranged for the cancellation of the signal from the remaining signal set. The optimal ordering for a ZF V-BLAST algorithm is given by [32]

$$k_i^{(\mathrm{ZF})} = \underset{j \notin \{k_1, k_2, \ldots, k_{i-1}\}}{\arg\min} \left\| (\boldsymbol{A}_i)_j \right\|^2, \tag{18}$$

where $(\boldsymbol{A}_i)_j$ is the j^{th} of $\boldsymbol{A}$ at the i^{th} iteration. The algorithm is repeated for each iteration until detecting all the signals is completed.

Algorithm 1 Proposed Massive MIMO Detection Technique Using V-BLAST Algorithm and Approximate Matrix Inversion Methods

1) Initialization:	
i=1	i is the iteration number.
$\boldsymbol{H}$	$\boldsymbol{H}$ is the channel matrix.
$\boldsymbol{y}$	$\boldsymbol{y}$ is the received signal.
2) Ordering and successive cancellation:	
$\boldsymbol{B}$	Find Gramian matrix for the ZF linear detector.
$\boldsymbol{B_{ZF}}$	For ZF, $\boldsymbol{B_{ZF}} = \boldsymbol{H}^H\boldsymbol{H}$.
$\boldsymbol{B}^{-1}$	Use NS and NI to obtain the approximate matrix inverse. For NS, $\boldsymbol{B}^{-1} = \sum_{n=0}^{\infty} \left(-\boldsymbol{D}^{-1}\boldsymbol{E}\right)^n \boldsymbol{D}^{-1}$. For NI, $\boldsymbol{X}_n^{-1} = \boldsymbol{X}_{n-1}^{-1}\left(2\boldsymbol{I} - \boldsymbol{B}\boldsymbol{X}_{n-1}^{-1}\right)$.
$\boldsymbol{A}$	Evaluate the linear detector. For ZF, $\boldsymbol{A}_{ZF} = (\boldsymbol{B_{ZF}})^{-1}\boldsymbol{H}^H$.
$\boldsymbol{G} = \boldsymbol{A}^H$	Obtain the Hermitian of the linear detector, $\boldsymbol{A}$.
k_i	Obtain the selection index k_i. For ZF, $k_i = \underset{j \notin \{k_1, k_2, \ldots, k_{i-1}\}}{\arg\min} \left\|(\boldsymbol{A}_i)_j\right\|^2$.
$\boldsymbol{m}_{k_i} = (\boldsymbol{G}_i)_{k_i}$	$\boldsymbol{m}_{k_i}$ is the i^{th} row of $\boldsymbol{G}$.
$\tilde{x}_i = \boldsymbol{m}_{k_i}\boldsymbol{y}$	Evaluate the estimated input signal $\tilde{x}_i$.
$\hat{x}_i = \hat{Q}[\tilde{x}_i]$	$\hat{Q}(.)$ is the quantization operation based on the used constellation.
$\boldsymbol{y}_{i+1} = \boldsymbol{y}_i - \boldsymbol{h}_{k_i}\hat{x}_{k_i}$	The interference due to $\hat{x}_{k_i}$ is cancelled.
$\boldsymbol{H}_{i+1} = \boldsymbol{H}_i^{\overline{k_i}}$	Update $\boldsymbol{H}$ at iteration i by zeroing the k_i column.
$i = i + 1$	Update i.

V. COMPLEXITY ANALYSIS

We investigate the computational complexity of the proposed massive MIMO detection techniques in this section. The computational complexity is dominant by the number of complex multiplications. The computational complexity for different massive MIMO detection algorithms in terms of $O(.)$ notation is presented in Table 3. We refer to the proposed detection techniques as ZF V-BLAST NS and ZF V-BLAST NI when the NS and NI are used, respectively, to approximate the matrix inverse for the ZF V-BLAST algorithm.

The computational complexity for matrix inverse using the Gaussian elimination method is provided by $O(ab^2)$ for an $a \times b$ matrix [55]. In the ZF V-BLAST algorithm, it is required to obtain the matrix inverse of $\boldsymbol{H}^H\boldsymbol{H}$ for the ZF

TABLE 3. Comparison of the computational complexity order for different linear and non-linear massive MIMO detection techniques.

MIMO detection technique	Complexity
ZF [57]	$O(N_r N_t^2)$
ZF V-BLAST [53]	$O(N_r N_t^3)$
ZF V-BLAST NS	$O(N_r N_t^3)$
ZF V-BLAST NI	$O(N_r N_t^3)$
MMNet [25]	$O(bN_r^2 L)$
MMNet-GS	$O(bN_r^2 L)$
MMNet-SOR	$O(bN_r^2 L)$

techniques. Hence, the matrix inverse in the ZF V-BLAST algorithm has a computational complexity of $O(N_t^3)$. Moreover, the same complexity order of $O(N_t^3)$ is required if Cholesky decomposition is utilized to compute the matrix inverse [56]. The matrix inversion causes high complexity for the ZF V-BLAST algorithm. The proposed technique reduces the complexity of the matrix inverse operation in the original ZF V-BLAST algorithm by avoiding the matrix inversion computation using the NS and NI methods which have a matrix inverse complexity of $O(N_t^2)$. Hence, matrix inversion complexity is reduced from $O(N_t^3)$ to $O(N_t^2)$. ZF V-BLAST and the proposed detection techniques have a computation complexity of $O(N_r N_t^3)$. However, the proposed detection algorithms that use the NS and NI approximate matrix inversion methods can lower the computational complexity by reducing the number of multiplications required by the ZF V-BLAST algorithm. Hence, the computational complexity required for matrix inversion using NS and NI methods is $O(N_t^2)$ when the number of iterations is equal to or less than 2 ($n \leq 2$).

The number of multiplications required for each massive MIMO detection technique is summarized in Table 4. In the conventional ZF V-BLAST algorithm, $\frac{N_r N_t (N_t+1)}{2}$ multiplications are required in order to obtain $\boldsymbol{H}^H\boldsymbol{H}$ and $\frac{N_t^3}{2} + \frac{3}{2}N_t^2$ multiplications are required in order to find $\left(\boldsymbol{H}^H\boldsymbol{H}\right)^{-1}$ [53], [56]. For example, the number of multiplications required at the first stage of the ZF V-BLAST algorithm is provided by $\frac{N_r N_t (N_t+1)}{2} + \frac{N_t^3}{2} + \frac{3}{2}N_t^2 + N_t^2 N_r + N_t N_r + 2N_r$. The same can be obtained for the second and third stages of the ZF V-BLAST algorithm by substituting $(N_t - 1)$ and $(N_t - 2)$, respectively. Hence, the number of multiplications required for the ZF V-BLAST algorithm is provided by $\frac{N_r \sum_{i=0}^{N_t-2}(N_t-i)(N_t-i+1)}{2} + \frac{\sum_{i=0}^{N_t-2}(N_t-i)^3}{2} + \frac{3}{2}\sum_{i=0}^{N_t-2}(N_t-i)^2 + \sum_{i=0}^{N_t-2}(N_t-i)^2 N_r + (N_t N_r + 2N_r(N_t-1) + N_r)$ [32]. For the NS and NI methods, the number of multiplications required to obtain the matrix inverse is $N_t^2 + Nt$ when the number of iterations is $n \leq 2$. Hence, the term required to obtain the matrix inverse $\frac{N_t^3}{2} + \frac{3}{2}N_t^2$ is reduced $N_t^2 + Nt$. For example, the number of multiplications required to obtain the exact matrix inverse for a massive MIMO system with $N_t = 32$ is 17920 multiplications. While using the NS and NI methods would require 1056 multiplications to approximate the matrix inverse. Hence, the number of multiplication required to obtain the matrix inverse is reduced considerably. It is also worth noting that the computational complexity of the MMNet [25] can be presented as $O(bN_r^2 L)$.

A. COMPLEXITY OF STAIR MATRIX IN APPROXIMATE MATRIX INVERSION METHODS

The number of the real multiplications required to obtain the inverse of the stair matrix is $3(N_t - 1)$. In comparison, the number of division operations required to obtain the inverse of the diagonal matrix is N_t. Although the initialization of the approximate matrix inversion methods with stair matrix increases the number of multiplications to $3(N_t - 1)$, this increase in complexity is considered negligible. For example, the NS detector with a stair matrix requires 17453 multiplications while the conventional NS detector with a diagonal matrix requires 17408 for a 16×64 massive MIMO system and $n = 2$. Consequently, the computational complexity to obtain $\boldsymbol{S}^{-1}$ is $O(N_t)$, which is equivalent to the computational complexity required to calculate $\boldsymbol{D}^{-1}$. Table 4 shows the number of multiplications required with and without stair matrix initialization for the proposed ZF V-BLAST NS, ZF V-BLAST NI, MMNet-GS, and MMNet-SOR detection techniques. It is shown that the increase in the number of multiplications by $3(N_t - 1)$ in the proposed algorithms is insignificant. Hence, the exploitation of the stair matrix initialization does not affect the complexity order for the proposed detection techniques.

In order to have a fair comparison between the proposed detection techniques and the recent work in [16], Table 5 illustrates the required number of flops and the estimated execution time when the computer is executing 1 million operation per second. A 8×64 MIMO system is employed where each complex multiplication requires 3 flops. It is clear that the work in [16] has the lowest execution time. However, the proposed detection techniques still have a very low execution time.

VI. SIMULATION RESULTS

The performance results of the proposed massive MIMO detection techniques based on the linear ZF detector and the V-BLAST algorithm in conjunction with approximate matrix inversion methods are presented in this section. We implement the proposed massive MIMO detection algorithms using two different architectures (ZF V-BLAST NS and ZF V-BLAST NI). We first compare the performance of the proposed detection techniques using the symbol error rate (SER) versus SNR. The main performance criteria will be the SNR required to achieve a range of $10^{-2} - 10^{-3}$ SER since, typically, the error correction schemes operate in this range for MIMO detection [25].

The ZF V-BLAST algorithm is implemented using the NS and NI methods for the inverse matrix approximation.

TABLE 4. Computational complexity of the proposed detection techniques and linear detection methods.

Detection technique	Complexity
ZF [53]	$0.5N_t\left(N_t^2+3N_t\left(N_r+1\right)+3N_r\right)$
ZF V-BLAST [33]	$\frac{N_r\sum_{i=0}^{N_t-2}(N_t-i)(N_t-i+1)}{2}+\frac{\sum_{i=0}^{N_t-2}(N_t-i)^3}{2}+\frac{3}{2}\sum_{i=0}^{N_t-2}\left(N_t-i\right)^2+\sum_{i=0}^{N_t-2}\left(N_t-i\right)^2N_r+(N_tN_r+2N_r(N_t-1)+N_r)$
ZF V-BLAST NS/NI	$\frac{N_r\sum_{i=0}^{N_t-2}(N_t-i)(N_t-i+1)}{2}+\sum_{i=0}^{N_t-2}\left(N_t-i\right)^2+\sum_{i=0}^{N_t-2}\left(N_t-i\right)^2N_r+(N_tN_r+2N_r(N_t-1)+N_r)$
ZF V-BLAST NS/NI with stair matrix	$\frac{N_r\sum_{i=0}^{N_t-2}(N_t-i)(N_t-i+1)}{2}+\sum_{i=0}^{N_t-2}\left(N_t-i\right)^2+\sum_{i=0}^{N_t-2}\left(N_t-i\right)^2N_r+\big(N_tN_r+2N_r(N_t-1)+N_r\big)+3(N_t-1)$
MMNet-GS	GS $(n=1)$: $(4N_r+2)N_t^2+2(N_r-1)N_t$
MMNet-SOR	SOR $(n=1)$: $(4N_r+2)N_t^2+2(N_r)N_t$
MMNet-GS with stair matrix	GS $(n=1)$: $(4N_r+2)N_t^2+2(N_r-1)N_t+3(N_t-1)$
MMNet-SOR with stair matrix	SOR $(n=1)$: $(4N_r+2)N_t^2+2(N_r)N_t+3(N_t-1)$
DLNet [16], [21]	$4N_t^3+9N_t^2-5N_t$

TABLE 5. Required number of flops and execution time if 8 × 64 MIMO system is employed.

Method	Number of flops	Execution time
MMNet-GS	52560	0.05256
MMNet-SOR	52608	0.052608
MMNet-GS with stair matrix	52623	0.052623
MMNet-SOR with stair matrix	52671	0.05261
DLNet [16]	7752	7.752×10^{-3}

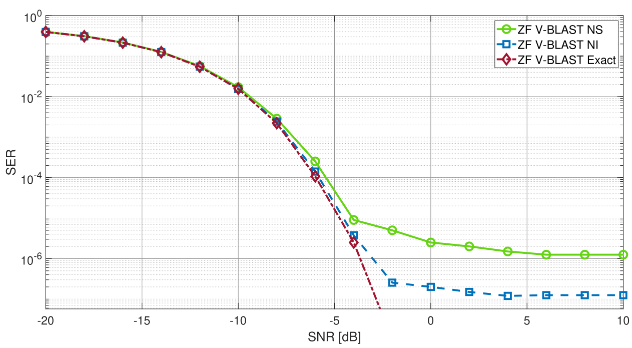


FIGURE 3. Performance comparison for different ZF V-BLAST massive MIMO detection algorithms using 8 × 64 MIMO system.

We refer to the ZF V-BLAST implemented with NS and NI as ZF V-BLAST NS and ZF V-BLAST NI, respectively. We test the ZF V-BLAST NS and ZF V-BLAST NI proposed techniques on different massive MIMO system configurations. In the first scenario, a system size ratio size N_t/N_r of 0.125 and 0.25 for $N_r = 64$. In the second scenario, the same system size ratios are tested for BS with $N_r = 32$.

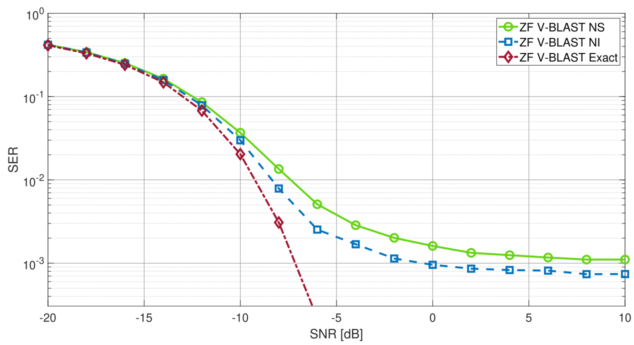


FIGURE 4. Performance comparison for different ZF V-BLAST massive MIMO detection algorithms using 16 × 64 MIMO system.

The detection techniques are tested in the third scenario for massive MIMO configurations with 0.125 and 0.25 massive MIMO system size ratios with $N_r = 128$. The modulation scheme used in the simulation is (quadrature amplitude modulation) QAM-4, with each simulation repeated for $100,000-1,000,000$ experiments. For brevity, the results for the first scenario are only shown in the paper.

In the first scenario, the ZF V-BLAST NS, ZF V-BLAST NI, and ZF V-BLAST with exact matrix inversion are tested using i.i.d. Gaussian channels. For a system size of 8 × 64, the proposed ZF V-BLAST NS and ZF V-BLAST NI detection techniques. As shown in Fig. 3 and Fig. 4, both of the proposed ZF V-BLAST NS and ZF V-BLAST NI detection techniques have attained excellent performance in an 8 × 64 massive MIMO system while having lower complexity than the original ZF V-BLAST algorithm. Similarly, the proposed detection techniques achieved satisfactory performance in the 16 × 64 massive MIMO system. It should be noted that ZF V-BLAST NI has better performance than ZF V-BLAST NS in massive MIMO systems with $N_r = 64$.

In the second scenario, the ZF V-BLAST NS, ZF V-BLAST NI, and ZF V-BLAST with exact matrix inversion are tested using i.i.d. Gaussian channels. The two massive MIMO system configurations considered for this scenario are 4×32 and 8×32 system sizes. The proposed ZF V-BLAST NS and ZF V-BLAST NI detection techniques achieve satisfactory SER performance in comparison to the exact ZF V-BLAST method while having lower complexity. It was also noticed that when the number of users increased to $N_t = 8$, the ZF V-BLAST NS and ZF V-BLAST NI still managed to achieve good performance compared with the original ZF V-BLAST algorithm with exact matrix inversion. Moreover, the ZF V-BLAST NI detector attains superior performance when compared to the ZF V-BLAST NS detector in both 4×32 and 8×32 massive MIMO system configurations. In the third scenario, the ZF V-BLAST NS, ZF V-BLAST NI, and ZF V-BLAST with exact matrix inversion are tested using i.i.d. Gaussian channels. The ZF V-BLAST NS and ZF V-BLAST NI achieved similar results to the first and second scenarios for both 16×128 and 32×128 system sizes. Both ZF V-BLAST NS and ZF V-BLAST NI algorithms had reliable SER with significantly lower complexity than the original ZF V-BLAST algorithm with exact matrix inverse for the mentioned system sizes. It should be noted that as the ratio between the transmitting users and the receiving antennas increases, the performance of the detection techniques will start to suffer. The reduction in performance is expected as the system is loaded with more users since this is the typical behavior for the ZF V-BLAST, NS, and NI methods. Accordingly, the proposed ZF V-BLAST NS and ZF V-BLAST NI massive MIMO detection techniques can achieve the best performance in massive MIMO systems where the number of receiving antennas is much larger than the number of users.

The simulation shows that the proposed detection techniques, which are ZF V-BLAST NS and ZF V-BLAST NI, have good performance in massive MIMO scenarios where the number of receiving antennas is much larger than the number of users. Moreover, the proposed detection techniques have excellent performance results that are similar to the original ZF V-BLAST algorithm when the system size ratio N_t/N_r is 0.125. In the case where the system size has a ratio of 0.25, the proposed detection techniques achieved SER of around 10^{-3} within a reasonable SNR.

A. STAIR MATRIX INITIALIZATION FOR THE PROPOSED MASSIVE MIMO DETECTION TECHNIQUES

The performance of the proposed massive MIMO detection techniques when implemented with the stair matrix initialization is presented in this section. In the first scenario, the ZF V-BLAST NS and ZF V-BLAST NI algorithms are implemented with a stair matrix and diagonal matrix for approximate matrix inversion methods. The ZF V-BLAST, ZF V-BLAST NS, and ZF V-BLAST NI algorithms are simulated using i.i.d. Gaussian channel model with massive MIMO system size of 8×32. The number of iterations used for both the NI and NS methods is 2. As shown in Fig. 5, the ZF V-BLAST NS and ZF V-BLAST NI algorithms implemented with stair matrix achieve better results than ZF V-BLAST NS and ZF V-BLAST NI algorithms implemented with the diagonal matrix, respectively.

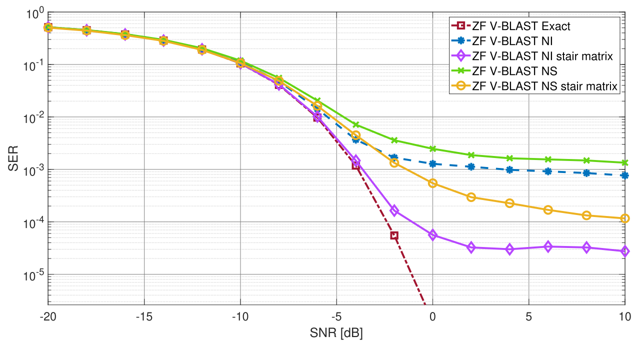


FIGURE 5. Performance comparison for different ZF V-BLAST massive MIMO detection algorithms with stair matrix initialization using 8×32 MIMO system.

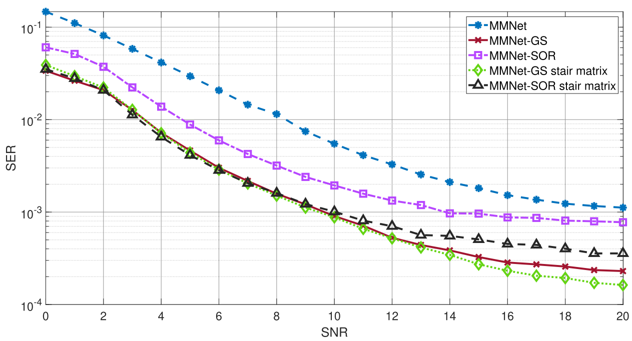
As discussed in [53], the SER is controlled by the cases when all the users have weak link powers. For instance, the interference sum is small. All users have weak link powers at low SNR; hence the total interference is slight. At higher SNR and in the early V-BLAST stages, the interference can be quite significant since the users do not have weak link powers. The most substantial user leads and faces a high SINR. On the other hand, this low SER scenario does not significantly influence the overall SER. In the later V-BLAST stages, the users face a lower SINR, and the involvement of the total SER is greater.

In the second scenario, the GS and SOR methods are utilized with a stair matrix using 1 iteration for the initialization of MMNet in offline training mode. The training of the MMNet model was carried out using a batch size of 500 samples, with 10000 iterations, and a test batch size of 5000 samples. We evaluate the performance of the algorithms using Quadiga channels for a massive MIMO system with $N_t = 8$ users and $N_r = 64$ antennas at the BS. Moreover, we assume a modulation scheme of 4-QAM in the simulation. The results for MMNet in addition to MMNet-GS, and MMNet-SOR initialized with diagonal and stair matrices are shown in Fig. 6a. Moreover, a comparison is provided for the performance of the MMNet-GS detector with stair matrix initialization using Gaussian and Quadriga channels in Fig. 6b. The results show that the realistic channels are significantly more challenging than Gaussian channels.

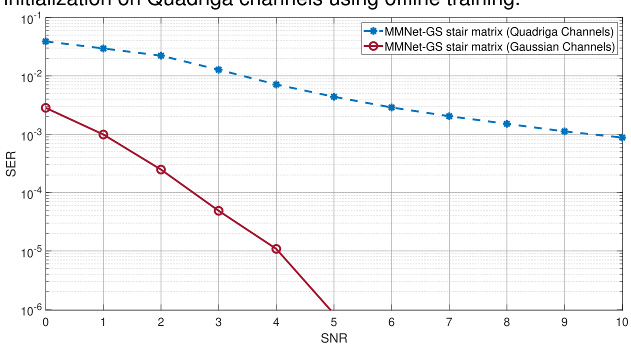
Table 6 shows a performance comparison for the massive MIMO detection techniques with stair matrix initialization at −5 dB, 0 dB, 5 dB, 10dB, and 20dB SNR values. The massive detection techniques are simulated using Quadriga channels with 8×64 system size. The DL algorithm initialized achieves better performance than the V-BLAST algorithms at the cost of higher computational complexity. For example, the MMNet algorithm initialized with approximate matrix inversion methods attains an SER performance of 10^{-4} at

TABLE 6. SER performance comparison for the proposed detection techniques with stair matrix initialization at −5 dB, 0 dB, 5 dB, 10dB, and 20dB SNR values using Quadriga channel models in an 8 × 64 massive MIMO system.

Massive MIMO detector	SER (-5dB)	SER (0dB)	SER (5dB)	SER (10dB)	SER (20dB)
MMNet-SOR	1.6×10^{-1}	6.1×10^{-2}	8.8×10^{-3}	2×10^{-3}	7.7×10^{-4}
MMNet-SOR stair matrix	1×10^{-1}	4×10^{-2}	5.4×10^{-3}	1×10^{-3}	3.5×10^{-4}
MMNet-GS	1.2×10^{-1}	6×10^{-2}	4.6×10^{-3}	9×10^{-4}	2.3×10^{-4}
MMNet-GS stair matrix	1.1×10^{-1}	3.9×10^{-2}	4.3×10^{-3}	8×10^{-4}	1.6×10^{-4}
ZF V-BLAST NS	5.2×10^{-2}	4.3×10^{-2}	4.1×10^{-2}	3.3×10^{-2}	2.6×10^{-2}
ZF V-BLAST NS stair matrix	9.3×10^{-3}	5.5×10^{-3}	4.5×10^{-3}	4×10^{-3}	3.7×10^{-3}
ZF V-BLAST NI	4.3×10^{-2}	3.5×10^{-2}	3.3×10^{-2}	2.7×10^{-2}	1.4×10^{-2}
ZF V-BLAST NI stair matrix	9.3×10^{-3}	4.9×10^{-3}	4.3×10^{-3}	3.4×10^{-3}	1.8×10^{-3}



(a) SER vs. SNR of different DL detection methods with stair matrix initialization on Quadriga channels using offline training.



(b) Performance comparison for the proposed MMNet-GS detector with stair matrix initialization using Gaussian and Quadriga channels.

FIGURE 6. Performance of different massive MIMO DL detection techniques.

20 dB. In comparison, the ZF V-BLAST algorithms initialized with approximate matrix inversion methods achieve an SER performance of 10^{-2} and 10^{-3}.

The massive MIMO detectors initialized with a stair matrix have significantly better performance than detectors initialized with approximate matrix inversion methods that use a diagonal matrix. For instance, at 10 dB ZF V-BLAST NI with stair matrix has an SER of 3.4×10^{-3} while ZF V-BLAST NI with a diagonal matrix has a 2.7×10^{-2}. Moreover, the use of the stair matrix provides an SER performance improvement with negligible added computational complexity. It should be noted that ZF V-BLAST NS and ZF V-BLAST NI algorithms have reliable performance with significantly lower complexity than the DL algorithms. Moreover, DL detectors with stair matrix initialization, such as MMNet-GS and MMNet-SOR, performed better than the same detector initialized with the diagonal matrix. Additionally, a low number of linear iterations is utilized to achieve satisfactory performance with stair matrix initialization. Hence, the computational complexity of the MMNet-GS and MMNet-SOR detection techniques is reduced.

B. THE PROPOSED MASSIVE MIMO DETECTION TECHNIQUES WITH IMPERFECT CSI

In this section, instead of assuming the $\boldsymbol{H}$ to be perfectly known on the receiver side, we evaluate the proposed V-BLAST algorithms with imperfect channel estimation. The proposed V-BLAST algorithms use the estimated CSI for the Gramian matrix of the ZF linear detector such that $\boldsymbol{B_{ZF}} = \hat{\boldsymbol{H}}^H \hat{\boldsymbol{H}}$. In Fig. 5, the proposed detectors which had the best performance in the scenario, i.e., ZF V-BLAST NS and ZF V-BLAST NI with stair matrix initialization, are tested with imperfect CSI using $r_0 = 0.99$ at the receiver. As shown in Fig. 7, the ZF V-BLAST NI was slightly affected by the imperfect channel estimation in the massive MIMO systems. Moreover, the technique attained satisfactory performance even with imperfect CSI, and channel correlation. It is worth noting that the ZF V-BLAST NS was more affected by imperfect channel information and the ZF V-BLAST NI algorithm outperformed ZF V-BLAST NS with both true channel and imperfect channel estimation scenarios.

In Fig. 8, we compare the performance of the ZF V-BLAST NS and ZF V-BLAST NI with stair matrix initialization using Gaussian channels and imperfect estimation of CSI as well as realistic channels in an 8 × 64 massive MIMO system. Similar results were obtained from the simulation in the

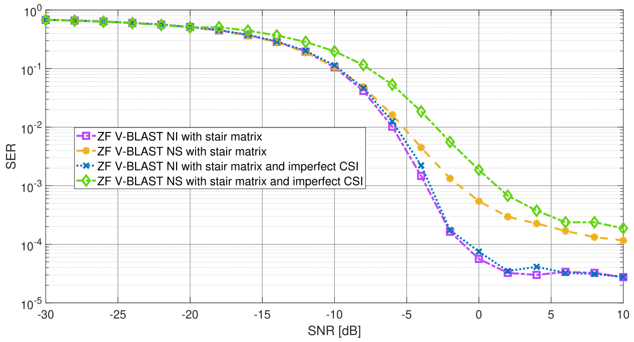


FIGURE 7. Performance comparison for different ZF V-BLAST algorithms with stair matrix initialization using 8 × 32 MIMO system under imperfect CSI.

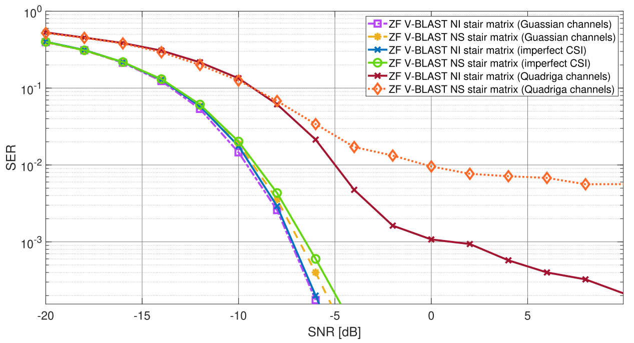


FIGURE 8. Performance comparison for different ZF V-BLAST algorithms with stair matrix initialization using 8 × 64 MIMO system under imperfect CSI and Quadriga channels.

4 × 32 massive MIMO system. The proposed algorithms' performance slightly degrades in case of imperfect CSI compared to Gaussian channel models. Moreover, it was shown that the most challenging scenario was using the realistic channels simulated using the Quadriga model. The proposed ZF V-BLAST NI with stair matrix initialization algorithm achieved good performance in realistic channels, followed by ZF V-BLAST NS with stair matrix initialization which achieved satisfactory results in the same scenario.

VII. CONCLUSION

In this paper, several low complexity massive MIMO detectors have been proposed based on approximate matrix inversion methods, stair matrix, DL approach, and V-BLAST. Proposed detectors have shown a significant performance gain and a remarkable complexity reduction. It is worth-noting that the NS and NI employment with ZF V-BLAST and stair matrix lead to a notable performance improvement while maintaining low computational complexity. In order to avoid any misleading conclusion, realistic channel scenario is considered and proposed detectors also achieved a satisfactory balance between performance gain and complexity reduction.

VIII. FUTURE RESEARCH DIRECTIONS

For future studies, different approximate matrix inverse techniques can be utilized in massive MIMO detection algorithms where exact matrix inversion is a computationally expensive task. Stair matrix initialization can be exploited as well to enhance approximate matrix inversion methods that are employed in various fields, such as machine learning and communication systems. For example, the approximate methods with the stair matrix can be used in various optimization problems in massive MIMO detection, beamforming, precoding, and channel estimation. In massive MIMO detection, the stair matrix can improve the performance and the convergence rate of linear iterative detectors while requiring a lower number of iterations. Accordingly, the stair matrix implementation can minimize the complexity of approximate matrix inversion methods without sacrificing performance for the same massive MIMO system.

Several parameters can be investigated to optimize the performance and reduce the computational cost in the neural network architecture of the DL-based massive MIMO detection techniques. Furthermore, the development of new machine learning and artificial intelligence algorithms specifically tailored for the unique characteristics of massive MIMO systems can be explored. The stair matrix can be used instead of the diagonal matrix with approximate matrix inversion methods to enhance efficiency in DL-based detectors. In addition, the effectiveness of the detection techniques using linear iterative approximate matrix inversion methods and DL can be studied for cell-free massive MIMO systems. The linear iterative and DL-based detectors have the potential to be adapted to the frequency selective channel, thereby opening up the possibility of their implementation in millimeter-wave (mmWave) massive MIMO systems. Moreover, approximate matrix inversion methods and DL can be employed in various fields to enhance performance and reduce complexity for real-time applications. Based on the channel hardening phenomenon, we believe that this work can be extended using different matrix structures such as the banded matrix, the Hankel matrix, and even the scaled identity matrix. Other initialization schemes can be exploited as well. For instance, the first iteration of NI can be exploited to initially approximate the matrix inversion.

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